# PREDOMINANT PERIODS OF MULTI-DEGREE-OF-FREEDOM-SYSTEM ANALYSIS AND DYNAMIC SOIL-STRUCTURE INTERACTION FOR BUILDING STRUCTURES

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**Abstract.** Coupled problems of the multi-degree-of-freedom-system (MDOF) or Soil-Structure Interaction (SSI) are usually translated to a series of the single-degree-of-freedom (SDOF) equations. In this paper, the predominant periods of MDOF analysis are analyzed without SDOF and the dynamic amplification factors for a sample data of a model building are shown. The analysis method is the assumption that, as well as SDOF analysis, the MDOF analysis is applied to the predominant periods by inelastic analysis. That can give the dynamic amplification factors of the MDOF. At the results, it is cleared that the predominant period of the MDOF is close to be the period by the eigenvalues and in the short period or in the high level modes, the dynamic amplification factors are high which should not be neglected.

Moreover, soil-structure interaction with SDOF are also analyzed by elastic analysis for the model building. Some effects to the building structure model by the rocking on the ground are discussed in the sway-rocking models.

## 1 INTRODUCTION

In the single-degree-of-freedom (SDOF) analysis, the mass, the damping coefficient and the stiffness can give the predominant periods and the response values. In the multi-degree-of-freedom-system (MDOF) analysis, after the mass matrix $[M]$ , the damping coefficient matrix $[C]$ and the stiffness matrix $[K]$ without any external forces can give the eigenvalues, the modes and others, the MDOF model is usually translated to a series of the SDOF models and the response values are analyzed. In the MDOF analysis, the predominant periods of the MDOF system with external forces are not usually discussed.

Therefore, in this paper, the predominant periods of the MDOF of the shear system of 3 mass with external forces of the periodic motions are analyzed. This analysis can give the response values of each mass in any periods of external forces, the predominant periods of each mass, by elastic and inelastic analysis. The dynamic amplification matrix is proposed.

Moreover, soil-structure interaction (SSI) with the SDOF is also analyzed by elastic analysis for the model building. Some effects of SSI to the building structure model are discussed in the sway-rocking model. The external force vector including moment is proposed.

## 2 PREDOMINANT PERIOD OF MDOF ANALYSIS

The predominant periods of the MDOF analysis are analyzed and the dynamic amplification ratios are shown. In this analysis, as well as the periodic motion $\ddot{y}$ (=$p\sin\omega t$) in the external force of SDOF analysis in equation (1), the periodic motion in the external force of MDOF analysis is applied in equation (2).

$$m\ddot{x} + c\dot{x} + kx = -mp\sin\omega t \tag{1}$$

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{\ddot{y}\} \tag{2}$$

Where
[$M$], [$C$], [$K$] : $n$-by-$n$ square matrix
$\{\ddot{x}\}$,$\{\dot{x}\}$,$\{x\}$: $n$-by-1 column matrix of response values of acceleration, velocity or displacement of MDOF
$\{\ddot{y}\}$ : $n$-by-1 column matrix for external force ($\ddot{y}$ =$p\sin\omega t$) (cm/sec$^2$)
$p$ : constant value (cm/sec$^2$)
$\omega$ : angular frequency of the periodic motion (rad./sec)

The system of the response values is also applied to be the periodic motion in equation (3). Equation (3) is the particular solution of equation (2) by the method of undetermined coefficient which has the assumption that $\{x\}$ on i-th storey is linear combinations of shaking functions <$\{x_i\}$= $\{c_{1i}\sin\omega t+c_{2i}\cos\omega t\}$ > (equation (3)), in which $\{c_{1i}\}$ and $\{c_{2i}\}$ are the undetermined coefficients. Equation (3) is substituted for equation (2) to make eqaution (4). Equation (4) is an identity in any time and can be solved to equation (5), (6), (7) or (8) which give the solution $\{x_i\}$ for the reponse displacement of MDOF.

The each element of $\{x_i\}$in equation (6) includes the periodic motion in $\{\ddot{y}_{\phi i}\}$ calculated by the shaking functions with the angular frequency $\omega$ and the phase lag $\phi_i$ on i-th storey. The dynamic amplification matrix [$A$], which doesn't include any shaking function, is calculated by [$M$], [$C$], [$K$] and $\omega$. The multiple of [$A$] and [$M$] can be amplitudes of the periodic motion $\{\ddot{y}_{\phi i}\}$. Therefore, equation (6) gives the predominant periods of the MDOF by the spectral analysis of $\omega$.

$$\begin{Bmatrix} \vdots \\ x_i \\ \vdots \end{Bmatrix} = \begin{Bmatrix} \vdots \\ c_{1i}\sin\omega t + c_{2i}\cos\omega t \\ \vdots \end{Bmatrix} \tag{3}$$

$$\begin{aligned} &-\omega^2[M](\{c_{1i}\}\sin\omega t + \{c_{2i}\}\cos\omega t) + \omega[C](\{c_{1i}\}\cos\omega t - \{c_{2i}\}\sin\omega t) \\ &+[K](\{c_{1i}\}\sin\omega t + \{c_{2i}\}\cos\omega t) = -[M]\{p\}\sin\omega t \end{aligned} \tag{4}$$

$$\begin{cases} \{c_{1i}\} = [A_1][M]\{p\} \\ \{c_{2i}\} = [A_2][M]\{p\} \end{cases} \tag{5}$$

$$\{x_i\} = [A][M]\{\ddot{y}_{\phi i}\} \tag{6}$$

where

$[A]$ : $n$-by-$n$ square matrix. $[A]$ is a part of the particular solution and also the dynamic amplification matrix, which is calculated by $[M]$, $[C]$, $[K]$ and $\omega$

$[A_1]$, $[A_2]$: $n$-by-$n$ square matrix. A part of $[A]$

$\{\ddot{y}_{\phi i}\}$: $n$-by-1 column matrix for a periodic motion part of the particular solution with the phase lag $\phi_i$ on i-th storey, $\{\ddot{y}_{\phi i} = p\sin(\omega t + \phi_i)\}$ (cm/sec$^2$)

$$\begin{Bmatrix} \vdots \\ x_i \\ \vdots \end{Bmatrix} = \begin{Bmatrix} \vdots \\ \sqrt{c_{1i}^2 + c_{2i}^2}\sin(\omega t + \phi_i) \\ \vdots \end{Bmatrix} \tag{7}$$

$$\begin{cases} \sin\phi_i = \dfrac{c_{2i}}{\sqrt{c_{1i}^2 + c_{2i}^2}} \\ \cos\phi_i = \dfrac{c_{1i}}{\sqrt{c_{1i}^2 + c_{2i}^2}} \end{cases} \tag{8}$$

In the analysis of SDOF, the complementary solution of equation (1), by the assumption that the external force is zero, can give the predominant period $\omega_{sgl} = \sqrt{k/m}$ and the particular solution by the method of undetermined coefficient of equation (1) can give the dynamic amplification ratio $\{\mu_S\}$ explicitly by the ratio of the dynamic amplification to the static one in equation (9). The spectral relationship between $\omega/\omega_{sgl}$ and $\mu_S$ of equation (9) is well known as a curve line with one peak.

$$\{\mu_S\} = \frac{k}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \tag{9}$$

However, in the analysis of MDOF, the dynamic amplification matrix $[A]$ of equation (6) doesn't explicitly show the dynamic amplification ratio, likely equation (9) of SDOF. Moreover, the complementary solution of equation (2), can give the eigenvalues and the eigenvectors. These eigenvalues are considered to be the square of the angular frequency in the solutions and usually the high level modes are negleted in the analysis.

In order to know what the eigenvalues are in the analysis of MDOF, one example of a stucture data is as follows.

## 3 EXAMPLE FOR PREDOMINANT PERIOD OF MDOF ANALYSIS

### 3.1 Sant'Agostino in L'Aquila, Italy

The structure is a 2 storeys masonry sotructure heritage with a lantern storey. İt is assumed to be the shear system of 3 mass.

In the 2009 Italy L'Aquila Earthquake, Abruzzo, Italy, many monuments are damaged. The complex monument and church of Sant'Agostino in L'Aquila, Italy is one of these monuments and the very valuable chance was given to see the damage of it in L'Aquila. The typical damage of Sant'Agostino is the lantern fallen down to the roof of the next building in the West and severe damages in the ellipse dome or the walls. In this analysis, some structural identification was made by the shear system of 3 mass (Figure 1.) [1].

The inelastic dynamic response analysis needs the weight, the stiffness and the shear coefficient of each storey. The size of Sant'Agostino were measured based on the Google satellite map and plan or elevation figures in some references of Sant'Agostino. The Guideline for the Construction Technical Law [2], Italy is also referred for the material properties. The specific gravity is assumed to be 19 (kN/m$^3$) and the shear stiffness is assumed to be 23.0 (kN/cm$^2$). The period of the 1st storey is assumed to be 1.0 (sec) and the structural identification for the appropriate period and shear coefficient of each storey were made to follow the collapse of the lantern fallen down in the West. According to the drawings, the height is considered to be 41m. The weight of each storey is based on the wall areas of planning and the height of each storey to calculate the wall volume of each storey, decreasing of the openings, multiplying the specific gravity. The periods of each storey are based on the period of the 1st storey and the assumption of the linear relationship between the period $T$ and the height of other storeys. The shear coefficients of each storey are decided by the assumption that the storey drift angles of the 1st and 2nd storey at the yielding point are nearly 1/1,000 (rad.) and the one of the 3rd storey (the lantern storey) is nearly 1/500. These storey drift angles, the periods or the stiffness give the shear coefficient.

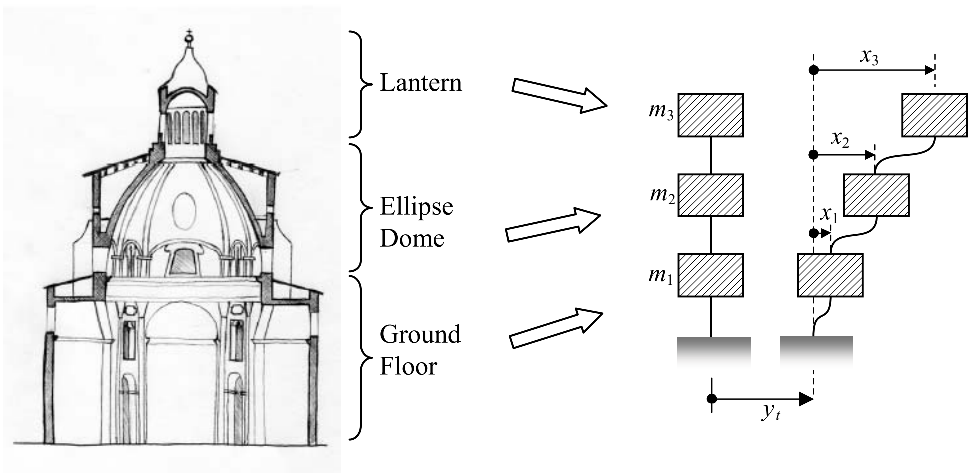



**Figure 1:** Cross section transverse in nearly EW direction and analysis model for shear system of 3 mass

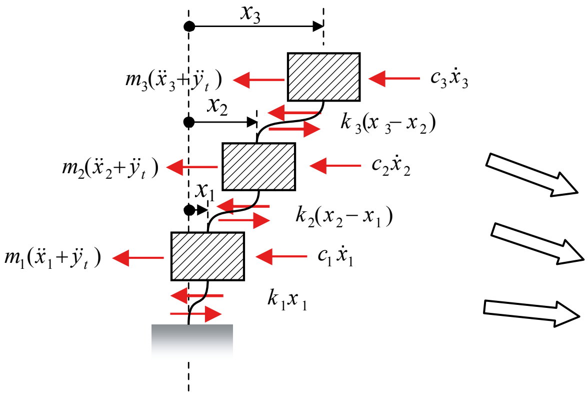



**Figure 2:** Analysis model for shear system of 3 mass

**Table 1:** Example values of stiffness and period of each storey for shear system of 3 mass

| Storey | Stiffness $k_i$ (kN/cm) | Period $T_i$ (sec) |
|---|---|---|
| 3 | 234 | 0.40 |
| 2 | 2,004 | 0.80 |
| 1 | 6,069 | 1.00 |

$$\begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix}\begin{Bmatrix} \ddot{x}_3 \\ \ddot{x}_2 \\ \ddot{x}_1 \end{Bmatrix}+\begin{bmatrix} c_3 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_1 \end{bmatrix}\begin{Bmatrix} \dot{x}_3 \\ \dot{x}_2 \\ \dot{x}_1 \end{Bmatrix}+\begin{bmatrix} k_3 & -k_3 & 0 \\ -k_3 & k_3+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_1 \end{bmatrix}\begin{Bmatrix} x_3 \\ x_2 \\ x_1 \end{Bmatrix}=-\sin\omega t\begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \quad (10)$$

where

Period of i-th storey $: T_i = 2\pi\sqrt{\dfrac{\sum_{j=i}^{n} m_j}{k_i}}$ [sec]

Damping coefficient of i-th storey $: c_i = 2h\omega_i m_i = 2hm_i\sqrt{\dfrac{k_i}{\sum_{j=i}^{n} m_j}}$ $\left[\text{kN}\dfrac{\text{sec}}{\text{cm}}\right]$

The matrix equation of motion in forced shakings with external dampings is shown in equation (10). The external forces are the periodic motions.

Figure 2 shows the analysis model. Table 1 shows the stiffness $k_i$ and the period $Ti$ of each 3 storeys. According to the notes of equation (10), the period of i-th storey is calculated by the stiffness $k_i$ of i-th storey and the summation of the mass upper than i-th storey. The damping coefficient of i-th storey is calculated by the damping factor $h$ , the stiffness $k_i$ and the mass $m_i$ of i-th storey and the summation of the mass upper than i-th storey. The damping factor $h$ is 0.05.

Equation (10) and (6) can give the dynamic amplification $\{x_{DA}\}$ and the static amplification $\{x_{SA}\}$ in equation (11-1) and equation (11-2). The ratio $\{\mu_D\}$ of the dynamic amplification to the static one in each storey is shown in equation (12).

Figure 3 shows the spectral analysis results for $\omega$ ($\omega$=2π/$T$) of the relationship between the dynamic amplification factor $\mu_D$ and the period $T$ (sec) for the spectral analysis. Figure 3 also shows the periods calculated by the eigenvalues which are given by equation (13). Equation (13) means the matrix equation of damped free motion with external damping.

$$\{x_{DA}\}=[A][M]\{1\}=\begin{Bmatrix} x_{DA3} \\ x_{DA2} \\ x_{DA1} \end{Bmatrix} \quad (11\text{-}1), \qquad \{x_{SA}\}=[K]^{-1}[M]\{1\}=\begin{Bmatrix} x_{SA3} \\ x_{SA2} \\ x_{SA1} \end{Bmatrix} \quad (11\text{-}2)$$

$$\{\mu_D\} = \begin{Bmatrix} \mu_{D3} \\ \mu_{D2} \\ \mu_{D1} \end{Bmatrix} = \begin{Bmatrix} x_{DA3}/x_{SA3} \\ x_{DA2}/x_{SA2} \\ x_{DA1}/x_{SA1} \end{Bmatrix} \tag{12}$$

$$\begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix} \begin{Bmatrix} \ddot{x}_3 \\ \ddot{x}_2 \\ \ddot{x}_1 \end{Bmatrix} + \begin{bmatrix} c_3 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_1 \end{bmatrix} \begin{Bmatrix} \dot{x}_3 \\ \dot{x}_2 \\ \dot{x}_1 \end{Bmatrix} + \begin{bmatrix} k_3 & -k_3 & 0 \\ -k_3 & k_3+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_1 \end{bmatrix} \begin{Bmatrix} x_3 \\ x_2 \\ x_1 \end{Bmatrix} = -\begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{13}$$

According to Figure 3, the spectra of $\mu_D$ has the curves lines with usually 3 peaks on each storey. The periods calculated by the 3 eigenvalues are “0.39(s), 0.64(s) and 1.11(s)”.

At the results, it is cleared that the predominant periods of MDOF are close to be the periods by the eigenvalues and in the short period or in the high level modes, the dynamic amplification ratios are at most 10. The dynamic amplification $\{x_{DA}\}$ at the predominant periods are usually different from the eigenvectors (Figure 4).

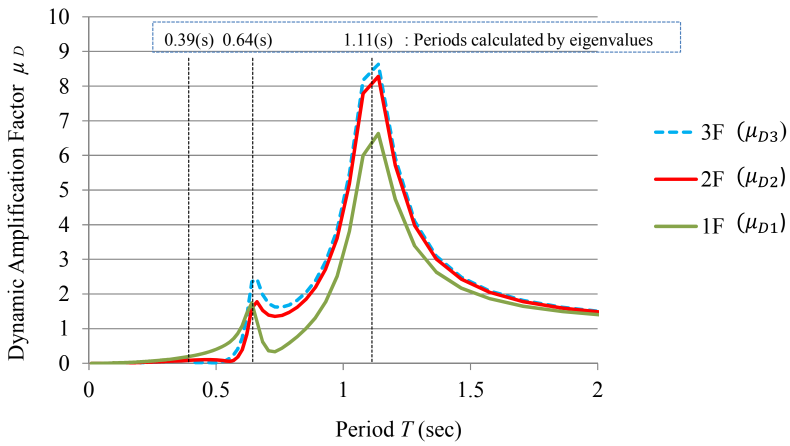



**Figure 3:** Spectra of dynamic amplification ratio $\mu_D$
(when the stiffness is the initial stiffness of the hysteresis property.) (Ref. : Section 4.2)

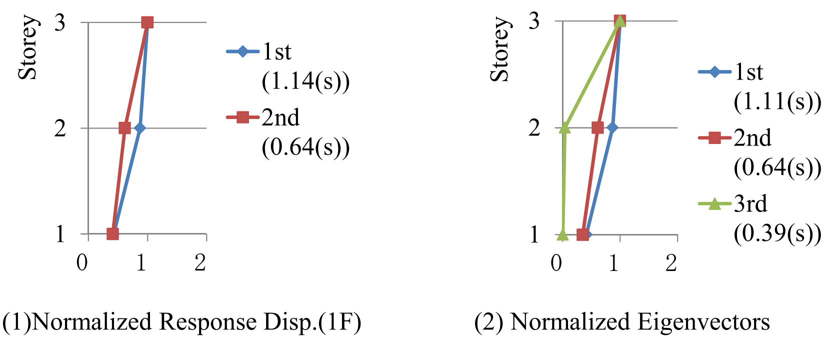



**Figure 4:** Comparison of normalized response displacement (1F) and normalized eigenvectors by the value on the 3rd storey, at the initial stiffness of the hysteresis property. (Ref. : Section 4.2)

Almost eigenvalues are complex numbers ($a \pm bi$, where $a$ and $b$ are real numbers, i: is imaginary number). But during the stiffness degrading, some of eigenvalues become real numbers. When the eigenvalues are real numbers, the motion is called as overdamping motion.

The number of these eigenvalues is even, the issued frequency $f\,(=1/T\,)$(Hz) is zero and the eigenvector of the 1st eigenvalue is similar to the one of the 2nd eigenvalue. All elements of the eigenvectors are also similar.

A part of the inelastic analysis results of Table 3.1 (shear coefficients at the yielding points qCyi are 0.07, 0.09 and 0.11), Section 4.2 is as follows. When the stiffness is decreased to 1/2,200 of the initial stiffness, the number of the eigenvalues which are real numbers is 2. The static amplification $\{x_{SA}\}$ in equation (11-2) is nearly 2.5x$10^4$ (cm) and the dynamic amplification $\{x_{DA}\}$ in equation (11-1) in the period $T$ =0.83 (sec), is from 19 ~ 15 (cm), when the eternal force vector is ${}^t\{1\ \ 1\ \ 1\}$ (transposed matrix) ($g$). Therefore the ratio $\{\mu_D\}$ in equation (12) is nearly 0.001 (Figure 5). The ratio $\{\mu_D\}$ is less than 1, because the stiffness on the 1st storey is decreased in the stiffness matrix [$K$] and the motion is an overdamping motion on the 1st storey.

Figure 6 shows the comparison of normalized response displacement (3F) and normalized eigenvectors by the value on 3rd storey, when the eigenvalue is a real number and the $qk_3$ is decreased. Figure 6.(1) shows the response displacement on the 1st storey is larger than the one on the upper storeys in the 2nd and 3rd mode. It seems to match the overdamping motion on the 1st storey and the higher level modes should not be neglected.

However, Figure 6.(2) seems to be similar to Figure 4.(2) at the the initial stiffness.

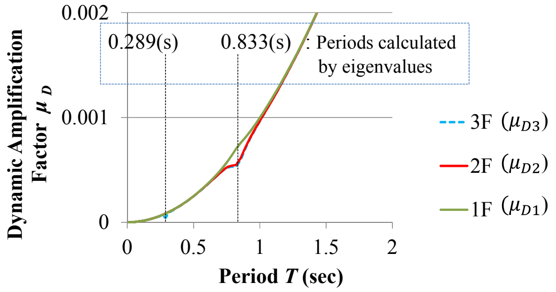
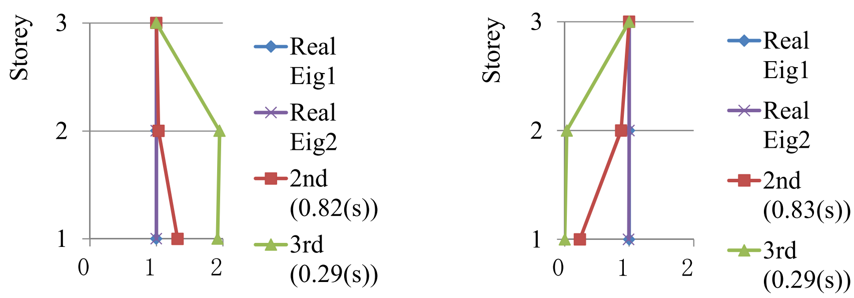



**Figure 5:** Spectra of dynamic amplification ratio $\mu_D$
( when the eigenvalues are real numbers and $qk_3$ = (1/1000)(qCy/$\delta y$) (Ref. : Section 4.2)




(1)Normalized Response Disp.(3F) (2) Normalized Eigenvectors

**Figure 6:** Comparison of normalized response displacement (3F) and normalized eigenvectors by the value on the 3rd storey, when the eigenvalues are real numbers and $qk_3$ = (1/1000)(qCy/$\delta y$) (Ref. : Section 4.2)

# 4 RESPONSE VALUES OF MDOF BY ACCELERATION RECORDS

## 4.1 Acceleration records

The earthquake waves are shown in Table 2. Not only [2009 AQV EW, Itaca], but also [2009 L'Aquila Parking Entrance (AQK) EW, Itaca] and [2009 L'Aquila Castle (AQU) EW, Itaca] are used because the observation point of AQK is closer to Sant'Agostino and AQU is closest to it. In order to make the structural identification to clarify the physical phenomenon of the lantern fallen down to the West, the EW direction of these earthquake waves were used.

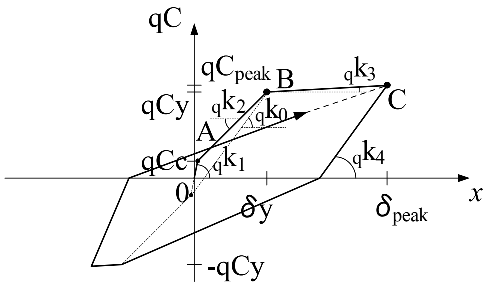
**Table 2:** Earthquake waves in EW direction of L'Aquila Main shock (Magnitude 5.8)

| Date and Time in local time | Station Code | Address (L'Aquila, Abruzzo, Italy) | Peak Acceleration in EW (cm/sec$^2$) | Epicentral Distance (km) | Distance to Sant'Agostino in L'Aquila (km) |
|---|---|---|---|---|---|
| 06 Apr. 2009 03:32:39 | AQV | Center of Valley Aterno | 662.6 | 4.9 | 5.5 |
| | AQK | L'Aquila Parking Entrance | 323.8 | 5.7 | 0.4 |
| | AQU*) | L'Aquila Castle | 258 | 6.0 | 1.0 |

Note) *) The acceleration records of AQU are corrected by subtracting the average of all data.

## 4.2 Inelastic dynamic analysis

Figure 7 shows the hysteresis property of Tri-linear Model for the inelastic dynamic analysis. In this analysis, Takeda Model is applied which has the oriented point before the yielding point is the crack point in the opposite. The stiffness ${}_qk_4$ of unloading after the




Takeda Model (The oriented point before the yielding point is the crack point in the opposite. The stiffness of unloading after the yielding point ; ${}_qk_4 = {}_qk_0 \left( \frac{\delta y}{\delta \text{ peak}} \right)^{0.4}$ )

$${}_qk_1 = 2.2 \cdot \frac{qCy}{\delta y} \quad {}_qk_2 = \frac{{}_qk_1}{3}$$

$${}_qk_3 = \frac{1}{1000} \frac{qCy}{\delta y}$$

$$qCc = 0.4 \cdot qCy, \quad \frac{qCy}{\delta y} = \frac{ky}{mg} = {}_qk_y$$

Notes)

(1) qC : Shear Coefficient

$$\left( qC = \frac{\text{Storey Shear Force (Q)}}{\text{Weight } (m \cdot g)} \right)$$

(2) qCy : Shear Coefficient at the yielding point of A ( qCy = 0.07, 0.11, etc.)

(3) $qC_{max}$ : Shear Coefficient at the peak of B

(4) Q : Storey Shear Force (kN) ( $= k \cdot x$ )

(5) ${}_qk_1$, ${}_qk_2$, ${}_qk_3$ : Stiffness Coefficient (1/cm)

$$\left( {}_qk_i = \frac{\text{Initial Stiffness } k_i}{\text{Weight } (m \cdot g)} \right)$$

(6) Stiffness Degrading Ratio : 1/1000

$$\left( {}_qk_3 = \frac{1}{1000} \times {}_qk_y \right)$$

(7) $h$ : Damping factor (= 0.05)

(8) $g$ : Gravity Acceleration (=980 (cm/sec$^2$))

**Figure 7:** Hysteresis property of Tri-linear Model

yielding point has some relationship with the stiffness ${}_{q}k_{0}$ between the yielding point and the crack point in the opposite side (${}_{q}k_{4} = {}_{q}k_{0}(\delta y/\delta \text{peak})^{0.4}$).

The other symbols of qCc, ${}_{q}k_{1}$, ${}_{q}k_{2}$ and ${}_{q}k_{3}$ are calculated by equations in Figure 7. Equations of qCc=0.4・qCy and $({}_{q}k_{2})=({}_{q}k_{1})/3$ are referred by the reference [3].

*Ty* is calculated by the mass *m* and the secant stiffness *ky* in equation (14).

$$Ty = 2\pi\sqrt{\frac{m}{ky}} = \frac{2\pi}{\sqrt{{}_{q}k_{y} \cdot g}} = 2\pi\sqrt{\frac{\delta y}{\text{qCy} \cdot g}} \tag{14}$$

### 4.3 Response values of MDOF by inelastic dynamic analysis

In the structural identification, some cases of the period and the shear coefficient on each storey are calculated and the one of these results are shown in Table 3.1 ~ Table 3.3. Table 3.1 shows that all of 3 cases (AQV, AQK and AQU) identified the larger displacement x3 on the lantern storey to the West. Table 3.2 and Table 3.3 show the results if the lantern storey would be retrofitted, when the $qCy_3$ is increased from 0.11 to 0.15 or 0.16. This retrofitting method would have valuable effects which would reduce the response displacement x3 on the lantern storey less than that before retrofitting.

This Section shows the analysis results for the effects of the shear coefficient $qCy_3$.

**Table 3.1:** Structural properties and analysis results before retrofitting

| Structure Properties of each Storey | | | | | | ACC. Records (EW) | Analysis Results of each Storey | | |
|---|---|---|---|---|---|---|---|---|---|
| Period (s) | | | Shear Coefficient (-) | | | | Response Relative Displacement (cm) | | |
| *Ty*1 | *Ty*2 | *Ty*3 | qCy1 | qCy2 | qCy3 | | *x*1 | *x*2 | *x*3 |
| 1.00 | 0.80 | 0.40 | 0.07 | 0.09 | 0.11 | AQV | -4.15 | -4.15 | +8.05 |
| | | | | | | AQK | -2.83 | -4.79 | -10.88 |
| | | | | | | AQU | 1.84 | 2.60 | +2.75 |

Notes)(1) *x*1, *x*2, *x*3 : the positive (+) means the displacement to the East and negative (-) to the West.
(2) Hysteresis property is the Tri-Takeda Model. Ref. : Figure 7.
(3) Period *Tyi* is calculated by the secant stiffness and the mass on i-th storey.
($Tyi = 2\pi\sqrt{m_i/kyi}$)
(4) These Notes are also applied to the Table 3.2 and Table 3.3.

**Table 3.2:** Structural properties and analysis results if retrofitting (qCy3 → 0.15)

| Structure Properties of each Story | | | | | | ACC. Records (EW) | Analysis Results of each Story | | |
|---|---|---|---|---|---|---|---|---|---|
| Period (s) | | | Shear Coefficient (-) | | | | Response Relative Displacement (cm) | | |
| *Ty*1 | *Ty*2 | *Ty*3 | qCy1 | qCy2 | qCy3 | | *x*1 | *x*2 | *x*3 |
| 1.00 | 0.80 | 0.40 | 0.07 | 0.09 | 0.15 | AQV | -4.17 | -4.00 | +14.23 |
| | | | | | | AQK | -2.89 | -3.91 | +5.89 |
| | | | | | | AQU | 1.80 | -2.56 | +5.73 |

**Table 3.3:** Structural properties and analysis results if retrofitting (qCy3 → 0.16)

| Structure Properties of each Story | | | | | | ACC. Records (EW) | Analysis Results of each Story | | |
|---|---|---|---|---|---|---|---|---|---|
| Period (s) | | | Shear Coefficient (-) | | | | Response Relative Displacement (cm) | | |
| *Ty*1 | *Ty*2 | *Ty*3 | qCy1 | qCy2 | qCy3 | | *x*1 | *x*2 | *x*3 |
| 1.00 | 0.80 | 0.40 | 0.07 | 0.09 | 0.16 | AQV | -4.11 | -4.27 | +1.41 |
| | | | | | | AQK | -2.84 | -4.51 | - 3.07 |
| | | | | | | AQU | 1.79 | -2.60 | +0.81 |

## 5 PREDOMINANT PERIOD OF SOIL-STRUCTURE INTERACTION

### 5.1 Sway Rocking (SR) model

The predominant period of soil-structure interaction is also analyzed.

The sway rocking (SR) model is applied. Figure 8 shows overview of sway-rocking model for soil-structure interaction with SDOF.

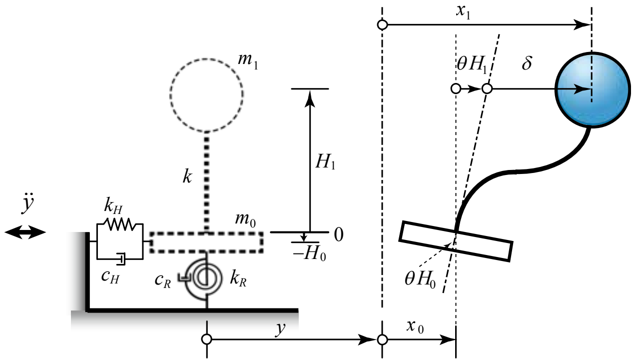



**Figure 8:** Overview of sway-rocking model for soil-structure ınteraction

Figure 8 shows SDOF, sway and rocking between the foundation and the ground. This SR model has a mass of $m_0$ and $m_1$ (kN/$g$) for mass of the structure and mass of the foundation. Stiffness of the structure and sway stiffness are $k$ , $k_H$ (kN/cm). Rocking stiffness is $k_R$ (kN・cm/rad.). Damping coefficient of the structure is $c = 2h\sqrt{m_1 k}$ (kN・sec/cm), the one of the sway in soil-structure interaction (SSI) is $c_H = 2hm_0\sqrt{k_H/(m_1+m_0)}$ (kN・sec/cm) and the one of the rocking in SSI is $c_R = 2h\sqrt{I\,k_R}$ (kN・cm・sec/rad.). Equivalent height at the center of gravity of the structure and the foundation are $H_1$ and $H_0$ (cm). $I\,(= m_1H_1^2 + m_0H_0^2)$ (kN・cm$^2$/$g$ /rad.) is the moment of inertia at the ground surface for the 2 mass of SDOF and the foundation. $h$ is damping facor ($h$ = 0.02). $g$ is gravity acceleration ( $g$ = 980 (cm/sec$^2$)).

When the external force $\ddot{y}$ (cm/sec$^2$) in acceleration records takes place on the ground level, SR model has the horizontal displacement of the ground $y$ (cm), the horizontal sway displacement $x_0$ (cm), the horizontal displacement of SDOF $\delta$ (cm) and the rotation angle of rocking $\theta$ (rad.). The relative horizontal displacement to the ground of SDOF $x_1$ (cm) is calculated by equation (15). The storey shear force of SDOF is $k\,\delta\,(= k\,(x_1 - x_0 - \theta H_1))$ (kN).

$$x_1 = x_0 + \theta H_1 + \delta \tag{15}$$

$$\begin{cases} m_1\ddot{x}_1 + c\,\dot{x}_1 + k(x_1 - x_0 - \theta H_1) = -m_1\ddot{y} \\ m_0\ddot{x}_0 + c_H\dot{x}_0 - k(x_1 - x_0 - \theta H_1) + k_H(x_0 - \theta H_0) = -m_0\ddot{y} \\ I\ddot{\theta} + c_R\dot{\theta} + k_R\theta = -m_1\ddot{y}H_1 - m_0\ddot{y}\,(-H_0) \end{cases} \tag{16}$$

These displacements have their velocities and accelerations. The motion equation is equation (16). In equation (16), the left sides are internal forces or moments and the right sides are external forces or moments. The 3rd equation in equation (16) has the external moment $\{m_1H_1 + m_0(-H_0)\}\ddot{y}$ (kN・cm) to SR model by the ground acceleration $\ddot{y}$ , which means the equilibrium of moments for the 2 mass around the ground surface and means that the 3rd equation includes implicitly the inertia forces, the damping forces and the storey shear forces in the 1st and 2nd equation in equation (16). Equation (16) is also described by equation (17) in matrix which has forced shakings and external dampings.

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & I \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_0 \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c & 0 & 0 \\ 0 & c_H & 0 \\ 0 & 0 & c_R \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_0 \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k & -k & -kH_1 \\ -k & k+k_H & kH_1 - k_H H_0 \\ 0 & 0 & k_R \end{bmatrix}\begin{Bmatrix} x_1 \\ x_0 \\ \theta \end{Bmatrix} = -\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & I \end{bmatrix}\begin{Bmatrix} 1 \\ 1 \\ \{m_1 H_1 + m_0(-H_0)\}/I \end{Bmatrix}\ddot{y} \tag{17}$$

The right side of equation (17) shows the unit external force vector ${}^t\{\ 1\ \ 1\ \ \{m_1 H_1 + m_0(-H_0)\}/I\ \}$ (transposed matrix). Similar unit external force vector in equation (10) shows that all elements are 1 (${}^t\{1\ \ 1\ \ 1\}$). But the 3rd equation of equation (17) is described for the response angle $\theta$ (rad.) or the moment by the horizontal force, the mass matrix [$M$] for the response acceleration vector is same as the one for the external force vector and the rightest value is $\ddot{y}$ (cm/sec$^2$). Therefore, this third element of the unit external force vector is "$\{m_1 H_1 + m_0(-H_0)\}/I$". This unit is "rad./cm" and 1 (rad./cm) is not appropriate for this third element. It is understandable when $m_0$ is 0 and $H_0$ is 0, the moment of inertia $I$ would be described to be $m_1 H_1^2$ and this third element is calculated to be $m_1 H_1/I = m_1 H_1 / m_1 H_1^2$ = $1/H_1$ (rad./cm), which is a reciprocal of an arm length of moment.

### 5.2 Example of elastic analysis for SR model

The predominat periods are also analyzed for SR model by elastic analysis.

The model building is still Sant'Agostino in L'Aquila and it is modeled to SDOF of Figure 8. SR model needs many values in equation (17). All values are listed in Table 4.

The period $T_1$ of SDOF of Figure 8 is assumed to be 1.14(sec), according to Figure 4, (1). Therefore, the stiffness $k$ of SDOF is 4.69x10$^3$ (kN/cm) ($k = (2\pi/T_1)^2 \cdot m_1$).

$k_H$ and $k_R$ are assumed to be 1.47x10$^6$ (kN/cm) and 2.77x10$^{11}$ (kN・cm/rad.), according to the test results in Japan before 1962 [4].

The height is modified to the equivalent height at the center of gravity for SDOF ($H_1$=1,130(cm)). Figure 1 doesn't show the foundation, but it is assumed to have the foundation of 100 (cm) depth, the equivalent height ($H_0$=50(cm)), the specific gravity (=15 (kN/m$^3$)) and the mass ($m_0$=27.4 (kN/$g$)).

Moreover, the period of the sway $T_H$ is 0.0697 (sec) ($T_H = 2\pi\sqrt{(m_1 + m_0)/k_H}$).

The period of the rocking $T_R$ is 0.167 (sec) ($T_R = 2\pi\sqrt{I/k_R}$).

**Table 4:** Data for SR model

| $m_1$ | $m_0$ | $I$ | $k$ | $k_H$ | $k_R$ | $c$ | $c_H$ | $c_R$ | $H_1$ | $H_0$ | $h$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 154 | 27.4 | 1.95x10$^8$ | 4.69x10$^3$ | 1.47x10$^6$ | 2.77x10$^{11}$ | 34.0 | 98.8 | 2.94x10$^8$ | 1,130 | 50 | 0.02 |

Figure 9 shows that the predominat periods of SR model are almost same as the periods calculated by the eigenvalues. The reason is predicted probably that the stiffness are far from other stiffness ($k \ll k_H \ll k_R$). All curves lines of SDOF, Sway and Rocking have the peaks at their periods and the curve of Sway has another peak at the same period as SDOF. The new period 0.0271 (sec) is very close to the period of only Sway without SDOF ($T_H'$ = $2\pi\sqrt{m_0/k_H}$ =0.0271 (sec)). The maximum of $\mu_D$ is nearly 25 in elastic analysis.

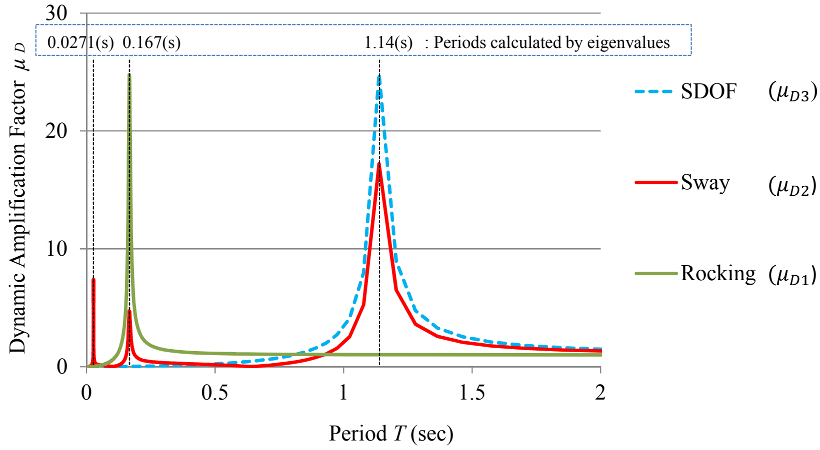



**Figure 9:** Spectra of dynamic amplification ratio $\mu_D$ in elastic dynamic analysis.

## 6 COUCLUSIONS

In this paper, the predominant periods of MDOF analysis are executed without SDOF and the dynamic amplification factors for a sample data of a model building are shown. The effects of soil-structure interaction are also analyzed in elastic analysis for the model building.

The main points of these results are as follows;

(1) According to the dynamic amplification ratio $\mu_D$ in the sample data of MDOF, the predominant periods of the MDOF for a model building are close to be the periods by the eigenvalues. The dynamic amplification ratios $\mu_D$ of MDOF are at most 10.

(2) The normalized response displacement of MDOF at the stiffness degrading shows that the higher level modes should not be neglected.

(3) In MDOF inelastic analysis, when some eigenvalues are real numbers, the motion seems to be overdamping.

(4) For the effects of soil-structure interaction (SSI) with SDOF, the elastic analysis of sway-rocking (SR) model, the predominant periods are close to the periods by the eigenvalues.

(5) The maximum of the dynamic amplification ratio $\mu_D$ of SR model in elastic analysis is nearly 25.

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