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PECULIARITIES OF AIR ENTRAINMENT WITH A LOOSE MATERIAL FLOW AT THE VARIABLE AERODYNAMIC RESISTANCE OF FALLING PARTICLES

O. A. AVERKOVA*, I.N. LOGACHEV* AND K. I. LOGACHEV*

* Belgorod State Technological University named after V.G. Shukhov
(BSTU named after V.G. Shukhov), 308012 Belgorod, Russia
e-mail: kilogachev@mail.ru, web page: http://www.bstu.ru

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Abstract. Gravity flows of loose-matter particles are accompanied with aerodynamic forces that produce ejection of air in the loading and unloading chutes. A flow of loose matter acts like a blower. Head created by this blower (which we choose to call ejection head) comprises a sum total of aerodynamic forces of falling particles divided by cross-sectional area of the flow [1].

Flows of loose material in loading and unloading chutes that arise during operation of high-performance bucket elevators feature elevated volumetric concentrations as high as $\beta_y = 0.01$. Estimates of air ejection caused by such flows should be based upon instantaneous rather than averaged aerodynamic drag coefficient ψ_y.

Within the range of significant volumetric concentrations, varying volumetric concentration of falling particles leads to fluctuations in instantaneous values of the coefficient ψ. These fluctuations cause the ejection head, even in the case of short chutes, to significantly diverge from the head determined using averaged coefficient ψ_y. In order to compute ejection heads inside loading and unloading chutes, it is necessary to introduce an adjustment coefficient K (K is the ratio of the true value of ejection head in an inclined chute to the mean value of this pressure) the value of which will noticeably diverge from one at small initial velocities of particle flow.

It is possible to view flows of particles in chutes at $B < 0.1$(B is the ratio of the average coefficient of drag encountered by a falling particle to the drag coefficient of an individual particle in the self-similarity area) as blowers with a performance curve determined with formula $p_e(Q) = K_0 z \frac{1}{3}\left[\left|1-\frac{Q}{Sv_k}\right|^3 - \left|n-\frac{Q}{Sv_k}\right|^3\right]$ (Q is the volumetric flow rate of air ejected through the loading chute of the elevator, z is a relative velocity of particles in a pipe, defined as the difference $v-u$, v is the velocity of falling particles, u is the velocity of ejected air, v_n, v_k is the fall velocity of particles at outlet of the chute, $n = v_n / v_k$, S is the cross-sectional area of the chute) in view of the resulting coefficient K_0 (K_0 is the ratio K absent ejection airflow).

For a flow of wheat ($d_e \approx 3$ mm) within the ranges $n > 0.5$ and $\beta_y = 0.001$, increasing drag coefficient along the fall height is only able to produce negligibly small changes in the intensity of ejection head. The adjustment coefficient may be dispensed with ($K = K_0 = 1$), and head value will then be determined using formula $P_{ey} = \frac{\psi_y K_m \varepsilon}{2} \frac{G_m v_k^3}{S a_\tau} \frac{|1-\varphi|^3 - |n-\varphi|^3}{3}$ (. G_m is mass flow rate of the material, a_τ is the acceleration which, for chutes installed at an angle α to horizontal surface, ε is the ratio of air density to particle density, φ is ejection coefficient).

1 INTRODUCTION

Gravity flows of loose-matter particles are accompanied with aerodynamic forces that produce ejection of air in the loading and unloading chutes. A flow of loose matter acts like a blower. Head created by this blower (which we choose to call ejection head) comprises a sum total of aerodynamic forces of falling particles divided by cross-sectional area of the flow [1]. Aerodynamic force may be expressed using the drag coefficient:

$$R = \psi F_m \frac{|v-u|(v-u)}{2} \rho , \tag{1}$$

where R is the aerodynamic force of a single particle (N),
ψ is the drag coefficient,
F_m is the mid-section area of a particle (m^2),
v is the velocity of falling particles (m/s),
u is the velocity of ejected air (m/s),
ρ is the density of air (kg/m^3).

Absolute value of relative velocity $|v-u|$ had to be isolated into a dedicated variable due to the need to vectorize the square of relative velocity for the case involving unidirectional motion of a two-component particle-air medium. This notation means that at $v > u$, R is positive and directed parallel with leading motion of solid particles. At $v < u$ this force is negative i.e. opposite to the falling particles.

The drag coefficient is determined not only by the geometrical shape of particles, but also by their concentration. The greater the volumetric concentration of falling particles, the more pronounced is the effect of aerodynamic shadow on a particle moving toward the “aft” side of a particle ahead of it. The following empirical relationship [53] has been identified for a flow of firm mineral particles sized between 2 and 20 mm falling in inclined chutes:

$$\psi_y = \psi_0 \exp\left(\frac{-1.8\sqrt{\beta_y} \cdot 10^3}{d_e} \right), \tag{2}$$

where ψ_0 is the drag coefficient for a single particle in turbulent surface flow mode (within the self-similarity area),
d_e is equivalent particle diameter (mm),

β_y is volumetric concentration, averaged over the length of the chute and equal to:

$$\beta_y = \frac{2G_m}{S\rho_m\left(v_n + v_k\right)}, \tag{3}$$

G_m is mass flow rate of the material (kg/s),
S is the cross-sectional area of the chute (m^2),
ρ_m is air density (kg/m^3),
v_n, v_k is the fall velocity of particles at inlet/outlet of the chute (m/s).

The majority of crushed mineral particles have a roughly-pointed granular shape and a coefficient $\psi_0 \approx 1.8$. In contrast, cereals are likely to produce oval-shaped fragments when crushed, therefore $\psi_0 \approx 1.0$ can be assumed. If the geometrical shape of these particles were to be evaluated by their dynamic form coefficient, then

$$K_d = \left.\frac{\psi_0}{\psi_b}\right|_{\mathrm{Re}=idem} \approx 2, \tag{4}$$

where ψ_b is the drag coefficient of a sphere within the self-similarity area $\left(\psi_b \approx 0.5\right)$.

Let's determine the ejection head p_e arising within unloading and loading chutes during handling of cereal grain at elevators.

2 VARIATION OF EJECTION HEADS CREATED BY FALLING PARTICLES IN CHUTES WITH VARYING FRONTAL DRAG COEFFICIENTS

By definition, the ejection pressure created in chutes as a result of interaction between components,

$$p_e = \frac{1}{S}\int_0^l R\frac{\beta S dx}{V_p}, \tag{5}$$

where V_p is the volume of a single particle (m^3),
β is the current volumetric concentration of falling particles,

$$\beta = \frac{G_m}{S\rho_m v}; \tag{6}$$

l is chute length (m)
With a small fall height, when the chute length is

$$l < l_y = \frac{1}{\psi_y K_m \varepsilon}, \tag{7}$$

the aerodynamic force is negligibly small compared to gravitational force of a particle and the flow of material can therefore be considered constantly accelerated:

$$\frac{dv}{dt} = \frac{vdv}{dx} = a_\tau\,, \tag{8}$$

where

$$K_m = \frac{F_m}{V_p} \approx \frac{1.5}{d_e}\,; \tag{9}$$

$$\varepsilon = \frac{\rho}{\rho_m}\,; \tag{10}$$

d_e is the equivalent (volumetric) particle diameter (m),

a_τ is the acceleration which, for chutes installed at an angle α to horizontal surface, is equal to

$$a_\tau = g\sin\alpha\left(1 - f_w \operatorname{tg}\alpha\right); \tag{10a}$$

g is gravitational acceleration (m/s^2),

f_w is the friction coefficient for particles rubbing against chute walls (for steel walls, $f_w \approx 0,5$).

In this case, the ejection head for prismatic chutes at $S = \text{const}$; $u = \text{const}$; $\psi = \psi_y = \text{const}$ is determined using the equation:

$$P_{ey} = \frac{\psi_y K_m \varepsilon}{2} \frac{G_m}{Sa_\tau} \frac{\left|v_k - u\right|^3 - \left|v_n - u\right|^3}{3} \tag{11}$$

or

$$P_{ey} = \frac{\psi_y K_m \varepsilon}{2} \frac{G_m v_k^3}{Sa_\tau} \frac{\left|1-\varphi\right|^3 - \left|n-\varphi\right|^3}{3} = \psi_y \frac{G_m v_k}{S} \Phi \frac{\left|1-\varphi\right|^3 - \left|n-\varphi\right|^3}{3}\,, \tag{12}$$

where φ is the component slip ratio (ejection coefficient)

$$\varphi = \frac{u}{v_k}\,; \tag{13}$$

$$n = \frac{v_n}{v_k}\,; \tag{14}$$

$$\Phi = \varepsilon \frac{K_m v_k^2}{2a_\tau}\,; \tag{15}$$

Velocity of cereal grains, considering their modest mass together with a large fall height $\left(l > l_y\right)$, is affected by the drag force. Motion equation for particles falling in an inclined chute in this case would appear as:

$$\frac{dv}{dt} = v\frac{dv}{dx} = a_\tau - \frac{R}{V_p \rho_m}\,. \tag{16}$$

Equations (5) and (16) enable the ejection head p_e to be discovered [52, 24] not only for heavier particles and lower heights $\left(l < l_y\right)$, but for light particles falling from great heights with an observable drag. However, changes in the ψ coefficient over the fall height was not taken into account in this case due to decreasing volumetric concentration of particles owing to (6)

$$\psi = \psi_0 \exp\left(\frac{-1.8\sqrt{\beta \cdot 10^3}}{d_e} \right). \tag{17}$$

Here ψ is a variable quantity that changes over the fall height due to decreasing volumetric concentration β as a result of increasing velocity of particles v.
Instantaneous drag coefficient will be expressed by a putative

$$\psi = \psi_0 B^{\sqrt{\frac{v_a}{v}}}, \tag{18}$$

with the following notational simplification:

$$B = e^{-1.8\frac{\sqrt{\beta_y \cdot 10^3}}{d_e}} = \frac{\psi_y}{\psi_0}; \qquad v_a = \frac{v_n + v_k}{2} \quad . \tag{19}$$

Let's now compare this effect for the case of short chutes when

$$a_\tau >> \frac{R}{V_p \rho_m}.$$

In view of (18) and (8), the following relation determines ejection pressure for a constantly accelerated flow of particles in a prismatic inclined chute (at $S = const$):

$$p_{eu} = \psi_0 \frac{G_m v_k}{S} \Phi \int_n^1 B^{\sqrt{\frac{1+n}{2\zeta}}} \left|\zeta - \varphi\right| \left(\zeta - \varphi\right) d\zeta. \tag{20}$$

This relation also accounts for change in the drag coefficient ψ along the trajectory of falling particles.

In order enable computational comparisons, we'll introduce the value of adjustment coefficient K that will determine the ratio of the true ejection head value to its putative value (12):

$$K = \frac{p_{eu}}{p_{ey}}. \tag{21}$$

Let's quantify it. In view of (12), (19) and (20) the expression appears as:

$$K = \frac{3}{\left(1-\varphi\right)^3 - \left|n - \varphi\right|^3} \frac{1}{B} \int_n^1 B^{\sqrt{\frac{1+n}{2\zeta}}} \left|\zeta - \varphi\right| \left(\zeta - \varphi\right) d\zeta. \tag{22}$$

Then the ratio of maximum ejection heads would be (at $u = \varphi = 0$)

$$K_0 = \frac{3}{1-n^3}\frac{1}{B}\int_n^1 B^{\sqrt{\frac{1+n}{2\zeta}}}\zeta^2 d\zeta = \frac{6}{1-n^3}\frac{1}{B}\int_1^{\frac{1}{\sqrt{n}}}\frac{e^{pt}}{t^7}dt\,, \tag{23}$$

where

$$p = \sqrt{\frac{1+n}{2}}\ln B\,. \tag{24}$$

Let's resolve the integral by representing the integrand function as a series:

$$\frac{e^{pt}}{t^7} = \sum_{k=0}^{\infty}\frac{p^k}{k!}\zeta^{k-7}\,, \tag{25}$$

which results in:

$$\int_1^{\frac{1}{\sqrt{n}}}\frac{e^{P_t}}{t^7}dt = F(t)\Big|_1^{\frac{1}{\sqrt{n}}} = F\left(\frac{1}{\sqrt{n}}\right) - F(1), \tag{26}$$

where the antiderivative function F is equal to:

$$F(t) = -\frac{1}{6}\frac{1}{t^6} - \frac{p}{5}\frac{1}{t^5} - \frac{p^2}{8}\frac{1}{t^4} - \frac{p^3}{18}\frac{1}{t^3} - \frac{p^4}{48}\frac{1}{t^2} - \frac{p^5}{120}\frac{1}{t} - \frac{p^6}{720}\ln t + \sum_{k\to 7}\frac{p^k}{k!}\frac{1^{k-6}}{k-6}\,, \tag{27}$$

then

$$K_0 = \frac{6}{1-n^3}\frac{1}{B}\left[F\left(\frac{1}{\sqrt{n}}\right) - F(1)\right]. \tag{28}$$

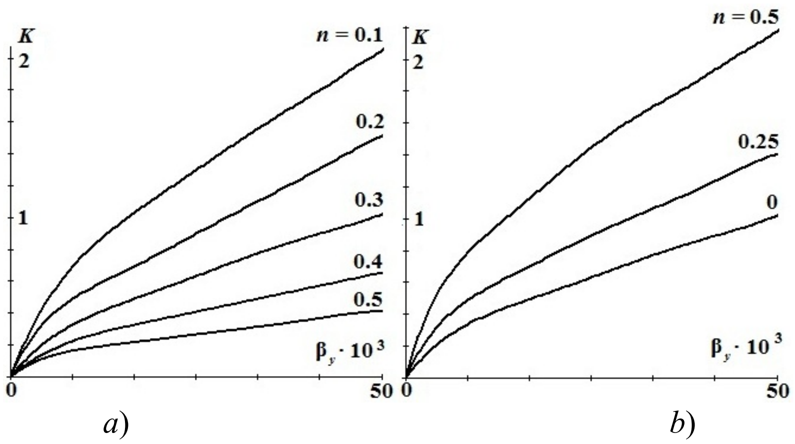

Figure 1: Variation of the adjustment coefficient as a function of volumetric concentration for a constantly accelerated flow of particles $d_e = 3$ mm (a – at $\varphi = 0$; b – at $\varphi \neq 0$ and $n = 0.3$)

Tables 1 and 2 summarize the values of the adjustment coefficient K_0 computed using formula (28) and K computed using formula (22). It is apparent from Table 1 that changes in the drag coefficient ψ along a trajectory travelled by a particle only cause a noticeable impact within the area of significant volumetric concentrations, when $B \leq 0.3$. For example, for grains of wheat ($d_e \approx 3$ mm) this corresponds to the averaged volume concentration $\beta_y = 0.004$. The adjustment factor is significantly higher for smaller initial velocities (at $n < 0.5$) of the flow (Fig. 1, a). Its value increases somewhat (Fig. 1, b) when air is flowing inside a chute (at $\varphi \neq 0$). However, considering that the ejection head

$$p_e = K\psi_y \frac{G_m v_k}{S} \Phi \frac{|1-\varphi|^3 - |n-\varphi|^3}{3} \tag{29}$$

diminishes with increasing φ as a result of braking action of the initial section of accelerated flow, the following assumption would be justified for computations

$$K \approx K_0 . \tag{30}$$

Table 1: Adjustment coefficient values for the case of still air in the chute

B	Values of K_0 at n equal to…								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	2.017	1.717	1.489	1.319	1.198	1.113	1.057	1.023	1.005
0.02	1.786	1.556	1.382	1.251	1.155	1.089	1.045	1.018	1.004
0.03	1.667	1.475	1.326	1.214	1.133	1.076	1.038	1.015	1.003
0.04	1.589	1.420	1.288	1.189	1.118	1.067	1.034	1.014	1.003
0.05	1.531	1.379	1.261	1.171	1.106	1.061	1.031	1.012	1.003
0.06	1.487	1.348	1.239	1.157	1.098	1.056	1.028	1.011	1.003
0.07	1.450	1.322	1.221	1.146	1.090	1.052	1.026	1.010	1.002
0.08	1.420	1.300	1.207	1.136	1.084	1.048	1.024	1.010	1.002
0.09	1.390	1.281	1.194	1.128	1.079	1.045	1.023	1.009	1.002
0.1	1.371	1.265	1.183	1.120	1.075	1.043	1.022	1.009	1.002
0.2	1.234	1.168	1.116	1.076	1.047	1.027	1.014	1.005	1.001
0.3	1.165	1.118	1.081	1.053	1.033	1.020	1.010	1.004	1.001
0.4	1.120	1.086	1.059	1.039	1.024	1.014	1.007	1.003	1.001
0.5	1.088	1.063	1.043	1.028	1.018	1.010	1.005	1.002	1.000
0.6	1.063	1.045	1.031	1.020	1.013	1.007	1.004	1.001	1.000
0.7	1.043	1.030	1.021	1.014	1.009	1.005	1.002	1.001	1.000
0.8	1.026	1.019	1.013	1.008	1.005	1.003	1.002	1.001	1.000
0.9	1.012	1.009	1.006	1.004	1.002	1.001	1.001	1.000	1.000
1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Keeping in mind that grain handling facilities in real-world elevator will have a flow velocity at duct inlet of ($0.3 \div 0.5 v_k$) ejection head can thus be determined using formula (12) by introducing an adjustment coefficient exclusively within the area of large volumetric concentrations, at

$$B < 0.1 \text{ and } n < 0.3 . \tag{31}$$

Thus, the flow of loose particulate matter can be considered as a kind of blower with the output described as:

$$p_e(Q) = K_0 z \frac{1}{3}\left[\left|1-\frac{Q}{Sv_k}\right|^3 - \left|n-\frac{Q}{Sv_k}\right|^3\right], \tag{32}$$

with the following assignment made to simplify the notation:

$$z = \psi_y \frac{G_m v_k}{S}\Phi = \psi_y \frac{G_m v_k}{S}\varepsilon\frac{K_m v_K^2}{2a_\tau}; \tag{33}$$

Q is the volumetric flow rate of air ejected through the loading chute of the elevator (m^3/s).

Table 2: Adjustment coefficient values for the case of moving air in the chute

B	Values of K at n equal to…								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
At $\varphi = 0.25$									
0.02	2.026	1.746	1.526	1.349	1.216	1.122	1.061	1.024	1.005
0.04	1.778	1.571	1.405	1.270	1.167	1.095	1.047	1.019	1.004
0.06	1.650	1.479	1.341	1.227	1.141	1.080	1.040	1.016	1.003
0.08	1.565	1.417	1.298	1.199	1.123	1.070	1.035	1.014	1.003
0.1	1.502	1.371	1.266	1.178	1.110	1.063	1.031	1.012	1.003
0.2	1.326	1.242	1.174	1.116	1.072	1.041	1.020	1.008	1.002
0.4	1.173	1.129	1.093	1.062	1.038	1.022	1.011	1.004	1.001
0.6	1.093	1.069	1.050	1.033	1.021	1.012	1.006	1.002	1.000
0.8	1.039	1.029	1.021	1.014	1.009	1.005	1.002	1.001	1.000
1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
At $\varphi = 0.5$									
0.02	4.507	2.455	1.822	1.526	1.342	1.198	1.097	1.037	1.008
0.04	3.840	2.152	1.644	1.412	1.270	1.157	1.077	1.030	1.006
0.06	3.482	1.989	1.548	1.351	1.230	1.134	1.066	1.025	1.005
0.08	3.239	1.878	1.483	1.309	1.203	1.119	1.058	1.022	1.005
0.1	3.056	1.794	1.435	1.277	1.183	1.107	1.053	1.020	1.004
0.2	2.499	1.547	1.292	1.185	1.123	1.072	1.035	1.013	1.003
0.4	1.926	1.311	1.161	1.101	1.067	1.039	1.019	1.007	1.002
0.6	1.556	1.173	1.089	1.055	1.037	1.021	1.010	1.004	1.001
0.8	1.261	1.075	1.039	1.023	1.016	1.009	1.005	1.002	1.000
0.9	1.131	1.034	1.019	1.011	1.007	1.004	1.002	1.001	1.000
1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

CONCLUSIONS

Flows of loose material in loading and unloading chutes that arise during operation of high-performance bucket elevators feature elevated volumetric concentrations as high as $\beta_y = 0.01$. Estimates of air ejection caused by such flows should be based upon instantaneous (relation (18)) rather than averaged aerodynamic drag coefficient ψ_y.

Within the range of significant volumetric concentrations, varying volumetric concentration of falling particles leads to fluctuations in instantaneous values of the coefficient ψ. These fluctuations cause the ejection head, even in the case of short chutes ($l < l_y$), to significantly diverge from the head p_{ey} determined using averaged coefficient ψ_y. In order to compute ejection heads inside loading and unloading chutes, it is necessary to introduce an adjustment coefficient K the value of which will noticeably diverge from one at small initial velocities of particle flow (at $n < 0.5$, Table 2, Fig. 1).

It is possible to view flows of particles in chutes at $B < 0.1$ as blowers with a performance curve determined with formula (32) in view of the resulting coefficient K_0. For a flow of wheat ($d_e \approx 3$ mm) within the ranges $n > 0.5$ and $\beta_y = 0.001$, increasing drag coefficient along the fall height is only able to produce negligibly small changes in the intensity of ejection head. The adjustment coefficient may be dispensed with ($K = K_0 = 1$), and head value will then be determined using formula (12).

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