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Computational modelling of three-dimensional bedform evolution

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ABSTRACT: We are developing a numerical model for simulating the development and migration of dunes in rivers in a 3D case. The numerical model consists of three steps: turbulent flow, sediment transport and morphology. We modelled the turbulent flow using state-of-the-art techniques for detailed hydrodynamics. A finite volume method combined with an isotropic unstructured Cartesian grid with local refining is developed for simulating time-dependent incompressible flow. The grid can be refined and adapt to the boundaries. The governing equations are discretized using a staggered grid and advance in time using the fractional step method. The Cartesian grid cells and faces are managed using an unstructured data approach. A ghost-cell immersed-boundary technique has been implemented for the cells which intersect the immersed boundaries. Because of the importance of coherent structures of turbulence on sediment transport, the turbulence regime is modelled by Large Eddy Simulation. The sediment is considered as rigid spheres in the turbulent flow. Models for pickup, transport and deposition are incorporated in the current study. The morphodynamics is determined by the rate of pickup and deposition of the sediment particles.

Keywords: Dunes, Sediment, Morphology, Large Eddy Simulation, Cartesian Grid

1 INTRODUCTION

The water levels during river floods, and hence the risk of flooding, depend on the hydraulic roughness of the river. One of the components of this roughness is produced by statistically periodic irregularities on the river bed called “dunes”. The development of dunes and the associated hydraulic roughness during a flood is complex. Initially dunes grow higher and make the river bed rougher, but in later stages the dunes grow longer with the opposite effect of making the river bed smoother. Subsequently, in a way still not well understood, new bedforms develop on top of the elongated dunes that make the river bed rougher again.

The sediment transport field over dunes and ripples in open-channel flows is strongly affected by the complex turbulence field caused by flow separation at the dune crest. The three-dimensionality of turbulence and the effect of turbulence on the sediment transport and morphological process form a complex problem which is not completely solved yet. At present, there is still limited knowledge about the effect of dunes on the hydraulic roughness of the rivers. Several researchers proposed methods to predict the dune dimensions based on parameterization methods, empirical relations (Julien & Klaasen 1995; Van Rijn 1984) and theoretical interpretations (Onda & Hosoda 2004). Existing experimental studies are limited to the formation of dunes on steady state flow regimes (Blom et al 2003). Wilbers (2004) has shown none of these predictors are able to predict correctly the dune dimensions during several floods in the River Rhine in the Netherlands. He developed an empirical method to predict dune development for unsteady flows. This method is applied successfully to three sections of the River Rhine branches, but it cannot be generalized. This method gives limited knowledge about the physical phenomena behind dune development during floods.

Existing numerical studies on the morphology are limited to the approximation of the two-dimensional fluid flow, with two-dimensional dunes development (Giri et al 2006, Shimizu et al 2001) or with fixed bed (Zedler et al 2001, Yue et al 2006, Yoon et al 1996). The nature of flow over 3D dunes is very different from the flow in many studies that have concerned 2D dunes; to the degree that the application of some of these 2D studies to the field requires careful attention (Best 2005). Field observations suggest an urgent requirement for a fuller analysis of dune three-dimensionality in both laboratory and numerical studies.

We are developing a numerical model for simulating the development and migration of dunes in rivers in a 3D case. The numerical model consists of three steps: turbulent flow, sediment transport and morphology. At the first step, we modelled turbulent flow using state-of-the-art techniques for detailed hydrodynamics. The development of dunes is directly influenced by the fluid flow. A correct prediction of dunes and migration of bedform requires an accurate prediction of fluid flow. The dunes have also a direct effect on the drag and hence on the fluid flow regime. It is important to calculate the fluid in its details close to the bedform. An accurate solution for the dunes can be achieved by a high-resolution grid close to the boundaries. Structured Cartesian grids are not suitable for this because the solution of a fully structured Cartesian grid can be very expensive. Here, a finite-volume method combined with an isotropic unstructured Cartesian grid with local refining is developed for simulating time-dependent incompressible flow. The grid can be locally refined and adapt to the bed form. The governing equations are discretized using a staggered grid and their solution is advance in time using the fractional step method. The Cartesian grid cells and faces are managed using an unstructured data approach. A ghost-cell immersed-boundary technique is implemented for the cells which intersect the immersed boundaries. Because of the importance of coherent structures of turbulence on sediment transport, the turbulence regime is modelled by Large Eddy Simulation.

The second step concerns the modelling of sediment pickup, transport and deposition. The sediment transport is modelled in a Lagrangian field, which may involve new concepts that is better suited for relatively small spatial and temporal scales. The sediment is considered as small rigid spheres in the flow. This method gives a better insight about the physical phenomena of the sediment transport, and makes it possible to simulate the detailed sediment motion such as jumping, sliding and rolling.

The third step concerns a morphology model for bedform growth, decay and migration. A discrete particles model is considered for pickup and deposition and hence for the evolution of bedform. This method yields one of the best predictions of immigration of sand dunes.

The model aims at giving better insight into the development of the hydraulic roughness, and hence the flooding risk, during river floods. The good understanding thus obtained will allow the development of parameterized models for larger spatial and temporal scales that can be used in operational models for flood early warning systems and the determination of design water levels.

2 GOVERNING EQUATIONS

2.1 *Fluid*

The governing equations for the fluid are the full three dimensional Navier-Stokes equations for incompressible flow with the Boussinesq approximation invoked. These equations are given below in terms of volume filtered variables.

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j}\left\{2\left(\nu + \nu_t\right) S_{ij}\right\} \qquad (2)$$

where x_i are the coordinates, t is the time, $\bar{P}$ the modified pressure, ρ_0 the density, $\bar{u}_i$ the filtered velocity component in x_i-direction, ν and ν_t the molecular and turbulent viscosities and S_{ij} is the resolved strain rate tensor:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) \qquad (3)$$

In Large Eddy Simulation, the large eddies is solved directly, ignoring the smaller eddies. The smaller eddies are then modelled. A volume filtering is used in LES, allows filter the eddies which are smaller than the grid cell volume. The effect of the small scales upon the resolved part of turbulence appears in the SGS stress term $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ which must be modelled. The SGS effect is modelled using a dynamics subgrid-scale model, averaged in Lagrangian field. The interested reader is referred to Meneveau et al (1996).

2.2 *Sediment*

A particle immersed in a fluid is subjected to the forces of gravity, and the fluid forces acting on it. The velocity of the individual sediment particle in the flow is calculated according to

$$\rho_s V_p \frac{d\mathbf{v}_p}{dt} = (\rho_S - \rho) V_p \mathbf{g}$$
$$+\rho V_p \frac{D\mathbf{u}_f}{Dt} + \frac{1}{2}\rho V_p \left(\frac{D\mathbf{u}_f}{Dt} - \frac{d\mathbf{v}_p}{dt} \right)$$
$$+\frac{3}{4}\frac{C_D}{d}\rho V_p \left|\mathbf{u}_f - \mathbf{v}_p\right| \left(\mathbf{u}_f - \mathbf{v}_p\right) \tag{4}$$
$$+\frac{3}{4}\frac{C_L}{d}\rho V_p \left(\left|\mathbf{u}_f - \mathbf{v}_p\right|_{top} - \left|\mathbf{u}_f - \mathbf{v}_p\right|_{bottom} \right)\mathbf{n}$$

v_p represents the velocity of the sediment particle, u_f is the fluid velocity at the particle location, C_D is the drag coefficient, C_L is the lift coefficient, d is the particle diameter, D/Dt = $\partial/\partial t + \mathbf{u}.\nabla$, $\mathbf{g}$ is the gravity vector, V_p is the particle volume and ρ and ρ_s are the densities of the fluid and particle respectively (Maxey & Riley 1983). The drag coefficient C_D can be found using the formulas proposed by Morsi & Alexander (1972). The lift coefficient C_L is a problematic issue, and it is very complicated to determine. There is limited knowledge regarding to the effect of solid boundaries on the particles. Moreover the velocity of fluid must be determined at the top and bottom of the particle. The size of particle is one or two order smaller than the grid cell, which makes the interpolated velocities on the top and bottom of the particle insignificant. Here we replace the term of lift force in equation (4) by the theoretical and experimental relations introduced by McLaughlin (1991) and Mei (1992), which are given as

$$\frac{C_L}{C_{L.Sa}} = 0.443J \tag{5}$$

where

$$J = J(\varepsilon) \approx$$
$$0.6765\left\{1 + \tanh\left[2.5\log_{10}(\varepsilon + 0.191)\right]\right\} \tag{6}$$
$$\left\{0.667 + \tanh\left[6(\varepsilon - 0.32)\right]\right\}$$

$$\varepsilon = \frac{\mathrm{Re}_\alpha^{0.5}}{\mathrm{Re}_p} \quad , \quad \mathrm{Re}_\alpha = \frac{\alpha d^2}{\nu} \quad , \quad \mathrm{Re}_p = \frac{\left|\mathbf{u}_f - \mathbf{v}_p\right| d}{\nu} \tag{7}$$

where α is the fluid shear rate. The index *Sa* denotes the corresponding result obtained by Saffman (1965), which is defined as

$$C_{L,Sa} = \frac{12.92}{\pi}\varepsilon \tag{8}$$

The lift force can be determined as,

$$F_L = \frac{\pi}{8} C_L \rho_f \left|\mathbf{u}_f - \mathbf{v}_p\right|^2 d^2 \tag{9}$$

Equation (4) is solved using an implicit scheme to avoid instabilities. Each particle position is then calculated according to

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{v}_p \tag{10}$$

until they deposit (or probably stay in suspension). The particles can have different behaviour when they interact with the bed. Schmeeckle et al. (2001) have shown that the appropriate physical scaling of this problem is a collision Stokes number. If $St \ll 1$, the viscous pressure will stop the particle before significant elastic energy can be stored in the deformation of the particles. In this case, there will be no initial rebound velocity. According to Schmeeckle et al. (2001), the transition between damped and un-damped particle collisions occurs at about the transition from medium to coarse sand. At a transport stage, the corresponding critical sediment diameter size is 2.7 mm. Therefore, for sediment larger than sand (> 2 mm), saltation impacts will almost always be partially elastic, and for sand and smaller particles, there is no significant normal rebound. When partially elastic collision occurs, the new velocities can be calculated as follows.

$$V_{n,new} = -\alpha_n V_{n,old}$$

$$V_{t,new} = \alpha_t V_{t,old}$$

Here V_n and V_t are the normal and tangential velocities respectively. The normal and tangential elastic coefficients α_n and α_t are set to the experimentally determined values of 0.65 and 0 (Schmeeckle and Nelson 2003). The partially elastic rebound occurs in Stokes numbers higher than 105. For Stokes numbers lower than 39, the particles are viscously damped (α_n = 0). In the range between 39 and 105, the behaviour of collision is not clear because of the negative pressure and cavitation between the particle and the bed. Schmeeckle and nelson (2003) set α_n = 0 for Stokes numbers between 39 and 105.

The bed of the river can be approximated by spherical particles lying in the form of layers. If the shear stress of the fluid on the bed exceeds a certain value, the particles begin to rotate and probably move from their position. Each spherical solitary sediment particle is resting over a closely packed three other spherical particles. It is the most stable three-dimensional configuration for spherical particles. The incipient motion of sediment can be determined by the vector summation of the mentioned forces. The initial motion of the spherical particles begins either by rolling, or by sliding. The most of particles begin their incipient motion by rolling and few numbers of particles by sliding. At the present work, the incipient motion

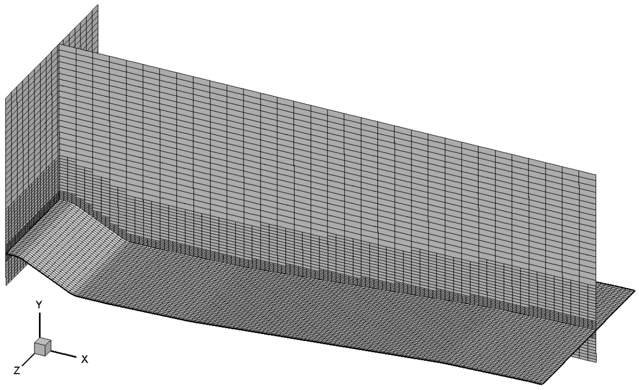

Figure 1. An isotropic Cartesian grid with local refining, over a dune.

by sliding is ignored. Only the incipient by rolling of particles is considered.

In our model, the pickup rate formula proposed by Nakagawa & Tsujimoto (1980) is incorporated to estimate the pickup rate. This approach was also effectively used by Onda and Hosoda (2004), and Giri and Shimizu (2006) for computation of bed form development. The dimensionless pickup rate is expressed as

$$P_S^* = \frac{P_S\sqrt{d}}{\sqrt{\left(\frac{\rho_s}{\rho}-1\right)g}} = F_0\tau_*\left(1-\frac{\tau_{*cr}}{\tau_*}\right)^3 \quad (11)$$

in which F_0=0.05 is an experimental constant. τ_* and τ_{*cr} are the dimensionless shear stress and critical shear stress for incipient motion of bed material particles, which are averaged on time and space. The number of particles which are picked up from the bed in time step Δt can be determined as

$$n_{pickup} = \frac{A_3}{A_2} P_S S d\, \Delta t \Big/ V_p \quad (12)$$

Here S is the area of bed-surface mesh, and A_2 and A_3 are shape coefficients of sand grains with two- and three-dimensional geometrical properties, and V_p is the volume of the particle.

2.3 *Bedform*

The bed of the river is approximated by a surface grid with equidistance Δx and Δz as shown in Figure 1. The pickup rate of each cell on the bed is calculated according to equations (11) and (12). The rate of deposition is calculated from the total number of particles which deposit in the current bed-cell in time step Δt. The difference in the number of the particles in pickup and deposition for any portion on the bed, indicates the amount of mass added to or decreased from the current area (here cells). The change in level for each bed-cell after time step Δt can be determined as,

$$\Delta y = \frac{A_2}{A_3}\frac{V_p\left(n_{depos}-n_{pickup}\right)}{S} \quad (13)$$

3 NUMERICAL METHODS

An approach which is gaining popularity in the recent years is the Cartesian grid method. At this method the governing equations are discretized on a Cartesian grid which cannot fit the immersed boundaries. Cut-cell technique and Ghost-cell methods are the most popular remedies for this problem. At the cut-cell technique, the intersecting cells are cut, yielding arbitrarily shaped cells, which add complexity to the computational model. The Ghost-cells method adds a force on the immersed boundaries. It is easy to implement and requires less computational efforts than the cut-cells techniques.

A simple structured Cartesian grid requires a large number of cells to capture the small eddies in a turbulent flow. In order to optimize the use of computational resources, we use an adaptive multi-level Cartesian mesh, with local refinement, in which more grids cells can be placed in high gradient regions such as boundaries. (Figure 1)

An isotropic Cartesian grid is used for this purpose. In isotropic Cartesian grids, the refining of each cell occurs in all directions. Any refined cell has 8 children in 3D or 4 children in 2D.

The set of refined Cartesian cells are commonly managed in two ways: the hierarchical tree data structure and the fully unstructured approach. The tree data structure (parent-child tree) requires a tree-traverse approach to determine neighbour connectivity based on logical recursive routines. The calculation time required for determining the neighbours can be considerably larger than the fully unstructured approach. Here we use a combination of unstructured approach and hierarchical tree data structure by defining local pointers to find the neighbours. By hierarchical tree data structure it is possible to apply multigrid technique to solve the desired equations.

The numerical method for the present contribution uses a staggered grid method, with the pressure located in the centre, and the velocities on the faces of cells. The governing Navier-Stokes equations for unsteady incompressible flow are discretized using finite-volume method with second order, linear discretizations for fluxes. Time advancement uses the frictional step method, which decouples the solution of the velocity field from the pressure. The second-order Adams-

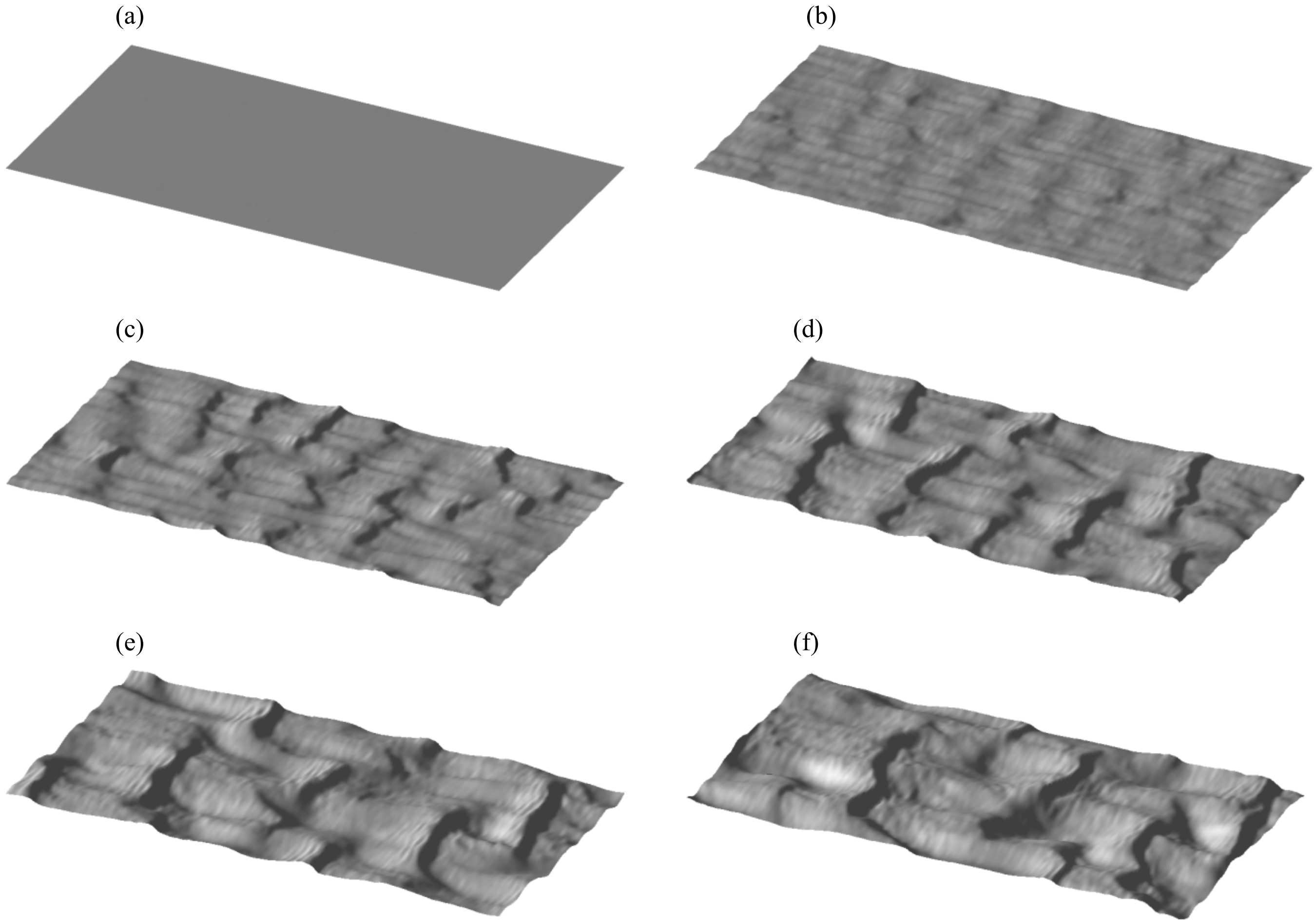

Figure 2 Simulation of morphology with sediment diameter of 245 μm. It starts from a flat bed (a), instabilities appear (b), then ripples are generated (c). The ripples grow (d), and lead to generation of dunes (e). The dunes grow and move as steady (f).

Bashforth-Crank-Nicolson method is used for the momentum equations.

The Poisson pressure correction equation is solved by Multigrid method. Ghost-cell immersed boundary techniques are used to conform the dune geometry (Fadlun et al 2000).

4 NUMERICAL EXPERIMENT

In this section, the generation and migration of dunes are simulated by using rigid spheres as sediment particles. The sediment is picked up from the bed, transported in the water, and settled in different locations. By considering the pickup and deposition as mass sources and sinks, the deformation of bed can be calculated.

The computational domain is set to be 60x120 cm^2. The wide of the domain is identical with the flume experiment which is carried out under the department of hydraulic engineering in Delft University of Technology. The total length of the flume is 25 m and width is 60 cm, has $3x10^{-4}$ slope. The bottom of the flume is covered with 20 cm thick sediment layer with 0.245 mm median diameter. The experiment begins from a flat bed, with water depth of 4.5 cm, and discharge of 6.8 lit/s.

For simulation purpose, the length is chosen as a part of the flume (1.2 m); hence the boundaries in streamwise direction are set to be periodic. For generalization and also simplicity, the boundaries in spanwise direction are also set to be periodic. The simulation begins from a flat bed, with the same water depth and discharge as experiment (Fig. 2a). The bed gets instabilities because of fluctuating shear stress (Fig. 2b). These instabilities lead to ripples (Fig. 2c). The ripples grow (Fig. 2d) and dunes are formed (Fig. 2e). Dunes move and finally they get a statistically steady state (Fig. 2f).

Figure 3 Distribution of sediment over the dunes (side view). The diameter of particles is 245 μm.

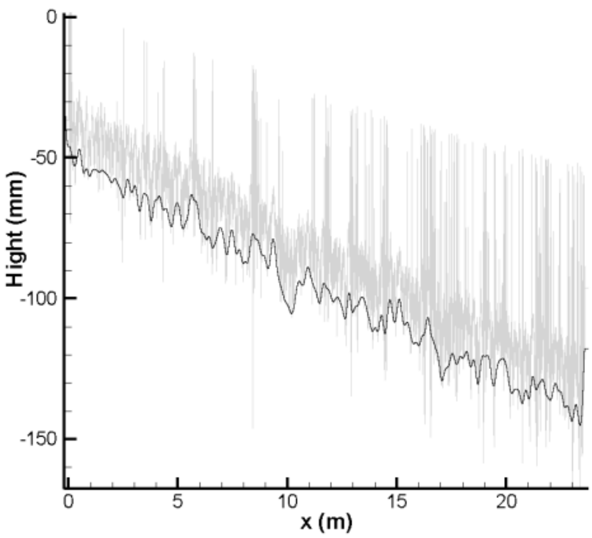

Figure 4 The measurement of dunes height and length (grey) and low-pass filtering of the measurements (black).

Figure 3 shows instantaneous sediment distribution (suspended) in the water above the dunes. Figure 4 shows a measuring of the dunes and its low pass filtering. The length of each dune is defined by the distance between two successive intersections of the curve defining 2D dune, with the baseline. Only the measurements for the centerline of the flume are considered in averaging, to minimize the effect of the walls. The baseline is found by taking linear least square method of all measured points. The height of each dune is defined as the difference between the maximum and minimum height between the intervals defined by the two successive intersections. The same methodology is applied for the simulated results and averaged in space and time. The length and height of dunes in our simulation are found to be 26.6 cm and 24 mm, which have good agreement with the experiment (25 cm and 23 mm).

The simulations are also applied for different grain sizes. Figures 5a-5f show the simulation with different sediment diameters (100, 120, 140, 160, 180 and 200 μm). As can be seen in these figures, the bed with fine sediment includes more ripples. The ripples are generated on the dunes and they are super positioned which leads to deformation of the dunes. This physics-based phenomenon is mentioned in Best 2005. Moreover, it can be seen that the bed with fine sediment, leads to two-dimensional dunes. By increasing the sediment diameter, the three-dimensionality dominates and the topology of the dunes changes to fully three dimension in the relatively larger sediment size.

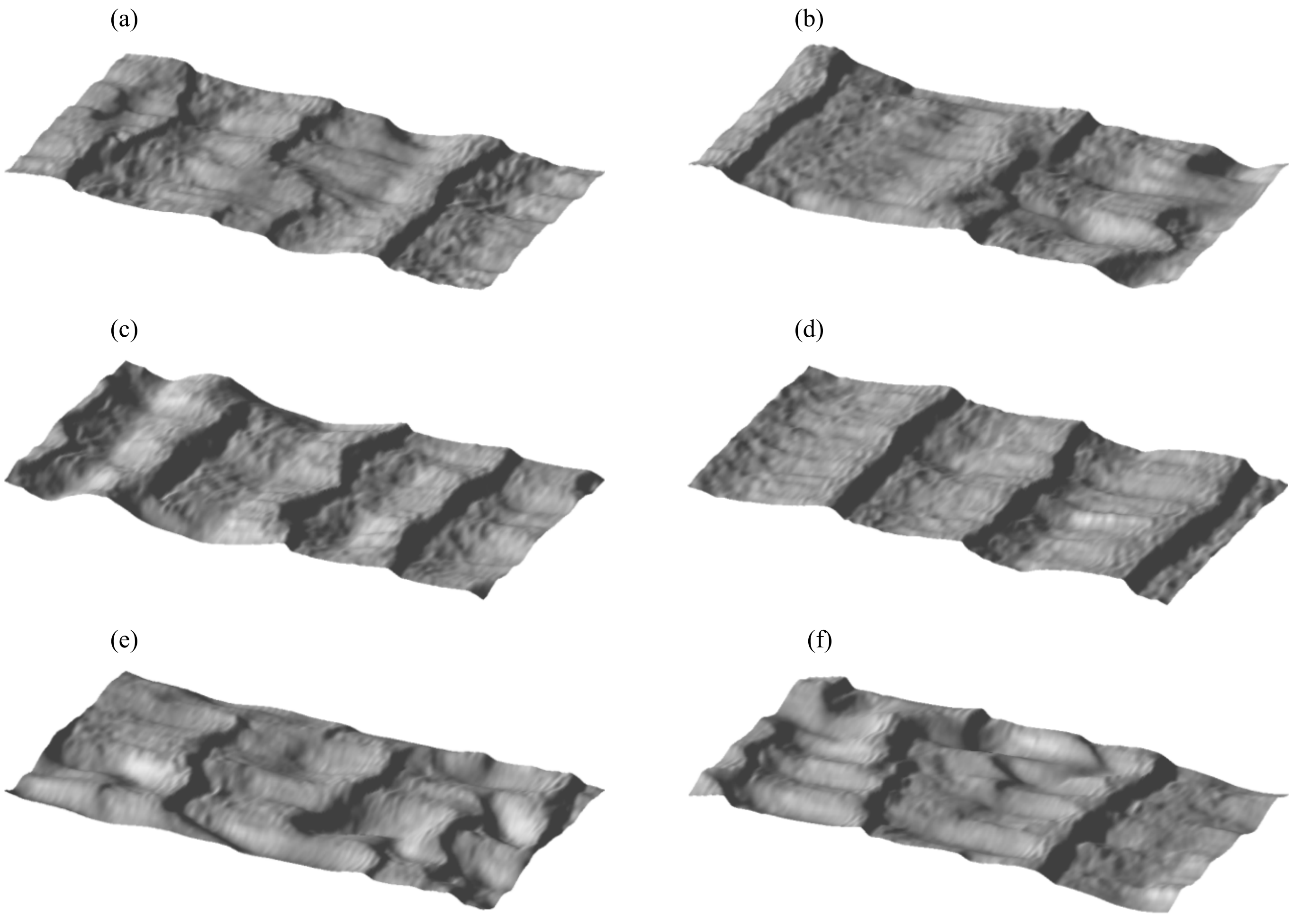

Figure 5 Simulation of dunes for different sediment diameters. (a) 100 μm, b) 120 μm c) 140 μm, d) 160 μm, e) 180 μm and f) 200 μm.

5 CONCLUSIONS AND FUTURE WORK

We are developing a model to simulate the migration of dunes in rivers. The model has three components, hydraulic model, sediment transport and morphology. The hydraulic model is based on a finite-volume method discretized on an unstructured Cartesian grid. The grid can be locally refined which makes the code more flexible and accurate. Additionally, the model takes less computational efforts than the simulation on structured grids for the same accuracy. Because of the large time and space scales of alluvial processes, the problem seems to be impossible to solve by DNS on nowadays computers. LES is an alternative technique which gives reasonably high accuracy.

To adapt the Cartesian grid to the boundaries, a Ghost-cells technique has been applied. The present Ghost-cell technique is based on the direct forcing approach that makes it possible to interpolate the velocities on the immersed boundaries and force the pressure with a cheap computational approach.

The sediment is considered as rigid spheres moving with the fluid. By considering the sediment pickup and deposition, the deformation of the bed is calculated in a physical way. This approach gives us accurate prediction of deformation of dunes and includes many physical aspects, which can be used to understand the physical behavior and the effect of turbulence on sediment transport and finally the generation of dunes.

The model is validated with an experiment which is carried out in Delft University of Technology. The results of simulation have good agreement with the experiment and it shows promisable futures for simulation of alluvial processes in more accurate and physical way.

The present simulations are expensive for applications. On basis of the results for morphology, the physical phenomena behind the dunes will be studied and empirical relations will be derived to be able to use in general and on any parts of the real field.

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