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NUMERICAL AND EXPERIMENTAL INVESTIGATIONS OF THE OSCILLATORY MOTION OF THIN PLATES IN A STILL VISCOUS FLUID

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Abstract. The paper is devoted to study of the aerodynamic forces acting on flat plates performing free flexural vibrations in a viscous fluid. The study consists of two parts. In the first part the experimental investigation of the aerodynamic loads based on analysis of damped flexural vibrations of cantilever test samples is conducted. In the second part the aerodynamics of the oscillating plates is investigated using direct numerical simulation.

1 INTRODUCTION

The past few decades have witnessed a rising interest in the study of mechanical vibrations of thin plates in viscous static fluids. The motivations for these studies stem from different practical applications covering diverse fields of knowledge such as atomic microscopy [1], sensors and actuators based on micromechanical oscillators [2], cooling devices [3], marine and offshore equipments [4, 5]. Our interest in this research area is connected with the development of the approach [6, 7] for determination of the damping properties of materials based on the study of the damped flexural vibration of cantilever flat beams.

When studying vibrations in air, it is necessary to accurately determine aerodynamic loads. In the general, the problem of evaluation of aerodynamic forces acting on a cantilever beam is extremely complicated, mainly because of the complexity of three-dimensional gas flows caused by vibrations of the beam. But when the length of the beam considerably exceeds its width and thickness at low structural vibration modes, the length of the vibrational wave is much greater than deviations of the beam, as a result it can be regarded as locally planar. In this case it is possible to use a simplified quasi-two dimensional model of interaction between the beam and a gas, according to which the aerodynamic forces acting on each cross section of the beam are caused by the planar flow around it. These assumptions form a theoretical core that is used for the study of aerodynamics around oscillating plates in the present research.

The study consists of two parts. In the first part the experimental investigation of the aerodynamic loads based on analysis of damped flexural vibrations of cantilever test samples is conducted. In the second part the aerodynamics of the oscillating plates is investigated using direct numerical simulation.

2 THE PROBLEM STATEMENT

Let us consider an elastic plate of length L, width b, and thickness h ($h << b << L$) (Fig. 1). One of its ends is rigidly fixed and the second is free. As soon as the plate is disturbed from the equilibrium, it starts to vibrate harmonically in the surrounding air. The frequency of these vibrations ω weakly varies in the vicinity of the basic natural frequency ω_0 of flexural vibrations of the plate, while the amplitude A weakly decays with time t because of air resistance and internal damping. The problem consists in determining the aerodinamic influence on this process. We will characterize the laws of slow variations of the amplitude and frequency by using the logarithmic decrement of vibrations (LD) δ(A) and the relative variation of frequency (RVF) Ω(A) as functions of the current amplitude of flexural vibrations of the plate:

$$\delta = -\frac{2\pi}{A\omega_0}\frac{dA}{dt}, \Omega = \frac{\omega_0 - \omega}{\omega_0}.$$

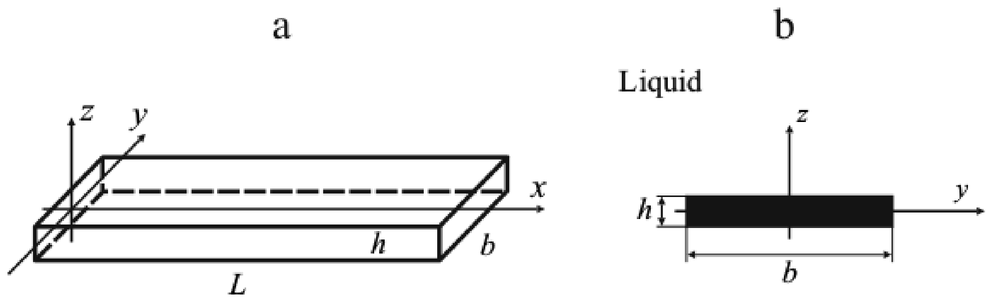

Figure 1: Schemes for the full beam vibration problem (a) and for the associated 2D fluid-structure interaction problem (b).

The equation describing vibrations of a plate according to a cylindrical flexural mode has the form

$$\frac{Ebh^3}{12} w^{IV} + \rho bh\ddot{w} = H + P. \quad (1)$$

Hereinafter, the Roman numerals designate differentiation with respect to spatial coordinate x and dots designate differentiation with respect to time t; w is the displacement of middle line of the plate along the z axis; H and P are the forces of internal friction and aerodynamic resistance; ρ and E are the effective density and Young's modulus of plate material. The boundary conditions correspond to a rigid fixation at $x = 0$ and to a free end at $x = L$.

The drag forces are smaller than the elastic one. Therefore, to a first approximation, we may assume that $H = P = 0$ .In this case, as is known, the basic vibration mode takes the form

$$w = A\cos(\omega_0 t)W(x/L) \quad (2)$$

The constants A and ω_0 represent the amplitude and natural frequency of the basic mode, and the profile W of vibrations ($W(1) = 1$) is described by the formula

$$W(x) = \frac{1}{2}(\cosh kx - \cos kx) - \frac{1}{2}\frac{\cosh kx + \cos kx}{\sinh kx + \sin kx}(\sinh kx - \sin kx).$$

The value of k = 1.8751 is the smallest positive root of the characteristic equation $\cos k \cosh k = -1$, and the frequency of natural vibrations is

$$\omega_0 = k^2 \frac{h}{L^2}\sqrt{\frac{E}{12\rho}}.$$

Owing to the presence of small forces ($\sim \varepsilon$) in the right-hand side of equation (1), the vibration amplitude A and frequency ω_0 in equation (2) do not remain constant, but slowly vary in time. An analysis of such a variation can be carried out by introducing, along with the fast time t, a slow time $\tau = \varepsilon\, t$ and performing a two-scale asymptotic expansion. Omitting details of this procedure, we present the final result:

$$\delta = 2\pi F_0^{-1}\frac{\{\sin \omega_0 t\langle (P+H)W\rangle\}}{\langle W^2\rangle},\ \Omega = F_0^{-1}\frac{\{\cos \omega_0 t\langle (P+H)W\rangle\}}{\langle W^2\rangle},\ F_0 = \rho bhA\omega_0^2. \qquad (3)$$

Hereinafter, the angular brackets designate averaging over the spatial coordinate x, while the braces mean time averaging.

It is obvious that, in view of linearity of the right-hand side parts of equations (3), the different components of forces can be calculated independently from each other. Thus we get the integral relationship between the aerodynamic forces and parameters of beam oscillations (LD and RVF).

3 AN EXPERIMENTAL INVESTIGATION OF AERODYNAMIC FORCES

3.1 Experimental setup

To get a damping parameters of the flexural oscillation of cantilever beam in the surrounded air the experiments for measurement the damped oscillation caused by the initial deflection of the free end of the console from equilibrium was carry out. As experimental samples duralumin beams with the next geometrical parameters are used: thickness h = 1, 2 mm, width b = 20, 30 mm and length L, which varies in the range from 200 to 400 mm with increments of 20 mm. The registration of vibrations is performed by the lightweight MEMS gyroscope mounted on the end of the beams.

As a result of the procedure, approximate relations for the vibration amplitude $A(t)$, frequency $\omega(t) \approx \omega_0$, LD $\delta(t)$ and RVF $\Omega(A)$ as functions of time are determined. From these approximation for each experiment in the range of the realized oscillation amplitudes the dependences $\delta(A)$, $\Omega(A)$ were built.

3.2 Extraction of aerodynamic damping parameters and calculation of force coefficients

In the general case, the parameters of the vibrations of the beam depend on the aerodynamic and mechanical components of damping. To separate of these components from each other in the experiments we use test samples of aluminum alloy. The internal damping of the beams of this material [8, 9] is almost independent of the oscillation amplitude up to very high strain. The mounting method of samples [9] provides independence of structural damping from oscillation amplitude. Thus, the change of the LD from oscillation amplitude is a consequence of aerodynamic effects.

The influence of the mechanical component of the damping on the RVF of the beam for the parameters of the present experiments is not known. In the subsequent discussion we assume that this influence is extremely small and RVF is completely determined by aerodynamic effects.

Ignoring the three-dimensional effects, consider the impact of aerodynamic flow as a result of quasi-two-dimensional flow around the beam. Also consider that the aerodynamic forces $P(x)$ at each cross section x of the beam can be described by approximation

$$P = -\frac{\pi}{4}\rho_a b^2 C_M \frac{du_0}{dt} - \frac{1}{2}\rho_a b C_D |u_0| u_0 \quad . (4)$$

Here $u_0(x, t) = \partial w/\partial t$ is the velocity of displacement of the current beam cross-section, ρ_a is the air density, C_M and C_D are local coefficients of added masses and drag respectively. Substituting (4) into (3), we get the next equations

$$\delta_P = \frac{\rho_a}{\rho}\frac{4}{3}\frac{A}{h}\frac{\langle C_D W^3\rangle}{\langle W^2\rangle}, \Omega_P = \frac{\pi}{8}\frac{\rho_a}{\rho}\frac{b}{h}\frac{\langle C_M W^2\rangle}{\langle W^2\rangle} \quad (5)$$

which connects the LD and RVF with the drag and added mass coefficients. It should be noted that the added mass force according to (5) affects only on the frequency of beam oscillations, while the drag force affects only on the amplitude of beam oscillations.

For transition from the oscillation parameters to the aerodynamic characteristics the nonlinear integral equations (5) must be solved. For this purpose we used the analytical method proposed in [10]. Thus, each experiment allows finding the dependence of the drag coefficient C_M and the coefficient C_D of added mass from the oscillation amplitude of the beam for the fixed frequency value.

4 NUMERICAL CALCULATION OF AERODYNAMIC FORCES

As an alternative method for determining aerodynamic characteristics numerical simulation is considered. The aerodynamic coefficients C_M and C_D are found during solving the planar fluid flow problem caused by vibrations of an infinitely extended thin rigid plate (Fig. 1b). Such a plate plays the role of a mobile solid boundary for the air surrounding it. In each given cross section x, the velocity of displacement of the boundary is given as

$$u_0(x,t) = -U_0(x)\sin(\omega_0 t),\ U_0(x) = \omega_0 A W(x/L).$$

The fluid flow around the plate generated by its motion is governed by the Navier-Stokes system of equations. Normalizing the spatial coordinates, time and velocity by b, bU_0^{-1}, U_0 respectively, we get the dimensionless formulation of the governing system in the following form:

$$\frac{\partial U}{\partial t} + U\cdot\nabla U = -\nabla p + \frac{1}{\text{Re}}\Delta U,\ \nabla\cdot U = 0 \quad (6)$$

Where $U = (u, v)$ is the dimensionless velocity, p is the dimensionless pressure, $\text{Re} = U_0 R/\nu$ is the Reynolds number.

For solving this problem numerically it is convenient to rewrite governing equations in a moving (non-inertial) coordinate system associated with the plate. To retain the governing system in the form (6), we determine a new pressure as

$$p = \tilde{p} + za.$$

Here the first term is the pressure in the fixed coordinate system, the second term is the inertial contribution, a is the acceleration of the moving coordinate system.

On the surface of the plate in the new coordinate system the no-slip conditions are fulfilled:

$$u\,|_{plate} = v\,|_{plate} = 0. \quad (7)$$

At infinity, the variation of velocity is determined by the following harmonic law:

$$u\,|_{\infty} = \sin(\pi t / \mathrm{KC}),\; v\,|_{\infty} = 0. \quad (8)$$

Here KC is the second dimensionless control parameter of the problem - the Keulegan-Carpenter number. A complex of two parameters (Re, KC) completely determines the flow of fluid near the oscillating plate. Sometimes it is also convenient to use their ratio

$$\beta = \frac{\mathrm{Re}}{\mathrm{KC}}$$

which plays the role of the oscillatory Reynolds number and has approximately the same value along the x direction of the flexural oscillating plate.

The forces acting on the plate by a viscous fluid in the dimensionless formulation are calculated as

$$C_P = \int_S pnds - \int_S \overline{\sigma} \cdot nds,$$

where $\overline{\sigma}$ is the viscous stress tensor, S is the surface of the plate and n is the surface normal vector. The force vector C_P can be expanded into a vertical component C_P^y (lift force) and a horizontal component C_P^z, which consists of the drag force and inertia force:

$$C_P^z = \pi (C_M + C_{FK}) \frac{du_\infty}{dt} + C_D |u_\infty| u_\infty$$

The inertial force acting on the plate due to the fluid acceleration can be split into two parts: the inertia force of added mass, arising due to the local acceleration near the cylinder, and the Froude-Krylov force (C_{FK}), which is related to the pressure gradient created in the fluid by the flow oscillations (associated with the transition to the moving coordinate system). The C_{FK} force coefficient for the considered case can be calculated as

$$C_{FK} = \frac{1}{\pi} \int_S znds.$$

To determine the quantities C_M and C_D as functions of the parameters β, KC, it is necessary to carry out a numerical solution of problem (6)-(8). The corresponding numerical calculations were performed in the OpenFOAM software package of computational hydrodynamics, based on model presented in [12]. It should also be noted that in the numerical simulations we accurately reproduce the shape of the plates used in the experiments, especially the ratio of width b to thickness h and the form of ends of the samples, which have the rounded corners.

5 RESULTS

As the main results of the study, we present here the dependences of the drag C_D and added mass C_M coefficients of flat plates on the dimensionless vibration amplitude KC.

The functions C_D(KC) are plotted in Fig. 2 for different values of β. Curves 1, 2, 3 represent experimental data obtained for β = 430, 970, 1290 respectively. Solid line with cross-shaped markers corresponds to numerical results for β = 55. As can be seen, present results (both numerical and experimental) are well agreed with the data presented in previous researches [10], [13], [14].

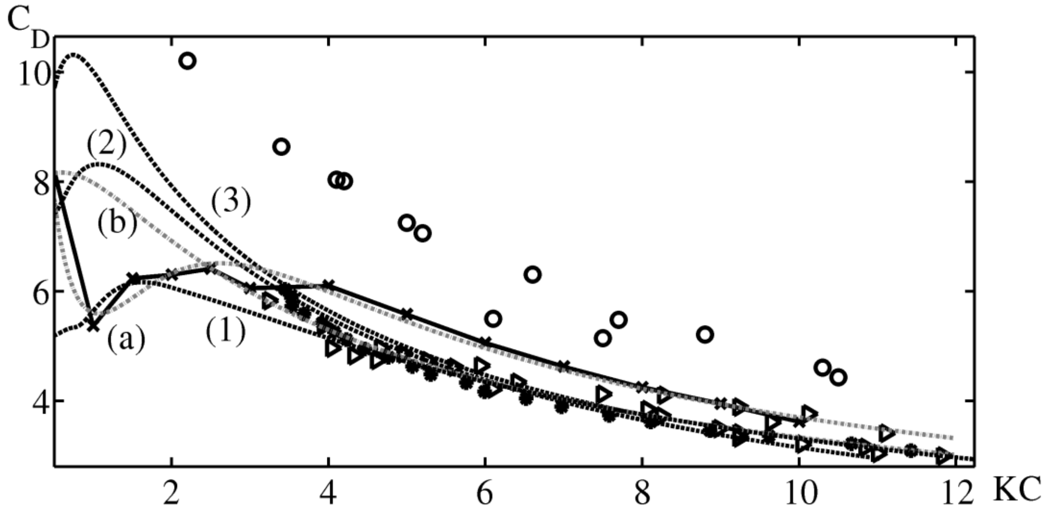

Figure 2: Drag coefficient C_D vs. dimensionless parameter KC. Black doted lines 1, 2, 3 represent experimental results and solid line with cross-shaped markers represent the numerical results of the present study. Gray doted lines a, b illustrate experimental results of [10]. Triangle markers correspond to experimental data of [13], black circles correspond to experimental data of [14] and white circles correspond to experimental data of [15].

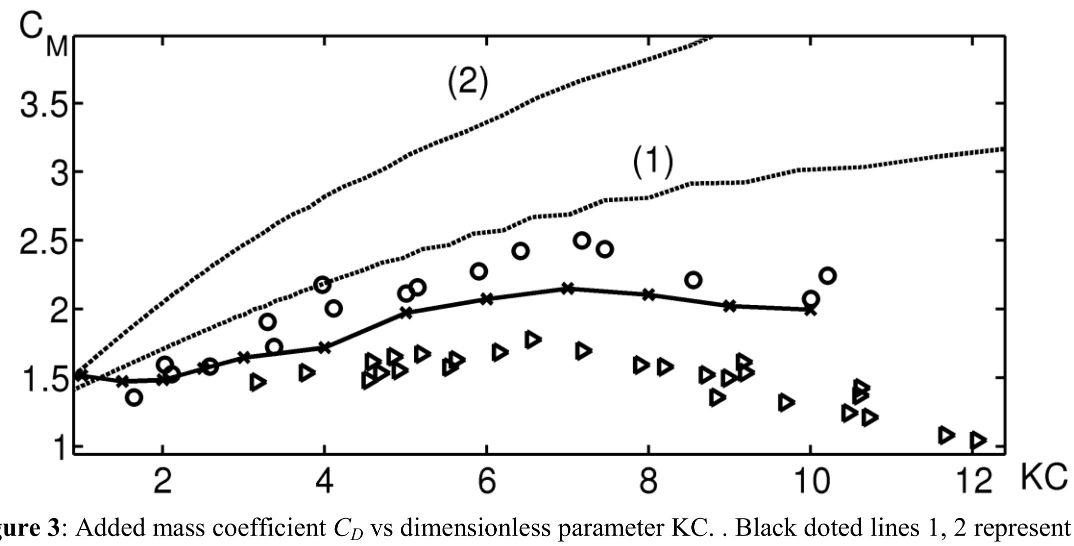

Figure 3: Added mass coefficient C_D vs dimensionless parameter KC. . Black doted lines 1, 2 represent experimental results and solid line with cross-shaped markers represent the numerical results of the present study. Triangle markers correspond to experimental data of [13] and white circles correspond to experimental data of [15].

The functions C_M(KC) for different values of β are plotted in Fig. 3. Curves 1, 2 represent experimental data obtained for β = 430, 970 respectively. As it can be seen, the present experimental results (espesualy for high values of β) lies higher than data from [13], [15] and the data of numerical results (obtained for β = 55). This can be caused by the influence of the non-zero mechanical component of the damping (see section 3.2).

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