Primordial Fluctuations in Loop Quantum Cosmology

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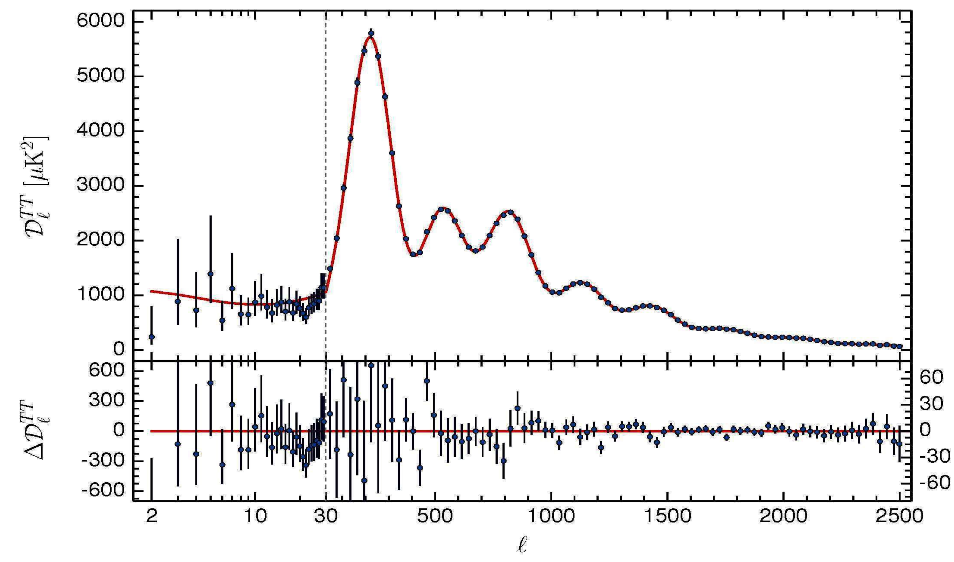
Introduction



- The Universe is approximately homogeneous and isotropic, with cosmological **perturbations**.

- In our era of **precision cosmology**, observational data can be used to falsify models.

- Observations might be indicating some **possible tensions**.
  Collections of observations may provide statistical significance.

Introduction

- Precision cosmology opens a window to observe genuine **QUANTUM COSMOLOGY** effects.

- Perturbations need a **gauge invariant** descriptions *(Bardeen, Mukhanov & Sasaki).*

- The passage to quantization asks for a **canonical formulation** *(Langlois, Pinto-Neto, Mena Marugán, Castelló Gomar, Olmedo, Fernández-Méndez, Lewandowski, Dapor, Puchta...)*.

- A complete **quantum** treatment should include the **background** *(Halliwell & Hawking, Shirai & Wada...)*.

Model

- We want to study quantum cosmology modifications to the **Mukhanov-Sasaki equations** for primordial fluctuations.

- We consider an **FLRW cosmology** coupled to a **scalar field**.
- For simplicity, we assume **compact flat** (three-torus) spatial topology.
- We focus the dicussion on **SCALAR pertubations**.
- We truncate the action at **quadratic** order in perturbations, with the background treated exactly up to that order.

Classical system

- The FLRW system is described by a **scale factor** and (the zero-mode of) a homogeneous **scalar field:** (a, φ). We set $G = 6\pi^2$.

- We expand the inhomogeneities in a (real) **Fourier** basis (sines and cosines).

- Modes are labeled by a wave vector $\vec{n} \in \mathbb{Z}^3$ (with positive first non-vanishing component). The eigenvalue of the Laplacian is $-\omega_n^2 = -\vec{n}\cdot\vec{n}$.

- **Scalar perturbations** are described by the Fourier coefficients of the scalar field, spatial metric (trace and traceless), lapse $(g_{\vec{n},\pm})$, and shift $(f_{\vec{n},\pm})$.

- The system as a whole is **symplectic**: zero modes + perturbations.

Classical system

- **<u>Constraints</u>**:
  Linear perturbative constraints (Hamiltonian constraint + diffeo constraint) + **Zero-mode** of the Hamiltonian constraint.

$$H = N_0\left[H_0 + \sum H_2^{\vec{n},\pm}\right] + a^{-3}\sum g_{\vec{n},\pm} H_1^{\vec{n},\pm} + a^{-2}\sum \omega_n^2 k_{\vec{n},\pm} H_{\uparrow 1}^{\vec{n},\pm}.$$

Homogeneous lapse

$$H_0 = \frac{1}{2a^3}\left(-a^2\pi_a^2 + \pi_\varphi^2 + 16\pi^3 a^6 V(\varphi)\right).$$

potential

π_i: momenta.

Gauge invariance

- We change the variables for the perturbations to a new canonical set:

⭑ The Mukhanov-Sasaki **gauge invariants** $v_{\vec{n},\pm}$.

⭑ Their **momenta** $\pi_{v_{\vec{n},\pm}}$, which are also **gauge invariants**.
A criterion is needed to fix the contribution of $v_{\vec{n},\pm}$ to them.

⭑ An **Abelianization** of the linear perturbative constraints (possible at the truncation order).

⭑ Suitable **momenta** of these, parametrizing possible gauge fixations.

Full system

- We extend the **canonical transformation** to the full system, at the considered **perturbative order**.

$$\tilde{w}_q^a = w_q^a + \frac{1}{2}\sum_{l,\vec{n},\pm}\left[X_{q_l}^{\vec{n},\pm}\frac{\partial X_{p_l}^{\vec{n},\pm}}{\partial w_p^a} - \frac{\partial X_{q_l}^{\vec{n},\pm}}{\partial w_p^a}X_{p_l}^{\vec{n},\pm}\right].$$

We call $\{w_q^a\} \equiv \{a, \varphi\}$, $\{w_p^a\}$ their momenta, and $\{X_{q_l}^{\vec{n},\pm}, X_{p_l}^{\vec{n},\pm}\}$ the old perturbative variables.

- Likewise for $\tilde{w}_p^a$, with a flip of sign in the corrections.
- The corrections are **QUADRATIC** in the perturbations.

New Hamiltonian

- Since the change of zero modes is **quadratic in the perturbations**, the new scalar constraint at our **truncation order** is

$$\boxed{H_0 + \sum_b \left(w^b - \tilde{w}^b\right) \frac{\partial H_0}{\partial w^b} + \sum_{\vec{n},\pm} H_2^{\vec{n},\pm} \quad at \quad \left(\tilde{w}^a, \tilde{X}_l^{\vec{n},\pm}\right),}$$

$$w^a - \tilde{w}^a = \sum_{\vec{n},\pm} \Delta \tilde{w}^a_{\vec{n},\pm} .$$

- So, the perturbative contribution to the new scalar constraint is

$$H_2^{\vec{n},\pm} + \sum_a \Delta \tilde{w}^a_{\vec{n},\pm} \frac{\partial H_0}{\partial w^a} \rightarrow \breve{H}_2^{\vec{n},\pm} \qquad \text{(up to } \textit{gauge}\text{).}$$

This gives precisely the Mukhanov-Sasaki Hamiltonian.

New Hamiltonian

- The **total Hamiltonian** of the system becomes

$$H=\bar{N}_0\left[H_0+\sum_{\vec{n},\pm}\breve{H}_2^{\vec{n},\pm}\right]+\sum_{\vec{n},\pm}G_{\vec{n},\pm}\breve{H}_1^{\vec{n},\pm}+\sum_{\vec{n},\pm}K_{\vec{n},\pm}H_{\uparrow 1}^{\vec{n},\pm}.$$

Redefined Lagrange multipliers. *Abelianized.*

- It should include **backreaction** at the considered perturbative order.

- The perturbative contribution to the scalar constraint is **quadratic** in the Mukhanov-Sasaki variables and momenta, and **linear** in $\pi_{\tilde{\varphi}}$.

Hybrid quantization

Approximation: Quantum geometry effects are especially relevant in the background.

- Adopt a **(loop) quantum** scheme for zero modes and quantize the perturbations à la **Fock**. The scalar constraint **couples** them.

- We assume:

  a) Zero modes **commute** with perturbations after quantization.

  b) Functions of $\tilde{\varphi}$ act by multiplication.

Fock representation

- A **Fock quantization** is fixed in QFT up to unitary equivalence by:
  - The background isometries.
  - The unitarity of the resulting Heisenberg evolution.
- The choice of representation does not fix the **vacuum**: any Fock state is valid.

Perturbative constraints

- We represent the **linear perturbative constraints** (or an integrated version of them) as derivatives (or as translations).

- Then, physical states are independent of their momenta (*gauge d.o.f.*).

- Physical states depend only on zero modes and gauge invariants (**no gauge fixing)**.

- They still must satisfy the **Hamiltonian (or scalar) constraint** given by the FLRW and the Mukhanov-Sasaki contributions.

Hamiltonian constraint

- This global **Hamiltonian constraint** can be written

$$H_S = \frac{1}{2}\left[\pi_{\tilde{\varphi}}^2 - H_0^{(2)} - \Theta_e - \Theta_o \pi_{\tilde{\varphi}}\right].$$

where

$$H_0^{(2)} = \tilde{a}^2 \pi_{\tilde{a}}^2 - 16\pi^3 \tilde{a}^6 V(\tilde{\varphi}), \qquad \Theta = \sum_{\vec{n},\pm} \Theta^{\vec{n},\pm}.$$

Even: $\Theta_e^{\vec{n},\pm} = -\left[\left(\vartheta_e \omega_n^2 + \vartheta_e^q\right)\left(v_{\vec{n},\pm}\right)^2 + \vartheta_e \left(\pi_{v_{\vec{n},\pm}}\right)^2\right],$ Odd: $\Theta_o^{\vec{n},\pm} = -\vartheta_o \left(v_{\vec{n},\pm}\right)^2,$

The same

$$\begin{cases} \vartheta_e = \tilde{a}^2, & \vartheta_e^q = \dfrac{H_0^{(2)}}{\tilde{a}^2}\left(19 - 18\dfrac{H_0^{(2)}}{\tilde{a}^2 \pi_{\tilde{a}}^2}\right) + 8\pi^3 \tilde{a}^4 \left(V'' - 4V\right), \\ \vartheta_o = -96\pi^3 \tilde{a}^3 \dfrac{V'}{\pi_{\tilde{a}}}. & \end{cases}$$

<u>All mode independent</u>

Hamiltonian constraint

$$H_S = \frac{1}{2}\left[\pi_{\tilde{\varphi}}^2 - H_0^{(2)} - \Theta_e - \Theta_o \pi_{\tilde{\varphi}}\right].$$

- Quantum constraint
  - QC: Factor ordering/regularization.
  - Symmetrization in the linear momentum.

- It is **quadratic** in the momentum of the zero mode of the scalar field.

- The linear **perturbative** term goes with the derivative of the potential.

Born-Oppenheimer ansatz

- Consider states for which the dependence on the FLRW geometry and the inhomogeneities (N) **split:**

$$\Psi=\xi(\tilde{a},\tilde{\varphi})\psi(N,\tilde{\varphi}).$$

- The FLRW state is normalized, and **evolves** in $\tilde{\varphi}$ as:

$$\xi(\tilde{a},\tilde{\varphi})=\hat{U}(\tilde{a},\tilde{\varphi})\chi(\tilde{a}).$$

$\hat{U}$ is an evolution **CLOSE** to the **unperturbed** one, with generator $\hat{\tilde{H}}_0$.

Born-Oppenheimer ansatz

- **Approximation**: Disregard transitions from ξ to other FLRW states.

  Taking expectation values in the **FLRW geometry**, we get a **quantum** constraint for the Mukhanov-Sasaki field:

$$\hat{\pi}_{\tilde{\varphi}}^{2}\psi+2\langle\hat{\tilde{H}}_{0}\rangle_{\xi}\hat{\pi}_{\tilde{\varphi}}\psi=\left[\left\langle\hat{\Theta}_{e}+\frac{1}{2}(\hat{\Theta}_{o}\hat{\tilde{H}}_{0}+\hat{\tilde{H}}_{0}\hat{\Theta}_{o})\right\rangle_{\xi}+\frac{1}{2}\langle[\hat{\pi}_{\tilde{\varphi}}-\hat{\tilde{H}}_{0},\hat{\Theta}_{o}]\rangle_{\xi}\right]\psi.$$

- If we can **neglect the first and last terms:**

$$\hat{\pi}_{\tilde{\varphi}}\psi=\frac{\langle 2\hat{\Theta}_{e}+(\hat{\Theta}_{o}\hat{\tilde{H}}_{0}+\hat{\tilde{H}}_{0}\hat{\Theta}_{o})\rangle_{\xi}}{4\langle\hat{\tilde{H}}_{0}\rangle_{\xi}}\psi.$$

Schrödinger-like equation for the gauge invariant perturbations

Mukhanov-Sasaki equations

- *Moreover*, **BY ONLY** assuming a direct effective dynamics for the inhomogeneities, we get the **modified** Mukhanov-Sasaki equations:

$$d^2_{\eta_\xi} v_{\vec{n},\pm} = -v_{\vec{n},\pm}\left[\omega_n^2 + \frac{\langle 2\hat{\vartheta}_e^q + (\hat{\vartheta}_o \hat{\widetilde{H}}_0 + \hat{\widetilde{H}}_0 \hat{\vartheta}_o) + [\hat{\pi}_{\tilde{\varphi}} - \hat{\widetilde{H}}_0, \hat{\vartheta}_o]\rangle_\xi}{2\langle \hat{\vartheta}_e \rangle_\xi}\right].$$

⬇

<u>Conformal time</u>: $\langle \hat{\widetilde{H}}_0 \rangle_\xi \, d\eta_\xi = \langle \hat{\vartheta}_e \rangle_\xi \, d\tilde{\varphi}$. Recall that $\vartheta_e = \tilde{a}^2$.

- The expectation values give the **quantum corrected mass,** which is **mode independent**.

- The effective equations are **hyperbolic in the ultraviolet** regime.

Example: LQC

- With the **standard variables** (v, b) and $\vee = 3(2\pi)^3 \gamma \sqrt{\Delta}|v|/2$,

$$\hat{\tilde{H}}_0^2 \approx \hat{H}_0^{(2)} = \frac{1}{(2\pi)^3}\left(\frac{\hat{\Omega}_0^2}{(2\pi)^3} - 2\hat{\vee}^2 \; V\right), \qquad \hat{\Omega}_0 = \frac{1}{2\gamma\sqrt{\Delta}} \hat{\vee}^{1/2}\left[\widehat{sgn(v)}\widehat{\sin(b)} + \widehat{\sin(b)}\widehat{sgn(v)}\right]\hat{\vee}^{1/2},$$

Neglecting backreaction

$(2\pi)^3 \tilde{a}\,\pi_{\tilde{a}}$

Area gap

Immirzi parameter

MMO prescription

$$\hat{\vartheta}_e = \frac{\hat{\vee}^{2/3}}{(2\pi)^2}, \quad \leftarrow \quad \tilde{a}^2$$

$$\Rightarrow \quad \hat{\vartheta}_e^q = (2\pi)^2 \left[\widehat{\frac{1}{\vee}}\right]^{1/3} \hat{H}_0^{(2)}\left(19 - 18(2\pi)^6 \hat{\Omega}_0^{-2} \hat{H}_0^{(2)}\right)\left[\widehat{\frac{1}{\vee}}\right]^{1/3} + \frac{\hat{\vee}^{4/3}}{(2\pi)^4}\left(V'' - (2\pi)^3 V\right),$$

$$\hat{\vartheta}_o = 12\sqrt{2\pi}\, V' \hat{\vee}^{2/3} |\hat{\Omega}_0|^{-1} \hat{\Lambda}_0 |\hat{\Omega}_0|^{-1} \hat{\vee}^{2/3} \quad \longleftarrow \quad 2\Lambda_0(b) \equiv \Omega_0(2b).$$

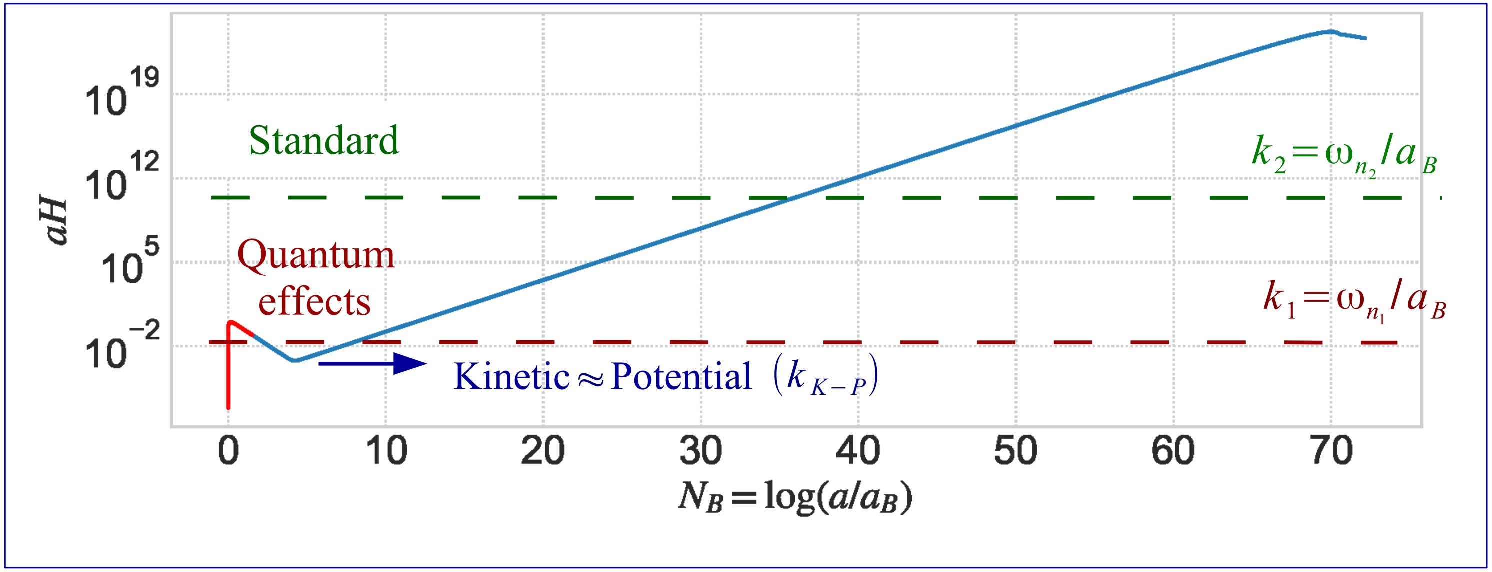
Example: LQC

Possible strategies:

- Compute the quantum expectation values **numerically**.
- Use an **interaction** picture around the massless or the de Sitter case.
- For suitable states, one often adopts the **effective LQC** description.

Initial conditions

- Initial conditions on the *background* within effective LQC:
- ➔ Quantum effects affect modes between the scale of LQC and k_{K-P}.
- ➔ The effects may be **relevant** and compatible with observations if those modes are entering the Hubble horizon today.

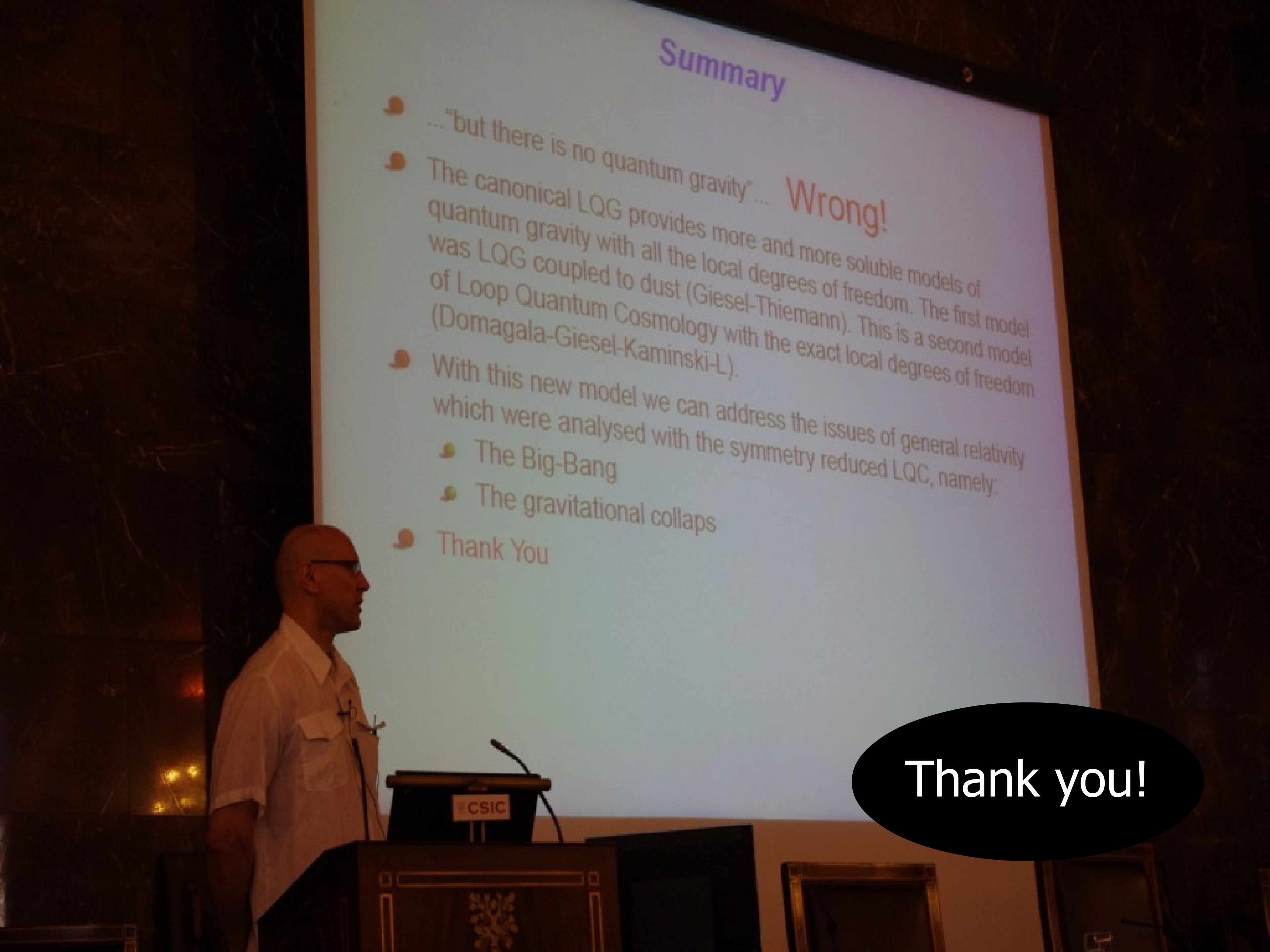


Initial conditions

- For backgrounds where this happens, one gets **short-lived** inflation.
- Modes affected by quantum effects do **not** first leave the Hubble horizon in the **slow-roll** regime.
- Those modes are not in a Bunch-Davies vacuum.
- The power spectrum is **modulated** by a factor that depends on the Bogoliubov coefficients of the **new vacuum** state.
- <u>Vacuum of the perturbations</u>: there are several proposals (*Martín-de Blas & Olmedo, Ashtekar & Gupt*...).

Conclusions

- We have studied (scalar) perturbations at **quadratic** order in the action.
- At this truncation order, we have found a canonical transformation for the **full system** leading to **Mukhanov-Sasaki** gauge invariants.
- In a **hybrid quantization**, physical states depend only on the *quantum background* and the Mukhanov-Sasaki field.
- We have derived **Mukhanov-Sasaki equations** modified with **quantum corrections** (beyond homogeneous effective descriptions).
- In order to extract predictions, it is essential to determine the **initial conditions** for the background and the vacuum of the perturbations.


Summary
..."but there is no quantum gravity"... Wrong!
The canonical LQG provides more and more soluble models of quantum gravity with all the local degrees of freedom. The first model was LQG coupled to dust (Giesel-Thiemann). This is a second model of Loop Quantum Cosmology with the exact local degrees of freedom (Domagala-Giesel-Kaminski-L).
With this new model we can address the issues of general relativity which were analysed with the symmetry reduced LQC, namely:
The Big-Bang
The gravitational collaps
Thank You
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Thank you!