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## PROBABILISTIC ANALYSIS OF A GAS STORAGE CAVITY MINED IN A SPATIALLY RANDOM ROCK SALT MEDIUM

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**Abstract.** In most engineering problems the material parameters spread over spatial extents but this variability is commonly neglected. Analyses mostly assign the mean value of a variable to the entire medium, while in the case of heterogeneous materials as geomaterials, this may lead to an unreliable design. The existing scatter in such materials can be represented in the design procedure using the random field concept.

In this paper, the random field method is used in a probabilistic analysis of a gas storage cavern in rock salt. The rock salt formation, as a porous media with low permeability and particular creep features, has been used for decades as the host rock for the hydrocarbon storage. To achieve a reliable design, a probabilistic model is presented to compute the failure probability of a cavern mined in a spatially varying salt dome. Here, the no-dilatant region around the cavity is regarded as the failure criterion. In this regard, a thermo-mechanical model of a natural gas storage in rock salt, employing BGRa creep law, is developed. Afterwards, the most effective input variable on the model response is identified, using global sensitivity analysis. The Karhunen-Loève expansion is introduced to generate random field. In the following, the subset simulation methodology is utilised to facilitate the execution of Monte-Carlo method. The findings of this study emphasize that considering spatial variability in rock properties significantly affects the reliability of a solution-mined cavity.

## 1 Introduction

Solution-mined cavities in rock salt represent an adequate opportunity for energy carriers' storage plants. Rock salt is nearly impermeable compared to other geomaterials and excavation in this rock costs relatively low due to the possibility of solution mining. For many decades, abandoned salt mines have been used for underground storage. The first cavern in rock salt for natural gas storage was excavated in Michigan, USA in the 1950s [1]. In the last 30 years, in Germany industry has made large investments in building energy storages in deep underground formations. For instance, the Etzel (IVG) cavern site currently includes 73 existing gas and oil storage caverns with a capacity of approximately 46 million cubic metres [2]. In addition to many caverns around the world storing hydrocarbons, there are also some new trends to provide long-term electrical storage capacities in such cavities in form of compressed air or Hydrogen.

The stability of rock salt caverns is the main concern of the design process of such massive underground structures. Rock salt is categorised as a soft rock and its non-linear time-dependent material behaviour (i.e. creep behaviour) makes rock salt different from other common host rocks. Thereupon, much effort has been paid to provide a consistent knowledge about the behaviour of rock salt enforced by different loading conditions. Moreover, in order to predict and explain the behaviour of the rock salt by mathematical equations, a wide range of constitutive laws have been proposed within different micro and macro observations, e.g., see [3, 4, 5]. Furthermore, numerous studies have been carried on to evaluate the response of the rock salt cavities under storage conditions using numerical simulation methods (e.g., [6, 7, 8, 9]).

However, a clear picture of the rock salt characteristics may only appear by gathering different information sources as laboratory analyses, geotechnical in-situ measurements, and on-site observations. In practice, for such particular structures which are extended vertically downwards more than hundred meters from the ground level, only limited experimental and in-situ data is available. On the other hand, the inherent randomness of natural materials as rocks causes a wide extent of spatial distribution in their physical properties. Considering these facts, the measure of involved uncertainties in the rock salt properties can not be neglected. Hence, a reliable design procedure can not rely merely on deterministic approaches. In order to provide an adequately accurate computational model, stochastic analysis approaches must be utilized, as well. Despite other geotechnical fields of study where the stochastic analyses are well established (e.g. [10, 11]), there are rare studies that investigate the involved uncertainties in the geotechnical design of rock salt cavities. For instance, in [12], the authors performed a probabilistic analysis on a compressed air rock salt cavity, using the subset simulation methodology as a modern Monte-Carlo approach. In that study, the involved input parameters were considered as the random variables, and the defined uncertainty measure applied to the entire medium and the spatial randomness was neglected.

To represent the real spatial variability of a field variable, a significant amount of infor-

mation must be gathered from the field, which is an expensive or even infeasible project. Also, if not enough amount of data is gathered then the parameter identification and design process can be inaccurate. In this regard, a probabilistic analysis concept may be employed to minimise sampling costs while it is still being able to provide informative data. For this, random fields are appropriate as models of the spatially distributed uncertainty and they can be utilised to produce probability measures regarding design criteria [13]. In the random field models, the characteristics of the spatial distribution of mechanical parameters are simulated as functions of spatial location. Various random field generator algorithms are available in the literature, which are shortly reviewed in Sect. 2. Among the others, the series expansion methods can approximate the random field by a finite sum of products of deterministic spatial functions and random variables. In this study, the Karhunen-Loève expansion as a series expansion method, introduced in Sect. 3, is applied to generate random field realisations.

In the following, a typical natural gas rock salt cavern is simulated by finite element method. Within the numerical model, the rock salt behaviour is described by BGRa [3], a well-known creep model. In Sect. 4 the numerical model, boundary and loading conditions are presented. Sect. 5 presents the probabilistic analysis of the considered cavity mined in spatially varying rock salt. The effect of the uncertainties in the input parameters on the system responses is inquired by carrying out a global sensitivity analysis. In the present paper, Sobol's method [14] as a variance-based sensitivity technique is employed. After ranking the importance level of parameters, random field discretization applied on the most governing one. In the following, the failure probability of the system considering dilatant behaviour is evaluated. At the end, a parametric study is conducted to investigate the effect of variations in the assumed autocorrelation lengths. The obtained results are concluded in Sect. 6.

## 2 Random field discretization

Because of the differences of mineral ingredients, stress history, and other geological factors, constitutive parameters of rocks or soils show spatial differences and correlations as well. The characteristics of the spatial distribution of constitutive parameters can be simulated as a geological parameter field which exhibits spatial variability. In general, a random field is characterised by its mean, variance, and its correlation structure. One of the major features of a random field representation of a material is the concept of statistical dependence between field values at different points, which is known as the correlation structure [15]. The correlation coefficient between two points  $x_1$  and  $x_2$ ,  $\rho(x, x')$  is decreasing gradually as the distance is increasing. On the other hand, when the distance between two realisation points approaches to zero, the material characteristics are identical. It should be stated that higher order moments of random field can also represent its characteristics, but due to difficulties in estimating them, random field models are often represented by information about the three mentioned measures. To accomplish this, firstly a relatively simple joint PDF (e.g., multivariate normal or lognormal distributions)

for the field should be adopted. The correlation structure is often assumed to be a simple function of the distance between points.

A continuous random field  $H(x, \theta)$  can be defined as a random function that describes a random quantity at each point  $x \in \Omega$  of a continuous domain  $\Omega \subset \mathbb{R}^n$ .  $\theta \in \Theta$  is a coordinate in the sample space  $\Theta$ . Hence,  $H(x_0, \theta)$  denotes the random variable associated with point  $x_0$ , and  $H(x, \theta_0)$  indicates  $\theta_0$  realisation of the field. As mentioned before, a random field can be defined by its mean  $\mu(x)$ , variance  $\sigma^2(x)$  and autocorrelation coefficient function that is

$$\rho(x_1, x_2) = \frac{C_H[H(x_1), H(x_2)]}{\sigma(x_1)\sigma(x_2)}, \quad (1)$$

where  $C_H(., .)$  is the autocovariance function. A random field can be discretised by approximating  $H(.)$  by  $\hat{H}(.)$ , described by means of a finite set of random variables  $\{\chi_i, i = 1, \dots, n\}$

$$\hat{H}(x) \xrightarrow{\text{Discretization}} \hat{H}(x) = F[x, \chi]. \quad (2)$$

Sudret & Der Kiureghian [15] classified the most commonly used methods of random field discretization into following main groups

- Point Discretization

In this group of methods, the random variables  $\chi_i$  are selected values of  $H(.)$  at specific points in a given spatial discretization (e.g., mesh in finite/difference element methods).

- Average Discretization,

In these methods, the random variable related to a given  $\chi_i$  is calculated as the weighted integrals of  $H(.)$  over a domain  $\Omega_e$

$$\chi_i = \int_{\Omega_e} H(x)\omega(x)d\Omega. \quad (3)$$

A comparative review on the above mentioned methods was provided in [16].

- Series Expansion methods

In the series expansion discretization methods, the random field is approximated by an expansion that involves deterministic and stochastic functions. The value of the random field is calculated on the basis of the coordinates of the point in deterministic functions. In these techniques, the field is represented as a series involving random variables and deterministic spatial functions. As examples of this group of methods, one can mention the Expansion Optimal Linear estimation method, proposed by [16]. The Karhunen-Loève expansion method is also a commonly used series expansion method. The Karhunen-Loève is utilised in the present study to discrete the random field and it is briefly described in the following.

### 3 Karhunen-Loève expansion

We consider  $H(x, \theta)$  as random process, when  $H$  denotes the expected value of the random field. The random field can be calculated by the Karhunen-Loève expansion as follow

$$\hat{H}(x, \theta) = \mu_H + \sum_{i=1}^{\infty} \sqrt{\lambda_i} \phi_i(x) \xi_i(\theta), \quad (4)$$

where  $\lambda_i$  and  $\phi_i$  are the eigenvalues and eigenfunctions of the autocovariance function, and  $\xi_i(\theta)$  is a vector of standard uncorrelated random variables.  $\mu_H$  is the mean function of the field. It should be noticed here that  $\xi_i(\theta) : \Theta \rightarrow \mathbb{R}$  are the stochastic variables that represent the random nature of the uncertain parameter. For practical purposes, the expansion in Eq. 4 can be truncated to a given number of terms,  $M$  as follow:

$$\hat{H}(x, \theta) \approx \mu_H(x) + \sum_{i=1}^M \sqrt{\lambda_i} \phi_i(x) \xi_i(\theta), \quad (5)$$

where  $M$  is the size of the series expansion,  $\lambda_i$  and  $\phi_i$  are the eigenvalues and eigenfunctions of the covariance function  $C(x_1, x_2)$ , and  $\xi_i(\Theta)$  is a vector of standard uncorrelated random variables. However, the eigenvalues and eigenfunctions  $\lambda_i$  and  $\phi_i$  are the deterministic functions of the Karhunen-Loève expansion. They can be evaluated as the solution of the following Fredholm integral equation:

$$\int_{\Omega} C_H[H(x_1), H(x_2)] \phi_i(x_2) dx_2 = \lambda_i \phi_i(x_1). \quad (6)$$

This integral can be solved analytically only for a few types of the autocovariance functions. Ghanem & Spanos [17] presented the detailed analytical solution of the integral in Eq. 6 for an exponential autocovariance function.

The choice of the number  $M$  of terms depends on the required accuracy of the considered problem. Sudret & Der Kiureghian [15] proposed the following error estimate ( $err(x)$ ) after truncating the expansion to  $M$  terms,

$$err(x) = \sigma_H^2 - \sum_{i=1}^M \lambda_i \phi_i^2(x), \quad (7)$$

where  $\sigma_H$  is the standard deviation of the random field.

### 4 Deterministic model of a rock salt cavern

Within this study, a gas cavern with the capacity of 368,000 m<sup>3</sup> is simulated by an axisymmetrical numerical model, its casing shoe is assumed to be located in the depth of 400 m. The shape of the cavern after excavation is idealised as a cylinder with the height of 150 m and 60 m diameter. The floor and roof of the cavern are considered as

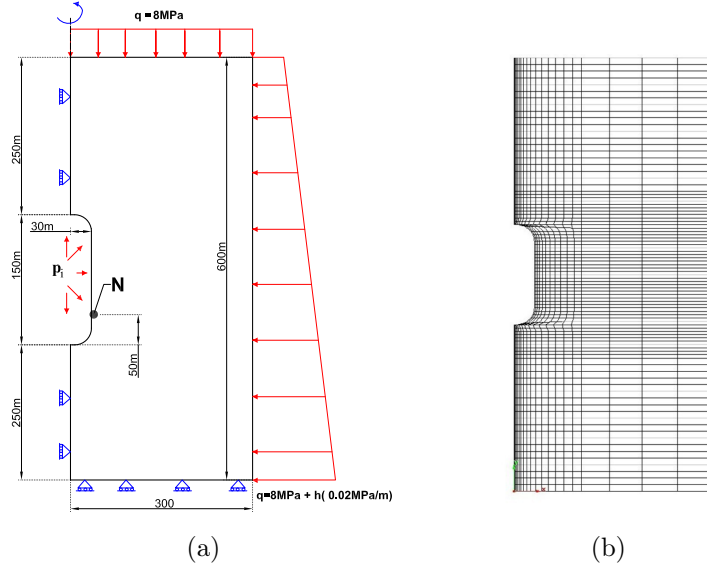


Figure 1: (a) Representative geometry and boundary conditions of the salt cavern model and (b) Finite element mesh discretization

semi-spheres with 30 m radius. Fig. 1a shows the geometry and boundary conditions of this cavity in detail. The considered mesh discretization is also shown in Fig. 1b. The uniform load at the top of the model substitutes the overburden weight. In order to model the solution mining procedure, the entire excavation phase is simplified by reducing the internal pressure of the cavern to the minimum gas pressure (First discharge phase) in a time interval of 300 days. In this study, the minimum inner pressure of the cavity is assumed to be equal to 4 MPa. In our simulation, the temperature of intact rock salt is assumed to be equal to 50°C. The thermal boundary condition of the wall of the cavity is decreased during the discharge phase to 30°C (for more details about the variation of thermal condition see [8]).

In this study, the creep behaviour of the rock salt is modelled on the basis of the BGRa constitutive model. In this constitutive model, the creep induced strain rate of rock salt is obtained using the following equation:

$$\dot{\epsilon}_{ij}^{cr} = A \exp\left(\frac{-Q}{RT}\right) \left(\frac{\sigma_{ij}}{\sigma_0}\right)^n, \quad (8)$$

where  $A$  denotes the value of fluidity at a reference temperature;  $R$  is the universal constant of perfect gas (8.314 e3 kJ/(mol.K)) and  $T$  is the absolute temperature (For more details see [3]). In the present study, the induced strain rate by the temperature change is also considered as an additional strain tensor in the mechanical model. This quantity is related to the linear thermal expansion coefficient  $\alpha$  and the temperature changing rate  $\dot{T}$ :

$$\dot{\epsilon}_{ij}^{th} = \alpha \dot{T} \delta_{ij} \quad (9)$$

The mechanical stability of storage cavities is the most important issue which should be assured in an accurate geotechnical design. Therefore, the dilatant zones which can be initiated by a specific stress state in rock salt should be avoided. The dilatancy boundary, which is defined by the beginning of the irreversible volumetric expansion, can be considered as a criterion which divides the stress space into compression and dilatancy regions. A Dilatancy boundary, known as compression/dilation (C/D) boundary can be identified on the basis of experimental data (for more details about different C/D boundaries, the reader is referred to [9]). In the present study, Desai C/D boundary [4] is used as no dilation criterion. In order to determine whether a region in the simulated rock salt cavity encountered dilatancy or not, the following quantity  $DF$  is defined.

$$DF = \sqrt{\frac{J_2}{J_2^{dil}(I_1)}}. \quad (10)$$

where  $J_2^{dil}(I_1)$  denotes the second stress invariant of C/D boundary corresponding with the value of first invariant ( $I_1$ ) in each observation point. When  $DF < 1$ , the stresses are inside the compressibility domain and opening of micro-cracks does not occur and subsequently, damage does not progress. In contrary, when  $DF \geq 1$ , the cavern may experience long-time failure due to the damage progress.

In the next step, the finite element numerical simulation of the typical cavern is conducted. Although in real cases the designer must assure that no-dilatant region around the entire cavern will take place, in the present study, the considered observation points are limited. The level of one-third of cavern's height from its bottom is generally treated as a benchmark region in measurements and numerical simulations as well. Hence, in this study the  $DF$  factor is evaluated in the region of  $N$ , illustrated in Fig. 1a.

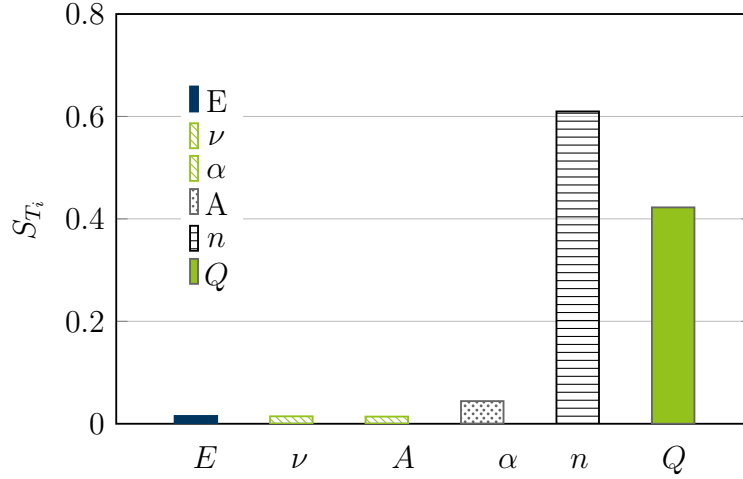
It is important to note that a stratigraphic unit may consist of several different homogeneous zones. The graphs provided by [18], which are derived from creep tests on two different shafts in Gorleben salt dome (north-east of Germany), indicate the existing of vertical heterogeneity in creep behaviour of this site. The present study considered a one-dimensional (vertical) spatially random field. It should be stated that the method of the Karhunen-Loève can cope with domains of arbitrary geometric mesh, different from the FEM mesh. However, the size of a given element in the deterministic mesh depends on the autocorrelation distances of the rock properties. Der Kiureghian & Ke [19] suggested that the length of the largest element of the deterministic mesh should not be less than half of the autocorrelation distance in that direction. In order to respect this criterion, the geometrical model is vertically divided to 10 m layers, with a specific random value assigned to each layer (with 10 m thickness).

## 5 Probabilistic analysis

Before generating the random field, the most important input parameter which makes the largest contribution to the variation of model response is identified by conducting

Table 1: Material parameters for rock salt

Elastic Parameters		Creep Parameters			Thermal Properties
E [GPa]	$\nu$ [-]	A [ $s^{-1}$ ]	n [-]	Q [kJ/mol]	$\alpha$ [ $1/^\circ C$ ]
[19 - 3]	[0.25 - 0.35]	[0.1e-5 - 0.5e-5]	[3 - 5]	[40000 - 70000]	[3e-5 - 5.5e-5]


 Figure 2: Estimation of total effect sensitivity index  $S_{T_i}$ , regarding FS

a global sensitivity analysis. The variance-based technique for evaluating the sensitivity indices of input parameters is proposed by [14]. This methodology was previously introduced to the geotechnical design of solution-mined cavities by the authors [20]. In this paper, Sobol's method (also known as Monte-Carlo based implementation) is utilised to evaluate the sensitivity measures of different constitutive parameters (the reader is referred to [21]). It should be mentioned that the sensitivity analysis is conducted on a random variable homogeneous model. Table. 1 represents the considered range of variation for each input variable. All the input variables follow a lognormal probability distribution function. Fig. 2 depicts the evaluated total-effect sensitivity index ( $S_{T_i}$ ) for each constitutive parameter. The most influential input parameter on the stress-state of the cavern's wall against dilation is  $n$ .

Hence, all parameters but  $n$  are fixed to their mean values, and the variable of  $n$  is discretized as a random field. The mean value and coefficient of variation of parameter  $n$  are respectively  $\mu = 4$  and  $COV = 10\%$ , and follows a lognormal probability density function. Fig. 3 shows a typical random field realisations of the variable  $n$  for two different autocorrelation distances. For larger values of autocorrelation distances, the model tends to a homogeneous field, while less autocorrelation lengths limit the correlations in a given simulation to smaller zones. In general, for a specific autocorrelation length, layers which are very close together tend to have similar  $n$  values and express a higher correlation.



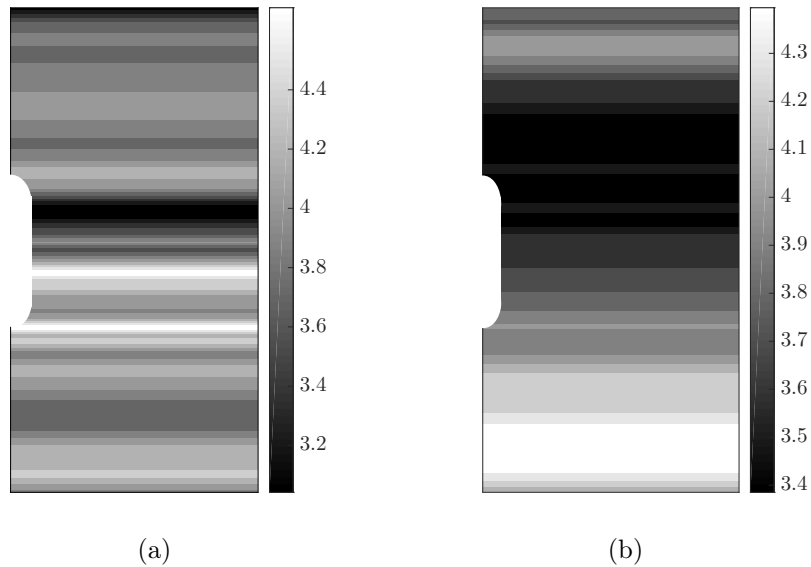


Figure 3: Random field realisations of the parameter  $n$  for autocorrelation distance (a)  $l_y=30$  m (b)  $l_y=650$  m

As mentioned earlier, the size of the Karhunen-Loève expansion  $M$  will effect on the accuracy of the approximated random field  $\hat{H}$ . Fig. 4a depicts the estimated error of random field approximation versus the assumed  $M$  value for different autocorrelation lengths. The error estimation converges to less than 5%, if  $M \geq 200$ . In the present study,  $M$  is assumed to be equal to 200 terms to ensure the accuracy level of random field approximation.

The crude Monte-Carlo simulation is a well-established methodology to evaluate the probability of failure which has been used for many decades. As conducting a Monte-Carlo simulation requires thousands of model evaluation and each run of the numerical model in this study takes more than two hours, employing Monte Carlo seems not feasible. In order to address this drawback, the authors applied the subset simulation technique, introduced previously in [22] which decreases the number of required simulation, drastically.

The probability analysis here is conducted for the region  $N$  in Fig. 1a and the failure probability for different autocorrelation lengths ( $l_y$ ) is computed against  $DF = 1$ . Fig. 4b illustrates the obtained results. In addition to different spatial variability scenarios, we also conduct a homogeneous simulation, where the constitutive parameters are assumed to be random variables. As Fig. 4b clearly shows, with decreasing the autocorrelation length in random field discretization, the probability of encountering a dilatant region at the considered observation point increases. Although in real cases, a local dilatant zone may not mainly endanger the stability of the entire structure, but in our case, we considered the no-dilatant region as the failure criterion, which is conservative.

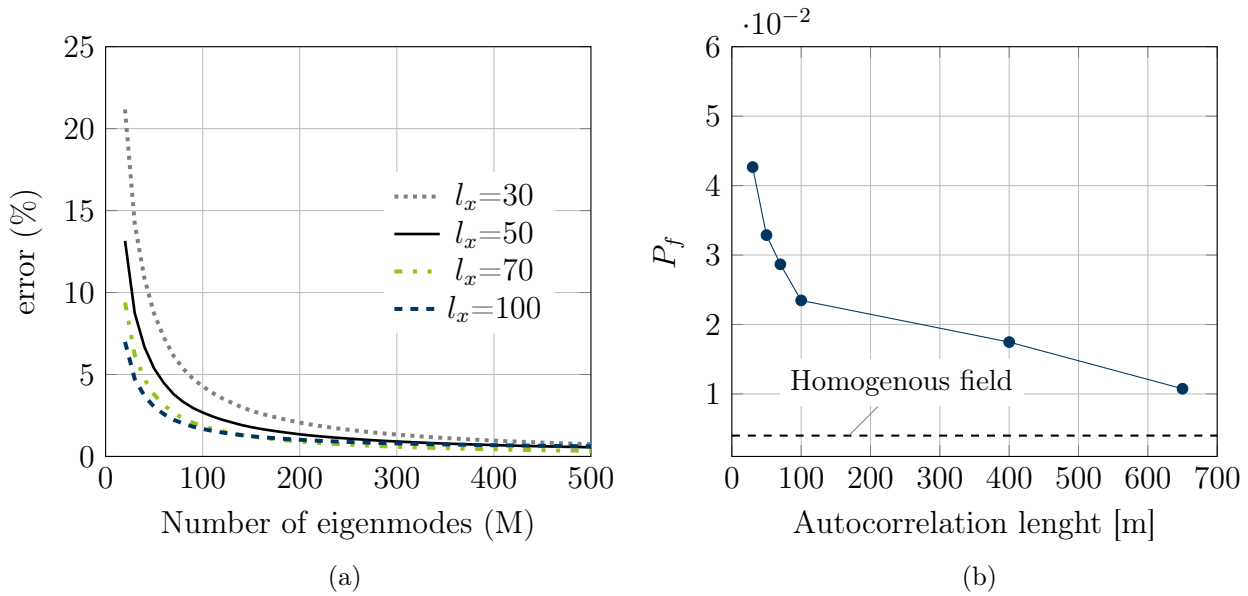


Figure 4: (a) Error estimation vs. number of eigenmodes for different values of autocorrelation lengths, (b) Probability of failure against dilatancy for different autocorrelation lengths

## 6 Conclusion

The inherent randomness of natural materials like rocks and soils causes a wide extent of spatial distribution in their physical properties. Thereupon, the spatial variability and consequently the induced uncertainty, have to be considered in the complex geotechnical problems. In this paper, the random field method is applied in a probabilistic analysis of a gas storage cavern in the rock salt. A random field discretization of constitutive parameters of the BGRa creep law using Karhunen-Loève is conducted. The failure probability of a rock salt cavity against the no-dilatant criterion is calculated for different spatial variability scenarios to present the effect of the autocorrelation lengths on the safety measures of the system against dilation. The obtained failure probabilities are compared with the corresponding results considering homogeneous rock in a random variable analysis. The comparison represents the necessity of considering spatial variability in the material properties.

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