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Internal erosion – state of the art and an approach with percolation theory –

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During groundwater flow fines can move inside the grain skeleton and suffosion occur. In previous investigation mostly the problem is reduced into a normal filtration or contact erosion problem. The criteria to control if suffosion take place or not don't take into account that the pore structure play an important role in suffosion processes. With percolation theory, which is a branch of probability theory dealing with properties of random media, it is possible to build up a model of the pore structure. Characteristic quantities can be determined to describe suffosion. Therefore certain input parameters are necessary. The determination of these will be topic of further research.

I. INTRODUCTION

The vulnerability of river basins as referring to extreme flood events has increased over the past years. This is a consequence of climatic changes and the more and more intensive usage of rivers and their environment, e.g. for industrialisation, land use and shipping. One of the negative results is that any change in flow conditions in river basin scale is a potential impact for erosion in subsurface. One kind of erosion is the internal erosion. Internal erosion of soil structures induced by seepage forces is also a main problem for the stability of water engineering structures. Dam failures statistical analysis taken from ICOLD [14] [15] show that the reasons for dam failures are 18 % internal erosion of the dam body and 12 % internal erosion of the subsurface. Only overtopping of dams has a higher responsibility for dam failures. River embankments and hydraulic structures are not considered in this statistic.

During groundwater flow fines in the grain skeleton can be displaced by seepage forces. The kind of erosion where the displacement of fines in the grain skeleton is taking place is called suffosion. When suffosion occur than the permeability and the porosity will increase while the bulk density decrease ([4] and [11]). The consequences are less resistance against external load and settlement as well as significant change in state of pore pressure. The probability for scour, landslides and hydraulic heave will increase.

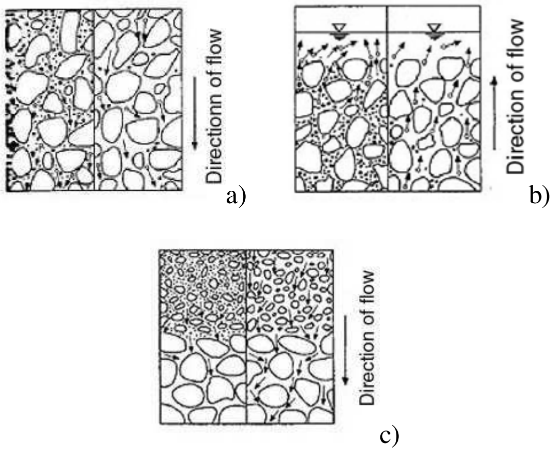


Figure 1. a) internal suffosion b) external suffosion and c) contact suffosion [25]

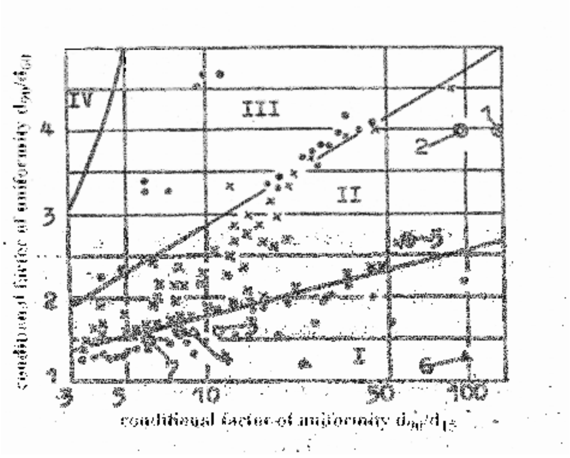
In dependency of the location where suffosion occur Ziems [25] distinguish between three kinds of suffosion i. e. internal suffosion, external suffosion and contact suffosion (Fig. 1). The focus in this paper is located at the phenomena of internal suffosion. Therefore external and contact suffosion will no longer be discussed. Good reviews to several kinds of internal erosion were published among others in [4] [14], [16] and [17]. Internal suffosion can be in the best case a local phenomena where the fines will be trapped in dependency of particle size and hydrodynamic forces (colmatation). But also suffosion can be maintained when contact erosion at different layers or external erosion follow.

To exclude that internal suffosion or internal erosion of soils can occur it is necessary to satisfy two criteria. The fundamental criterion is the proof if it is possible that fine material can pass through void throats without clogging (geometrical criteria). The sufficient criterion is satisfied when it can be excluded that the hydrodynamic load in the void structure provides a critical energy needed to mobilize the fines inside the void structure (hydraulic criteria). The most important criteria for suffosion used in German engineering practice will be summarized.

II. GEOMETRICAL CRITERIA

The proof of the geometrical criteria, especially for non-uniform soils with steady and concave grain size distributions or soils with an omitted-size fraction, are based mostly on studies focused on filter materials for embankment dams [3]. Burenkova [3] found a empirical solution (Fig. 2) after numerous studies on different soils. Schneider et al. [20] formulated that this empirical relationship is very useful to predict if a risk for suffosion exist or not.

The Federal Waterways Engineering & Research Institute (BAW) in Germany recommend in [13] to first separate the grain size distribution into a finer and coarser part and to proof the geometrical criterion of Cistin/Ziems (Fig. 3) afterwards. The criterion of Cistin/Ziems were initially developed to analyse contact erosion phenomena. The geometrical criterion – i. e. no filtration – is satisfied if the d50-relation $A50 = d_{50II}/d_{50I}$ is less than the ultimate-relation A50,ult. given at the y-axis of the chart in Fig. 3.



Zone I and III:	suffosive
Zone II:	non-suffosive
Zone IV:	zone of artificial soils

Figure 2. Criterion of Burenkova [3]

Other fundamental investigations to geometrical criteria were performed by e.g. Terzaghi (1948) [22], Patrašev (1938), Sichardt (1928), Istomina (1957), Pavčič (1961), (cited in [25]), Lubočkov [12] and Kenney and Lau [10]. The study of these criteria permit to characterise in advance which soils are definitely not at risk that suffosion occur and which kind of soil has to be analysed. Therefore characteristical non-suffosive soils are ([4], [17]):

- Soils with a factor of uniformity $C_U = d_{60}/d_{10} \approx 1$ (d_{60} and d_{10} - diameters of particles for which 60 % or 10 % are smaller by weight).

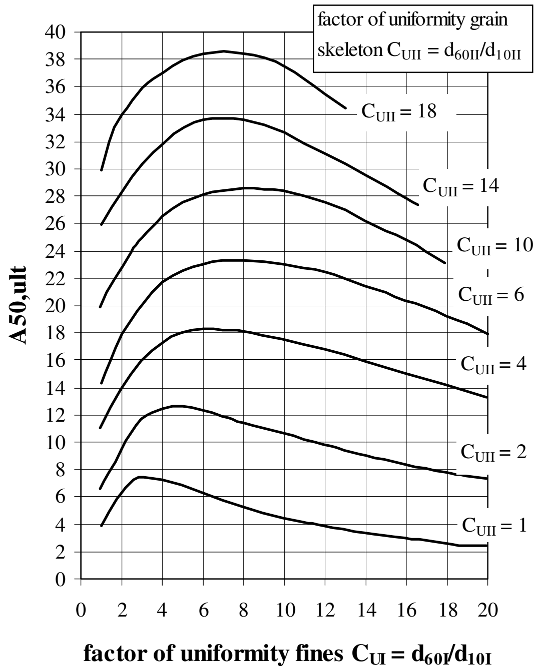


Figure 3. Criterion of Cistin/Ziems (cited in [4])

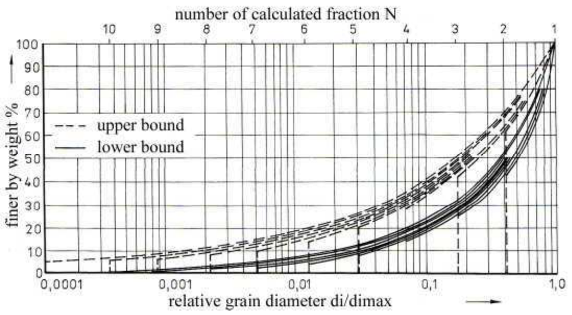


Figure 4. Limits of grain size distributions for non-suffosive soils [12]

- Soils with a rather linear grain size distribution in semi-logarithmic scale with $C_U < 10$ irrespective of density index

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}}. \quad (1)$$

- Non-uniform soils with $C_U > 10$ and $D_r < 0.6$.
- Steady curved grain size distribution with $C_u < 8$ irrespective of D_r.
- Non-uniform soils which are very close to the Fuller or Talbot grain size distribution.
- After Lubočkov [12] non-uniform soils with $D_r = 0.3$ till 0.6 and steady curved grain size distribution in border area of Fig. 4 .

III. HYDRAULIC CRITERIA

The hydraulic criteria for suffosion compare the existing hydraulic gradient i to the allowable hydraulic gradient i_{ult}. The hydraulic criterion is exceeded when the fines inside the grain skeleton begins to move. To guarantee that the suffosion criteria are satisfied the BAW MSD 2005 [13] defined the ultimate condition

$$\eta = \frac{i_{ult}}{i} \geq 2. \quad (2)$$

For the determination of the ultimate hydraulic gradient Busch et al. [4] and the BAW MSD 2005 [13] recommend, for the case that the grain size distribution is steady, the following formula:

$$i_{ult} = \varphi_0 \cdot \sqrt{\frac{n \cdot g \cdot d_s^2}{\nu \cdot k}} \quad (3)$$

$$\varphi_0 = 0.6 \cdot \left(\frac{\gamma_d}{\gamma_w} - 1\right) \cdot \mu^* \cdot \sin\left(30 + \alpha/8\right) \quad (4)$$

$$\mu^* = 0.82 - 1.8 \cdot n + 0.0062 \cdot (C_U - 5) \quad (5)$$

$$d_s = 0.27 \cdot \sqrt[6]{C_U} \cdot \frac{n}{1-n} \cdot d_{17} \quad (6)$$

The definition of the angle α which characterize the direction of flow is:

$$\alpha = \begin{cases} \rightarrow 90° \\ \downarrow 0° \\ \uparrow 180° \end{cases} . \tag{7}$$

α	angle between acceleration of gravity and direction of flow
n	Porosity
k	Permeability
ν	kinematic viscosity
γ_d	dry specific weight of soil
γ_w	specific weight of water
d_s	largest suffosive grain diameter

For a non-steady grain size distribution and α= 180° (upward directed flow) the BAW MSD 2005 [13] and Busch et al. [4] recommend the approximate formula of Istomina (1957).

$$i_{crit} = \frac{n \cdot k_F}{n_F \cdot k} + f\left(\frac{d_{10,Sk}}{d_{10,F} \cdot \tan \varphi_F} \right). \tag{8}$$

n, k	Porosity and permeability of soil sample
n_F, k_F	Porosity and permeability of filling
$d_{10,Sk}$	d_{10} of grain skeleton (matrix)
$d_{10,F}$	d_{10} of filling

The border of filling and matrix is chosen by Istomina at the 1 mm grain size diameter. For the second term of the criterion there exist a graphical description (Fig. 5) and the value tan φ_F can be taken from Tab 1.

Another criterion for two component mixtures of sand and gravel and α = 180°can be found in the dissertation of Wittmann [24]. He put the velocity of the fluid inside the voids in opposite to the decantation rate after Zanke (cited in [24]). This criterion allows also the examination of suffosion under turbulent flow in the void structure.

IV. VALUATION OF THE CRITERIA

In the existing criteria there is assumed that only a transport of the skeleton filling obey while the grain skeleton don't have further structural change. This assumption stands in contradiction to observations in nature. Already first investigations of Leussink et al. [11] at artificial soils show clearly a interrelationship of structural change and soil-mechanical characteristics. The fundamental of geometrical criteria are the simplification that the grains are spheres. The determination of the controlling grain diameter is going back to a sphere packing in two dimensions without an extrapolation to 3-D (e. g. [14], [18], [19] or [24]). An observation in randomly chosen two dimensional slices don't represent the true void or void throat size distribution (Fig. 6). In two dimensions it is not possible to differentiate if the space between the grains are voids or void throats. The void size distribution determined with these approach represents a mean opening size but not the distribution of the minima, i.e. the throats, which are responsible for filtration.

TABLE I.
VALUES TAN φ_F [17]

Filling	**Silt**	**Fine Sand**	**Medium Sand**
tan φ_F	0,57	0,6	0,7

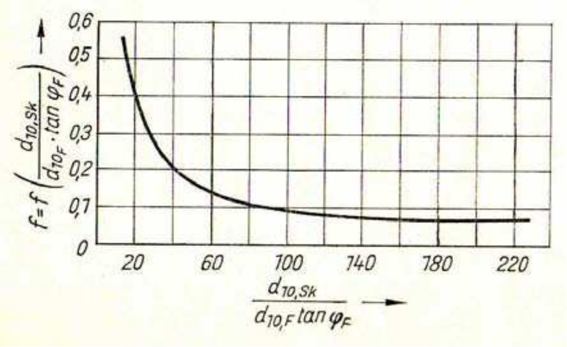


Figure 5. Graphical description of the Istomina-function f=f() [4]

The criteria are mostly practicable in a short range inside a factor of uniformity between C_U = 3 – 65. Without Schuler [18] and Witt [23] the investigations for geometrical criteria don't take into account that with increasing thickness of the layers the probability that a particle can be trapped increase, too. Therefore the factor of safety is very high. New investigation were not developed because the mathematical and physical relationships and the technical facilities to rebuild and understand the transport and fluid flow inside the complex void structure were only partially given or not at all.

V. PERCOLATION THEORY

A modern approach to comprehend flow and transport mechanisms inside a void structure is the percolation theory which is a branch of probability theory dealing with properties of random media ([1]). Large advances to understand complex relationships concerning transport and flow in porous media were developed in the last 15 years with this theory. Broadbent and Hammersley [2] can be named as originators for the use of percolation theory to study fluids in a maze. Percolation theory is used in several disciplines dealing with complex structures like petrophysics, hydrology, chemistry and statistical physics. Hence there were published several reviews to percolation theory in general (e. g. [6] and [21]) and particular in relation to porous media (e. g. [5], [7] and [15],). Schuler [18] used a percolation model to simulate the penetration length into a filter. He realised for his studies a very sim-

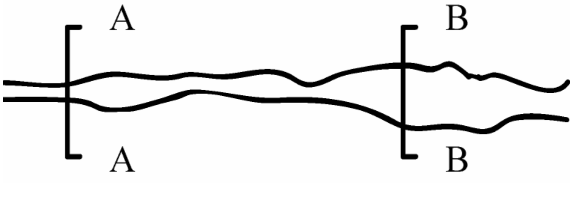


Figure 6. Void channel with random slices A-A and B-B

ple percolation model. He simulated the void throat sizes on a simple square lattice without correlations between voids, void throats and their interrelationship in the void structure. The consequence is a misinterpretation of the penetration depth. All investigations with percolation theory concerning flow and transport processes in porous media demonstrate that percolation theory is a helpful tool to analyse also internal erosion phenomena like suffosion.

The huge advantage of percolation models to previous investigations concerning geometrical suffosion criteria is that a transformation of the void structure, very close-to-the reality, is possible. This allows detailed analysis of transport phenomena inside the void structure. The rearrangement and transportation of fines can be analysed. Percolation theory deals with three kinds of models namely bond, site and continuum percolation models. Exemplary and most important to suffosion processes bond percolation is explained below.

In this approach the void structure will be transferred randomly to a given two or three dimensional grid (lattice). In this lattice the knots (sites) represent voids and the connections (bonds) represent the void-throats.

A very simple percolation model is the bond percolation model on a two dimensional square lattice (Fig. 7). Bond percolation means that only the void throats were simulated on a square lattice. A grain can pass from one void (site) to a neighbouring void if the void throat (bond) is big enough. In this case the bond is called occupied and the sites are connected. The connected nearest neighbour sites form a cluster. A spanning cluster is a cluster of occupied bonds from one border of the lattice to the opposite border. All other clusters are called finite clusters. Each void has a coordination number $z = 4$ which means that each void has four neighbouring voids. The randomly placed void throats are with occupation probability p occupied or with $q = 1 - p$ not. If $p = 1$ all bonds are occupied and if the lattice is very large and the occupation probability p is small there are only small finite clusters (Fig. 7). Dependent of the kind of lattice (e. g. cubic, square, triangular) one well defined value p_c (p_c = percolation threshold), below which there is no spanning cluster, exist [21]. In the case that no correlations are allowed several exact solutions for different lattices of the bond percolation threshold p_c can be denoted (Tab 2).

The determination of the percolation threshold is identical with the determination of the biggest suffosive grain diameter which can be transported by seepage forces irrespective of the layer thickness. Additionally suffosion properties can be characterized by several other quantities. The most important of which are:

TABLE II.
SELECTED BOND PERCOLATION THRESHOLD P_C BOND FOR DIFFERENT UNCORRELATED LATTICES [15]

Lattice Type	**Coordination Number z**	**p_c bond**
Honycomb	3	$1-2\sin(\pi/18)$
Square	4	0.5
Triangular	6	$2\sin(\pi/18)$
Diamond	4	0.388
Simple Cubic	6	0.2488
BCC	8	0.1795
FCC	12	0.119

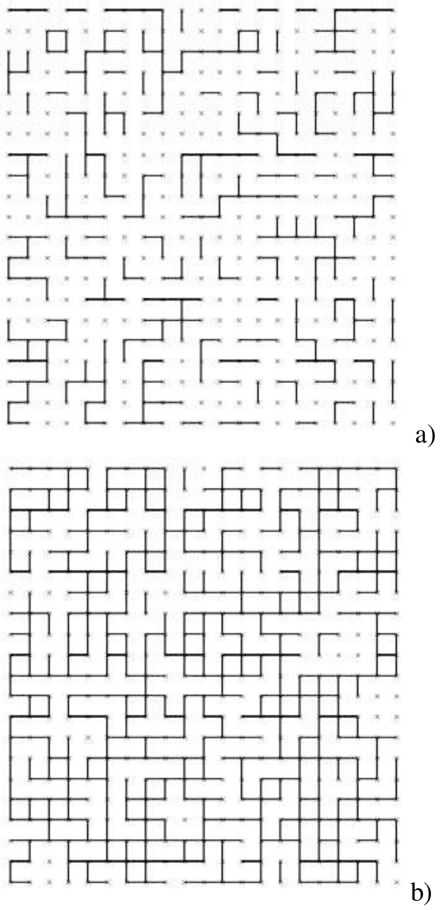


Figure 7. Bond percolation clusters on a square lattice with bond occupation probability a) $p = 0.30$ and b) $p = 0.60$.

- The percolation probability P (p) , which describes the probability that a site belongs to the spanning cluster when the occupation probability for bonds is p.
- The backbone fraction X^B (p), which is the fraction of occupied bonds participating in the transport of fines in the examined lattice at bond occupation probability p. This fraction take into account that some voids are dead-end i.e. that suffosion processes will be stopped as a reason of clogging.
- The correlation length ξ (p), which is a factor for the Representative Element Volume (REV). It describes the typical radius of a finite cluster for $p < p_c$ and the length scale over which a lattice is macroscopically homogeneous for $p > p_c$ [15]. The discretisation L of the lattice, i. e. the number of sites per direction in space, have to be larger than ξ (p) to be independent of L.
- The average size of a finite cluster per site, which is the weighted average of cluster sizes. The average cluster size S (p) is the average cluster size per open bond. This fraction is an information about local structural changes in the examined lattice.
- The fractal dimension D, which is a factor of self-similarity of the system.

A very important characteristic of the percolation theory are the universal scaling laws. Independent of the particular lattice some of the percolation properties obey scaling laws near the percolation threshold. They depend only on the Euclidian Dimensionality d of the system. Referring to the above mentioned percolation quantities the following scaling laws can be specified.

$$P(p) \propto (p - p_c)^{\beta} \text{ für } p > p_c \tag{9}$$

$$S(p) \propto (p - p_c)^{-\gamma} \tag{10}$$

$$\xi(p) \propto (p - p_c)^{-\nu} \text{ if } p < p_c \tag{11}$$

$$X^B(p) \propto (p - p_c)^{-\beta B} \text{ if } p < p_c \tag{12}$$

The exponents are universal constants, which can be used as control parameters for the lattice considered.

A first simple bond percolation model has been developed at the Bauhaus – University Weimar. The general suitability to use percolation theory to understand filtration phenomena could be demonstrated. For a broader explication realistic pore structures are needed. The determination of the relevant largest suffosive grain diameter and the segregation behaviour is not analysed till now. This will be the topic of current research.

VI. CONCLUSIONS

A further development of geometrical suffosion criteria is necessary because previous investigations do have numerous assumptions which are in parts even false.

The percolation theory provides a new approach for a realistic analyse of suffosion. To use a realistic percolation model and to determine a realistic percolation threshold and the important quantities it is necessary to simulate adequate the real pore-structure of natural and manmade soil layers. Therefore secured void and void throat size distributions as well as their correlations in the void structure have to be determined. Important correlations are e. g. the number of void-throats per void, the number of voids at unit length or the dependency between void size and size of the adjacent void throats. The density dependency of the void structure and their change during suffosion have to be understood in detail. It is important to study the void structure by e. g. structure images (e. g. Synchrotron Tomography or X Ray tomography) or with modelling of 3 D sphere packing. The segregation and the derive of the relevant largest suffosive grain diameter have to be analysed. The determination of these important input parameters are topic of the current research at the Professorship Geotechnical Engineering of the Bauhaus – University Weimar.

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