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**TELEMAC**

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# A new gravity-driven runoff and erosion model for TELEMAC

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**Abstract**— With the aim of representing hydraulic transfers and upstream erosion sources on mountain watershed, the representation of the infiltration, immature debris flows and shallow landslides is important because it can be the largest contributor to the amount of sediment exported at the outlet. To model these phenomenon, a hydrological and a gravity-driven erosion model have been implemented in TELEMAC2D. The infiltration is a 1D vertical model. The motion of the granular flows is described with a fully dynamic system. A Coulomb like bottom friction treatment, more adapted to the properties of the flow, is also added. These new developments are complementary to the use of SISYPHE for the erosion, deposition and transfers in the hydraulic network.

The infiltration model is confronted to field data on a real catchment (Draix, in the Southern French Alps). Then, the new erosion model is qualitatively evaluated on a theoretical test case: a plot with a steep slope upstream and a break in slope to evaluate the form of the erosion and deposition. Then, on a real catchment, the upstream hillslope erosion and the deposition in the hydraulic network is observed.

## I. INTRODUCTION

In mountain watersheds, extreme events can lead to the exportation of a large amount of sediment at the outlet. Indeed, the combination of steep slopes and highly erodible soil in badlands is responsible for this phenomenon. The presence of a large mass of sediment in the river flows can fill the dam reservoirs which affects the hydro-electricity production and causes safety issues. At the outlet of small watersheds, the filling of tunnel built to bypass water channels is also observed. The prediction of the sediment transfers is important to dimension some structures or to prevent erosion by hillslope management. A physically based modeling gives explicit values of the hydraulic and sedimentary parameters distributed in time and space. It offers the possibility of identifying the critical erosion zones but also anticipating watershed responses to a management strategy like slopes revegetation or dike construction.

There are several processes involved in the sediment dynamics at the catchment scale. The erosion and deposition processes being heavily reliant on the flow characteristics, the choice of the hydraulic model is crucial. In a precedent paper [18], the selection of the numerical scheme used in TELEMAC2D has been justified. The hydrology of a watershed needs to be taken into account to represent the correct hydraulic transfers. Concerning the sedimentary

processes, the model used by SISYPHE is well adapted in the hydraulic network. However, the upstream erosion sources on the hillslopes are not well represented in the code. Then, a focus is on these gravity-driven erosion mechanisms, and particularly the immature debris flows. Plenty of models can be found in the literature to represent the detachment, deposition and dynamic of muddy or granular flows [6, 19, 16, 13], but none of them is integrated to a global model with a hydraulic and hydrological description. The key is to build a model capable of representing the hydrology of a watershed with TELEMAC2D, the upstream gravity-driven erosion sources that supply the classical erosion and sedimentation model in the hydraulic network already present in SISYPHE.

In this paper, the modelling of all the processes above-mentioned and the coupling method are presented. Then, a theoretical test case and a field application are used to assess qualitatively the model.

## II. MATERIALS AND METHODS

### A. Presentation of the test cases

The first test case is a straight channel with a length of 5 m and a width of 1 m. This channel is divided in two parts: an upstream part with a 50% slope and a downstream part with a 5% slope. The break in slope is in the middle of the channel. On this domain, a steady rain, with an intensity of 100 mm/h, is applied during 10 s. The spatial discretization of the channel is a triangular mesh with a mean space step of 10 cm. The upstream and lateral boundaries are considered as walls. The flow being supercritical at the outlet, the boundary is treated with a free condition.

Thanks to data provided by [11], it is possible to evaluate the model by comparing the results to measured discharges from field campaigns on a real catchment. The Laval watershed is a sub-catchment of the Bouinenc watershed, located on the Draix site in the Southern French Alps. Its total area is about 86.4 ha and the mean slope is 58%. The soil is mostly constituted of black marls and 68% of the surface is a bare soil. At the outlet of the catchment, the discharge are available for many rainfall events with a time step of 60 s. The rainfall intensity associated to the discharge is also measured every 60 s. Two different events are selected, one spring rain following three rainy days with a high soil moisture initial condition, and a summer storm with a dry initial soil state.

### B. Overland flow simulation

To simulate rain induced runoff, TELEMAC2D solves the Shallow Water equations which are

$$\partial_t U + \partial_x F(U) = S,$$

where  $U = \begin{pmatrix} h \\ hu \end{pmatrix}$ ,  $F(U) = \begin{pmatrix} hu \\ hu^2 + gh^2/2 \end{pmatrix}$  and  $S = \begin{pmatrix} R-I \\ -gh(\partial_x z + S_f) \end{pmatrix}$  with  $h$  the water height in m,  $u$  the flow velocity in m/s,  $g$  the gravity constant in m/s<sup>2</sup>,  $R$  the rain intensity in m/s,  $I$  the infiltration in m/s,  $z$  the bottom elevation in m and  $S_f$  the friction slope. For that, the following explicit finite volume scheme is used:

$$U_i^{t+1} = U_i^t - \frac{\Delta t}{\Delta x} \left( F_{i+\frac{1}{2}}^t - F_{i-\frac{1}{2}}^t \right) + \frac{\Delta t}{\Delta x} S_i,$$

where  $F_{i+\frac{1}{2}}^t = F(U_{i+\frac{1}{2}+}, U_{i+\frac{1}{2}-})$  is the numerical flux calculated at the interface  $i + \frac{1}{2}$  with the HLLC method [20]

and  $S_i = \begin{pmatrix} R_i - I_i \\ s_{i+\frac{1}{2}-} - s_{i-\frac{1}{2}+} \end{pmatrix}$  are the source terms. The intermediate

states  $U_{i-\frac{1}{2}+} = \begin{pmatrix} h_{i-\frac{1}{2}+} \\ h_{i-\frac{1}{2}+} u_i \end{pmatrix}$ ,  $U_{i+\frac{1}{2}-} = \begin{pmatrix} h_{i+\frac{1}{2}-} \\ h_{i+\frac{1}{2}-} u_i \end{pmatrix}$ ,  $s_{i+\frac{1}{2}-}$  and  $s_{i-\frac{1}{2}+}$  are defined as:

$$z_{i+\frac{1}{2}} = \min(\max(z_i, z_{i+1}), \min(h_i + z_i, h_{i+1} + z_{i+1})),$$

$$h_{i-\frac{1}{2}+} = \min(h_i + z_i - z_{i-\frac{1}{2}}, h_i),$$

$$s_{i-\frac{1}{2}+} = \frac{g}{2} (h_i - h_{i-\frac{1}{2}+}) (z_{i-\frac{1}{2}} - z_i),$$

$$h_{i+\frac{1}{2}-} = \min(h_i + z_i - z_{i+\frac{1}{2}}, h_i),$$

$$s_{i+\frac{1}{2}-} = \frac{g}{2} (h_i + h_{i+\frac{1}{2}-}) (z_i - z_{i+\frac{1}{2}}),$$

according to the Chen and Noelle's scheme [3].

The friction slope, defined as:

$$S_f = \frac{q|q|}{C^2 h^3},$$

with  $q = hu$ , is then added to the scheme by a semi-implicit treatment (like in [1]), with a Chézy coefficient  $C = 30 \text{ m}^{1/2}/\text{s}$ .

This scheme correspond to the 5<sup>th</sup> finite volume scheme with the 2<sup>nd</sup> hydrostatic reconstruction option of the V7P2 version of TELEMAC2D.

### C. Infiltration model

A derivation of the Green and Ampt's [8] model, presented in [5] and [4] is used to represent the infiltration in the model. It is a vertical 1D model computed at each cell of the domain. The infiltration velocity is described like

$$I = K \left( 1 + \frac{h_f + h}{z_f} \right),$$

with  $K$  the soil conductivity under less than 1 cm of hydraulic head in m/s,  $h_f$  the capillarity head in m and  $z_f$  the wetting

front in m. Then, the wetting front is updated following the equation:

$$z_f = \frac{I_c}{\theta_s - \theta_i},$$

where  $I_c$  the cumulated height infiltrated since the beginning of the event,  $\theta_s$  the saturated soil moisture or the porosity and  $\theta_i$  the initial soil moisture. The soil is vertically divided in two layers, a first with an associated width  $Z_C$  and conductivity  $K_C$  and a second with a width considered as infinite and a conductivity  $K_S$ . The conductivity  $K$  varies with the wetting front evolution according to this equation:

$$\begin{cases} K = K_C \text{ if } z_f \leq Z_C \\ K = \frac{z_f}{z_f - Z_C} \frac{Z_C}{K_S} \text{ else.} \end{cases}$$

### D. Debris flow modelling

The detachment criterion is calculated in accordance to [19]. At each cell interface, the slope of the ground  $\phi$  between two nodes of the mesh is calculated. Then, depending on the water depths and the soil properties of these nodes, a stability angle  $\phi_1$  is evaluated and compared to  $\phi$ . The soil is considered stable if  $\phi_1 < \phi$  and unstable if  $\phi_1 \geq \phi$ . The formula to calculate the critical angle is:

$$\tan \phi_1 = \frac{F_0}{F_1} \left( 1 + \frac{c\sqrt{1 - c^2 k^{-2} F_1^{-2} g^{-2} h^{-2}} + F_0^2 F_1^{-2}}{k F_0 g h (1 - c^2 k^{-2} F_1^{-2} g^{-2} h^{-2})} \right),$$

where:

$$F_0 = C_*(\rho_s - \rho) \tan \varphi,$$

$$F_1 = C_*(\rho_s - \rho) + \rho \left( 1 + \frac{1}{k} \right),$$

with  $c$  the apparent cohesive strength of the sediment layer in Pa,  $k$  a numerical coefficient near unity,  $C_*$  the maximal volumetric concentration allowed in the flow,  $\rho_s$  the sediment density in kg/m<sup>3</sup>,  $\rho$  the water density in kg/m<sup>3</sup> and  $\varphi$  is the internal friction angle of the sediment. The Fig. 1 shows the behaviour of the detachment criterion depending on the slope and the water depth for  $C_* = 0.65$ ,  $c = 35 \text{ Pa}$ ,  $\rho_s = 2650 \text{ kg/m}^3$ ,  $\rho = 1000 \text{ kg/m}^3$  and  $k = 0.85$ . If the rainfall intensity is large enough to generate a runoff with a water depth in the unstable zone, a layer with a thickness  $e$  is eroded. This thickness is proportional to the water depth following

$$e = \frac{C_e}{C_* - C_e} h,$$

with  $C_e = \frac{\rho \tan \phi}{(\rho_s - \rho)(\tan \phi - \tan \phi)}$  the equilibrium concentration.

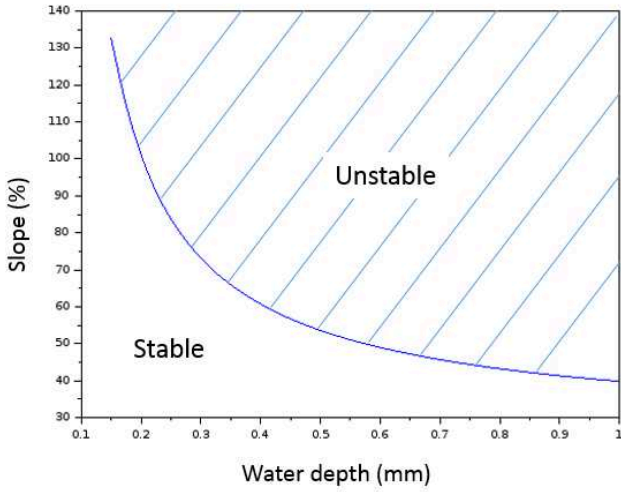


Figure 1. Detachment criterion for the debris depending on the slope and the water depth

Concerning the deposition velocity, the formula is similar to the one used for the cohesive sediment deposition in SISYPHE:

$$D = \left(1 - \left(\frac{u_s}{u_c}\right)^m\right)_+ C V_s,$$

where  $u_s$  is the debris flow velocity in m/s,  $u_c$  is the critical deposition velocity in m/s,  $m$  is a coefficient less than 1,  $C$  is the sediment concentration in the flow,  $V_s$  is the settling velocity in m/s and  $(\cdot)_+ = \max(0, \cdot)$ .

The debris flow motion is described by its velocity  $u_s$  in m/s and depth  $h_s$  in m. The evolution of these variables is governed by the shallow water equation with  $U = \begin{pmatrix} h_s \\ h_s u_s \end{pmatrix}$ ,  $F(U) = \begin{pmatrix} h_s u_s \\ h_s u_s^2 + g h_s^2 / 2 \end{pmatrix}$  and  $S = \begin{pmatrix} E-D \\ -g h_s (\partial_x z + S_f) \end{pmatrix}$ . The source term is modified with the erosion  $E = e/\Delta t$  and deposition velocities, and the friction becomes:

$$S_f = \frac{q|q|}{C^2 h^3} + \cos \phi \frac{|u|}{u},$$

the turbulent part is treated semi-implicitly and the coulomb friction is added explicitly to the scheme (see [15]).

### E. Coupling method

The coupling between the infiltration and the runoff is made through the source term  $I$  in the mass conservation equation. Concerning the debris flow dynamic, it interact with the runoff equations by modifying the bottom elevation like:

$$(1 - \theta_s) \partial_t z = E - D.$$

The entire model is presented in the Fig. 2. The available eroded layer is limited by the wetting front calculated with the infiltration model. Indeed, only the saturated part of the soil is considered available for the debris flow simulation.

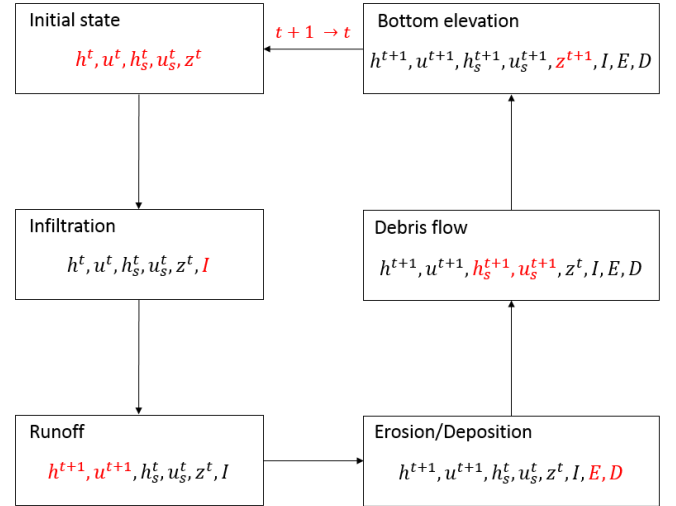


Figure 2. Schematic representation of the model

## III. RESULTS

### A. Channel test case

To fulfill the condition of instability, we set  $c = 1$  Pa,  $k = 0.85$ ,  $\rho_s = 2650$  kg/m<sup>3</sup>,  $\rho = 1000$  kg/m<sup>3</sup>,  $C_* = 0.65$ ,  $\tan \phi = 0.8$ ,  $u_c = 1$  m/s,  $m = 0.3$  and  $V_s = 0.01$  m/s. The saturated zone is initially set at the constant value 0.1 m and the infiltration is not considered. The Fig. 3 presents the bottom profile along the channel at the end of the simulation. In the upstream part, the erosion starts after 20 cm, when the depth of the water is sufficient to reach the unstable state. Then, a fully eroded zone is observed until the deposition starts, when the velocity decrease with the gentlest slope. This results are consistent with what is expected from the model. Moreover, the shape of the deposition is close to the experimental observations of [19] shown in Fig. 4.

### B. Watershed hydrology

The hydrological model is confronted to field data by comparing the measured and simulated outlet discharges on two events. The events are selected because they are the most erosive of the 2012 year, in terms of sediment volume exported at the outlet. The first event is recorded the 29<sup>th</sup> of May succeeding six rainy events from 21<sup>st</sup> to the 27<sup>th</sup> of May. The maximal value of the rain intensity is 84 mm/h and the peak discharge is 3 m<sup>3</sup>/s. Concerning the other event, it is a summer storm of the 25<sup>th</sup> august with an initial dry state because the last recorded event is the 25<sup>th</sup> July. The maximal intensity of the rain is 156 mm/h and the peak discharge is 6.6 m<sup>3</sup>/s.

To simulate these events, we set the properties of the soil constant, modifying only the initial soil moisture ( $\theta_i$  in the infiltration model). The calibration gives the following soil parameters:

- first layer thickness:  $Z_C = 8$  cm,
- first layer porosity:  $\theta_1 = 0.35$ ,
- first layer conductivity:  $K_C = 20$  m/s,

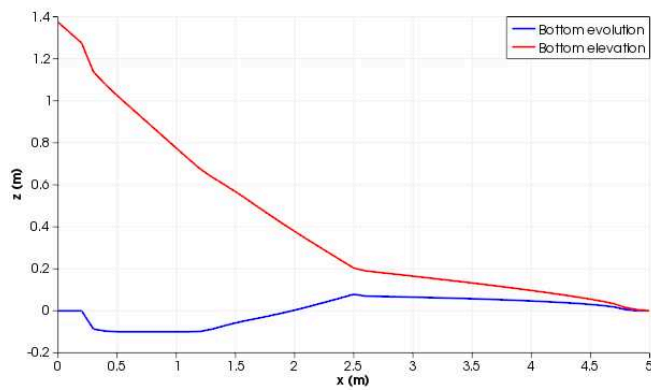


Figure 3. Bottom elevation and total evolution at the end of the simulation

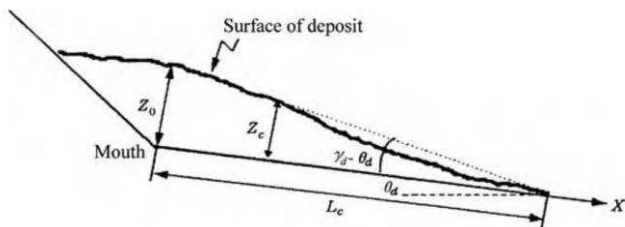


Figure 4. Side view of the shape of the deposition, experimental results from [19]

- second layer conductivity:  $K_s = 1$  m/s,
- second layer porosity:  $\theta_2 = 0.15$ ,
- capillarity head:  $h_f = 5$  cm.

Inspired by [12], the parameters are chosen to have a porous surface layer and a more structured base layer. The initial soil moisture is set to  $\theta_i = 0.22$  for the spring event and  $\theta_i = 0.03$  for the summer storm. The Fig. 5 presents the measured and simulated outlet discharges for the spring event and the Fig. 6 shows the results for the summer event.

The simulated results are in good agreement with the observations. Despite some important simplifications of the model, the outlet hydrograph are well represented. Indeed, the soil properties are considered as uniform in space, as well as the friction coefficient. In addition, the exfiltration is not represented and compensated by a low conductivity of the base layer. The representation of the vertical structure of the soil is sufficient to have a satisfactory reproduction of the hydrographs.

### C. Watershed gravity-driven erosion

The debris flow model is now applied to the Laval watershed. The goal is to observe quantitatively if the upstream sources are well represented and if the sediment are deposited in the hydraulic network. The model is tested on the summer event, because of its high rain intensity. The parameters used for the detachment and deposition are:

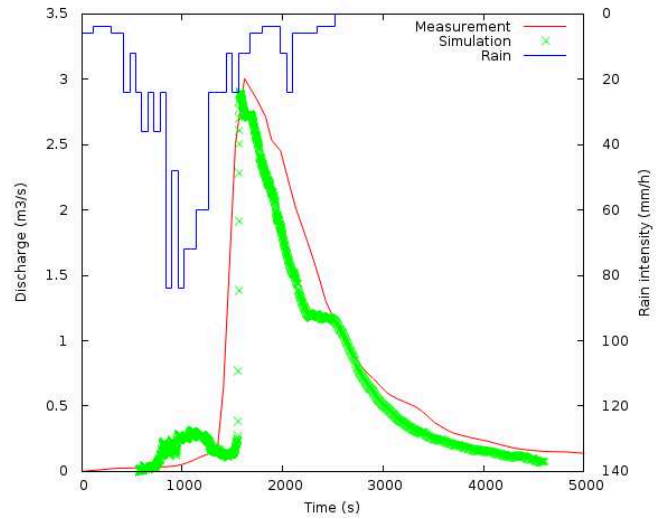


Figure 5. Discharge at the outlet of the Laval catchment, 29 May 2012 event: measurement vs simulation

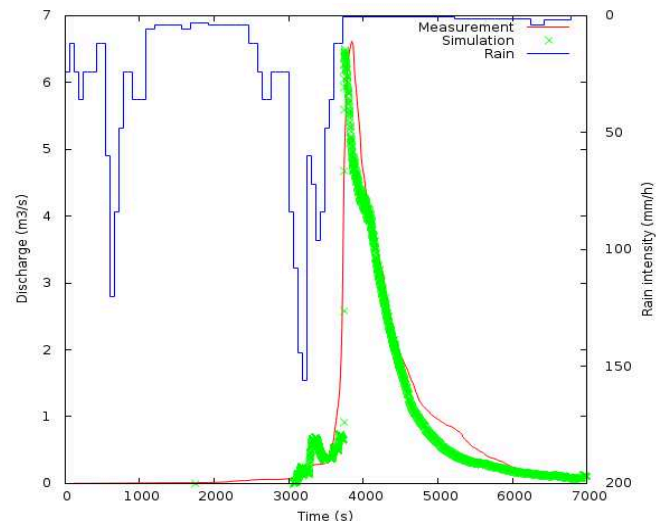


Figure 6. Discharge at the outlet of the Laval catchment, 25 August 2012: measurement vs simulation

- cohesive strength of the soil:  $c = 35$  Pa
- density of the sediment:  $\rho_s = 2650$  kg/m<sup>3</sup>
- density of the water:  $\rho = 1000$  kg/m<sup>3</sup>
- parameters:  $k = 0.85$  and  $m = 0.3$
- internal friction angle:  $\tan \varphi = 0.8$
- maximal concentration:  $C_* = 0.65$
- critical deposition velocity:  $u_c = 1$  m/s

The Fig. 7 shows the erosion and the deposition in the Laval catchment. The erosion is mainly localized at the upstream of the watershed, in the small gullies. Then the sediments are deposited in the main channels, distributed in the entire watershed.



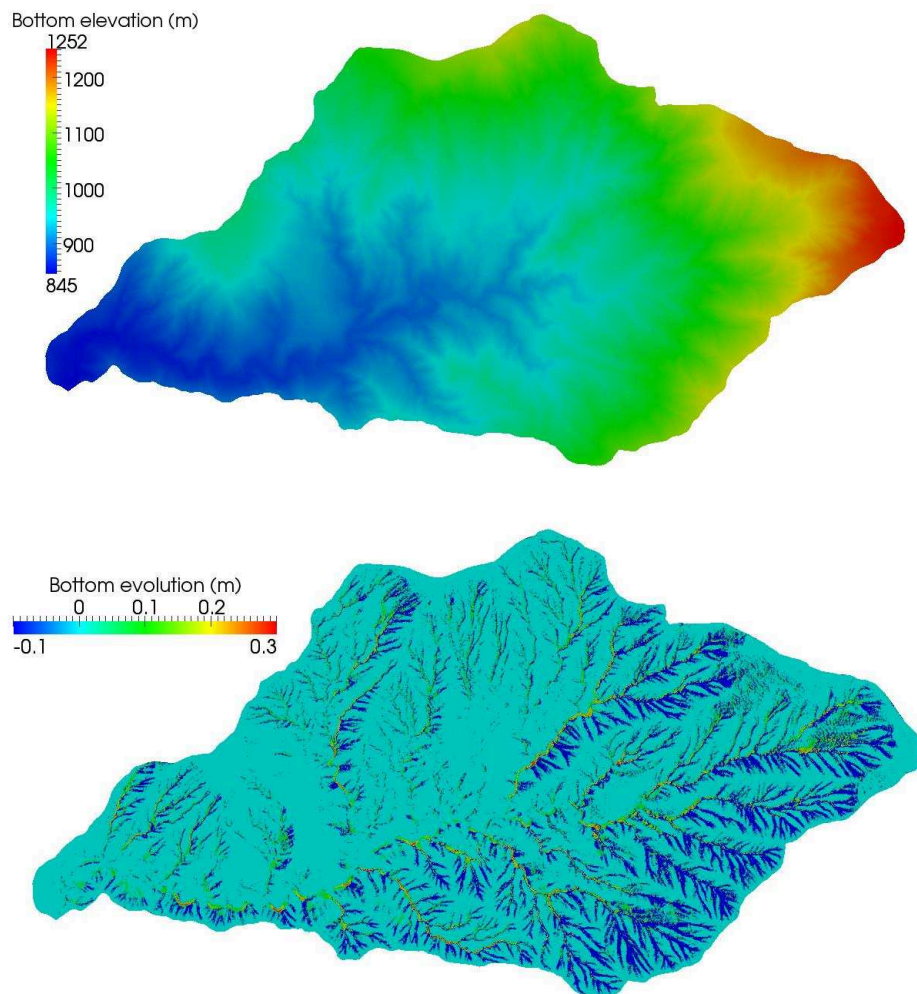


Figure 7. Bottom elevation and evolution at the end on the simulation in the Laval catchment

#### IV. DISCUSSION

The model presented above has many benefits to represent the erosion in mountain watershed. It is composed of models widely used, whose efficiency has been proven. The Green-Ampt infiltration model [8] can be found in a lot of physically-based hydrological models: CASC2D [10], WASIM [17], FullSWOF2D [4]. Furthermore, the accuracy of this model has been demonstrated at the plot scale [5], but also at the catchment scale, in comparison with other infiltration formulas [2]. Concerning the gravity-driven erosion model, the detachment formula is based on experiments with immature debris flows, which is the main phenomenon observed in small mountain watersheds [14]. Indeed, this study shown that for events with a rain intensity greater the 55 mm/h, it can contribute to more than 60% of the total erosion in the Draix catchments, including the Laval watershed. The shallow water equations have also shown their efficiency to simulate the motion of muddy or granular flows ([21], [7]), adding a Coulomb term to represent the friction of the solid fraction of the flow with the bottom. The

coupling method between the water runoff and the debris flow ensures that the hydrology of the watershed is slightly affected, and the infiltration and overland flow model stay efficient. Indeed, the system has the same properties as the Saint-Venant/Exner one, with a different dynamic in the solid discharge formulation. Another advantage of this method is that the deposition is located in the hydraulic network and can be provided to SISYPHE as amount susceptible to be eroded.

However, the main assumption of the model is the fact that the runoff on the upstream and the debris flows are moving in the same theoretical volume. Indeed, the bottom elevation considered for the water and the muddy flow is the same. This approximation raises a question concerning the validity of this hypothesis. This is justified by [9], saying that the aquifer responses can be described with the shallow water approximation. The shallow water equations are governing the aquifer responses while the debris flows is considered as a surface process on the upstream slopes. This approach still needs to be validated with the concentration measurement on the Draix watersheds.

## V. CONCLUSION

The TELEMAC computation code is adapted to river simulations and need to be adapted to simulate hydraulic and sedimentary transfers in mountain watersheds. In a precedent paper [18], a numerical scheme has been proposed to simulate the runoff on steep slope.

A derivation of the Green-Ampt hydrological model [8] is first presented. It is a 1D vertical model which give a precise representation of the properties of the soil. This model have been tested on the Laval watershed, on two different rainy events. The simulated and measured outlet discharges are in good agreement, keeping the same parameters of the soil properties and modifying only the initial soil moisture between the two events.

Then, a gravity-driven erosion model has been tested. It consists in simulated the debris flow formation, evolution and deposition in the model. On a simple test case, the behaviour of the debris flow is expected and the form of the deposition is close to what can be observed in the experiments. Then, on the Laval watershed, it is interesting to see that the debris flows are generated in the gullies upstream and deposited in the hydraulic network. The amount deposited can be consider as an input for SISYPHE to model erosion and sedimentation in the network.

This model still need to be validated. The concentration at the outlet are available on several events on the Laval watershed. Data on other catchment can also be used, like tracking of deposition and erosion on the main river event by event.

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