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ABSTRACT: When performing classical reliability analysis in practical geotechnical engineering subjective assumptions about the probability density function of parameters are often made because in many cases the results of geotechnical investigations are set valued rather than being precise and point valued. Alternatively, imprecise probability theories can be employed and Random Set Theory could be considered as a possible attractive candidate. To demonstrate the applicability and efficiency of Random Set Theory in combination with finite element analysis (RS-FEM) in geotechnical practice, a case study has been chosen, namely a tunnel excavation located in the south of Germany. It is shown that RS-FEM is an efficient tool to determine most likely bounds of the performance of a geotechnical structure being complementary to the observational method. Comparison between calculation and field measurements prove the applicability of the presented approach. In addition, the same problem has been analysed using a point estimate method. It is shown, that under certain assumptions, similar conclusions with respect to the expected behaviour of the tunnel can be obtained.

Keywords: finite element method, random set theory, tunnel, point estimate method

1 INTRODUCTION

Uncertainties in determination of the in situ soil profile and material parameters for individual soil or rock layers are one of the problems geotechnical engineers have to face. In this context it is important to realise that different sources of uncertainty exist, material parameters varying in a known range may be one of them but simply the lack of knowledge may be the more pronounced one. A rigorous mathematical treatment of all aspects of uncertainties is not straightforward and thus is commonly replaced in practice by engineering judgment. Recent theoretical developments and advances made in computational modelling allow for a more formal consideration of uncertainties supporting engineering judgment. It can be expected that theories and models taking uncertainty into account in the design of geotechnical structures, which could form a basis for comprehensive risk analyses, will be more appreciated in near future.

The random set theory developed by several authors (e.g. Dempster (1967), Kendall (1974), Shafer (1976), Dubois & Prade (1991)) has provided an appropriate mathematical model to cope with uncertainty overcoming some of the drawbacks of "classical" probability theory. Tonon et al. (1996, 2000a,b) demonstrated the application of Random Set Theory in rock mechanics and reliability analysis of a tunnel lining. Peschl (2004), Schweiger & Peschl (2007) have extended Random Set Theory to be combined with the finite element method, called Random-Set-Finite-Element-Method (RS-FEM). They illustrated the applicability of the developed framework to practical geotechnical problems and showed that RS-FEM is an efficient tool for reliability analysis in geotechnics in early design phases being highly complementary to the so-called observational method. For further details about basic concepts of RST and RS-FEM procedure the reader is referred to the work of e.g. Tonon & Mammino (1996), Schweiger & Peschl (2007) and therefore only a brief summary of the basic steps to be performed is given in the following for continuity. Finally a comparison with the point estimate method is presented.

2 RANDOM SET FINITE ELEMENT PROCEDURE

An advantage of the RS-FEM is that there is no need to modify the available Finite Element Codes, and any commercial FE software can be used for performing the required "deterministic" calculations. The steps that have to be followed in a RS-FEM procedure are summarized below and graphically illustrated in Figure 1.
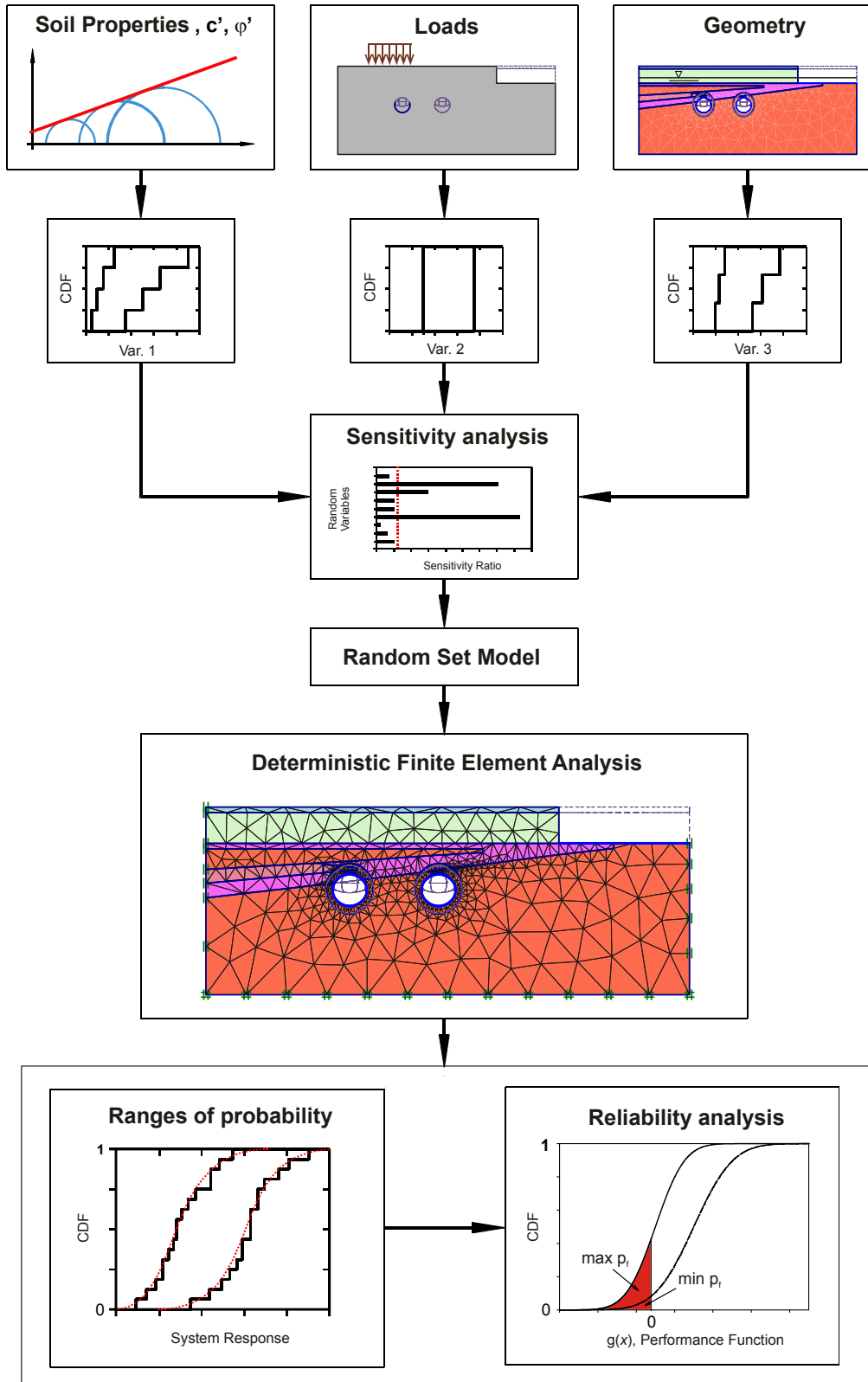

Figure 1. RS-FEM procedure (modified from Peschl, 2004)

Basic steps in setting up a Random Set Finite Element Model:

1. Definition of the geometry or geometries, in case a geometric feature is one of the basic variables, of the problem, preparation of the respective master file(s) and the FE model, selection of an appropriate constitutive model for soil/rock and support elements.

2. Selection of input parameters that should be considered as basic variables in the random set analysis providing the expected ranges from different sources of information (random sets).

3. Modification of the selected random sets, which show spatial variability considering the length of the possible failure mechanism in the model. This step requires the determination of spatial correlation length as well as an estimation of the length of the possible failure surface.

4. The computational effort of RS-FEM increases proportional to 2^n (n is the number of basic random variables). Thus employing a sensitivity scheme to identify the variables that have negligible effect on desired results even if there is a wide uncertainty on them is essential in order to reduce the computational effort.

5. Computation of the calculation matrix, which includes the defined parameter combinations, the preparation of deterministic FE input files and the relevant probability share of the individual calculation considering dependencies (correlation coefficients) between the basic random variables involved (see Peschl 2004).

6. Finite element calculations and determination of results such as stresses, strains, displacements and internal forces in support elements in terms of bounds of discrete cumulative probability functions, which may be compared to measured data once they become available. Subsequently fitting the resulted CDF's using best-fit methods, in order to achieve a continuous distribution function. For this step, commercial software such as the package @RISK® (Palisade, 2008) can be employed.

7. Definition of suitable performance functions. This definition is of paramount importance and is a crucial step in the analysis. For example a function can be defined over the critical deformations such as tunnel crown displacement to control the required clearance of the tunnel and/or maximum stresses carried by shotcrete lining. The performance function can be evaluated with results from the finite element calculations (bounds on continuous distribution functions of the evaluated system parameters), in order to obtain a range for the probability of failure or unsatisfactory performance. This is not considered in this paper, see e.g. Schweiger & Peschl (2005).

3 APPLICATION TO TUNNEL EXCAVATION

A tunnel application was chosen to demonstrate the applicability of the RS-FEM for these types of problems. The 460 m long tunnel located in Germany with a typical horse-shoe shaped section and dimension of 15x12.3 m width and height respectively, is constructed according to the principles of the New Austrian Tunnelling Method (NATM), and is divided into three main excavation stages top-heading, bench and invert. The overburden along the tunnel axis starts from 7.5 m in the portal region to a maximum of 25 m. A section with the overburden of 25 m was selected to apply the RS-FEM. The relevant 2D model geometry and finite element mesh including some model specifications are depicted in Figure 2. Approximately 900 15-noded triangular elements were employed in the model. The finite element software Plaxis has been used for all calculations (Brinkgreve et al. 2008) and the Mohr-Coulomb failure criterion is chosen as constitutive model in order to be able to compare results with conventional design analysis, which is however not topic of this paper.

   The Mohr-Coulomb parameters are derived from a Hoek-Brown classification which is based on test results performed on rock samples described in the site investigation report. The choice of the method for determining equivalent cohesion and friction angle is largely a matter of experience (Merifield et al. 2006) and there are two options to derive equivalent cohesion and friction angle from HB parameters. First, by fitting the MC failure line to the HB failure curve tangentially at a specific minor principal stress or normal stress or secondly a regression method can be applied over a dominant stress range of the problem to obtain average values of MC strength parameters. The latter has been adopted here and more details on the parameter determination can be found in Nasekhian (2011). The resulting parameter sets are summarized in Table 1 and a graphical representation of the random sets is provided in Figure 3. E_{rm} is the elastic modulus of the rock mass, φ and c are the usual strength parameters. R_f-values in Table 1 are pre-relaxation factors in order to account for 3D-effects in a 2D analysis. It should be noted that extreme values of R_f for top heading, bench and invert are used simultaneously.

Table 1. Random Set parameters

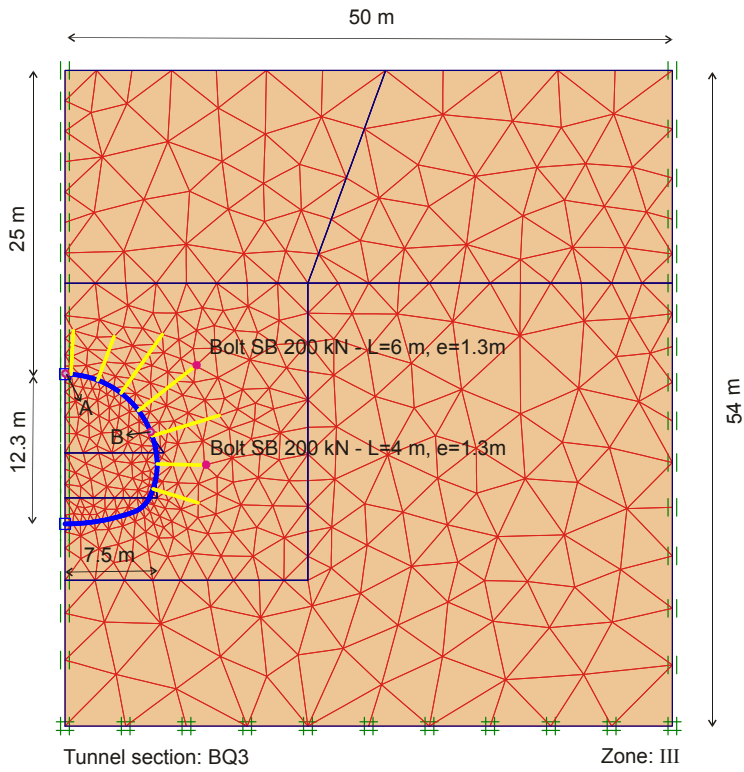| Variable	E_{rm}	φ	c	$R_{f,\ top\ heading}$	$R_{f,\ bench}$	$R_{f,\ invert}$
Unit	[MPa]	[°]	[kPa]	[-]	[-]	[-]
Set 1	1300-2300	22-32	450-750	0.4-0.6	0.3-0.5	0.2-0.4
Set 2	1900-3400	24-34	1000-1600	0.3-0.5	0.2-0.4	0.1-0.3

Figure 2. Finite element mesh used in RS-FEM analysis

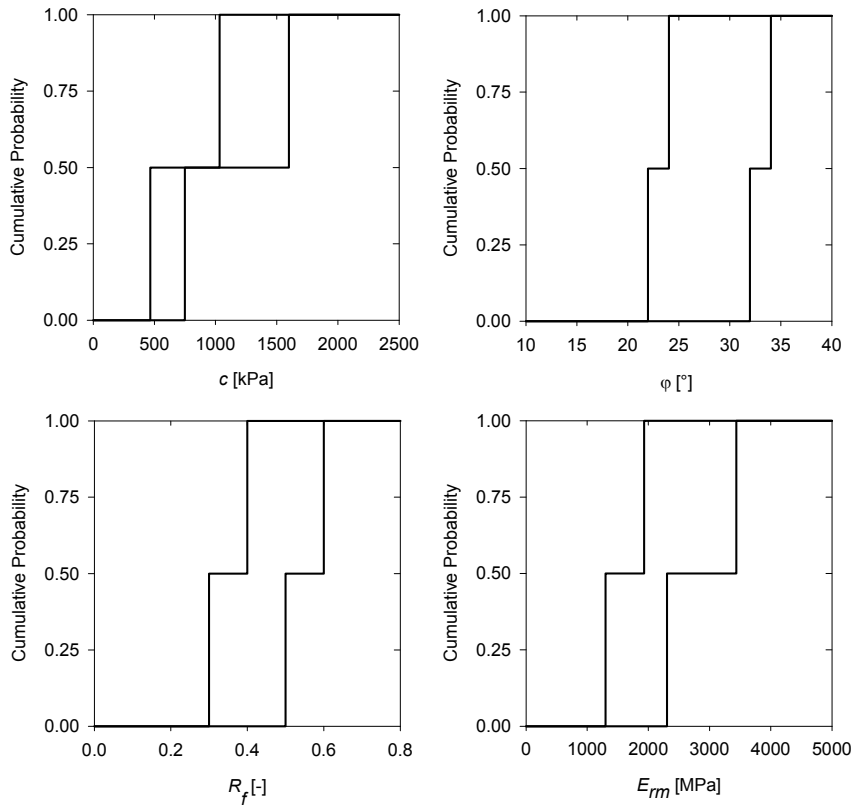Figure 3. Random sets: cohesion c, friction angle φ, R_f-value for top heading, elastic modulus of rock mass E_{rm}

4 RESULTS FROM RANDOM SET ANALYSIS

Some selected results of the system response are depicted in terms of discrete cumulative probability distributions in Figures 4 and 5. Points A and B denote points where measurements during construction have been obtained, A being at the tunnel crown and B at the tunnel sidewall respectively (see Figure 2). The measured values have been added in Figure 4. Since two observations were available within the considered "homogeneous" section, measurements can be presented in terms of discrete cumulative probabilities too, comprising of two steps. Figure 5 shows calculated maximal normal forces and bending moments in the lining, again in form of discrete cumulative probability distributions. It follows that measured values for crown displacements (Figure 4a) fall well inside the predicted range, but for point B measurements are slightly outside. However, considering the very small displacements this could possibly be attributed to uncertainties or inaccuracies in the measurements. Generally speaking, measurements have to be within the calculated bounds if the range assumed for the input parameters covers the true range of material behaviour and the chosen geotechnical and numerical model is appropriate. Leaving aside the one measurement which is outside the bounds it can be observed that an analysis in terms of worst case parameters would lead to very conservative results (upper bounds of displacements in Figure 4). However, the best case would be too optimistic. From other case histories (see e.g. Schweiger & Peschl 2005) it seems that measurements of displacements would tend to fall in the "lower third" of the predicted range.
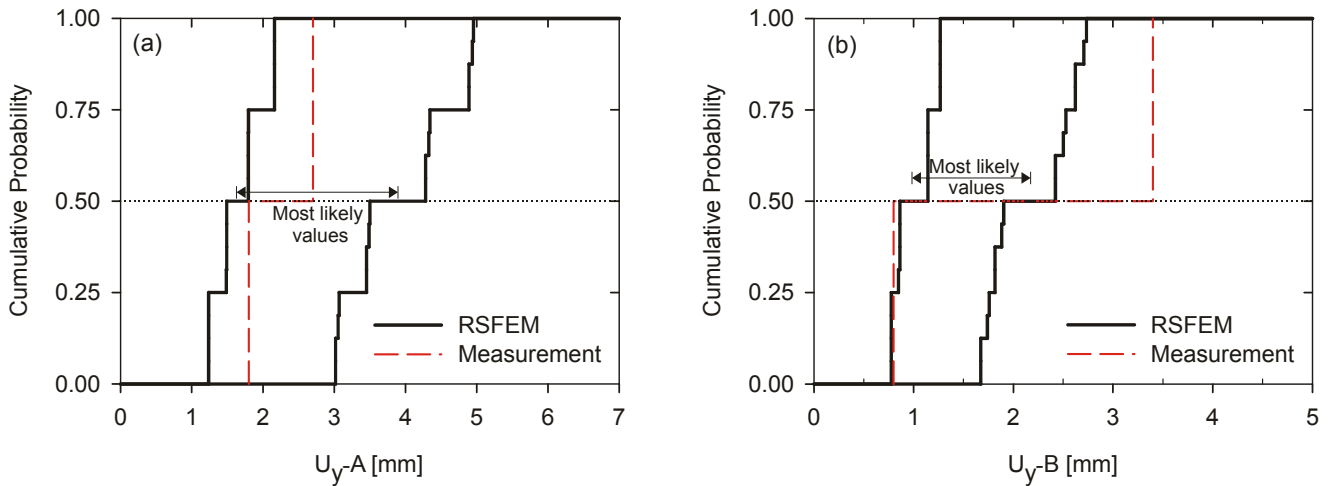

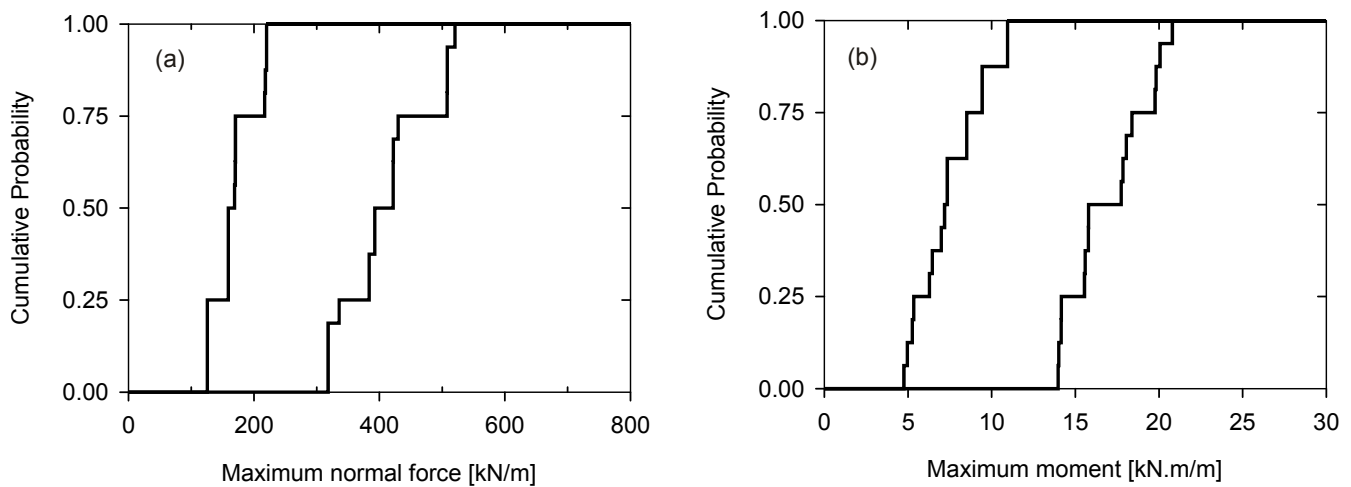Figure 4. RS-FEM results: a) vertical displacement crown (point A), b) vertical displacement side wall (point B)

Figure 5. RS-FEM results: a) maximum normal force in lining, b) maximum bending moment in lining

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5 COMPARISON WITH POINT ESTIMATE METHOD

In this section the results of two uncertainty models, namely the Point Estimate Method (PEM) and the Random Set Method, having different theoretical background, are compared with the measurements of the tunnel problem presented earlier. PEM has been first introduced by Rosenblueth (1975), but further developments have been published by a number of authors thereafter.

In the Point Estimate Method the continuous density distribution function $f_x(x)$ is replaced by specially defined discrete probabilities which are supposed to model the same low-order moments of $f_x(x)$. The determination of these moments is done by adding up the weighted discrete realisations. In Figure 6 this relationship is shown in a simplified way by the two realisations at x+ and x- represented with the corresponding weights w+ and w-. In the work presented here the approach suggested by Zhou and Nowak (1988) has been applied, employing a $2n^2+1$ (n is the number of basic variables) integration rule, which is considered as an optimum compromise between accuracy and computational effort (Thurner 2000). The method is extensively covered in the literature, e.g. Rosenblueth (1975), Lind (1983), Harr (1989), Hong (1998), Zhou and Nowak (1988) and therefore not described in further detail here.
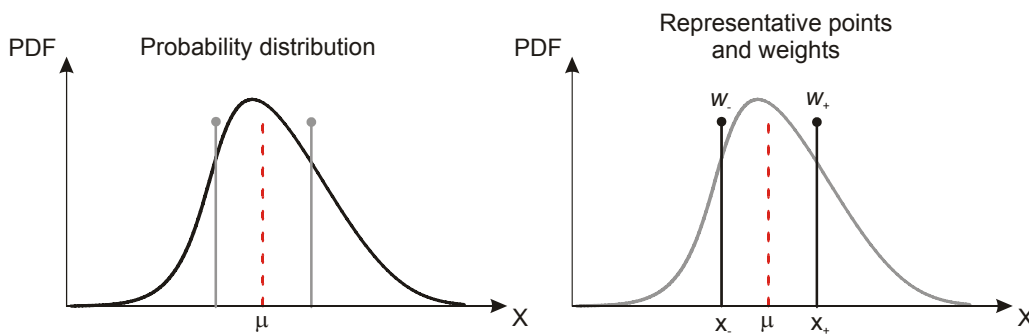

Figure 6. Simplified graphical representation of Point Estimate Method (after Thurner 2000)

In order to compare results from RS-FEM and PEM analyses a probability distribution function which reflects approximately the input data of the RS-FEM analysis is required. Due to the significant differences in the underlying concepts of the two approaches there is no rigorous way of doing this and some assumptions have to be made. The approach utilized here is as follows (for more details see Nasekhian 2011).

First a uniform distribution is constructed whose left and right extreme values are medians of left and right random set bounds respectively. Then, typical distributions are fitted and depending on the shape of the random bounds and the variable, one can select an appropriate distribution for further analysis. For instance, if the approach is applied to the friction angle using Triangular, Normal and Lognormal distributions as depicted in Figure 7 a considerable discrepancy between the different distribution types occurs; therefore, engineering judgment is necessary.

Although the random set exhibits some kind of symmetry on the right and left bounds, the selection of Lognormal looks more reasonable since it covers the whole range of random set values and it is also a commonly used distribution for friction angle always yielding positive values. In a similar way the distributions for the other random variables of the problem have been defined, they are summarized in Table 2.

Table 2. Basic random variables for PEM analysis

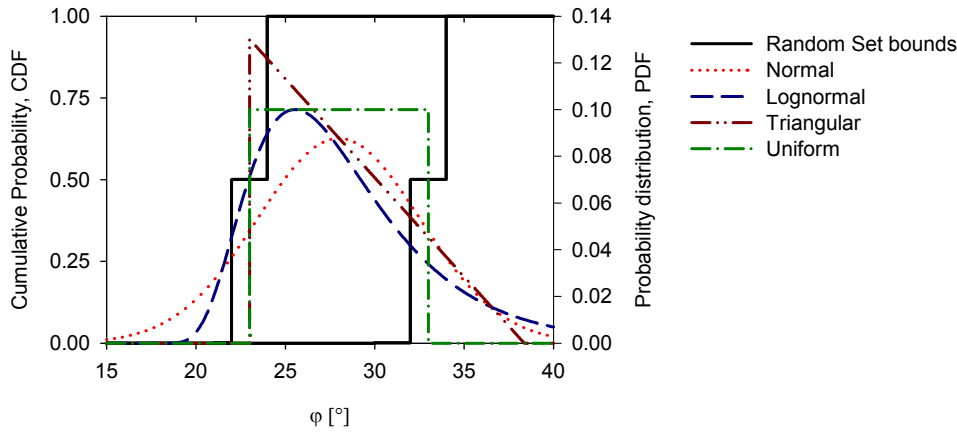| Variable	E_{rm}	φ	c	R_f
Unit	[MPa]	[°]	[kPa]	[-]
Distribution type	Lognorm.	Lognorm.	Normal	Normal
Mean	2256	28	950	0.45
Standard dev.	631	5	205	0.09
COV %	28	17.8	21.6	20

Figure 7. Comparison of assumed distribution for friction angle equivalent to the random set

A comparison of the results from PEM and RS-FEM analyses for crown settlements and maximum normal forces in the shotcrete lining is provided in Figure 8. It follows that displacements and normal forces show a good agreement with the RS-FEM results in the sense that PEM's results are supposed to present the 'true' distribution of the system response and should not exceed the random set bounds. In Figure 8a measured values and in addition intervals $\{-1.8\sigma < x < +1.8\sigma\}$ and $\{-3.0\sigma < x < +3.0\sigma\}$ obtained from PEM results are also shown. PEM only approximates the statistical moments of the system response and therefore a subjective probability distribution function should be selected for individual target values. On the other hand, it follows from probability theory (Pukelsheim 1994) that the interval $\pm 1.8\sigma$ represents the 86% confidence interval irrespective of the distribution of the target variable. Additionally, the range of results obtained from RS-FEM presenting the most likely behaviour is identified here where the cumulative probability of the lower and upper bounds is 0.5. It follows from this Figure that consistent conclusions can be drawn from both analysis in the sense that the "most likely" behaviour can be extracted from both approaches despite their differences in the underlying theories.
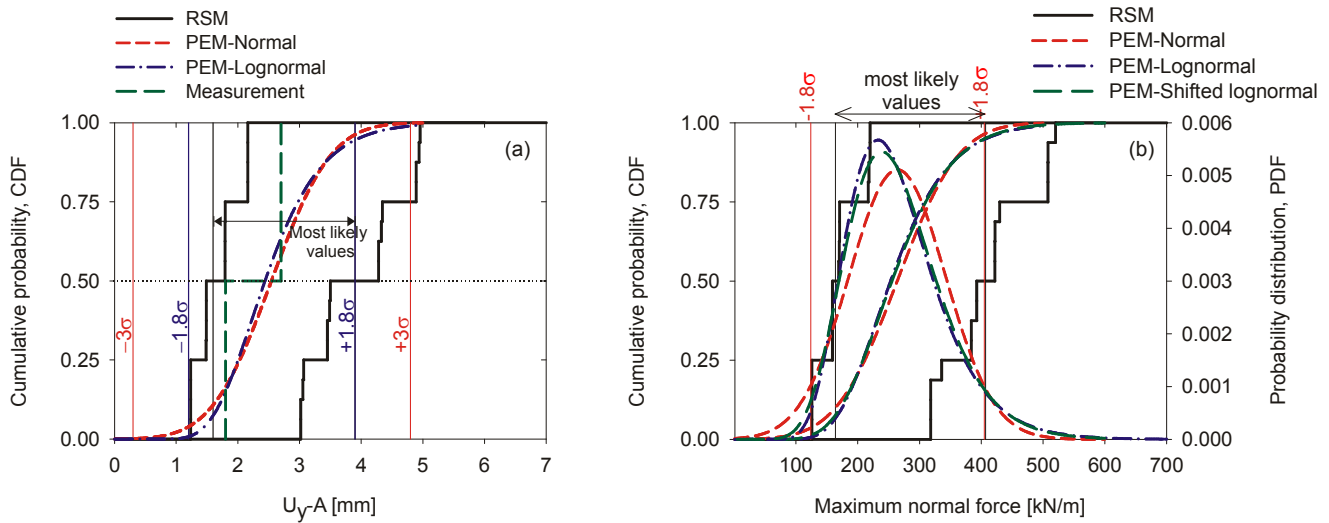

Figure 8. Comparison of results a) vertical displacement crown (point A), b) normal force in lining

6 SUMMARY AND CONCLUSIONS

A tunnel project was used to demonstrate the applicability of the RS-FEM framework in geomechanics where engineers have to face not only imprecise data but also lack of information. RS-FEM proved its capability of capturing uncertainty in a complex geotechnical application producing bounds of the expected system behaviour which compares reasonably well with measurements obtained during construction. The RE-FEM can account for epistemic and aleatory uncertainty and requires less computational effort as compared to fully probabilistic methods such as the Monte-Carlo simulation.

In this paper a comparison of RS-FEM and a point estimate method has been attempted and it could be concluded that despite the fundamentally different underlying theories comparable conclusions with respect to the expected system behaviour could be drawn for this particular problem.

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