Dorfmann, Clemens; Moser, Konrad; Zenz, Gerald

2D Numerical Simulations of Embankment Dam Failure Due To Overtopping

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Clemens Dorfmann, Konrad Moser, Gerald Zenz
Institute of Hydraulic Engineering and Water Resources Management, Graz University of Technology
Graz, Austria
clemens.dorfmann@tugraz.at

Abstract—**In this paper two cases regarding the failure of homogenous embankment dams due to overtopping are presented. The simulations of the breaching processes were performed with the 2D depth-averaged hydrodynamic model TELEMAC-2D in coupled mode with the sediment transport model SISYPHE. The first case deals with the numerical simulation of a laboratory experiment in which the failure of a homogenous sandy dam due to overtopping was investigated. In a small sensitivity analysis the effects of different hydrodynamic and sedimentological parameters are tested. Comparing the experimental with the numerical results, the study shows that especially the type of transverse deviation correction has a definite influence on the shape of the erosion. The implementation of an alternative formulation for the factor beta in Talmon's deviation formula indicates a reliable improvement. Furthermore the transport stage dependent alpha coefficient in the Meyer-Peter & Müller bed load transport equation was investigated which gives an additional improvement. In the second case the findings of the first case are applied to the modelling of the breaching process in prototype scale of a water reservoir for snow production. The computed breach hydrographs are compared to the hydrograph calculated by a commercial breach parameter model. The study involving the two test cases reveals the excellent ability of the open source TELEMAC suite for the simulation of homogenous embankment breaching processes due to overtopping.**

I. INTRODUCTION

For various functions, dams are built for the retention of water. Usually much attention is paid to the security of the dam. Cases in history show, that dam breaks e.g. caused by overtopping can lead to danger for humans and enormous physical damages. According to the analysis given in [1], from more than 900 dam failures, more than 65% are earth dam failures. Especially smaller earth dams with a height less than 15 meters statistically break more often than larger dams. The main causes for earth dam failures are quality problems and overtopping [1]. The investigation of the physical breaching processes and the predicting of the breach discharge and its peak discharge due to dam failure are of vital importance for dam failure prevention and mitigation. In the recent years a lot of research efforts have been made regarding the numerical simulations of embankment dam failures, e.g. [2] or [3].

In Austria there are more than 500 small earth embankment reservoirs for snow production. For these artificial lakes the public authority regulates the preparation of emergency plans for the case of hypothetical dam failure. Motivation of this work is the assessment of the open TELEMAC suite for the calculation of embankment dam failure due to overtopping.

II. COLEMAN CASE

Coleman, Andrews and Webby presented in [4] laboratory experiments about noncohesive homogeneous embankment failure due to overtopping flow. They provided an overview and analysis of breach development processes for noncohesive embankment materials. The results are particularly significant in that they provide data about breach shape and erosion rates as a breach develops. Previously breach development patterns were typically conjectured based solely on final breach profiles [4]. That said the data are of particular value for testing, calibration and verification of parametric breach models or higher dimensional numerical models like the open TELEMAC suite.

A. Experimental Setup

The experiments reported in [4] were carried out in a 2.4 m wide by 12 m long flume. The flume was equipped with a flow straightener at the inlet, a side-spill weir, a flap gate, a water-level probe and a weir downstream of the dam (Fig. 1).

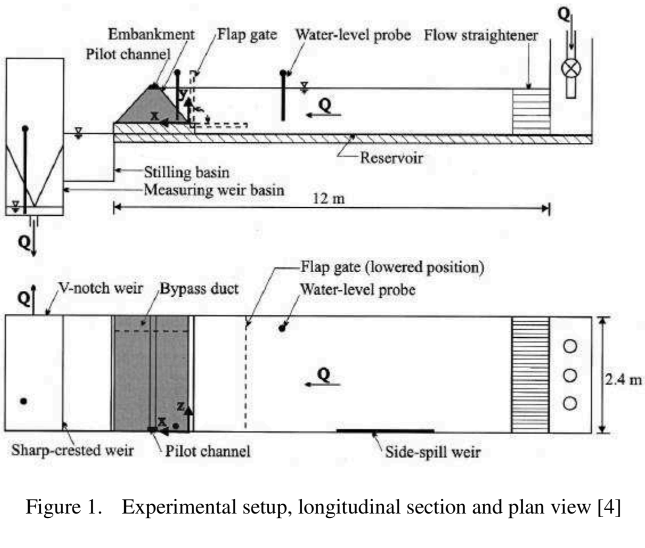

Figure 1. Experimental setup, longitudinal section and plan view [4]

The embankment measured 0.3 m high, with upstream and downstream slopes of the dam of 1:2.7 (V:H). The crest width and length were 0.065 m and 2.21 m, respectively. With the reservoir raised to a predetermined depth of approximately 0.3 m, the breaching process was initiated by the cutting of a triangular pilot channel in the embankment crest (Fig. 1). The invert of the pilot channel of fixed geometry was located on the left flume side wall 0.02 m below the embankment crest level, with the flume wall simulating the breach channel centerline. The initial width of the pilot channel was approximately 0.02 m. Flow entering the flume was adjusted to ensure a constant reservoir level of 0.3 m as the breach developed, the tests thereby simulating the failure of an embankment dam impounding a very large upstream reservoir. Thus, the discharge over the dam was only influenced by the erosion process itself. The discharge was measured via the V-notch weir downstream of the dam.

The laboratory experiments were carried out using uniform sediment material for the embankment dam and assuming no cohesive effects. The investigations were performed using three different sediment materials: d_{50} = 0.5 mm, d_{50} = 0.9 mm and d_{50} = 1.6 mm.

B. Experimental Results

Coleman, Andrews and Webby in [4] focused on the analysis of the experiment with medium sand d_{50} = 0.5 mm regarding the analysis of the breach erosion process. For the medium sand the authors specified a solid sediment density of 2630 kg/m³, a geometric standard deviation of particle size $\sigma_g = (d_{84}/d_{16})^{0.5} = 1.6$ and a friction angle of 32°.

The investigations of the erosion processes included the measurement of the longitudinal profiles along the breach channel centerline at six time steps as shown in Fig. 2. Furthermore the analysis involved the measurement of the breach channel cross sections at the embankment crest at different time steps and the measurement of the time dependent flow rates (Q_b) through the breach. The interested reader is referred to [4] for the detailed analysis of the experimental results.

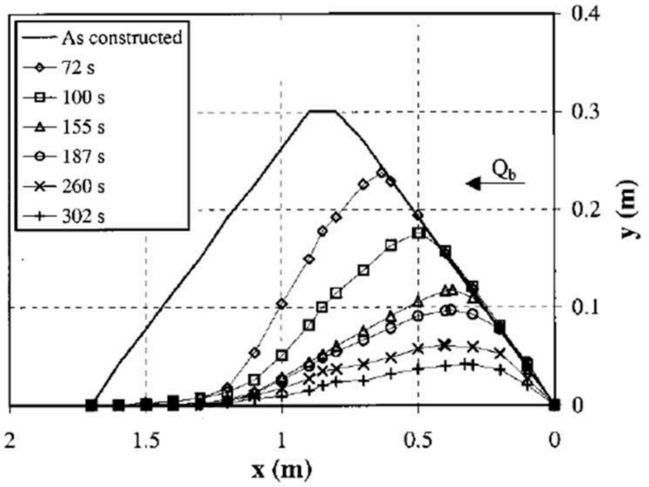

Figure 2. Measured longitudinal profiles along breach channel centerline for medium-sand embankment (d_{50} = 0.5 mm) [4]

III. 2D Simulations Of The Coleman Case

A. Overview

Purpose of the numerical simulations was the testing of the ability of the open source TELEMAC suite (www.opentelemac.org) to simulate highly unsteady dam breaching processes and to reproduce the experimental results from the Coleman case described in the previous chapter. The numerical simulations were performed with the 2D depth-averaged hydrodynamic model TELEMAC-2D in coupled mode with the sediment-transport module SISYPHE. The TELEMAC version v6p3r2 was used.

The main comparison of the numerical results with the experimental results is carried out using the measured longitudinal profiles for medium-sand (d_{50} = 0.5 mm) as shown in Fig. 2. For the calibration process of the numerical model only the measured profiles of the first two time steps after 72 sec and 100 sec are used. Subsequently, the effects of different parameter variations are validated for the later time steps. Further numerical investigations, results and quantitative analysis can be found in [5].

B. Mesh and Hydraulic Boundary Conditions

The geometry and the mesh were created using the data provided in [4]. On the upstream side of the dam the flume was reproduced only to where the water level probe in the laboratory case was placed. Downstream of the dam base an artificial outflow region with a steep slope was modelled in order to guarantee a backwater-free outflow of water and sediments. The mesh was triangulated with the free software Blue Kenue [6] using for the inflow and outflow areas an edge length of 10 cm, for the dam itself an edge length of 2 cm and for the region around the breach channel centerline an edge length of 0.5 cm. The triangular mesh had 42200 elements. The bottom was defined generally as non-erodable bed besides the region of the contact area of the dam.

At the upstream hydraulic boundary condition a constant water level was defined as in the experiment in order to simulate a very large upstream reservoir. For the downstream boundary condition a free outflow condition far away from the zone of interest was applied. As initial condition a constant rest water surface level corresponding to the dam crest height was defined as in the experiment. Fig. 3 shows the significant geometrical and hydraulic related design.

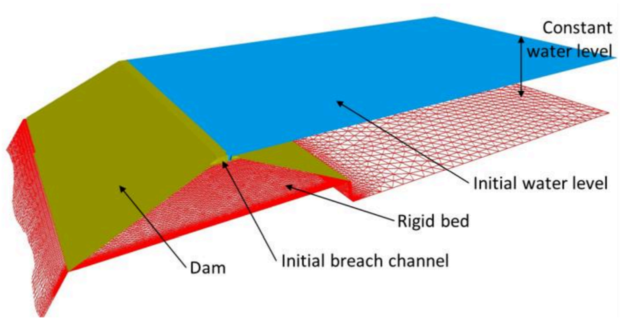

Figure 3. 3D view, numerical setup, initial conditions and upstream hydraulic boundary condition

C. Hydrodynamic and Sedimentological Parameters

In the first simulations the investigations were limited to only a few hydrodynamic and sedimentological parameter variations. For the comparison of the numerical simulations with the experimental results only the first two measured longitudinal profiles over time (after 72 and 100 seconds) were used.

1) Hydrodynamic parameters:

Energy losses due to bottom friction were parameterized using a Strickler roughness value k_{ST} of 92 $m^{1/3}/s$ according to the relationship $k_{ST} = 26/k_S^{1/6}$ and assuming a flat bed with $k_S = d_{50} = 0.5$ mm. For the lateral wall boundaries a fully slip condition was applied. For turbulence closure a constant eddy viscosity of 1.E-6 m²/s was applied. In the case of Finite Element (FE) simulations the time step was set to 0.001 seconds in order to guarantee the Courant numbers lower than 1. When simulating supercritical flow with Finite Element schemes this criterion appears necessary for convergent simulation results; independent of the employed implicitation coefficients in the time discretization. In the case of Finite Volume (FV) simulations a variable time step was applied to satisfy the required Courant criterion.

First simulation attempts revealed a lot of numerical instabilities when using the Finite Element schemes since they have poor shock-capturing abilities. The instabilities originate from the very high Froude numbers reaching values of up to 10 during the dam break process. Stable simulations could be achieved applying Mass Lumping for the depth and fully SUPG upwinding for the depth. Further details about the used methods and formulas can be found in the TELEMAC-2D User Manual [7].

2) Sedimentological parameters:

For the SISYPHE sediment transport simulations most of the default keywords were used. Table 1 shows the embankment materials used in the simulations. It is to say that in [4] the values for the bed porositiy were not provided, so a value of 0.40 was assumed.

TABLE I. EMBANKMENT MATERIALS USED IN THE SIMULATIONS

Sediment material	Grain size d_{50} [mm]	Sediment density ρ_S [kg/m³]	Friction angle φ [° deg]	Bed porosity
Medium sand	0.5	2630	32	0.40

The sediment transport process was modelled as only bed load transport using the Meyer-Peter and Müller bed load equation. The Soulsby formula was used to account for the bed slope effect. The influence of the transverse deviation effect was considered using Talmon's formula. In order to prevent the bed slope to become greater than the maximum friction angle the sediment slide effect model implemented in SISYPHE was used. Further details about the used methods and formulas can be found in the SISYPHE User Manual [8].

Fig. 4 shows exemplary two simulation results and points out the significance of the slide effect model. The simulation of the breaching process without slide effect model creates a very narrow and unnatural channel with steep side banks. On the other hand, the simulation with the slide effect model produces a much more realistic breach channel.

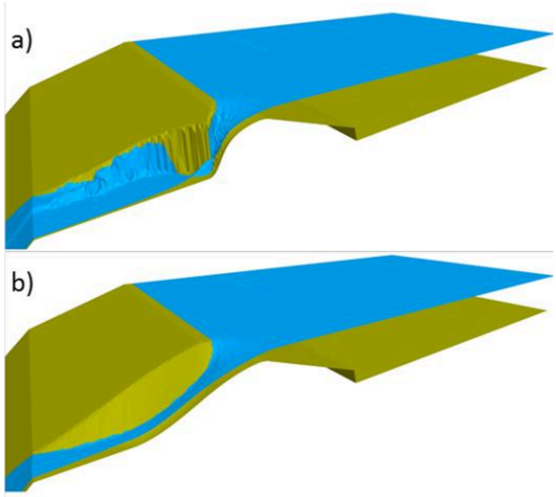

Figure 4. Simulated bottom and water surface evolution after 100 seconds, 3D view, a) without sediment slide effect; b) with sediment slide effect

D. Hydrodynamic Investigations

The hydrodynamic investigations involved the testing of two Finite Element advection schemes for the velocity, namely the Method of Characteristics (MOC) and the Non-conservative PSI (Positive Streamwise Invariant) scheme (PSI) as well as the Finite Volume HLLC (Harten–Lax–van Leer Contact) scheme (FV - HLLC). In Fig. 5 and Fig. 6 the experimental results and the numerical results after 72 and 100 seconds are compared, respectively. At the time of 72 seconds the FE schemes and the FV scheme match the experimental results very well, however at the time of 100 seconds all the schemes simulate too less erosion. A possible explanation for the somewhat better performance of the FV - HLLC scheme could be that in this case the FV scheme produces lower numerical diffusion compared to the MOC scheme and PSI scheme. Generally it can be stated that all the schemes perform very well and that the breaching process by itself can be modelled by using any of the schemes investigated and presented here.

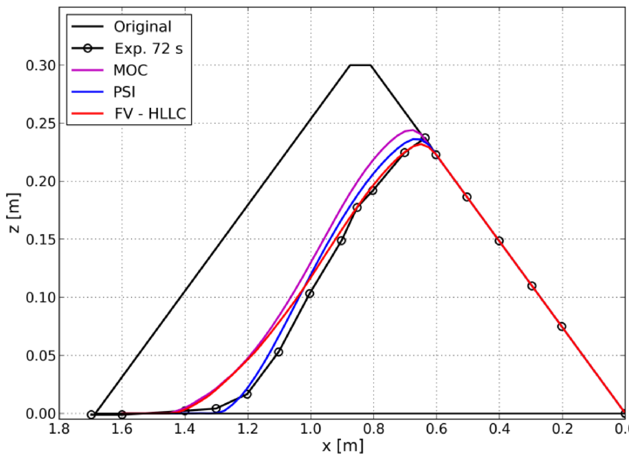

Figure 5. Variation of hydrodynamic FE and FV schemes, experimental and numerical results after 72 seconds

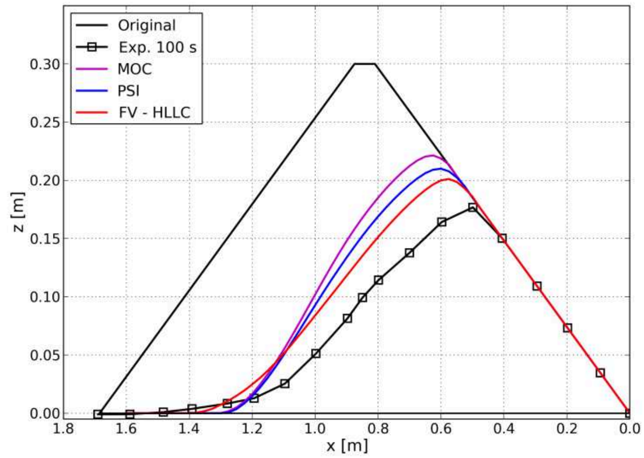

Figure 6. Variation of hydrodynamic FE and FV schemes, experimental and numerical results after 100 seconds

E. Sediment Parameter Investigations

Principal aim of the sediment parameter investigations was to achieve a better agreement between experimental and numerically calculated longitudinal profiles after 100 seconds (Fig. 6).

The investigations focused on two main directions: first, the modification of the calibration factor α in the Meyer-Peter and Müller bed load equation and second, the modification of the β coefficient in the Talmon's formula for the transverse deviation effect.

1) Modifying the α coefficient in the Meyer-Peter and Müller bed load equation:

The motivation for this investigation was the fact that a dam break process as presented here is characterized by highly unsteady flow and sediment transport conditions. The highly variable conditions produce large variations in the emerging shear stresses and thus large variations in the emerging intensities of sediment transport (transport stage parameter T).

The Meyer-Peter and Müller bed load transport equation [9] includes a calibration factor α for which Meyer-Peter and Müller determined a value of 8 (1).

$$\phi = \alpha \cdot (\theta - \theta_{Cr})^{3/2} \tag{1}$$

where ϕ is the non-dimensional sediment flux, θ is the Shields stress and θ_{Cr} is the critical Shields stress.

Different researchers, e.g. [10], [11], determined different α coefficients based on different datasets from laboratory or field investigations. Based on the analysis of datasets, Wiberg and Smith [12] point out that the observed variation in the α coefficient is well explained by the different prevailing transport stage parameters $T = \theta / \theta_{Cr}$. Accordingly the variable α coefficient can be well captured by a dependence on the Shield's stress (θ), giving a transport stage T dependent α coefficient (2) [12]. This relationship was tested for the here presented dam break case.

$$\alpha = 1.6 \cdot \ln(\theta) + 9.8 \approx 9.64 \cdot \theta^{1/6} \tag{2}$$

Fig. 7 shows the relationship (2) between the α coefficient and the transport stage parameter T.

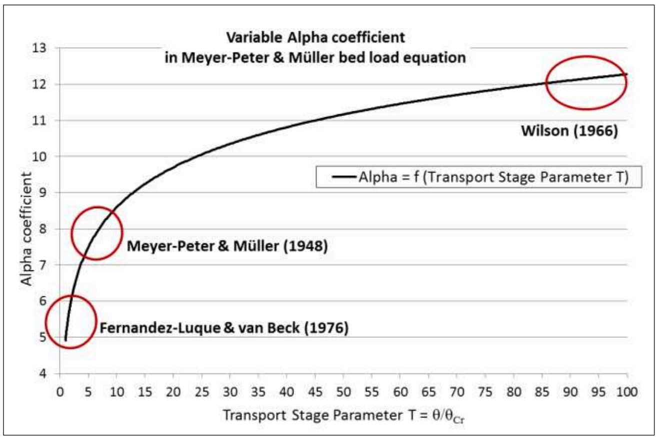

Figure 7. Varying α coefficient depending on the transport stage parameter T (2) in Meyer-Peter and Müller formula

2) Modifying the β coefficient in the Talmon's formula for the transverse deviation effect:

The breaching process of embankment dams is governed not only by erosion in the streamwise direction but also highly by the erosion in the crosswise direction due to the evolving bank slopes during the breaching process. This implies a correction of the direction of the bed load transport rate in relation to the computed flow direction. In SISYPHE the change in the direction of solid transport is taken into account by (3) [8].

$$\tan(\alpha) = \tan(\delta) - \frac{1}{f(\theta)} \frac{\partial z_b}{\partial n} \tag{3}$$

where α is the direction of solid transport in relation to the flow direction, δ is the direction of bottom stress in relation to the flow direction, f(θ) is the weigh-function depending on the Shields stress θ, zb is the bed elevation and n is the coordinate along the axis perpendicular to the flow.

Based on bed-levelling experiments and collection of auxiliary dataset, Talmon et al. [12] relate the weigh-function to an empirical coefficient β and the Shields stress θ (4).

$$f(\theta) = \beta \cdot \sqrt{\theta} \tag{4}$$

A small literature research regarding the determination of β coefficients is summarized in Table 2. Talmon et al. [13] determined the β coefficient of 0.85 for natural rivers and the β coefficient of 1.7 for laboratory conditions. Furthermore Talmon et al. provide a relationship between the β coefficient and the ratio of the grain size d50 and the water depth h, based on the analysis of numerical computations of large scale river-bed deformations, with A = 9.0 and C = 0.3 (5).

$$\beta = \mathrm{A} \cdot \left(\frac{d_{50}}{h}\right)^{C} \tag{5}$$

Equation (5) was tested for the here presented dam break case with A = 9.0 and C = 0.3. References [14] and [15] specified β coefficients also based on laboratory experiments. In this context it is to state that the β coefficients found in the literature were mainly developed based on investigations of bed form formations. This means usually steady or quasi-steady flow conditions with small variations in the water depths and slow sediment transport processes, whereas dam break processes are characterized by highly unsteady flow and sediment transport conditions.

TABLE II. OVERVIEW: SOME BETA VALUES IN THE LITERATURE

β	Source
0.85 (natural rivers)	Talmon et al. [13]
1.7 (laboratory conditions)	Talmon et al. [13]
$\mathrm{A} \cdot (d_{50}/h)^{\mathrm{C}}$ with A = 9.0 and C = 0.3	Talmon et al. [13]
1.0	Wiesemann et al. [14]
$\mathrm{A} \cdot (d_{50}/h)^{\mathrm{C}}$ with A = 22.3 and C = 0.3	Schoonen [15]

F. Results

Based on the preliminary hydrodynamic investigations the FV - HLLC scheme was used for the final hydrodynamic calculations. In the sediment transport model SISYPHE the variable α formulation according to (2) was implemented in the Meyer-Peter and Müller bed load equation (MPM mod). The parametrization of the transverse deviation effect was modified by implementing the variable β formulation according to (5) in the Talmon's formula (Beta var.).

As shown in Fig. 8 after 100 seconds the simulated profiles agree very well with the measured profiles compared to the simulated profile obtained without the modifications. Applying only the variable α formulation (MPM mod) gives nearly the same results as if both the two modifications are considered. The investigations show that the speed of erosion depends on the β coefficient, with lower erosion speed at higher β values and vice versa. The numerical results reflect the dependency of the β coefficient on the relative submergence h/d_{50} and demonstrate the principal very good ability of the relationship given in (5) to account for the transverse deviation effect in the numerical modelling of dam break processes. Thereby the coefficients A and C in (5) can be seen as calibration parameters for which further investigations are needed.

Fig. 9 shows the comparison between experimental and numerically simulated longitudinal profiles for all time steps. In the numerical simulations both the modifications (MPM mod and Beta var.) were applied. The comparison shows the good agreement between experimental and numerical results and demonstrates the excellent performance of the TELEMAC system in calculating the time dependent breaching process. Fig. 10 exemplifies via the three-dimensional view the temporal development of the simulated bottom and water surface evolution.

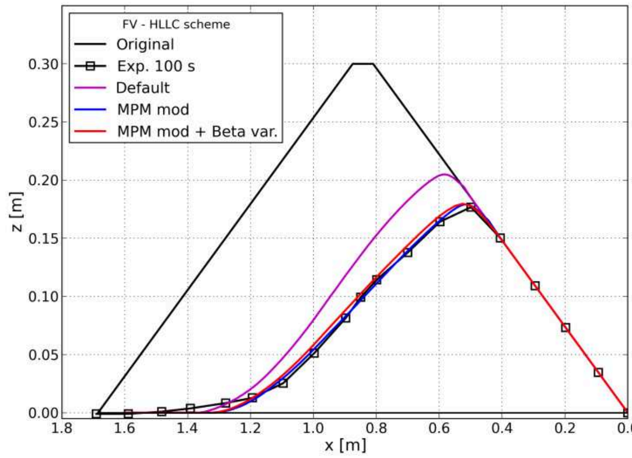

Figure 8. Variation of sediment parameters, experimental and numerical results after 100 seconds

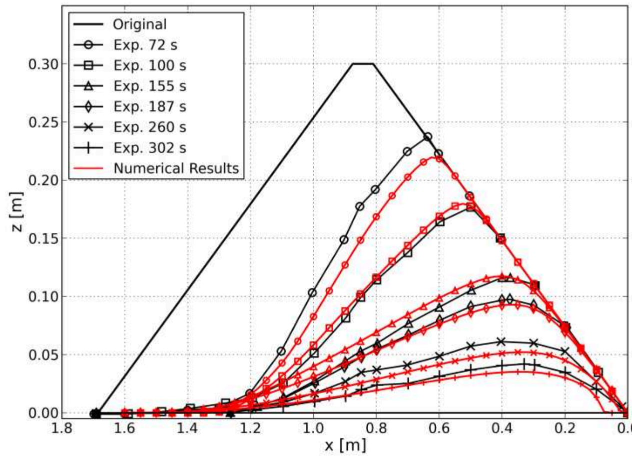

Figure 9. Comparison between experimental and numerical simulation results, all time steps

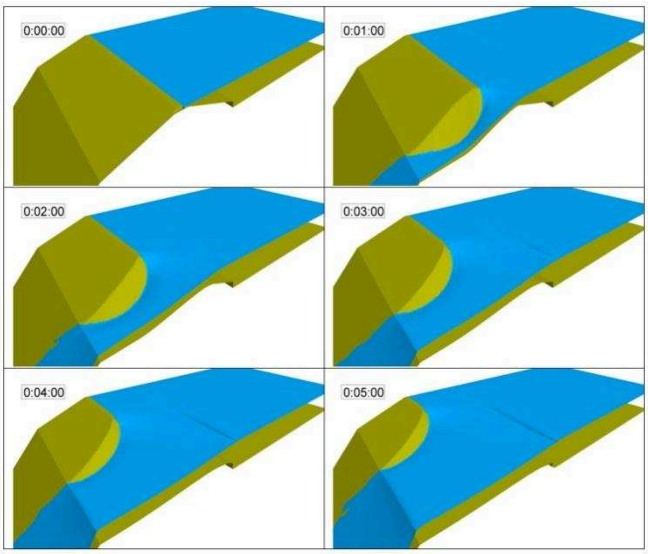

Figure 10. 3D view, bottom and water surface evolution over time

IV. 2D Numerical Modelling Of A Prototype Case

In this part, the experiences collected from the numerical modelling of the Coleman case are applied to a real case by simulating the earth dam break process of an alpine water storage for snow production. As there are no measurement data available for the comparison with the numerical results, the computed breach hydrographs by TELEMAC are compared with the breach hydrograph obtained by the breach parameter model Deich [16]. The geometry data and the breach hydrograph from the parameter model were provided by the engineering office Ingenieurbüro Moser GmbH.

A. Overview of the Prototype Case

The dam height of the snowmaking reservoir is 11 m, the crest width is 3.5 m, the slope of the upstream face is 1:1.8 and the slope of the downstream face is 1:1.75. The water volume at capacity level amounts to 30000 m³, the water area at capacity level is 5000 m². The dam is equipped with a bottom outlet, a membrane at the upstream face and various other safety devices. All this safety arrangements were not considered in the numerical simulations. An initial pilot channel was modelled to start the erosion process, which of course does not exist in the real dam.

The mesh with 50900 elements was triangulated using edge lengths from 0.5 m to 10 m. As initial condition the water level in the reservoir was set to the capacity level. Since the valley downstream of the dam has steep slopes, the outlet boundary was modelled as free outflow condition. First simulations showed that in contrast to the rectangular pilot channel geometry assumed in the breach model Deich, in the TELEMAC simulations a trapezoidal pilot channel has to be designed. Reason is that when using the slide effect model in SISYPHE and a rectangular pilot channel, the slope of the side wall is higher than the friction angle of the sediments, thus the material slides down and the pilot channel is closed just after some time steps. Fig. 11 shows the mesh, the location of the initial pilot channel and the hydraulic conditions.

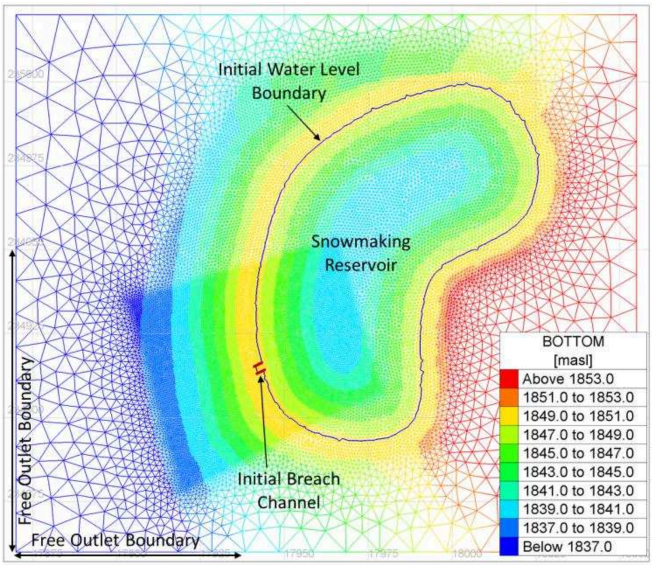

Figure 11. Plan view, mesh, numerical setup, initial conditions and hydraulic boundary condition

B. Breach Parameter Model Deich

The program Deich, developed by the engineering company Broich, calculates the breach widening and the breach hydrograph employing an explicit algorithm [16]. The discharge is calculated employing a modified weir equation. The water surface level is calculated by means of the volume balance. The dam erosion is calculated using the Exner equation in combination with assumptions for the kinematic and spatial description of the breach development (6).

$$\frac{\mathrm{dV}_{erod}}{\mathrm{dt}} + \frac{1}{1-\mathrm{p}}(V_{s2} - V_{s1}) = 0 \qquad (6)$$

where dVerod is the volume eroded in one time step dt and p ist the bed porosity. Vs1 and Vs2 are the sediment volumes, which cross the upstream and downstream control area, respectively. They are calculated using a sediment transport formula, e.g. via the Meyer-Peter and Müller equation.

C. 2D Numerical Simulations

In the 2D numerical simulations the same hydrodynamic and sediment parameters were used as in in the simulations of the Coleman case described in the previous chapter 3. Using the Finite Volume HLLC scheme for the hydrodynamic part, the investigation focused on the comparison between SISYPHE'S default parameters and the modified α coefficient (2) in the Meyer-Peter and Müller bed load equation (MPM mod) as well as the modified β coefficient (5) in the transverse deviation formula (Beta var.).

The sediment material properties shown in Table 3 were adopted from the breach parameter model Deich assuming likewise a uniform sandy dam without cohesive effects.

TABLE III. Embankment Materials Used In The Simulations

Grain size d_{50} [mm]	**Sediment density ρ_S [kg/m³]**	**Friction angle φ [° deg]**	**Bed porosity**
5	2000	35	0.25

D. Results

The breach hydrographs computed by the TELEMAC simulations shown in Fig. 12 reveal generally a slower erosion process of the embankment compared to the hydrograph calculated by the breach model Deich. Clearly to observe is the difference between the simulation with the default sediment parameters and the modified ones. The combined usage of the modified α coefficient according to (2) (MPM mod) and the modified β coefficient (5) (Beta var.) leads to a faster erosion process resulting in a remarkably higher peak outflow. In consideration of the fact that the hydrograph by Deich is also merely a computed solution without the claim to be more realistic, the TELEMAC hydrographs agree qualitatively well with the Deich hydrograph, especially the one using the modified sediment parameters. The results demonstrate the ability of TELEMAC-2D & SISYPHE to simulate the dam break erosion processes in prototype scale and to compute realistic breach hydrographs.

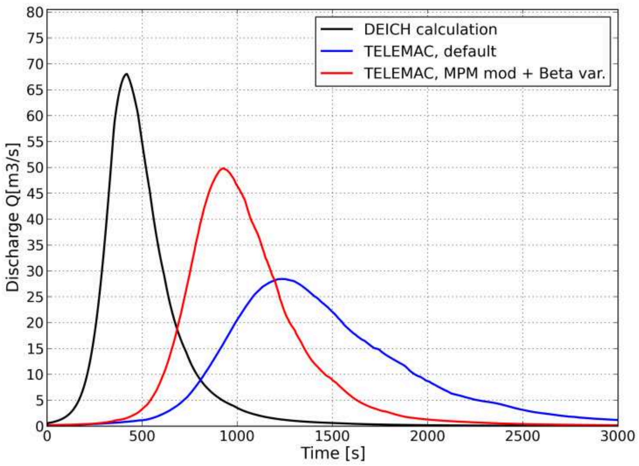

Figure 12. Calculated breach hydrographs by TELEMAC and Deich

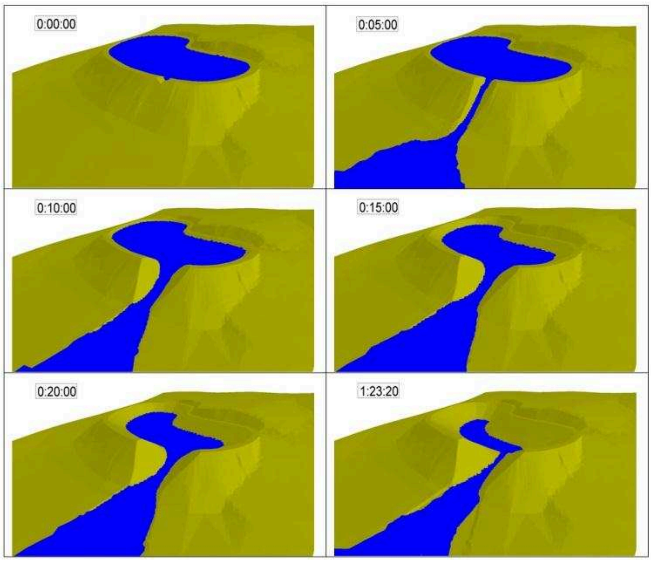

Figure 13. 3D view, bottom and water surface evolution over time

V. CONCLUSION AND OUTLOOK

We presented in this paper the numerical simulations of dam breaching processes due to overtopping using TELEMAC-2D coupled with the sediment transport model SISYPHE.

The first part deals with the simulations of laboratory experiments and the analysis of relevant hydrodynamic and sedimentological parameters in order to reproduce the laboratory results. The investigation focused on the alternative formulation for the α coefficient in the Meyer-Peter and Müller bed load equation and for the β coefficient in the transverse deviation formula. The results show the excellent ability of the open TELEMAC suite for the simulation of the evolving erosion processes in time during an embankment failure.

In the second part some understandings from the first part were applied to the prototype scale by simulating the dam breaching of a snowmaking reservoir. The simulated breach hydrographs were compared to the calculated outflow hydrograph by a breach parameter model. The hydrographs are qualitatively in very good agreement. The prototype case proves the ability of the open TELEMAC suite to simulate realistic breach hydrographs, at least assuming homogenous dam material. It can be stated that the test case demonstrates the applicability of the open TELEMAC suite in the engineering practice for the use in project studies concerning the estimation of breach hydrographs due to dam failure.

As outlook one interesting experiment would be the combination of the simulation of the breaching process with the simultaneous simulation of the flood propagation downstream of the dam. Further research investigations could involve the physical and numerical simulation of the erosion processes of non-uniform embankment materials.

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