Conference Paper, Published Version

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River Flow 2010 - Dittrich, Koll, Aberle & Geisenhainer (eds) - © 2010 Bundesanstalt für Wasserbau ISBN 978-3-939230-00-7

Comparison between 2D and 3D modelling of sediment transport: application to the dune evolution

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ABSTRACT: In most 1D or 2D depth-averaged sediment transport models, the sediment in suspension is assumed to be advected by the depth-averaged velocity. This contribution highlights the fact that the depth-averaged velocity must be weighted by the concentration profile to take into account the fact that the largest part of the sediment is transported near the bed. For this reason, a correction factor is introduced and an analytical formulation of this factor is provided. Through comparison with 3D computations, the efficiency of this correction factor is evaluated on a test case representing a gentle dune (with different dune steepnesses) propagating downstream under the action of a steady flow. For small dune steepness, the correction factor enables results from 2D computation to be closer to 3D simulation.

Keywords: Suspended load, Advection velocity, Dune evolution.

1 INTRODUCTION

Numerical morphodynamic models are now being extensively used by the engineering or scientific community in order to predict the bed evolution in various complex in-situ applications. Most existing modeling systems offer a 1D, 2D or 3D version. The choice of the model dimension is often guided by the scale of the domain. Considered as a good compromise, 2D depth averaged modelling is widely applied to medium scale domains. Besides the economic reasons discussed above, the most important scientific question remains whether all models should be equivalent in relatively simple applications.

In most 1D or 2D depth-averaged models (Cao and Carling 2002), the sediment is assumed to be advected by the depth averaged velocity, which is calculated by coupling with the hydrodynamic module via the shallow water equations. As discussed in this paper, this assumption is not valid for sediment transport in suspension. Therefore, a correction factor has to be introduced to take into account the fact that most sediment is transported near the bed.

A general expression of this correction factor is first derived. Since this general expression would require numerical integration and would thus increase the computational time, an analytical formulation is also provided for simplified theoretical profile of concentration and velocity. This analytical formulation has been implemented into a 2D model and is compared to 3D modelling using a steady uniform non-recirculating flow above a gentle dune bed.

The framework is the finite element Telemac system developed at EDF R&D, with both Telemac-2D and -3D hydrodynamic models, which can be internally coupled to the Sisyphe morphodynamics module (Hervouet 2007, Villaret 2010). Sediment can be transported as bed load and suspended load, the bed load being calculated by a classical sand transport formula, and the suspended load, by solving an additional 2D or 3D transport equation

The objective of this contribution is to illustrate that the proposed correction factor enables results from 2D computation to be closer to the 3D computation for such gentle configuration. If the dune steepness is increased above a certain value, the assumption of theoretical profiles for the velocity and concentration, namely a logarithmic velocity profile and a Rouse concentration profile, breaks down. A full 3D simulation is then requested to capture the recirculation cells in the lee of the dune. To test the robustness of this correction factor, different dune steepnesses are presently considered.

2 COUPLING BETWEEN HYDRODYNAMIC AND SEDIMENT TRANSPORT

2.1 *Coupling differences between 2D and 3D modeling*

Both Telemac -2D and -3D models are applied for the comparison. Concerning the sediment transport, only the suspension load is treated here. As discussed later, the suspension load can be indeed reckoned as the dominant transport mode for this flow configuration and bed material.

For 2D modeling, the Telemac-2D hydrodynamic model is internally coupled to the 2D morphodynamic model Sisyphe, which calculates the suspension load. For 3D modeling, the suspended load is directly calculated by Telemac 3D, which solves an additional 3D transport equation for the sediment concentration, and calculates the bed evolution.

2.2 *Suspended load treatment in Sisyphe*

For 2D modeling, the following depth-averaged 2D suspended load equation is solved:

$$\frac{\partial \overline{C}}{\partial t} + U_{conv}\frac{\partial \overline{C}}{\partial x} + V_{conv}\frac{\partial \overline{C}}{\partial y} = \frac{\partial}{\partial x}\left(\gamma_t \frac{\partial \overline{C}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\gamma_t \frac{\partial \overline{C}}{\partial y}\right) + \frac{(E-D)_{z=a}}{h} \quad (1)$$

where $\overline{C}$ is the depth-averaged concentration, U_{conv} and V_{conv} are the advective velocities, which are less or equal to the depth-averaged velocities (as explained later in following section). γ_t is the horizontal turbulent diffusion coefficient. E and D are respectively the erosion and deposition rates, calculated at the reference elevation z = a.

The resulting bed-evolution due to the suspension is simply:

$$(1-n)\frac{\partial z_f}{\partial t} = (D-E)_{z=a} \quad (2)$$

The erosion and deposition fluxes are obtained from the Celik and Rodi (1988) formulation, which is based on an equilibrium concentration C_{eq}:

$$(E-D)_{z=a} = w_s(C_{eq} - C_{z=a}) \quad (3)$$

where w_s is the particle settling velocity.

The equilibrium concentration is calculated by the empirical formula of Zyserman and Fredsoe (1994), where the reference elevation is taken as proportional to the mean grain size (a = $2d_{50}$). This formula has been applied consistently in both 2D and 3D simulations without skin friction correction even though this relation should be based on the stress acting on the grain rather than the total bed shear stress. Since the same assumption is used for both 2D and 3D computations, this will not influence the comparison.

Telemac 3D solves the same equations for the bed elevation (Eq. 2), where the erosion and deposition fluxes are also calculated based on Eq. (3).

3 ADVECTION VELOCITY FOR THE SUSPENSION

3.1 *Discussion about the advection velocity*

For the 2D transport equation of the suspended load, it is generally assumed that the advection velocity corresponds to the depth-averaged velocity. In fact, this assumption should only be valid for homogeneous vertical concentration profile, which is generally not the case for the suspension. Indeed, let us consider a simple 3D advection equation:

$$\frac{\partial C}{\partial t} + div\left[C\vec{U}\right] = 0 \quad (4)$$

The 2D equation is obtained by integrating Eq. (4) from the reference elevation z = a to the water depth z = h:

$$\frac{\partial \overline{C}}{\partial t} + div\left[\overline{C\vec{U}}\right] = 0 \quad (5)$$

Eq. (5) can be rewritten to transform it into an advection equation for the depth-averaged concentration:

$$\frac{\partial \overline{C}}{\partial t} + div\left[\frac{\overline{C\vec{U}}}{\overline{C}}\overline{C}\right] = 0 \quad (6)$$

Therefore, the advection velocity does not correspond to the depth-averaged velocity but to an averaged concentration weighted by the concentration.

$$U_{conv} = \frac{\int_a^h U(z)C(z)\,dz}{\int_a^h C(z)\,dz} \quad (7a)$$

and $$V_{conv} = \frac{\int_a^h V(z)C(z)\,dz}{\int_a^h C(z)\,dz} \quad (7b)$$

3.2 *Estimation of the correction factor*

During the coupling between the hydrodynamic and sediment transport modules, Telemac 2D sends the depth averaged velocities to Sisyphe. Consequently, it is necessary to correct these values by a factor α:

$$\alpha = \frac{U_{conv}}{\overline{U}} = h\frac{\int_a^h U(z)C(z)dz}{\int_a^h U(z)dz \int_a^h C(z)dz} \quad (8)$$

To determine the value of the correction factor α, integration of the velocity and concentration profiles are thus necessary. Since these profiles are unknown with 2D computation, these integrations are performed using the assumption of logarithm profile for the velocity and Rouse profile for the suspension.

$$\frac{C(z)}{C(a)} = \left(\frac{(h-z)}{(h-a)}\frac{a}{z}\right)^R \quad (9)$$

The Rouse number R is defined as: $R = \frac{w_s}{\kappa u_*}$

where u_* is the shear velocity and κ the von Karman constant.

Implicitly, the influence of the turbulent Schmidt number is thus not reckoned in the present analysis.

For the velocity profile:

$$\frac{U(z)}{u_*} = \frac{1}{\kappa}Ln\left(\frac{30z}{k_s}\right) \quad (10)$$

with k_s the equivalent bed roughness. The depth-averaged velocity is thus given by:

$$\overline{U} = \frac{u_*}{\kappa}Ln\left(\frac{30h}{k_s e}\right) \quad (11)$$

Rearranging the term within the integrals, the correction factor can be written as:

$$\alpha = -\frac{I_2 - Ln\left(\frac{B}{30}\right)I_1}{I_1 Ln\left(\frac{eB}{30}\right)} \quad (12)$$

with

$$I_1 = \int_B^1 \left(\frac{(1-u)}{u}\right)^R du$$

and

$$I_2 = \int_B^1 Ln\,u \left(\frac{(1-u)}{u}\right)^R du \quad (13)$$

introducing u = z / h and the normalized roughness B, which is defined as:

$$B = \frac{k_s}{h} = \frac{a}{h} = Z^{-1} \quad (14)$$

Here we further assume that the bed roughness and reference elevation are equal, which is generally the case for flat bed or rippled bed.

3.3 *Simplified equation*

In order to avoid numerical integration, the equations for I_1 and I_2 can be simplified by assuming an exponential concentration profile as:

$$C = C_a\left(\frac{a}{z}\right)^R \quad (15)$$

I_1 and I_2 then become:

$$I_1 = \int_Z^1 \left(\frac{1}{u}\right)^R du \quad (16)$$

$$I_1 = \frac{1}{(1-R)}\left(1 - Z^{(1-R)}\right) \quad for\ R \neq 1$$

$$.\ I_1 = -Log(Z) \quad for\ R = 1 \quad (17)$$

where Z is defined by Eq. (14)

$$I_2 = \int_Z^1 Log\,u\left(\frac{1}{u}\right)^R du \quad (18)$$

$$I_2 = \frac{I_1 + LogZ\,Z^{(1-R)}}{(R-1)} \quad for\ R \neq 1$$

$$I_2 = -\frac{1}{2}\left(LogZ\right)^2 \quad for\ R = 1 \quad (19)$$

The dependence of the correction factor to the Rouse number and the relative roughness B is plotted on Figure 1.

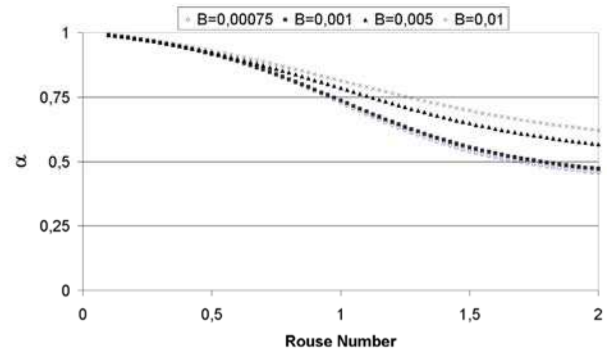

Figure 1 Correction factor α versus the Rouse number for different values of the relative roughness B.

The correction factor is thus weak for fine sediment (small value of the Rouse number) and becomes more significant for coarser material. The values of the correction factor calculated from Eqs (17-19) can be compared, with values obtained from the approximated solutions of the Einstein integral provided by Guo et al. 1996 (Figure 2). For Rouse number larger than 1, their approximated solutions are based on a recurrence formula.

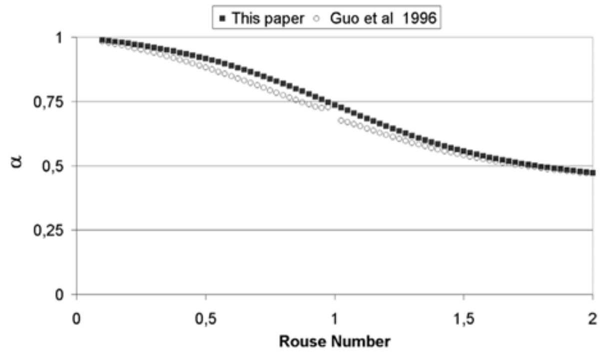

Figure 2 Comparison with the approximated solution provided by Guo et al. 1996

Figure 2 illustrates that both approximations are relatively close: differences between the approximated solution given in this paper and the one of Guo et al. 1996 are less than 7% for all values of the Rouse number.

4 TEST CASES

The test cases considered correspond to a 2D sinusoidal sand dune propagating downstream under the action of a uniform steady flow. Here, our goal is to test the robustness of the correction factor on progressively steeper configurations. Three configurations are considered where the dune height is increased for a constant dune length.

The initial dimensions of the dune (length L_D and height H_D) have been chosen according to the initial water depth (1,1 m). The initial length is imposed to 8 m for the three configurations. The ratio between dune length and water depth is thus equal to 7,3, which corresponds to the value obtained by van Rijn (1984b).

Different dune heights have been selected: 10%, 20% and 30 % of the water depth, which is the range generally observed (van Rijn 1984b). The characteristics of the three test cases are summed up in Table 1.

Table 1: Characteristics of the test cases

Cases	H_D (m)	H_D / L_D
Case 1	0,11	0,015
Case 2	0,22	0,03
Case 3	0,33	0,045

Such large symmetric dunes have already been observed on field configuration, as for instance in the Fraser River (Kostaschuck and Villard 1996). Even if the dune height of case 3 has is relatively high compared to the water depth, the lee slope is still relatively low (<5°). Therefore even for the steepest dune, no flow separation should be observed.

5 MODELS SET-UP

5.1 *Initial and boundary conditions*

The computational domain represents a rectangular channel: the width of the channel is 1,1 m and its length 16 m. The dune top is initially located at 6 m downstream the inlet.

The 2D mesh is a structured triangular grid (0,1 m x 0,2 m) with 1600 elements. The 3D mesh is obtained by extrusion of the 2D mesh through 16 planes. The distance between different planes is increasing progressively from the bottom to the water surface. The first plane near the bottom is located at z = 0,006m

For the hydrodynamic boundary conditions, a flow rate is imposed at the inlet (0,5 m^3/s) and a water depth at the outlet (1,1 m). A logarithmic profile for the velocity is imposed at the inlet for 3D computation.

For the suspension, equilibrium concentrations are imposed at the inlet in both 2D and 3D simulations. For 2D computation, the equilibrium depth-averaged concentration is calculated assuming a Rouse concentration profile and near bed equilibrium concentration as obtained from the Zyserman and Fredsoe equation. In order to avoid unwanted erosion at the inlet recirculating conditions have been imposed in the 3D model: the imposed concentration profile at the inlet has been extracted from the concentration profile obtained at the outlet of the domain (without dunes). This concentration profile can be considered as in equilibrium.

5.2 *Physical parameters*

For the friction term, a constant Strickler coefficient K has been imposed (K = 50 $m^{1/3}/s$) for both 2D and 3D computations. For the initial flow conditions, the value of the Strickler coefficient would stand for a bed with a roughness about 0,02 m. It would mean that the choice of the Strickler coefficient accounts for the presence of small-scale bedforms such as ripples, which are -smaller than the grid size and therefore need to be parameterized.

The numerical time step is set equal to1 s whereas the final time reaches 8 h.

The bed material is assumed uniform with a median grain diameter equal to 0,15 mm and corresponding settling velocity of 1,623 cm/s.

The turbulence is modeled in 2D by a constant viscosity whereas a mixing length model has been selected in 3D since no separation flow is expected. A k-ε turbulence model would allow a more accurate representation of the turbulent flow structure in the lee side of the dune. However, the length of the domain is not sufficient to reach fully developed of turbulence conditions with the k-ε model and the results would therefore highly depend on input conditions, which are generally based on mixing length model. Therefore, a simple mixing length turbulence model has been here considered sufficient for the present work.

For both 2D and 3D, the diffusion coefficient for the velocity is set equal to 0,1 m^2/s and 0,01 m^2/s for the sediment.

5.3 *Discussion about the advection schemes*

A special care must be brought to the numerical scheme for the suspended sediment advection-diffusion equation. We use a simple first order explicit upwind finite volume scheme, the finite volumes being centred on the finite element degrees of freedom (Postma and Hervouet 2006). The volumes are then the integral of the test-functions in finite elements, which gives a compatibility for the computation of mass. Such schemes ensure mass-conservation and monotonicity if the velocity field obeys the continuity equation. However this latter condition is not guaranteed when the advection velocity field is corrected and in this case theory shows that the monotonicity cannot be ensured. As a matter of fact the depth-averaged sediment concentration is not in this case subjected to the maximum principle and may increase locally.

6 INFLUENCE OF THE CORRECTION FOR THE ADVECTION VELOCITY

For the considered configuration, the Rouse number is initially about 1,52 and the relative roughness B reaches 0,0004 which finally conducts to a correction coefficient of α = 0,55. It thus means that the advected velocity is nearly half reduced compared to the depth-averaged velocity.

According to Chien (1954) criteria, the sediment transport for this value of the ratio u_*/ws (1,6) is located outside the dominant bed load (< 0,33) and rather into the domain for which the suspension can already be met from above the mid water level to the water level.

For the first test case, which has the lower bedform steepness, the correction on the advection velocity is illustrated on Figs 3 and 4. Figure 3 compares the suspension load for 2D computation (with and without the modification) with the 3D computation where the suspended load has been depth integrated. The comparison is achieved in the beginning of the computation (after 500 s) when the deformation of the dune does not significantly influence the results.

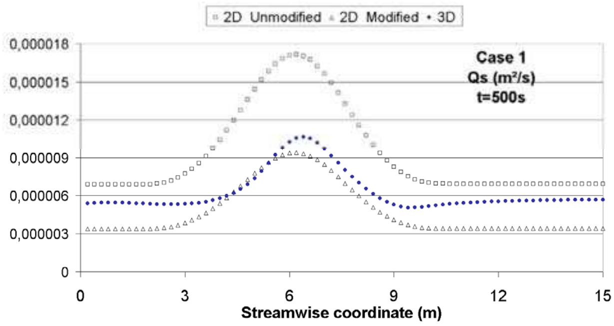

Figure 3 Influence of the correction factor on the suspension load (Q_s)

Figure 3 illustrates that the 2D model without correction (2D unmodified) over-estimates the suspension load. Figure 3 points out that the correction is particularly efficient on the dune (from x = 4 to 9 m). The difference between the modified 2D and the 3D near the inlet and the outlet probably comes from an influence from treatment of the boundary condition for the concentration as discussed previously (see section 5.1). The concentration profile extracted at the outlet section is about 0,07 g/l whereas the equilibrium concentration imposed by Sisyphe at the inlet is 0,04 g/l.

On Figure 3, the comparison between the computations on case 1 is extended to the time evolutions of the bed level (zf).

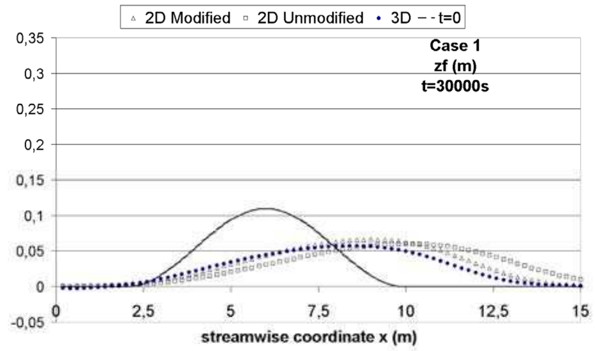

Figure 4 Influence of the correction factor on the bed evolution (zf)

On Figure 4, it is effectively observed that the dune is migrating too fast for 2D computation without the correction factor. The bed evolution from the corrected 2D computation nicely follows the 3D computation on the stoss slope of the dune. After the dune crest, the evolution of the dune is still over-estimated even with the correction. It may come from the fact that even with this gentle dune steepness, the 3D flow effects already influence the sediment transport on the lee side, and departure from the logarithmic velocity profile and equilibrium Rouse profile assumption.

7 ROBUSTNESS OF THE CORRECTION FACTOR

To test the robustness of the correction factor, computations have been performed on two steeper dune configurations (case 2 and case 3). Flow visualization has confirmed that the flow stays attached along the dune curvature. The comparison between both 2D approaches and the 3D calculation, are plotted in Figure 5 (case 2 until t = 20000 s) and Figure 6 (case 3 until t = 10000 s)

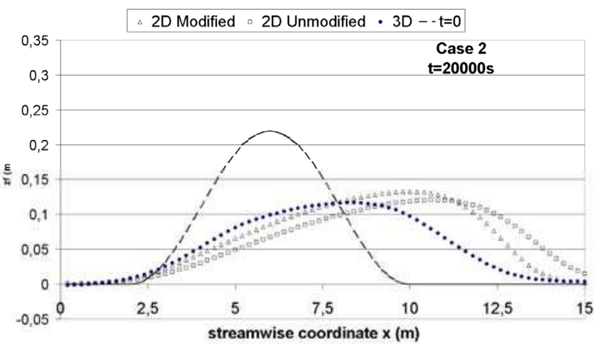

Figure 5 Comparison of the bed evolution for test case 2

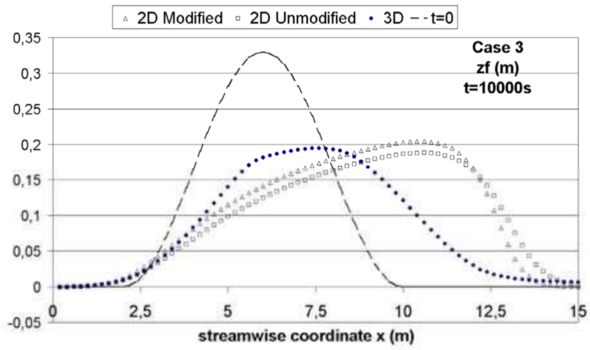

Figure 6 Comparison of the bed evolution for test case 3

Figs 5 and 6 point out the correction coefficient stays efficient at the inlet of the channel and in the beginning of the stoss slope. Nevertheless when the dune steepness is increasing, the magnitude of the vertical velocity and the 3D effects are growing. The difference between the results from the 2D modified simulation and the 3D is starting more upstream from the dune crest (on the dune stoss slope) and earlier (for test case 3, the results are already very different at t = 10000 s).

The correction factor integrates into depth averaged model information relative to the vertical distribution. Compared to classical depth-averaged model, this approach improves the quality of the results for gentle configurations and should thus also improves the accuracy of the results on classical configuration on which the 2D depth averaged are generally applied (medium scale). However compared to a full 3D flow, the assumptions about the velocity profile and the concentration profile are of course not debatable. For instance as shown by Lyn (1988), information related to the Schmidt number should be integrated too or the feasibility to include “two layers” model (as proposed for instance by van Rijn 1984a, Verbanck 2000 or Verbanck et al. 2002) should be considered.

8 CONCLUSION AND PERSPECTIVES

Whereas most of the 1D and 2D depth averaged models consider the sediment to be advected by the depth-averaged velocity, this contribution has highlighted that the depth-averaged velocity should be weighted by the concentration profile. A correction factor has been defined and an analytical formulation using simplified theoretical profiles for both concentration and velocity has been provided. It can therefore be easily implemented into the model without increasing the computational time. On a gentle dune configuration (dune steepness less than 0,015), it has been illustrated that the correction allows the 2D results to tend to the full 3D computation with a large gain in computer time.

However when the dune steepness is increasing, the correction factor becomes less efficient due to the growing importance of the vertical velocity.

In a near future, the comparison will be extended to an asymmetrical dune configuration (with steeper lee side) and should be also confronted to experimental data.

The influence of the correction has to be evaluated on other test cases for which the flow can be reckoned as one- or bi-dimensional. Application to a field configuration with larger space scale as for instance the Gironde estuary in France is also planned.

ACKNOWLEDGEMENTS

We greatly acknowledge Pr A.G . Davies from University College of Wales (Bangor, UK) for his input on the formulation.

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