

HENRY

Hydraulic Engineering Repository

Ein Service der Bundesanstalt für Wasserbau

Conference Paper, Published Version

Sadaghiani, Mohamad R. Salehi; Witt, Karl Josef

Variability of the grain size distribution of a soil related to suffusion

Verfügbar unter/Available at: <https://hdl.handle.net/20.500.11970/99571>

Vorgeschlagene Zitierweise/Suggested citation:

Sadaghiani, Mohamad R. Salehi; Witt, Karl Josef (2011): Variability of the grain size distribution of a soil related to suffusion. In: Vogt, Norbert; Schuppener, Bernd; Straub, Daniel; Bräu, Gerhardt (Hg.): Geotechnical Safety and Risk. ISGSR 2011. Karlsruhe: Bundesanstalt für Wasserbau. S. 239-248.

Standardnutzungsbedingungen/Terms of Use:

Die Dokumente in HENRY stehen unter der Creative Commons Lizenz CC BY 4.0, sofern keine abweichenden Nutzungsbedingungen getroffen wurden. Damit ist sowohl die kommerzielle Nutzung als auch das Teilen, die Weiterbearbeitung und Speicherung erlaubt. Das Verwenden und das Bearbeiten stehen unter der Bedingung der Namensnennung. Im Einzelfall kann eine restriktivere Lizenz gelten; dann gelten abweichend von den obigen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Documents in HENRY are made available under the Creative Commons License CC BY 4.0, if no other license is applicable. Under CC BY 4.0 commercial use and sharing, remixing, transforming, and building upon the material of the work is permitted. In some cases a different, more restrictive license may apply; if applicable the terms of the restrictive license will be binding.



Variability of the grain size distribution of a soil related to suffusion

M. R. Salehi Sadaghiani & K. J. Witt
Bauhaus-Universität Weimar, Germany

ABSTRACT: A purely deterministic approach in geotechnical engineering usually is chosen despite there is enough knowledge about the uncertainties and spatial variability of the soil parameters. The geotechnical design and recommendations are based on some field observations, measurements and calculations with homogenized material and simplified models. These affect the results and important decisions in the design process. With the knowledge of the spatial variation and autocorrelation of the parameters, statistical methods can be applied in geotechnical design as well as in the risk assessment of the earthen structures. The Paper delivers statistical data and methods to handle these, which are investigated systematically in a river dike. The importance of the spatial variation and autocorrelation of the grain size distribution is demonstrated regarding suffusion phenomena.

Keywords: spatial variability, grain size distribution, internal erosion, earth structure

1 INTRODUCTION

Homogeneity has become a very important issue in geotechnical engineering in recent years. This is a fact that homogeneity is an important characteristic of geological and respectively geotechnical systems and affects a wide range of theoretical and practical issues (Saucke et al. 1999; Witt and Brauns 1984). While in the last decades major progress toward a geotechnical theory of homogeneity is being made, the concept is frequently misused because homogeneity nowadays means different things to different specialist. Quantification of homogeneity is done without a clear notation of what is exactly being quantified. A researcher must explicitly answer the question: homogeneity of what? This has not been the common practice.

To overcome these serious problems, we need a quantitative, effective definition of homogeneity. In this paper we extend the discussion of the definition and quantification of geotechnical homogeneity advocated by Witt and Brauns 1984 and Li and Reynolds 1995. We extend the operational definition of homogeneity proposed by Li and Reynolds and suggest an approach for quantifying homogeneity of a granular soil in respect of sensitivity against suffusion which is consistent with this definition and provide one example that illustrates how this scheme can be applied in practice.

The suffusion is primarily a function of the grain size distribution (geometrical criterion) and secondly depends on the hydraulic impact (hydraulic criterion). Due to the natural interactions, the soil placement or soil treatment the composition of the grains in a soil changes, particularly wide graded soils show a significant degree of variation. Though, in current internal stability design for the earthen structure, usually based on grain size ratios, the variance of the soil parameters is not taken explicitly into account. If the variation of the grain size distribution is neglected and the average grain size distribution or just upper and lower band is proved against suffusion, it can be possible that either the average parameters satisfy the suffusion criteria but in some places of the earthen structure suffusion can be occurred.

This effect of parameter variation on the probability of local failure has been investigated and will be discussed in the following. Along with the new approach for the calculation of suffusion failure probability, the degree of parameter variation as well as the limits of homogeneity in respect of grain size distribution (GSD) will be statistically examined, with help of the field inquiries.

For a probabilistic design, it is necessary to have a suitable equation of limit state to consider the appropriate parameters. Consequently, we go over the main points of the suffusion phenomena and the failure conditions.

2 SUFFUSION PHENOMENA AND CURRENT DESIGN PROCEDURE

“Suffusion” is the migration of the soil particles through the soil skeleton. The physical understanding of stability against suffusion is the ability of the voids of soil skeleton (or more precisely, of their pore constrictions) to hold its smaller particles, which are considered as mobile. With this definition, suffusion can be reduced to a filter/base problem, in which the filter is spatially created by the soils structure and the base conform with the mobile parts of the soil grains, the filling. An ideal limit state equation which defines the failure boundary should have the following general form:

$$Z = f(GSD, CSD, g, n, C_h, F_{Seepage}, \sigma_{eff}) \quad (1)$$

where GSD = grain size distribution of mobile particles of the soil, g = the shape of the grains, CSD = the effective pore constriction size distribution of the skeleton, n = porosity, C_h = degree of homogeneity, $F_{seepage}$ = seepage forces within the soil, σ_{eff} = effective stress

For investigating the worse conditions, it is assumed that sufficient hydraulic conditions exist and the effective stresses do not have any effect on the process because the mobile particles are not stressed. The effect of grain shape (g) and degree of homogeneity (C_h) are neglected. It is assumed that the grain shape has no big effect on the local process and the soil is locally homogenous ($C_h = 1$). With these assumptions the ideal limit state equation reduces to:

$$Z = f(GSD, CSD) \quad (2)$$

In this model only the GSD and CSD of the soil dominate the suffusion process. It may appear crude, but this is a rational way of using engineering simplifications as a guide to sophisticated primary models. In other words, the suffusion problem is reduced to a simple problem of geometry.

It is known that the CSD and the porosity are direct function of GSD. The GSD can be obtained by sieve analysis. However, the determination of the CSD is quite difficult. Several theoretical and experimental approaches have been tried to give a relative exact method for the determination of CSD. The different methods can be mentioned:

- Measurement of void characteristics based on saturation-capillary pressure tests (Payatakes 1973)
- Probabilistic model of randomly packed spherical filter particles (Silveira 1975)
- Measurement of distribution of void areas in specially prepared sample sections (Wittmann 1980)
- Mathematical procedure to determine the controlling constriction size (Schuler 1999, Indraratna 2007, Reboul 2008)

Each of these methods shows its limit of applicability, especially for wide and gap graded soils. Till now there is no possibility to measure the CSD in an appropriate way.

The GSD can be obtained by sieve analysis. It is a fact that the GSD-Curve of a soil has an obscure mathematical characteristic so that simple but exact criteria for stability against suffusion can not be formulated. For that reason in current practice we tend to use more simplifications such as substituting the GSD with a characteristic diameter and inserting a factor regarding to its degree of uniformity. The CSD is also replaced with a characteristic diameter of the GSD and the coefficient of uniformity. These Simplifications lead to a so called “Grain Size Criteria”, of which the limit state equation can be written in the following general form:

$$Z = f(C_u) \cdot d_{85,F} - d_{15,S} \quad (3)$$

where $d_{85,F}$ = is a diameter such that 85% of the grain’s diameter of the filling are smaller than this size, $d_{15,S}$ = is a diameter such that 15% of the grain’s diameter of the skeleton are smaller than this size, $f(C_u)$ = is function of uniformity coefficient.

The positive value of Z gives the stage of the safety of this simple system. The function of non-uniformity coefficient $f(C_u)$ has to be determined experimentally, empirically or by means of theoretical considerations. Such design criteria based on experimental investigations or empirical knowledge are still used in practice. One famous example is the Terzaghi criterion (Terzaghi and Peck 1948) with $f(C_u) = \text{const.} = 4 \sim 5$, including some factor of safety and without safety factor $C(c_u) = 9$, which is only valid for uniform base and filter combinations (In this paper it is assumed that the CSD-Curve of Filling and Skeleton are uniform).

$$Z = 9 \cdot d_{85,F} - d_{15,S} \quad (4)$$

The advantage of the method of GSD separation of the soil into two separate filling and skeleton GSD-Curves is that the produced GSDs are not wide graded soils anymore, i.e. after separation the GSD; the coefficient of uniformity will be extremely reduced. However choosing the separation point is an important part of the whole analysing procedure. The separation point gives the information about the skeleton and filling of soil (according to Kenny and Lau et al. 1985, primary fabric and loose particles).

3 LABORATORY METHODS FOR FINDING THE SKELETON AND FILLING OF A SOIL

The main objective of the laboratory tests which was accomplished in the Bauhaus-Universität Weimar is to finding the skeleton or primary fabric of the chosen soil. The local and global transport of mobile particles of a wide-gap-graded soil was measured by suffusion tests. During each test steady flow conditions with different hydraulic gradients at the embankment dams were simulated. The local structural changes of the soil were determined by measuring the changes of the pore water pressures. The measurement of eroded material (effluent) at the outlet was a primary criterion for the determination of stability of the sample. After each test, changes of the grain size distribution in 5 various layers of the soil column were measured. This allowed a conclusion about global and local structural changes of the soil.

The tests described here, were carried out with one soil sample (figure 1) with 500 mm height. The specimen was placed in 100-mm layers and compacted. Each layer was built in the test apparatus with the same specified grain size distribution which in one hand ensured the homogeneity of the soil and in the other hand, it was possible to measure the transported fractions from each layer. A reference layer of glass balls with the diameter of 16 mm were built at the base of the sample. A mesh grid were used below and above of the reference layer. The reference layer was to avoid losing fine particles of the first layer of the sample during the compaction of this layer.

Four different hydraulic gradients ($i = 0,1; 0,2; 0,4; 1,0$) were applied for each suffusion test to evaluate the suffusion stability. The weight of the entire dry material from effluent (Washout) was the value between 2,92% and 6% of the total weight of the soil specimen. It is obvious that the selected soil is to be classified as a soil in the border of stability against suffusion. These tests emphasized again that the degree of erodibility of a suffusive soil is correlated to the homogeneity of its structure. The results show that the preferred flow paths and material transport along these paths were the reason of local segregations. By these tests no global washout could be observed. Nevertheless the observation of the soil column during percolation of water showed that there were mobile particles which were not fixed in the skeleton and there were suffusive particles which were transported from the soil column. The mobile fractions were moved in direction of flow and were captured within the structure after passing through a certain path. In the other hand the suffusive particles were moved through the whole column and the skeleton of the soil was not able to capture them. The flow caused a disarrangement of the original structure into sometimes a more stable one or sometimes an unstable one dependent to the direction of the flow. This resulted in a randomly distributed micro stratification however any changes in flow conditions and direction were able to influence a remobilisation of the mobile particles.

The suffusive particles were determined directly from effluent. The biggest suffusive particle was the soil fractions between 0,125 and 0,5 mm. Moreover the mobile particles were determined by balancing the weight of the different fractions of each soil layer with total weight and the mass of washout fractions. The biggest mobile particle within the soil column was the soil fractions between 0,5 and 1 mm, and these results are in good agreement with the values reported by Binner et al. 2010. According to the grain size distribution of the soil, the fractions with diameter of 1 mm correlate with the value of d_{22} . All of accomplished suffusion tests delivered the same result with consideration of the suffusive and mobile particles. Due to the fact that the soil samples which were used for the tests was a mixture of several samples

from different locations along the dikes, it can be considered that the used grain size distribution is an average grain size distribution and it is assumed that the d_{22} conforms an average value in regarding to point of separation for all of the dike materials (figure 1).

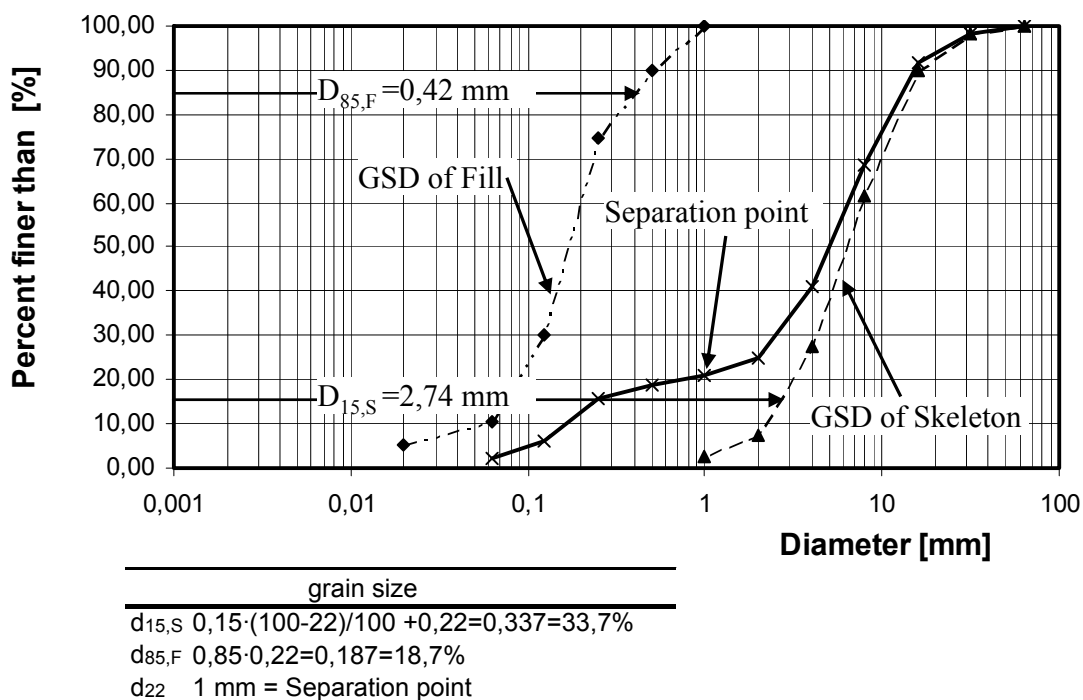


Figure 1. Grain size distribution of the soil specimen used for suffusion tests

Figure 1 shows the separation point which correlates with $d_{22} = 1$ mm and the other two important grain sizes $d_{85,F}$ which correlates with $d_{18,7}$ and respectively the $D_{15,S}$ correlates with $d_{33,7}$ of the original average grain size distribution. In the following section the descriptive statistical values of these parameters were discussed.

4 HOMOGENEITY AND PARAMETER VARIATION

The homogeneity was defined based on two components (Li and Reynolds 1995): the system property of interest and its complexity or variability. The system property can be anything that is of geotechnical interest, e.g., cohesion, permeability and so on. Complexity refers to qualitative or categorical descriptors of the property. Therefore homogeneity is complexity and/or variability of a system in the space or time. It has to be mentioned that here the time homogeneity is not considered and just the spatial homogeneity is the interest of this paper. It is also obvious that the homogeneity is a function of scale (Witt & Brauns 1985). Li and Reynolds 1995 proposed two factors which are called grain and extent, are the primary scaling factors that affect complexity or variability. Grain is the finest resolution of the data (e.g. the volume of the soil sample) and extent is the area encompassed by a study. The observational scale (i.e. grain or extent) is dependent on the sampling scheme used, which in turn is determined by the nature of the phenomenon and the research objective. The observed data and the treatment of the data determine what kind of homogeneity may be measured. From a data analysis viewpoint, data treatment or resampling (e.g. sieve analysing of the soil or hydrometer analyse) may modify grain or extent or both and therefore plays a role in quantification of homogeneity.

As far as the problem of suffusion is concerned, homogeneity is mainly a matter of similarity in regard to the parameter of the soil which has an influence on the vulnerability of the soil against suffusion. A part of a soil in a dike or natural soil may be considered as homogenous, if the variation in gradation from a place to place is within a certain limit which is still to be defined for each case.

In order to get more insight into the actual conditions under practical circumstances, a systematic sampling on Rhein River dikes was performed. The mean GSD band of all samples has been shown in Figure 2. For this material the theoretical sample size (mass) was equal to ca. 15 kg (Witt 1984). In the study area, 158 samples were taken with high resolution within a grid of different distances from 25 cm to 8 m. According to the homogeneity definition, with this kind of sampling the sampling effect is reduced and the evaluation gives a better answer for homogenous distances. On the basis of the 158 samples, the

construction of different groups was performed. For every group, we plotted a histogram and calculated the frequency distribution for the relevant characteristic grain size such as $d_{18,7}$ and $d_{33,7}$.

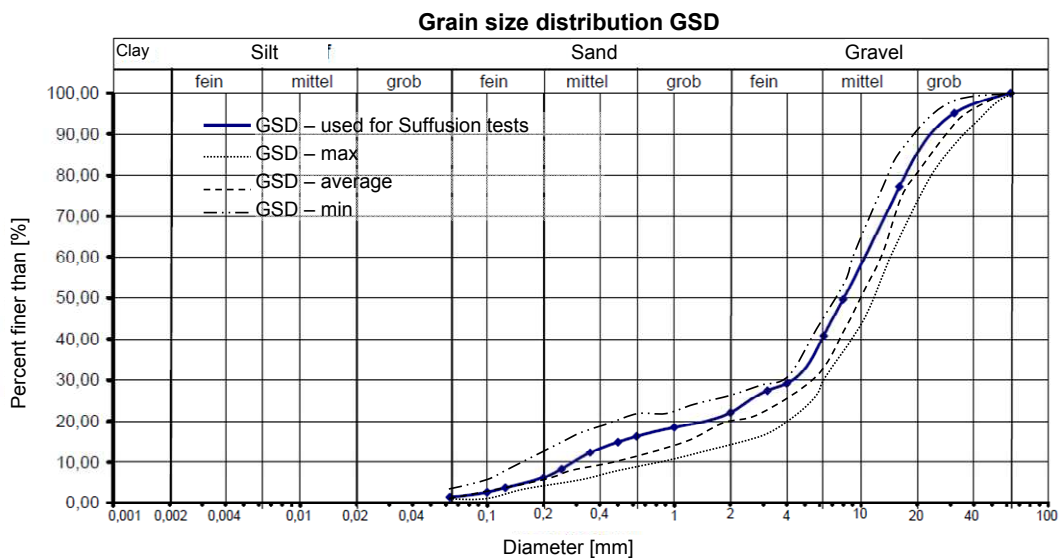


Figure 2. Band of grain size distribution of the soil specimens in the study area

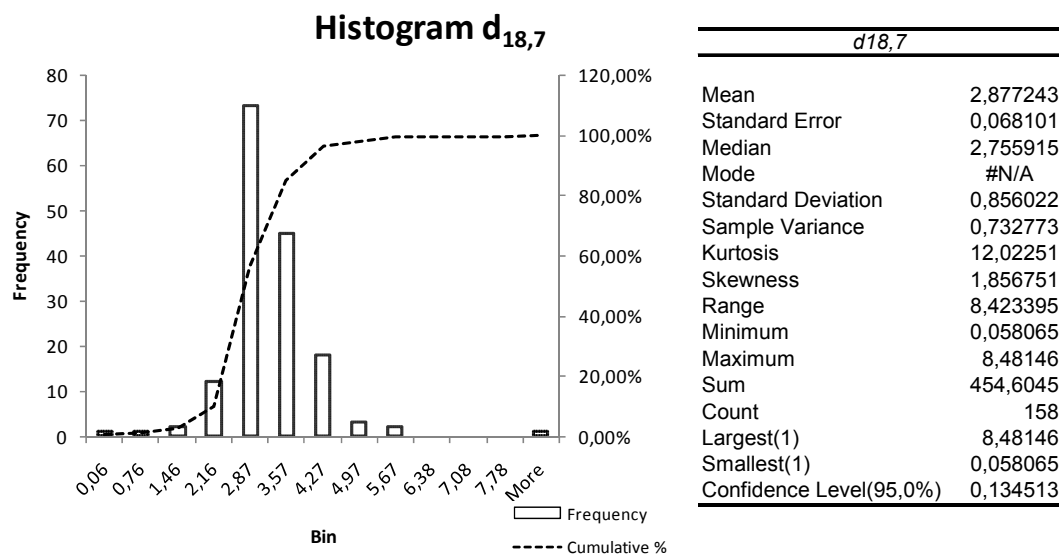


Figure 3. Histograms and frequency distributions of $d_{18,7}$ for 158 samples

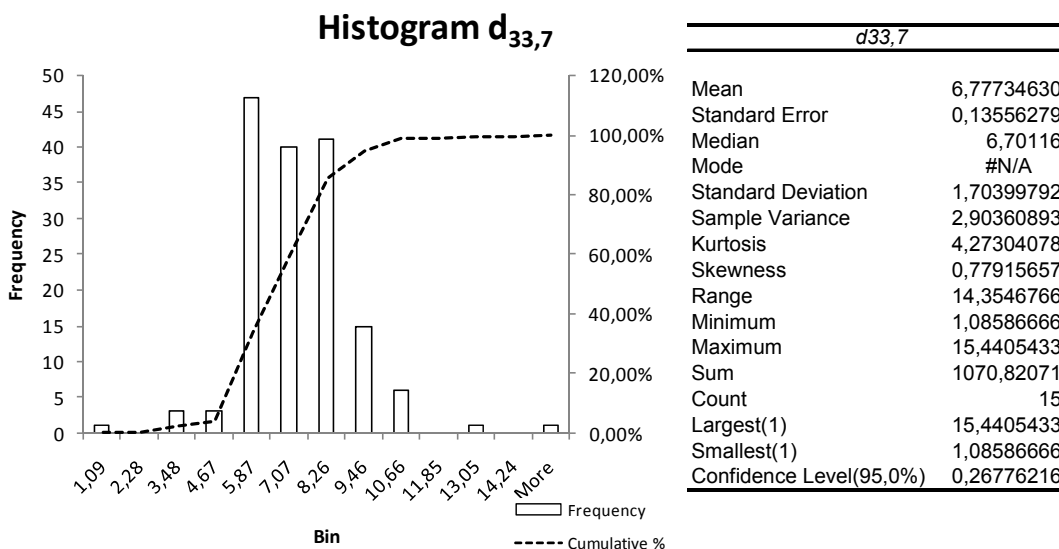


Figure 4. Histograms and frequency distributions of $d_{33,7}$ for 158 samples

In figure 3, it is interesting to see that the variability of $d_{18,7}$ is larger than that of $d_{33,7}$. This result is maybe typical for such a soil which was a mixture of the sand and gravel with meadow silt. It could already be seen from the figure 2. which shows grain size distribution band with a smaller upper grain sizes and a broad lower grain sizes of $d_{33,7}$. The normal transformation shows a very good fit of both $d_{18,7}$ and $d_{33,7}$. The corresponding statistical values can be taken from figure 4. The coefficient of variation COV ($d_{18,7}$) is equal to 29,75 % and this value is for COV ($d_{33,7}$) equal to 25,14%. A precise quantitative definition of homogeneity seems difficult. But a coefficient of variation of 15% may be allowable with considering that in the probabilistic, normally distributed random variables with a COV < 10% are regarded as deterministic value. In fact soil with a COV < 15% can be seen as a homogenous material. However the definition of statistically homogenous seems very appropriate for the soil materials.

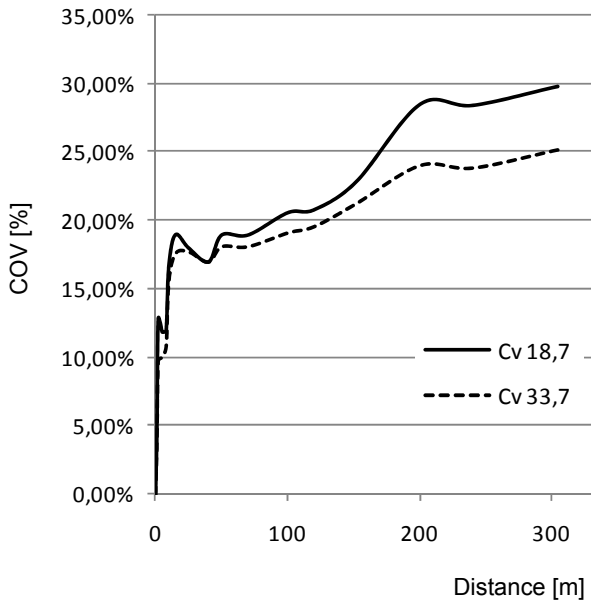


Figure 5. Coefficient of variation in dependence on sample distance

As it is to be expected, the coefficient of variation in figure 5 increases with sample distance. If a distance of 9,3 m for $d_{18,7}$ is chosen the diagram gives a result of 15% for coefficient of variation COV .

This clearly shows that in this area the variation of the $d_{18,7}$ is relatively small and in each 9,3 m there is a statistically homogenous distribution of $d_{18,7}$ and respectively for $d_{33,7}$ it will be the homogenous area of 9,8 m.

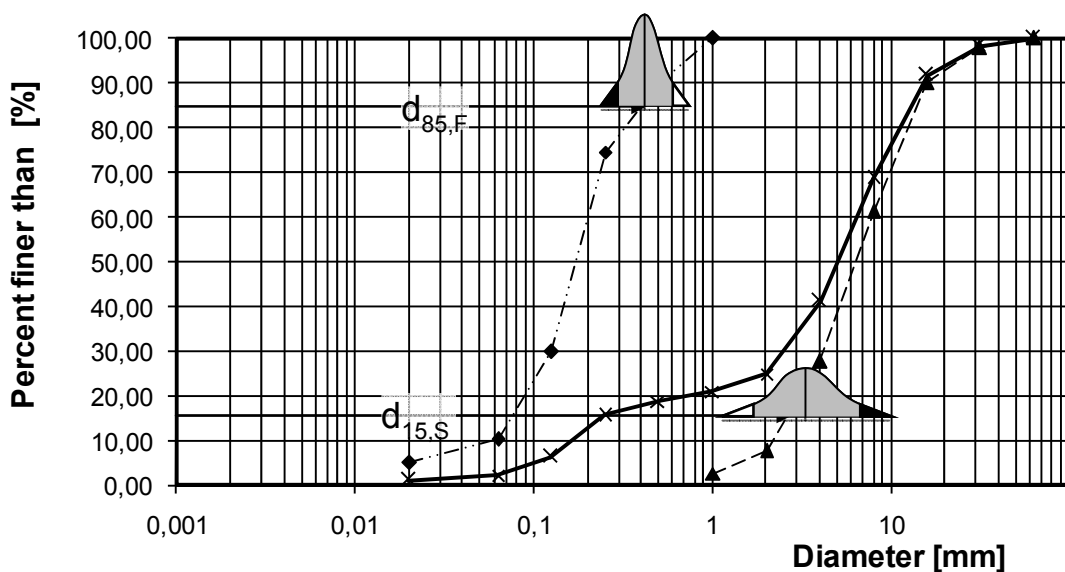


Figure 6. The variation of the $d_{18,7}$ and $d_{33,7}$ of the original grain size distribution

Figure 6 shows the normal distribution of the related grain sizes for the equation 4. Figure 7 gives some other information about mean value and standard deviation of this distribution. The shaded area contains 95% of the area and extends from 3,37 to 10,19 mm for $d_{18,7}$ and the shaded area of the normal distribu-

tion of the $d_{33,7}$ extends from 1,16 to 4,59 mm. For all normal distributions, 95% of the area is within 1.96 standard deviations of the mean. If the upper band of 95 percentile of $d_{33,7}$ and the lower band of the 95 percentile of $d_{18,7}$ are considered, the Z (equation 4) can be calculated as:

$$Z = 9 \cdot 1,16 - 10,19 = 0,25 \approx 0 \quad (4)$$

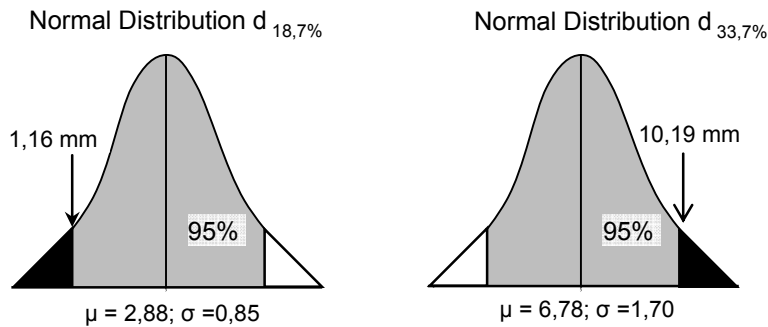


Figure 7. The normal distribution of the grain sizes $d_{18,7}$ and $d_{33,7}$ – 95% of the area is within 1,96 standard deviations of the average value.

This can be interpreted as 5% of the whole samples with mentioned assumptions are suspected to suffusion and the rest of samples seems to be stable (without any safety factor). From this point of view easily the failure probability of the dikes and dams can be evaluated. In the following section some aspect of variation in grain size distribution according to satiability against suffusion are discussed.

5 CONSIDERATION OF GRAIN SIZE VARIATION FOR SUFFUSION CRITERIA

A reliable suffusion criterion has to take into account the random nature of the relevant soil parameters. For getting a reliable criterion, variability of the parameters must be induced into the limit state equation. Here the writers try to induce the variability into the geometrical grain size criterion (eq. 3).

In equation 3, the distribution of Z has to be find with consideration of the relevant parameters (C_u , d_{15} and d_{85}), knowing that the failure is defined by $Z = 0$. If the suffusion has been occurred, the soil probability failure is equal to the probability of the Conditions which Z is less than zero. For simplification, as it was mentioned in section 2, it is assumed that C_u of the separated GSD shows small range of variation so that we can take it as a deterministic function. The random variables in the limit state equation are then: $9 \cdot d_{85}$ (resisting part or R) and d_{15} (active part or S). If we substitute R for $9 \cdot d_{85}$ and S for d_{15} , the equation takes on the well known form used in probabilistic failure considerations:

$$Z = R - S \quad (5)$$

The probability of failure

$$P_f = P(Z < 0) = P(R - S < 0) = P(R / S < 1) \quad (6)$$

Then is (Freundthal et al. 1966)

$$P_f = \int_0^{\infty} F_R(x) \cdot f_s(x) \cdot dx \quad (7)$$

where F_R = cumulative distribution function of $C_u \cdot d_{85}$, f_s = cumulative distribution function of d_{15}

As shown in section 2 the distribution of both characteristic diameters can be well fitted with normal distribution or log-normal distribution, and in the special case the solution of equation 7 is according to Witt et al. 1993:

$$P_f = P(R/S < 1) = \Phi\left(\frac{\ln \frac{\tilde{r}}{\tilde{s}}}{\sqrt{\sigma_{\ln R}^2 + \sigma_{\ln S}^2}}\right) = \Phi(-\beta) \quad (8)$$

where \tilde{r} = median value of the R-distribution, \tilde{s} = median value of the S-distribution, $\sigma_{\ln R}$ = Standard deviation of $\ln R$, $\sigma_{\ln S}$ = Standard deviation of $\ln S$, $\Phi(-\beta)$ = Standardized normal distribution function (Gaussian-distribution function), β = safety index

By substituting the characteristic diameters of the grain size distribution and using the coefficient of variation of their frequency distributions, the probability failure can be defined as.

$$P_f = P(R/S < 1) = \Phi\left(\frac{\ln \frac{C_u \cdot \tilde{d}_{85,F}}{\tilde{d}_{15,S}}}{\sqrt{\sigma_{\ln d_{15,S}}^2 + \sigma_{\ln d_{85,F}}^2}}\right) = \Phi(-\beta) \quad (9)$$

The equation 9 gives the correlation between the failure probability of the soil against suffusion, the central safety factor $\frac{C_u \cdot \tilde{d}_{85,F}}{\tilde{d}_{15,S}}$ and the standard deviation of $\ln(d_{15,S})$ and $\ln(d_{85,F})$.

The Overall failure probability of an embankment dam, in regard to internal stability, is on the one hand a function of the parameter variation of the materials involved, and on the other, of the spatial arrangement of the constituting elements. In designing of the embankment dam, this fact is taken into account by providing adequate minimum dimensions of a zone depending on the segregation of the material used. Nevertheless the risk related to the parameter variation in embankment materials can not be quantified up to now and design criteria are mainly empirical. Unfortunately there is a big lack of information on homogeneity, autocorrelation of parameters and relevant sample size.

6 DISCUSSION

The challenge in assessing the erosive processes is to determine or to estimate the relevant parameters, i.e. the pore constriction size of wide graded soils under consideration of special uncertainty due to heterogeneity as well as the effective size of mobile particles which are able to block the pores of the soil skeleton to create stable conditions. If these relations cannot be determined with an appropriate accuracy or if there are some soils which are geometrically prone to suffusion and the limit range of the hydraulic impact have to be defined, laboratory tests as described above should be carried out to allow a quantitative assessment of the risk of suffusion.

Minor differences of the particle size composition affect whether a soil is internally stable. It is recommended that for important structures laboratory tests should be carried out on the soils which are tested in the marginal areas of limit state equations or other criteria. The method suggested here is just for the used grain size distribution band and for the other grain size distribution are not be examined by the mentioned method.

For using this approach we need to separate a soil into base-filter grain size. The separation method was described and the experimental way of determining this separation point is a very important topic to discuss. Also the statistical distribution of the parameter has been investigated experimentally by systematic sampling on an embankment dam. It was indicated, that the relevant $d_{15,S}$ and $d_{85,F}$ parameters follow a log normal or normal distribution. The coefficient of variation shows an effect, which are depending on the point of the sampling and its distance. Theoretical considerations lead to a distance for considering of a homogenous area in regard to each parameter.

Considering the relative variation of the separation point, another probability of failure could be achieved with this method. With a systematic sampling of each site with different dike profiles and con-

stitution materials, it is necessary to make some tests to determine the separation point for calculating of failure probability.

7 CONCLUSIONS

This paper deals with the effect of spatial variability of the grain size distribution on internal stability of a gap graded soil. The physical background of suffusion is discussed. The grain size criteria which can be used in practical design are described as simplifications of a more general and more complex function. Such criteria describe both resisting and active parts by a characteristic grain size (d_{15} or d_{85}) and a function considering the uniformity of the soil. This equation is very suitable for a probabilistic approach of a design for reliability against suffusion.

The procedure for predicting and determining the mobile and suffusive particles of sand gravel soils with silty fines based on particle size distribution are proposed. It is shown that the degree of erodibility of a wide-gap-graded soil is close correlated to the homogeneity of its structure and material transport along the preferred paths which often result in local segregation. Even with a very high degree of homogeneity different results were obtained. The observations of the soil column during percolation of water have shown that there are mobile particles which are not fixed in the skeleton and there are suffusive particles which were transported through the soil column.

For using such a criterion we need to separate a soil into base-filter grain size. The separation method was described and the experimental way of determining the separation point was discussed. Also the statistical distribution of the parameter has been investigated experimentally by systematic sampling on an embankment dam. It has been indicated, that the relevant d_{15} and d_{85} parameters follow a normal or log normal distribution. The coefficient of variation shows an effect, which is dependent on the point of the sampling and its distance. Theoretical considerations lead to a distance for considering of a homogenous area in regard to each parameter.

With taking the random nature of the grain size parameters regarding to segregation problem into account the limit state equation allows calculating the failure probability. Thus the relation between the central safety factor, parameter variation and failure probability could be established. The theoretical approaches showed that it can be very dangerous to use the current suffusion criteria, calculating with the average or lower grain size distribution and neglecting the parameter variation, or even underestimate the degree of variation without enough samples.

This contribution has been prepared in order to excite other engineers to investigate dam fills as systematically as possible and publish test data which can form a better basis for the application of the geo-statistical methods to quantify the probability of erosion and failure in embankment dams.

8 ACKNOWLEDGMENT

The writers started to investigate the factors affecting internal erosion processes in embankment dams and their foundations as part of a research project funded by German Waterways and Shipping Office Freiburg (WSA). The authors wish to thank the WSA-Freiburg and BAW-Karlsruhe for the financial and technical supports.

REFERENCES

- Binner, R., Homberg, U., Prohaska, S., Kalbe, U., Witt, K.J. 2010. Identification of descriptive parameters of the soil pore structure using experiments and CT. Proc. of the fifth international conference on scour and erosion. Geotechnical special publication No. 210, November 7-10, 2010, San Francisco, California, USA, 397-407.
- Freudenthal, A.M., Gerrelts, J.M. and Shinozuka, M. 1966. The analysis of structural safety. Journal of the structural Division. ASCE, Vol.92, No. St 1.
- Indraratna, B., Raut, A.K., Khabbaz, H. 2007. Constriction-Based retention criterion for granular filter design. Journal of geotechnical and geoenvironmental engineering, ASCE, 266-276, March 2007.
- Kenny T. C., Lau D., 1985. Internal stability of granular filters. Canadian geotechnical journal 22, 215-225.
- Li H., Reynolds J.F., 1995. On definition and quantification of heterogeneity. Blackwell Publishing on behalf of Nordic Society Oikos, Vol.73, No. 2 (Jun.,1995), 280-284.
- Payatakes, C.A., Tien, C. and Turian, R.M. 1973. A new model for granular porous media, Part 1, AICHE Journal, Vol. 19 No. 1.

- Reboul, N. 2008. Transport de particules dans les milieux granular. Application à l'érosion interne, L'Ecole Centrale de Lyon, Diss., Nov. 2008.
- Saucke, U., Brauns J., Schuler U. 1999. Zur Entmischungsneigung körniger Schüttstoffe. Geotechnik Zeitschrift, V22, Nr.4, 259-268
- Schuler, U. 1997. Bemessung von Erdstoff-Filtern unter besonderer Berücksichtigung der Parameterstreuung. Veröffentlichung des Inst. F. Bodenmech. u. Felsmech., Universität Karlsruhe, Heft 143.
- Silveira, A. Peixoto Jr. T.L. and Nogueira, J.B. 1975. On void-size distribution of granular materials. Proc. 5th Panam. Conference Soil mechanic Fdn. Engng, Buenos Aires, Vol.3.
- Terzaghi, K. and Peck, R.B. 1948. Soil mechanics in engineering practice. John Wiley & Sons Inc., New York.
- Wittman, L. 1980. Filtrations- und Transportphänomene in porösen Medien. Veröffentlichung des Inst. F. Bodenmech. u. Felsmech., Universität Karlsruhe, Heft 86.
- Witt, K.J., Brauns, J. 1984. The influence of parameter variation on the reliability of the filters. Proc. of the international conference on safety of dams, University of Coimbra.
- Witt, K.J. 1993. Reliability study of granular filters. Filters in Geotechnical and hydraulic engineering , Brauns, Heibaum & Schuler. Balkema, Rotherdam, The Netherlands, 35-41.