

HENRY

Hydraulic Engineering Repository

Ein Service der Bundesanstalt für Wasserbau

Conference Paper, Published Version

Arnold, Patrick; Hicks, Michael

A stochastic approach to rainfall-induced slope failure

Verfügbar unter/Available at: <https://hdl.handle.net/20.500.11970/99555>

Vorgeschlagene Zitierweise/Suggested citation:

Arnold, Patrick; Hicks, Michael (2011): A stochastic approach to rainfall-induced slope failure. In: Vogt, Norbert; Schuppener, Bernd; Straub, Daniel; Bräu, Gerhardt (Hg.): Geotechnical Safety and Risk. ISGSR 2011. Karlsruhe: Bundesanstalt für Wasserbau. S. 107-116.

Standardnutzungsbedingungen/Terms of Use:

Die Dokumente in HENRY stehen unter der Creative Commons Lizenz CC BY 4.0, sofern keine abweichenden Nutzungsbedingungen getroffen wurden. Damit ist sowohl die kommerzielle Nutzung als auch das Teilen, die Weiterbearbeitung und Speicherung erlaubt. Das Verwenden und das Bearbeiten stehen unter der Bedingung der Namensnennung. Im Einzelfall kann eine restriktivere Lizenz gelten; dann gelten abweichend von den obigen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Documents in HENRY are made available under the Creative Commons License CC BY 4.0, if no other license is applicable. Under CC BY 4.0 commercial use and sharing, remixing, transforming, and building upon the material of the work is permitted. In some cases a different, more restrictive license may apply; if applicable the terms of the restrictive license will be binding.



A stochastic approach to rainfall-induced slope failure

P. Arnold

Delft University of Technology, Delft, The Netherlands
University of Manchester, Manchester, United Kingdom

M. A. Hicks

Delft University of Technology, Delft, The Netherlands

ABSTRACT: This paper considers the influence of spatial variability of soil properties on the stability of an unsaturated soil slope during and antecedent to a rainfall event. With water tending to follow a rather tortuous flow path during the infiltration process, slope failures may occur locally due to loss in matric suction with increasing degree of saturation. An elasto-viscoplastic finite element program combined with random field theory is used to analyse the influence of the heterogeneity of the subsoil, as characterised by the point and spatial statistics of the property values. Using a Monte Carlo framework, the results of multiple realisations have been evaluated in terms of reliability as a function of both global factor of safety and time.

Keywords: Heterogeneity, rainfall, reliability, slope stability, unsaturated soil

1 INTRODUCTION

Analysing the stability of soil slopes is one of the oldest tasks in geotechnical engineering. However, even this relatively “simple” task of modelling and analysing the performance of a slope, in a residual soil or as part of a man-made embankment, will become challenging when accounting for the unsaturated state of the soil in interaction with the soil-atmosphere boundary at the ground surface. Changes in moisture content and matric suction as a function of the atmospheric condition directly influence the permeability, stiffness and strength of the subsoil. Thus, in order to address the stability of a soil slope, this boundary needs careful attention. However, the continuing occurrence of slope failures and landslides during or antecedent to rainfall events discloses the need for further investigations.

With the increasing demands of a fast growing world population and developing countries for suitable infrastructures, the design of engineered soil slopes in urban and industrial areas is becoming more required than ever. Since the risk associated with a slope failure may be interpreted as the product of the probability and the consequence, reliability-based methods should be used in order to account for the uncertainties involved within the slope design and construction process. Hence, rather than using the usual “cautious estimate”, the risks involved can be individually addressed and quantified.

The degree of uncertainty involved will be influenced by both the *epistemic* (subjective) uncertainty, accounting for the lack of knowledge, e.g. as in sampling, testing and modelling, as well as by the *aleatory* (objective) uncertainty, representing the inherent spatial variability of the subsoil (Helton, 1997). This paper aims to investigate the influence of the second type of uncertainty, that is, due to the subsurface heterogeneity, on the reliability of an unsaturated soil slope subjected to a rainfall event.

Various numerical analyses of rainfall-induced slope failure, in homogeneous unsaturated deposits, have been conducted in recent years, e.g. Cho and Lee (2001), Cai and Ugai (2004), Rahardjo et al. (2007) and Huang and Jia (2009), with reliability-based investigations on unsaturated slope stability being limited to first- and second-order analyses, e.g. Babu and Murthy (2005) and Zhang et al. (2005).

However, even within a moderately heterogeneous soil deposit, failure tends to propagate through the inherent weaker zones. Using random field methodology, the influence of the spatial variability of soil properties on the stability of saturated soil slopes has been investigated for undrained conditions, e.g. by Paice and Griffiths (1997), Hicks and Samy (2002a, 2002b, 2002c, 2004), Griffiths and Fenton (2004) and Hicks and Spencer (2010), for slopes under drained conditions, e.g. Szyrakiewicz et al. (2002) and

Griffiths et al. (2009), and for soil slope liquefaction, e.g. Hicks and Onisiphorou (2005). Random field methodology has also been used by Arnold and Hicks (2010a, 2010b), to analyse the influence of spatial variability of matric suction on unsaturated slope stability under steady state conditions. This paper aims to extend these previous investigations to transient conditions, accounting for the spatial variability of both the soil properties influencing the effective shear strength, as well as those soil properties controlling the infiltration capacity, hydraulic conductivity, water content and thus the local matric suction.

2 METHOD OF ANALYSING RAINFALL-INDUCED SLOPE FAILURE

2.1 Constitutive model formulation

In very recent years, several constitutive frameworks have been developed accounting for the direct coupling between the mechanical, hydraulic and thermal behaviour of unsaturated soils. However, the use of sophisticated models is generally accompanied by an increasing number of model parameters, in many cases with a decreasing physical meaning. Due to the scarcity of information on the in-situ variability of soil property data, especially for unsaturated conditions, a rather simple constitutive model formulation has here been applied in order to capture the implications of inherent spatial variability on unsaturated slope failure. For this purpose, the hydraulic model has been implicitly (weakly) coupled with the mechanical model. This means that a change in water content will affect the matric suction and thus the shear strength within the subsoil; however, a change in mechanical properties, such as of the porosity due to a collapse upon wetting and therefore of the hydraulic conductivity, is disregarded in this paper.

Darcy's law is valid for describing flow through an unsaturated soil stratum (Buckingham, 1907), and has been presented in terms of the total head H as the driving potential by Richards (1931). The mass balance equation in mixed form, that is, using the head potential on the driving side and the water content on the residual side, is given by

$$\nabla \cdot K \nabla H + Q = \frac{\partial \theta_w}{\partial t} \quad (1)$$

where K is the hydraulic conductivity, Q is the boundary flux per unit time t , θ_w is the volumetric water content and H is the total head which is the driving potential in moving the water. By assuming a constant gas potential, with the pore-air pressure being equal to the atmospheric condition, and by neglecting the osmotic potential by assuming pure water as the liquid phase, the total head is the sum of the suction head ψ and the elevation head z ($H = \psi + z$). Using the relationships proposed by van Genuchten (1980), in combination with the statistical pore-size distribution relationship by Mualem (1976), θ_w and K can both be computed as functions of the suction head ψ , that is

$$\theta_w = \theta_r + \frac{\theta_s - \theta_r}{\left[1 + (\alpha\psi)^n\right]^{1-1/n}} \quad \text{and} \quad K = K_s \frac{\left[1 - (\alpha\psi)^{n-1} \left[1 + (\alpha\psi)^n\right]^{\frac{(1-n)}{n}}\right]^2}{\left[1 + (\alpha\psi)^n\right]^{\frac{(n-1)}{2n}}} \quad (2)$$

where θ_s is the volumetric water content at saturation which is equal to the porosity ϕ , θ_r is the residual volumetric water content, α is the inverse of the air-entry suction head ψ_{ae} below which the soil is assumed to behave in a saturated manner, K_s is the saturated hydraulic conductivity and n is the slope of the soil water retention curve about the inflection point.

Bishop's (1959) effective stress concept, combined with a linear elastic, perfectly plastic Mohr-Coulomb type soil model extended to unsaturated conditions, provides the mechanical framework of the model. Hence,

$$\tau_f = c' + (\sigma - u_a) \tan \phi' + \chi (u_a - u_w) \tan \phi' \quad (3)$$

where τ_f is the soil shear strength, c' is the effective cohesion, ϕ' is the effective friction angle, σ is the total stress, u_a and u_w are the pore air and pore water pressure respectively, and $(\sigma - u_a)$ is the net stress. $(u_a - u_w)$ is the matric suction, which is equal to the suction head times the unit weight of water ($s = -\psi\gamma_w$), and χ is the suction stress parameter, which is a scalar relating to the suction induced effective stress; the product of both, χs , is often referred to as the suction stress. There is an ongoing discussion regarding the definition of χ , since it cannot be measured directly. Of the numerous existing empirical equations,

$$\chi = S_e = \frac{\theta_w - \theta_r}{\theta_s - \theta_r} \quad (4)$$

proposed by Vanapalli et al. (1996) and representing the effective degree of saturation S_e , has been shown to give in most cases an appropriate estimate of the suction stress parameter.

2.2 Numerical framework

The constitutive framework has been implemented within a finite element program based on Smith and Griffiths (2004). The suction head values ψ at the nodes are computed by solving Equation 1, using the modified Picard iteration method (Celia et al., 1990) within an implicit Crank-Nicholson time integration scheme. An advantage of this weakly coupled constitutive framework is that the time steps within the seepage and slope stability analyses are independent. At user specified times, the suction head values ψ are used to compute the suction stress χ_s at the Gaussian integration points of the finite element mesh, for analysing the slope stability. Gravitational loading is applied to the soil slope, in order to generate the in situ stresses. The strength reduction method is utilised to determine the point of failure, which is obtained by gradually reducing the shear strength. The slope stability analysis is thereby performed, whereas the seepage analysis is continuously running in parallel until reaching the next time step specified for a stability analysis.

The net flux applied to the soil-atmosphere boundary is a function of the precipitation, evaporation and run-off. Modelling the evaporation effects is generally quite important when analysing long term and seasonal events, in order to accurately predict the initial conditions prior to a rainfall event, since these have a significant influence on the infiltration capacity of the soil. However, in this investigation a single rainfall event is analysed and the effect of evaporation has not been accounted for. As a function of the moisture content, the infiltration capacity of the surface nodes is calculated interactively. The difference between the precipitation and the net flux is assumed to flow down the slope as run-off and may infiltrate at nodes where the actual infiltration capacity is not utilised by the precipitation. Assuming an efficient drainage system at the right-hand boundary of the domain analysed in this paper, the remaining accumulated run-off is removed at this point from the system.

2.3 Reliability-based methods

The local reduction in shear strength accompanying the movement of the wetting front through the sub-soil, during and antecedent to a rainfall event, leads to a time dependent factor of safety. Furthermore, the local advancement of this wetting is clearly a function of the spatial variability of the soil properties. As stated by Duncan (2000): “Through regulation or tradition, the same value of safety factor is often applied to conditions that involve widely varying degrees of uncertainty. This is not logical.”

The suction stress, water content and hydraulic conductivity are intrinsically coupled, and are time dependent variables influenced by the changes at the soil-atmosphere boundary. Thus, even the use of a “cautious estimate” of the soil property values within a deterministic analysis may lead to an overestimation of the slope safety. Consequently, the understanding of unsaturated soil mechanics in general and unsaturated slope stability in particular, would benefit from the use of reliability-based design methods.

In order to quantify the uncertainty, approximate first- and second-order probabilistic methods such as the First Order Reliability Method (FORM), Second Order Second Moment (SOSM) method and Point Estimate Method (PEM), as well as the Monte Carlo Method (MCM), are gaining increasing attention in engineering practice. However, by using only the point statistics, usually the mean μ_X as a measure for the central tendency and the variance σ_X^2 as a measure for the variability, of a parameter X_i , the spatial nature of the soil variability is either accounted for in a simplistic manner or possibly not at all. However, since the changes in suction stress are not only a function of time in combination with the applied soil-atmosphere boundary conditions, but also a spatially variable parameter with the water tending to avoid less permeable zones by flowing in a rather torturous manner to follow the path of least resistance, it is important to account for this property within the analysis.

2.4 Accounting for spatial variability

The spatial statistics are the scale of fluctuation in the vertical and horizontal directions, $\theta_{X,v}$ and $\theta_{X,h}$ respectively. Vanmarcke (1983) defined the scale of fluctuation as the distance over which X_i is strongly correlated. Thus, the larger the value of θ_X , the more homogeneous the soil deposit.

In this investigation, the heterogeneity within the soil is considered to be moderate. Hence, the aim is to analyse the effect of the spatial variability within what seems to be a homogeneous soil stratum, so that a slope failure will be influenced by local weak zones, rather than by cracks, fractures and layer boundaries implying a strong heterogeneity. Furthermore, the use of the finite element method in analysing unsaturated flow is then straight forward, whereas, in a strongly heterogeneous deposit, more advanced constitutive flow formulations, e.g. double porosity models, would need to be implemented to account for steep hydraulic gradients between, for instance, a crack and the surrounding soil.

In this investigation, fields of random properties are generated using an algorithm based on Local Average Subdivision (Fenton and Vanmarcke, 1990). Based on the spatial statistics, n isotropic standard normal random fields are generated for a square domain through a process of uniform subdivision. Using Cholesky decomposition, the parameters $X_1 \dots X_n$ are pointwise cross-correlated. A certain degree of anisotropy of the heterogeneity, $\xi_X = \theta_{X,h} / \theta_{X,v}$, may be introduced by squashing and, if required, stretching of the isotropic field, as described by Hicks and Samy (2002b, 2004). The cell values are then transformed to the designated distributions according to μ_X and σ_X^2 .

3 EXAMPLE PROBLEM

A 45° slope of height 5m, founded on a firm base at a depth of 10m (Figure 1), has been analysed in 2D assuming plane strain conditions. The problem domain has been discretised using 0.25×0.25m eight-node quadrilateral finite elements for the stability analysis. For the computation of the suction head at any given time, 0.125×0.125m four-node quadrilateral elements have been used. At every user defined time level chosen for analysing the slope stability, the suction stress values are mapped onto the Gaussian integration points of the slope mesh, as shown in Figure 2.

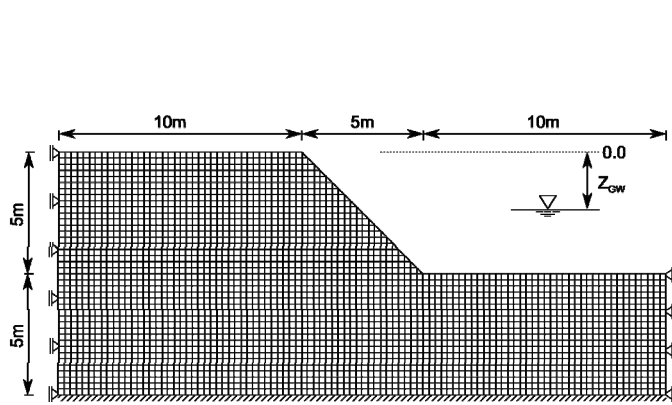


Figure 1. Problem domain, mesh and boundary conditions

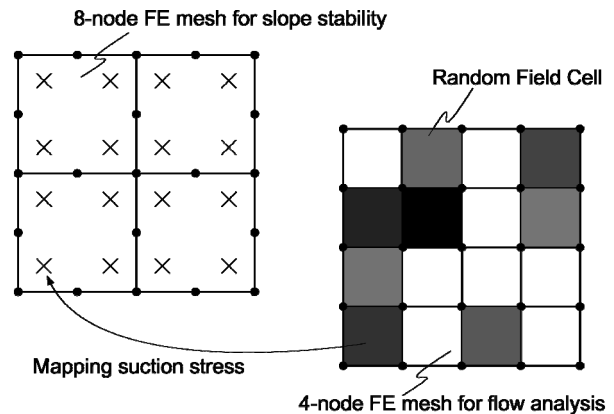


Figure 2. Mapping of the suction stress χ_s

Although it is generally possible to describe all soil parameters by their point and spatial statistics, in using the implemented constitutive framework for analysing the stability of the unsaturated slope, five predominant parameters have been selected as spatially varying. Specifically, it is assumed that $\tau_f(\mathbf{x}) = f(c'(\mathbf{x}), \phi'(\mathbf{x}), \phi(\mathbf{x}), K_s(\mathbf{x}), \alpha(\mathbf{x}))$, with the effective shear strength parameters c' and ϕ' directly influencing the shear strength τ_f , and ϕ , K_s and α indirectly via the suction stress χ_s . The point statistics and distributions are summarised in Table 1 and are representative of a sandy clayey loam, see e.g. Rawls et al. (1982) and Carsel and Parrish (1988).

It is assumed that c' , ϕ' and ϕ are log-normally distributed, although the low coefficients of variation, $V_X = \sigma_X / \mu_X$, of 0.2, 0.3 and 0.15 respectively suggest that a normal distribution might also be used.

The saturated hydraulic conductivity K_s , as well as the inverse of the air-entry pressure α , are also log-normally distributed, with coefficients of variation of 1.75 and 0.9 respectively.

Table 1. Point statistics and distributions

Material parameter		μ_X	V_X	Distribution
Effective cohesion	c' [kPa]	10.0	0.2	Log-normal
Effective friction angle	ϕ' [°]	25.0	0.3	Log-normal
Porosity	ϕ [-]	0.4	0.15	Log-normal
Saturated hydraulic conductivity	K_s [m h ⁻¹]	0.0036	1.75	Log-normal
Inverse of the air-entry suction head	α [m ⁻¹]	-1.0	0.9	Log-normal

The “representative” deterministic values assumed for the remaining material parameters are the soil unit weight, $\gamma = 20\text{kN/m}^3$; Young’s modulus, $E = 1 \times 10^5\text{kPa}$; Poisson’s ratio, $\nu = 0.3$; dilation angle, $\psi_d = 0^\circ$; slope of the soil-water retention curve, $n = 2.0$; and residual volumetric water content, $\theta_r = 0.08$.

The definition of the covariance structure is one of the key issues in stochastic modelling and is especially difficult for soils; not only the definition of the parameter variance σ_X^2 , but mainly the definition of the cross-correlation coefficients ρ_{X_i, X_j} between the parameters is a complex challenge. This is due to the scarcity of data relating to the in situ variability, as well as the difficulty in interpreting the cross-correlative effects on the outcome, e.g. the interpretation of the contributions of c' and ϕ' to the saturated shear strength.

Based on test results found in literature, a typical correlation matrix \mathbf{R} (see Equation 5) has been set up to define the covariance structure for this boundary value problem. Test results mainly show a negative correlation between $\ln(c')$ and $\ln(\phi')$, here assumed to be -0.5 in the underlying standard normal field; however, this is not always the case, as some results have shown, e.g. Lumb (1970). The strong positive correlation of $\rho_{\ln(\phi), \ln(K_s)} = 0.8$ is reasonable, since the larger the porosity the larger the saturated hydraulic conductivity. Moreover, an increasing porosity is associated with a decreasing air-entry suction head ψ_{ae} , and so a positive correlation for $\rho_{\ln(\phi), \ln(\alpha)} = 0.6$ and implicitly for $\rho_{\ln(K_s), \ln(\alpha)} = 0.5$ is assumed.

Looking at the heterogeneity within a soil layer, for instance, the effective friction angle ϕ' and the air-entry pressure ψ_{ae} are both more likely to increase in denser zones, that is, with a decreasing porosity ϕ ; in contrast, the saturated hydraulic conductivity K_s will tend to decrease. Thus, despite the absence of experimental test data, it seems reasonable to assume some negative correlation between $\ln(\phi')$ and the inverse of the air-entry pressure $\ln(\alpha)$, as is the case between $\ln(\phi')$ and $\ln(K_s)$. However, since the correlations between the effective cohesion $\ln(c')$ and $\ln(\phi)$, $\ln(K_s)$ and $\ln(\alpha)$ are not clearly evident from literature, these parameters are assumed to be uncorrelated.

$$\mathbf{R} = \begin{bmatrix} \begin{matrix} 1.0 & -0.5 \\ \rho_{\ln(\phi'), \ln(c')} & 1.0 \end{matrix} & \begin{matrix} 0.0 & 0.0 & 0.0 \\ -0.3 & -0.2 & -0.2 \end{matrix} \\ \rho_{\ln(\phi), \ln(c')} & \rho_{\ln(\phi), \ln(\phi')} & \begin{matrix} 1.0 & 0.8 & 0.6 \\ \rho_{\ln(K_s), \ln(\phi)} & 1.0 & 0.5 \\ \rho_{\ln(\alpha), \ln(\phi)} & \rho_{\ln(\alpha), \ln(K_s)} & 1.0 \end{matrix} \end{bmatrix} \quad (5)$$

The scale of fluctuation is a function of the geological deposition process and thus it seems reasonable to assume that $\theta_{\ln(c')} = \theta_{\ln(\phi')} = \theta_{\ln(\phi)} = \theta_{\ln(K_s)} = \theta_{\ln(\alpha)}$ in the underlying standard normal field. A vertical scale of fluctuation of $\theta_{\ln(X), v} = 2\text{m}$ has here been assumed. The influence of the degree of anisotropy of the heterogeneity will be investigated by analysing both isotropic and anisotropic soil property fields, with $\xi_{\ln(X)} = 6$ for the latter.

For the current investigation, the effect of a 48h rainfall event, on the stability of the soil slope shown in Figure 1, is analysed. Using a *Dirichlet-type* boundary condition, a constant suction head of $\psi_{init} = -7.0\text{m}$ is applied to the soil-atmosphere boundary, representing an initially “dry” condition. A constant head of $\psi_{gw} = 0.0\text{m}$ defines the groundwater table, which, in this example, is fixed at the firm base at a depth of $z_{gw} = -10.0\text{m}$. A continuous surface flux of $q_{rain} = 18.0\text{mm h}^{-1}$ is applied as a *Neuman-type* boundary condition for 48h, which is representative of a heavy rainfall event, and this is followed by an antecedent light rainfall event of $q_{ant} = 1.0\text{mm h}^{-1}$, which is representative for a final “wet” condition.

In order to compute the reliability of the soil slope, multiple Monte Carlo simulations are performed in order to obtain a converged solution. The air saturated c' - ϕ' slope is analysed first: that is, *Case 1*, using only the 2×2 correlation sub-matrix at location North-West in Equation 5.

For a given factor of safety, the reliability of the slope is given by

$$R = 1 - P_f = 1 - \frac{N_f}{N} \quad (6)$$

where P_f is the probability of failure, N is the total number of realisations and N_f is the number of realisations in which the slope fails. Two sets of reliability analyses have been performed for the slope under unsaturated conditions. First, c' and ϕ' are kept constant at their mean values to analyse only the effect of the spatially variable suction stress on slope stability: that is, *Case 2*, using only the 3×3 correlation sub-matrix at location South-East in Equation 5. Thereafter, for *Case 3*, a complete analysis with $\tau(\mathbf{x}) = f(c'(\mathbf{x}), \phi'(\mathbf{x}), \phi(\mathbf{x}), K_s(\mathbf{x}), \alpha(\mathbf{x}))$ is performed. Since the factor of safety is variable in time, multiple reliability distributions have been computed in order to quantify reliability R as a function of time t . For this preliminary investigation, 300 Monte Carlo realisations per time step were found to be sufficient to analyse the time dependent structural response in a qualitative manner.

4 DISCUSSION OF RESULTS OF EXAMPLE PROBLEM

Assuming the soil is completely air saturated above the groundwater level, the computed traditional factor of safety based on the mean property values is $FOS_{sat}(\mu) = 1.39$. Figure 3 shows the influence of scale of fluctuation on reliability R versus global factor of safety F for *Case 1*, where F is computed for every Monte Carlo realisation by dividing the traditional factor of safety based on the mean strength values by the factor of safety based on the heterogeneous property field (i.e. $F = FOS_{sat}(\mu) / FOS_{sat}$). It is seen that, for the isotropic and anisotropic fields, most responses are weaker than the deterministic solution based on the property means; that is, with $R < 0.5$ for $F = 1.0$, thereby implying that failure is attracted to the weaker zones. Also, the response distribution becomes wider as the degree of spatial correlation increases, due to each field having a more uniform appearance which leads to a wider range of possible solutions.

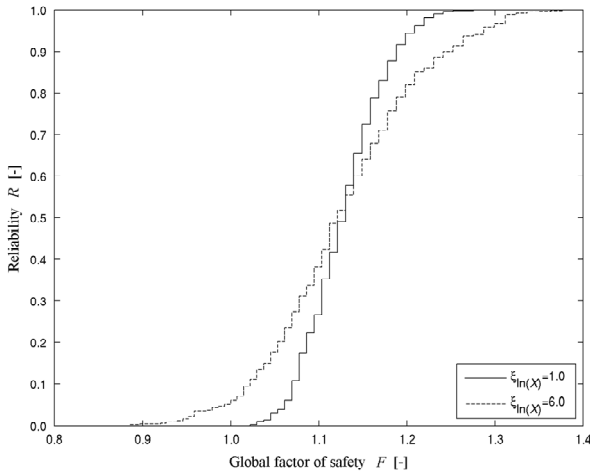


Figure 3: Reliability versus global factor of safety for *Case 1*

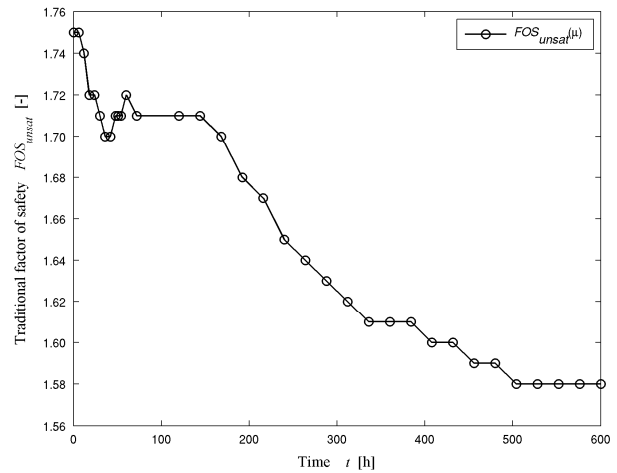


Figure 4: Time dependent factor of safety based only on the mean property values

Based on the deterministic mean property values, Figure 4 shows $FOS_{unsat}(\mu)$ for the soil slope accounting for the unsaturated state as a function of time. At the initial state there is a maximum suction stress of $\chi_s \approx 9.71$ kPa within the soil, causing the factor of safety to increase from $FOS_{sat}(\mu) = 1.39$ for the air saturated case to $FOS_{unsat}(\mu) = 1.75$. During the 48 hours of heavy rainfall the stability of the slope is only slightly reduced. This is because, firstly, for this sandy clayey loam, the wetting front is moving only slowly through the ground, reaching zones critical for defining the slope failure only antecedent to the rainfall event itself. Secondly, the infiltration capacity reduces quickly as the area close to the surface becomes saturated, leading to run-off and thus to a reduced net influx. Note that, for the current boundary value problem, the matric suction is only temporarily reduced to zero during the heavy rainfall in the local region of the moving wetting front and recovers partly thereafter due to drainage of the soil water. A minimum factor of safety of $FOS_{unsat}(\mu) = 1.58$ for the final wet condition is reached 500h after the start of the heavy rainfall.

For the first 288 hours Figures 5 and 6 summarise, in the form of four contour plots, the reliability $R(t)$ for *Case 2* and *Case 3*. The global factor of safety is now computed by $F(t) = FOS_{unsat}(\mu, t) / FOS_{unsat}(t)$, at every time step specified for the stability analysis. Figure 5 indicates that, for *Case 2*, that is, with only $\chi_s(\mathbf{x})$ varying and c' and ϕ' fixed to their means, the soil response is stronger relative to the deterministic solution, that is, with $R(t) > 0.5$ for $F(t) = 1.0$. Evaluating the influence of the heterogeneity on the suction stress profile is difficult, since, besides the dependency on the soil properties themselves, χ_s is largely dependent upon the relative location to the soil-atmosphere boundary and on the elevation above the groundwater level.

Due to the non-linearity of the soil-water retention curve, as well as the positive correlation between $\ln(\phi)$ and $\ln(\alpha)$, the degree of saturation is most likely to be higher in a zone with a lower porosity, that is, for a similar location under the same initially dry steady state conditions. This means that, for a specific suction head value, the effective degree of saturation $S_e = \chi$ (Equation 4) is likely to be higher in a denser zone, thus leading to a higher shear strength than for a soil with mean properties in the same location.

For both the isotropic and anisotropic analyses in *Case 2*, the reliability tends to slightly decrease with time for a certain global factor of safety F , starting from the beginning of the heavy rainfall. This is a consequence of water tending to infiltrate faster through more permeable flow paths, as well as going into storage in the denser zones causing a strength reduction due to the decrease of the initially high suction stresses χ_s . As for *Case 1*, the variance of the response increases with increasing correlation length.

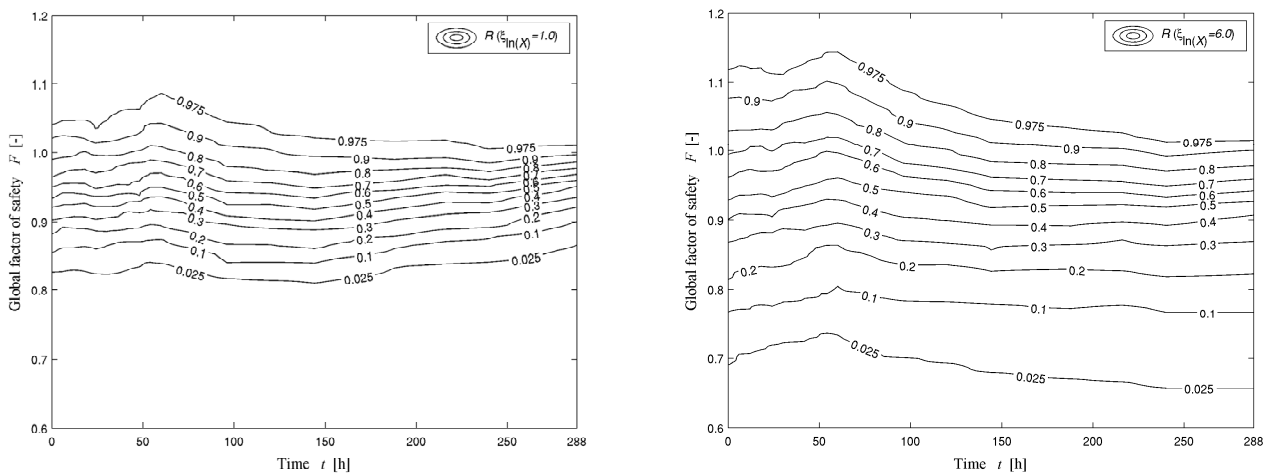


Figure 5. Contour plot of reliability for *Case 2*, for isotropic and anisotropic heterogeneity

From the *Case 3* results in Figure 6, it is evident that, although the soil is gaining some strength due to the spatially variable suction stress (*Case 2*), the response distribution for this example is mainly governed by the spatial variability of the effective shear strength parameters c' and ϕ' . That is, the response based upon the mean properties tends to overestimate the stability of the slope, with R being relatively lower for $\xi_{\ln(\chi)} = 6$. However the influence of $\chi_s(\mathbf{x})$ is evident, with R increasing over time for a certain F .

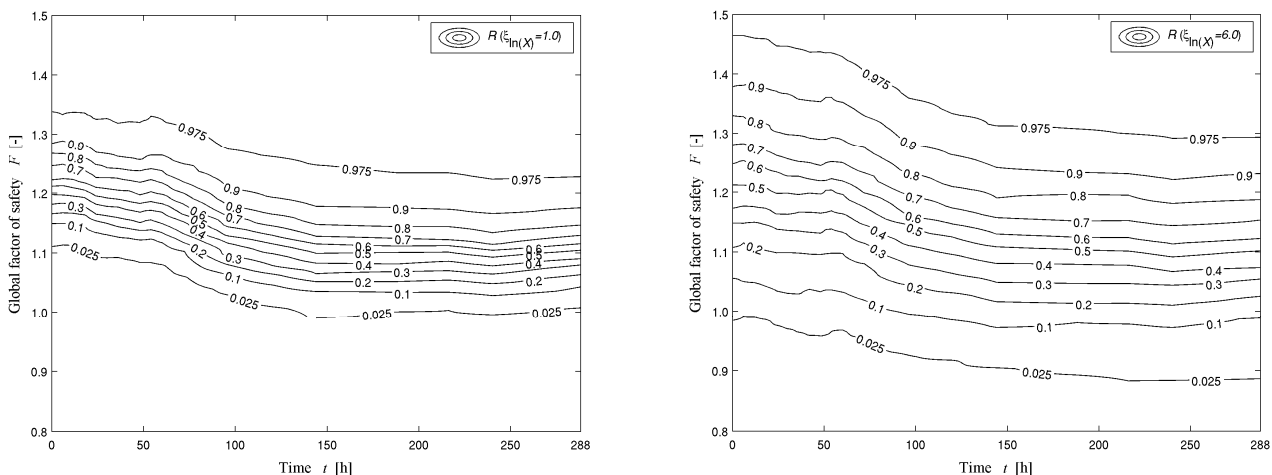


Figure 6. Contour plot of reliability for *Case 3*, for isotropic and anisotropic heterogeneity

5 CONCLUSIONS

The influence of the matric suction on the stability of an unsaturated soil slope has been evaluated accounting for the soil heterogeneity. Based on an example problem it has been shown that, although in this instance the failure is driven by the spatial variability of the effective shear strength parameters, neglecting the influence of the suction stress may lead to an erroneous assessment of the slope reliability.

ACKNOWLEDGEMENTS

This work has been funded by the European Union FP7 Projects IRIS and GEO-INSTALL.

REFERENCES

- Arnold, P. and M. A. Hicks (2010a). A stochastic investigation of unsaturated slope stability. In A. Zervos, editor, *Proc.: 18th UK National Conference on Computational Mechanics in Engineering (ACME-UK)*, Southampton, UK, pages 9–12.
- Arnold, P. and M. A. Hicks (2010b). Stochastic modelling of unsaturated slope stability. In E. Alonso and A. Gens, editors, *Proc.: Fifth International Conference on Unsaturated Soils (UNSAT)*, Barcelona, Spain, pages 1237–1242.
- Babu, S. G. L. and D. S. N. Murthy (2005). Reliability analysis of unsaturated soil slopes. *Journal of Geotechnical and Geoenvironmental Engineering*, 131(11), 1423–1428.
- Bishop, A. W. (1959). The principle of effective stress. *Teknisk Ukeblad*, 106(39), 859–863.
- Buckingham, E. (1907). Studies on the movement of soil moisture. Technical report, Bulletin 38, USDA, Bureau of Soils, Washington, DC, USA.
- Cai, F. and K. Ugai (2004). Numerical analysis of rainfall effects on slope stability. *International Journal of Geomechanics*, 4(2), 69–78.
- Carsel, R. F. and R. S. Parrish (1988). Developing joint probability distributions of soil water retention characteristics. *Water Resources Research*, 24(5), 755–769.
- Celia, M. A., E. T. Bouloutas, and R. L. Zarba (1990). A general mass-conservative numerical solution for the unsaturated flow equation. *Water Resources Research*, 26(7), 1483–1496.
- Cho, S. E. and S. R. Lee (2001). Instability of unsaturated soil slopes due to infiltration. *Computers and Geotechnics*, 28(3), 185–208.
- Duncan, J. M. (2000). Factors of safety and reliability in geotechnical engineering. *Journal of Geotechnical and Geoenvironmental Engineering*, 126(4), 307–316.
- Fenton, G. A. and E. H. Vanmarcke (1990). Simulation of random fields via local average subdivision. *Journal of Engineering Mechanics, ASCE*, 116(8), 1733–1749.
- Griffiths, D. V. and G. A. Fenton (2004). Probabilistic slope stability analysis by finite elements. *Journal of Geotechnical and Geoenvironmental Engineering*, 130(5), 507–518.
- Griffiths, D. V., J. Huang, and G. A. Fenton (2009). Influence of spatial variability on slope reliability using 2-D random fields. *Journal of Geotechnical and Geoenvironmental Engineering*, 135(10), 1367–1378.
- Helton, J. C. (1997). Uncertainty and sensitivity analysis in the presence of stochastic and subjective uncertainty. *Journal of Statistical Computation and Simulation*, 57(1–4), 3–76.
- Hicks, M. A. and C. Onisiphorou (2005). Stochastic evaluation of static liquefaction in a predominantly dilative sand fill. *Géotechnique*, 55(2), 123–133.
- Hicks, M. A. and K. Samy (2002a). Influence of anisotropic spatial variability on slope reliability. In G. N. Pande and S. Pietruszczak, editors, *Proc.: 8th International Symposium on Numerical Models in Geomechanics (NUMOG)*, Rome, Italy, pages 535–539.
- Hicks, M. A. and K. Samy (2002b). Influence of heterogeneity on undrained clay slope stability. *Quarterly Journal of Engineering Geology and Hydrology*, 35(1), 41–49.
- Hicks, M. A. and K. Samy (2002c). Reliability-based characteristic values: A stochastic approach to Eurocode 7. *Ground Engineering*, 35(12), 30–34.
- Hicks, M. A. and K. Samy (2004). Stochastic evaluation of heterogeneous slope stability. *Italian Geotechnical Journal*, 38(2), 54–66.
- Hicks, M. A. and W. A. Spencer (2010). Influence of heterogeneity on the reliability and failure of a long 3D slope. *Computers and Geotechnics*, 37(7–8), 948–955.
- Huang, M. and C.-Q. Jia (2009). Strength reduction FEM in stability analysis of soil slopes subjected to transient unsaturated seepage. *Computers and Geotechnics*, 36(1–2), 93–101.
- Lumb, P. (1970). Safety factors and the probability distribution of soil strength. *Canadian Geotechnical Journal*, 7(3), 225–242.
- Mualem, Y. (1976). A new model for predicting the hydraulic conductivity of unsaturated porous media. *Water Resources Research*, 12(3), 513–522.
- Paice, G. M. and D. V. Griffiths (1997). Reliability of an undrained clay slope formed from spatially random soil. In J.-X. Juan, editor, *Proc.: Ninth International Conference on Computer Methods and Advances in Geomechanics (IACMAG)*, Volume 1, Wuhan, China, pages 543–548.
- Rahardjo, H., T. H. Ong, R. B. Rezaur and E. C. Leong (2007). Factors controlling instability of homogeneous soil slopes under rainfall. *Journal of Geotechnical and Geoenvironmental Engineering*, 133(12), 1532–1543.

- Rawls, W. J., D. L. Brakensiek and K. E. Saxton (1982), Estimating soil water properties. *Transactions of the American Society of Agricultural Engineers*, 25(5), 1316–1320, 1328.
- Richards, L. A. (1931). Capillary conduction of liquids through porous mediums. *Journal of Applied Physics*, 1(5), 318–333.
- Smith, I. M. and D. V. Griffiths (2004). *Programming the Finite Element Method*. Chichester: John Wiley & Sons.
- Szynakiewicz T., D. V. Griffiths and G. A. Fenton (2002). A probabilistic investigation of c' , ϕ' slope stability. In C. Muller-Karger et al., editors, *Proc.: 6th International Congress on Numerical Methods in Engineering and Scientific Applications (CIMENICS)*, Caracas, Venezuela, pages 25–36.
- van Genuchten, M. T. (1980). A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal*, 44(5), 892–898.
- Vanapalli, S. K., D. G. Fredlund, D. E. Pufahl, and A. W. Clifton (1996). Model for the prediction of shear strength with respect to soil suction. *Canadian Geotechnical Journal*, 33(3), 379–392.
- Vanmarcke, E. H. (1983). *Random Fields: Analysis and Synthesis*. Cambridge, Massachusetts: MIT Press.
- Zhang, L. L., L. M. Zhang and W. H. Thang (2005). Rainfall-induced slope failure considering variability of soil properties. *Géotechnique*, 55(2), 183–188.