Received: 18 July 2018 | Revised: 22 January 2019 | Accepted: 27 March 2019

DOI: 10.1002/asjc.2144

REGULAR PAPER

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Transient response comparison of feedback and feed-forward compensation methods in systems with zero steady state error

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Funding information
Ministerio de Economia y Competividad, Grant/Award Number: DPI 2017-86915-C3-3-R; Programa de Actividades de I+D+i de la Comunidad de Madrid, Grant/Award Number: S2018/NMT-4331

Abstract

Proportional–Integral–Derivative (PID) controllers with integral action are conventionally used as feedback controllers. They are used to obtain zero steady-state error when the reference input or the disturbance are steps and where zero-type systems (i.e. with no poles at the origin) are controlled. The controller meets its objective when the controlled system is non linear, but usually introduces undesirable changes in the dynamics that must be compensated by readjusting the proportional gain. In this paper, we compare PID with two alternative techniques based on the use of a feed-forward system and a multiple feedback system, respectively. A detailed comparison of the transient response obtained with these methods is presented and validated with some simulation examples.

KEYWORDS
control systems, feed-forward compensation, feedback systems, PID controllers, steady-state errors

1 | INTRODUCTION

When designing a linear controller, two main aspects of the closed-loop system are taken into account: its dynamics and its steady-state behavior. The focus of this paper is on the second.

Many recent studies have demonstrated that linear control systems still have some margin for improvement. The following examples are all worth mentioning on different techniques for linear systems: learning [1], pole placement [2], optimal control [3], anti-windup [4], dynamic analysis [5,6], signal saturations [7,8], cascade control [9] and state feedback control [10]. In fact, PID controllers for non-linear systems can be found in the recent literature [11]. In the present work, the steady-state error of feedback linear systems is analyzed.

It is well known that adding an integral action to a P or PD controller eliminates the steady-state error both when the input is a step in the loop reference or in the disturbance [12–15]. However, although the method is a guarantee that the objective will be reached, it usually worsens the transient response. The PI and PID controllers add a pole-zero pair that increases the system order, complicating its dynamic analysis. They also introduce a dominant real pole close to the origin which greatly smooths the transient response and forces the compensation of this effect by empirically readjusting the proportional gain.

Additionally, many applications of feed-forward compensators have been implemented, to achieve an input-output closed-loop transfer function with a concrete transient and steady-state behavior as in [16–20]. The use of feed-forward control to compensate for measured disturbances was described in detail in [21] and [22]. A mixed feed-forward compensation technique for trans-conductance amplifiers was presented in [23]. Feed-forward compensation for the design of high-frequency trans-conductance amplifiers was also used in [24], in an attempt to reduce the settling-time. Disturbance compensation methods were analyzed in [25], focusing on time delays, in the case of both PID and Model Predictive Control structures.

Besides, other authors have used feed-forward compensation when the input is the reference. For instance, a Preisach model was employed in [26] to describe piezoelectric actuator hysteresis and an inverse model to build a feed-forward controller, which proved the solution in the case of PID, fuzzy control, and fuzzy-PID control. A trial test using feed-forward control to compensate quadrant glitches occurring with bearings for rolling/sliding was described in [27].

A tuning procedure for PID plus a feed-forward controller was proposed in [28], assuming a first order plus a dead time model of the process. Finally, a Smith predictor in which dead-time models were used to feed-forward the disturbance was discussed in [29].

However, most of these compensators use the inverse transfer function of the system under control, something that could work in theory, but that could generate severe implementation problems. Hence, in this work we propose a feed-forward compensation block and focus on the steady-state behavior, regardless of the above-mentioned inverse transfer function, which is referred to in [30] as an approximate inverse and that is likewise known as a proportional anticipative control. We introduce a block in two situations: (i) when the input is the set-point; and (ii) when the input is a disturbance.

This paper will have the following structure. In Section 2, three methods of obtaining zero steady-state error will be presented: adding an integral action (subsection 2.1), adding a feed-forward compensator (subsection 2.2), and adding a multiple feed-back compensator (subsection 2.3). Then, in Section 3, a feed-forward method will be presented for use when the input is a disturbance instead the reference. Subsequently, in Section 4, the previous methods are compared with different simulation experiments. Finally, the conclusions will be drawn in Section 5.

2 | METHODS TO OBTAIN ZERO STEADY-STATE ERROR

Assuming a block diagram and using the Laplace transform corresponding to a linear feedback system shown in Figure 1, where G(s) is the system under control, R(s) is the controller, and h is a constant gain sensor.

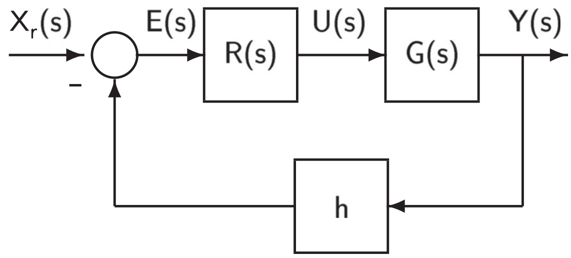

FIGURE 1 Basic feedback control loop

We will also assume that we have type-zero G(s) and R(s), which means that they have no poles at the origin, so all the following parameters are finite values:

$$K_R = \lim_{s\to 0} R(s) \tag{1}$$

$$K_G = \lim_{s\to 0} G(s) \tag{2}$$

$$K_P = h \lim_{s\to 0} R(s)G(s) = hK_R K_G \tag{3}$$

As we know, when the reference input is a unitary step, this control schema has a non-zero steady-state error, e_p. In the following sections, this control schema will be used as a departure point, for comparisons with other proposed schemas with zero steady-state error.

2.1 | Controller with integral action

It is well known that assuming zero-type G(s), zero steady-state error can be obtained by adding an integral action (pole at the origin) to the controller R(s), for instance:

$$R'(s) = R(s)\left(1 + \frac{1}{T_i s}\right) = R(s)\frac{1 + T_i s}{T_i s} \tag{4}$$

hence:

$$K_R' = \lim_{s\to 0} R'(s) = \infty \tag{5}$$

$$K_G = \lim_{s\to 0} G(s) \neq \infty \tag{6}$$

$$K_P' = h \lim_{s\to 0} R'(s)G(s) = \infty \tag{7}$$

so when the reference input is a unitary step, we obtain:

$$e_p = \frac{1}{1 + K_p'} = 0 \tag{8}$$

On the other hand, the following transfer functions have changed:

$$\frac{Y(s)}{X_r(s)} = \frac{R(s)G(s)(1 + T_i s)}{T_i s + hR(s)G(s)(1 + T_i s)} \tag{9}$$

$$\frac{E(s)}{X_r(s)} = \frac{T_i s}{T_i s + hR(s)G(s)(1 + T_i s)} \tag{10}$$

$$\frac{U(s)}{X_r(s)} = \frac{R(s)(1 + T_i s)}{T_i s + hR(s)G(s)(1 + T_i s)} \tag{11}$$

The main difference is that the feedback system is now of a higher order, so the system dynamics will have changed. This effect in the dynamic response will be analyzed with an experimental example that will be presented later on.

2.2 | Feed-forward control

The classical feed-forward schema is shown in Figure 2.

The goal is to obtain a constant input-output transfer function such that $Y(s) = \frac{X_r(s)}{h}$. In this case we have:

$$\frac{Y(s)}{X_r(s)} = \left(1 + \frac{1}{hR(s)G(s)}\right) \frac{R(s)G(s)}{1 + hR(s)G(s)} = \frac{1}{h} \tag{12}$$

In theory, this expression guarantees that $e_p = 0$, but forces the closed-loop dynamic to disappear and uses the inverse of G(s), which has no physical interpretation. In addition, we can propose an innovative method to obtain zero steady-state error by using a feed-forward action, as in Figure 3.

Here, as in the original loop, G(s) and R(s) are of the zero type, and so all the following parameters are finite values:

$$K_R = \lim_{s \to 0} R(s) \tag{13}$$

$$K_G = \lim_{s \to 0} G(s) \tag{14}$$

$$K_P = h \lim_{s \to 0} R(s)G(s) \tag{15}$$

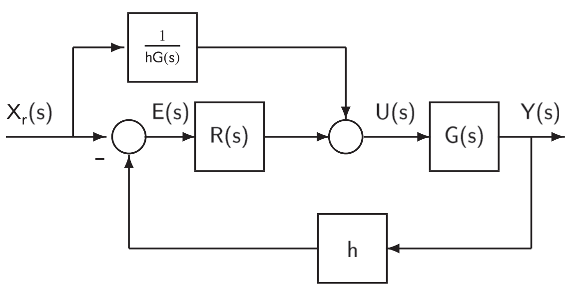

FIGURE 2 Classical feed-forward compensation

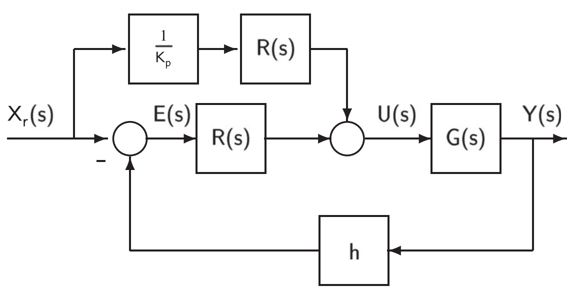

FIGURE 3 Proposed feed-forward compensation

In this case we obtain the following transfer function relations:

$$\frac{Y(s)}{X_r(s)} = \left(1 + \frac{1}{K_p}\right) \frac{R(s)G(s)}{1 + hR(s)G(s)} \tag{16}$$

$$\frac{E(s)}{X_r(s)} = 1 - \left(1 + \frac{1}{K_p}\right) \frac{hR(s)G(s)}{1 + hR(s)G(s)} \tag{17}$$

$$\frac{U(s)}{X_r(s)} = \left(1 + \frac{1}{K_p}\right) \frac{R(s)}{1 + hR(s)G(s)} \tag{18}$$

We will now prove that, when the reference input is a unitary step, this control schema will also have a zero steady-state error:

$$\begin{aligned} e_p &= \lim_{s \to 0} sE(s) \\ &= \lim_{s \to 0} s \left[1 - \left(1 + \frac{1}{K_p}\right) \frac{hR(s)G(s)}{1 + hR(s)G(s)}\right] \frac{1}{s} \\ &= 1 - \left(1 + \frac{1}{K_p}\right) \frac{K_p}{1 + K_p} = 0 \end{aligned} \tag{19}$$

A core improvement in comparison with the previous case (controller with integral action) is that there is now no change to the transient response. Nevertheless, in the presence of modelling errors or non-linearities in the process under control, we would have $h\lim_{s \to 0} R(s)G(s) \neq K_P$, and the steady-state error would be small although non-zero. Despite which we can argue that it also happens in the classical feed-forward compensation with $G^{-1}(s)$ and that many modelling errors affect the dynamic behavior rather than the static component alone.

2.3 | Multiple feedback control

A second innovative method to obtain zero steady-state error is further modification of the original simple control loop by means of introducing a second feedback action, as in Figure 4.

Then, as in the original loop, zero-type G(s) and R(s) are used, and so all the previous parameters will again have finite values:

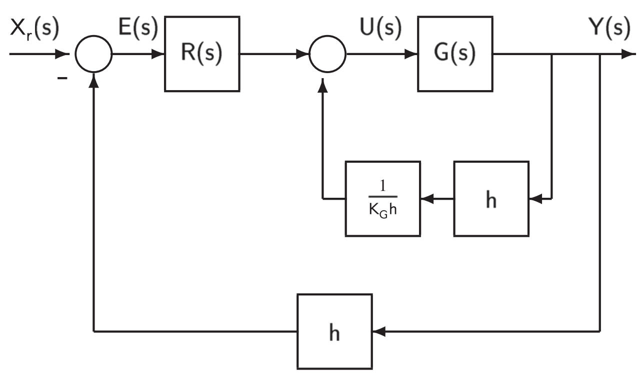

FIGURE 4 Proposed multiple feedback compensation

$$K_R = \lim_{s\to 0} R(s) \tag{20}$$

$$K_G = \lim_{s\to 0} G(s) \tag{21}$$

$$K_P = h \lim_{s\to 0} R(s)G(s) \tag{22}$$

Taking into account that the internal feedback transfer function: is

$$G^*(s) = \frac{G(s)}{1 - \frac{G(s)}{K_G}} = \frac{K_G G(s)}{K_G - G(s)} \tag{23}$$

we have the following closed-loop transfer function relations:

$$\frac{Y(s)}{X_r(s)} = \frac{K_G R(s) G(s)}{K_G - G(s) + hR(s)G(s)K_G} \tag{24}$$

$$\frac{E(s)}{X_r(s)} = \frac{K_G - G(s)}{K_G - G(s) + hR(s)G(s)K_G} \tag{25}$$

$$\frac{U(s)}{X_r(s)} = \frac{K_G R(s)}{K_G - G(s) + hR(s)G(s)K_G} \tag{26}$$

It is also possible to prove that, when the reference input is a unitary step, this control schema also has a zero steady-state error:

$$\begin{aligned} e_p &= \lim_{s\to 0} sE(s) \\ &= \lim_{s\to 0} s \frac{K_G - G(s)}{K_G - G(s) + hR(s)G(s)K_G} \frac{1}{s} \\ &= \frac{K_G - K_G}{K_P K_G} = 0 \end{aligned} \tag{27}$$

The main difference with the two previous systems (integral action controller and feed-forward controller) is that, although changes are now introduced in the system dynamics, the feedback system order remains unchanged. Nevertheless, in the presence of modelling errors or non-linearities in the process under control, we would have $\lim_{s\to 0} G(s) \neq K_G$, so the steady-state error, although of the non-zero type, would still be small.

2.4 | Equivalence of methods

Multiple feedback compensation is equivalent to the feed-forward compensation method when in the first one we replace the controller with the following expression:

$$R'(s) = R(s)\left(1 + \frac{1}{K_P}\right) \tag{28}$$

The demonstration is presented as follows. In the multiple feedback compensation schema we have:

$$G_3(s) = \frac{K_G G_1(s) G_2(s)}{K_G - G_1(s)G_2(s)} \tag{29}$$

$$\begin{aligned} \frac{Y(s)}{X_r(s)} &= \frac{R'(s)G_3(s)}{1 + R'(s)G_3(s)h} \\ &= \frac{R'(s)\frac{K_G G_1(s)G_2(s)}{K_G - G_1(s)G_2(s)}}{1 + R'(s)\frac{K_G G_1(s)G_2(s)}{K_G - G_1(s)G_2(s)})h} \\ &= \frac{R'(s)K_G G_1(s)G_2(s)}{K_G + G_1(s)G_2(s)\,(R'(s)K_G h - 1)} \\ &= \frac{R'(s)G_1(s)G_2(s)}{1 + G_1(s)G_2(s)\left(R'(s)h - \frac{1}{K_G}\right)} \\ &= \frac{R'(s)G_1(s)G_2(s)}{1 + G_1(s)G_2(s)\left(R'(s)h - \frac{R(s)h}{K_P}\right)} \\ &= \frac{R'(s)G_1(s)G_2(s)}{1 + G_1(s)G_2(s)h\left(R'(s) - \frac{R(s)}{K_P}\right)} \end{aligned} \tag{30}$$

Taking the $R'(s)$ as proposed above, will yield:

$$\begin{aligned} \frac{Y(s)}{X_r(s)} &= \frac{R(s)\left(1 + \frac{1}{K_P}\right)G_1(s)G_2(s)}{1 + G_1(s)G_2(s)h\left[R(s)\left(1 + \frac{1}{K_P}\right) - \frac{R(s)}{K_P}\right]} \\ &= \left(1 + \frac{1}{K_P}\right) \\ &\quad \frac{R(s)G_1(s)G_2(s)}{1 + R(s)G_1(s)G_2(s)h\left[\left(1 + \frac{1}{K_P}\right) - \frac{1}{K_P}\right]} \\ &= \left(1 + \frac{1}{K_P}\right)\frac{R(s)G_1(s)G_2(s)}{1 + R(s)G_1(s)G_2(s)h} \end{aligned} \tag{31}$$

which is the same transfer function as in the feed-forward compensation method. Nevertheless, in the comparative examples from section 4, this modification will not be applied to $R(s)$, in order to obtain and to compare different transient responses.

3 | STEADY-STATE ERROR IN THE PRESENCE OF DISTURBANCE

We will now analyze the case of a disturbance instead of a reference input. The block diagram is shown in Figure 5.

As there are zero-type $G_1(s)$ and $G_2(s)$ here, and

$$K_R = \lim_{s\to 0} R(s) \tag{32}$$

$$K_{G_1} = \lim_{s\to 0} G_1(s) \tag{33}$$

$$K_{G_2} = \lim_{s\to 0} G_2(s) \tag{34}$$

$$K_P = h \lim_{s\to 0} R(s)G_1(s)G_2(s) \tag{35}$$

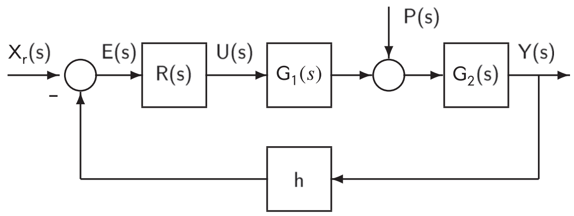

FIGURE 5 Basic control loop with disturbances

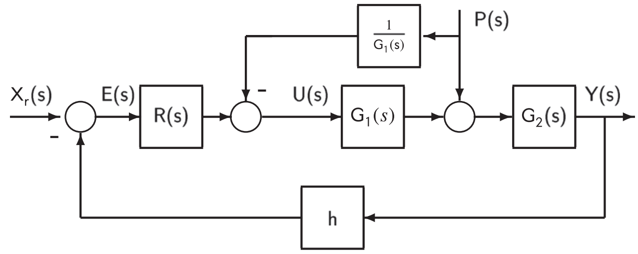

FIGURE 6 Classical feed-forward loop for disturbance compensation

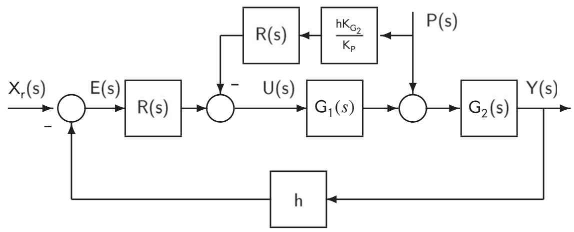

FIGURE 7 Proposed feed-forward loop for disturbances compensation

we therefore obtain the following transfer function relations:

$$\frac{Y(s)}{P(s)} = \frac{G_2(s)}{1 + hR(s)G_1(s)G_2(s)} \tag{36}$$

$$\frac{E(s)}{P(s)} = \frac{-hG_2(s)}{1 + hR(s)G_1(s)G_2(s)} \tag{37}$$

$$\frac{U(s)}{P(s)} = \frac{-hR(s)G_2(s)}{1 + hR(s)G_1(s)G_2(s)} \tag{38}$$

So, when the disturbance is a unitary step, then $P(s) = \frac{1}{s}$, so the error is therefore non-zero and in the steady-state:

$$\begin{aligned} e_s &= \lim_{s\to 0} sE(s) \\ &= \lim_{s\to 0} s\frac{-hG_2(s)}{1 + hR(s)G_1(s)G_2(s)}\frac{1}{s} \\ &= \frac{-hK_{G_2}}{1 + K_P} \neq 0 \end{aligned} \tag{39}$$

3.1 | Controller with integral action

Again, it is well known that assuming type-zero G(s), zero steady-state error can be obtained adding an integral action (pole at the origin) to the controller R(s),

$$R'(s) = R(s)\left(1 + \frac{1}{T_i s}\right) = R(s)\frac{1 + T_i s}{T_i s} \tag{40}$$

So:

$$K'_R = \lim_{s\to 0} R'(s) = \infty \tag{41}$$

$$K_{G_1} = \lim_{s\to 0} G_1(s) \neq \infty \tag{42}$$

$$K_{G_2} = \lim_{s\to 0} G_2(s) \neq \infty \tag{43}$$

$$K'_P = h \lim_{s\to 0} R'(s)G_1(s)G_2(s) = \infty \tag{44}$$

and, then, when the disturbance is a unitary step, we have:

$$e_s = \frac{-hK_{G_2}}{1 + K'_P} = 0 \tag{45}$$

The previous transfer functions will have changed as follows:

$$\frac{Y(s)}{P(s)} = \frac{G_2(s)T_i s}{T_i s + hR(s)G_1(s)G_2(s)(1 + T_i s)} \tag{46}$$

$$\frac{E(s)}{P(s)} = \frac{-hG_2(s)T_i s}{T_i s + hR(s)G_1(s)G_2(s)(1 + T_i s)} \tag{47}$$

$$\frac{U(s)}{P(s)} = \frac{-hG_2(s)(1 + T_i s)}{T_i s + hR(s)G_1(s)G_2(s)(1 + T_i s)} \tag{48}$$

The feedback system is now of higher order and the system dynamics are no longer the same. The effect on the dynamic response will be analyzed at a later stage.

3.2 | Feed-forward control

The classical feed-forward schema is shown in Figure 6.

In this case the goal is to obtain a null transfer function between the reference and the output, such that $Y(s) = 0$:

$$\begin{aligned} \frac{Y(s)}{P(s)} &= \left(1 - \frac{G_1(s)}{G_1(s)}\right)\frac{G_2(s)}{1 + hR(s)G_1(s)G_2(s)} \\ &= 0 \end{aligned} \tag{49}$$

In theory, this function guarantees that $e_s = 0$, but it once again forces the closed-loop dynamics to disappear and applies the inverse of G(s), causing the problem that has previously been mentioned. Again, we can obtain zero steady-state error by using a feed-forward action as in Figure 7.

Both here and in the original loop, zero-type $G_1(s)$, $G_2(s)$ and R(s) are found, so all the following parameters will have finite values:

$$K_R = \lim_{s\to 0} R(s) \tag{50}$$

$$K_{G_1} = \lim_{s\to 0} G_1(s) \tag{51}$$

$$K_{G_2} = \lim_{s\to 0} G_2(s) \tag{52}$$

$$K_P = h\lim_{s\to 0} R(s)G_1(s)G_2(s) \tag{53}$$

and, we obtain the following transfer function relations:

$$\frac{Y(s)}{P(s)} = \left(1 - \frac{hK_{G_2}}{K_P}R(s)G_1(s)\right) \cdot \frac{G_2(s)}{1 + hR(s)G_1(s)G_2(s)} \tag{54}$$

$$\frac{E(s)}{P(s)} = \left(1 - \frac{hK_{G_2}}{K_P}R(s)G_1(s)\right) \cdot \frac{-hG_2(s)}{1 + hR(s)G_1(s)G_2(s)} \tag{55}$$

$$\frac{U(s)}{P(s)} = \left(1 + \frac{hK_{G_2}}{K_P}R(s)G_1(s)\right) \cdot \frac{R(s)}{1 + hR(s)G_1(s)G_2(s))} \tag{56}$$

We will now prove that, when the disturbance is a unitary step, this control schema will also have zero steady-state error:

$$\begin{aligned} e_s &= \lim_{s\to 0} sE(s) \\ &= \lim_{s\to 0} s\left[\left(1 - \frac{hK_{G_2}}{K_P}R(s)G_1(s)\right) \cdot \frac{-hG_2(s)}{1 + hR(s)G_1(s)G_2(s)}\right]\frac{1}{s} \\ &= \left(1 - \frac{K_P}{K_P}\right)\frac{-hK_{G_2}}{1 + K_P} = 0 \end{aligned} \tag{57}$$

Of course, once again in the presence of modelling errors and non-linearities in the control process, we would have $h\lim_{s\to 0}R(s)G_1(s)G_2(s) \neq K_P$, and the non-zero steady-state error would be small.

4 | COMPARISON OF METHODS

4.1 | Process control example

The previous methods will be tested with chains of tanks.

Figure 8 shows a tank where A is the tank section, g is gravity, the liquid height $h(t)$ is an internal variable, and

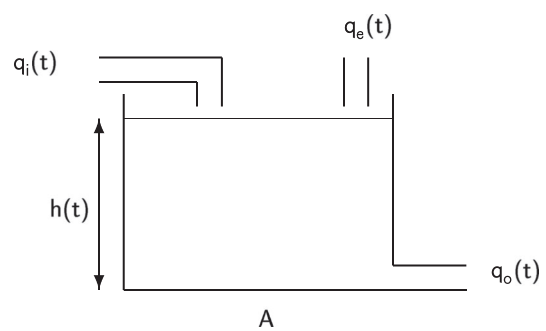

FIGURE 8 Example of a tank

the inputs are the input flow $q_i(t)$ and the external flow disturbance $q_e(t)$, and, finally, the output is the output flow $q_o(t)$.

The differential equations of the tank are as follows:

$$\begin{cases} A\frac{dh}{dt} = q_i(t) + q_e(t) - q_o(t) \\ q_o(t) = C\sqrt{2gh(t)} \end{cases} \tag{58}$$

Using Laplace transform, we have:

$$Q_i(s) + Q_e(s) = (1 + Cs)Q_o(s) \tag{59}$$

with which the open loop block diagram is obtained shown in Figure 9, which corresponds to a first-order system.

In the following experiment, we present the case of two serial tanks which corresponds to a second-order system:

$$G_1(s)G_2(s) = \frac{1}{(s+1)(s+2)} \tag{60}$$

4.2 | Experiments with two serial tanks

In this case, we will suppose a P controller with $R(s) = 10$ and constant sensor $h = 1$, as in Figure 10.

$X_r(s)$ is the second tank output flow set-point, $U(s)$ is the first tank input flow, and $P(s)$ is the second tank external flow disturbance. We will consider that the controller includes a valve with a constant transfer function.

4.2.1 | Reference input

When the set-point is the input, we obtain the following closed-loop transfer functions:

$$\frac{Y(s)}{X_r(s)} = \frac{10}{s^2 + 3s + 12} \tag{61}$$

$$\frac{E(s)}{X_r(s)} = \frac{(s+1)(s+2)}{s^2 + 3s + 12} \tag{62}$$

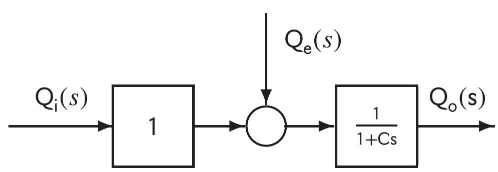

FIGURE 9 Block diagram of the tank

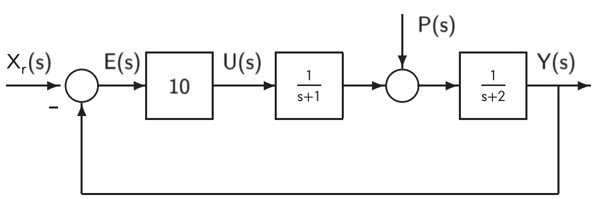

FIGURE 10 Proportional controller (two tanks)

$$\frac{U(s)}{X_r(s)} = 10\frac{(s+1)(s+2)}{s^2+3s+12} \tag{63}$$

Adding an integral action to the controller, for example, $R(s) = 10\left(1+\frac{1}{2s}\right)$ the transfer functions become:

$$\frac{Y(s)}{X_r(s)} = \frac{10(s+0.5)}{s(s+1)(s+2)+10(s+0.5)} \tag{64}$$

$$\frac{E(s)}{X_r(s)} = \frac{s(s+1)(s+2)}{s(s+1)(s+2)+10(s+0.5)} \tag{65}$$

$$\frac{U(s)}{X_r(s)} = 10\frac{(s+0.5)(s+1)(s+2)}{s(s+1)(s+2)+10(s+0.5)} \tag{66}$$

Now, using the proposed feed-forward compensation method, and keeping $R(s) = 10$, we can obtain the compensation block $\frac{1}{K_P} = 0.2$ and the following transfer functions:

$$\frac{Y(s)}{X_r(s)} = \frac{12}{s^2+3s+12} \tag{67}$$

$$\frac{E(s)}{X_r(s)} = 1.2\frac{s(s+1)}{s^2+3s+12} \tag{68}$$

$$\frac{U(s)}{X_r(s)} = 12\frac{(s+1)(s+2)}{s^2+3s+12} \tag{69}$$

And, finally, using the proposed multiple feedback compensation method, and keeping $R(s) = 10$, we have the compensation block $\frac{1}{K_G h} = 0.5$ and the transfer functions:

$$\frac{Y(s)}{X_r(s)} = \frac{10}{s^2+3s+10} \tag{70}$$

$$\frac{E(s)}{X_r(s)} = \frac{s(s+3)}{s^2+3s+10} \tag{71}$$

$$\frac{U(s)}{X_r(s)} = 10\frac{(s+1)(s+2)}{s^2+3s+10} \tag{72}$$

The unitary step evolution of both $y(t)$ and $u(t)$ are shown for the four cases in Figures 11A and 11B, respectively.

The analysis of this figure can lead to the following conclusions (Integral Absolute Error is also provided for each case):

1. The **original P controller** (in black) has steady-state error, while the other three improvements eliminate it (final value equal to the unitary step input).
2. The **controller with integral action** (in red) is a bit smoother than the others, slightly changing the transient response in comparison with the original one. IAE = 0.62.
3. The **feed-forward controller** (in blue) simply multiplies the output by a scale factor to guarantee the appropriate final value. This operation produces a higher control action value than in the other cases (a 20 percent more). IAE = 0.53.

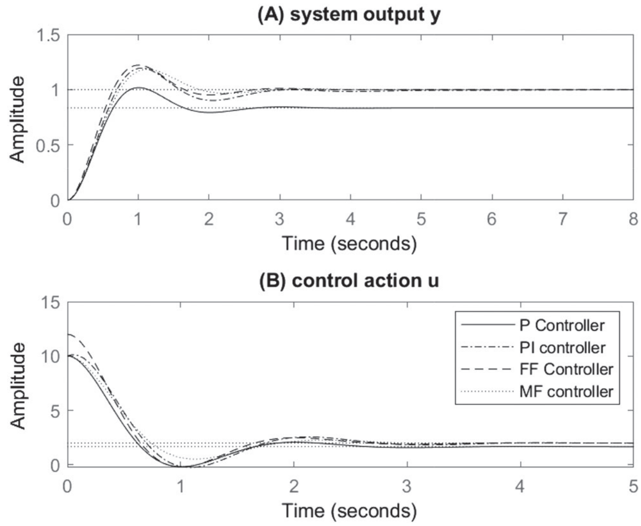

FIGURE 11 System output and control action evolution for the four controllers when the input is a step in the reference

4. The **multiple feedback controller** (in green) transient response is a bit smoother than the others, changing the closed-loop system dynamics as in the case of the integral action controller. IAE = 0.55.

In all cases, the three responses with zero steady-state error are very similar.

4.2.2 | Disturbance input

When the disturbance is the input, we obtain the following closed-loop transfer functions:

$$\frac{Y(s)}{P(s)} = \frac{s+1}{s^2+3s+12} \tag{73}$$

$$\frac{E(s)}{P(s)} = \frac{-(s+1)}{s^2+3s+12} \tag{74}$$

$$\frac{U(s)}{P(s)} = \frac{-10(s+1)}{s^2+3s+12} \tag{75}$$

When adding an integral action to the controller, for example, the same as in the previous case, $R(s) = 10\left(1+\frac{1}{2s}\right)$ the transfer functions become:

$$\frac{Y(s)}{P(s)} = \frac{s(s+1)}{s(s+1)(s+2)+10(s+0.5)} \tag{76}$$

$$\frac{E(s)}{P(s)} = \frac{-s(s+1)}{s(s+1)(s+2)+10(s+0.5)} \tag{77}$$

$$\frac{U(s)}{P(s)} = \frac{-10(s+0.5)(s+1)}{s(s+1)(s+2)+10(s+0.5)} \tag{78}$$

And in the case of using the proposed feed-forward compensation method, keeping $R(s) = 10$, we have the compensation block $\frac{hK_{G_2}}{K_P} = 0.1$ and the transfer functions:

$$\frac{Y(s)}{P(s)} = \frac{s}{s^2 + 3s + 12} \tag{79}$$

$$\frac{E(s)}{P(s)} = \frac{-s}{s^2 + 3s + 12} \tag{80}$$

$$\frac{U(s)}{P(s)} = \frac{-10s}{s^2 + 3s + 12} \tag{81}$$

Evolution of $y(t)$ and $u(t)$ when the disturbance is a unitary step are shown in Figure 12A and 12B, respectively.

The analysis of this figure can lead to the following conclusions:

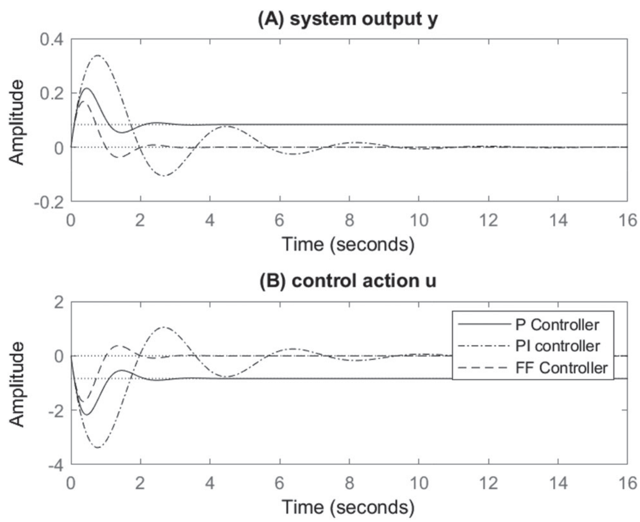

FIGURE 12 System output and control action evolution for the three controllers when the input is a step in the disturbance

1. The **original P controller** (in black) has steady-state error, while the error is eliminated by the other two improvements (final value equal to the zero reference input).
2. The **controller with integral action** (in red) is more oscillatory than the others, changing the transient response in comparison with the original one, and with a much more aggressive control action due to the zero that appears in the transfer function numerator. IAE = 0.23.
3. The **feed-forward controller** (in blue) also modifies the transient response, with a control action smoother than in the other cases. IAE = 0.13.

4.3 | Comparison

A comparative table of the methods when the input consists of both the reference and the disturbance is shown in table 1.

All the alternative methods to simple feedback with P or PD like controllers eliminate steady-state error, provided that no modelling errors are present.

Including an integral action increases system order and therefore significantly changes the system dynamics, so the controller gain (PI or PID) must then be tuned; furthermore, when the input is a disturbance the control action is greatly increased. On the other hand, it can still function in the presence of modelling errors and non-linearities.

Feed-forward controller increases control action when the input is the set-point, but neither modifies the system dynamics nor increases the system order. Nevertheless, it cannot achieve zero steady-state error when high modelling errors are present.

Multiple feedback controller changes the system dynamics and will not eliminate the steady-state error when high modelling errors are present, but will not increase the control action value.

TABLE 1 Comparison of methods

Method	Steady state error	Increases system order	Increases control action	Changes transient response	Works with modelling errors	Works with non linearities
Simple feedback	Yes	—	—	—	—	—
Integral action	No	Yes	Yes	Yes	Yes	Yes
Feed-forward	No	No	Yes	No	No	No
Multiple feedback (set-point)	No	No	No	Yes	No	No

5 | CONCLUSION

In this study, feed-forward and multiple feedback compensation methods have been proven to have reasonable transient responses in comparison with the use of integral action controllers, both when the input is the set-point and a disturbance. We have also proven that both methods are equivalent under some conditions when the input is the set-point. In conclusion, when there are very small or no modelling errors at all, these methods can be considered alternatives to the use of integral action in P and PD controllers.

ACKNOWLEDGEMENTS

This research is in receipt of funding from the Spanish Ministry of Science, Innovation and Universities (COGDRIVE project: Artificial Intelligence Techniques and Assistance to Autonomous Navigation, reference DPI 2017-86915-C3-3-R). It has also received funding from the RoboCity2030-DIH-CM Madrid Robotics Digital Innovation Hub project (Robótica aplicada a la mejora de la calidad de vida de los ciudadanos. fase IV; S2018/NMT-4331), funded by Programas de Actividades I+D en la Comunidad de Madrid and cofunded through EU Structural Funds.

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How to cite this article: Matía F. Transient response comparison of feedback and feed-forward compensation methods in systems with zero steady state error. *Asian J Control*. 2019;1–10. https://doi.org/10.1002/asjc.2144