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## **UNCERTAINTY QUANTIFICATION ON A REAL CASE WITH TELEMAC-2D**

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# UNCERTAINTY QUANTIFICATION ON A REAL CASE WITH TELEMAT-2D

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**Abstract**— In this study the software TELEMAT-2D ([www.opentelemat.org](http://www.opentelemat.org)) is used with the OpenTURNS library ([www.openturns.org](http://www.openturns.org)) to quantify the uncertainty on a real hydraulic case. The used approach is based on the chaining of OpenTURNS and TELEMAT-2D using the SALOME platform ([www.salome-platform.org](http://www.salome-platform.org)) in order to implement a Monte Carlo-like algorithms. Each uncertain parameter (inlet discharge, friction coefficient) is associated to a statistical distribution (defined using OpenTURNS). A sufficient number of TELEMAT-2D runs are achieved with respect to the pre-defined random entries in order to guarantee the convergence of the studied Monte Carlo-like algorithms. EDF's cluster has been used to run the simulations.

Indeed, to handle the uncertainty with the Monte Carlo method, it is important to run a lot of simulations in order to have reliable results. The obtained results are analysed twofold: On one hand, the effect of variability of random inputs is assessed at some specific points (assumed to be around a fictive point of interest). On the other hand, a global statistical analysis all over the domain is done. A spatial distribution of the mean water depth and its variance is obtained. These results are of utmost importance for dimensioning of protecting dykes. Furthermore, they are very useful when establishing scenarios for flood managing.

However, Monte Carlo technique that while generic and robust is also computationally expensive. Ways to lower the cost typically require to replace the pure random sampling that form the backbone of the Monte Carlo method by alternative sampling methods such as the Latin Hypercube Sampling approach and the quasi-Monte Carlo method based on low discrepancy sequence. The present work aims to compare the behavior of these Monte Carlo-like algorithms.

This work shows that, thanks to the availability of important computer resources and to an optimized software, we are able to consider Monte Carlo-like algorithms for uncertainty quantification of real hydraulic models. This critical conclusion was, even an unfeasible dream, couple of years ago.

## I. INTRODUCTION

Water resource management and flood forecasting are crucial societal and financial stakes that require a solid capacity of flow depth estimation that is often limited by

uncertainties in hydrodynamic numerical models. In order to overcome these limits, uncertainties should be analyzed. Uncertainty analysis means the quantification of the uncertainty in the model outputs due to uncertainty in the input data, parameters, model structure and modelling assumptions.

In this study, we investigate the effect of two uncertainty sources on water level calculation for extreme flood event, the roughness coefficient and the upstream discharge. Indeed, the hydraulic roughness is uncertain because flow measures are not available or reliable for calibration and validation. Discharge is also uncertain because it results from extrapolation of discharge frequency curves at very low exceeding probabilities.

A variety of statistical methods can be used to propagate input uncertainties through the model into output uncertainties. Most classical method to propagate the uncertainty through the dynamical model is the Monte Carlo technique. This approach requires random generation of the ensemble of inputs from their probability distributions and successive deterministic model simulations to generate a lot of realizations of the output. The main drawback of this is the computational cost. A way to lower the computationally demanding is to replace the pure random sampling that form the backbone of the Monte Carlo method by alternative sampling methods such as the Latin Hypercube sampling approach and the quasi-Monte Carlo method based on low discrepancy sequence of Sobol. The present work aims to compare the behaviour of these Monte Carlo-like algorithms.

This work has been carried out using the SALOME platform in which the hydraulic software TELEMAT-2D is coupled with the uncertainty library OpenTURNS.

The paper is organized as follows: in the first section, the numerical tools used during the study are presented. In section 3, the model is presented with a description of the study area, the hydraulic model and the uncertainty study. Then the results of the simulations are described in the section 4. Finally, in the last section we discuss the results and we draw some conclusions.

## II. NUMERICAL TOOLS

As already mentioned, this study was performed by coupling the hydrodynamic model TELEMAC-2D and the uncertainty library OpenTURNS in the SALOME platform. These numerical tools are presented in this section.

### A. Hydraulic modelling system TELEMAC-2D

The modeling system TELEMAC is a hydro-informatic software developed by the LNHE (National Laboratory for Hydraulics and Environment) from the research and development department of EDF. It is an open source software ([www.opentelemac.org](http://www.opentelemac.org)) which can be used to perform numerical simulation in two and three dimensions. Several modules can be used to solve different problems such as tidal wave (Artemis, Tomawac), current (TELEMAC-2D, TELEMAC-3D), sediment transport (Sisyphé) and water quality (Delwaq, developed by Deltares).

In this work, hydrodynamic is provided using TELEMAC-2D depth-averaged hydrodynamic model. It solves the shallow water equations in two dimensions. In each point of the mesh, TELEMAC-2D gives the water depth and the vertically average horizontal velocity field [4].

### B. Uncertainty treatment library OpenTURNS

OpenTURNS is an open source library for uncertainty treatment coded in C++ ([www.openturns.org](http://www.openturns.org)) used through python scripts. OpenTURNS stands for “Open source initiative to Treat Uncertainties, Risks’N Statistics”. It is co-developed since 2005 by EADS IW, EDF R&D and PHIMECA Engineering. It is used according the uncertainty method describes as follow by EDF R&D (see Fig. 1) [8].

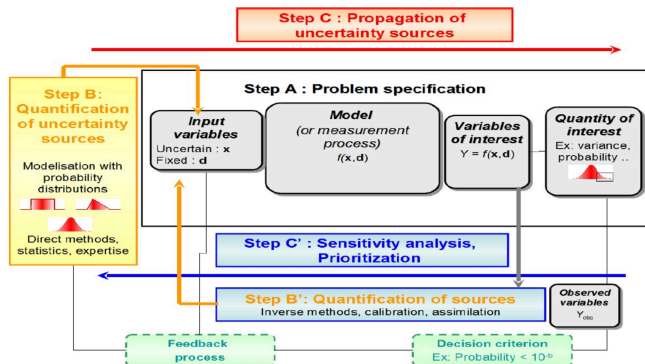


Figure 1. Steps for an uncertainty study [8]

### C. The SALOME platform

Salome is an open source software ([www.salome-platform.org](http://www.salome-platform.org)) which is a platform for pre and post processing for numerical simulation and where it is possible to define a chain or a coupling of computer codes. It is based on an open and flexible architecture with reusable components. SALOME is developed by EDF, the CEA and OPENCASCADE S.A.S. with the GNU LGPL license as the source code can be downloaded and modify from the website. All the components within SALOME can be used

together with the YACS module which build a computation scheme and call each module and make them communicate. In our case TELEMAC-2D and OpenTURNS are working together within this platform as shown in Fig. 2.

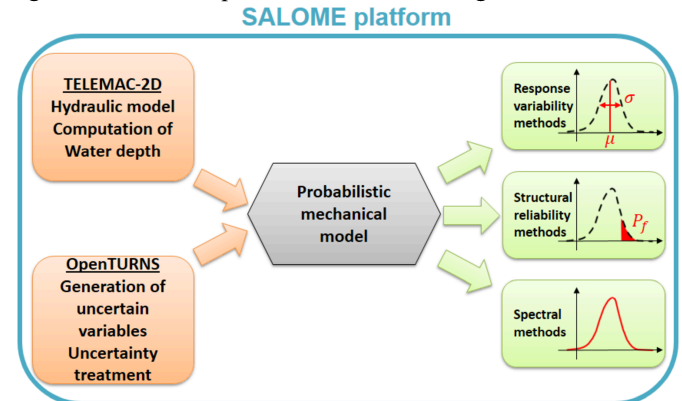


Figure 2. The SALOME principle for uncertainty quantification (inspired from [9])

## III. PROBLEM SPECIFICATION

In this part the study area is presented with its global location and general overview of the situation. Then the model itself with its input parameters are introduced to bring the uncertainty study. Finally the method used to propagate and quantify the uncertainty are presented.

### A. Study area

The area chosen for this study extends over a reach of the Garonne river measuring about 50 km, between Tonneins (upstream), downstream of the confluence with the Lot river, and La Réole (downstream) (see Fig. 3).



Figure 3. Study area of the Garonne [1]

This part of the valley was equipped in the 19<sup>th</sup> century with infrastructure to protect against floods of the Garonne river which had heavily impacted local residents. A system of longitudinal dykes and weirs was progressively built after that flood event to protect the floodplains, organize submersion and flood retention areas. This configuration is also similar to the characteristic of other managed rivers such as the Rhone and the Loire.

### B. The hydraulic model

#### 1) Boundary conditions

The 2D Telemac model, constituted by a triangular mesh of some 41 000 nodes with an extremely small mesh size around the dykes, has a constant discharge upstream imposed at Tonneins and downstream, a stage-discharge relationship corresponding to the stream gauge at La Réole. This model has been realized by Besnard and Goutal (2008) [1].

In this work, the upstream discharge is set up to 8 790 m<sup>3</sup>/s corresponding to a very low exceeding probabilities (a thousand return period discharge) in order to model an extreme flood event with points affected around the floodplain.

### 2) Roughness coefficient

The models were calibrated in [1] using steady-state water surface profiles at high discharge, from bank-full discharge in the main channel (2 500 m<sup>3</sup>/s) to bank-full discharge in the overbank flow channel between dykes.

For the main channel, the Strickler roughness coefficient was split into three different areas:

- Tonneins – upstream of Mas d’Argenais: 45
- Upstream of Mas d’Argenais – upstream of Marmande: 38
- Upstream of Marmande – La Réole: 40

In floodplain, the roughness coefficient is selected as an area with cultivated fields all around the river with a Strickler coefficient of 17.

## C. Uncertainty study

### 1) Variable of interest

As already mentioned, the quantity of interest considered in this study is the flow depth all over the computational domain.

### 2) Uncertainty quantification

In this study, we investigate the effect of two uncertainty sources on water level calculation for extreme flood event, the roughness coefficient and the upstream discharge. In fact, the hydraulic roughness is uncertain because flow measures are not available or reliable for calibration and validation. Discharge is also uncertain because it results from extrapolation of discharge frequency curves at very low exceeding probabilities. The quantification of these uncertainty sources is given the following subsections.

#### a) Probability density function of roughness coefficient

Classically, according to the available expert knowledge, the friction coefficient is contained in an interval bounded by physical values depending on the roughness of soil material. Consequently, using the principle of maximum entropy [9], the distribution of the bounded Strickler roughness coefficient is uniform. The boundaries of the uniform distribution are arbitrarily chosen  $\pm 5$  from the calibrated value given in the section B.2). Fig. 4 shows the probability density function of the Strickler coefficient in the floodplain.

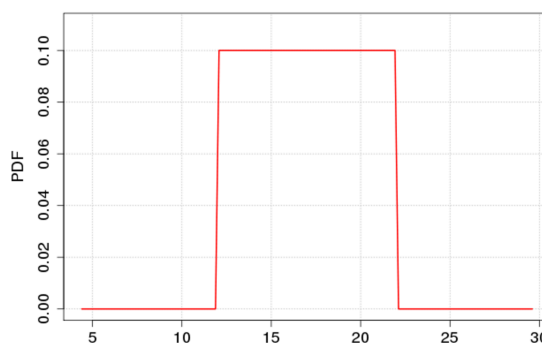


Figure 4. Probability density function of the Strickler coefficient in the floodplain

#### b) Distribution of the discharge

As already mentioned, the upstream discharge is estimated using an extrapolation of discharge frequency curves at very low exceeding probabilities corresponding to a thousand year return period event. Confidence intervals on the extrapolated value can be derived. In that case, when the mean value (discharge of the thousand year return period) and the standard deviation (extrapolated from the confidence intervals) are known, the maximum entropy distribution is Gaussian [9]. The mean and standard deviation are set to, respectively, 8 490 m<sup>3</sup>/s and 900 m<sup>3</sup>/s. Moreover, to avoid too high or too low values, the probability density function is truncated at 5 790 m<sup>3</sup>/s and 11 190 m<sup>3</sup>/s which means the probability to have a discharge outside these boundaries is zero (see Fig. 5).

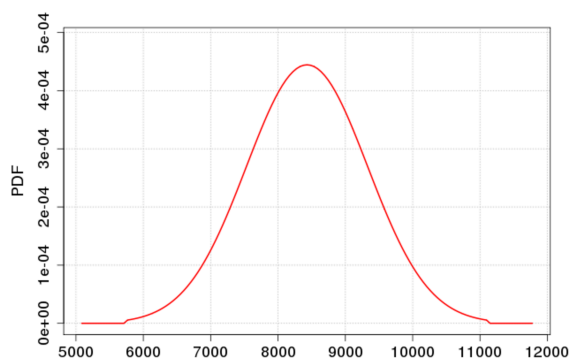


Figure 5. Probability density function of the upstream discharge

## 3) Uncertainty propagation methods

### a) The Monte Carlo method

The Monte Carlo method requires random generation of the ensemble of input random variables from their probability distributions. The resulted sampling form a matrix composed by  $n$  (number of simulations)  $\times$   $s$  (number of variables). Each row of the matrix represents a configuration that is used as an input for the hydraulic simulation. A lot of realizations of the output is generated by



successive deterministic model simulations corresponding to each configuration of the sampling matrix. Then, some statistical estimators can be computed on the output sample. For example, the mean value  $\bar{\mu}_Y$  and the standard deviation  $\bar{\sigma}_Y$  of a response quantity  $Y = f(x_k)$  are given by (1) and (2).

$$\bar{\mu}_Y = \frac{1}{n} \sum_{k=1}^n f(x_k) \quad (1)$$

$$\bar{\sigma}_Y^2 = \frac{1}{n-1} \sum_{k=1}^n [f(x_k) - \bar{\mu}_Y]^2 \quad (2)$$

With  $x_k$  the input sample of uncertain variables.

The statistics computed on sample sets are random quantities in nature. Therefore, confidence intervals on the results should be provided. Monte Carlo method easily give a confidence intervals for the estimation by using the central limit theorem.

It implies that if a random variable  $Y$  have a mean  $\mu_Y$  and a variance  $\sigma_Y^2$  which are finite, then the distribution of the mean of  $n$  independent realizations  $Y_i$  converge toward a Gaussian distribution when  $n$  tends towards infinity. More precisely, if  $n \rightarrow \infty$ ,

$$\frac{\bar{\mu}_Y - \mu_Y}{\sigma_Y / \sqrt{n}} \xrightarrow{L} \mathcal{N}(0,1). \quad (3)$$

As show in (3), the convergence speed of the method is then, on average,  $o(\sqrt{n})$  independent of the dimension  $s$  of the problem. The Monte Carlo method is theoretically applicable whatever the complexity of the deterministic model or the desired statistical estimator. However, its computational cost makes it rather impracticable when the computational cost of each run of the model is non negligible and when the statistical estimator requires a lot of realization to be converged. One way to lower the computationally demanding is to replace the pure random sampling that form the backbone of the Monte Carlo method by alternative sampling methods such as the Latin Hypercube sampling approach and the quasi-Monte Carlo method based on low discrepancy sequence of Sobol. These sampling methods are developed in the next two sections. In these sections, it is assumed that the sampling space is the unit cube  $I^s = [0,1]^s$ . In fact, even if each uncertain parameter can take values in a certain finite range, it is always possible to rescale them appropriately to obtain a unit cube.

#### b) The Latin Hypercube sampling

The Latin Hypercube Sampling (or LHS) is a sampling method enabling to better cover the domain of variations of the input variables, thanks to a stratified sampling strategy.

The sampling procedure is based on dividing the domain of each variable into several intervals of equal probability. A unique random value is chosen in each interval and then the values obtained for the variables  $x_i$  and  $x_j$  are randomly combined. This step is repeated for all the random variables to give a  $n \times s$  matrix which can be used as an input sample.

Fig. 6, extracted from [2], shows the comparison between the two sampling methods of two random variables ( $x_1, x_2$ ) taken from a uniform distribution in the interval  $[0,1]$ . This figure demonstrates the sampling

strategy of the LHS as each row and column are filled with points instead of the Monte Carlo sampling in which some rows and columns does not have points.

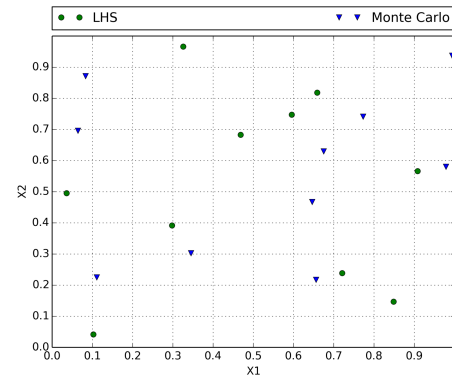


Figure 6. Comparison between Latin Hypercube and Monte Carlo sampling with 10 values [2]

According to [7], if the function  $f$  is monotonic in each of its arguments, then the variance of the estimator of the LHS is lower than the Monte Carlo one given by (4).

$$\text{Var}(\bar{\mu}_{Y_{LHS}}) \leq \text{Var}(\bar{\mu}_{Y_{MC}}) \quad (4)$$

The expression of the estimator is defined by (1) where the input samples  $x_k$  are generated according the LHS and Monte Carlo techniques.

Therefore, by (4) the LHS technique is supposed to be more efficient in term of convergence rate than the Monte Carlo method.

#### c) The quasi-Monte Carlo method

Quasi-Monte Carlo (or QMC) techniques are deterministic methods that have been designed by analogy with Monte Carlo simulation. In quasi-Monte Carlo, the random sample of Monte Carlo is replaced by a sequence of well distributed points called a low discrepancy sequence [7]. Fig. 7 presents the comparison between the sampling from the Sobol sequence and Monte Carlo. It demonstrates that the Monte Carlo does not fill the domain as the Sobol sequence does.

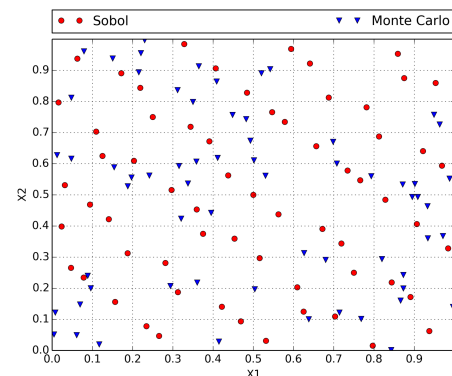


Figure 7. Comparison between low discrepancy sequence and Monte Carlo sampling with 64 values

A low-discrepancy sequence is a sample whose points are in a way that approximates the uniform distribution as close as possible. The discrepancy is a measure of deviation

from uniformity of a sequence of points in  $D = [0,1]^s$ . The discrepancy of a quasi-Monte Carlo sampling is known and given by (5).

$$D^*(P_n) \leq c \frac{[\ln(n)]^s}{n} \quad (5)$$

With  $c$  a constant which depends on the sequence used.

Moreover, the discrepancy contributes to the error in quasi-Monte Carlo methods. The deterministic error bounds, through the Koksma-Hlawka theorem, can be estimated by (6).

$$\left| \frac{1}{n} \sum_{i=1}^n f(u_i) - \int_{I^s} f(u) du \right| \leq V(f) D^*(P_n) \quad (6)$$

With  $V(f)$  the variation in the sense of Hardy and Kraus of the  $f$  function in the mono dimensional case on  $I = [0,1]$  given by (7).

$$V(f) = \sup_{P \in \mathcal{P}} \sum_{i=0}^{n_p-1} |f(u_{i+1}) - f(u_i)| \quad (7)$$

Where  $\mathcal{P}$  is a set of all the partitions  $P$  of  $I = [0,1]$  and  $P_n$  a low discrepancy sample.

Thus, when  $V(f) < \infty$  and  $P_n = \{\mathbf{u}_1, \mathbf{u}_2, \dots\}$  is based on a low discrepancy sequence, the control of the variance of the approximation is about  $o\left(\frac{[\ln(n)]^s}{n}\right)$  [7]. Comparing this with the probabilistic Monte Carlo error that is in  $o\left(\frac{1}{\sqrt{n}}\right)$ , one can argue that for a fixed dimension  $s$ , the quasi-Monte Carlo method converges faster than with Monte Carlo. So, for function that are smooth enough and if you are willing to take  $n$  sufficiently large, the error with quasi-Monte Carlo technique will be smaller than the Monte Carlo one.

#### d) Quantity of interest

The objective of an uncertainty study is to assess some characteristics of interest of the uncertain output variable distribution, such as, probability of exceeding a threshold, quantile, or expectation and variance. In this study, the considered characteristics of interest are the first four statistical moments (mean, variance, skewness and kurtosis) of the water depth.

To ensure the relevance of the comparison of the different Monte Carlo-like algorithms, the ‘‘bootstrap’’ method is used to estimate confidence intervals on the Monte Carlo results.

#### CONFIDENCE INTERVAL BY BOOTSTRAP

The ‘‘bootstrap’’ method is the practice of estimating properties of an estimator (such as its variance) by measuring these properties when sampling from an approximating distribution. This technique is easily implemented and rely on few hypothesis [5]. In this work, the non-parametric bootstrap is used.

Let  $x = (x_1, \dots, x_n)$  denote a sample of  $n$  independent realizations and identically distributed according to the probability density function  $F$ . The statistical estimator  $\theta = T(F)$  (mean, variance...) is sought. To estimate  $\theta$ ,  $\hat{\theta} = T(\hat{F}_n)$  is calculated where  $\hat{F}_n$  is the empirical cumulative density function defined by (8).

$$\hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^n 1_{u_i \leq x} \quad (8)$$

The idea of the non-parametric bootstrap is to simulate data from the empirical cumulative density function  $\hat{F}_n$ . Here  $\hat{F}_n$  is a discrete probability distribution that gives probability  $\frac{1}{n}$  to each observed value  $x_1, \dots, x_n$ . A sample of size  $n$  from  $\hat{F}_n$  is thus a sample size  $n$  drawn with replacement from the collection  $x_1, \dots, x_n$ . Once the bootstrap samples done, the properties of the estimator  $\hat{\theta}$  can be determined as shown in Fig. 8.

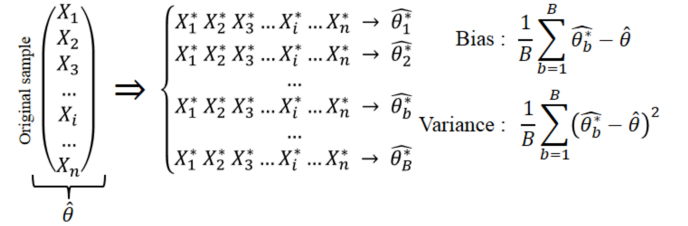


Figure 8. Bootstrap algorithm [5]

## IV. RESULTS

Firstly, in this section, results induced by the Monte Carlo technique are presented. In fact, a sufficient number of TELEMAC-2D runs has been carried out with respect to the pre-defined random entries in order to guarantee the convergence of the method. The obtained results provide the reference statistical estimators used to compare the efficiency of the Monte Carlo-like methods which constitute the second part of this section.

#### A. Monte Carlo results

To handle the uncertainty with the Monte Carlo technique, it is important to run a lot of simulations in order to have reliable results. In this work, around 70 000 Monte Carlo computations have been carried out. EDF’s cluster has been used to run these simulations. MPI library was used for launching and managing the uncertainty quantification study. Post-processing of the huge amount of results files is tackled through some Python scripts specifically developed within OpenTURNS.

The obtained results are analyzed twofold: On one hand, the effect of variability of random inputs is assessed at some specific points (assumed to be around an industrial plant, for example). On the other hand, a global statistical analysis all over the domain is done, as shown in Fig. 9.

A spatial distribution of the mean water depth and its variance is obtained. These results are of utmost importance for dimensioning of protecting dykes. Furthermore, there are very useful when establishing scenarios for flood managing.

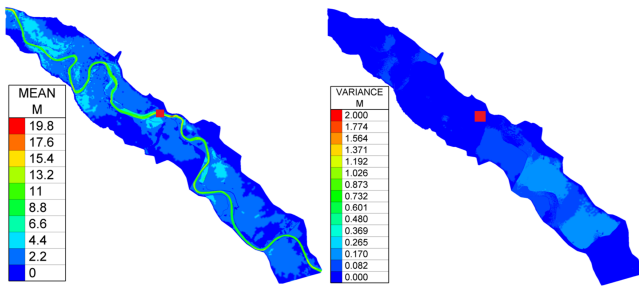


Figure 9. Mean and variance all other the domain (■ node 37 242)

To be sure that the obtained results are reliable, it is important to verify the convergence of them, especially by plotting the graph of the dispersion coefficient ( $\sigma/\mu$ ) as a function of  $N$ : if the convergence is not visible, it is necessary to increase  $N$  or if needed to choose another propagation method to estimate the uncertainty [8].

Fig. 10 shows the convergence of the dispersion coefficient and the mean of the water depth at the node number 37 242 located on Fig. 9.

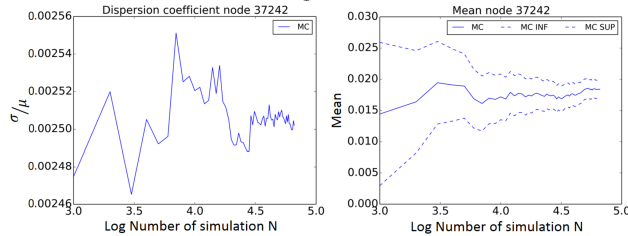


Figure 10. Convergence graphs of the dispersion coefficient and the mean according to the logarithm of the number of simulations

These graphics shows that the convergence of results are guaranteed from 30 000 simulations of Monte Carlo Technique. These results are then used to provide reference statistical estimators in the comparison of the efficiency of the Monte Carlo-like methods.

### B. Comparison of Monte Carlo-like algorithms

As shown in IV.A, thanks to the availability of important computer resources and to an optimized software, we are able to consider Monte Carlo uncertainty propagation algorithm for real hydraulic models. This conclusion was, even an unfeasible dream, couple of years ago.

However, the growing complexity of studies (such as coupled waves and hydrodynamics or hydro-sedimentological simulations, for instance) and the ever-greater needs in terms of precision in results (very fine mesh simulations) tend to encourage the use of techniques requiring less computation time.

As mentioned previously, a way to lower the computationally demanding of the Monte Carlo method is to replace the pure random sampling by alternative sampling methods such as the Latin Hypercube sampling approach and low discrepancy sequences. The section presents the comparison of Monte Carlo, quasi-Monte Carlo and Latin Hypercube Simulation on fourth first statistical estimators (mean, variance, skewness and kurtosis) of the distribution of the water depth.

The characteristic of interest of the output distribution is considered as stabilized when its variation are contained in the confidence interval of the reference solution. This confidence interval is calculated using the bootstrap technique, described in III.C.3)d), on the 70 000 Monte Carlo simulation results. The comparison was carried out at some points all over the computational domain. Since the obtained results are similar to each other, only one node results (node 37 242) are presented in Fig. 11. Firstly, the response variability limited to the mean value and the variance is studied. This constitutes the central part of the model response. In that case, the quasi-Monte Carlo algorithm has a faster convergence rate than the others techniques. In fact, from the beginning (about 1 024 runs), the mean value and the variance estimates are in the reference confidence

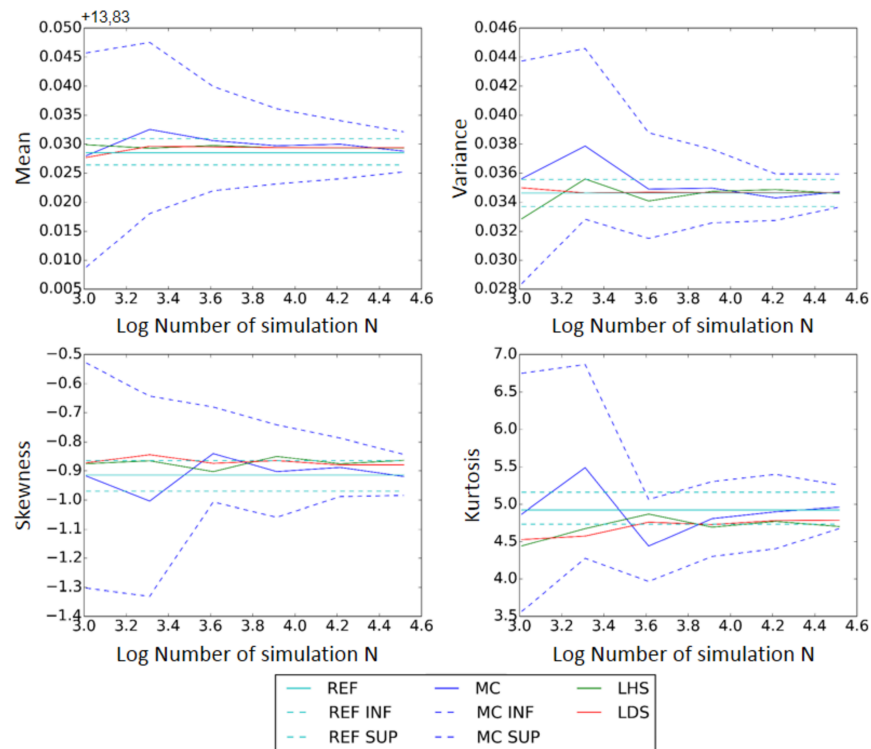


Figure 11 Comparison of the first four statistical moments on the node 37 242

intervals in contrary of the Latin Hypercube simulation and the Monte Carlo technique. The Latin Hypercube Sampling technique is more efficient than the Monte Carlo technique. Respectively, these techniques needs 2 000 and 4 000 simulations in order to obtained results contained in the reference confidence intervals.

The computation of higher order moments (skewness and kurtosis) do not converge as fast as for the mean and standard deviation since the variation of the related estimators of these moments is large. However, as observed for the central part of the model response, the quasi-Monte Carlo is the technique more efficient to determine these moments. In fact, about 4 000 runs are sufficient to reach the reference interval with the quasi-Monte Carlo method. It is more complicated for the Monte Carlo and Latin Hypercube simulation. In fact, the skewness estimated by the Monte Carlo converge as fast as the quasi-Monte Carlo whereas the estimation provided by the Latin Hypercube Sampling is oscillating at the upper bound of the reference confidence interval. At the opposite, the kurtosis estimation based Latin Hypercube Sampling has the same behavior that the low discrepancy sequence of Sobol and the Monte Carlo estimation needs more runs (about 10 000 runs).

As expected, sampling techniques, with their better exploration of the uncertain variable domain of variation, are more efficient than the brute random sampling. However, among the two techniques tested in this work, the quasi-Monte Carlo method is more effective. In fact, according to [3], unlike the quasi-Monte Carlo method, the LHS does not

control the quality of the joint distribution of samples when the dimension is higher than two.

## V. CONCLUSION AND DISCUSSION

In this paper, the feasibility of an uncertainty propagation with the Monte Carlo method on a two dimensional real case with TELEMAT-2D has been presented. In order to improve the converge speed of the Monte Carlo method, the Latin Hypercube Sampling and the quasi-Monte Carlo method are tested. In all cases, the more efficient technique is the quasi-Monte Carlo method. The improvement of the convergence speed induced by this method opens the doors of uncertainty studies with more complicated cases and bigger meshes where the computation time is crucial. However one of the drawbacks of the quasi-Monte Carlo method is that it does not possess a confidence interval of the results which is essential in practice. In fact, the error estimation, possible in theory using (6), is intractable in practice in contrary of the Monte Carlo method which easily provides a statistical confidence intervals [11]. In order to get the error estimates, the randomized quasi-Monte Carlo method can be used. This method, which constitutes an outlook of the current study, applies a randomization technique to the low discrepancy sequence [7].

Moreover, in the spirit of decreasing the computation cost of uncertainty studies, some techniques can be applied on the sensibility analysis too. The sensibility analysis intends to quantify the relative importance of each input

parameter of a model. The variance-based methods aim at decomposing the variance of the output to quantify the participation of each variable. Generally, these techniques compute sensitivity indices called Sobol indices. In practice, these sensibility indices are calculated using the Monte Carlo simulation. However, as for the uncertainty propagation, this technique requires a lot of computation time. So, in order to decrease the computational cost, some techniques such as the polynomial chaos method and derivative-based global sensitivity measures can be tested:

- The polynomial chaos method is a spectral method which gives a representation of the random response of the experiment. Based on this technique, it is possible to obtain sensitivity indices [10].
- When the derivatives of a computer program are known (Adjoint code for example), it is possible to apply the derivative-based global sensitivity measures (DGSM) [6] to perform sensitivity analysis.

These methods reduce drastically the number of runs needed for the sensitivity indices estimation and should be applicable to more complicated studies. Therefore, in the same way as this work, the sensitivity analysis can be optimized as well with further research.

#### REFERENCES

- [1] A. Bernard and N. Goutal, "Comparison between 1D and 2D models for hydraulic modeling of a floodplain: case of Garonne river", proceedings of River Flow conference, 2008, in press.
- [2] EDF-EADS-PHIMECA, "Reference guide", OpenTURNS version 1.1, 2013.
- [3] R. Faivre, B. Iooss, S. Mahévas, D. Makowski, H. Monod, "Analyse de sensibilité et exploration de modèles", 2013, éditions Quæ, pp.67.
- [4] J-M. Hervouet, "Hydrodynamics of Free Surface Flows", Wiley, 2007, pp. 83–130.
- [5] M. Kadiri Ottmani, "Traitement statistique d'un échantillon", 2002, Technical report of CEA.
- [6] M. Lamboni, B. Iooss, A.-L. Popelin, F. Gamboa, "Derivative-based global sensitivity measures: General links with sobol' indices and numerical test", ELSEVIER, February 2014, in press.
- [7] C. Lemieux, "Monte Carlo and Quasi-Monte Carlo Sampling", 2009, Edition Springer Series in Statistics, pp. 201–203.
- [8] A. Pasanisi, M. Couplet and A-H. Dutfoy Lebrun, "Guide méthodologique pour le Traitement des Incertitudes", Note EDF-MRI, reference H-T57-2013-02207-FR, 2013.
- [9] B. Sudret, "Uncertainty propagation and sensitivity analysis in mechanical models. Contributions to structural reliability and stochastic spectral methods", Accreditation to supervise research report, 2007.
- [10] B. Sudret, "Global sensitivity analysis using polynomial chaos expansions, Reliab. Eng. Sys. Safety, in press.
- [11] B. Tuffin, "Randomisation of quasi-Monte Carlo methods for error estimation: survey and normal approximation", in press.