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Complete and incomplete fusion cross-sections for $^{7}Li+^{119}Sn$ reaction using different approaches

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In reference to complete fusion (CF) and incomplete fusion (ICF) processes, the analysis of $^{7}Li+^{119}Sn$ reaction forming ^{126}I compound nucleus (CN) is carried out at incident energies spread across the Coulomb barrier. Firstly, the total fusion (TF) cross-sections are calculated using the Wong formula. Since, it overestimates the experimental data, so ℓ-summed Wong approach is employed to address the TF cross-section, which limits the contribution of partial waves up to ℓ_{max} value. Within ℓ-summed Wong model, the energy dependent selection function is used to separate out the contributions of CF and ICF from the TF cross-sections. This phenomenological selection function seems to give adequate distribution of CF and ICF cross-sections at higher energies. Beside this, the CF and ICF contributions are also separated out on the basis of angular momentum window and by using the energy correction formula. In the angular momentum distribution case, CF and ICF cross-sections are estimated in view of ℓ-windows, $0\leq\ell<\ell_{critical}$(for CF) and $\ell_{critical}\leq \ell \leq\ell_{max}$(For ICF). Finally, in energy normalization case, the incident energy of ^{7}Li beam ($^{7}Li\rightarrow{}^{4}He + {}^{3}H$) is distributed among alpha and tritium fragments, and the ICF cross-sections for $^{3}H+^{119}Sn$ ICF channel are estimated using the ℓ-summed Wong model.

Keywords: Compound nucleus, Incomplete fusion, Loosely bound projectile, Fusion cross-section

1 Introduction

In heavy ion collisions, the availability of light exotic beams with Z<10 provide an opportunity to investigate the incomplete fusion (ICF) process at energies above the Coulomb barrier. Both the CF and ICF processes undergo compound nucleus formation but in the former case, the projectile fuses completely with the target nucleus. Whereas, in the later case, only a fraction of the projectile interacts with the target nucleus and the residual non-interacting part moves in the forward direction[1]. The first evidence of ICF was provided by Britt and Quinton[2], when performing the experiment with ^{12}C, ^{14}N and ^{16}O projectile beams at energy 10MeV/A[2]. Later on, Galinet *et al.*[3] and Morgensten *et al.*[4] analysed the break-up of projectile nucleus and termed such reactions as ICF reactions or the breakup fusion reaction. The study of ICF reactions got impetus after different experiments carried out by Morgensten[4] using various combinations of projectile and target nuclei, where it was observed that ICF has higher probability in case of mass asymmetric reactions.

Theoretically, various models/approaches have been proposed to explain the mechanism of ICF reactions, but none of them succeeded in explaining the explicit features of ICF process[5]. Hence, it continues to be an active area of research. In the present work, we intended to examine the fusion cross-sections of loosely bound projectile (^{7}Li) undergoing $^{7}Li+^{119}Sn\rightarrow{}^{126}I$ reaction[6] at energies around the Coulomb barrier. Experimentally[6], it has been observed that the projectile nucleus breaks as $^{7}Li\rightarrow{}^{3}H+^{4}He$, where ^{3}H fuses with ^{119}Sn target as: $^{3}H+^{119}Sn\rightarrow{}^{122}Sb$, providing the cross-sections for incomplete fusion process. Here, an effort is made to distinguish the contribution of CF, ICF and TF cross-section for the chosen reaction. The investigation is carried out on the basis of three different approaches, (i) Energy Dependent Selection Function (EDSF)[7], (ii) by opting different angular momentum ℓ-windows for CF and ICF (iii) Energy correction formula for breakup fragments. These approaches are tested in the framework of Wong[8] and ℓ-summed Wong model[9], wherein the deformation effects are duly incorporated.

2 Wong and ℓ-Summed Wong Model

In Wong formula[8], an analytical expression for total fusion cross-sections in terms of center of mass energy $E_{c.m.}$, for two colliding nuclei A_1 and A_2 is given as:

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$$\sigma(E_{c.m.},\beta_i,\theta_i)=\frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)P;k=\sqrt{\frac{2\mu E_{c.m.}}{\hbar^2}} \quad \ldots (1)$$

where $\mu=mA_1A_2/(A_1+A_2)$ is the reduced mass, and P is the penetration probability through the barrier $V_B=V_C+V_N+V_\ell$, where V_C, V_N and V_ℓ stand for Coulomb, nuclear and centrifugal potential, respectively. Here the nuclear interaction part is calculated using the Proximity potential Prox 77 and the penetration probability is determined using the Hill-Wheeler approximation[10]as:

$$P_\ell=\left[1+\exp\left(\frac{2\pi\left(V_B^\ell(E_{c.m.},\theta_i)-E_{c.m.}\right)}{\hbar\omega_{\ell(E_{c.m.},\theta_i)}}\right)\right]^{-1} \quad \ldots (2)$$

where V^ℓ_B, R^ℓ_B and $\hbar\omega_\ell$ are the barrier height, barrier position and curvature (barrier characteristics) that are calculated at ℓ=0 wave. Using this approximation, and replacing the summation by an integral, Eq. (1) on integration over orientation angle θ_i gives the total fusion cross sections as:

$$\sigma(E_{c.m.})=\int_{\theta_i=0}^{\pi/2}\sigma(E_{c.m.},\beta_i,\theta_i)\sin\theta_i d\theta_i \quad \ldots (3)$$

The extension of Wong formula is referred as ℓ-Summed Wong model[9]in which the summation over ℓ-values is limited to ℓ_{max} as:

$$\sigma(E_{c.m.},\beta_i,\theta_i)=\frac{\pi}{k^2}\sum_{\ell=0}^{\ell_{max}}(2\ell+1)P \quad \ldots (4)$$

Within ℓ-Summed Wong method, the ICF component is analyzed using three different approaches, which are explained below:

(i) Energy dependent selection function

In order to extract the contribution of CF and ICF from TF, one can multiply σ_{TF} by energy dependent selection function $f(E/V_B)$[7]as follows:

$$f\left(\frac{E}{V_B}\right)=\begin{cases}0.08, \ for\ \frac{E}{V_B}<0.91,\\ \left[-13.34\left(\frac{E}{V_B}\right)^2+29.99\left(\frac{E}{V_B}\right)-16.16\right], for 0.91\le\frac{E}{V_B}\le1.13,\\ 0.70, \ for\ \frac{E}{V_B}>1.13,\end{cases}$$

for CF, $\sigma_{CF}=f\left(\frac{E}{V_B}\right).\sigma_{TF}$

and ICF,

$$\sigma_{ICF}=\left[1-f\left(\frac{E}{V_B}\right)\right].\sigma_{TF} \quad \ldots (5)$$

where, σ_{TF} is determined using the Wong and ℓ-Summed Wong method.

(ii) Angular momentum ℓ-windows

In this approach, the two competing reaction processes are distinguished on the basis of angular momentum ℓ window as

For CF cross section, ℓ-window is defined as$0<\ell<\ell_{crit}$.

In case of ICF, ℓ-window is taken as $\ell_{crit}<\ell<\ell_{max}$

For TF cross section, summation is carried out for ℓ-values lying in the range, $0<\ell<\ell_{max}$.

where, ℓ_{crit} and ℓ_{max} are calculated[11]as follows:

$$\ell_{max}=R_B\sqrt{2\mu(E_{c.m.}-V_B)/\hbar^2} \quad \ldots (6)$$

$$\frac{\sigma_{CF}}{\sigma_{TF}}=\left(\frac{\ell_{Crit}+1}{\ell_{max}+1}\right)^2$$

(iii) Energy correction formula

To distinguish both the processes, energy correction formula[12] is also applied. It states that the projectile energy is equally distributed among all of its nucleons and if A_{ICF} is the mass of the fragment, which interacts with the target after projectile breakup. Then, energy of the fragment participating in ICF channel is given as:

$$E_{ICF}=\frac{E_{lab}}{A}A_{ICF}, \quad \ldots (7)$$

where, E_{lab} is the original beam energy.

3 Results and Discussion

In the present work, we are aiming to address the fusion dynamics of loosely bound projectile ($^7Li+^{119}Sn\rightarrow^{126}I$) by employing the Wong and ℓ-summed Wong model at energies around the Coulomb barrier. Firstly, the total fusion cross sections for $^7Li+^{119}Sn\rightarrow^{126}I$ reaction are analyzed using the standard Wong formula at center of mass energies $E_{c.m.}$ =15 – 27 MeV. The comparison of calculated cross-sections with the experimental data is shown in Fig. 1(a), which depicts that the results obtained using Wong formula overestimate the experimental data. To resolve this issue, ℓ-summed Wong model is applied, where the summation of cross-sections is taken up to ℓ_{max} only. The ℓ_{max} is calculated in view of[11] and the calculated cross-sections are presented in Fig. 1(b), which depict that ℓ-summed Wong model based calculations address the total fusion cross-section nicely. Therefore, further calculations are done using the ℓ-summed Wong model.

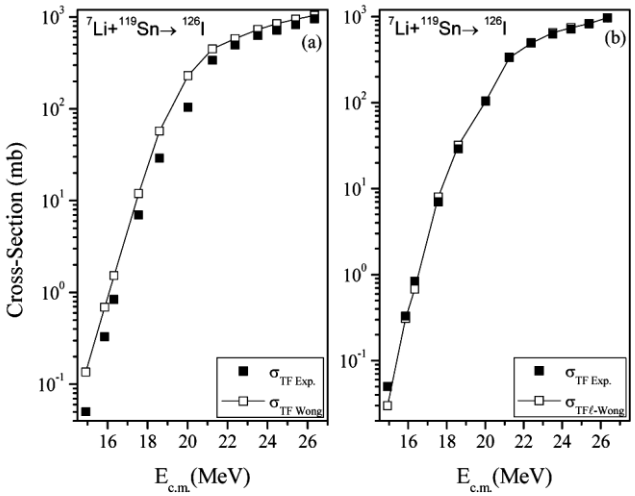

Fig. 1 — Fusion cross section for $^{7}Li+^{119}Sn \rightarrow ^{126}I$ reaction determined using the (a) Wong and (b) ℓ-Summed Wong formula at energies around the Coulomb barrier.

In case of loosely bound projectile, the incomplete fusion process starts competing with complete fusion process at higher incident energies. Here, we intended to distinguish the CF and ICF processes by using three different approaches. Based upon this, the analysis is divided into three sub sections. In section 3.1, the energy dependent selection function is applied on total fusion cross section to separate out the contribution of CF and ICF from TF cross sections. In section 3.2, these processes are distinguished on the basis of angular momentum ℓ- windows associated with them. Finally, in section 3.3, the contribution of CF and ICF components is analyzed by distributing the incident beam energy between the breakup fragments.

3.1 Use of Energy dependent selection function

Firstly, in this section, the energy dependent selection function is calculated using Eq. (5), that helps to separate out the CF and ICF from the TF (examined theoretically using the ℓ-summed Wong model) cross sections. In this approach, the function $f(E/V_B)$ termed as selection function is evaluated in terms of observables of barrier characteristics and then the conditions for CF and ICF are imposed on the TF cross sections, which finally gives the CF and ICF cross sections[7]. The comparison of estimated cross-sections with the respective experimental data is shown in Fig. 2. The figure clearly depicts that CF contribution obtained from the selection function gives decent result with the experimental data across the barrier. This implies that this empirical approach

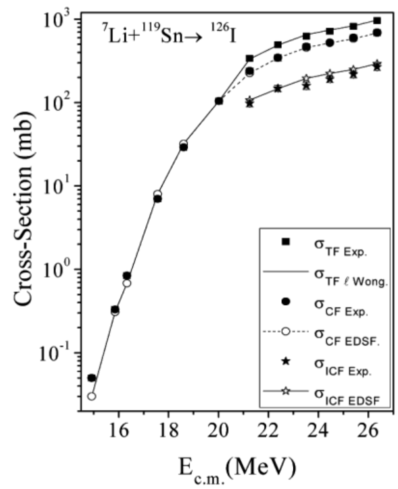

Fig. 2 — Comparison of CF, ICF and TF cross sections with the respective experimental data, determined using the energy dependent selection function.

obtained using the least square fitting procedure is good to address the CF cross-section. On the other hand, the calculated ICFcross-sections show slight deviation with the available data at higher incident energies. This means that energy dependent selection function works reasonably well to address dynamics of loosely bound reactions. Next, the CF and ICF processes are analyzed on the basis of angular momentum and energy correction formula.

3.2 Use of Angular momentum windows

It is well known fact that the ICF owes its origin due to higher angular momentum ℓ values. This implies that CF process occurs at lower ℓ-values ($0 \leq \ell < \ell_{critical}$), however at higher ℓ-values ($\ell_{critical} \leq \ell \leq \ell_{max}$), ICF mechanism start emerging. In view of this, an attempt to distinguish the both processes (CF and ICF) is made on the basis of ℓ-window criteria. In the framework of ℓ-summed Wong model, the complete fusion cross- sections (σ_{CF}) are summed up to critical angular momentum ($\ell_{crit.}$), while the contribution of σ_{ICF} is taken for $\ell_{crit.} \leq \ell \leq \ell_{max}$ and the summation ($\sigma_{CF}+\sigma_{ICF}$) is termed as total fusion cross-sections (σ_{TF}). The results are shown in Fig. 3(a), which shows that the contribution of CF and ICF calculated using ℓ-window criteria gives decent agreement with the experimental data. Further, the variation in ℓ-values with respect to energy is also shown for all the three processes, i.e., CF, ICF and TF in Fig. 3(b). It is evident from the figure that for all these three processes (CF, ICF and TF), contributing ℓ-window increases with increase in the incident energy.

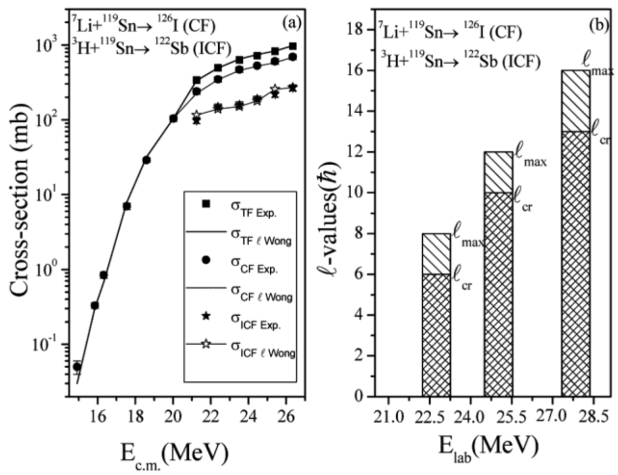

Fig. 3 — (a) Comparison of total fusion, complete fusion and incomplete fusion cross sections with respect to experimental data and (b) variation in angular momentum values for CF and ICF channels as a function of E_{lab}.

Next the energy correction formula is employed to examine the contribution of CF and ICF in the present reaction.

3.3 Use of energy correction formula

Here, it is assumed that the breakup fragments carry away some fraction of mass and energy of the original projectile beam and strike the target nucleus with normalized beam energy. In present case, ^{7}Li breaks in to ^{4}He and ^{3} H and it is observed[6] that ^{3}H interacts with the target nucleus i.e. ICF channel proceeds as $^{3}H+^{119}Sn\rightarrow {}^{122}Sb$. Thus, the energy of interacting fragment (^{3}H) in ICF channel is calculated as per Eq. (7) and by using this energy, the incomplete fusion cross sections are analyzed in the frame work of ℓ-summed Wong model. Both complete and incomplete fusion cross-sections for $^{7}Li+^{119}Sn$ reaction, calculated using energy correction formula are then compared with the experimental data and shown in Fig. 4. The CF are presented in Fig. 4(a) and Fig. 4(b) depicts the ICF cross-sections calculated at normalized beam energies. It is evident from the figure that the calculated cross sections are in agreement with experimental data. All the approaches seem to provide nice agreement with the experimental data, particularly for CF channel. For better insight of ICF analysis, the comparisons of theoretically estimated ICF cross-sections obtained using three different approaches are plotted in Fig. 5.

A careful look of this figure suggests that the ICF cross sections determined using the ℓ-window approach give relatively better agreement than the

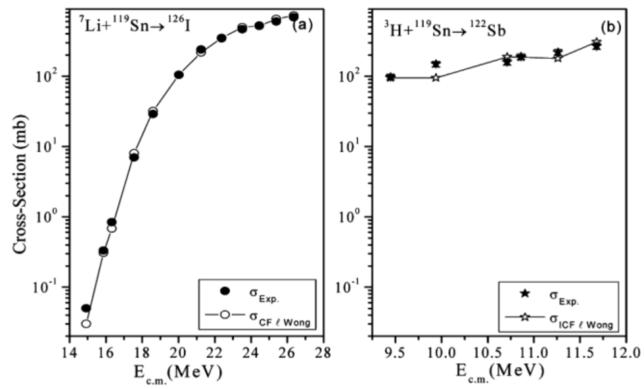

Fig. 4 — Comparison of experimental data with (a) CF cross sections, (b) for ICF cross sections determined using the energy normalization approach.

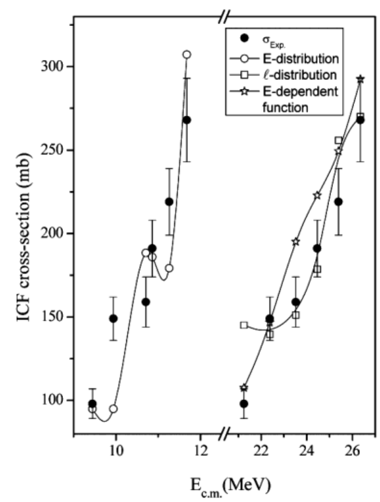

Fig. 5 — Comparison of ICF cross section estimated using the energy selection function, energy distribution approach and the angular momentum distribution approach.

ones calculated using the energy dependent selection function and the energy correction formula.

It may be noted that, this conclusion is based on the investigation of CF and ICF components for one reaction ($^{7}Li+^{119}Sn$). It will be of further interest to see the utility of these three approaches on a larger set of loosely bound reactions.

4 Conclusions

In this manuscript, the fusion dynamics of $^{7}Li+^{119}Sn\rightarrow{}^{126}I$ reaction is analysed at energy spread across the Coulomb barrier. The total fusion cross sections are estimated using the Wong and ℓ-summed Wong model. Due to loosely bound characteristics of the ^{7}Li projectile, the incomplete fusion seems in operation at higher incident energies along with usual complete fusion process.

To account the contribution of ICF, three different methods have been applied within the ℓ-summed Wong model. It has been observed that the ICF contribution gives relatively better agreement with experimental data by using ℓ-window criteria of CF and ICF channels. Other two approaches, energy dependent selection formalism and the energy correction formula also give reasonable addressal of CF and ICF data. It will be of further interest to investigate the role of considered approaches by opting a wider range of loosely bound reactions.

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