# GRAPHICAL FITTING OF THE VON BERTALANFFY EQUATION

P.L. CADWALLADER

*Department of Zoology, University of Canterbury,
Christchurch, New Zealand

ABSTRACT

The parameters of the von Bertalanffy equation were estimated graphically using mean annual length data derived from the common river galaxias, *Galaxias vulgaris* Stokell. For males:

$$l_t = 115.7\ (1 - e^{-0.3705\ (t\ +\ 1.091)})$$

and for females:

$$l_t = 156.9\ (1 - e^{-0.2366\ (t\ +\ 1.106)})$$

These equations adequately described annual growth in length up to the end of the fourth year. The estimates of the parameters were similar to those obtained using Allen's least squares method. It is considered that, although the Allen method is more exact, the graphical method provides an adequate alternative means of estimating the parameters of the von Bertalanffy equation.

## INTRODUCTION

Estimation of fish production is facilitated by the generalization of growth data in the form of growth curves or models. One such model is the von Bertalanffy equation (von Bertalanffy 1938), which is represented by:

$$l_t = l_\infty\ (1 - e^{-K\ (t\ -\ t_o)}) \qquad (1)$$

where $l_t$ is the length at age t, $l_\infty$ is the average "maximum" or asymptotic length, K is a constant determining the rate of change in the length increment, t is age in years, and $t_o$ is the hypothetical age when length is zero. This equation has been used widely in fisheries investigations to describe both annual (Kohler 1959, May *et al.* 1965) and seasonal growth (Lockwood 1974). However, growth of all fish is not represented satisfactorily by the equation (Allen 1969) and many other growth models are to be found in the literature (Dickie 1968, Rafail 1971). The physiological basis of the von Bertalanffy equation has been attacked by a number of workers, e.g. Ricker (1958) and Hemmingsen (1960), and the concept of asymptotic growth has been questioned by Larkin *et al.* (1957), Paloheimo

*Present address: Fisheries and Wildlife Division, Ministry for Conservation, 632 Bourke Street, Melbourne, Victoria, Australia 3000.

and Dickie (1965) and Knight (1968). Also, Silliman (1969) showed that for some species in which the von Bertalanffy equation adequately describes growth, an asymptote has no biological meaning. For example, in chum salmon, *Oncorhynchus keta*, the asymptote would not be approached until several years beyond the maximum age of the species, since all chum salmon die after spawning at or before five years of age. Nevertheless, while the biological interpretation of growth models and their parameters still presents problems (Dickie 1968), there is no doubt of the general value of expressions such as the von Bertalanffy equation for descriptive purposes.

The parameters of the equation may be estimated either graphically, using the method described by Beverton and Holt (1957) and Ricker (1958), or mathematically, using the least squares method developed by Allen (1966). The Allen method is more exact, but requires a great deal of computation. It uses data from individual fish, whereas the graphical method uses mean values. Also, it provides estimates of the limits of error of the parameters, which the graphical method does not provide. This paper describes the fitting of the equation by the graphical method, and compares the estimates of the parameters so obtained with those obtained using the Allen method.

TABLE 1. MEAN TOTAL LENGTHS (TL) AND THEIR 95% CONFIDENCE LIMITS (CL) FOR FOUR YEAR CLASSES OF *GALAXIAS VULGARIS* FROM THE GLENTUI RIVER.

| AGE (years) | MALES | | | FEMALES | | |
|---|---|---|---|---|---|---|
| | Mean TL (mm) | 95% CL | Sample size | Mean TL (mm) | 95% CL | Sample size |
| 1 | 61.33 | ±0.7156 | 157 | 61.66 | ±0.7307 | 208 |
| 2 | 79.06 | ±1.2872 | 85 | 81.52 | ±1.0791 | 122 |
| 3 | 91.12 | ±1.9425 | 32 | 97.45 | ±1.8622 | 45 |
| 4 | 97.81 | ±4.3493 | 8 | 110.07 | ±3.8940 | 14 |

Data were derived from the common river galaxias, *Galaxias vulgaris* Stokell, taken from the Glentui River (172°16'E, 43°13' S), Canterbury, New Zealand (Cadwallader in press). Average longevity in the Glentui was three years. Mean lengths at each age were obtained by back-calculation from otolith readings (Table 1). Lengths achieved by male and female fish at the end of the first year of life were not significantly different ($t = 0.6351$, d.f. = 363, $P > 0.05$). However, after the first year, growth of males lagged behind that of females.

## GRAPHICAL ESTIMATION OF THE PARAMETERS OF THE VON BERTALANFFY EQUATION

Firstly, $l_\infty$ was calculated for males and females using the expression developed by Ford (1933) and Walford (1946):

$$l_{t+1} = l_\infty (1 - k) + k l_t \qquad (2)$$

Walford graphs, fitted by eye (Fig. 1b), of $l_{t+1}$ (length at age t+1) against $l_t$ (length at age t) yielded values for k (slope) of 0.63 and 0.8 for males and females respectively. The intercepts (males = 42.8, females = 31.38) were equated to $l_\infty$ (1 - k) and thus values of $l_\infty$ were calculated. These gave 115.7 for males and 156.9 for females. Values of $l_\infty$ can also be obtained directly from the graph, being the intercepts on the 45° diagonal.

Secondly, K and $t_o$ were calculated using the natural logarithmic form of equation (1):

$$\log_e (l_\infty - l_t) = \log_e l_\infty - K(t - t_o) \qquad (3)$$

Removal of the bracket on the right hand side of the equation gives:

$$\log_e (l_\infty - l_t) = \log_e l_\infty + Kt_o - Kt \qquad (4)$$

The graph, fitted by eye, of $\log_e (l_\infty - l_t)$ against t was a straight line of negative slope (Fig. 1a) from which K (slope) and the Y-axis intercept were obtained. For males, K = 0.3705 and the Y-axis intercept = 4.3471, and for females, K = 0.2366 and the Y-axis intercept = 4.7941. Values of $t_o$ were calculated by equating the Y-axis intercept to $\log_e l_\infty + Kt_o$. For males $t_o$ = -1.091, and for females $t_o$ = -1.106.

The von Bertalanffy equation describing growth in length of males was therefore:

$$l_t = 115.7\ (1 - e^{-0.3705(t + 1.091)})$$

and for females:

$$l_t = 156.9\ (1 - e^{-0.2366(t + 1.106)})$$

These equations adequately describe annual growth in length of *G. vulgaris* up to the end of the fourth year (Fig. 2).

TABLE 2. PARAMETERS OF THE VON BERTALANFFY EQUATION OBTAINED BY THE GRAPHICAL AND ALLEN METHODS, DESCRIBING ANNUAL GROWTH IN LENGTH OF *GALAXIAS VULGARIS* UP TO THE END OF THE FOURTH YEAR.

| | MALES | | | FEMALES | | |
|---|---|---|---|---|---|---|
| | $l_\infty$ | K | $t_o$ | $l_\infty$ | K | $t_o$ |
| Graphical estimates | 115.7 | 0.3705 | -1.091 | 156.9 | 0.2366 | -1.106 |
| Allen estimates (Cadwallader in press) | 112.6 | 0.4271 | -0.8405 | 158.9 | 0.2295 | -1.1397 |

The graphically-estimated parameters, particularly those for females, are similar to those obtained by the Allen method (Table 2), and annual lengths calculated from the graphically-derived equations are very similar to those calculated from the mathematically-derived equations (Table 3). Thus, although the Allen method is more exact, the graphical method provides an adequate alternative means of estimating the parameters of the von Bertalanffy equation, and is particularly useful if computing facilities are not available.

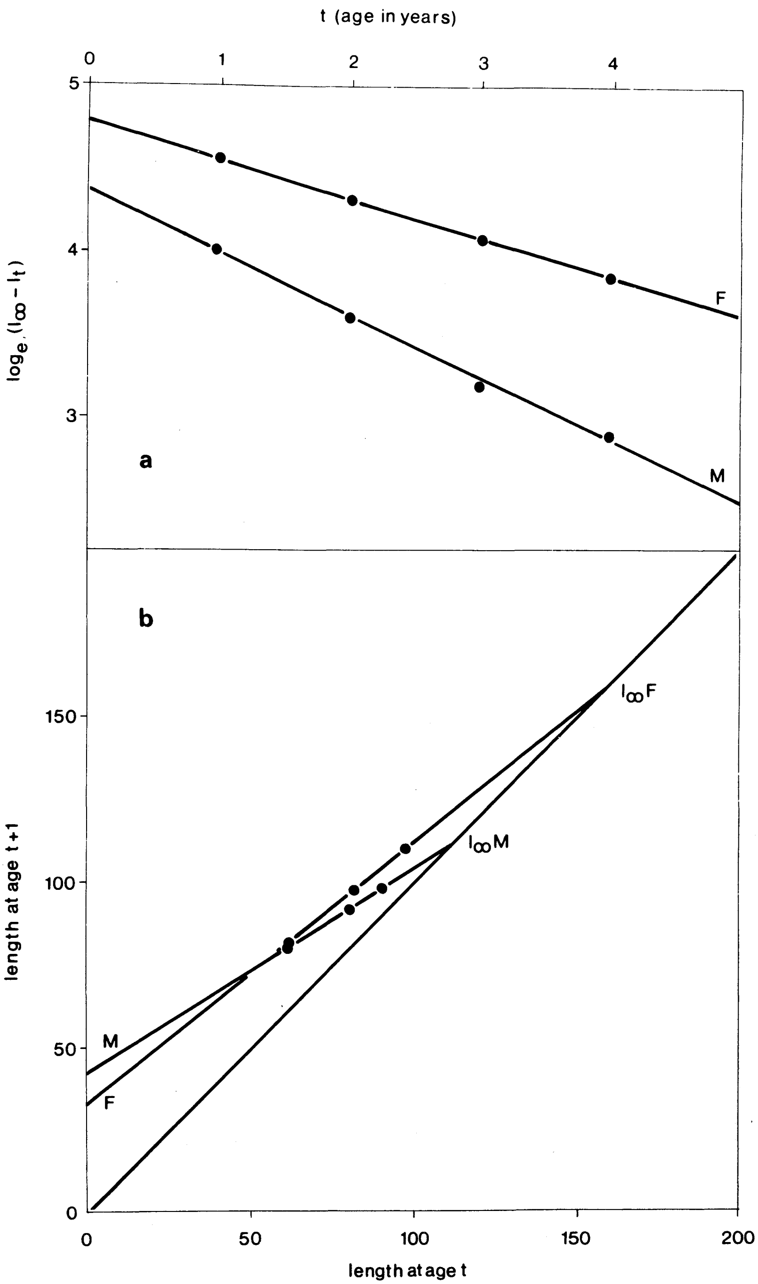


Fig. 1. Graphical estimation of the constants of the von Bertalanffy equation, (M = males, F = females). See text for explanation.

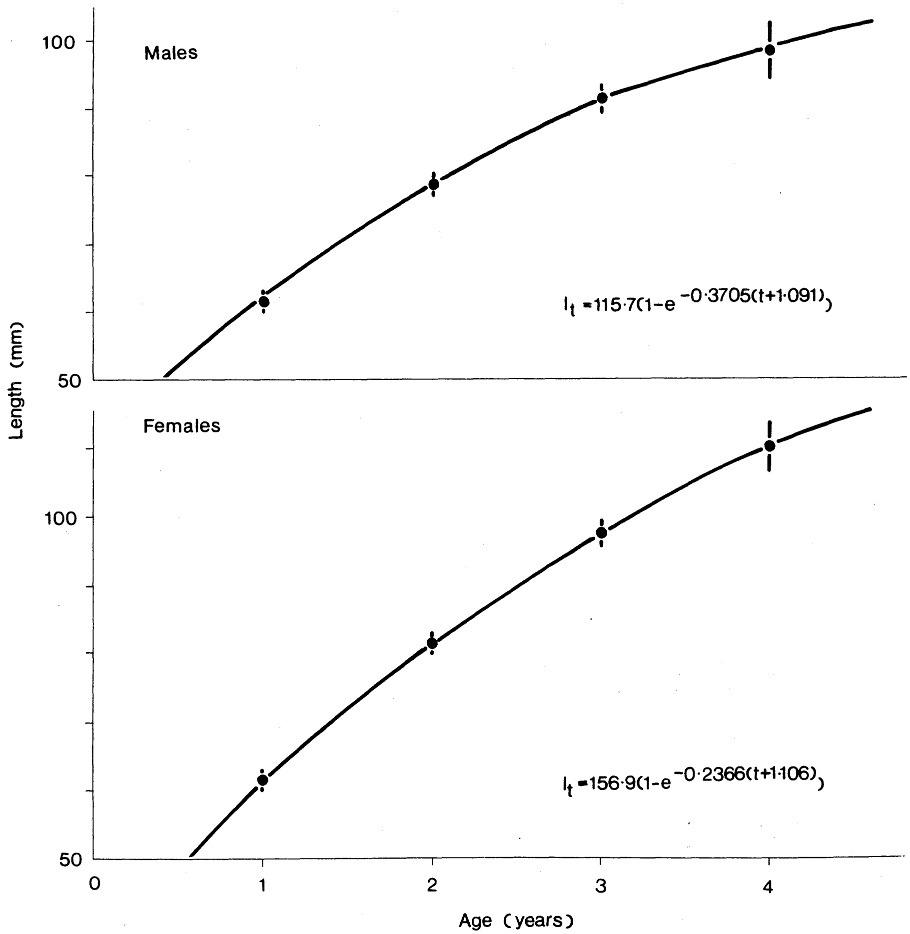


Fig. 2. Annual growth in length of *Galaxias vulgaris*. The mean length of each age group, together with its 95% confidence interval (Table 1), is superimposed on the von Bertalanffy curve, the parameters of which were estimated graphically.

TABLE 3. ANNUAL LENGTHS (TL) OF *GALAXIAS VULGARIS* CALCULATED FROM VON BERTALANFFY EQUATIONS WHOSE PARAMETERS WERE DERIVED GRAPHICALLY (G) AND BY THE ALLEN METHOD (A).

| Age (years) | MALES TL (mm) | | FEMALES TL (mm) | |
|---|---|---|---|---|
| | G | A | G | A |
| 1 | 62.37 | 61.32 | 61.55 | 61.65 |
| 2 | 78.88 | 79.14 | 81.65 | 81.59 |
| 3 | 90.28 | 90.77 | 97.50 | 97.45 |
| 4 | 98.15 | 98.36 | 110.02 | 110.05 |

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