MODELLING, SIMULATION AND EXPERIMENTAL VALIDATION OF NONLINEAR DYNAMIC INTERACTIONS IN AN ARAMID ROPE SYSTEM

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Abstract. *Vibration phenomena taking place in lifting and hoist installations may influence the dynamic performance of their components. For example, in an elevator system they may affect ride quality of a lift car. Lateral and longitudinal vibrations of suspension ropes and compensating cables may result in an adverse dynamic behaviour of the entire installation. Thus, there is a need to develop reliable mathematical and computer simulation models to predict the dynamic behaviour of suspension rope and compensating cable systems. The aim of this paper is to develop a model of an aramid suspension rope system in order to predict nonlinear modal interactions taking place in the installation. A laboratory model comprising an aramid suspension rope, a sheave/ pulley assembly and a rigid suspended mass has been studied. Experimental tests have been conducted to identify modal nonlinear couplings in the system. The dynamic behaviour of the model has been described by a set of nonlinear partial differential equations. The equations have been solved numerically. The numerical results have been validated by experimental tests. It has been shown that the nonlinear couplings may lead to adverse modal interactions in the system.*

1 INTRODUCTION

Vibration may influence all lift components and affect the ride quality of a lift. In particular, lateral and longitudinal vibrations of hoist, compensating ropes and travelling cables can occur and car vibrations may take place [1]. A lift car – suspension rope assembly is a nonlinear system subject to various excitations. These excitations are external and/or parametric/autoparametric terms. The external excitation appears as an inhomogeneous term in the equation of motion and/or a boundary condition [2]. The parametric excitation is formed by the time dependent coefficients of the equation which describes the system. Systems that experience autoparametric excitation cannot be modeled by linear equations and boundary conditions. Such excitation leads to instability due to the exponential growth of the response [3]. In systems described by nonlinear equations, the external resonance may not be as detrimental as if the system had been described by linear equations. Also when the external excitation is an additive of two or more of the system's natural frequencies, an external combination resonance may occur in which the peak of amplitude is surprisingly higher to that predicted by the linear theory [4]. The nonlinear behaviour of the ropes and cables have been studied extensively. Also various aspects of the dynamic behaviour of the lift car-rope suspension systems have been investigated by many researchers. However, a strong need exists to develop comprehensive models that account for the nonlinear couplings existing in the car – suspension rope installation. This is particularly important in lift systems used in high-rise applications due to the need to predict and to eliminate unwanted nonlinear resonances that affect the ride quality and the structural integrity. The aim of this paper is to develop a model of an aramid suspension cable system in order to predict nonlinear modal interactions taking place in the installation.

2 PROBLEM FORMULATION

2.1 Aramid cable system

The system under investigation is shown in Figure 1. In this system, an aramid cable of the modulus of elasticity E, cross-sectional area A and mass per unit length m is attached to a support at the right hand side and passes over a pulley of radius R forming an inclined section of length L_1. The cable then forms a vertical section of length L_2 and is tensioned by a suspended mass M_2. The second moment of inertia of the pulley is given as J and the angle of inclination of the cable is denoted as α.
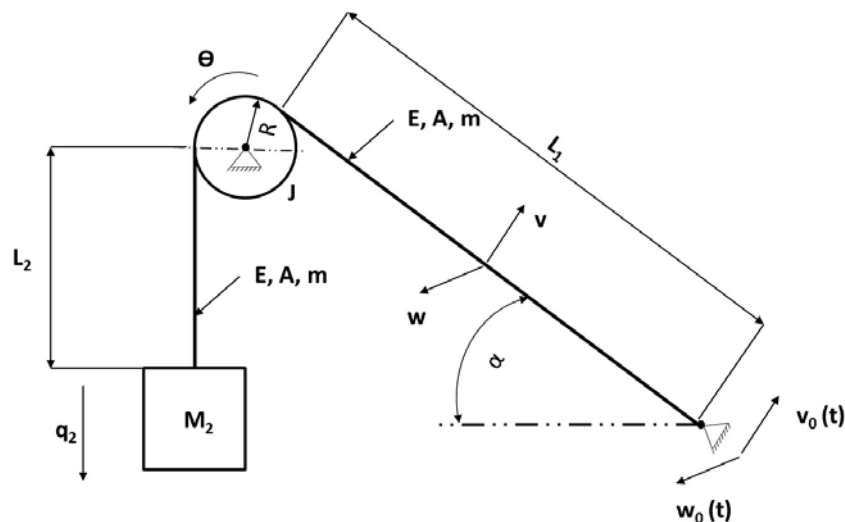

Figure 1: Aramid cable system.

2.2 Theoretical model

Hamilton's principle has been used to derive equations governing the undamped dynamic motions of the system. The following assumptions are made:

- The inclined cable section is taut and flat in the equilibrium position.

- The cable has negligible torsional rigidity and small but not negligible bending rigidity (with only small changes in the curvature taking place).

- Nonlinear stretching of the mid layer of the inclined cable is accounted for.

- No interaction takes place between the lateral modes and the longitudinal modes of the inclined cable section and the longitudinal inertia of the inclined cable can be neglected.

- The pulley is perfectly rigid and there is no slip of the cable across its surface.

- Only longitudinal motions are admitted in the vertical section of the system and the vertical section of the cable is treated as a linear spring.

The following differential equations of motion result:

$$mv_{tt} + EIv_{xxxx} - \left(T^i v_x\right)_x - \frac{EA}{L_1}\left[q_1 + \frac{1}{2}\int_0^{L_1}\left(v_x^2 + w_x^2\right)\right]v_{xx} = 0 \tag{1}$$

$$mw_{tt} + EIw_{xxxx} - \left(T^i w_x\right)_x - \frac{EA}{L_1}\left[q_1 + \frac{1}{2}\int_0^{L_1}\left(v_x^2 + w_x^2\right)\right]w_{xx} = 0 \tag{2}$$

$$M_1\ddot{q}_1 - k\left(q_2 - q_1\right) + \frac{EA}{L_1}\left[q_1 + \frac{1}{2}\int_0^{L_1}\left(v_x^2 + w_x^2\right)\right] = 0 \tag{3}$$

$$M_2\ddot{q}_2 + k\left(q_2 - q_1\right) = 0 \tag{4}$$

where $v(x,t)$ and $w(x,t)$ denote the in-plane and out-of-plane displacements of the inclined cable, x is a spatial coordinate measured from the bottom support along the cable, $(\ )_x \equiv \partial(\ )/\partial x$, $(\ )_t \equiv \partial(\ )/\partial t$, I is the second moment of area of the cable, M_1 denotes the effective linear mass of the pulley, $q_1(t) = R\theta(t)$, where $\theta(t)$ represents the angle of rotation of the pulley, $q_2(t)$ is the displacement of the suspended mass and T^i represents the mean tension in the cable given as

$$T^i(x) = M_2 g + m\left(x - L_1\right)g\sin\alpha \tag{5}$$

where g is the acceleration of gravity. Furthermore, $k = EA/L_2$ is the effective longitudinal stiffness coefficient of the vertical cable. The incline cable is subjected to periodic base excitation and the lateral displacements at $x = 0$ are defined as:

$$v(0,t) = v_0(t); \ w(0,t) = w_0(t) \tag{6}$$

where the out-of-plane base motion $w_0(t)$ is assumed to be small and at the pulley end the lateral displacements are zero. The lateral displacements of the cable are then expressed as

$$v(x,t) = V_0(x,t) + \sum_{n=1}^{N} \Phi_n(x)\, \xi_n(t)$$

$$w(x,t) = W_0(x,t) + \sum_{n=1}^{N} \Phi_n(x)\, \eta_n(t)$$

(7)

where $V_0(x,t)$, $W_0(x,t)$ are functions that satisfy the nonhomogeneous boundary conditions (6), $\xi_n(t)$, $\eta_n(t)$ are the nth modal coordinates and $\Phi_n(x)$ is the nth trial function assumed as $\Phi_n = \sin\dfrac{n\pi x}{L_1}$. Using (7) in Eqs. (1-3), treating the inclined cable as a simply supported tensioned beam, and applying the Galerkin method the following set of ordinary differential equations is obtained

$$\ddot{\xi}_r(t) + \lambda_r^2 \left\{ \hat{c}_r^2 + \bar{c}^2 + c^2 \left[\frac{q_1(t)}{L_1} + \frac{1}{2}\left(\left(\frac{v_0(t)}{L_1}\right)^2 + \left(\frac{w_0(t)}{L_1}\right)^2 \right) \right] \right\} \xi_r(t) =$$

$$-\sum_{n=1}^{N} \tilde{K}_{rn}\xi_n(t) - \left(\frac{\lambda_r}{2}c\right)^2 \xi_r(t) \sum_{n=1}^{N} \lambda_n^2 \left[\xi_n^2(t) + \eta_n^2(t) \right] + Q_r^v(t), \quad r = 1,2,\ldots,N$$

(7)

$$\ddot{\eta}_r(t) + \lambda_r^2 \left\{ \hat{c}_r^2 + \bar{c}^2 + c^2 \left[\frac{q_1(t)}{L_1} + \frac{1}{2}\left(\left(\frac{v_0(t)}{L_1}\right)^2 + \left(\frac{w_0(t)}{L_1}\right)^2 \right) \right] \right\} \eta_r(t) =$$

$$-\sum_{n=1}^{N} \tilde{K}_{rn}\eta_n(t) - \left(\frac{\lambda_r}{2}c\right)^2 \eta_r(t) \sum_{n=1}^{N} \lambda_n^2 \left[\xi_n^2(t) + \eta_n^2(t) \right] + Q_r^w(t), \quad r = 1,2,\ldots,N$$

(8)

$$\ddot{q}_1 + \omega_1^2 q_1 = \bar{\omega}_1^2 q_2 - \frac{EA}{4M_1} \sum_{n=1}^{N} \lambda_n^2 \left[\xi_n^2(t) + \eta_n^2(t) \right] + Q_1(t)$$

(9)

$$\ddot{q}_2(t) + \bar{\omega}_2^2 q_2(t) = \bar{\omega}_2^2 q_1(t)$$

(10)

where $\lambda_r = \dfrac{r\pi}{L_1}$, $\hat{c}_r^2 = \dfrac{EI}{m}\lambda_r^2$, $c^2 = \dfrac{EA}{m}$, $\bar{c}^2 = \dfrac{(M_1 - mL_1\sin\alpha)g}{m}$, $\omega_1^2 = \dfrac{\dfrac{EA}{L_1}+k}{M_1}$, $\bar{\omega}_1^2 = \dfrac{k}{M_1}$,

$\bar{\omega}_2^2 = \dfrac{k}{M_2}$ and the coefficients $\tilde{K}_{rn}$ together with the excitation functions acting upon the cable and the pulley, respectively, are defined as

$$\tilde{K}_{rn} = \frac{2g\sin\alpha}{L_1}\begin{cases} \dfrac{r^2\pi^2}{4}, & n = r \\[2ex] \left[\dfrac{nr\left(n^2+r^2\right)}{\left(n^2-r^2\right)^2}\right]\left[(-1)^{r+n}-1\right], & n \neq r \end{cases}$$

$$Q_r^v(t) = -\frac{2}{L_1}\left\{\left[\mathrm{X}_r - \frac{1}{L_1}\Pi_r\right]\ddot{v}_0(t) + \frac{g\sin\alpha}{L_1}\mathrm{X}_r v_0(t)\right\} \tag{11}$$

$$Q_r^w(t) = -\frac{2}{L_1}\left\{\left[\mathrm{X}_r - \frac{1}{L_1}\Pi_r\right]\ddot{w}_0(t) + \frac{g\sin\alpha}{L_1}\mathrm{X}_r w_0(t)\right\}$$

$$Q_1(t) = -\frac{EA}{2M_1}\left[\left(\frac{v_0(t)}{L_1}\right)^2 + \left(\frac{w_0(t)}{L_1}\right)^2\right]$$

where $\mathrm{X}_r = -\dfrac{L_1}{r\pi}\left[(-1)^r - 1\right]$ and $\Pi_r = -(-1)^r\dfrac{L_1}{r\pi}$.

It is evident from Eqs (1-2) that the in-plane and out-of-plane motions of the cable are coupled through cubic geometric nonlinear terms arising due to the effect of stretching. The pulley response q_1 and the base excitation motions v_0, w_0 affect the stiffness of the cable and appear as autoparametric and parametric excitation terms, respectively. The pulley response is coupled with the cable motions through the quadratic terms ξ_n^2, η_n^2, respectively. There is a linear coupling between the response of the end mass M_2 and the response of the pulley.

Component	Symbol	Value	Unit
Effective pulley mass	M_1	5.45	kg
Suspended mass	M_2	55.5	kg
Cable diameter	d	9.53	mm
Cable mass density	ρ	1437.5	kg/m^3
Length of inclined cable	L_1	2.98	m
Length of vertical cable	L_2	2.46	m

Table 1: Fundamental parameters of the system.

3 EXPERIMENTAL SETUP

An experimental rig was fabricated to carry out experimental tests, The configuration of the experimental rig is shown in Figure 2. The rig comprises a rigid metal structure hosting the aramid cable suspension system. The aramid cable passes over the pulley which is mounted within the structure. The cable is excited in the lateral in-plane direction by an electrody-

namic shaker (the LDS Model V406 with PA100E power amplifier). The shaker is attached to a steel base which is mounted on the floor. The shaker table is attached to the cable termination at the lower boundary. The B&K Pulse^{TM} DAQ system is used to generate the excitation signal and to acquire data from four acceleration sensors. The accelerometers (B&K Model 4507/8) are mounted on the cable, the pulley and the suspended mass structure in order to measure the response of the system.. The experimental setup is depicted in Figure 2.
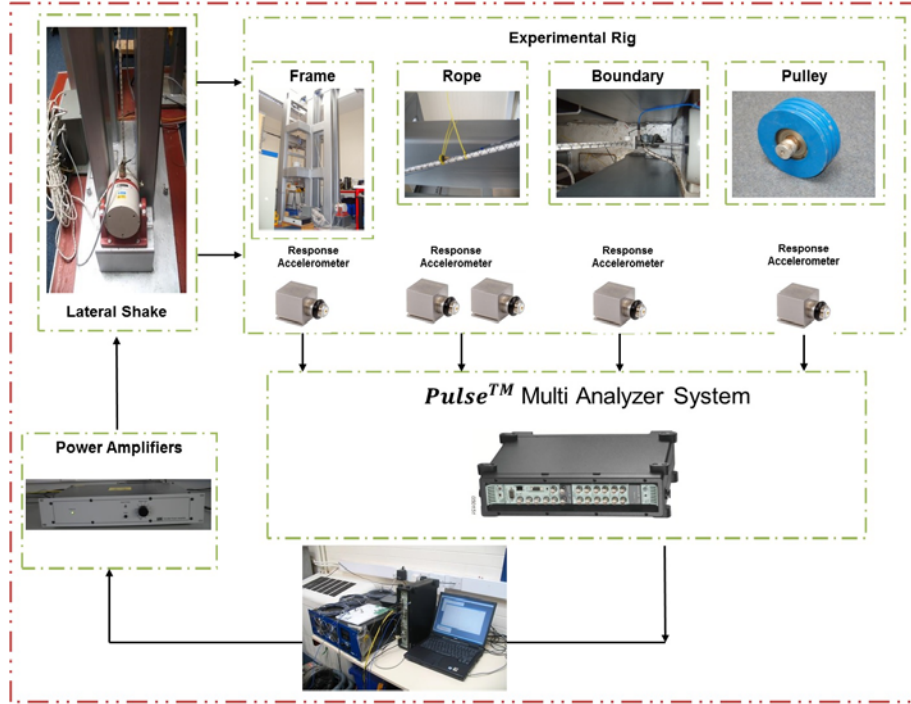
Figure 2: Rig and Testing Arrangement

4 RESULTS

The response of the system has been investigated through numerical simulation and experimental tests. The numerical simulation has been based on Eqs. (7-10). The main parameters of the system used in the simulation are presented in Table 1. The system is subjected to the in-plane base motion excitation functions $v_0(t)$, $\ddot{v}_0(t)$ shown in Figure 3(a) and 3(b), respectively. The base motion applied in the tests is of the frequency 13.8 Hz equal to the fundamental frequency of the inclined cable. The excitation has been measured during the experimental tests and then used in the numerical simulation. An explicit Runge-Kutta (4,5) formula has been applied to integrate the differential equations Eqs. (7-10). The simulations have been carried out assuming linear damping, with the damping ratios for the cable assumed as 0.75% across all modes and the damping ratios at the pulley - suspended mass are taken as 4%. The diagrams shown in Figure 4(a) and 4(b) represent the in-plane and out-of-plane displacements of the cable calculated at $x = L_1/3$ (blue curve) superimposed on the displacements determined from the experimental tests. The calculated (simulated) response of the cable agree well with the displacements measured during the experimental tests. The presence of cubic nonlinear coupling, as evident from the nonlineaar terms in Eqs (7-8), promotes conditions for autoparametric (internal) 1:1 interactions between the in-plane and out-of-plane modes. Initially the response of the cable is planar but due to the internal resonance the planar motions become unstable resulting in large non-planar tubular (whirling) motions of the cable as shown in Figure 5.
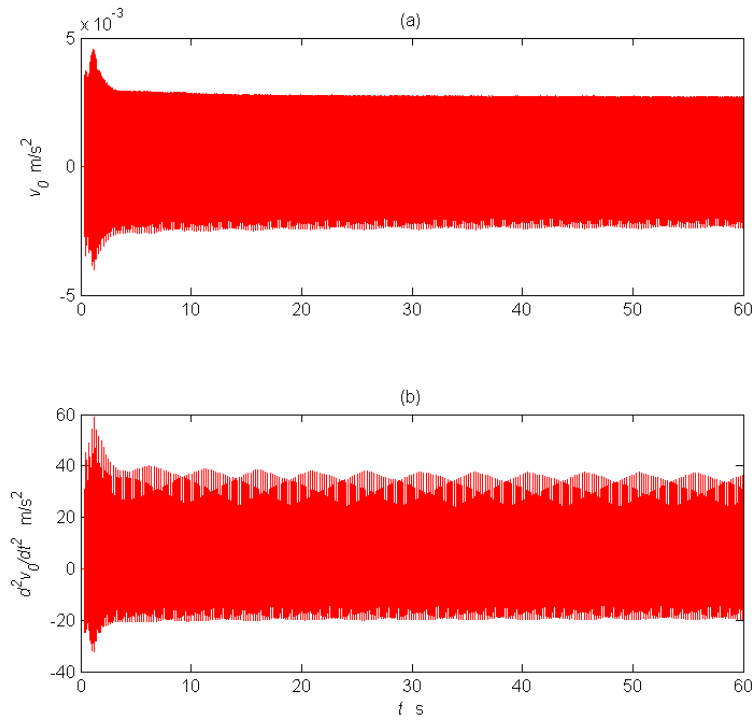
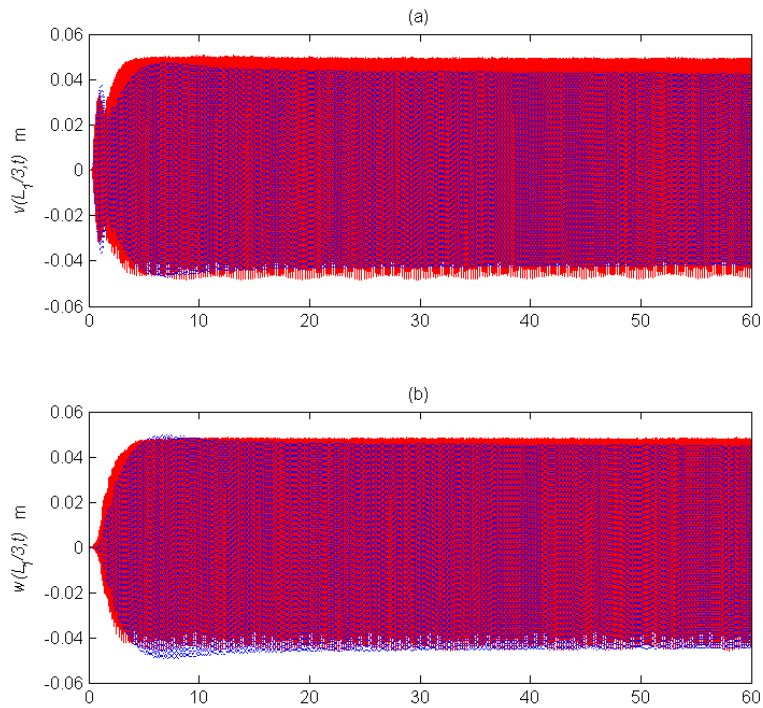Figure 3: Base excitation (a) displacement (b) acceleration.

Figure 4: Measured and simulated cable responses at $x = L_1/3$ (a) in-plane displacements and (b) out-of-plane displacements.
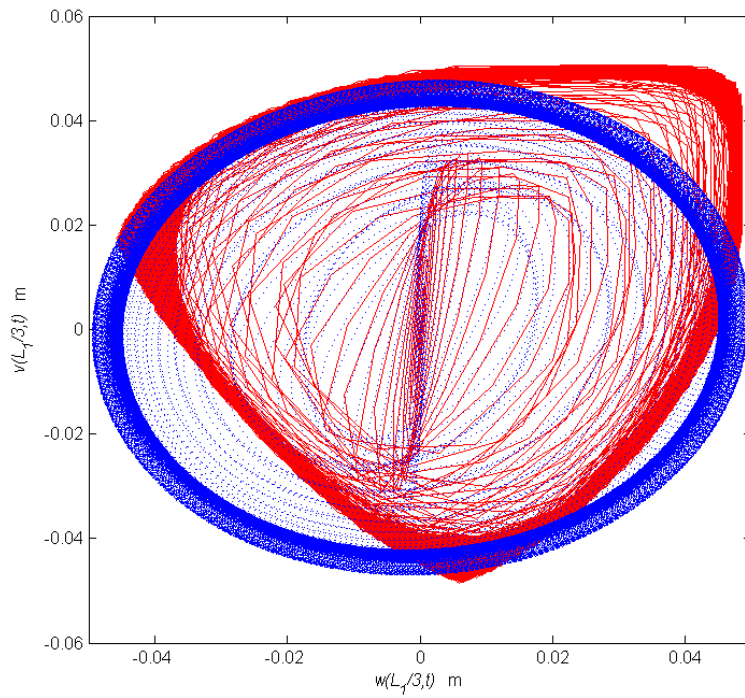
Figure 5: Non-planar tubular (whirling) motions of the cable.
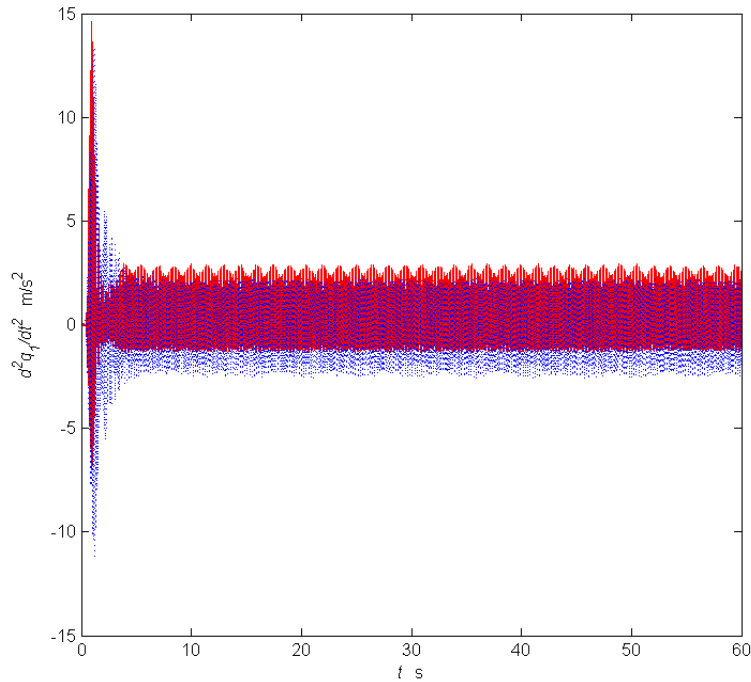

Figure 6: Pulley acceleration time response, measured (red line) and simulated (blue line).

Figure 6 and 7 show the time records and the corresponding FFT spectra of the tangential ac-celeration of a point on the circumference of the pulley (measured response - red line and

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simulated response - blue line). It is evident that the main frequency component of the response of the pulley (approximately 28Hz) is of twice the frequency of the fundamental frequency of the inclined cable.
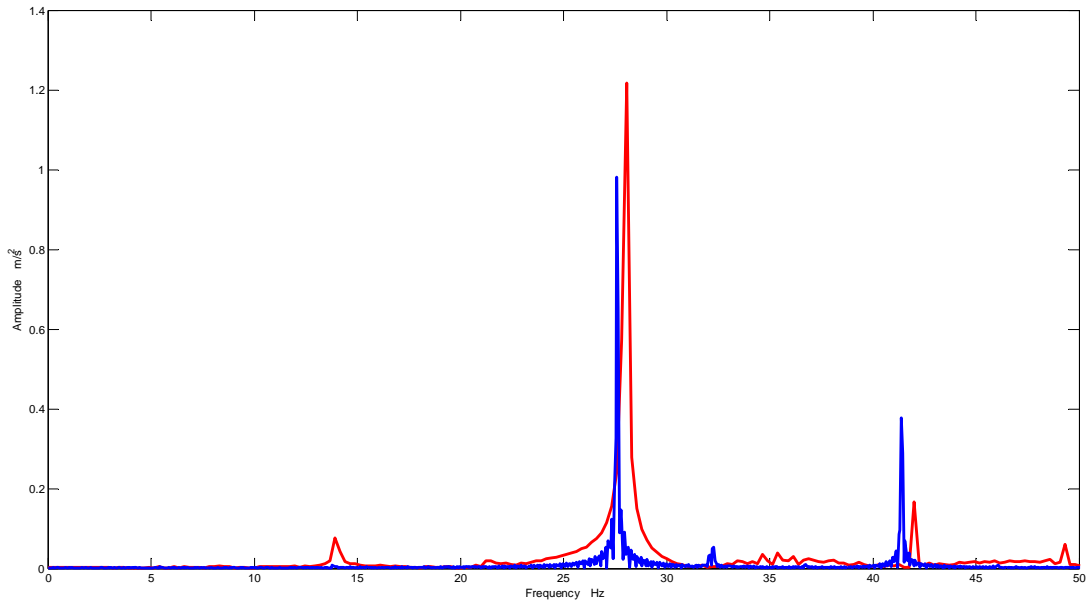

Figure 7: FFT spectra of the measured acceleration signal (red line) and simulated acceleration (blue line) of a point on the circumference of the pulley.
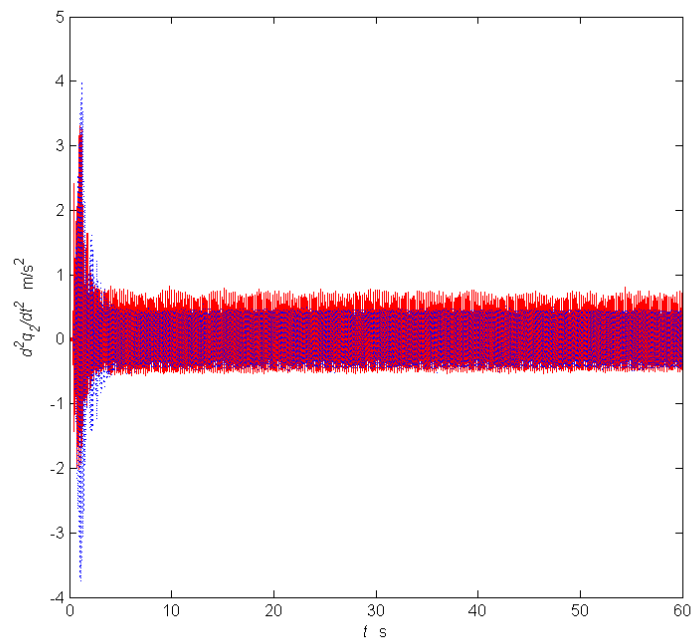

Figure 8: Suspended mass response $\ddot{q}_2(t)$.

The measured (read line) and simulated (blue line) acceleration time responses of the suspended mass $\ddot{q}_2(t)$ are presented in Figure 8. There is a good agreement between the theoretical predictions and the experimental results.

5 CONCLUSIONS

Numerical simulation studies combined with experimental tests have been used to investigate the dynamic behaviour of the aramid cable system subjected to a strong excitation acting in one plane. There is a good agreement between the numerical simulation predictions, based on a nonlinear model of the system, and the results of experimental tests. Due to the nonlinear couplings in the system the inclined cable develops whirling motions under external resonance excitation conditions and autoparametric resonance interactions that exist in the system. The pulley responds at the frequency equal to twice the fundamental frequency of the cable which results in the principle parametric resonance. The model developed in the course of this work can be applied to design a suitable vibration control algorithms to mitigate the effects of nonlinear resonance interactions in the system.

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