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***Abstract*—One of the most critical issues in the modelling of graded sediment transport is the vertical discretization of the bed into different layers and their interaction, particularly between the active layer and active stratum. By applying the TELEMAC - SISYPHE system to study the influence of an open stone ramp on flood events of a river stretch in Germany we had often faced challenges related to unphysical simulation and numerical instability. To improve the sediment transport module SISYPHE concerning this matter, some parts of the FAST computer code (developed by KIT and TUM) are adapted into the TELEMAC environment. The present paper shows the fundamentals of a new layer subroutine and modifications required for the SISYPHE environment. Special treatments for nonerodible grid points are also presented. The calculated results of the developed model are compared with laboratory measurements conducted by Günter (1971) to analyse the behaviour of new implementation.**

I. Introduction

Modelling of graded sediment transport is quite a challenging task. The mixing of different soil layers with different sediment classes below the surface is not trivial. The module SISYPHE, part of the TELEMAC-MASCARET modelling environment, includes an algorithm for this task - the *layer.f*-subroutine. Applying this code to a fractionized sediment model some instabilities and errors are observed. Therefore, at the Chair of Hydraulic Research and Water Resources Management, Technical University of Munich (TUM) is a new version for SISYPHE implemented. The main idea is to adapt the *layer.f* and related subroutines based on the FAST computer code, which has been developed at the Institute for Hydromechanics, University of Karlsruhe, Germany (KIT) and TUM. As usual for graded sediment transport models a so called size-fraction method is used, in which the bed is divided into different layers and size-fractions, each characterised by a certain diameter and volumetric percentage of occurrence in the river bed. The effect of fractional sediment transport leads to an exchange of grains between the layers, and so a grain sorting process can be approached. A special treatment of nonerodible parts within a calculation domain comes up during the code development. Nonerodible regions, like concrete walls, bridge piers or large stone settings are typical structures in river engineering cases. In the present paper the structure for vertical layer discretization, fractional grain exchange within layers and nonerodible treatment is presented. Almost all variables in the new version remain the same as ones used before in the SISYPHE source code. The new approach is validated by modelling two of the well documented laboratory experiments performed in 1971 by Günter at the Laboratory of Hydraulics, Hydrology and Glaciology, Eidgenössischen Technischen Hochschule (ETH) Zürich, Switzerland [2]. Finally, brief remarks of the model application for a real case study are also given.

II. Sisyphe

A. Background and theoretical aspects

The existing and the new codes are both based on the so-called size-fraction method, where bed material is divided into a certain number of grain classes, which are different in size and percentage of occurrence. Furthermore, the bed is discretised in vertical direction into several layers. The first one is the active layer, which is directly exposed to the flow. Below this one are several subsurface layers, which are only in exchange with the surrounding layers. Due to evolution of the river bed, the thickness of the layers changes as well as the available percentages of each grain-class in each layer [4].

The bed-level change due to a fraction i is calculated from a mass-balance (1):

$$(1\text{-}p)\,\frac{\partial Z_{b,i}}{\partial t}+\nabla\overrightarrow{Q_{b,i}}=0 \qquad (1)$$

using p = porosity of the bed material; and $\overrightarrow{Q_{b,i}}$ = fractional bed load flux, determined by an empirical transport function. The total bed deformation is then determined in the following equation:

$$\frac{\partial Z_b}{\partial t}=\sum_{i=1}^{NSICLA}\frac{\partial Z_{b,i}}{\partial t} \qquad (2)$$

using NSICLA = number of all size classes [1].

B. River bed representation in the numerical model

In the TELEMAC-MASCARET modelling environment the calculation domain is represented by a grid consisting of nodes connected to unstructured triangular elements. To perform a simulation it is necessary to provide initial conditions all over the domain for each node. For the morphologic simulation information about the bottom (e.g. the river bed level ZF and the rigid bed elevation ZR, with ZF ≥ ZR) is necessary. Furthermore, the initial composition of the river bed has to be specified by the number of vertical soil layers NOMBLAY, the number of grain size classes NSICLA, the availability of each class i within the layer k $AVAIL_{k,i}$, and the thickness of each layer ES_k [4].

III. NEW IMPLEMENTATION

By applying the SISYPHE modelling environment from version v6p3 to a large, complex real river application in Germany some errors and numerical problems arouse. A common error message after several time steps was "Error in layer" and the simulation stopped. Using the newer release version v7p0 it was not even possible to start the simulations. From the User-Forum of TELEMAC it seems that many users face these problems. In order to solve this issue, the layer concept from FAST was adapted and integrated into SISYPHE. In the following parts, another treatment of the interaction of the layers to each other and of a nonerodible part is presented. Furthermore, the existing bedload formula after Hunziker [3] is modified and the transport function after Wu [5] is implemented in *qsform.f* subroutine. The new code was initially developed for the version v6p3r2 of the TELEMAC-SISYPHE system, but it is also integrated in the newer releases.

C. Treatment of nonerodible nodes

Modelling nonerodible parts in a calculation domain is a quite common task in river engineering problems. The river bed is commonly very thick until bedrock is reached, however, in some locations (e.g. stone ramps, concrete walls at embankment structures or at weirs, etc.) the river bed is nonmovable. In numerical models, a node is classified as nonerodible when the thickness of its layers is zero $ES_k = 0$. However, it should be noted, that during the simulation period deposition can occur at these places and the deposited materials can be eroded depending on the local hydromorphological conditions. This process should be considered in the numerical model. Furthermore, the condition

$$\sum_{i=1}^{NSICLA} AVAIL_{k,i} = 1 \qquad (3)$$

has to be fulfilled in any case, to avoid mass inconsistencies and division by zero.

The new developed code includes an additional size class in addition to the actual available ones to represent nonerodible structures. So that a high stability, consistency and flexibility of the model could be achieved. This additional size class is independent per se from the defined bed grain sizes, as the transport rate of this additional class is defined to be zero and it is excluded from most of the internal calculations. This additional grain class occurs only at nonerodible layers. Following equations can be formulated for any layer k:

$$\text{if } ES_k = 0 \text{ then } \begin{cases} AVAIL_{k,NSICLA} = 1 \\ \sum_{i=1}^{NSICLA-1} AVAIL_{k,i} = 0 \end{cases} \qquad (4)$$

Equation (4) states that in case of a layer with zero thickness, its material contains up to 100 % of the additional grain class. Vice versa in case of erodible layers the additional grain class does not occur. This is formulated in (5), which claims that in this case the sum of residual grain classes must be 100 %.

$$\text{if } ES_k > 0 \text{ then } \begin{cases} AVAIL_{k,NSICLA} = 0 \\ \sum_{i=1}^{NSICLA-1} AVAIL_{k,i} = 1 \end{cases} \qquad (5)$$

From physical point of view this additional grain class can be compared to a large boulder which cannot be moved by the flow, which is quite close to reality. The implementation of this treatment requires modification in some relevant subroutines showed in the following list:

- *bedload_formula.f*
- *bedload_hunz_meyer.f*
- *bedload_main.f*
- *init_avai.f*
- *init_compo.f*
- *init_sediment.f*
- *init_transport.f*
- *layer.f*
- *mean_grain_size.f*
- *qsform.f*
- *tob_sisyphe.f*

In fact, the subroutine *noerod.f* to define the rigid bed is not needed anymore, as this function is now fully integrated into *init_compo.f*. In case of using the bed roughness predictor, suitable values for Nikuradse grain roughness k_s must be specified, since SISYHPEs bed roughness predictor options might not work proper on nonerodible nodes.

D. River bed decomposition

In the SISYPHE system, the river bed is decomposed into vertical layers, initially in the *init-compo.f* and *init_avai.f* subroutines and during the simulation in the *layer.f* subroutine. It is important to note that the initially defined number of layers at each node NOMBLAY remains the same during the calculation. Furthermore, for each layer a maximum possible thickness has to be defined. In case of the first layer, the active layer, this is named ELAY0, which can be either constant or depending on the diameter of the material in the active layer. The second layer, the active stratum thickness is named ESTRAT0 and must be also defined. The last layer has no thickness limit. Otherwise, it could happen that in case of high deposition the defined number of layers are not capable to represent the total sediment thickness. Vice versa it is not possible that a layer can get a negative value. It is determined as follows:

$$ZF - ZR = \sum_{k=1}^{NOMBLAY} ES_k \qquad (6)$$

The river bed elevation ZF is determined in a geometry file, which includes the information BOTTOM. The rigid bed level can be defined either constant or varying for each node depending on the river structures. Here an algorithm is implemented to read the information ZR from the same file. This function works the same as for BOTTOM or BOTTOM FRICTION and is therefore not explained here further.

The new layer treatment considers the following three different options, depending on the defined number of layers:

- One layer case
- Two layer case
- Multilayer case

In case of only one layer (NOMBLAY = 1) the total available thickness is equal to the thickness of the active layer after:

$$ES_{NOMBLAY} = ZF - ZR \quad (7)$$

A two layer case (NOMBLAY = 2) includes an active layer with a maximum defined thickness and one residual layer below, as shown in (8).

$$ES_1 = \min(ELAY0; ZF - ZR) \quad (8)$$

$$ES_{NOMBLAY} = ZF - ZR - ES_1$$

The following lines describe the code sequence for decomposition of a multilayer case (NOMBLAY ≥ 3):

$$ES_1 = \min(ELAY0; ZF - ZR) \quad (9)$$

$$ES_2 = \min(ESTRAT0; ZF - ZR - ES_1)$$

$$\ldots.$$

$$ES_k = \min\left(ESTRAT0; ZF - ZR - \sum_{k=1}^{k-1} ES_k\right)$$

$$ES_{NOMBLAY} = ZF - ZR - \sum_{k=1}^{NOMBLAY-1} ES_k$$

After that, the available percentages of each class i in each layer k has to be defined ($AVAIL_{k,i}$). This can be done explicitly for each layer in *init_compo.f*. Via mass balance the volumetric amount of sediment VOL in the domain is calculated using (10).

$$VOL = \sum_{k=1}^{NOMBLAY} ES_k * \sum_{i=1}^{NSICLA} AVAIL_{k,i} \quad (10)$$

In fig. 1 the discretization of the river bed surface and the nonerodible level is schematized for an exemplary case with maximum five layers at three nodes. Node one is initially nonerodible and so the bottom surface is equal to the nonerodible level (ZF=ZR) and all layer thicknesses are zero. At node two the nonerodible level is lower than the surface and the difference is distributed to into layers, starting from the top. Layer one to four attains their maximum defined thickness and the last one reaches to the rigid bed. The third nodes rigid bed is at a medium height and only four layers are necessary to distribute the river bed. The thickness of layer 5 is zero.

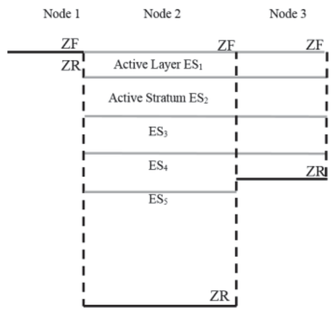

Figure 1. Scheme of vertical river bed discretization at three different nodes.

E. Vertical layer interaction

Based on the initial discretised bed, the model calculates the interaction of layers to each other and to the flow. The key concept is the existence of an active layer, where the flow picks up the transportable sediment and receives the grains that the flow is unable to transport [1].

For erosion of the river bed the temporal change of the volumetric percentage of a fraction i in the active layer is calculated considering the taken material from the flow and the available material in the stratum below. This is done via a mass balance, given in (11).

$$\frac{\partial AVAIL_{1,i}}{\partial t} * ES_1 = \frac{\partial Z_{b,i}}{\partial t} - \frac{\partial Z_b}{\partial t} * AVAIL_{2,i} \quad (11)$$

using $\frac{\partial AVAIL_{1,i}}{\partial t}$ = change of fraction i in the active layer, ES_1 = active layer thickness; $\frac{\partial Z_{b,i}}{\partial t}$ = bed level change of fraction i; $\frac{\partial Z_b}{\partial t}$ = total bed level change; $AVAIL_{2,i}$= available percentage of fraction i in the active stratum layer. The active stratum is capable of an exchange with the stratum below, balanced in (12). If the layer below is nonerodible or the maximum number of layer is reached, no interaction will take place.

$$\frac{\partial AVAIL_{k,i}}{\partial t} * ES_k = \frac{\partial Z_b}{\partial t} * (AVAIL_{k,i} - AVAIL_{\min(NOMBLAY,k+1),i}) \quad (12)$$

For deposition case, the material enters the top element, so no relation with lower layers has to be considered here, see (13).

$$\frac{\partial AVAIL_{1,i}}{\partial t} * ES_1 = \frac{\partial Z_{b,i}}{\partial t} - \frac{\partial Z_b}{\partial t} * AVAIL_{1,i} \quad (13)$$

Due to the deposition the active stratum gets some upward directed movement and material is in exchange with the layer above, the same for other substrate layers:

$$\frac{\partial AVAIL_{k,i}}{\partial t} * ES_k = \frac{\partial Z_b}{\partial t} * (AVAIL_{k-1,i} - AVAIL_{k,i}) \quad (14)$$

After updating the available percentages of each fraction in each layer, the thickness of each layer is new distributed according to the procedure shown in part *D* (see (7) – (9)). Finally via a counter check mass balance is ensured and the total amount of sediment within this time step is reached.

F. *Empirical transport functions*

The transport function after Hunziker has been developed in 1995 using the data conducted by Günter. This equation is already implemented in SISYPHE. However, the hiding function has to be adapted for the additional grain class treatment and the code is rewritten to solve the equations within one loop over all grid nodes. The basic of this transport function is the concept of equal incipient motion for all sediments. Sediment transport starts only if the dimensionless shear stress of the flow is higher than the dimensionless threshold. The determining parameters are here the critical shields parameter θ_c and a relation between the mean grain diameter of the surface layer d_m and subsurface layer d_{mo}. The critical shear stress is then modified according to the following equation.

$$\theta_{cm}=\theta_c * \left(\frac{d_{mo}}{d_m}\right)^{0.33} \quad (15)$$

According to the Günter experiments a hiding/exposure function is evaluated and parametrized in order to describe which sediments are more or less exposed to the flow. The sediment discharge after Hunziker is given in (16).

$$Q_{b,i}=\sqrt{(s-1)*g*d_m^3}*AVAIL_{1,i}*5*(\varphi_i(\mu*\theta_{dm}-\theta_{cm}))^{3/2} \quad (16)$$

using s = relative density, g = gravity, d_m = mean diameter of the surface layer, φ_i = hiding factor, μ = parameter for skin friction correction, θ_{dm} = dimensionless shear stress parameter depending on the mean diameter and flow condition, θ_{cm} = modified critical shields parameter considering mean diameters of surface and subsurface layers [4].

The empirical transport function after Wu assumes that the probability of a grain to be exposed to the flow is depending on the diameter of the grain and the surrounding grains as well as the availability. Including a correlation parameter m = 0.6, which can be used in the calibration, the hiding and exposure function is formulated in (17) with

$$\theta_{cm}=\theta_c * \left(\frac{p_{e,i}}{p_{h,i}}\right)^{m} \quad (17)$$

using the critical shields parameter θ_c and the probability of exposure $p_{e,i}$ and hiding $p_{h,i}$ of a grain i at the surface layer. The transported bedload discharge is given as

$$Q_{b,i}=\sqrt{(s-1)*g*d_i^3}*AVAIL_{1,i}*0.0053*\left(\mu*\theta_{d_i}/\theta_{cm}-1\right)^{2.2} \quad (18)$$

using θ_{d_i} = dimensionless shear stress parameter depending on the diameter of each grain and flow condition, θ_{cm} = modified critical shields parameter including the hiding factor. For more details and full description of the formulas after Wu see [3].

IV. Calculation results

G. *Günter experiment – grain sorting*

The new developed subroutines are validated by modelling the laboratory experiments conducted by Günter in 1971 at the ETH Zürich. The experiments were performed in a 40 metres long and 1 meter wide rectangular channel. Sediment mixtures of a certain defined composition were installed in this channel according to a defined slope. Running the experiment with constant flow conditions after around 40 days, erosion leads to a development of new slopes and armoured layers by wash out of fine materials [2].

It was decided to recalculate the laboratory experiments with the bed load transport formula after Hunziker and Wu. For the validation two experiments were numerically modelled, experiment #3 and #9. The initial river bed composition in case #3 according to Günter is close to a typical river bed composition in an alpine river bed. The second case #9 is rather unnatural, with high amount of fine and coarse grains and less intermediate ones [3]. For each test case are the determining parameters given, in table I hydrodynamic quantities and in table II the morphodynamic ones.

The numerical mesh consist of around 900 elements with an average edge length of 33 centimetres. This mesh allows with an average time step of 0.5 seconds the simulation of 40 days in an acceptable duration. The boundary conditions for the hydrodynamic part are constant discharge at the inlet and fixed water level at the outlet, 1 centimetre lower than estimated water depth at the end of the experiment h_G. River bed roughness is defined after Nikuradse with a temporal bed roughness predictor, depending linearly on the ratio between skin friction and mean dimeter of the active layer, with $k_s = \alpha d_m$ [4]. The ratio coefficient α is used for calibration.

Morphological boundary conditions are defined as free, so that no material enters the domain and the river bed can evolve without constraints. The river bed is discretised into three layers, with a constant active layer thickness of three times the initial d_{90}. Active stratum is defined to be three times the active layer. Shields parameter θ_c and the hiding-factor of Wu transport function are assumed to be most influencing the result and are used in the calibration, too.

TABLE I. BOUNDARY AND FINAL FLOW CONDITIONS

Case	Q_{in}	I_0	h_G	I_G
[/]	*[l/s]*	*[‰]*	*[cm]*	*[‰]*
#3	56.0	2.50	9.91	2.327
#9	39.4	4.00	6.87	4.176

TABLE II. INITIAL SEDIMENT COMPOSITION

size class i	1	2	3	4	5	6
$d_{m,i}$ [cm]	0.051	0.151	0.255	0.360	0.465	0.560
#3-Initial	0.359	0.208	0.119	0.175	0.067	0.072
#9-Initial	0.336	0.117	0.099	0.139	0.129	0.180

The simulations were performed using the existing codes, named “old”, insofar as it was possible, and the new developed codes, named “new”. The calibration parameters are adjusted in order to get good results in all versions with the same parameter set for each test case. The simulations are analysed regarding to grainsize distribution in the surface layer, water depth and river bed inclination. All values of the domain are considered and averaged. Table III shows the defined parameters and results together with the corresponding bed load functions and different versions of the program. The development of the grainsize distribution in the surface layer is shown in fig. 2 to fig. 5 separately for each experiment and bed load function. Important is, that with the “old” codes of SISYPHE no simulation could be performed using version v7p1, as in all cases the simulation stops after a few time steps with “Error in layer”. Using the new code structures the crashes does not occur, but the gained results are unrealistic, which points to a deeper error in the source code of the program. However, this error seems to be corrected in the newest unreleased version of TELEMAC, the trunk-version, and more realistic results are gained for old and new layer treatment. This topic was also discussed in the TELEMAC-MASCARET user forum.

TABLE III. RESULTS OF THE SIMULATIONS

		#3				#9			
		Hunziker		*Wu*		*Hunziker*		*Wu*	
α		2.5		2.5		2.0		2.0	
θ_c		0.044		0.044		0.047		0.047	
m		/		0.7		/		0.7	
		***h_G* [cm]**	***I_G* [‰]**	***h_G* [cm]**	***I_G* [‰]**	***h_G* [cm]**	***I_G* [‰]**	***h_G* [cm]**	***I_G* [‰]**
v6p3	***old***	10.42	1.8	9.87	2.5	6.75	3.9	6.75	3.9
	new	9.99	2.2	9.97	2.5	6.75	3.9	6.81	4.2
v7p1	***old***	/	/	/	/	/	/	/	/
	new	10.51	1.6	11.45	1.0	7.01	3.1	7.63	2.3
trunk	***old***	10.11	1.8	9.87	2.5	6.74	3.9	6.82	4.1
	new	9.98	2.4	10.0	2.4	6.75	3.9	6.82	4.1

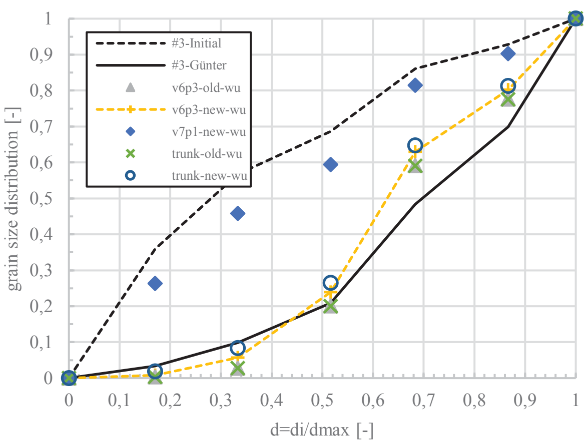

Figure 2. Case #3 - grain size distribution using Wu’s function

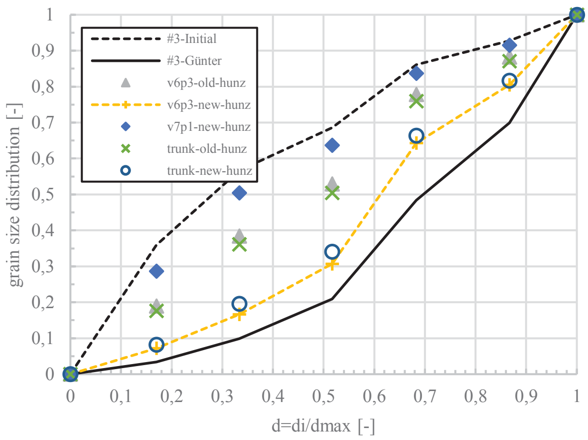

Figure 3. Case #3 – grainsize distribution using Hunziker’s function

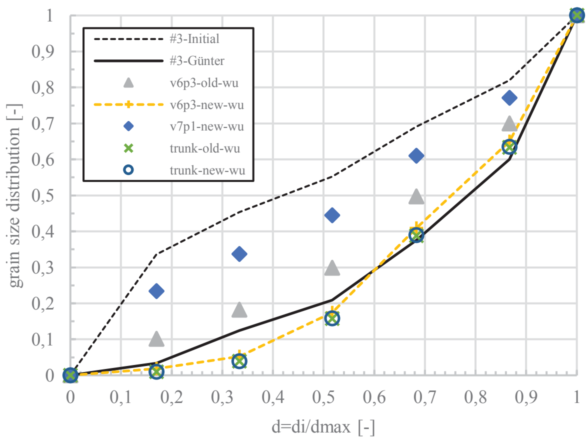

Figure 4. Case#9 – grainsize distribution using Wu’s function

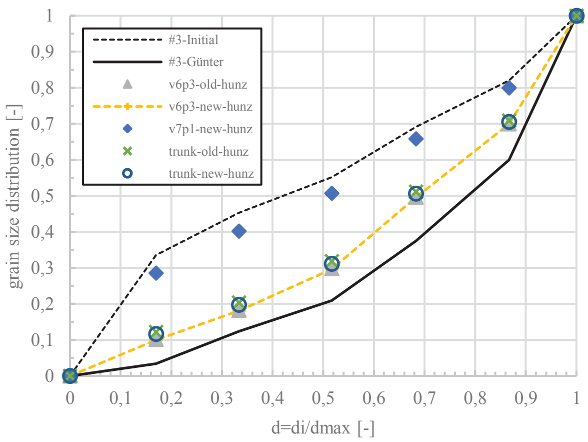

Figure 5. Case#9 – grainsize distribution using Hunziker’s function

The bed load function after Wu shows very good agreement with the measurements conducted by Günter for test case #3 and #9 (fig. 2 and fig. 4). The water depths are close to the measurements with an absolute difference lower than 1 millimetre and the inclination has an absolute maximum deviation of 0.2 per mille. However, the discrepancy between old and new layer treatment is rather small and in both cases the armouring of the river bed is well represented. The small differences are a result that the Wu bedload function takes the diameter of the subsurface layer not directly into account. In the Wu's formula only the available percentage of a grain in the surface layer is considered. The Hunziker's function (abbr. hunz) instead uses the diameter of the substrate layer directly to modify the critical shields parameter, see (15). Therefore, the exchange between layers gets more important. In fig. 3 the differences of the grain sorting process are more significantly visible. With the new layer treatment a better sorting is achieved for test case #3 applying version v6p3 and the unpublished trunk version. Also the water depth and bed slope are more accurate simulated with the new codes. For simulation test case #9 only slightly better results are gained with the new layer treatment compared to the existing codes (fig. 5).

The new layer treatment allows to simulate the Günter experiments with a numerical model with high accuracy. But it must be considered that also the existing codes can be used to simulate the experiment, in most cases. However, when the calculation domain contains nonerodible nodes, the existing codes shows their weaknesses. With the new layer treatment this problem can be solved, as it can be seen the following part.

H. River case – nonerodible treatment

The functionality of the new treatment for nonerodible areas is now tested by the application to the real case, where the problems arises first by applying the original SISYPHE codes. The test case is a three kilometres long river stretch with floodplain in the southern part of Germany, which includes an open stone ramp to limit the erosion in this region. Furthermore, in the river stretch exists a ground sill below a bridge to prevent scour. The ramp, the ground sill and the floodplain with embankment dams are initially classified as nonerodible. The domain consist of around 130'000 nodes and 250'000 elements, which does not allow a manual identification of nonerodible nodes via node number.

Applying the new subroutines for nonerodible areas and layer treatment, this river stretch is finally analysed by a quite accurate and stable hydromorphological model. The simulations are also performed successfully on a server in parallel mode. In fig. 6, a longitudinal section along the river channel is given, with flow from left to right. From the initial river bed (black line) with the fixed parts at the ramp rkm 4.6 and the ground sill at rkm 2.975 the simulation of a flood event over six days leads to significant bed level change. The model is able to simulate the observed water levels along the domain in a very good manner and the shape of the flooded area is close to the expected one. The initial nonerodible ramp is after the flood event covered with sediments, which is a problem for the maintenance. The ground sill keeps the river bed upstream of it on a similar level, but downstream of the ground sill large erosion is observed due to the weir operating during the flood event at the outflow boundary.

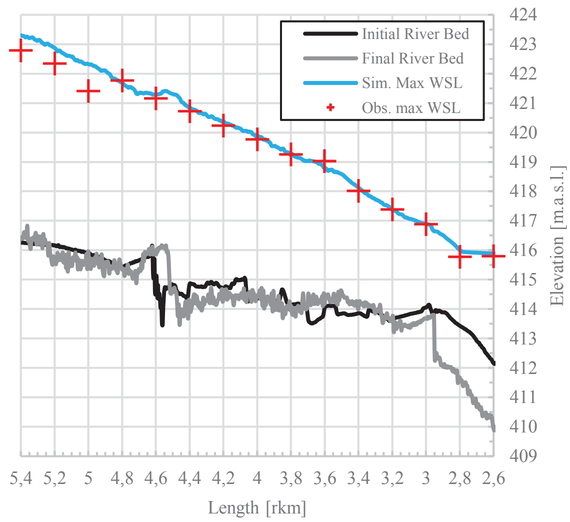

Figure 6. Longitudinal section of the river case for initial (back line) and final (grey line) river bed and for simulated (blue line) and observed (red crosses) water surface levels.

With the developed model several scenarios and modifications were analysed to increase the flood safety for the surrounding cities. By a modification at the ramp and the ground sill the water levels of a 100-year flood could be halved to a maximum height of 1 m at the floodplains, which offers in combination with a flood protection dam a feasible protections system.

V. Conclusion

The modelling environment TELEMAC-MASCARET is a powerful tool to analyse river engineering issues. The problem of numerical errors regarding fractional sediment transport leads to the implementation of an alternate treatment of grain sorting processes and nonerodible structures on the river bed. The newly developed code increases the stability and flexibility of the TELEMAC-SISYPHE system. The code was validated by the numerical modelling of laboratory test cases. The measurements of the bed armouring, flow depth and final river bed slope were accurately represented. The final adaption to a real case study shows the model capacity for long river stretches with complex bed structures. This model provides a promising tool to analyse the impact of sediment transport during a flood event in fluvial rivers.

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