

HYDROMECHANICAL ANALYSIS OF A HYDRAULIC FRACTURING PROBLEM

LETICIA P. MORAIS^{*}, JORDANA G. S. SABA[†] AND

MANOEL P. CORDÃO-NETO^{††}

^{*} University of Brasília (UnB) FT/ENC/SG 12
Post-Graduation Program of Geotechnical Engineering
e-mail: leticia.12.morais@gmail.com web page <http://www.geotecnia.unb.br/>

[†]University of Brasília (UnB) FT/ENC/SG 12
Post-Graduation Program of Geotechnical Engineering
e-mail: jordana.quinosan@gmail.com web page <http://www.geotecnia.unb.br/>

^{††}University of Brasília (UnB) FT/ENC/SG 12
Post-Graduation Program of Geotechnical Engineering
e-mail: mporfirio76@gmail.com web page <http://www.geotecnia.unb.br/>

Key words: Hydraulic Fracturing, Hydro mechanical, XFEM

Abstract. Hydraulic Fracturing is a well stimulation technique which recently has been widely used for shale gas extraction. Hydraulic fracturing is when a fluid is injected into the wellbore under controlled pressure and flow. The differential pressure generated by the injection of fluid initiates cracks that will propagate into the deep-rock formations, so that it allows the extraction of hydrocarbons trapped into the rock. The technique is used in conventional and unconventional reservoirs of hydrocarbons. In the first case, in conventional reservoirs, the technique is applied in order to increase the production of the well, while in unconventional reservoirs (shale gas) the technique is used to enable the extraction of the gas due to its very low permeability. Furthermore, the process of fracturing the rock at great depths involves the control over the type of fracture created or reactivated, as this will depend on a number of factors. The study of the technique is important to improve the control over the execution of this procedure and also to avoid possible contingencies and accidents. A formulation was implemented in this work capable of representing discontinuities in a continuous mesh using a finite element code. The Extended Finite Element Method (XFEM) was implemented in a hydro-mechanical coupled formulation. Additionally, analyses were performed to identify how the permeability of the rock and the permeability of the fracture influence the hydraulic fracturing. As a result, it was observed that maintaining all of the mechanical properties constant, the hydraulic properties have a great impact on the hydraulic fracturing process. Also, the velocity of propagation of the fracture is affected by the permeability of the rock, and its ratio is inversely proportional.

1 INTRODUCTION

Oil and gas have played a major role in the energy industry for many years and a lot of effort were put into the development of techniques to improve the extraction of oil from non-conventional reservoirs. One of these techniques is the Hydraulic Fracturing, which made possible the extraction of oil from shale reservoir, and increased the production of conventional reservoirs of oil. The technique consists of injecting a high-pressured fluid in the well, which will fracture the rock and will create preferential flow paths for the oil to flow to the well.

The Hydraulic Fracturing is a Hydro-mechanical coupled problem with high complexity because it needs to include the presence of a discontinuity in the medium and consider the intricacy of the flow within the fracture, which even under laminar flow regime is complex. In this work was used a home-made XFEM code to simulate the problem of hydraulic fracture. Darcy's law was used for the flow within the fracture because of its simplicity and because it is widely use in Geotechnical problems.

2 FORMULATION

2.1 COUPLED HYDROMECHANICAL FORMULATION

The Coupled Hydro mechanical saturated problems are solved using two equations, the first to describe the mechanical behavior of the solid material (static equilibrium) and the last to describe the fluid behavior (continuity). The equilibrium equation for a soil volume is:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0 \quad (1)$$

where σ_{ij} is the total stress tensor, b_i are the body forces and x_j is direction of the Cartesian coordinate system.

In the Equation 1 the tensor σ_{ij} is the sum of the effective stress σ'_{ij} and the pore water pressure u_w . The effective stress in a deformable porous media relates with the strain with a constitutive relation. The media is linear elastic and the problem is consider to be plane strain, where are given stresses in σ_x and σ_y with $\sigma_z \neq 0$. Also, the strain is computed in $\varepsilon_x, \varepsilon_y$ e γ_{xy} and $\varepsilon_z = 0$.

The constitutive relation is given by the D plain strain matrix:

$$D = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} (1 - \nu) & \nu & 0 \\ \nu & (1 - \nu) & 0 \\ 0 & 0 & \frac{(1 - 2\nu)}{2} \end{bmatrix} \quad (2)$$

The continuity of the liquid phase is described by the mass conservation equation. Assume that water is incompressible, the equation for the water is given:

$$\frac{\partial(v_i)}{\partial x_i} + \frac{\partial\theta_w}{\partial t} = 0 \quad (3)$$

where $\theta_w = n \cdot S$ is the volumen of water in the soil, n is the porosity, S is the saturation degree and v_i is the water velocity. The velocity can be defined by the Darcy's law as:

$$v_i = -k_{ij} \frac{dh}{dx_j} \quad (4)$$

where k_{ij} is the permeability matrix and $\frac{dh}{dx_j}$ is the hydraulic gradient vector. If the medium is saturated, homogenous and isotropic, the continuity equation of water can be simplified, assuming that the water volume variation in a porous media is equal to the porous media volumetric variation. Therefore, the Equation (3) is written as:

$$\frac{\partial(v_i)}{\partial x_i} + \frac{\partial\varepsilon_v}{\partial t} = 0 \quad (5)$$

where ε_v is the volumetric variation of a soil element.

2.2 XFEM FORMULATION

XFEM (Extended Finite Element Method) is a numerical method developed to model weak and strong discontinuities within a finite element mesh. In this method, a fracture can be represented in the mesh without explicitly adding it to the finite element mesh.

The Extended Finite Element Method was developed by [1] and [3] to study the propagation of elastic fractures. They used functions to enrich the mesh and describe the presence of a fracture in the displacement field.

According to [2] the approximation of the displacement can be described in a general form as:

$$u(x) = \sum_{i=1}^n N_i(x)\bar{u} + enrichment\ terms \quad (6)$$

where n is the set of all nodes and N_i are the shape functions. The enrichment terms uses the Partition of Unity Method, and the shape functions are multiplied by the enrichment functions defined in the nodes of the element located on the discontinuity. The enriched domain is written as

$$u^h(x) = u^{EF} + u^{enr} = \sum_{i=1}^n N_i(x)\bar{u} + \sum_{j=1}^m N_j(x)\psi(x)\bar{a}_j \quad (7)$$

in which the first term is related to FEM interpolation and the second is the enriched interpolation, m is the set of nodes cut by the discontinuity, \bar{a}_j are the nodal degrees of freedom corresponding to the enrichment function, $\psi(x)$ is the enrichment function.

The type of the function used to enrich the problem will depend on the condition of the problem. In this work it was chosen the Heaviside function and the *crack tip* to simulate the problem of Hydraulic Fractures. The Heaviside function is a step function defined as:

$$H(\xi) = \begin{cases} 1 & \forall \xi > 0 \\ 0 & \forall \xi < 0 \end{cases} \quad (8)$$

The crack tip function is used to approximate the behavior of the tip of the fracture within the element. The asymptotic solution of Williams (1957) for the two-dimensional isotropic media was used to define the four enrichment functions:

$$F(r, \theta) = \sqrt{r} \left\{ \cos \frac{\theta}{2}, \sin \frac{\theta}{2}, \sin \frac{\theta}{2} \cdot \sin \theta, \cos \frac{\theta}{2} \cdot \sin \theta \right\} \quad (9)$$

These functions are defined in a local polar coordinate system (r, θ) at the crack tip. Some transformations are made for the Cartesian coordinate.

3. RESULTS

The XFEM code was validated with some problems of fracture known in the literature and it is presented in [4]. After that, some simplified simulations of the Hydraulic fracturing were made. In these simulations, a fluid was injected in a linear, homogenous and elastic media to open and propagate a fracture. It was applied a flow rate in the nodes of the mesh (Figure 1) and it was observed the propagation of the fracture under different conditions. The properties used can be observed on Table 1.

Table 1: Properties used in the simulation

Parameter	Value
Flow rate	2e-5 m ³ /s
Poisson	0,3
Elastic Modulus (E)	2Mpa
K _{Ic}	140 KPa.m ^{1/2}

In the Figure 2 it is illustrated the opening of the fracture with the fluid injection. In the simulation was observed that: the process of fracturing is strongly influenced by the permeability of the media and of the fracture, the influence of the permeability of the media and of the fracture in the relief of porewater pressure inside the fracture and the lost of fracturing fluid to the media (leak-off).

Due to the need of defining the permeability inside the fracture different from the permeability of the surrounding media, it was added in the code a factor to relate the permeability in the porous media with the permeability of the elements cut by the fracture. That was a first approach to deal with this problem. Later it will be used a cubic law to simulate the flow inside the fracture.

Figure 1: Finite Element mesh used in the simulation

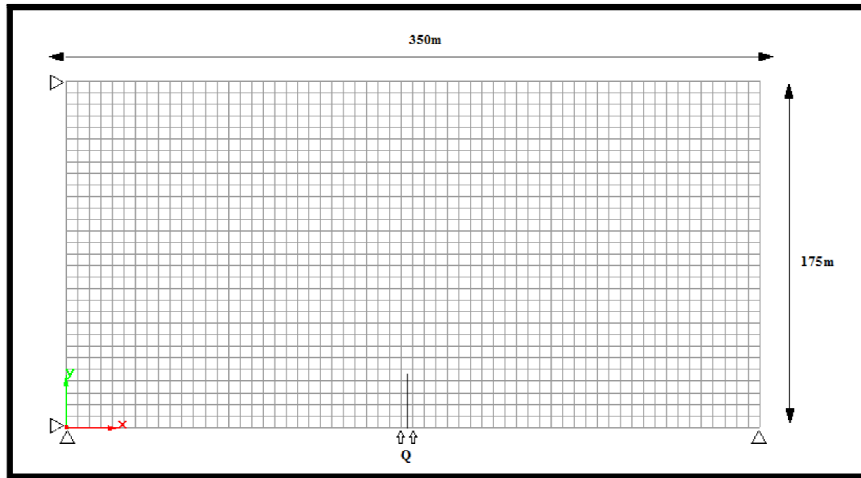
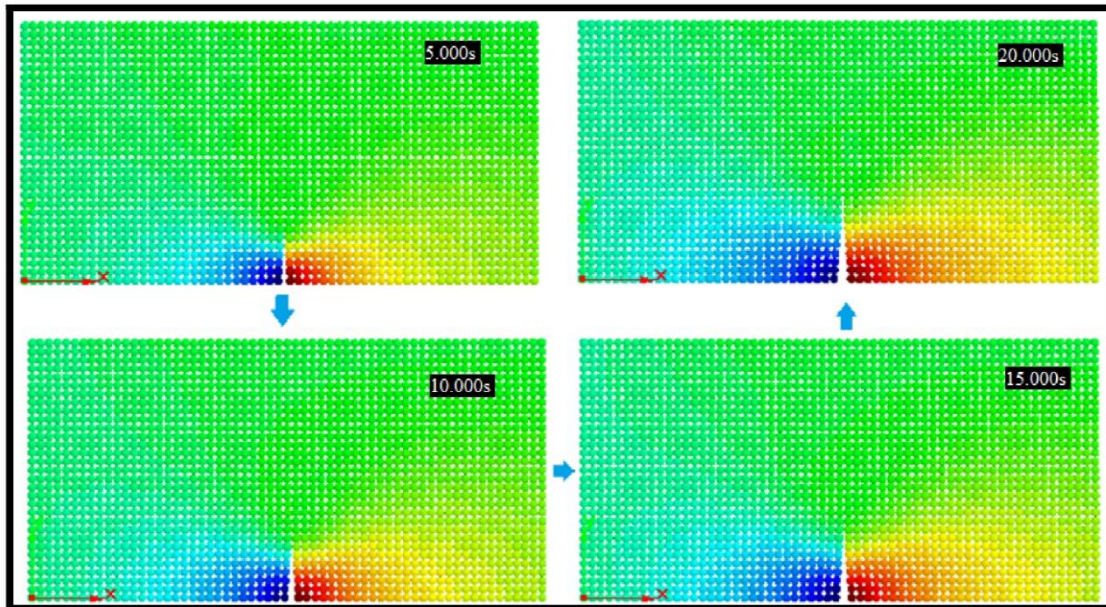


Figure 2: Fracture opening with fluid injection for increasing time

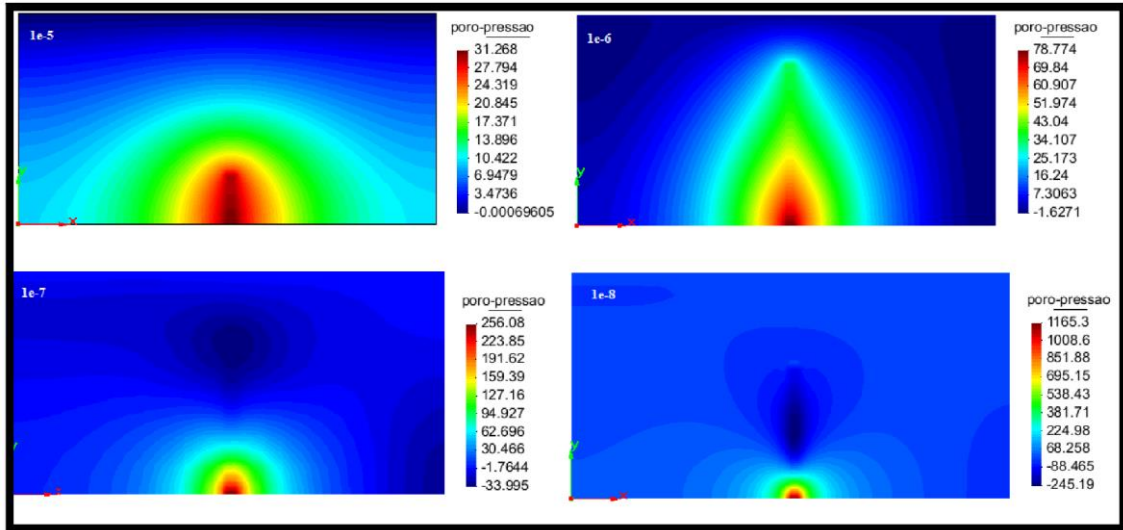


The first study made was a qualitative simulation to observe the relief of pore water pressure. In the Figure 3 it is possible to see the pore water pressure field after injecting the liquid in the media. The values for the permeability of the media and for the permeability of the fracture are on Table 3. In the media with permeability of $1e-5$ m/s occurs the relief of the pore water pressure, and due to the boundary conditions and the time of the simulation the pore water pressure field reach a stationary regimen, and the length of the fracture remains constant.

Table 2: Permeability of the simulation

Medium permeability	Fracture permeability
1e-5 m/s	1e-3 m/s
1e-6 m/s	1e-4 m/s
1e-7 m/s	1e-5 m/s
1e-8 m/s	1e-6 m/s

Figure 3: Pore pressure field for different medium permeability (Pore pressure kPa)



The next simulation was made to observe the relation between the permeability of the media and the loss of fracturing fluid to the media (leak-off). It was injected a fluid with the same rate in medias with different permeabilities, and the permeability of the fracture was constant. The permeability of the media were: 1e-3 m/s, 1e-4 m/s, 1e-5 m/s, 1e-6 m/s, 1e-7 m/s e 1e-8 m/s. The results show that for the media with high permeability (1e-3 m/s and 1e-4 m/s), the injection rate was not sufficient to propagate the fracture because was occurring leak-off with a higher rate than the injection of the fluid and the increase in the pore water pressure (Figure 4).

In the media with lower permeability the process of fracturing was higher because the leak-off process was lower (Figure 5). Despite the large number of variables affecting the process of hydraulic fracturing, when you maintain all other parameters constant and change only the permeability, the result of the fracturing will be different for each permeability value of the media.

Figure 4: Simulation of Hydraulic Fracturing for medias with permeability higher than $1e-5$ m/s

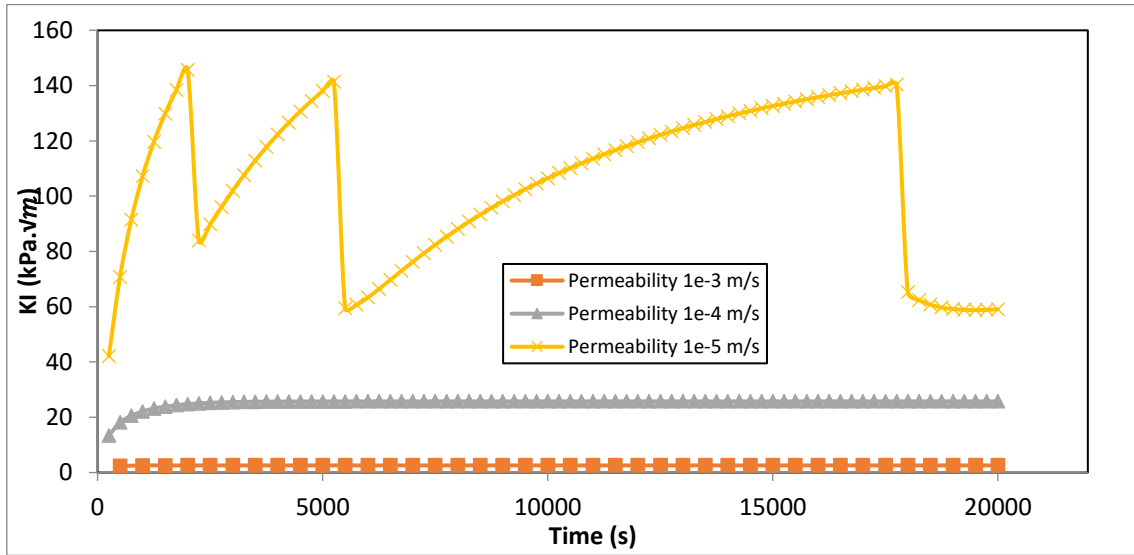
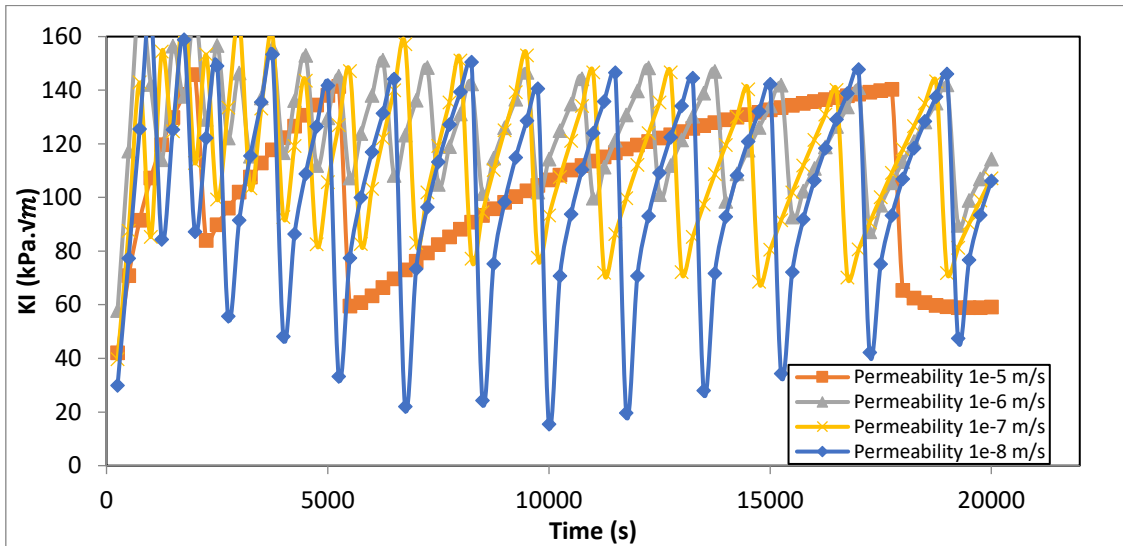


Figure 5: Simulation of Hydraulic Fracturing in medias with permeability lower than $1e-5$ m/s



It was observed in the tests that the length of the fracture and the opening of the fracture are directly proportional when the flow rate to open the fracture is constant. In Figure 6 the graphic shows the increase of the length of the fracture with decrease of the permeability of the media, and in Figure 7 the graphic shows the increase of the opening of the fracture with the decrease of the permeability of the media.

The length of the fracture increase with the decrease of the permeability because when the media is very permeable lots of fracturing fluid is lost (leak-off), decreasing the pressure and hence the expansion of the fracture. The same occurs with the opening of the fracture, in the case where the media has high permeability, the leak-off occurs and the pressure of the fluid inside the fracture decreases, with a lower opening of the fracture.

The last test was made varying the permeability inside the fracture and keeping the permeability of the media constant. The permeability of the media adopted was $1e-7$ m/s and the permeability inside the fracture was $1e-4$ m/s, $1e-5$ m/s and $1e-6$ m/s. The results are in Figure 8 and 9. This tests showed that when the permeability inside the fracture is very low, the porous pressure is not dissipated and it value is high inside the fracture. Also, due to the low permeability inside the fracture, the pressure is high and the opening of the fracture is high.

Figure 6: Crack grow by Hydraulic Fracturing for medias with permeability lower then $1e-5$ m/s

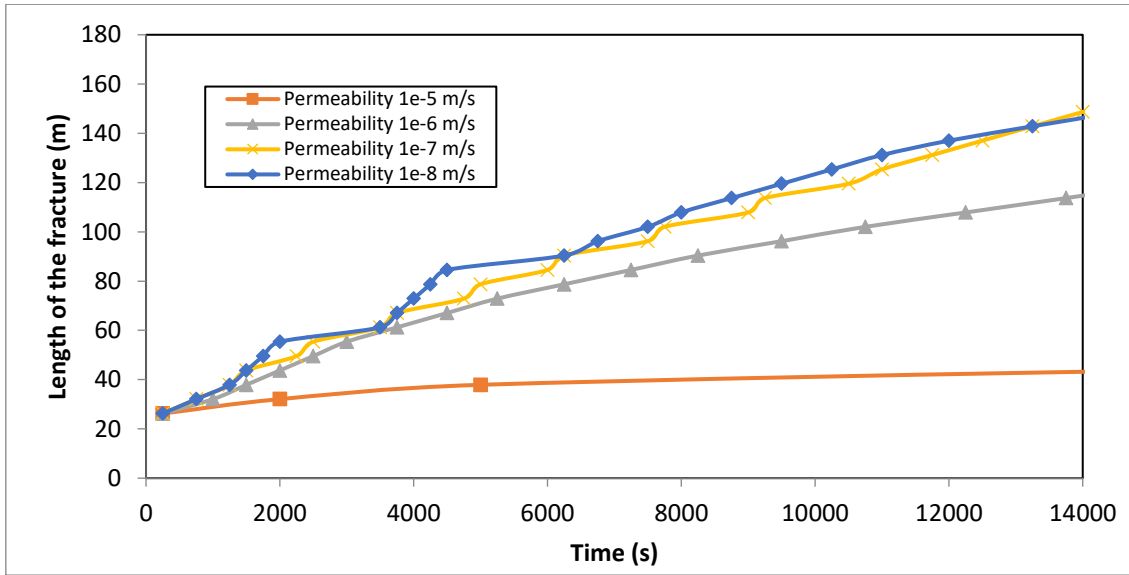
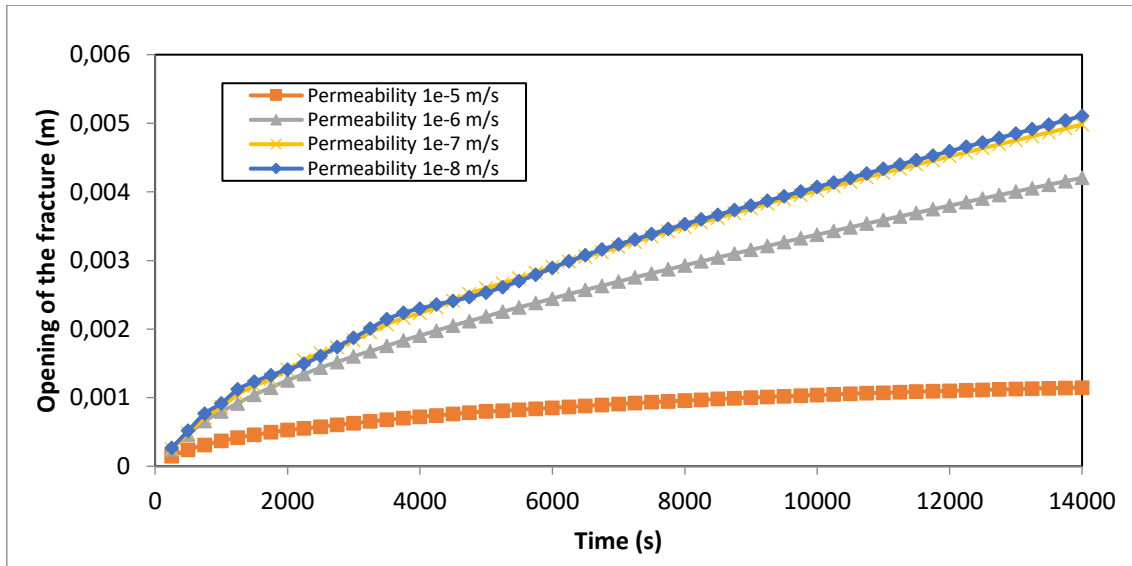


Figure 7: Opening of the fracture by Hydraulic Fracturing for medias with permeability lower then $1e-5$ m/s



The last test was made varying the permeability inside the fracture and keeping the permeability of the media constant. The permeability of the media adopted was $1e-7$ m/s and the permeability inside the fracture was $1e-4$ m/s, $1e-5$ m/s and $1e-6$ m/s. The results are in Figure 8 and 9. This tests showed that when the permeability inside the fracture is very low, the porous pressure is not dissipated and it value is high inside the fracture. Also, due to the low permeability inside the fracture, the pressure is high and the opening of the fracture is high.

Although the simplification of the law to consider the flow inside the fracture, the simulation showed that the permeability has big influence in the hydraulic fracture process. The next step is to implement a more accurate law to describe the flow inside the fracture, for example, the cubic law.

Figure 8: Variation of pore pressure inside the fracture for different permeability inside the fracture

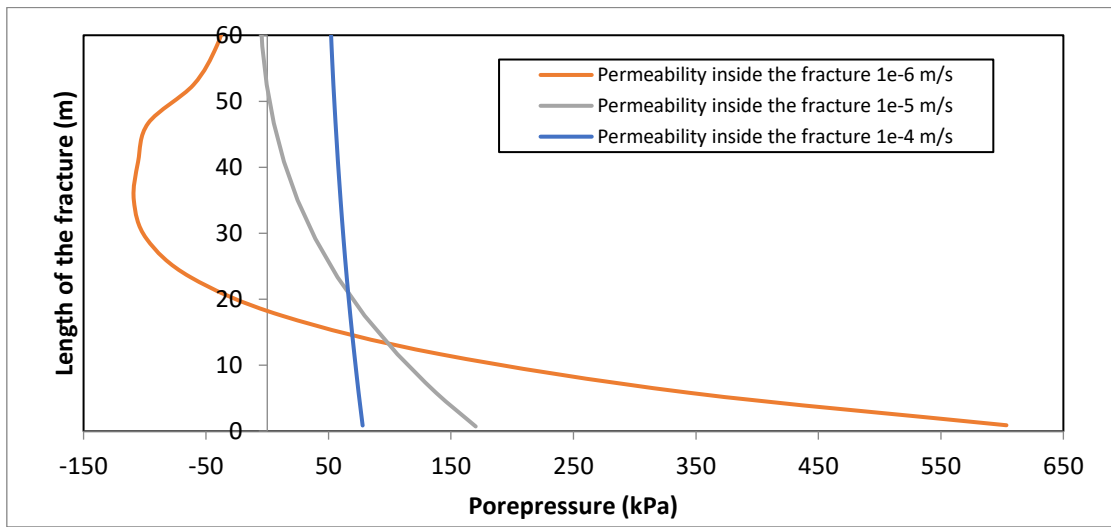
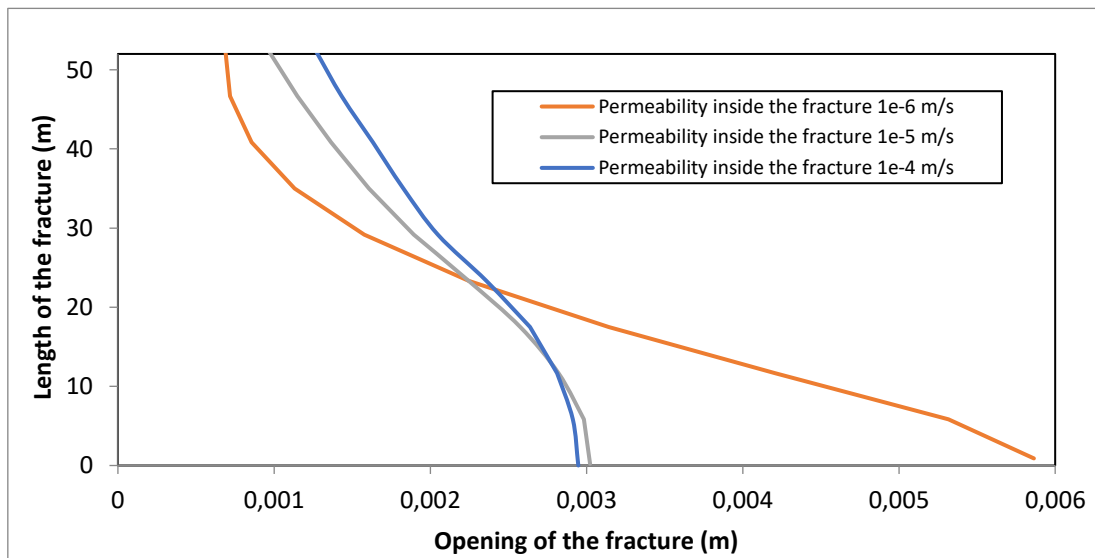


Figure 9: Variation of the opening of the fracture for different permeabilities inside the fracture



3 CONCLUSIONS

- XFEM method is a good alternative to deal with problems involving Hydraulic Fracturing. The implementation of the XFEM allow the research to use part of existing FEM codes.
- The numerical results show that the permeability of the media and inside the fracture has great influence in the Hydraulic Fracturing process.
- The results of the numerical simulation are in agreement with the results obtained by [5].

REFERENCES

- [1] BELYTSCHKO T., BLACK T. (1999). Elastic crack growth in finite elements with minimal remeshing. *International Journal for Numerical Methods in Engineering*, 45, p. 601–620.
- [2] KHOEI, A. R. (2015). *Extended Finite Element Method, Theory and Applications*, John Wiley, (584 pages) 2015, ISBN: 978-1-118-45768-9
- [3] MOËS, N., DOLBOW, J., BELYTSCHKO, T. (1999). A finite element method for crack growth without remeshing. *International Journal for Numerical Methods in Engineering*, 46, p.131–150.
- [4] Morais, L. P. (2016) *Análise Hidromecânica do Problema de Fraturamento Hidráulico* Dissertação de Mestrado, Publicação G.DM- 267/2016, Departamento de Engenharia Civil, Universidade de Brasília, Brasília, DF, 125p.
- [5] Zielonka, M. G., Searles, K. H., Ning, J., Buechler, S. R., “Development and Validation of Fully-Coupled Hydraulic Fracturing Simulation Capabilities”, In *Proceedings of Simulia Customer Conference*. Providence, RI, USA. May 19-22, 2014.