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SEMI-ANALYTICAL HYBRID APPROACH FOR MODELLING WAVE MOTION EXCITED BY A PIEZOELECTRIC TRANSDUCER IN A LAMINATE WITH MULTIPLE CRACKS

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Abstract. A semi-analytical hybrid approach is presented here to simulate the dynamic behaviour of a multi-layered elastic waveguide with a system of delaminations and a piezoelectric transducer mounted on the surface of the waveguide. The proposed hybrid approach combines the advantages of the frequency domain spectral element method, which is applied to discretize a complex-shaped piezoelectric structure, and the boundary integral equation method employed to simulate wave propagation in multi-layered waveguide with multiple delaminations. The proposed method is applicable to the multi-parameter analysis of the phenomena related to elastic wave scattering and excitation. The advantages of the presented extended semi-analytical hybrid approach method along with the results of the parametric analysis of wave propagation in the considered structures are discussed.

1 INTRODUCTION

A considerable number of methods employed in non-destructive evaluation and structural health monitoring (SHM) for damage identification are related to elastic waves. Any structural defect reflects the guided waves and the response signal is used to obtain the position and the size of the defect. Among the latter, cracks and interface disbonds are main goals of inspection, due to their potential role in possible breaks of integrity and catastrophic failures of the structures employed, for instance, in aircraft and spacecraft, and the rotor blades of wind turbines. For this reason, the scattering of elastic waves by cracks has been extensively studied applying various methods ranging from purely numerical ones to more analytically oriented approaches. A number of built-in or on-board piezoelectric transducers and sensors are used to excite wave motion and record structural responses afterwards. Refinement of the corresponding SHM systems demands understanding and simulation of the interaction between transducers and multi-layered structures with defects. Development of practical techniques for damage detection is impossible without efficient tools for simulation and numerical analysis. In order to simulate piezo-induced guided waves propagation and scattering from the internal delaminations accurate and reliable mathematical models should be developed.

This paper presents a semi-analytical hybrid approach (SAHA) for dynamic behaviour simulation of a multi-layered elastic waveguide with system of delaminations and piezoelectric actuators mounted on the surface. The presented method extends the hybrid approach proposed in [1] for modelling of the dynamic interaction of perfectly bonded or partially debonded piezoelectric structures with a layered elastic waveguide. The SAHA combines the advantages of the frequency domain spectral element method (FDSEM) [2] to discretize complex-shaped piezoelectric structures and the boundary integral equation method (BIEM) [3, 4] to simulate wave propagation in multi-layered waveguides with a set of horizontal delaminations. The coupling of these two methods is performed in the contact area between waveguide and transducer via the introduction of an unknown traction vector-function. To the authors' knowledge, only the hybrid method [6] is rather similar to the SAHA in this sense. However, a solution in a perfectly bonded rectangular transducer was approximated in [6] via Chebyshev polynomials without discretization into finite elements, and it is not applicable to simulate debonded transducers and internal delaminations. The proposed method is applicable to the multi-parameter analysis of the phenomena related to elastic wave scattering and excitation. The advantages of the presented extended SAHA along with the results of the parametric analysis of wave propagation in the considered structures are discussed.

2 STATEMENT OF THE PROBLEM

The piezoelectric wafer active sensor (PWAS) exciting wave motion occupies domain $V^{(\mathrm{a})} = \{0 \le x_1 \le w_a, 0 \le x_2 \le h_a\}$, while the laminate structure occupies the domain $V^{(0)} = \{|x_1| < \infty, -H \le x_2 \le 0\}$. Accordingly, w_a and h_a denotes width and height of the PWAS. The composite consists of N elastic layers $V^{(n)} = \{|x_1| < \infty, -h_n \le x_2 \le -h_{n-1}$ with Lame constants $\mu^{(m)}$, $\lambda^{(m)}$ and mass density $\rho^{(m)}$. M strip-like cracks with stress-free surfaces occupy domains $\Omega^{(m)} = \{|x_1 - c_m| \le l_m, x_2 = -d_m\}$, see Figure 1.

Constitutive equations for the PWAS are given as follows

$$\sigma_{ij} = C_{ijkl}s_{kl} - e_{kij}E_k,$$

$$D_i = e_{ikl}s_{kl} + \varepsilon_{ij}E_j.$$

Here σ_{ij} is stress tensor, s_{ij} is strain tensor, D_i is electric displacements vector and E_i is electric field vector, while C_{ijkl}, e_{kij} and ε_{ij} are matrices of the elastic constants,

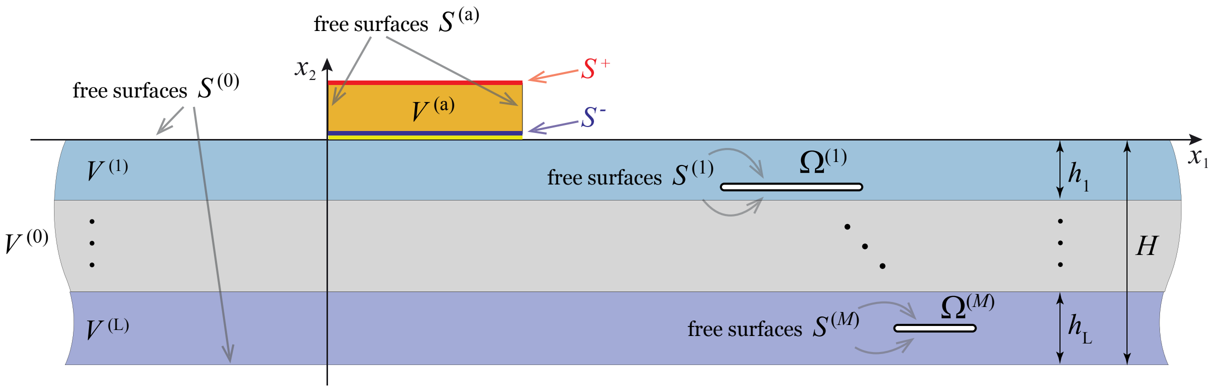

Figure 1: Geometry of the problem

piezoelectric constants and dielectric constants measured with zero strain respectively $(i, j, k, l = \overline{1,3})$. Laminate $V^{(0)}$ with the boundaries $(S^{(0)} = \{|x_1| < \infty, x_2 = \{-H, 0\}\})$ is assumed elastic, so electric components are absent. The dependence between strain s_{kl} and mechanical displacements u_k is expressed with relation

$$s_{kl} = \frac{1}{2}\left(u_{k,l} + u_{l,k}\right),$$

where $u_{k,l}$ denotes derivatives of u_k with respect to x_l. The components of electric field vector are expressed in terms of electric potential ϕ as follows:

$$E_i = -\phi_{,i}.$$

The two-dimensional time-harmonic vibrations with the circular frequency ω are considered with plane strain assumption. Accordingly, governing equations for the piezoelectric media $V^{(a)}$ have the following form:

$$C_{ijkl}u_{k,lj} + e_{kij}\phi_{,kj} + \rho\omega^2 u_i = 0, \qquad (1)$$

$$e_{ikl}u_{k,li} - \varepsilon_{ik}\phi_{,ki} = 0. \qquad (2)$$

Electric potentials $V^{\pm}$ are applied at the lower $(S^- = \{0 \le x_1 \le w_a, x_2 = 0\})$ and the upper $(S^+ = \{0 \le x_1 \le w_a, x_2 = h_a\})$ surfaces of the PWAS:

$$\phi = V^{\pm}, \quad \boldsymbol{x} \in S^{\pm}. \qquad (3)$$

At the side boundaries of the PWAS $S^{(a)} = \{x_1 = \{0, w_a\}, 0 \le x_2 \le h_a\}$, zero electric displacements are given:

$$D_i = 0, \quad \boldsymbol{x} \in S^{(a)}. \qquad (4)$$

The lower boundary $S^- = V^{(0)} \cap V^{(a)}$ is common for the PWAS and the waveguide. Therefore, the continuity of the displacement and the traction vector composed of normal and tangential stresses $\boldsymbol{\tau} = \{\sigma_{12}, \sigma_{22}\}$ is assumed at S^-:

$$[\boldsymbol{\tau}] = [\boldsymbol{u}] = 0, \quad \boldsymbol{x} \in S^-. \qquad (5)$$

Here square brackets denote a jump. All the other surfaces of the PWAS and the waveguide with normal η_j are assumed stress-free:

$$\sigma_{ij}\eta_j = 0, \quad \boldsymbol{x} \in S^{(\mathrm{a})} \cup S^{+} \cup \left(S^{(0)} \backslash S^{-}\right). \tag{6}$$

At the surfaces of cracks the stress free boundary conditions are assumed:

$$\boldsymbol{\tau} = 0, \quad \boldsymbol{x} \in \Omega^{(m)}. \tag{7}$$

3 THE FREQUENCY DOMAIN SPECTRAL ELEMENT METHOD

The FDSEM [2] is employed to describe motion of the PWAS. In order to implement the SAHA, an unknown traction vector $\boldsymbol{q}$ at the contact area is introduced. Accordingly, an auxiliary boundary value problem for domain $V^{(\mathrm{a})}$ is considered. The governing equations (1) and (2) are supplemented with the boundary conditions (3), (4), (6) and the condition at the bottom surface of the PWAS

$$\boldsymbol{\tau} = \boldsymbol{q}, \quad \boldsymbol{x} \in S^{-}, \tag{8}$$

where the traction vector $\boldsymbol{q}$ is assumed to be given.

It is convenient to introduce the decision vector $\boldsymbol{y} = \{u_1, u_2, \phi\} \in Y$ being assumed from the Sobolev space H^2 of square-integrable functions and their derivatives of orders $k < 2$:

$$Y = \left\{\boldsymbol{y}(\boldsymbol{x}) | y_i \in H^2(V^{(\mathrm{a})}), y_3(\boldsymbol{x}) = V^{+}(\boldsymbol{x}), \, \boldsymbol{x} \in S^{+}, \quad y_3(\boldsymbol{x}) = V^{-}(\boldsymbol{x}), \, \boldsymbol{x} \in S^{-}\right\}$$

The variational formulation of the equations (1) and (2) can be written using test functions $\boldsymbol{v} = \{v_1, v_2, v_3\}$ from the space

$$W = \left\{\boldsymbol{v}(\boldsymbol{x}) | v_i(\boldsymbol{x}) \in L^2(V^{(\mathrm{a})}), v_3(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in \ S_1 \cup S_3\right\}$$

as follows

$$\iint\limits_{V^{(\mathrm{a})}} \sigma_{ij,j} v_i \mathrm{d}S + \rho\omega^2 \iint\limits_{V^{(\mathrm{a})}} u_i v_i \mathrm{d}S = 0, \tag{9}$$

$$\iint\limits_{V^{(\mathrm{a})}} D_{i,i} v_i \mathrm{d}S = 0. \tag{10}$$

After integration by parts, equations (9)–(10) are rewritten:

$$\int\limits_{\partial V^{(a)}} q_i v_i \mathrm{d}s - \iint\limits_{V^{(\mathrm{a})}} \sigma_{ij} v_{i,j} \mathrm{d}S + \rho\omega^2 \iint\limits_{V^{(\mathrm{a})}} u_i v_i \mathrm{d}S = 0, \tag{11}$$

$$\int\limits_{\partial V^{(a)}} D_i v_3 \eta_i \mathrm{d}S - \iint\limits_{V^{(a)}} D_i v_{3,i} \mathrm{d}S = 0. \tag{12}$$

According to the FDSEM, the solution of the auxiliary problem can be approximated using Lagrange interpolation polynomials $C^I(x_1, x_2)$ at Gauss–Lobatto–Legendre points

$$y_k = \sum_I y_k^I C^I(x_1, x_2), \tag{13}$$

here special index I used to identify each node in the PWAS (for more details see [1]). Test functions $\boldsymbol{v} \in W$ are chosen the same $v_k(\boldsymbol{x}) = C^{I'}(x_1, x_2)$ as basis functions except another index is used below to distinguish from I.

4 THE BOUNDARY INTEGRAL EQUATION METHOD

Let us consider second auxiliary problem, where a surface load function $\boldsymbol{q}(x_1)$ is assumed known at the part $S^{(0)}$ of the upper surface of the waveguide $V^{(0)}$, while M cracks $\Omega^{(m)}$ are situated in the waveguide itself. The corresponding boundary value problem is stated as the governing equation (1), given for pure elastic material, and the boundary conditions (6), (8) and (7). The total wave-field is then a sum of the wave-field $\boldsymbol{u}^{(\mathrm{in})}$ incident by the load and wave-fields $\boldsymbol{u}^m$ scattered by each of M cracks.

$$\boldsymbol{u} = \boldsymbol{u}^{(\mathrm{in})} + \sum_{m=1}^{M} \boldsymbol{u}^{(\mathrm{m})}.$$

The BIEM can be applied to obtain the integral representation [4]:

$$\boldsymbol{u}^{(\mathrm{in})}(\boldsymbol{x}) = \frac{1}{2\pi} \int\limits_{\Gamma} \mathbf{K}^{(0)}(\alpha, x_2) \boldsymbol{Q}(\alpha) \mathrm{e}^{-\mathrm{i}\alpha x_1} \mathrm{d}\alpha. \tag{14}$$

Here $\mathbf{K}^{(0)}(\alpha, x_2)$ is the Fourier transform of Green's matrix of the waveguide, $\boldsymbol{Q}(\alpha)$ is the Fourier transform of the surface load function $\boldsymbol{q}(x_1)$ with respect to x_1 coordinate, while Γ is a contour along real axis surrounding poles of the integrand. For the scattered wave-fields the following integral representation is valid:

$$\boldsymbol{u}^{(m)}(\boldsymbol{x}) = \frac{1}{2\pi} \int\limits_{\Gamma} \mathbf{K}^{(m)}(\alpha, x_2) \boldsymbol{W}^{(m)}(\alpha) \mathrm{e}^{-\mathrm{i}\alpha x_1} \mathrm{d}\alpha, \tag{15}$$

where $\mathbf{K}^{(m)}(\alpha, x_2)$ is the Fourier transform of Green's matrix and $\boldsymbol{W}^{(m)}(\alpha)$ is the Fourier transform of the unknown crack opening displacement (COD) $\boldsymbol{w}^{(m)}(x_1)$ for m-th crack. Similar integral representations can be written for traction vectors:

$$\boldsymbol{\tau}^{(\mathrm{in})}(\boldsymbol{x}) = \frac{1}{2\pi} \int\limits_{\Gamma} \mathbf{T}^{(0)}(\alpha, x_2) \boldsymbol{Q}(\alpha) \mathrm{e}^{-\mathrm{i}\alpha x_1} \mathrm{d}\alpha, \tag{16}$$

$$\boldsymbol{\tau}^{(m)}(\boldsymbol{x}) = \frac{1}{2\pi}\int\limits_{\Gamma} \mathbf{T}^{(m)}(\alpha, x_2)\boldsymbol{W}^{(m)}(\alpha)\mathrm{e}^{-\mathrm{i}\alpha x_1}\mathrm{d}\alpha, \tag{17}$$

here $\mathbf{T}^{(0)}$ and $\mathbf{T}^{(m)}$ are matrices obtained after differential stress operator has been applied to representations (14)-(15).

The COD for m-th crack is expanded via Chebyshev polynomials of the second kind with square-root weight $p_n^{(m)}(x_1)$:

$$\boldsymbol{w}_k^{(m)}(x_1) = \sum_{n=0}^{N_m} \boldsymbol{\gamma}_{kn}^{(m)} p_n^{(m)}(x_1). \tag{18}$$

Following [4], the unknown COD can be determined using Galerkin method if the surface load is known.

5 HYBRID METHOD

The solution of the coupled problem (1)–(7) is constructed below in the assumption that the traction vector $\boldsymbol{q}(x_1)$ as well as CODs $\boldsymbol{w}^{(m)}(x_1)$ and values of $\boldsymbol{y}(x_1, x_2)$ at the nodal points within the PWAS domain $V^{(\mathrm{a})}$, i.e. $\boldsymbol{y}^I(x_1, x_2)$, are unknown. The traction vectors $\boldsymbol{q}$ introduced at the intersection S^- of domains $V^{(\mathrm{a})}$ and $V^{(0)}$ are assumed equal for both domains and expressed in terms of splines

$$s^+(x) = \begin{cases} 1-x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}, \qquad s^-(x) = \begin{cases} 1+x, & -1 \le x \le 0 \\ 0, & \text{otherwise} \end{cases} \tag{19}$$

based at the nodal points belonging to the contact area S^-. Employing the special index J to number nodes χ_J laying at S^-, $\boldsymbol{q}(x_1)$ is expanded as follows

$$\boldsymbol{q}(x_1) = \sum_J \boldsymbol{q}^J s_J(x_1), \tag{20}$$

Here $s_J(x_1)$ are the combinations of splines (19) with basis at nodal point χ_J. The Fourier transform of $\boldsymbol{q}(x_1)$ is obtained via the application of the Fourier transform to (20):

$$\boldsymbol{Q}(\alpha) = \sum_J \boldsymbol{q}^J S_J(\alpha), \tag{21}$$

where S_j are Fourier transforms of the splines (19).

Thus, a system of the linear algebraic equations with respect to the vector of unknowns

$$\boldsymbol{g} = \{y_1^I, y_2^I, y_3^I, q_1^J, q_2^J, \gamma_{1l}^{(1)}, \gamma_{2l}^{(1)}, \ldots, \gamma_{1l}^{(M)}, \gamma_{2l}^{(M)}\}$$

composed of expansion coefficients can be written:

$$\begin{pmatrix} \mathbf{A}_{ij} & \mathbf{B}_{ik} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{C}_{lj} & \mathbf{D}_{lk} & \mathbf{F}^{(1)}_{lk} & \cdots & \mathbf{F}^{(M)}_{lk} \\ \mathbf{0} & \mathbf{G}^{(1)}_{lk} & \mathbf{H}^{(11)}_{lk} & \cdots & \mathbf{H}^{(1M)}_{lk} \\ \vdots & \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{G}^{(M)}_{lk} & \mathbf{H}^{(M1)}_{lk} & \cdots & \mathbf{H}^{(MM)}_{lk} \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{y} \\ \boldsymbol{q} \\ \boldsymbol{\gamma}^{(1)} \\ \vdots \\ \boldsymbol{\gamma}^{(M)} \end{pmatrix} = \begin{pmatrix} \boldsymbol{V} \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix} \tag{22}$$

The first set of equations in the system is obtained via the substitution of (13) and (20) into (11) and (12). The formulae for $\mathbf{A}_{ij}$ and $\mathbf{B}_{ik}$ are the same as given in [1]. The second set arises from the condition of the continuity of the displacement vector

$$\boldsymbol{u}_i^{(\mathrm{in})} + \sum_{m=0}^{M} \boldsymbol{u}_i^{(m)} = \boldsymbol{y}_i, \quad \boldsymbol{x} \in S^-. \tag{23}$$

In order to satisfy (23), two methods have been applied: collocation method with points χ_J and Bubnov-Galerkin. Matrices forming the system have different values. In the case of the collocation method, substitution of (14) and (15) into equality (23) gives

$$C_{J'I;nj} = \delta_{J'I}\delta_{nj},$$

$$D_{J'J;lk} = -\frac{1}{2\pi}\int_{\Gamma} K^{(0)}_{lk}(\alpha,0)S_J(\alpha)\mathrm{e}^{-\mathrm{i}\alpha\chi_{J'}}\mathrm{d}\alpha,$$

$$F^{(m)}_{J'n;lk} = -\frac{1}{2\pi}\int_{\Gamma} K^{(m)}_{lk}(\alpha,0)P^{(m)}_n(\alpha)\mathrm{e}^{-\mathrm{i}\alpha\chi_{J'}}\mathrm{d}\alpha,$$

while employment of Bubnov-Galerkin method leads to the following relations:

$$C_{J'I;nj} = B_{IJ';nj},$$

$$D_{J'J;lk} = -\frac{1}{2\pi}\int_{\Gamma} K^{(0)}_{lk}(\alpha,0)S_J(\alpha)S^*_{J'}(\alpha^*)\mathrm{d}\alpha,$$

$$F^{(m)}_{J'n;lk} = -\frac{1}{2\pi}\int_{\Gamma} K^{(m)}_{lk}(\alpha,0)P^{(m)}_n(\alpha)S^*_{J'}(\alpha^*)\mathrm{d}\alpha.$$

The representations (16)–(17) are substituted into boundary conditions (7) and Bubnov-Galerkin method only is applied. The resulting equations are components of the system (22):

$$G^{(m)}_{n'J;lk} = -\frac{1}{2\pi}\int_{\Gamma} T^{(0)}_{lk}(\alpha,-d_{m'})S_J(\alpha)P^*_n(\alpha^*)\mathrm{d}\alpha,$$

$$H^{(mm')}_{n'n;lk} = -\frac{1}{2\pi}\int_{\Gamma} T^{(m)}_{lk}(\alpha,-d_{m'})P_n(\alpha)P^*_{n'}(\alpha^*)\mathrm{d}\alpha.$$

6 NUMERICAL ANALYSIS

In this section, some examples of computations provided by the SAHA are given for the piezoelectric transducer of width $w_a = 10$ mm and height $h_a = 1$ mm made of PIC155 material and the aluminum elastic structure (material properties can be found in [1]). The input voltage signal is applied at the lower boundary of the PWAS $V^- = 30$ V, while the upper one is grounded ($V^+ = 0$ V).

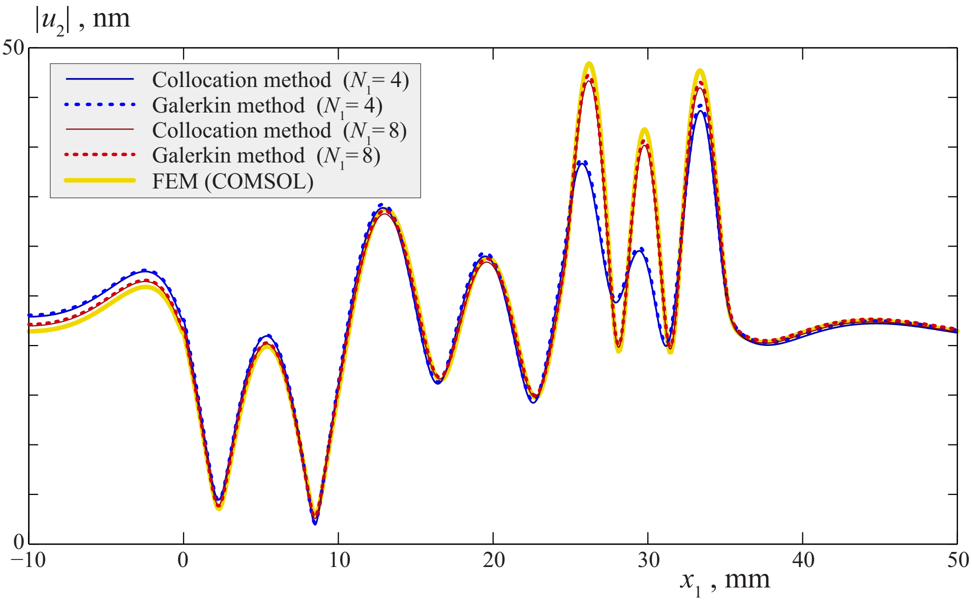

Figure 2: Amplitudes of out-of-plane displacements $|u_2(x_1, 0)|$ at the surface of the waveguide with $M = 1$ crack at frequency $f = 100$ kHz

In [1], the semi-empirical formulae for the number of elements for the SAHA employed for one transducer depending on frequency was derived. According to the estimation, the number of nodal points should be more than 12 nodes per wavelength. The results presented here are obtained using this estimation, when the PWAS has been discretized via the FDSEM. The main focus here in on the number of Chebyshev polynomials necessary to compute solution with good accuracy. Figures 2 and 3 demonstrate amplitudes of out-of-plane displacements at the upper surface $x_2 = 0$ of the waveguide with $M = 1$ crack at frequencies $f = 100$ kHz and $f = 500$ kHz respectively ($\omega = 2\pi f$). The dimensions of the crack are $l_1 = 5$ mm, $d_1 = 0.5$ mm, $c_1 = 30$ mm, the thickness of the host structure is $h_1 = 2$ mm. The results presented at these plots are computed via the SAHA using the collocation method and Galerkin scheme. The results calculated by the standard FEM software (COMSOL Multiphysics) [7], where PML layers [8] were used at the edges of the finite plate, are exhibited in the figures as well. One can observe a good agreement between the FEM solution and the developed hybrid method. Number of Chebyshev

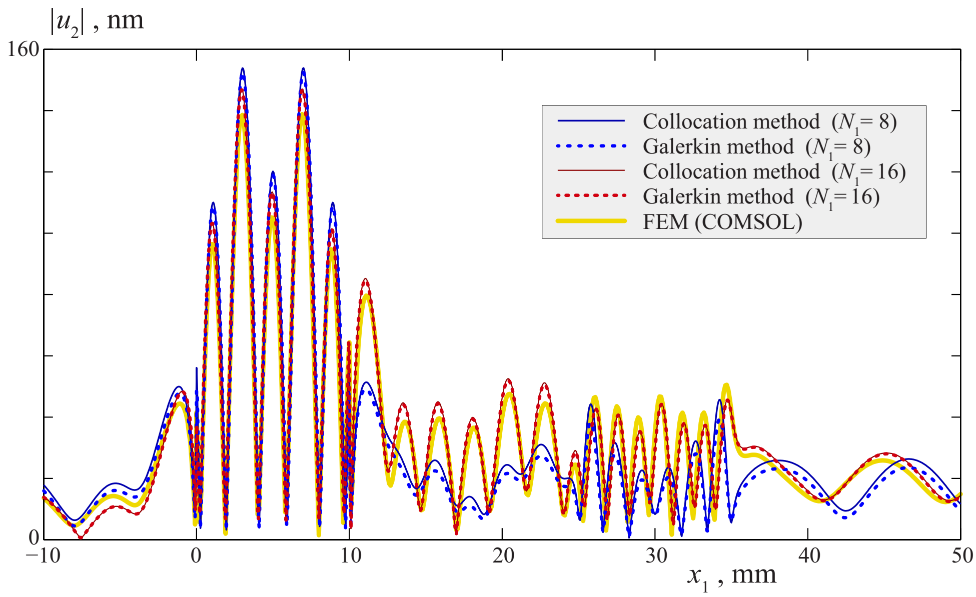

Figure 3: Amplitudes of out-of-plane displacements $|u_2(x_1, 0)|$ at the surface of the waveguide with $M = 1$ crack at frequency $f = 500$ kHz

polynomials necessary for the accurate solution calculation has been determined as $N_1 = 10$ for frequency $f = 100$ kHz and $N_1 = 16$ for frequency $f = 500$ kHz.

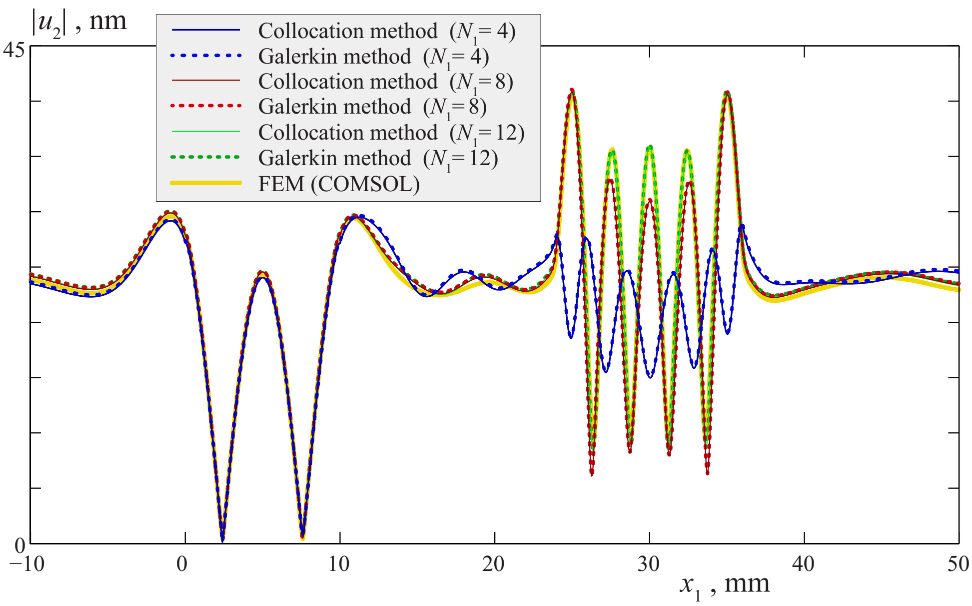

Figure 4: Amplitudes of out-of-plane displacements $|u_2(x_1, 0)|$ at the surface of the waveguide with $M = 3$ cracks at frequency $f = 100$ kHz

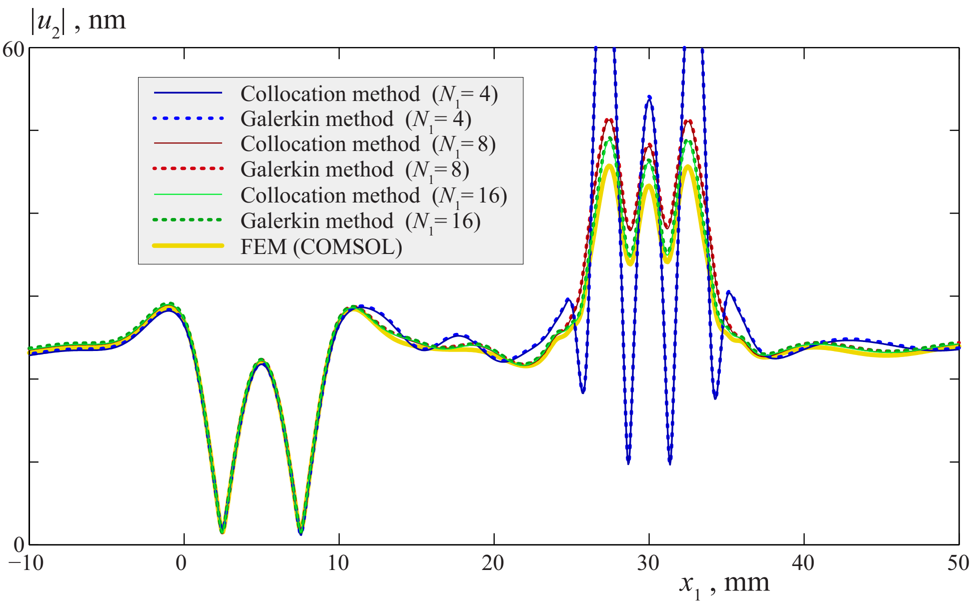

Figure 5: Amplitudes of out-of-plane displacements $|u_2(x_1, -H)|$ at the surface of the waveguide with $M = 3$ cracks at frequency $f = 100$ kHz

The out-of-plane displacements on the upper ($x_2 = 0$) and lower ($x_2 = -h_1$) boundaries of an elastic layer at frequency $f = 100$ kHz in the case of impact-induced damage modelled as a stack of three cracks are shown in Figures 4 and 5. The distances between the centres of delaminations and the origin of coordinates have been chosen the same ($c_1 = c_2 = c_3 = 30$ mm), while their depth and sizes were different: $l_1 = 6$ mm, $l_2 = 5$ mm, $l_3 = 4$ mm; $d_1 = 0.25$ mm, $d_2 = 0.5$ mm, $d_3 = 0.75$ mm. Collocation and Galerkin methods have been again used to perform coupling of the two methods in the contact area. Good agreement with the FEM is clearly seen outside the damaged zone. A certain discrepancy in the solutions is observed right under and above the cracks. This disagreement can be explained by a certain difference in eigenfrequencies calculated by the BIEM and the FEM [9].

7 CONCLUSIONS

The coupled hybrid mathematical model based on the FDSEM and the BIEM has been developed. The model can be applied to simulate guided waves excitation by piezoelectric transducer as well as wave propagation and scattering by multiple horizontal delaminations. Two different approaches for coupling two methods in the contact area have been used. The obtained solution has been compared for two variation of the hybrid method with the results calculated with the standard FEM software (COMSOL Multiphysics). Good agreement has been demonstrated, though Galerkin method provides less discrepancy with the FEM.

8 ACKNOWLEDGEMENTS

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REFERENCES

[1] Golub, M.V. and Shpak, A.N. Semi-analytical hybrid approach for the simulation of layered waveguide with a partially debonded piezoelectric structure, *Appl. Math. Model.* (2019) **65**:234–255.

[2] Shi, L., Zhou, Y., Wang, J.M., Zhuang, M., Liu, N. and Liu, Q.H. Spectral element method for elastic and acoustic waves in frequency domain. *J. Comp. Phys.* (2016) **327**:19—38.

[3] Glushkov, E.V. and Glushkova, N.V. On the efficient implementation of the integral equation method in elastodynamics. *J. Comp. Acoust.* (2001) **9(3)**:889–898.

[4] Glushkov, E.V., Glushkova, N.V., Golub, M.V. and Zhang, C. Resonance blocking of travelling waves by a system of cracks in an elastic layer. *Acoust. Phys.* (2009) **55**:8–16.

[5] Komatitsch, D. and Tromp, J. Introduction to the spectral element method for three-dimensional seismic wave propagation, *Geophys. J. Int.*, (1999) **139**:806–822.

[6] Vivar-Perez, J.M., Duczek, S. and Gabbert, U. Analytical and higher order finite element hybrid approach for an efficient simulation of ultrasonic guided waves I: 2D-analysis. *Smart, Struct. Syst.* (2014) **13**(4):587–614.

[7] Comsol Multiphysics v. 5.4a. www.comsol.com. COMSOL AB, Stockholm, Sweden.

[8] Hastings, F.D., Schneider, J.B. and Broschat, S.L. Application of the perfectly matched layer (PML) absorbing boundary condition to elastic wave propagation. *J. Acoust. Soc. Am.* (1996) **100**(5):3061–3069.

[9] Eremin, A.A., Golub, M.V., Glushkov, E.V., and Glushkova, N.V. Identification of delamination based on the Lamb wave scattering resonance frequencies. *NDT & E Int.* (2019) https://doi.org/10.1016/j.ndteint.2019.03.001.