

Ein Service der Bundesanstalt für Wasserbau

Conference Paper, Published Version

Petrie, J.; Diplas, Panayiotis; Nam, S.; Gutierrez, M. S. Local boundary shear stress estimates from velocity profiles measured with an ADCP

Verfügbar unter/Available at: https://hdl.handle.net/20.500.11970/99838

Vorgeschlagene Zitierweise/Suggested citation:

Petrie, J.; Diplas, Panayiotis; Nam, S.; Gutierrez, M. S. (2010): Local boundary shear stress estimates from velocity profiles measured with an ADCP. In: Dittrich, Andreas; Koll, Katinka; Aberle, Jochen; Geisenhainer, Peter (Hg.): River Flow 2010. Karlsruhe: Bundesanstalt für Wasserbau. S. 1749-1758.

#### Standardnutzungsbedingungen/Terms of Use:

Die Dokumente in HENRY stehen unter der Creative Commons Lizenz CC BY 4.0, sofern keine abweichenden Nutzungsbedingungen getroffen wurden. Damit ist sowohl die kommerzielle Nutzung als auch das Teilen, die Weiterbearbeitung und Speicherung erlaubt. Das Verwenden und das Bearbeiten stehen unter der Bedingung der Namensnennung. Im Einzelfall kann eine restriktivere Lizenz gelten; dann gelten abweichend von den obigen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Documents in HENRY are made available under the Creative Commons License CC BY 4.0, if no other license is applicable. Under CC BY 4.0 commercial use and sharing, remixing, transforming, and building upon the material of the work is permitted. In some cases a different, more restrictive license may apply; if applicable the terms of the restrictive license will be binding.



# Local boundary shear stress estimates from velocity profiles measured with an ADCP

## J. Petrie, P. Diplas & S. Nam

Baker Environmental Hydraulics Laboratory, Department of Civil & Environmental Engineering, Virginia Tech, USA

## M. S. Gutierrez

Division of Engineering, Colorado School of Mines, USA

ABSTRACT: The acoustic Doppler current profiler (ADCP) has become an important tool in the study of river processes. When measurements are obtained at a fixed location within the river channel, time-averaged velocity profiles can be calculated. These profiles have the potential to quantify flow properties such as secondary currents and boundary shear stress. Velocity profiles from ADCP measurements obtained on the lower Roanoke River in the USA are used to estimate local mean boundary shear stress. The procedure combines the well known log-law with visually establishing the region within the flow depth where this law is valid. Additionally, methods are presented to (i) determine if movement of the ADCP adversely affects the measured velocity profile, (ii) test whether the recorded data is stationary, and (iii) calculate the depth-averaged velocity.

Keywords: Acoustic techniques, Boundary shear, Field tests, Stationary processes, Velocity profile

## 1 INTRODUCTION

Field measurements of river velocity can provide a valuable contribution to understanding morphological processes, contaminant transport, and stream ecology. The acoustic Doppler current profiler (ADCP) is used increasingly in river-related studies to measure velocity and determine flow rate (Simpson 2001), turbulence characteristics (e.g. Stacey et al. 1999, Nystrom et al. 2007), boundary shear stress (Sime et al. 2007), and sediment transport (Rennie & Millar 2004). Additionally, the high spatial resolution data from ADCPs may provide a useful tool for calibrating and validating computational fluid dynamics models. This study presents ADCP measurements from the lower Roanoke River, a regulated river in eastern North Carolina, USA and describes a procedure for determining local boundary shear stress. Fixed-vessel measurements (Muste et al. 2004) were obtained at a location within a meander bend for two flow rates, one close to the mean annual flow (flow rate,  $Q = 220 \text{ m}^3 \text{ s}^{-1}$ ) and the other at near bankfull conditions ( $Q = 565 \text{ m}^3 \text{ s}^{-1}$ ). Maintaining a fixed location within the river for the entire measurement duration presented a challenge. The effect of the ADCP motion on the measured velocity profiles is assessed.

Velocity profiles are often used as an indirect method to determine mean boundary shear stress in natural rivers (Wilcock 1996). While several methods are available to determine the time-averaged local boundary shear stress (e.g. Biron et al. 2004, Dietrich & Whiting 1989), this study employs the theoretical log-law, given as:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \left( \frac{z}{z_0} \right) \tag{1}$$

where  $u = \text{velocity}, u_* = \text{shear velocity} (u_* =$  $(\tau_0/\rho)^{0.5}$ , where  $\tau_0$  = boundary shear stress and  $\rho$ = fluid density),  $\kappa$  = von Karman's constant ( $\kappa$  = (0.40), z = position perpendicular to the channelbed, and  $z_0$  = roughness height. Following the approach of Raupach et al. (1991), the perpendicular position above the bed is defined as z = Z + d, where Z = position above the origin as defined by the top of the roughness elements and d = zerodisplacement plane. When a measured velocity profile is available, a least squares error approach can be used to fit a linear equation to the profile of u vs. ln(z). This approach has two primary advantages: (i) no knowledge of the roughness height is required to determine the shear velocity and (ii) a measure of the goodness of fit for the data is available through the coefficient of determination, also known as the R<sup>2</sup>-value. The least square error approach has been used both in the laboratory (e.g. Dancey & Diplas 2008) and the field (e.g. Papanicolaou & Hilldale 2002, Stone & Hotchkiss 2007).

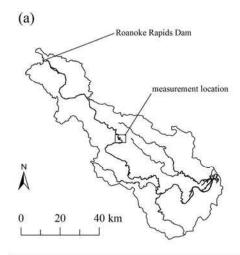
#### 2 STUDY SITE

The lower Roanoke River is a regulated river located on the coastal plain of eastern North Carolina, USA. The study site, shown in Figure 1, is approximately 77 river kilometers downstream of the Roanoke Rapids Dam. Due to a lack of major tributaries, the flow rate is largely controlled by dam releases. The ADCP measurements were obtained near the channel centerline at the bend apex (Fig. 1b) on May 28 and June 14, 2009. The dam releases were constant for at least a week prior to each measurement date as necessitated by different operational modes. The field work in May occurred during spawning operations, which require steady flow releases for a duration of at least one week. This operational mode resulted in a flow rate close to the mean annual flow  $(Q = 220 \text{ m}^3 \text{ s}^-)$ 1) for several weeks. The June field work followed a period of heavy rainfall in the watershed surrounding the reservoir resulting in flood control operations which produced a sustained release of  $Q = 565 \text{ m}^3 \text{ s}^{-1}$ . The flood control releases generated bankfull conditions at the measurement site. These flow rates resulted in flow depths at the measurement location of 5.84 m ( $Q = 220 \text{ m}^3 \text{ s}^{-1}$ ) and 9.21 m ( $Q = 565 \text{ m}^3 \text{ s}^{-1}$ ) as determined by the ADCP.

#### 3 BACKGROUND

A brief review of relevant ADCP operational principles is provided here, while more complete coverage can be found in Simpson (2001). Velocity is measured in each of four beams emitted from the ADCP and the results are averaged at discrete intervals throughout the flow depth known as either bins or depth cells. For the current measurements, a bin size of 0.25 m was used. Near-bed and near-surface measurements are not possible with an ADCP. A variety of settings are available for the ADCP depending on flow characteristics such as depth and velocity. For the measurements reported here, water mode 12, a high ping rate mode, was used with 20 sub-pings sent 50 milliseconds apart, then averaged together to create a single measurement. Further details on the water modes available for RDI ADCPs can be found in Simpson (2001). The resulting sampling frequency was approximately 10 Hz. Measured velocities include contributions from both the flowing river

water and the motion of the boat. To isolate the river velocity, the ADCP software can subtract the boat velocity from the total measured velocity using a velocity reference which defines the boat motion. This velocity reference can be found using GPS data or bottom-tracking. Bottom-tracking is a feature available with many ADCP models that determines the boat velocity relative to the channel bottom. Additionally, the velocity reference may be set to none, which retains the original measured velocities.



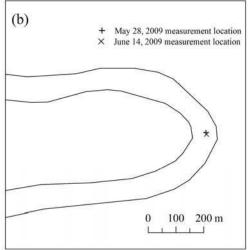


Figure 1. (a) The Roanoke River watershed below the Roanoke Rapids Dam. (b) The measurement locations on the lower Roanoke River.

The ADCP velocity output is defined in an earth coordinate system with components in the north, east, and vertical directions. These components can be transformed using the average flow direction determined by an internal compass to a curvilinear river coordinate system with components in the streamwise (tangential), spanwise (normal), and vertical directions, shown in Figure 2. The streamwise component represents the primary flow direction, while the spanwise component represents the flow across the channel, such as secondary circulation.

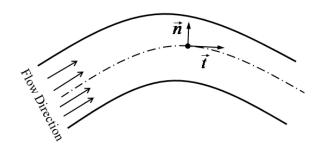


Figure 2. Curvilinear river coordinate system showing streamwise (tangential) and spanwise (normal) directions. The vertical axis is directed outward from the page.

## 4 MEASUREMENT PROCEDURE

A 1200 kHz Workhorse Rio Grande ADCP manufactured by Teledyne RD Instruments (Poway, CA) and a Trimble DSM 232 GPS (Sunnyvale, CA) were mounted to a Riverboat tethered boat (Oceanscience, Oceanside, CA). The tethered boat was attached using rope to a motor boat (length = 6 m) for all measurements. The ADCP and GPS data were recorded with WinRiver II software (Teledyne RD Instruments). The GPS data is used to determine the boat position only and has no bearing on the velocity measured by the ADCP. The horizontal accuracy of the GPS is approximately 1.0 m. The measurement locations were recorded using HYPACK LITE (HYPACK, Inc, Middletown, CT) so that the locations from May could be revisited in June.

The boat was secured within the river channel using anchors at the bow (front) and port-side stern (left-side rear) similar to the approach of Stacey et al. (1999). The typical procedure to secure the boat involved driving upstream some distance of the desired measurement point and releasing the bow anchor. The boat was then slowly steered to the measurement location where the stern anchor was released and both anchor lines were secured. The bow anchor required a rope with a length approximately three times the flow depth. The total set up time ranged from 10 minutes to one hour depending on flow conditions and the desired location within the channel. The tethered boat was placed near the middle of the motor boat which prevented the anchor lines from intersecting with the ADCP beams. Once the boat was stabilized in the measurement location, ADCP velocity measurements were recorded for approximately 20 minutes. During the measurement period, the boat motor was turned off and movement by the technicians was minimized. This approach to fixing the boat within the channel is preferred to mooring, due to the excessive wear on the motor that mooring may cause.

The sample record length of 20 minutes was selected to be conservative. Measurement durations as small as 6 minutes have been shown to be sufficient for determining mean quantities in larger rivers than the lower Roanoke River (Muste et al. 2004). To demonstrate that 20 minutes is a sufficient sample record length, Taylor's hypothesis may be used to convert between length and time scales (Soulsby 1980):

$$T = \frac{H}{U} \tag{2}$$

where T = integral time scale, H = flow depth, and U = depth-averaged velocity. While this approximation is not strictly correct for all flows, it provides a useful tool prior to performing ADCP measurements. Applying Taylor's hypothesis results in integral time scales of 8.52 s (Q = 220 m<sup>3</sup> s<sup>-1</sup>) and 9.30 s (Q = 565 m<sup>3</sup> s<sup>-1</sup>) for the conditions encountered on the lower Roanoke River. The sample record length is then at least 130 times the integral time scale, which is judged to be sufficient.

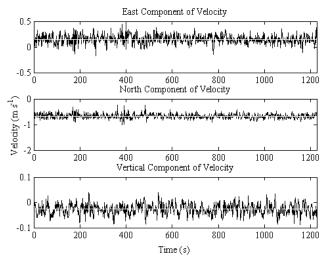


Figure 3. Times series of velocities in earth coordinates for bin 10 (z/H = 0.49) from May 28, 2009 ( $Q = 220 \text{ m}^3 \text{ s}^{-1}$ ). The mean of each component is shown with a white line.

## 5 RESULTS

The data output from WinRiver II was analyzed using codes developed in MATLAB© at the Baker Environmental Hydraulics Laboratory. All velocity, location, and depth measurements marked as "bad" by WinRiver II were removed. No additional smoothing or filtering of the data was performed. The preliminary data analysis involves establishing that the boat motion during each measurement was not significant so that the results may be considered a fixed-vessel measurement and that the measured velocity time series are stationary.

## 5.1 Establishing Stationarity

A time series may be considered stationary when the mean value and autocorrelation function are constant (Bendat & Piersol 1986). As many analysis techniques for turbulent flow data are limited to stationary data (Tennekes & Lumley 1972), whether or not the data is stationary should be established at the outset. The approach used here follows the previous work presented in Soulsby (1980) which employs the run test, a nonparametric test of statistical independence (Bendat & Piersol 1986). In summary, the 20 minute sample record is divided into subsamples, each with a duration of 30 seconds, and the run test is performed on the mean and variance of the subsamples. This procedure is performed on each velocity time series, i.e. north, east, and vertical components, for all bins in the profile. The run tests failed to reject the hypothesis of stationarity for all velocity records in both profiles at the  $\alpha = 0.05$  level of significance, indicating that the data is stationary. Visual confirmation of stationarity can be seen in the sample velocity record provided in Figure 3.

## 5.2 Depth-averaged Velocity

The depth-averaged velocity, *U*, is used to scale the velocity profile measurements in the following sections. If the velocity profile can be described as a function of the height above the channel bottom, then:

$$U = \frac{1}{H} \int_{d}^{H} u(z) dz \tag{3}$$

where U = depth-averaged velocity, u(z) = streamwise velocity profile as a function of height above the channel bed, and H = flow depth. The flow depth is determined by averaging the sample records for depth measured by the four ADCP beams. Stone & Hotchkiss (2007) applied this approach by fitting a logarithmic profile to velocity profiles measured with an ADCP. However, if the entire flow region cannot be described by a single function, determining U from ADCP profiles is not straightforward due to the lack of velocity measurements in the near-bed and near-surface regions of the flow depth.

To overcome this issue, the flow depth is divided into the three regions illustrated in Figure 4: (1) the near-bed region, (2) the middle region containing velocity measurements, and (3) the near-surface region. Equation (3) then becomes:

$$U = \frac{1}{H} \left[ \int_{R_1} u_1(z) dz + \int_{R_2} u_2(z) dz + \int_{R_3} u_3(z) dz \right]$$
 (4)

where R1 and  $u_1(z)$  correspond to Region 1 and the streamwise velocity profile in Region 1, respectively.

The integral for Region 2 is calculated by numerically integrating the velocity measurements from the ADCP. Separate logarithmic profiles were then fit to Regions 1 and 3 using several velocities measured by the ADCP adjacent to each region. The logarithmic equations were then integrated over the appropriate region of the flow depth. For example, the near-surface logarithmic equation is integrated from the height of the top of the bin closest to the free surface to the free surface height above the channel bed.

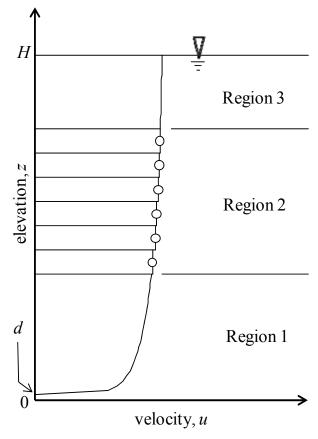
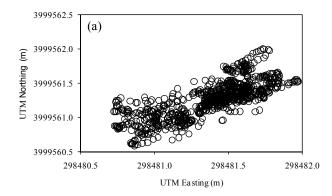


Figure 4. Demonstration sketch for depth-averaged velocity calculations with measured ADCP velocities shown with open circles (not to scale).

## 5.3 Effect of ADCP Motion on Velocity Profiles

During the duration of each measurement, some ADCP movement is unavoidable as demonstrated by the GPS measurements provided in Figure 5. As one would expect, the bankfull conditions (Fig. 5b) produce increased ADCP motion resulting in a range of 4.28 m in the east direction and 6.51 m in the north direction. The range for the mean annual flow rate (Fig. 5a) is 1.25 m in the east direction and 1.42 m in the north direction. As noted previously, the measurement location in May was recorded in HYPACK so that the same location could be revisited. In June, it was not

possible to return to the exact location due to the strong current within the river at the bankfull flow rate. The mean locations of the two different measurements are separated by 5.9 m.



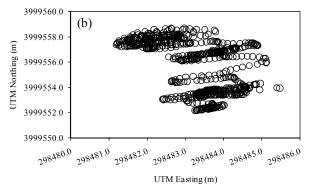


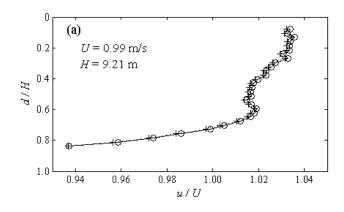
Figure 5. ADCP locations recorded with GPS during velocity measurements for (a) May 28, 2009 ( $Q = \text{m}^3 \text{ s}^{-1}$ ) and (b) June 14, 2009 ( $Q = 565 \text{ m}^3 \text{ s}^{-1}$ ).

To determine if the boat motion had an impact on the measured mean velocity profiles, the velocity profiles were compared using different velocity references. As discussed above, when the GPS velocity reference is used, the boat motion as determined by the GPS is removed from the total velocities measured by the ADCP, leaving only the river water velocity. When no velocity reference is used, the ADCP velocities are not corrected for boat motion. Therefore if the velocity profiles using none and GPS as the velocity references are in agreement, the measurement may be consider a true fixed-vessel measurement. The maximum pitch and roll measured by the ADCP system was 1.46° which has an insignificant effect on horizontal velocities (Simpson 2001). Figure 6 provides the streamwise and spanwise velocity profiles measured in June with the mean velocity for each bin, u, normalized by the depth-averaged velocity, U, and the bin depth, d, normalized by the total flow depth, H. It can be seen that the streamwise profiles are in good agreement with a maximum percent difference at any bin of 0.56%. The spanwise component also show good visual agreement but result in a maximum percent difference of 14%. However, this maximum difference occurs for measured velocities on the order of 0.005 m s<sup>-1</sup>

which is well below the reliable range for the ADCP. The mean percent difference for the entire spanwise profile is less than 2%. The results from May demonstrate similar agreement and the profiles are not shown here. Based on the comparison of the two velocity references it can be concluded that the measurements represent fixed-velocity measurements where the ADCP motion did not affect the measured velocities. The remainder of the reported velocities will use GPS for the velocity reference.

### 5.4 Boundary Shear Stress

The log-law is likely only valid for the bottom 20% of the flow depth (Nezu & Nakagawa 1993), however a logarithmic distribution is often assumed to approximate the velocity profile for the entire flow depth (Bathurst 1997). Figure 7 shows plots of the velocity and the natural logarithm of the height above the channel bed and clearly demonstrates that the log-law is not valid for the entire flow depth at this measurement location for either flow rate. One possible explanation for the deviation from the log-law is the presence of secondary currents as demonstrated by Figure 6b. The empirical evidence supporting a logarithmic velocity profile was obtained under dimensional flow conditions (Nezu & Nakagawa 1993).



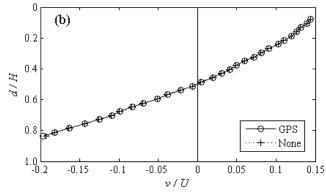
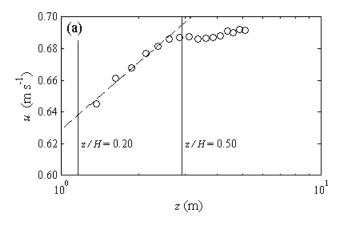


Figure 6. Velocity profiles using both none and GPS for the velocity reference for (a) streamwise and (b) spanwise velocity components from June 14, 2009 ( $Q = 565 \text{ m}^3 \text{ s}^{-1}$ ).



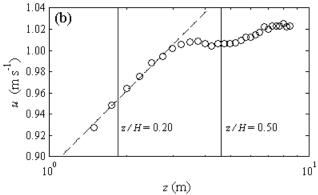


Figure 7. Velocity measurements (open circles) plotted along with regression equations (dashed lines) for profiles measured on (a) May 28, 2009 ( $Q = 220 \text{ m}^3 \text{ s}^{-1}$ ) and (b) June 14, 2009 ( $Q = 565 \text{ m}^3 \text{ s}^{-1}$ ).

While a logarithmic profile is not seen for the entire flow depth, a linear region is observed for both profiles within the bottom half of the flow depth. While measurements were not obtained within the bottom 20% of the flow depth in May, velocity was measured at two locations within this range for the higher flow rate in June. Both of these measurements are contained in the linear region described above. While near-bed measurements are not available with the ADCP, these profiles suggest that a logarithmic velocity profile may be appropriate through a region between approximately 20 and 50% of the flow depth.

Using the least square errors approach, an equation was fit to the linear portion of each profile. The shear velocity and boundary shear stress determined from the regression equation as well as the coefficients of determination are provided in Table 1. One difficulty encountered when presenting boundary shear stresses is that the results have been shown to be dependent on the method of calculation (e.g. Biron et al. 2004). To gauge if the results presented here are reasonable, the boundary Reynolds number  $(R_* = u_* d_s / v)$ , where  $d_s = u_* d_s / v$ representative grain size, v = fluid kinematic viscosity) and dimensionless shear stress ( $\tau_* = \tau_o/(\gamma_s - \tau_o)$  $\gamma d_s$ , where  $\gamma_s$  = specific weight of sediment,  $\gamma$  = specific weight of water) were calculated. To perform these calculations, a representative grain size,  $d_s$ , of 1.0 mm was selected based on visual evidence near the measurement site. When the resulting dimensionless variables,  $R_*$  and  $\tau_*$  (provided in Table 1), were plotted on the Shield's diagram, both points were located above the threshold for sediment motion. The dimensionless shear stress for the May measurement fell just above the critical shear stress value,  $\tau_{*c} \approx 0.032$ , while the June measurement value was significantly larger than the critical value,  $\tau_{*c} \approx 0.038$ . This finding is in agreement with observed sediment motion documented by the ADCP bottom tracking feature.

#### 6 DISCUSSION

Two issues regarding the ADCP velocity profiles and the implications for the calculated shear stress require further discussion. First, near-bed velocity measurements are not possible due to side lobe interference (Simpson 2001). While it has been thought that near-bed measurements are required for an accurate determination of shear stress (e.g. Biron et al. 2004), Yu and Tan (2006) demonstrate that using points from the upper portion of the logarithmic region for the regression analysis yield improved estimates of boundary shear stress.

The other issue arises from the fact that the ADCP beams diverge from each, resulting in a large measurement volume for the profile. For the conditions encountered on the lower Roanoke River, the maximum distance between two beams is about 4.2 m ( $Q = 220 \text{ m}^3 \text{ s}^{-1}$ ) and 6.7 m ( $Q = 565 \text{ m}^3 \text{ s}^{-1}$ ). Despite this large sample volume, good agreement has been found between mean velocity profiles from an ADCP and acoustic Doppler velocimeter in both the laboratory (Nystrom et al. 2007) and field (Stone & Hotchkiss 2007). When the least square error method is used to determine the boundary shear stress, the velocity profile is the only required input.

Table 1. Summary of Velocity Profiles

	,	
Date	May 28, 2009	June 14, 2009
$ \frac{Q (\text{m}^3 \text{s}^{-1})}{U (\text{m s}^{-1})} $ $ R^2$	220	565
$U(\text{m s}^{-1})$	0.68	0.99
$R^2$	0.98	0.99
$u_* \text{ (m s}^{-1})$	0.025	0.042
$\tau_{o}$ (Pa)	0.61	1.80
$R_*$	25	42
$ au_*$	0.037	0.111

#### 7 CONCLUSIONS

Techniques have been presented to: (1) establish that measured velocity time series data are statio-

nary, (2) determine the depth-averaged velocity, (3) confirm that ADCP motion during the measurement does not adversely affect the mean velocity profile, and (4) calculate the local mean boundary shear stress. These techniques have been tested at a location on the lower Roanoke River for two different flow rates. The procedures proposed for both the depth-averaged velocity and boundary shear stress do not require the velocity profile to follow the log-law for the entire flow depth. Rather, these techniques benefit from the visual determination of the region where the loglaw is valid. It was found that for conditions up to bankfull, the boat could be secured within the river channel so that the ADCP motion during sampling did not adversely affect the mean velocity profiles. It should be noted that during the higher flow rate, significantly more time and effort was required to secure the boat.

While no direct method to measure boundary shear stress in a natural river channel is currently available, the technique presented here offers a relatively simple technique to indirectly estimate the local boundary shear stress for flows where the log-law does not apply over the entire flow depth. Verification of boundary shear stress estimates remains a difficult issue in both the laboratory and field. However, comparing the observed bed load motion with the calculated  $R_*$  and  $\tau_*$  values provides some confidence in the results. Finally, it should be noted that the results presented here are limited to the conditions encountered on the lower Roanoke River and may not be directly applicable to different field conditions.

# **ACKNOWLEDGEMENTS**

The authors gratefully acknowledge the support of Dominion, U.S. Army Corps of Engineers, and the Sussman Fund. Assistance with the field measurements was provided by Ozan Celik and Nikos Apsilidis.

#### **REFERENCES**

- Bathurst, J.C. 1997. Environmental river flow hydraulics. In: Applied Fluvial Geomorphology for River Engineering and Management, C. R. Thorne, R. D. Hey, and M. D. Newson, eds., Wiley, Chichester, 69-94.
- Bendat, J.S., Piersol, A.G. 1986. Random data: analysis and measurement procedures, 2nd Ed., Wiley, New York. 566 p.
- Biron, P.M., Robson, C., Lapointe, M.F., Gaskin, S.J. 2004. Comparing different methods of bed shear stress estimates in simple and complex flow fields. Earth Surface Processes and Landforms, 29(11), 1403-1415; DOI: 10.1002/esp.1111.

- Dancey, C.L., Diplas, P. 2008. Statistical uncertainty and the estimation of log law parameters. Journal of Hydraulic Engineering, 134(9), 1353-1356; DOI: 10.1061/(ASCE) 0733-9429(2008)134:9(1353).
- Dietrich, W.E., Whiting, P. 1989. Boundary shear stress and sediment transport in river meanders of sand and gravel. In: River Meandering, S. Ikeda and G. Parker, eds., American Geophysical Union, 1-50.
- Muste, M., Yu, K., Spasojevic, M. 2004. Practical aspects of ADCP data use for quantification of mean river flow characteristics; Part II: fixed-vessel measurements. Flow Measurement and Instrumentation, 15(1), 17-28; DOI: 10.1016/j.flowmeasinst.2003.09.002.
- Nezu, I., Nakagawa, H. 1993. Turbulence in open-channel flows, Balkema, Rotterdam. 281 p.
- Nystrom, E.A., Rehmann, C.R., Oberg, K.A. 2007. Evaluation of mean velocity and turbulence measurements with ADCPs. Journal of Hydraulic Engineering, 133(12), 1310-1318; DOI: 10.1061/(ASCE)07339429(2007)133: 12(1310).
- Papanicolaou, A.N., Hilldale, R. 2002. Turbulence characteristics in gradual channel transition. Journal of Engineering Mechanics, 128(9), 948-960; DOI: 10.1061/(ASCE) 0733-9399(2002)128:9(948).
- Raupach, M. R., Antonia, R. A., and Rajagopalan, S. 1991. Rough-wall turbulent boundary layers. Applied Mechanics Reviews, 44(1), 1-25.
- Rennie, C.D., Millar, R.G. 2004. Measurement of the spatial distribution of fluvial bedload transport velocity in both sand and gravel. Earth Surface Processes and Landforms, 29(10), 1173-1193; DOI: 10.1002/esp.1074.
- Sime, L.C., Ferguson, R.I., Church, M. 2007. Estimating shear stress from moving boat acoustic Doppler velocity measurements in a large gravel bed river. Water Resources Research, 43(3); DOI: 10.1029/2006WR005069.
- Simpson, M.R. 2001. Discharge measurements using a broad-band acoustic Doppler current profiler. Sacramento: United States Geological Survey Open-file report 01-1
- Soulsby, R.L. 1980. Selecting record length and digitization rate for near-bed turbulence measurements. Journal of Physical Oceanography, 10(2), 208-219.
- Stacey, M.T., Monismith, S.G., Burau, J.R. 1999. Measurements of Reynolds stress profiles in unstratified tidal flow. Journal of Geophysical Research-Oceans, 104(C5), 10933-10949.
- Stone, M.C., Hotchkiss, R.H. 2007. Evaluating velocity measurement techniques in shallow streams. Journal of Hydraulic Research, 45(6), 752-762.
- Tennekes, H., Lumley, J.L. 1972. A first course in turbulence, MIT Press, Cambridge, Mass. 300 p.
- Wilcock, P.R. 1996. Estimating local bed shear stress from velocity observations. Water Resources Research, 32(11), 3361-3366.
- Yu, G.L., Tan, S.K. 2006. Errors in the bed shear stress as estimated from vertical velocity profile. Journal of Irrigation and Drainage Engineering, 132(5), 490-497; DOI: 10.1061/(ASCE)0733-9437(2006)132:5(490).