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Housing Prices and Credit Constraints in Competitive Search*

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Abstract

We embed a competitive search model of the real estate market into a heterogeneous agent setting where households face credit constraints and idiosyncratic turnover shocks. Households can accumulate a risk-free asset to build a down payment and to smooth non-housing consumption. There is an inelastic supply of identical homes. The model is "block recursive". In equilibrium wealthier home buyers sort into submarkets with higher prices and shorter buying times. We identify a novel amplification mechanism, arising from sorting, by which demand shocks can substantially affect housing prices. In particular, lowering down payment requirements induces entry of new buyers in the market and higher asset accumulation by current searchers, as these agents target more expensive (less congested) submarkets. This affects the distribution of prices and trading probabilities, and thereby the wealth distribution. Our quantitative results suggest that the effects on the long-run level and dispersion of housing prices can be significant.

Keywords: housing prices, credit constraints, competitive search, price dispersion, inelastic housing supply, wealth inequality, sorting.
JEL Classification: D31, D83, E21, R21, R30.

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1 Introduction

In the expansion prior to the Great Recession housing prices experienced significant increases at the same time that credit conditions eased greatly. It is a widely held view that the increase in the availability of credit fueled the housing boom (e.g. Glaeser et al., 2012; Landvoigt et al., 2015; Favara and Imbs, 2015). However, changes in financial constraints typically do not have significant quantitative effects in traditional Walrasian macroeconomic models. This paper identifies a novel amplification mechanism by which changes in credit constraints can substantially affect housing prices when real estate transactions are subject to search and matching frictions.

We embed a competitive search model of the real estate market into a heterogeneous agent setting where households face credit constraints. The equilibrium features price dispersion, as wealthier buyers sort into submarkets with higher prices and shorter average buying times. In this setting, loosening credit constraints induces not only the entry of new buyers in the market but also higher asset accumulation by current searchers, as these agents direct their search to more expensive submarkets, where they are more likely to trade. The entire distribution of prices and trading probabilities, and thereby the wealth distribution, are affected by these changes in the extensive and intensive margins generated by increased buyer competition. We calibrate the model to illustrate its numerical comparative static properties and find that the implied effects on the long-run level and dispersion of housing prices can be significant (relative to a Walrasian model).

Search models constitute a powerful mechanism for demand shocks to affect aggregates; see, for instance, Díaz and Jerez (2013). A growing number of quantitative studies uses these models to account for several dynamic and cyclical features of housing markets (e.g. Díaz and Jerez, 2013; Ngai and Tenreyro, 2014; Head et al., 2014; Hedlund, 2016b; Garriga and Hedlund, 2017). Yet most of this literature assumes that households are risk neutral and ignores their savings decisions. The recent contributions by Hedlund (2016a 2016b), Garriga and Hedlund (2017) and Eerola and Maattanen (2018) are notable exceptions which we discuss below.

We consider a partial equilibrium setting where households consume a nondurable good and housing services. Households face idiosyncratic turnover shocks which make them want to change residence, and they can either own their home (an indivisible durable good) or rent. Home purchases can be partially financed with non-defaultable mortgage loans, and buyers must search for a home in a decentralized market where prices and trading probabilities are determined by competitive/directed search. Households can accumulate a risk-free asset both to put a down payment on a home and to smooth non-housing consumption. To keep things simple, we abstract away from idiosyncratic income risk and assume a fixed stock of owner-occupied housing consisting of symmetric units (which do not depreciate). All price dispersion arising in equilibrium is then purely frictional. We focus our analysis on stationary equilibria.

Our model is "block recursive", as those of Shi (2009) and Menzio and Shi (2010), meaning that the agents' value and policy functions do not depend on the distribution of households across individual states.[1] Instead, they depend on a finite-dimensional variable which summarizes all the relevant information regarding the terms trade in the housing market. In the seminal models of Shi (2009) and Menzio and Shi (2010), "block recursivity" arises from the combination of directed search and free entry of job vacancies created by risk-neutral firms under constant returns. Here, it arises because we assume that buyers and sellers (both of whom are risk averse) do not trade directly with each other.[2] Instead, trades are intermediated by risk-neutral agents with transferable utility who freely enter the market (see also Hedlund, 2016a; Karahan and Rhee, 2019). To keep the model as stylized as possible, we assume that sellers face no trading delays, and focus on the frictions faced by buyers. Specifically, homeowners can sell their homes in a Walrasian market to intermediaries, who then look for potential buyers in the decentralized market.

The model's block-recursive structure allows us to derive several properties of the households' value and policy functions, and to bring to light the underlying amplification mechanism. We first show that the value functions exist and are differentiable along the optimal paths. This suffices to obtain the Euler equations. These results are not trivial since, due to the lack of concavity of the household's problem, the theorems of Mirman-Zilcha and Benveniste-Scheinkman do not apply to our setting. Menzio et al. (2013) circumvent the technical difficulties arising from the non-concavity by introducing lotteries. This makes the model tractable, but obviously not equivalent to the original problem since the optimal policy functions differ. In this paper, we do not need to introduce lotteries but work directly within the non-concave framework. We show that the value functions are concave (on the range of assets that corresponds to participation and non-participation in the decentralized market, respectively) and their optimal choices are unique

[1] This is unlike random search models with an endogenous asset distribution, which are known to be intractable precisely for this reason (e.g. Molico, 2006).

[2] Dealing with a setting with two-sided heterogeneity and two-sided risk aversion would be highly involved.

provided the optimal consumption policies are monotone.[3] These theoretical results are derived by adapting the approach recently introduced in Rincón-Zapatero (2020). This approach does not directly apply to the Bellman equations of our model (due to their particular structure), so additional work is required. To the best of our knowledge, these results are novel and provide a new benchmark for analyzing similar block-recursive search models with an endogenous asset distribution without the need of introducing lotteries.

In equilibrium home buyers with higher financial wealth (who have a lower marginal utility of wealth) direct their search to submarkets with higher prices, where buying times are shorter on average. In turn, more expensive homes take longer to sell. These results are consistent with empirical work in the real estate literature which finds that, after controlling for housing attributes and location, buyers with higher income tend to pay higher prices (see Elder et al., 1999; Qiu and Tu, 2018), and search for a shorter period of time on average (see Elder, Zumpano, and Baryla 1999, 2000). There is also widespread evidence of a positive relationship between the price of real estate property and its average time on the market (e.g. Merlo and Ortalo-Magné, 2004; de Wit and van der Klaauw, 2013). Frictional house price dispersion is hard to measure since houses are hedonic goods which are heterogeneous along many dimensions (some of which are unobservable). Yet several hedonic studies find variations in house prices after controlling for house characteristics and location using data for different countries (e.g. Malpezzi et al., 1980; Elder et al., 1999; Leung et al., 2006; Yiu et al., 2008).[4]

To compute the equilibrium, we adapt the endogenous grid method used by Fella (2014) in a model with exogenous non-convex adjustment costs to our search environment (where trading delays are endogenous). Fella's algorithm yields substantial gains in accuracy and computational time relative to standard techniques used to solve non-concave problems. In particular, this method is much more efficient and accurate than standard value function iteration, which is the approach used in the literature to compute related search models with an endogenous distribution of assets (e.g. Hedlund, 2016b; Chaumont and Shi, 2018; Eeckhout and Sepahsalari, 2018).

To illustrate how the model works and what its numerical comparative statics properties are, we calibrate it to reproduce selected statistics for the U.S. economy. In our setting, any shock

[3]This is the case in all our numerical exercises.

[4]Search theory has long been used to rationalize the existence of frictional price dispersion. Recent related work by Piazzesi et al. (2020) documents differential search patterns by buyers at the ZIP code level using data from California's website Trulia, and argues that these patterns can explain differences in the prices of houses with similar characteristics across ZIP codes. Yet their model assumes risk-neutral searchers and hence no wealth effects.

affecting demand affects the distribution of prices paid by home buyers and thus the mean and variance of housing prices. It also affects the trading delays buyers face and the degree of housing market liquidity.

Our comparative statics exercises show that price dispersion, market liquidity, and wealth accumulation are tightly linked and that the interaction between these equilibrium objects is crucially affected by credit conditions. Take the case of a highly liquid market, where demand is high and average buying times are long. In this scenario buyers who do not find a trading opportunity (a likely event for poor households) accumulate more assets and, in the next period, they direct their search towards more expensive submarkets, where they are more likely to trade. Hence, when credit constraints are relaxed, these buyers can afford to enter submarkets with higher prices. In addition, buyer participation increases, as some households who were not searching for a home (e.g. because they were saving to build a down payment) enter the market. The increase in participation exacerbates the overall congestion effects buyers face, thus reinforcing the aforementioned asset accumulation process. This amplification mechanism, which operates through the inherent heterogeneity of the economy generated by search and matching frictions, leads to substantial increases in the average housing price. Also, price dispersion and average selling times fall when credit is eased, which is intuitive given the increase in competition among buyers. A rise in labor income has similar effects, the effect on average prices being even stronger. In general, increases in housing demand increase average housing prices and market liquidity and reduce price dispersion, whereas negative demand shocks have the opposite effect. The effect on the wealth distribution is complex due to the interaction between changes in prices and trading probabilities. To the best of our knowledge these amplification effects, arising from directed search, have not been identified before.

The model's amplification mechanism critically relies on the inelasticity of the housing supply. In particular, with a fixed housing stock a reduction in down payment requirements has a substantial impact on the average housing price but leaves the homeownership rate almost unaltered. Yet, in the opposite extreme case where the housing supply is infinitely elastic, the average price does not change but the homeownership rate rises substantially.

1.1 Related work

Eerola and Maattanen (2018) study a quantitative random search and bargaining model of the housing market with heterogeneous agents and a fixed supply of identical homes, calibrated to Finish data. These authors also find that tightening credit constraints reduces housing market liquidity and increases frictional price dispersion, and present empirical evidence which is consistent with this finding. Their model is more involved, as they allow for idiosyncratic income risk and assume that buyers and sellers trade directly with each other. Its main drawback is its lack of tractability (as it is not block recursive). In particular, these authors cannot study how changing credit conditions affect the homeownership rate and the wealth distribution. Instead, all their exercises are performed by keeping the homeownership rate fixed (as they cannot perform actual across-steady state comparisons). The mechanism generating frictional price dispersion is also different. Since search is random, wealthier and poorer traders are equally likely to meet potential trading partners. Yet, conditional on a match, wealthier buyers are more likely to complete a transaction and pay higher prices on average because they have higher reservation prices. We view the two mechanisms as complementary explanations of why wealth inequality may generate frictional housing price dispersion.

Hedlund (2016a 2016b) develops a related quantitative model with several features which we abstract from, such as trading delays affecting homeowners, idiosyncratic income risk, heterogeneous housing units, an endogenous supply of mortgage credit, and equilibrium default. He introduces a slightly different intermediation sector which renders the model block-recursive and assumes that new homes can be built instantaneously within a period under decreasing returns.[5]

Hedlund (2016a) studies how the interaction between search frictions and credit constraints affects the cyclical behavior of housing market aggregates. His key finding is that the increasing illiquidity of housing assets characteristic of a slowdown tightens credit constraints for borrowers (who find it harder to sell their homes). This creates a vicious circle that increases foreclosures and further depresses the housing market. Hedlund (2016b) uses a model variant with aggregate productivity shocks to study cyclical housing dynamics and to assess the effects of stabilization policies. Garriga and Hedlund (2019) study another variant where a combination of higher downside labor earnings risk and tighter credit conditions can explain a substantial share of the fall in prices during the Great Recession. The emphasis in these quantitative papers is then on the frictions

[5] There are two competitive search markets where intermediaries trade with homeowners and buyers, respectively. There is also a Walrasian market where these intermediaries trade with each other and with home builders. The fact that this market clears every period implies that, if the number buyers who trade with an intermediary (total home purchases in the first search market) exceeds the number of homeowners who trade with an intermediary (total sales in the second search market) in a given period, new homes must be built during the period to meet demand.

sellers face, whereas we focus on those faced by buyers. Whereas the former are more relevant during a bust, the latter should be more relevant during a boom, as pointed out by Garriga and Hedlund (2019). We thus view our work as complementary.

The paper is organized as follows. In Section 2 we describe the environment and the problems solved by agents and define a stationary equilibrium. Section 3 presents our theoretical results. Section 4 discusses our computational method, the calibration, and our main comparative-statics results. Section 5 concludes. Proofs and computational details are relegated to the Appendix.

2 The model economy

2.1 Preferences and endowments

Consider a location populated by a continuum of households who live forever. Time is discrete. Households derive utility from a divisible consumption good and the service flow provided by an indivisible durable good which we refer to as *housing*. Their lifetime utility is $\sum_{t=0}^{\infty} E_0 \beta^t u(c_t, h_t)$, where $c_t, h_t \in \mathbf{R}_+$ are the amounts of the divisible good and housing services consumed each period, respectively, and β is the discount factor. The function u is strictly increasing, strictly concave and $\mathcal{C}^2$, with $u_{ch} \geq 0$ and $\lim_{h \to 0} u(c, h) = -\infty$. Each period households have a fixed endowment w of the consumption good (the numeraire) and can choose to either own or rent a (single) housing unit. The stock of owner-occupied housing consists of H symmetric units that do not depreciate.

Each period homeowners face i.i.d. preference shocks and can be in two states, $\mu \in \{0, 1\}$. Owners in state 1 consume $\hbar > 0$ units of housing services, whereas owners in state 0 consume none. Owners in state 1 are then matched with their home, and owners in state 0 are mismatched. The state μ follows a Markov process with transition probabilities $P(\mu' = 1|\mu = 1) = 1 - \pi_\mu \in (0, 1)$ and $P(\mu' = 0|\mu = 0) = 1$. So $\mu = 0$ is an absorbing state; owners can only transit out of this state by selling their home and moving to a new unit. By contrast, renters consume an exogenous amount h_r of housing services, where $0 < h_r \leq \hbar$.[6]

Households face also idiosyncratic moving shocks (which are realized jointly with the owners' preference shocks). Owners and renters are hit by these independent shocks with different time-

[6]We thus allow for a taste for ownership.

invariant probabilities, denoted by $\pi_{\xi_0}, \pi_{\xi_r} \in (0,1)$, respectively.[7] Households hit these shocks migrate to a *symmetric* location in the rest of the world at no cost, and are instantaneously replaced by an equal mass of immigrants. The details on these entry flows are specified below. We normalize the constant measure of households in the location to one.

Our model abstracts away from idiosyncratic labor income risk and aggregate risk. We just introduce the minimum amount of idiosyncratic risk needed to generate turnover in the housing market. The omission of idiosyncratic income risk is very restrictive, in particular, in terms of the wealth distribution generated by the calibrated model. Yet this omission allows us to keep the model tractable to derive our theoretical results and to bring to light the underlying amplification mechanism. We elaborate on this in Section 5.

2.2 Market arrangements and real estate intermediation

Households can save by means of a risk-free asset with return R. Their home purchases can be partially financed with a non-defaultable mortgage loan. Specifically, a household can borrow up to a fraction $(1-\delta)$ of the home's liquidation value, so it must save to meet the corresponding down payment. The mortgage is a loan in perpetuity with no costs associated if there is early repayment. Houses also serve as collateral for loans: homeowners can obtain a home equity loan for up to a fraction $(1-\delta)$ of the home's value (i.e., they can always remortgage). There are indirect taxes on real estate transactions. Home sellers pay taxes on the value of the house at the rate τ_s, whereas the buyers' tax rate is τ_b. For simplicity, we assume that tax revenues are thrown away. Also, there is no spread between borrowing and lending rates, and households who do not own residential assets cannot borrow.[8]

Real estate transactions are intermediated by agents with linear transferable utility and deep pockets who are free to enter the economy. These agents purchase homes from mismatched homeowners, and then look for potential buyers. Intermediaries are also infinitely-lived with discount factor β and can hold at most one unit each period, and they do not pay taxes. We do not model the rental market explicitly, and assume that households who rent do so at a fixed price r_h.

Below we specify the timing of the model, and describe the market structure in detail. Each

[7]We introduce the moving shocks for quantitative purposes, specifically to match migration flows. In the data, renters move more often than owners (e.g. Head et al. (2014)).

[8]These financial market arrangements are as in Díaz and Luengo-Prado (2008).

period is divided into three subperiods: *morning*, *afternoon*, and *night*.

2.2.1 Morning

At the start of a period, there are two types of households in the economy depending on their tenure status: *owners* and *renters*. First, preference and moving shocks are realized. Then, a Walrasian market opens where the owners who have been hit by these shocks supply their homes inelastically.[9] Intermediaries can freely enter this market to purchase a unit at the market clearing price, $\overline{p}$. Once the market closes, the households who were hit by the moving shock migrate and are replaced by an equal measure of immigrants who do not own residential assets.

Note that homeowners do not face trading delays in our model; it is intermediaries who face the inventory risk, as we shall see. Yet this risk will be priced into $\overline{p}$. We introduce the Walrasian market because it highly simplifies the analysis, allowing us to focus on the frictions buyers face.

2.2.2 Afternoon

During the afternoon, those households who sold their home in the morning and did not migrate, those who were renters in the previous period, and the newly arrived immigrants decide whether or not to search for a home to buy. We refer to these households as *potential buyers*. Matched homeowners make no economic decisions in this subperiod, so we refer to them as *non-traders*.

A competitive search market operates in the afternoon where intermediaries put their vacant homes up for sale at cost $\kappa_s > 0$.[10] Buyers may borrow up to a fraction $1 - \delta$ of the home's value in the Walrasian morning market; i.e., their borrowing limit is $(1 - \delta)\,\overline{p}$. The implicit assumption (as in Kiyotaki and Moore, 1997) is that banks lend the amount they can recover in the Walrasian market if they seized the house. Potential buyers may choose not to participate in the afternoon market (e.g. if they have not accumulated enough assets to meet the corresponding down payment).

As in Moen (1997), buyers and intermediaries can participate in different submarkets where they meet bilaterally and at random, and where each trader experiences at most one bilateral match.

[9] This is optimal for mismatched owners as $\lim_{h\to 0} u(c, h) = -\infty$. It is also optimal for owners who are hit by the moving if they face a sufficiently high cost of leaving their home unsold.

[10] We assume that buyers search for a home at a negligible cost. This rules out equilibria where some households participate in the frictional market even though they do not plan to trade there (because doing so is costless).

The matching probabilities in a given submarket depend on the associated buyer-seller ratio θ (or market tightness). Specifically, an intermediary is matched to a buyer with probability $m_s(\theta)$, and a buyer is matched to an intermediary with probability $m_b(\theta) = m_s(\theta)/\theta$.[11] As is standard, $m_s(\theta)$ is strictly increasing, strictly concave and $\mathcal{C}^2$, with $m_s(0) = 0$ and $\lim_{\theta\to\infty} m_s(\theta) = 1$, and $m_b(\theta)$ is strictly decreasing and $\mathcal{C}^2$, with $\lim_{\theta\to 0} m_b(\theta) = 1$ and $\lim_{\theta\to\infty} m_b(\theta) = 0$. In words, the higher the buyer-seller ratio θ, the easier it is for intermediaries to contact buyers, and the harder it is for buyers to locate a home for sale (due to congestion externalities). As θ goes to infinity (zero) the intermediary's matching probability goes to one (zero), and the buyer's matching probability goes to zero (one). The elasticity $\eta(\theta) \equiv \frac{m_s'(\theta)\theta}{m_s(\theta)} \in [0,1]$ is assumed non increasing, and $\hat{m}_s(m_b) \equiv m_s(m_b^{-1}(\cdot))$ is such that $\ln \hat{m}_s$ is concave.[12]

To describe the price determination process in the competitive search market, we adopt the price-taking approach in Jerez (2014). The idea is to think of houses traded in submarkets with different tightness levels $\theta \in \mathbf{R}_+$ as different commodities, which are characterized by different degrees of trading uncertainty. The prices of these differentiated commodities are described by a continuous function $p : \Theta \to \mathbf{R}_+$. That is, $p(\theta)$ is the housing price in a submarket with tightness $\theta \in \mathbf{R}_+$. Buyers and intermediaries choose the submarkets they enter taking $p(\theta)$ as given and have rational expectations about the tightness level prevailing in active submarkets. To model market participation, we introduce a "fictitious submarket" $\theta_0 \in \mathbf{R}_-$, and extend the functions m_b, m_s and p to $\Theta \equiv \mathbf{R}_+ \cup \{\theta_0\}$ by setting $m_b(\theta_0) = m_s(\theta_0) = p(\theta_0) = 0$. Households who choose submarket θ_0 do not participate in the afternoon market.

As shown in Jerez (2014), our price-taking equilibrium notion is equivalent to that of directed search. In particular, $p(\theta)$ is the inverse of the schedule $\theta(p)$ describing the agents' beliefs (about the tightness level in submarkets with different prices) in directed search models. We choose the price-taking formulation because it makes the connection with the standard notion of recursive competitive equilibrium more direct and transparent. The crucial difference with the standard notion is that the frictional afternoon market does not clear in equilibrium.

[11] The underlying assumption is that the total number of bilateral trading meetings is determined by a matching function with constant returns to scale and that the Law of Large Numbers holds.

[12] Equivalently, $-\hat{m}_s{}'(m_b)/\hat{m}_s(m_b)$ is non decreasing. This assumption guarantees that the problem solved by potential buyers is concave and has a unique solution (see Sections B-D in the Appendix), and can be further relaxed (see Section E.1). See also Menzio and Shi (2010) where $\hat{m}_s$ is assumed concave (a slightly stronger assumption).

2.2.3 Night

Households who bought a home in the afternoon are *owners* at night, just as the non-traders. The rest of the households are *renters*. At night households receive the endowment, w, and decide their non-housing consumption and the level of assets to be carried to the next period.

2.3 Stationary equilibrium

Below we state the problems of the agents in each subperiod, starting at night and going backward. A stationary equilibrium is then defined.

2.3.1 Night

Recall that intermediaries are inactive at night. Let $A = [\underline{a}, \infty)$ be the set in which the households' financial assets can take values, and let $a \in A$ be the household's assets at the start of the night. The afternoon value functions of potential buyers and non-traders are $W_b : A \to \mathbf{R}$ and $W_n : A \to \mathbf{R}$, respectively. The night value function of an *owner* is given by

$$\begin{aligned} W_o(a) = \quad & \max_{c, a' \in \mathbf{R}} \Big\{ u\left(c, \hbar\right) + \beta \left(1 - \pi_{\xi_o}\right)\left(1 - \pi_\mu\right) W_n\left(a'\right) \\ & \qquad\qquad + \beta \left[1 - \left(1 - \pi_{\xi_o}\right)\left(1 - \pi_\mu\right)\right] W_b\left(a' + \left(1 - \tau_s\right)\overline{p}\right) \Big\} \\ \text{s.t.} \quad & c + \tfrac{1}{R}\, a' \leq w + a, \\ & a' \geq -(1 - \delta)\, \overline{p}, \\ & c \geq 0, \end{aligned} \tag{2.1}$$

where c and $\hbar$ are the amounts of the divisible good and housing services consumed, and a' is the level of financial assets carried to the next period. Owners choose the values of c and a' to maximize their expected lifetime utility subject to a standard intertemporal budget constraint and also face a borrowing limit equal to $(1 - \delta)\, \overline{p}$. (As described earlier, they can remortgage their home, in which case the price of reappraisal is the Walrasian market price.) With probability π_{ξ_o}, owners will be hit by a moving shock on the next morning. If not, there is still a probability π_μ that they will move to another location. An owner who is not hit by any of these shocks will be a non-trader in the next afternoon, with continuation value $W_n(a')$. An owner who is hit will sell her home at price $\overline{p}$ in the next morning and pay the corresponding indirect taxes. Note that the assumption that agents hit by a moving shock migrate to a symmetric location at no cost implies that the owner's continuation value is the same regardless of the kind of shock that hits her. Owners hit by the preference (but not by the moving) shock will be potential buyers in their current location, whereas owners hit by the moving shock will be potential buyers elsewhere. In both cases, their continuation value is $W_b(a' + (1-\tau_s)\,\overline{p})$. Denote the owners' optimal policies by $g_o^c(a)$ and $g_o^a(a)$.

The night value function of a *renter* is defined in a similar way:

$$\begin{aligned} W_r(a) = \quad & \max_{c,a' \in \mathbf{R}} \left\{ u\left(c, h_r\right) + \beta\, W_b\left(a'\right) \right\} \\ \text{s.t.} \quad & c + \tfrac{1}{R}\, a' \leq w - r_h + a, \\ & a' \geq 0, \\ & c \geq 0, \end{aligned} \tag{2.2}$$

and $g_r^c(a)$ and $g_r^a(a)$ denote the optimal decision policies. The main difference is that these households pay the rent, r_h, consume h_r units of housing services, and are not allowed to borrow. Also, their continuation value is not affected by the moving shocks they face. These agents will be potential buyers in the next afternoon (either in the current location or in a symmetric one).

2.3.2 Afternoon

Let a be the household's financial assets at noon. *Non-traders* are inactive during the afternoon, so their value function is given by

$$W_n(a) = W_o(a). \tag{2.3}$$

Potential buyers choose the submarkets they join taking as given the price schedule, $p(\theta)$, and the maximum loan they can obtain, $(1-\delta)\,\overline{p}$. Their value function is given by

$$\begin{aligned} W_b(a) = \quad & \max_{\theta \in \Theta} \left\{ m_b\left(\theta\right)\; W_o\left(a - (1+\tau_b)\, p(\theta)\right) + \left(1 - m_b\left(\theta\right)\right)\; W_r\left(a\right) \right\} \\ \text{s. t.} \quad & a - (1+\tau_b)\, p(\theta) \geq -(1-\delta)\,\overline{p} \text{ if } \theta \in \mathbf{R}_+, \end{aligned} \tag{2.4}$$

and $g_b^\theta(a)$ denotes their optimal decision rule. The collateralized borrowing constraint in problem (2.4) ensures that buyers who join submarket $\theta \in \mathbf{R}_+$ have enough assets to pay for the corresponding down payment and the associated taxes. With probability $m_b(\theta)$ these households buy a home and enter the night with financial assets $a-(1+\tau_b)\,p(\theta)$.[13] With complementary probability, they do not trade and carry their full assets into the night, when they will be renters (just as those potential buyers who choose not to participate in the afternoon market).

Likewise, *intermediaries* choose the submarkets they join in order to maximize their expected lifetime value given $p(\theta)$, so their expected value in the afternoon is

$$J=\max_{\theta\in\mathbf{R}_+}\Big\{-\kappa_s+m_s\left(\theta\right)\,p(\theta)+\left(1-m_s\left(\theta\right)\right)\,\beta\,J\Big\}. \tag{2.5}$$

Intermediaries who join submarket $\theta\in\mathbf{R}_+$ pay the cost κ_s and sell their unit with probability $m_s(\theta)$ at price $p(\theta)$, in which case they exit the location. With complementary probability, they do not trade and must wait until the next afternoon, when they will continue to search for a buyer. We denote the set of optimal solutions for problem (2.5) by Θ_J.[14]

2.3.3 Morning

Recall that, during the morning, owners hit by a shock sell their home at price $\overline{p}$ to the intermediaries that enter the Walrasian market (whereas the rest of the households are inactive). Since entry is free, the expected profits of these intermediaries are zero in equilibrium:

$$\overline{p}=J. \tag{2.6}$$

2.3.4 Stationary equilibrium definition

The distribution of non-traders and potential buyers at noon is described by the Borel measures $\psi_n,\psi_b\in M_+(A)$, respectively. Similarly, $\psi_o,\psi_r\in M_+(A)$ represent the distribution of owners and

[13] Households with a mortgage have negative assets at night and pay interests on that debt, as implied by the intertemporal budget constraint in (2.1).

[14] As we shall see, in equilibrium, Θ_J includes a continuum of elements, intermediaries being indifferent between all $\theta\in\Theta_J$ (see Section 3).

renters at night. Since the total mass of households is one,

$$\psi_n(A) + \psi_b(A) = \quad 1, \tag{2.7}$$

$$\psi_r(A) + \psi_o(A) = \quad 1. \tag{2.8}$$

We describe the asset distribution of immigrants by an exogenous probability measure $\zeta_i \in P(A) \subset M_+(A)$. Since net migration flows are zero, the inflow of immigrants is given by $\psi_i \in M_+(A)$ with

$$\frac{\psi_i}{\psi_i(A)} = \quad \zeta_i, \tag{2.9}$$

$$\psi_i(A) = \quad \pi_{\xi_r}\psi_r(A) + \pi_{\xi_o}\psi_o(A). \tag{2.10}$$

The law of motion of $\{\psi_o, \psi_r, \psi_n, \psi_b, \psi_i\}$ is described in Appendix A.

A measure $b \in M_+(\mathbf{R}_+)$ describing the distribution of buyers across submarkets $\theta \in \mathbf{R}_+$ is easily constructed from ψ_b and g_b^θ:

$$b(\Xi) = \psi_b\left(\{a \in A : g_b^\theta(a) \in \Xi\}\right), \qquad \text{for all Borel } \Xi \subset \mathbf{R}_+. \tag{2.11}$$

In words, $b(\Xi)$ is the measure of buyers who participate in a submarket $\theta \in \Xi$. Similarly, we describe the distribution of intermediaries across submarkets by $s \in M_+(\mathbf{R}_+)$. The support of s contains elements $\theta \in \Theta_J$ which solve problem (2.5). The precise distribution s on Θ_J will be determined by rational expectations (see equation (2.16) below). The set of active submarkets (which attract both buyers and intermediaries) is given by the intersection of the supports of b and s.

It remains to describe the law of motion of the vacancy stock held by intermediaries. Let $\widetilde{V}$ be the stock at the start of a period, which is equal to the mass of intermediaries who did not sell their units in the previous period. The mass of new intermediaries who enter the Walrasian morning market, ΔV, is equal to the number of units supplied in this market (by the owners who are hit by either a moving or a preference shock) since this market clears in equilibrium. That is,

$$\Delta V = [\pi_{\xi_o} + \pi_\mu\ (1 - \pi_{\xi_o})]\ \psi_o(A). \tag{2.12}$$

The vacancy stock held by intermediaries at noon is then

$$V = \widetilde{V} + \Delta V. \tag{2.13}$$

These are the units which are up for sale in the afternoon:

$$V = \int_{\theta \in \mathbf{R}_+} ds = s(\Theta_J). \tag{2.14}$$

Those intermediaries who do not trade in the afternoon carry their inventories into the next period:

$$\widetilde{V}' = \int_{\theta \in \mathbf{R}_+} [1 - m_s(\theta)] \, ds. \tag{2.15}$$

We are now ready to define a stationary equilibrium.

***Definition* 1.** *A recursive stationary equilibrium for this economy, given the interest rate $R-1$, the rental housing price r_h and the probability distribution of the immigrants' asset holdings ζ_i, is a list of value functions and optimal decision policies for the households $\left\{W_o, W_r, W_n, W_b, g_o^c, g_o^a, g_r^c, g_r^a, g_b^\theta\right\}$, a value J and a set Θ^J of optimal decisions for intermediaries in the afternoon, prices $(\overline{p}, p(\cdot))$, Borel measures $\{\psi_o, \psi_r, \psi_n, \psi_b, \psi_i, b, s\}$, and positive real numbers $(V, \Delta V)$ such that:*

1. $\left\{W_o, W_r, W_n, W_b, g_o^c, g_o^a, g_r^c, g_r^a, g_b^\theta\right\}$ *solve the households' problems in* (2.1)–(2.4) *given* $(\overline{p}, p(\cdot))$.
2. *J and Θ_J solve the intermediary's problem in* (2.5) *given $p(\cdot)$, and the zero profit condition* (2.6) *holds.*
3. *All agents have rational beliefs about the tightness levels prevailing in active submarkets during the afternoon:*

   $$\int_{\theta \in \Xi} db = \int_{\theta \in \Xi} \theta \, ds, \qquad \text{for all Borel } \Xi \subset \mathbf{R}_+, \tag{2.16}$$

   where b is given by (2.11), *and* $\operatorname{supp} s \subset \Theta_J$.
4. *The Walrasian morning market clears: ΔV is given by* (2.12).
5. *The vacancy stock at the start each period is stationary: $\Delta V = \int_{\theta \in \mathbf{R}_+} m_s(\theta) \, ds$.*

6. *The total number of homes that are either owner-occupied or for sale in the afternoon is equal to the housing stock:* $V + \psi_n(A) = H$.

7. *The stationary probability measures* $\{\psi_o, \psi_r, \psi_n, \psi_b, \psi_i\}$ *satisfy equations* (2.7)–(2.10) *and the associated laws of motion.*

The only equilibrium condition that is not self-explanatory is (2.16). This rational expectations condition ensures that the measures of buyers and intermediaries in each active submarket are consistent with the tightness levels that agents take as given when they make their optimal afternoon decisions. Intuitively, ds represents the density of intermediaries and db represents the density of buyers in the set of active submarkets. If the traders' conjectures about θ are correct then db should be equal to $\theta\, ds$. Formally, (2.16) says that b is absolutely continuous with respect to s, with Radon-Nikodym derivative θ. The supports of b and s then coincide almost everywhere, and this common support gives the set of submarkets which are active in equilibrium.

3 Block-recursivity and equilibrium characterization

In this section, we show that the equilibrium price schedule $p(\cdot)$—an infinite-dimensional object which is used to calculate the households' value and policy functions—is pinned down by the value of $\overline{p}$. The equilibrium is block recursive because the problems solved by individual households do not depend on the distribution of households over financial assets. They only depend on the Walrasian price (a scalar). Given $\overline{p}$, households know the price schedule $p(\theta)$, which is all they need to know to make their optimal decisions. The households' asset distribution only affects their decisions through its effect on $\overline{p}$. This block-recursive structure arises because (1) search is competitive, and (2) intermediaries have linear transferable utility and can freely enter the economy.[15]

[15]Even without free entry, our arguments go through provided the mass of intermediaries who seek to buy homes in the Walrasian market in equilibrium is higher than the mass of sellers in that market, so intermediaries make zero expected profits. Yet the argument would break down if there where excess supply in this market. In this case, intermediaries would make a positive expected profit which would depend on the sellers' asset distribution.

3.1 Equilibrium price schedule

The intermediary's Bellman equation, (2.5), and the zero profit condition, (2.6), imply:

$$p(\theta) \leq \frac{\kappa_s + (1-\beta)\,\overline{p}}{m_s\,(\theta)} + \beta\,\overline{p},\ \text{for all } \theta \in \mathbf{R}_+,\ \text{with strict equality if } \theta \in \Theta_J. \tag{3.1}$$

In active submarkets, (3.1) then holds with equality, so $p(\theta)$ decreases with θ. Intuitively, since intermediaries get the same expected payoff J in all active submarkets and the probability $m_s\,(\theta)$ of completing a sale increases with θ, prices are lower in those submarkets where θ is higher. Prices in inactive submarkets imply a weakly lower expected payoff for intermediaries.

In fact, there is no loss of generality in assuming that equation (3.1) holds with equality *for all* $\theta \in \mathbf{R}_+$, so intermediaries get the same expected payoff in all submarkets, whether active or not. A standard feature of general equilibrium models with a continuum of commodities is that prices in inactive markets are indeterminate. Assuming that (3.1) holds with equality for all $\theta \in \mathbf{R}_+$ is equivalent to selecting the highest prices that support the equilibrium allocation.[16] With this selection rule, $p(\theta)$ is pinned down by $\overline{p}$. By the zero profit condition, $J = \overline{p}$, so $\overline{p}$ is the average return from a sale in the frictional market. As shown in Figure 1(a), $p(\theta)$ is strictly convex and $\mathcal{C}^2$ (since m_s is strictly concave and $\mathcal{C}^2$). It is also bounded below by $p_{min} \equiv \kappa_s + \overline{p}$ (the sum of the Walrasian price and cost of posting a vacancy in the frictional market). The lower bound p_{min} is the price intermediaries would charge if the probability of completing a sale was one (to break even). Since trade is subject to rationing, no intermediary would trade at a lower price.

3.2 Properties of the value functions

In Appendix B we show that, given the selected price function, the dynamic programming problems (2.1)–(2.4) admit continuous solutions W_o, W_r, W_n, and W_b which are unique in a suitable class of functions (under quite general conditions).[17] Also, W_o, W_r, and W_n are strictly increasing and W_b is non-decreasing. Whereas these functions need not be concave and differentiable in general, in Appendix C we show that they are differentiable along the optimal paths. This is all we need to establish the validity of the Euler equations used in our computation. We also show that, if we

[16] As discussed in Jerez (2014), this price selection rule is equivalent to the restriction typically imposed on out-of-equilibrium beliefs in directed search models, known as the market utility property.

[17] The method of proof is classical, and is based on a contraction mapping theorem. See Theorem 1.

restrict to the range of assets of the households who participate in the frictional market, W_o, W_r, and W_o are strictly concave and W_b is concave provided the renters' consumption policy function, $g_r^c(a)$, is non-decreasing on this range. The latter condition holds in all our numerical exercises. We exploit these results to characterize the equilibrium sorting pattern and establish the existence of a participation threshold in the next section.

3.3 The household's afternoon problem: Sorting and participation in the frictional market

The optimal decision rule of a buyer who participates in the afternoon market is

$$\begin{aligned} g_b^\theta(a) \in \quad & \underset{\theta \in \mathbf{R}_+}{\arg\max} \left\{ W_r(a) + m_b(\theta)\, \left[W_o\left(a - (1+\tau_b)\, p(\theta)\right) - W_r(a) \right] \right\} \\ \text{s. t.} \quad & a - (1+\tau_b)\, p(\theta) \geq -(1-\delta)\, \overline{p}. \end{aligned} \tag{3.2}$$

The buyer's ex-post gains from trading at price p are given by

$$S(a,p) = W_o\left(a - (1+\tau_b)\, p\right) - W_r(a)\,. \tag{3.3}$$

Buyers join the submarket $\theta \in \mathbf{R}_+$ where their expected gains are highest. Since $W_o(a)$, $m_b(\theta)$ and $p(\theta)$ are differentiable, $g_b^\theta(a)$ satisfies the first-order condition for problem (3.2):

$$m_b'(\theta)\, S(a, p(\theta)) - m_b(\theta)\, W_o'\left(a - (1+\tau_b)p(\theta)\right)\, (1+\tau_b)\, p'(\theta) = \lambda(a)\, (1+\tau_b)\, p'(\theta), \tag{3.4}$$

where $\lambda(a)$ is the Lagrange multiplier of the borrowing constraint. If this constraint is not binding, (3.4) simplifies to

$$\left(\frac{1}{1+\tau_b}\right)\left(\frac{1-\eta(\theta)}{\theta}\right)\left(\frac{S(a,p(\theta))}{W_o'\left(a-(1+\tau_b)\, p(\theta)\right)}\right) = -p'(\theta), \tag{3.5}$$

where $\eta(\theta)$ is the elasticity of $m_s(\theta)$. The left-hand side of (3.5) represents the buyer's marginal rate of substitution of θ for p. Equation (3.5) says that the buyer's optimal choice is characterized by a tangency between her indifference curve on the space (θ, p) and the equilibrium price function $p(\theta)$. The optimal choice of an unconstrained buyer then attains her highest indifference curve at those prices (see Figure 1(a)).[18]

If the borrowing constraint binds,

$$p\left(g_b^\theta(a)\right) = \frac{a + (1-\delta)\,\overline{p}}{(1+\tau_b)}. \tag{3.6}$$

Constrained buyers join the submarket where homes are sold at the maximum price they can afford to pay given the assets they have accumulated, the taxes involved in the transaction, and the borrowing limit (see Figure 1(b)). These buyers start the night with a negative asset position equal to $-(1-\delta)\,\overline{p}$. As one would expect, for constrained buyers the shadow price of the borrowing constraint, $\lambda(a)$, decreases with a (see Lemma 1 in Appendix D). There are then three possible cases. Either all buyers are unconstrained, they are all constrained, or the constraint only binds below a threshold. The last case is the relevant one in our numerical exercises, as we shall see.

For a given $\overline{p}$, the problem of a constrained buyer has a unique solution, characterized by (3.6). The same is true for unconstrained buyers provided $g_r^c(a)$ is non decreasing (and so W_o is concave) on the range of assets that correspond to participation. In this case, there is a single tangency point between the buyers' indifference curves and the price schedule $p(\theta)$, meaning that buyers with identical financial assets join the same submarket in equilibrium.[19]

***Proposition* 1.** *A solution for problem* (3.2) *exists. Moreover, if $g_r^c(a)$ is non decreasing on the range of a for which $\theta_0 \notin g_b^\theta(a)$ then $g_b^\theta(a)$ is single-valued on this range.*

We now show that buyers with different financial assets sort themselves out across submarkets with different prices and different trading probabilities. For constrained buyers, (3.6) implies that $p(g_b^\theta(a))$ increases and $g_b^\theta(a)$ decreases with a. Under the assumption in Proposition 1, this is also the case for unconstrained buyers provided the gains from trading at a given price p, $S(a,p)$, increase with a.[20] In this case, the buyers' indifference curves are steeper when a is higher (see Figure 1(c)). Intuitively, buyers with higher financial wealth are willing to accept a larger price

[18]Since the schedule $p(\theta)$ corresponds to the intermediaries zero isoprofit curve on the space (θ, p), the buyer's indifference curve is tangent to this isoprofit curve. This is the standard characterization of a competitive search equilibrium in the absence of borrowing constraints (e.g. Moen, 1997; Acemoglu and Shimer, 1999).

[19]One cannot conclude from (3.5) that the buyer's marginal rate of substitution increases along an indifference curve as θ rises (as depicted in Figure 1(a)), since $\eta(\theta)$ is non-increasing. Yet we may assume that traders choose m_b rather than θ, since there is a one-to-one mapping between both variables. Under the assumption in Proposition 1, the indifference curves of buyers and the zero isoprofit curve of intermediaries in the space (m_b, p) have a strictly convex shape, so they are tangent at most one point.

[20]This is the case in all our computations. It is direct to check that a sufficient condition for this requirement is that the purchase of a home always implies a lower night consumption level: $g_o^c\left(a - (1+\tau_b)p_{min}\right) < g_r^c(a)$.

increase to increase their trading probability (while remaining indifferent) relative to buyers with lower assets. This implies that in equilibrium wealthier buyers trade in more expensive submarkets where average buying times are shorter. This endogenous separation of different agent types across different submarkets is a typical property of directed search models.

***Proposition* 2.** *(Sorting by financial assets). For constrained buyers,* $g_b^\theta(a') < g_b^\theta(a)$ *if* $a < a'$*. If* $g_r^c(a)$ *is non decreasing the range of* a *for which* $\theta_0 \notin g_b^\theta(a)$ *and* $S(a, p)$ *increases with* a *for each* $p \geq p_{min}$ *then* $g_b^\theta(a)$ *is strictly decreasing on this range (whether or not buyers are constrained).*

Regarding the participation decision, under the assumptions in Proposition 2, there is a threshold $a_{part} \in A$ such that potential buyers with assets $a > a_{part}$ strictly prefer to participate, those with assets a_{part} are indifferent between participating or not, and the rest do not participate. Thus $W_b(a) > W_r(a)$ for all $a > a_{part}$, and $W_b(a) = W_r(a)$ for $a \leq a_{part}$. The price that buyers with assets a_{part} pay if they participate is the lowest price in the frictional market.

***Proposition* 3.** *(Participation) Suppose that* $W_b(a) > W_r(a)$ *for some* a*. Under the assumptions in Proposition 2, there exists* $a_{part} \in A$ *such that* $g_b^\theta(a) \in \mathbf{R}_+$ *if* $a > a_{part}$*,* $g_b^\theta(a) = \theta_0$ *if* $a < a_{part}$*, and* $g_b^\theta(a_{part}) = \{\theta_0, \overline{\theta}\}$*, where* $\overline{\theta}$ *is the tightness level in the cheapest active submarket.*

4 Computation and calibration

To illustrate the quantitative properties of the model, we calibrate it to reproduce selected statistics for the U.S. economy. We start by briefly discussing the computation method and our calibration strategy. We then present the main statistics of the benchmark economy, as well as some comparative statics results.

4.1 Computation

Before describing our computation method it is useful to compare our framework to related directed search models studying the interaction between wealth accumulation and frictional wage dispersion, such as Chaumont and Shi (2018) or Eeckhout and Sepahsalari (2018). These models assume, as we do here, that only one side of the market (workers in their setting and households in ours) is risk averse, whereas the other (firms seeking to fill job vacancies there and intermediaries here) has linear transferable utility and is free to enter the economy. Our sorting result parallels that in Chaumont and Shi (2018), whereby wealthier workers earn higher wages and take longer to switch employment states.[21] There are some important differences, though. First, these models assume that firms may create new vacancies at a fixed cost. The equivalent of this assumption in our setting would be to assume that intermediaries can build new homes each period at an exogenous cost. By free entry, the Walrasian morning price, $\overline{p}$, would be equal to this cost in equilibrium.[22] We will return to this issue in Section 4.4, where we show that movements in the Walrasian price are crucial for changes in aggregate (e.g. credit) conditions to affect the distribution of frictional prices in our calibrated model. A second difference is that, in our setting, households use financial assets not only to smooth (non-housing) consumption but also to build equity to pay the down payment. This introduces a not trivial participation margin, which implies that the household's problem has a non-concavity which is not present in the above models. Nevertheless, as shown in Section 3.2, the value functions have standard properties on the range of assets that corresponds to participation and non-participation, respectively.

The computation of the model is not straightforward. One possibility is to discretize the choice of financial assets and use value function iteration to solve the household's problem. This is the procedure used by Chaumont and Shi (2018), Eeckhout and Sepahsalari (2018) and Hedlund (2016b), for instance. The drawback of this approach is that, by discretizing the choice of financial assets, we would be limiting the number of submarkets that are active in equilibrium ex ante, which may bias our results on price dispersion (especially when we conduct policy exercises). We instead apply the results in Section 3.2 and compute the policy functions by solving the Euler equations.

Our computation uses the endogenous grid method (EGM hereafter). Specifically, we extend the procedure in Fella (2014) to our framework. Fella (2014) modifies the EGM to include a discrete control variable subject to exogenous non-convex adjustment costs, in addition to the standard continuous variable. His algorithm yields substantial gains in accuracy and computational time relative to standard techniques used to solve non-concave problems. In our model, households make a discrete choice (participating or not in the frictional market) and two continuum choices (the choice of submarket and savings). Instead of exogenous adjustment costs, home buyers face

[21]See also Eeckhout and Sepahsalari (2018) and Herkenhoff (2018). Menzio et al. (2013) obtain a similar sorting result in the context of a monetary search model.

[22]Transactions in labor and housing markets are fundamentally different, in particular, because households may participate in both sides of the housing market (both buying and selling houses). Also, interactions between workers and employers are inherently dynamic, while the housing transactions we describe are one-time transactions.

endogenous trading delays (as the choice of submarket determines the probability of buying). In Appendix E we describe in detail the algorithm we use to solve the household's problem as well as to find the stationary equilibrium.

4.2 Calibration

The model period is a month. We use the additively-separable felicity function

$$u(c,h) = \frac{c^{1-\sigma}}{1-\sigma} + \log(h). \tag{4.1}$$

Recall that matched owners consume $\hbar$ housing services, and renters consume h_r. As in Chaumont and Shi (2018), the matching technology is given by

$$m_s(\theta) = \left(1+\theta^{-\gamma}\right)^{\frac{-1}{\gamma}}, \quad m_b(\theta) = m_s(\theta)/\theta, \tag{4.2}$$

with $\gamma > 0$. We use this function instead of the standard urn-ball matching function because it has an extra degree of freedom in that γ governs the elasticity of $m_b(\theta)$ with respect to θ. With the urn-ball matching process, this elasticity is constant. The parameter γ also determines the severity of search and matching frictions. As γ increases, frictions are reduced. Since our computation method requires a one-to-one mapping between θ and m_b, we cannot use the standard (truncated) Cobb-Douglas matching function (which implies $m_b = 1$ for θ sufficiently low).

Table 1 shows the calibration of the benchmark economy. We interpret the endowment of the divisible good as labor income and set $w = 1000$. We calibrate the preference and mobility shocks to match the following observations on household turnover. According to the National Association of Realtors (NAR), the average tenure length for US homeowners is around ten years. As in Head et al. (2014), we have assumed that households move across locations and target the annual frequency of owners and renters moving across counties in the US, which is about 3.2 and 12 percent, respectively, according to the Census Bureau. The three targets combined are used to calibrate the probabilities of the three shocks, π_μ, π_{ξ_o}, and π_{ξ_r}. The risk aversion parameter is set equal to $\sigma = 2$, which is a standard value. Regarding the value of housing services, the ratio $h_r/\hbar$ is the key parameter that determines the homeownership rate in our model. We target this rate for working-age households in the Survey of Consumer Finances (SCF). The average homeownership rate for the 1989-2007 waves is 69.43 percent, and the implied value for $h_r/\hbar$ is 0.99. We set the discount factor to match the median wealth to earnings ratio for working-age renters observed in the data. The average of this statistic in the SCF is 0.34.

According to the NAR the median time to buy (TTB) is between 10 to 12 weeks. Thus, we set γ so that the median TTB in the steady state is 11 weeks. We choose the owner-occupied housing stock, H, to match the median housing wealth to earnings ratio for working-age owners in the SCF, which is 2.72.

The real return to the risk-free asset, R, is such that its implicit annual return is 3.91 percent, as calibrated in Díaz and Luengo-Prado (2010). The American Housing Survey reports that the median housing cost-to-income ratio for renters for the last 10 years is about 28 percent. This includes the cost of maintenance and utilities. Thus, we take r_h to be 25 percent of the monthly wage. We follow Díaz and Luengo-Prado (2008) and set the tax on home purchases, τ_b, to 2.5 percent, and the tax on sales, τ_s, to 6 percent. The cost of posting a vacancy, κ_s, is set to zero.

The down payment parameter, δ, is set to 0.5 to match the median loan-to-value ratio observed in the SCF for working-age households. The average of this statistic for the 1989–2007 waves is 41.04 percent. The reason for this calibration choice (instead of setting $\delta = 0.2$, which is a more typical number in the literature) is to have a meaningful tenure choice. Díaz and Luengo-Prado (2008) show that, in the absence of risk, all households make the same tenure choice regardless of their wealth; i.e., they all either rent or own, depending on the value of the user cost of owner occupied-housing relative to the rental price of housing. Here, households do face idiosyncratic risk but this risk is so small that they all participate in the afternoon market and end up owning a home for low values of δ. We will return to this issue in Section 4.4.

We have assumed that immigrants own no residential assets. Since we do not have a sensible way to calibrate the distribution of their financial assets, we assume that they all enter the location with zero assets.

4.3 The benchmark economy

Next, we discuss the properties of the household's policy functions, as well as some key aggregate and distributional implications of our benchmark.

4.3.1 Household's policy functions

Panel (a) of Figure 2 depicts the price paid by buyers with different financial assets, which is given by $g^p(a) \equiv p(g^\theta(a))$. Panel (b) depicts the associated trading probabilities. Potential buyers with financial assets below 1.40 times annual earnings do not participate in the afternoon market. We refer to a buyer with this threshold level of assets, a_{part}, as the *marginal buyer*. This buyer faces a binding borrowing constraint and directs her search to a submarket where the price of a house is 2.66 times her accumulated wealth. Upon completing a transaction, her mortgage amounts to 1.32 times her annual earnings, which is the collateralized borrowing limit, $(1-\delta)\,\overline{p}$. The probability that she buys a home is only 0.07 though. The borrowing constraint binds for any buyer with financial wealth below 1.46 times annual earnings. For constrained buyers, the probability of buying a home rises very rapidly with a. For instance, it reaches 0.35 for the threshold level of assets above which the borrowing constraint no longer binds. Any buyer whose wealth is greater than 2.87 times annual earnings buys a home upfront and does not get a mortgage loan. A buyer whose wealth is equal to this level trades with probability 0.49 and pays 2.78 times her annual earnings for the house.

Let us turn to the renters' savings decision. As shown in panel (c) of Figure 2 (red line), consumption falls with assets for poor renters (as they save to buy a home). Yet, as soon as they have accumulated enough assets to participate in the frictional market, consumption starts rising with assets. For instance, a renter with zero assets has to wait for 7.58 years to become a marginal buyer. If she does not trade (the most likely event for a marginal buyer), she will increase her savings further and will direct her search to a submarket with a higher price in the following period, where the probability of trading is higher. This probability rises slowly with a for high wealth levels though, as shown in panel (b) of Figure 2. This and the fact that households discount the future imply that the savings policy function of a renter is smooth and standard for levels of financial assets above a_{part}. This function has a fixed point: renters whose financial assets are 1.47 times their annual earnings consume their income and role over their wealth. Any renter with more assets depletes them as the probability of buying is not large enough to compensate for lower non-housing consumption. The renter at the fixed point (whose assets are 1.47 times her annual earnings) directs her search to the submarket where the price is 2.72 times her annual earnings and gets a mortgage below her borrowing limit. This also implies that, at a steady state, the support of the financial wealth distribution of renters is the interval $[0, 1.47 \times 12\,w]$. The support is so compressed because the probability of buying, m_b, rises very slowly with a for high wealth levels.

For instance, the largest point of the grid is equal to 20 times annual earnings. A renter so rich directs her search to a submarket where the price is 3 times her annual earnings and m_b is 0.69. The elasticity of $m_b(\theta)$ is key to determine the support of the financial wealth distribution of renters and, therefore, that of the price distribution in the frictional market.

Consider now the owners' savings decisions. We do not show their savings policy function because it is very smooth and concave, as owners are hit by a mismatch or a moving shock only every 10 years on average. Since the model period is a month, their behavior is very similar to the case in which they face no idiosyncratic risk at all. The fixed point of the policy function corresponds to a value of a equal to -1.04 times their annual earnings. Hence, in equilibrium all owners hold debt. Yet their net worth—the value of all assets minus liabilities—is positive, for the following reason. Take a home buyer who exhausts the borrowing limit and becomes an owner with a mortgage equal to $(1-\delta)\,\overline{p}$. In equilibrium, the Walrasian price equals 2.65 times annual earnings, and recall that $\delta = 0.5$. Valuing her home at this price, the owner's net worth is equal to $\delta\,\overline{p}$, which amounts to 1.32 times her earnings. Yet, if she sells her home, she will have to pay $\tau_s\,\overline{p}$ in taxes. Her liquid wealth is then $(\delta - \tau_s)\,\overline{p}$, which is 1.16 times her earnings. An owner whose assets equal -1.04 times her earnings (the fixed point of the policy function) holds a net worth, $a + \overline{p}$, equal to 1.60 times her earnings, whereas her liquid wealth is 28.09 percent of her earnings.

To sum up, in this economy renters hold positive financial assets and all owners hold debt. Renters accumulate assets to finance a down payment. This is so because the probability that owners are hit by a shock is low and so is the interest rate of mortgages, $R-1$ (given the discount factor β). In terms of net worth, owners' wealth is much more concentrated than that of renters (who only hold financial wealth).

4.3.2 Aggregate implications

Table 2 shows some selected targeted statistics of the benchmark economy. The homeownership rate is 69.69 percent, a bit higher than in the data. The median of the ratio of housing wealth to annual earnings for working-age owners in the data is 2.70. We take as the model counterpart for median housing wealth the median price paid by home buyers; the ratio of this median price to annual earnings equals 2.72. The median loan-to-value ratio in the model (and in the data) is calculated in the following way. We take the (working age) owners who hold negative financial assets (mortgages in the data). We then calculate the median of their financial assets and divide them by the median housing wealth. This statistic is 41.04 percent in the data and 38.62 percent in the model. The median wealth-to-earnings ratio for renters is 0.35 in the data, and 0.29 in the model, which is a bit low. But recall that we are assuming away any income risk that would result in precautionary savings. The median TTB is 11.55 weeks in the frictional market, which is within the range of about 10–12 weeks reported by NAR.[23]

Next, we analyze the implications of the model for some key non-targeted statistics. Take the rent-to-price ratio, a typical index used to measure the return to housing. According to Sommer and Sullivan (2018), in the data, this ratio is between 8 and 15 percent. To calculate the equivalent statistic in the model we need to take a stand on which is the reasonable statistic for house prices. On the one hand, houses are sold at price $\overline{p}$ in the Walrasian market. On the other hand, we have the cross-sectional house price distribution in the frictional market. Note that, regardless of its purchasing price, the liquidation value of a house is $\overline{p}$. This is why we report the rent-to-price ratio as the annual rent divided by $\overline{p}$. This ratio is 9.43 percent in the steady state, which is within the range reported by Sommer and Sullivan (2018).

Consider now average time on the market (TOM), a typical index of housing market liquidity. Recall that the probability that a house is sold during the afternoon and thus average TOM varies across submarkets. Also, intermediaries who do not sell their units can always choose to join a different submarket in the next period. All these intermediaries face the same (ex-ante) expected probability of selling in the next period. Therefore, we can calculate the expected TOM associated with the decision to join a particular submarket. The median of the distribution of this variable is 10.19 weeks, which is within the range of 4 to 17 weeks reported by NAR. Another index of market liquidity is months supply–the ratio of vacancies over sales in a given month (e.g. Hedlund, 2016b). In the data, the average of this ratio was 5.47 in 2017 according to NAR, whereas in our model it is 2.62. Finally, the model's vacancy rate matches that in the data, although we have not used this statistic as a target.In sum, the model does a fairly good job in matching aggregate features of the housing market and household's portfolio in spite of its simplicity.

[23]For each potential buyer with assets $a \geq a_{part}$, we calculate TTB taking into account the fact that, if the buyer does not trade this period, she will have more assets in the next period and will then direct her search to a submarket with a higher trading probability.

4.3.3 The distribution of prices and wealth

In this subsection, we analyze the distributional implications of the model. As explained in Section 4.3.1, the equilibrium price distribution in the afternoon market is very compressed. The associated standard deviation is around 1 percent (see Table 2). This is mainly because agents face a tiny amount of idiosyncratic risk. In particular, there are no differences in labor earnings across households, which would propagate to the wealth distribution (in the absence of complete markets) and, from the latter to the price distribution.

Many empirical studies suggest that frictional dispersion in housing prices is significant, but we do not have many estimates of it. For instance, Lisi and Iacobini (2013) estimate a hedonic pricing model using Canadian data controlling for residual price dispersion (i.e., not explained by hedonic housing attributes). They find that the mean of the prediction error falls from 16.50 to 14.15 percent when taking into account residual dispersion, whereas its standard deviation falls from 14.62 to 12.37 percent. We thus take the difference from 16.50 to 14.15 as an estimate of the mean prediction error due to frictional price dispersion; this number is 2.35 percent. Likewise, we take as an estimate of the standard deviation of the prediction error the difference 14.62 − 12.47, which is 2.25. The mean prediction error in our benchmark economy is 0.41 percent (see Table 2), which accounts for 36.12 percent of the estimate by Lisi and Iacobini (2013). The coefficient of variation of prices is 0.51 percent, which is about 42 percent of the dispersion not explained by observables in their paper.[24]

Another source of information on frictional price dispersion comes from Zillow, the online US real estate database. Zillow's methodology is applied to homogeneous sets of homes in a given geographical segment and combines information about physical attributes of the home and the land, among other things.[25] As reported on their website, Zillow's accuracy has a median error rate of 1.9 percent at the national level. Median errors are also reported at the county and MPA level, with substantial variation across segments for which enough data are available. For instance, for top metropolitan areas, median errors lie between 1.2 and 3.6 percent. They report that about 16.3 percent of the transaction prices are our of the 5 percent interval of their estimate, called

[24] Lisi and Iacobini (2012) apply the same methodology to a rich Italian dataset which includes information about both buyers and sellers. They find that the standard error that can be attributed to frictional dispersion is about 4.56 percent for a sample of selected Italian cities.

[25] Guerrieri et al. (2013) use data from Zillow to study the link between within-city migration, different house price dynamics across neighborhoods and gentrification.

Zestimate. Thus, if a home's sale price is different from the Zestimate, we can attribute part of the difference to the presence of search and matching frictions in the housing market.

To construct a model counterpart for this estimate, we calculate the average price in the afternoon market, which is 2 percent higher than the Walrasian price. Since the distribution of prices is very compressed, all house prices lie within a 5 percent range of the average price. Yet 5.69 percent lie out of the 1 percent range. The fact that we cannot match the magnitude of Zillow's error is consistent with the widely held view that unobserved heterogeneity must explain part of the residual dispersion in housing prices. Yet our comparable statistic is about the same order of magnitude, which leads us to conjecture that frictional dispersion may be a substantial part of the overall residual dispersion.

Recall that all immigrants enter the economy with zero assets. Because of sorting, poorer buyers trade with a lower probability and accumulate more assets to access a submarket where they are more likely to trade. This behavior and the turnover due to the preference and mobility shocks generate a non-degenerate distribution of financial assets. The Gini coefficient for renters' wealth is 0.53, which is pretty high; Díaz and Luengo-Prado (2010) report a coefficient of 0.89 in the 1998 SCF. Valuing homes at price $\overline{p}$, the Gini of wealth for the total population is 0.21. This is significant for a setting with such a tiny amount of uncertainty if we consider that the typical Gini coefficient for household wealth in the US is about 0.8 (see Díaz and Luengo-Prado, 2010).

It will be useful to compare our results with those of Eerola and Maattanen (2018) at this point. This should be done with caution since their environment is quite different. Specifically, their model features two-sided risk aversion as buyers and sellers trade directly with each other, as well as additional idiosyncratic income risk. Also, since search is random, all traders meet an agent on the other side of the market with the same exogenous probability, but not all matches lead to trade.[26] Average TTB and TOM thus depend on the fraction of successful matches. Whether a match leads to trade or not depends on the traders' asset positions, and so do transaction prices in successful matches. Wealthier buyers are more likely to trade and pay higher prices on average because they have higher reservation prices than poorer buyers. In turn, poorer sellers who are close to being borrowing constrained are more likely to trade than richer sellers, but at relatively low prices because of their lower reservation prices. The coefficient of variation of prices in our benchmark is twice the level reported in Eerola and Maattanen (2018). Yet we do not know how

[26]By contrast, in our model matching probabilities are endogenous and all matches lead to trade.

much of this difference is due to random search and how much is due to two-sided risk aversion.

4.4 Comparative statics

We now discuss the steady-state effects of changes in some key parameters. In doing so, it is instructive to compare our economy with an alternative one where the housing stock is allowed to respond to market conditions. The alternative economy is constructed in the following way. We fix $\overline{p}$ at its stationary value in our model and assume that intermediaries who enter the economy can build a house at cost $\overline{p}$ before the decentralized market opens. We refer to this as the *construction* economy. As discussed in Section 4.1, this economy parallels directed search models of the labor market where new vacancies are created at an exogenous cost. This cost in turn determines the relationship between tightness levels and equilibrium prices (the equivalent of our price schedule, $p(\theta)$). The difference is that here the cost, $\overline{p}$, is endogenous.

4.4.1 Changes in credit conditions

A reduction in the down payment eases credit constraints for two reasons. First, it reduces the amount of equity needed to participate in the frictional market. Second, it increases the liquidity of residential assets allowing owners to smooth non-housing consumption against their housing collateral. Note that, since there is no capital in our setup, the size of the economy is not affected.

Table 3 describes the effects of a 10 percent reduction (increase) in δ, from 0.5 to 0.45 percent (0.55 percent, respectively). This produces about a 10 percent change of the opposite sign in the Walrasian price (see columns two and five), which in turn shifts the equilibrium price schedule, $p(\theta)$. Take the case of a down payment reduction. Figure 3(a) plots the prices paid by the buyers who participate in the frictional market as a function of their assets. First, note that the function shifts upwards (from the benchmark blue curve marked as $\delta = 50\%$ to the red one marked as $\delta = 45\%$) so, for a given level of financial assets, buyers now pay higher prices. Second, its domain widens as poorer potential buyers now participate (a_{part} falls). Third, its shape changes, becoming more concave for high levels of assets. This is so because the probability of buying is an increasing concave function of the price and the households' marginal utility is decreasing, which implies that price increases are smaller for wealthier buyers. The three effects, combined with the endogenous change in the wealth distribution, determine the Walrasian price as well as the mean and variance of prices in the frictional market in the new steady state.

Figure 3(b) shows the average TTB as a function of the buyers' assets. This function also shifts (from the blue to the red curve). Specifically, there is an upward shift for high asset levels, so wealthier buyers face a slightly longer TTB (relative to the benchmark). That is, the fact that more buyers participate and all buyers can afford to pay higher prices results in an increase in congestion in more expensive submarkets.

The overall effect of a 10 percent reduction in δ is a 9.81 percent increase in the Walrasian price, a rise in the median TTB from 11.56 to 12.05 weeks, and a fall in price dispersion. Table 3 includes two different measures of price dispersion. The first is the percentage of transaction prices out of the 1 percent range of the average price, which falls a bit from 5.69 to 5.15 percent. The second one is the price range–the ratio of the highest to the lowest price. This statistic falls slightly. Price dispersion falls because of the aforementioned change in the shape of the price function in Figure 3(a) and also because the renters' wealth distribution changes. As we can see, the Gini coefficient of renters' wealth falls a bit from 0.53 to 0.52. The Gini coefficient for owners' wealth, however, does not change. While all owners hold more debt (the median loan-to-value ratio rises from 38.69 to 43.73 percent), their housing wealth is higher too (the median housing wealth to earnings ratio increases from 2.71 to 2.96), compensating their higher indebtedness. Finally, the median TOM and the vacancy rate fall, so the afternoon market becomes more liquid. The homeownership rate rises slightly from 69.69 to 69.75. This is consistent with the fact that the participation rate rises moderately and buying times increase.

The case of an increase in the down payment is symmetric in terms of its aggregate effects (see Table 3). The Walrasian price falls by 10% and the homeownership rate falls slightly. The asymmetries show up in Figures 3(a) and 3(b). When δ rises, the function which gives prices that buyers with different assets pay shifts down (as buyers pay lower prices) and its support narrows (due to a fall in participation). The function also becomes steeper. This and the fact that the Gini coefficient of renters' wealth rises results in an increase in price dispersion. The market becomes less liquid, with more vacancies, higher median TOM, and lower buying times.

To have a sense of the importance of assuming a fixed housing stock, we also report the corresponding effects in the *construction economy* in columns 3 and 6 of Table 3. There, a 10 percent decrease in δ does not affect the Walrasian price (by assumption), but there is a large effect on distributional statistics. The homeownership rate is also very sensitive to credit conditions. This rate rises from 69.42 to 76.08 percent, as housing becomes more affordable (the rent-to-price ratio increases). Yet neither the median TTB nor the median TOM change. This is because the infinitely elastic supply of housing is effectively undoing the congestion effect brought about by the increase in housing demand. The Gini coefficient of renters' wealth falls significantly relative to the benchmark. Consider now to the case where δ rises to 55%. In this case, no renter wants to buy a house anymore, since they do not want to sacrifice non-housing consumption to build the high down payment. Hence, there are no homeowners in the new steady state and the Gini coefficient of renters' wealth is zero since renters do not face any risk. This exercise shows in a stark way that the congestion buyers face in the frictional market combined with an inelastic housing supply endogenously generate a significant amount of wealth heterogeneity.

4.4.2 The role of search and matching frictions

In the previous discussion, we have taken as given the matching technology. The importance of the elasticity of the probability of buying with respect to the market tightness in determining the range of equilibrium prices was emphasized in Section 4.3.1. As noted there, buyers accumulate more assets because this allows them to avoid tighter submarkets by paying a higher price. In this section, we investigate the importance of this channel by increasing the value of γ from 0.65 to 1.5. This higher value implies a very large elasticity of m_b with respect to θ (on the relevant range). It also implies that search frictions are reduced relative to the benchmark.

The main statistics of the new steady state are shown in column 9 of Table 4 (labeled as '$\gamma = 1.5$, fixed H'). A first key difference relative to the benchmark is that market liquidity is higher; both the median TTB and TOM are now lower. The Walrasian price rises by 12 percent and the participation rate falls from 7.29 to 4.33 percent due to the increase in prices. Yet the percentage of buyers who trade in equilibrium doubles its counterpart in our benchmark. This is so buyers trade with a higher probability. The increase in the probability of buying is particularly important for poor buyers, who become owners at a higher rate than in the benchmark. Thus, the average wealth of potential buyers rises, pushing prices up and reducing participation. In particular, renters never accumulate financial assets above 1.67 of their annual wage. This level of assets is so low that, even if these agents get a mortgage, they cannot afford a home in the afternoon. Since any renter who is hit by the moving shock is replaced by a migrant who enters the economy with zero assets, the only households who participate in the frictional market are the homeowners who were hit by a preference shock in the morning. The wealthiest owner holds negative financial wealth but, when she is hit by the shock, her house is liquidated at the Walrasian price and her total financial wealth is high enough to buy a new home. These forces imply that the range of prices in the frictional market is negligible (implying that the deviation of all prices from the average price is less than 1 percent) and the average price rises by almost 12 percent relative to the benchmark. The high elasticity of m_b with respect to θ is the key factor underlying these results. Nevertheless, the effect of increasing γ on the price level is not monotone (as we show below). For sufficiently high γ, the reduction in search and matching frictions implies that the price level goes down relative to the benchmark.

The matching function in (4.2) converges to the efficient matching function as $\gamma \to \infty$; i.e., $\lim_{\gamma\to\infty} m_s(\theta) = \min\{\theta, 1\}$ and $\lim_{\gamma\to\infty} m_b(\theta) = \min\{1, \theta^{-1}\}$. This allows us to compare our setting to one where search frictions are absent. For γ large (say $\gamma = 100$), the matching function is a good approximation to the efficient one.[27] In this (almost) frictionless economy, a single submarket is active in equilibrium where all homes are sold at the Walrasian price, so there is no price dispersion. Buyers still face trading delays, since houses are indivisible objects and there are more buyers than intermediaries (homes for sale) in the afternoon market. By contrast, intermediaries trade with probability one and they appropriate all the gains from a bilateral transaction (the buyers' gains being negligible). In this economy, the probability of buying a home does not rise with its price. Instead, all buyers pay the same price and trade with identical probability. Therefore, renters do not have any incentive to accumulate assets above the threshold that allows them to participate in the afternoon market—our amplification mechanism is shut down. The main statistics of this economy are shown in the last column of Table 2 (labeled as 'Walrasian economy'). Note that the Walrasian price is less than one fourth of its counterpart in our benchmark. While intermediaries sell their homes instantaneously in the afternoon (houses are fully liquid assets), buyers face very long buying times (68.8 weeks). The homeownership rate is equal to the per capita housing stock, 70.6447 percent. There are no vacancies overnight since the homes of all the owners who became mismatched in the morning are sold in the afternoon by intermediaries. Even though there is no price dispersion in this economy, wealth is nor equally distributed. This is so because renters still have the incentive to save to participate in the housing market. Our computations show that

[27] We do not study the limiting economy because the matching function is not smooth (so our proofs do not apply).

changes in the down payment have a negligible impact on the Walrasian price in this economy, whereas the effects on the average TTB are substantial.

4.4.3 Other comparative static exercises

To give a more complete picture, we also report the steady-state effects of a permanent change in the wage, the interest rate, transaction taxes, and the rental housing price in Table 4. The effect of a 5 percent increase in the wage is reported in the third column. The first thing that stands out is the 9.43 percent increase in the Walrasian price. The effect on the rest of the variables is very similar to that of a down payment reduction. This may be surprising since (unlike a change δ) a change in the wage does affect the size of the economy. The reason why the effect is similar is that ours is a partial equilibrium economy: we are ignoring the general equilibrium effects on production of the non-housing good and the interest rate. There is a difference, though, as there is a negligible effect on the median loan-to-value ratio (which depends directly on the credit constraint). Also, a permanent change in the wage has, in relative terms, a larger effect on prices. Our model then suggests that increases in aggregate productivity, combined with looser credit constraints, should have a significant impact on housing prices when the housing supply is inelastic.

It is worth mentioning that changes in transaction taxes—especially taxes on home purchases—have also a sizable effect on the loan-to-value ratio. Finally, a reduction in the real interest rate produces a small reduction in the Walrasian price as well as a corresponding increase in the home-ownership rate. The effect on the price seems counterintuitive, but recall that agents do not face any income risk. Thus, if the interest rate falls, they change the composition of consumption, increasing their non-housing consumption. As a result, they save less and housing prices fall.

4.4.4 Discussion

In the expansion period prior to the Great Recession required down payments fell from about 20 to 1 percent, whereas the price-to-rent ratio increased between 30 and 49 percent, depending on the price index used; see Favilukis et al. (2017). Thus, a 95 percent fall in the down payment was accompanied by a 40 percent increase in the price-to-rent ratio. In our benchmark, a 10 percent reduction in the down payment produces about a 9 percent change in the price-to-rent ratio. These numbers may suggest that our mechanism over-amplifies the effects of changes in credit constraints, but we should bear in mind that there is neither labor income risk and nor construction in our benchmark. It is still useful to compare our results with those in Eerola and Maattanen (2018), who also assume a fixed housing stock but do allow for idiosyncratic income risk. They find that a 10 percent reduction in the down payment from 95 to 85 percent brings about a rise of 6 percent in the average housing price. Yet the homeownership rate is kept fixed in their analysis, so one should think of this number as an upper bound.

Standard heterogeneous agents models of housing assume Walrasian markets for real estate. Kiyotaki et al. (2011) and Sommer et al. (2013) find that changes in financial conditions have negligible effects on housing prices in these models. This is so despite the fact that Sommer et al. (2013) also assume a fixed housing stock. These authors find that the homeownership rate varies significantly, though. The reason for these results is that easing financial conditions affects only constrained agents, which represent a small fraction of the households in the economy. Favilukis et al. (2017) stress the importance of housing risk affecting all agents regardless of their wealth, along with sufficiently high heterogeneity, for financial conditions to have a sizable aggregate effect on housing prices. They find that in a model economy with these ingredients, a down payment reduction from 25 to 1 percent produces a 20 percent increase in the price-to-rent ratio. In Favilukis et al. (2017) housing risk is a byproduct of aggregate risk. Their main insight is that the combination of housing risk and high heterogeneity produces a significant number of "constrained" agents, which is key for changes in financial conditions to affect prices.

Housing risk is indeed an essential part of our environment: buyers purchase a home with a certain probability and all homeowners face a probability of realizing capital losses. Since search is directed, agents can affect the amount of risk they face through their savings and search decisions. In particular, the severity of search and matching frictions affects the households saving decisions and thus the wealth distribution. Conversely, as we have seen, there is also a feedback effect from the wealth distribution to the frictions agents face in the housing market. The key point is that, when given additional credit, all home buyers direct their search towards a submarket with a higher price, even if they do not face a binding credit constraint. This is why changes in credit constraints have such a large impact on the average housing price in our benchmark. This mechanism operates through the inherent heterogeneity of the economy which search and matching frictions generate. This is clear when we look at the exercises where no agent wants to buy a home; when all households are renters, they all hold the same level of wealth.

Our amplification mechanism critically relies on the inelasticity of the housing supply. Our results are consistent with Favara and Imbs (2015), who are able to identify a credit supply shock using US county and bank branches data for the period 1994-2005, and find that the response of prices to a credit shock depends on the response of the housing stock. There are many studies estimating housing supply elasticities. For instance, Green et al. (2005) or Gyourko et al. (2013) show that these estimates vary widely across cities depending on the degree of land abundance and particular regulation in each area.[28] Moreover, they find that the cities which experienced a higher price boom were those with a low elasticity of housing supply and high-income growth.

5 Final comments

The message of this paper is that changes in credit conditions and other demand shocks can be highly amplified in the presence of search and matching frictions. In our setting, the interaction between credit constraints and search frictions is particularly important for the response of housing prices to a loosening of credit constraints. Our analysis suggests the effects of changes in credit conditions –and, in particular, the effects on housing prices and market liquidity and the distributional effects– critically depend on whether or not the housing stock can adjust to those changes.

We have made some strong simplifying assumptions to bring to light the underlying amplification mechanism. The most important omission is the absence of idiosyncratic income risk. Also, we have assumed that homeowners can sell their property in a Walrasian housing market (where there is neither price dispersion nor trading delays). This eliminates part of the congestion that search and matching frictions create, which is why owners hold so little wealth in our model. Hedlund (2016a 2016b) argues that this margin is important to understand how the joint interaction between tighter credit standards and decreasing housing market liquidity affect housing markets during a slowdown (see also Head et al. (2019)). Finally, we have studied two extreme cases: an economy with a fixed housing stock and one with an infinitely elastic housing supply. Both economies deliver strikingly different effects of changes in credit constraints and other shocks. The key question is what the empirically relevant elasticity is.

We have also abstracted away from differences in real estate properties. In a variant of the

[28] Housing supply restrictions are key to understand housing markets (e.g. Davis and Heathcote, 2005) and the increase in the overall housing price dispersion in the US, in particular (e.g. Nieuwerburgh and Weill, 2010; Gyourko et al., 2013; Albouy and Zabek, 2016).

model with heterogenous houses, say of different quality or size, wealthier buyers would seek to buy higher quality homes. Our conjecture is that buyers seeking to buy homes of a given type will still sort by their level of wealth (so our mechanism should still be at work within each market segment). Introducing a life cycle component and the possibility of a property ladder would be particularly interesting. Ortalo-Magné and Rady (2006) find that these elements may lead to amplified effects of relaxing credit constraints in a model with a Walrasian housing market.[29]

Finally, we have also focused our attention on steady states. Studying the transitional dynamics of our model is not trivial, for the following reason. The key state variable in our benchmark is the Walrasian price. Out of the steady state this price, which is pinned down by the intermediary's zero profit condition, depends not only on current economic conditions but also on expectations about future conditions. If intermediaries expect higher future prices, the current Walrasian price will increase, shifting the distribution of prices paid by home buyers upwards and to the right. Thus, expectations should play a key role out of the steady state. In the construction economy we have studied —which parallels the labor models of Menzio and Shi (2010) and Chaumont and Shi (2018)— this effect is absent because, in equilibrium, the Walrasian price equals the fixed cost of building a house.[30] In any case, the problem amounts to finding a sequence of prices, which is much easier than finding sequences of higher dimensional objects. We leave all these interesting extensions for future work.

[29]Another interesting question is how binding leverage constraints are throughout the wealth distribution as there are various programs that effectively relax down payment constraints for low income households, first-time home buyers,... Wealthier households who want to buy better properties are limited in terms of the leverage they can have, though.

[30]This effect is also absent in Hedlund (2016a), where the key state variable is the price which clears the Walrasian market where intermediaries trade with homebuilders (see footnote 5). This price depends only on current conditions since intermediaries are not allowed to carry inventories to the next period.

A Law of motion of the distribution of households

Let $\mathcal{A}$ denote the Borel σ-algebra on A. Define the transition function $Q_o : A \times \mathcal{A} \to [0,1]$ which gives the probability that an owner holding $\tilde{a} \in A$ assets at night will carry assets $a \in X \in \mathcal{A}$ into the next morning. Likewise, Q_r denotes the corresponding transition function for renters. That is, $Q_j(a, X) = \psi_j(\{a \in A : g_j^a(a) \in X\})$ for $j \in \{o, r\}$.

We use primes to denote the corresponding measures in the next period. The laws of motions from the night to the following afternoon are

$$\psi_n'(X) = (1-\pi_\mu)(1-\pi_{\xi_o}) \int_{a\in A} Q_o(a, X) d\psi_o, \tag{A.1}$$

$$\psi_b'(X) = (1-\pi_{\xi_r}) \int_{a\in A} Q_r(a, X) d\psi_r + \pi_\mu (1-\pi_{\xi_o}) \int_{a\in A} Q_o(a, X) d\psi_o + \psi_i(X), \tag{A.2}$$

for each $X \in \mathcal{A}$. Similarly, the laws of motion from the afternoon to the night are

$$\psi_o'(X) = \psi_n(X) + \int_{a\in A} \Pi_o(a, X)\, d\psi_b, \tag{A.3}$$

$$\psi_r'(X) = \int_{a\in A} \Pi_r(a, X)\, d\psi_b, \tag{A.4}$$

where the transition functions $\Pi_o : A \times \mathcal{A} \to [0,1]$ and $\Pi_r : A \times \mathcal{A} \to [0,1]$ give the probability that a potential buyer holding a assets at the start of the afternoon will be an owner or a renter with assets in X at night, respectively. These probabilities are related to the probability that the buyer purchases a home in the afternoon, which depends on the submarket θ she joins. A successful trade implies, not only a change in tenure status, but also a change in the financial assets (which again depends on θ). Specifically,

$$\Pi_o(a, X) = \begin{cases} m_b\left(g_b^\theta(a)\right), & \text{if } a - (1+\tau_b)\, p\left(g_b^\theta(a)\right) \in X, \\ 0, & \text{otherwise,} \end{cases}$$

$$\Pi_r(a, X) = \begin{cases} 1 - m_b\left(g_b^\theta(a)\right), & \text{if } a \in X, \\ 0, & \text{otherwise.} \end{cases}$$

B Properties of the value functions

Let a denote the household's assets in a given subperiod (either night or afternoon). Recall that $A = [\underline{a}, \infty)$ and let $C(A)$ be the space of continuous functions $f : A \to \mathbf{R}$, and $E = C(A) \times C(A)$.

Define the Bellman operator T on E by $T = (T_o, T_r)$, where

$$
\begin{aligned}
T_o(f_o, f_r)(a) = \quad & \max_{c,a'} \Big\{ u\,(c, \hbar) + \beta\ (1-\pi_{\xi_o})\,(1-\pi_\mu)\ f_o\,(a') \\
& \qquad\qquad + \beta\,[1-(1-\pi_{\xi_o})\,(1-\pi_\mu)]\ T_b(f_o, f_r)\,(a' + (1-\tau_s)\,\overline{p}) \Big\} \\
\text{s.t.} \quad & c + \tfrac{1}{R}\,a' \leq w + a, \\
& a' \geq -(1-\delta)\,\overline{p}, \\
& c \geq 0,
\end{aligned}
\tag{B.1}
$$

$$
\begin{aligned}
T_r(f_o, f_r)(a) = \quad & \max_{c,a'} \Big\{ u\,(c, h_r) + \beta\,T_b(f_o, f_r)\,(a') \Big\} \\
\text{s.t.} \quad & c + \tfrac{1}{R}\,a' \leq w - r_h + a, \\
& a' \geq 0, \\
& c \geq 0,
\end{aligned}
\tag{B.2}
$$

and where $T_b(f_o, f_r)$ is defined by

$$
T_b(f_o, f_r)(a) = \max\left\{ \max_{\theta \in D(a)} \Big\{ m_b\,(\theta)\ f_o\,(a - (1+\tau_b)\,p(\theta)) + (1 - m_b\,(\theta))\ f_r\,(a) \Big\}, f_r(a) \right\}. \tag{B.3}
$$

The feasible correspondence D of the inner maximization problem in (B.3) is defined by

$$
D(a) = \{\theta \in \mathbf{R}_+ \,:\, a - (1+\tau_b)p(\theta) + (1-\delta)\overline{p} \geq 0\} \quad \text{for } a \in A. \tag{B.4}
$$

If $D(a) = \emptyset$, we attach the value $-\infty$ to participation, and thus $T_b(f_o, f_r)(a) = f_r(a)$ in this case. Also, since

$$
p(\theta) = \frac{\kappa_s + (1-\beta)\overline{p}}{m_s\,(\theta)} + \beta\overline{p} \quad \text{for all } \theta \in \mathbf{R}_+, \tag{B.5}
$$

$\lim_{\theta\to\infty} p(\theta) = p_{min}$. Since p is decreasing, $D(a) \neq \emptyset$ if and only if $a > (1+\tau_b)p_{min} - (1-\delta)\overline{p} \geq 0$. Since p is continuous in $\mathbf{R}_{++}$, D has closed sections. However, $D(a)$ is not compact. To circumvent this problem and be able to apply Bergé's Maximum Theorem, we assume that agents choose m_b rather than θ. Let

$$
\hat{p}(m_b) = \frac{\kappa_s + (1-\beta)\,\overline{p}}{\hat{m}_s\,(m_b)} + \beta\,\overline{p} \quad \text{for } m_b \in (0,1), \tag{B.6}
$$

and $\hat{p}(0) = p_{min}$. The function $\hat{p}$ is continuous in $[0,1)$, since it is the composition of two continuous functions when $0 < m_b < 1$, and, for $m_b = 0$, $\lim_{m_b\to 0^+} \hat{p}(m_b) = \lim_{\theta\to\infty} p(\theta) = p_{min}$. Also, since $\hat{m}_s$ is strictly decreasing and $-\hat{m}_s{}'/\hat{m}_s$ is non decreasing, $\hat{p}$ is strictly increasing and strictly convex. Finally, $\lim_{m_b\to 1^-} \hat{p}(m_b) = \lim_{\theta\to 0^+} p(\theta) = \infty$. By choosing m_b as the new decision variable, the feasible correspondence D becomes $\overline{D}$, as defined by

$$
\overline{D}(a) = \{m_b \in [0,1) \,:\, a - (1+\tau_b)\,\hat{p}(m_b) + (1-\delta)\,\overline{p} \geq 0\}. \tag{B.7}
$$

The sections of $\overline{D}$ are nonempty and compact for $a - (1+\tau_b)\,\hat{p}(m_b) + (1-\delta)\,\overline{p} > 0$. In fact, when nonempty, $\overline{D}(a)$ is the bounded and closed interval $\left[0, \hat{p}^{-1}\left(\frac{a+(1-\delta)\,\overline{p}}{1+\tau_b}\right)\right]$. Problem (B.3) thus transforms into

$$T_b(f_o, f_r)(a) = \max\left\{ \max_{m_b \in \overline{D}(a)} \left\{ m_b\, f_o\left(a - (1+\tau_b)\,\hat{p}(m_b)\right) + (1-m_b)\ f_r\left(a\right) \right\}, f_r(a) \right\}. \quad \text{(B.8)}$$

In Theorem 1 below, we assume that a positive level of consumption is always possible for both owners and renters. Since an initial wealth of $a = \underline{a}$ is admissible at the initial state, a positive consumption in the first period for owners and renters implies $w + \underline{a} + \frac{(1-\delta)\overline{p}}{R} > 0$ and $w - r_h + \underline{a} > 0$, respectively. In particular, the first inequality implies $w - \left(1 - \frac{1}{R}\right)(1-\delta)\overline{p} > 0$, which means that the owner can sustain a strictly positive level of consumption at the borrowing limit. Another consequence of the above inequalities is that $n_o = u\left(w + \underline{a} + \frac{(1-\delta)\,\overline{p}}{R}, \hbar\right) > -\infty$ and $n_1 = u(w - r_h + \underline{a}, h_r) > -\infty$. This follows because the utility function $u(c,h)$ is finite if $c > 0$, for $h \in \{\hbar, h_r\}$. On the other hand, utilities may be unbounded from above. Hence, we need to control for their rate of growth on the feasible correspondence, as well as for the size of β, to guarantee that the dynamic programming equations define a contraction operator. Consider the number sequence $\{a_0, a_1, \ldots, a_j, \ldots\}$, where

$$a_j = \left(\frac{Rw}{R-1} + \underline{a}\right) R^j - \frac{Rw}{R-1}, \quad j = 0, 1, 2, \ldots. \quad \text{(B.9)}$$

Note that $\underline{a} \leq a_j \leq a_{j+1}$, $a_j \to \infty$ as $j \to \infty$, and $a_0 = \underline{a}$. Let

$$u_j^o = \max_{a \in [\underline{a}, a_j]} \left| u\left(w + a + \frac{(1-\delta)\overline{p}}{R}, \hbar\right) \right|,$$
$$u_j^r = \max_{a \in [\underline{a}, a_j]} \left| u\left(w - r_h + a, h_r\right) \right|,$$

and $u_j = \max\{u_j^o, u_j^r\}$. Note that both u_j^o and u_j^r are well defined because $n = \min\{n_o, n_r\} > -\infty$. Define

$$v_j := \sum_{i=j}^{\infty} \beta^{i-j} u_j, \quad \text{for } j = 0, 1, 2, \ldots. \quad \text{(B.10)}$$

The following theorem establishes the existence of a unique solution to the Bellman equation in a suitable class of functions. The result covers both the bounded and unbounded from below cases.

Theorem 1. *Suppose that $n > -\infty$ and that*

$$\lim_{j \to \infty} \frac{u_{j+1}}{u_j} := \overline{u} < \frac{1}{\beta}. \quad \text{(B.11)}$$

Then, the dynamic programming equations (B.1), (B.2) *and* (B.3) *admit unique continuous solutions W_o, W_r and W_b, respectively, in the class of functions F defined by*

$$F = \left\{ f \in C(A) \,:\, f(a) \geq \frac{n}{1-\beta},\ \text{for all } a \in A,\ \max_{a \in [\underline{a}, a_j]} f(a) \leq v_j,\ \text{for all } j = 0, 1, \ldots \right\}. \quad \text{(B.12)}$$

Moreover, both W_o and W_r are strictly increasing and W_b is non decreasing.

Proof. Let $(f_o, f_r) \in E$. If $a \leq (1+\tau_b)\,p_{min} - (1-\delta)\overline{p}$ then the agent's optimal choice is θ_0, and so $T_b(f_o, f_r)(a) = f_r(a)$, which is continuous. When $a > (1+\tau_b)\, p_{min} - (1-\delta)\,\overline{p}$, the function $(m_b, a) \mapsto m_b\, f_o\,(a - (1+\tau_b)\,\hat{p}(m_b)) + (1-m_b)\; f_r\,(a)$ is continuous on $(a, m_b) \in A \times [0,1)$ and the correspondence $\overline{D}$ defined in (B.7) is nonempty valued, compact valued, and continuous. Hence, by the Theorem of the Maximum, the value function

$$\max_{m_b \in \overline{D}(a)} \left\{ m_b\, f_o\,(a - (1+\tau_b)\,\hat{p}(m_b)) + (1-m_b)\; f_r\,(a) \right\} \tag{B.13}$$

is continuous. Since $T_b(f_o, f_r)$ is defined as the maximum between this value function and f_r, it is also continuous. It follows that the functions defining the right hand side of $T_o(f_o, f_r)$ and $T_r(f_o, f_r)$ given in (B.1) and (B.2), respectively, are continuous. Moreover, the feasible correspondence is nonempty valued, continuous and compact valued in both cases. Hence, by the Theorem of the Maximum, both $T_o(f_o, f_r)$ and $T_r(f_o, f_r)$ are continuous. Let us see that $T_i(F \times F) \subseteq F$, for $i = o, r, b$. Let $(f_o, f_r) \in F \times F$. By the definition of T_b as the maximum of a convex combination of f_o and f_r, it is clear that $T_b(f_o, f_r) \geq \frac{n}{1-\beta}$. Also,

$$T_o(f_o, f_r)(a) \geq \max_{c, a'} u(c, \hbar) + \beta \frac{n}{1-\beta} \geq u\left(w + \underline{a} + \frac{(1-\delta)\overline{p}}{R}, \hbar\right) + \beta \frac{n}{1-\beta} \geq n + \beta \frac{n}{1-\beta} = \frac{n}{1-\beta}, \tag{B.14}$$

and

$$T_r(f_o, f_r)(a) \geq \max_{c, a'} u(c, \hbar) + \beta \frac{n}{1-\beta} \geq u(w - r_h + \underline{a}, h_r) + \beta \frac{n}{1-\beta} \geq n + \beta \frac{n}{1-\beta} = \frac{n}{1-\beta}. \tag{B.15}$$

On the other hand,

$$T_b(f_o, f_r)(a) \leq m_b\, f_o\,(a - (1+\tau_b)\,\hat{p}(m_b)) + (1-m_b)\; f_r\,(a) \leq m_b v_j + (1-m_b) v_j = v_j \tag{B.16}$$

and $T_b(f_o, f_r)(a) \leq f_r(a) \leq v_j$, for all $a \in [\underline{a}, a_j]$, for all $j = 0, 1, \ldots$. Hence, given that for any $a \in [\underline{a}, a_j]$, $\overline{D}(a) \subseteq [\underline{a}, a_{j+1}]$ by the definition of v_j, we have

$$T_o(f_o, f_r)(a) \leq u_j + \beta v_{j+1} = v_j, \quad \text{for all } a \in [\underline{a}, a_j]. \tag{B.17}$$

By a similar computation, $T_o(f_o, f_r)(a) \leq v_j$ for all $a \in [\underline{a}, a_j]$. It thus follows that $T_i(F \times F) \subseteq F$, for all $i = o, r, b$. Consider now $C(A)$ with the topology generated by the countable family of seminorms $\|f\|_j = \max_{a \in [\underline{a}, a_j]} |f(a)|$, for all $j = 0, 1, \ldots$. This family is separated ($\|f\|_j = 0$ for all j implies that f is the null function). Since the compact intervals $[\underline{a}, a_j]$ form an increasing family that covers A and they have nonempty interiors, the space $C(A)$ is complete with this topology (see Rincón-Zapatero and Rodríguez-Palmero, 2003). Consider the product space $E = F \times F$ with the seminorms $\|(f_o, f_r)\|_j = \max\{\|f_o\|_j, \|f_r\|_j\}$, for $j = 0, 1, \ldots$ and $(f_o, f_r) \in E$. It is clear that E is complete with this topology, and that the set E is closed. Consider the series $\sum_{j=0}^{\infty} c^{-j} u_j$, with $c > \overline{u}$, where $\overline{u}$ was defined in (B.11). By the ratio test and by (B.11)

$$\lim_{j \to \infty} \frac{c^{-(j+1)} u_{j+1}}{c^{-j} u_j} = \frac{\overline{u}}{c} < 1, \tag{B.18}$$

so the series converges. Moreover, since $\beta\,\overline{u} < 1$, it is possible to choose $c > \overline{u}$ with $\beta\, c < 1$. Following Theorem 4 in Rincón-Zapatero and Rodríguez-Palmero (2003), $T = (T_o, T_r)$ is a local contraction on $F \times F$, so T admits a unique fixed point in $F \times F$, that is, there are unique $W_o \in F$,

$W_r \in F$ such that $T_o(W_o, W_r) = W_o$ and $T_r(W_o, W_r) = W_r$. Also, $T_b(W_o, W_r) = W_b$ is the buyer's value function.

To prove that W_o and W_r are strictly increasing, let $a_1 < a_2$. Then $\overline{D}(a_1) \subseteq \overline{D}(a_2)$, since $\hat{p}$, as the composition of two decreasing functions, is increasing. Let $(f_o, f_r) \in E \times E$, where both f_o and f_r are non decreasing. Then $m_b\, f_o\,(a - (1+\tau_b)\,\hat{p}(m_b)) + (1-m_b)\; f_r\,(a)$ is non decreasing in a, since $0 \leq m_b < 1$. Hence,

$$\begin{aligned}
&\max_{m_b \in \overline{D}(a_1)} \left\{ m_b\, f_o\,(a_1 - (1+\tau_b)\,\hat{p}(m_b)) + (1-m_b)\; f_r\,(a_1) \right\} \\
&\qquad \leq \max_{m_b \in \overline{D}(a_1)} \left\{ m_b\, f_o\,(a_2 - (1+\tau_b)\,\hat{p}(m_b)) + (1-m_b)\; f_r\,(a_2) \right\} \\
&\qquad \leq \max_{m_b \in \overline{D}(a_2)} \left\{ m_b\, f_o\,(a_2 - (1+\tau_b)\,\hat{p}(m_b)) + (1-m_b)\; f_r\,(a_2) \right\}.
\end{aligned}$$

It follows that $T_b(f_o, f_r)$ is continuous and, being the maximum of two non decreasing functions, it is also non decreasing. Plugging this result into the definitions of T_o and T_r, we get, by the same reasoning, that both $T_b(f_o, f_r)$ and $T_r(f_o, f_r)$ are non decreasing, since the feasible correspondence of both problems is increasing in a. Actually, both $T_b(f_o, f_r)$ and $T_r(f_o, f_r)$ are strictly increasing, since the utility function $u(w + a - a'/R, h)$ is increasing with respect to a, for $h \in \{h_r, \hbar\}$. Finally, the subset of F of non decreasing functions is closed in F, so the fixed points W_o, W_r and W_b are non decreasing. However, in the case of W_o and W_r, they are increasing by the previous argument, as they satisfy $T_o(W_o, W_r) = W_o$ and $T_b(W_o, W_r) = W_b$, respectively. □

It is direct to show that the general theorem above applies, among others, to the utility functions used in the calibration of the model.

Corollary 2. *The conclusions of Theorem 1 hold under the same hypotheses, in the following cases.*

1. $u(c,h) = \dfrac{c^{1-\sigma}}{1-\sigma} + v(h)$, *with* $\sigma > 1$.
2. $u(c,h) = \dfrac{c^{1-\sigma}}{1-\sigma} + v(h)$, *with* $\sigma \leq 1$ *and* $R^{1-\sigma}\beta < 1$,

where $v(h) < v(\hbar)$. *Note that* $\sigma = 1$ *corresponds to* $u(c,h) = \log(c) + v(h)$.

Proof. We only need to show that (B.11) holds. Note that $u(\cdot, h)$ is increasing in cases 1 and 2. When $\sigma > 1$, u is negative and bounded. The sequence $\{u_j\}$, being increasing and bounded, then converges and $\overline{u} = 1 < \frac{1}{\beta}$. When $\sigma < 1$, u is positive but unbounded. In fact,

$$u_j^o = u\left(w + a_j + \frac{(1-\delta)\overline{p}}{R}, \hbar\right). \tag{B.19}$$

Given the definition of a_j, it is direct to see that

$$\lim_{j\to\infty} \frac{u_{j+1}^o}{u_j^o} = \lim_{j\to\infty} \frac{\phi\left(w + a_{j+1} + \frac{(1-\delta)\overline{p}}{R}\right)^{1-\sigma} + v(\hbar)}{\phi\left(w + a_j + \frac{(1-\delta)\overline{p}}{R}\right)^{1-\sigma} + v(\hbar)} = R^{1-\sigma}. \tag{B.20}$$

In the logarithmic case, where $\sigma = 1$, u_j^o is bounded by $\left|\log\left(w + a_j + \frac{(1-\delta)\overline{p}}{R}\right)\right| + |v(\hbar)|$ for large enough j. The ratio

$$\frac{|\log\left(w + a_{j+1} + \frac{(1-\delta)\overline{p}}{R}\right)| + |v(\hbar)|}{|\log\left(w + a_j + \frac{(1-\delta)\overline{p}}{R}\right)| + |v(\hbar)} \tag{B.21}$$

tends to 1 as $j \to \infty$, so (B.11) is satisfied. A similar computation holds for u_j^r. □

C Differentiability, Euler equations and concavity

In this section we prove the differentiability of the value functions along the optimal paths. This suffices to obtain the Euler equations; differentiability in the entire domain is not required. Other approaches to prove differentiability of the value function in a non-concave framework are due to Dechert and Nishimura (1983), Milgrom and Segal (2002), or Clausen and Strub (2016)), but do not apply to our setting (for the same reasons they do not apply to the model of Menzio et al. (2013)). Thanks to the results that we introduce in this section, we do not need to introduce lotteries but work directly within the non concave framework. We show that the Euler equations still hold as necessary conditions of optimality, so they can be used to compute the optimal policies. We are also able to establish a link between the concavity of the value functions and the monotonicity of the optimal consumption policies. Our results are based on the approach recently introduced in Rincón-Zapatero (2020). However, this approach does no apply directly to the Bellman equations satisfied by W_o, W_r and W_b, due to their particular structure, so we need to elaborate a bit more.

We introduce the concepts of Fréchet super– and subdifferentials of a function (F-superdifferential and F-subdifferential, henceforth) to simplify the presentation and the proofs that follow. For a continuous function $f : \Omega \subseteq \mathbf{R}^n \to \mathbf{R}$, where Ω is an open set, the vector $p \in \mathbf{R}^n$ belongs to the F-superdifferential of f at $x_0 \in \Omega$, $D^+f(x_0)$, if and only if there exists a continuous function $\varphi : \Omega \longrightarrow \mathbf{R}$ which is differentiable at x_0 with $D\varphi(x_0) = p$, $f(x_0) = \varphi(x_0)$ and $f - \varphi$ has a local maximum at x_0. Similarly, $p \in \mathbf{R}^n$ belongs to the F-subdifferential of f at $x_0 \in \Omega$, $D^-f(x_0)$, if and only if there exists a continuous function $\varphi : \Omega \longrightarrow \mathbf{R}$ which is differentiable at x_0 with $D\varphi(x_0) = p$, $f(x_0) = \varphi(x_0)$ and $f - \varphi$ has a local minimum at x_0. $D^+f(x_0)$ and $D^-f(x_0)$ are closed convex (and possible empty) subsets of $\mathbf{R}^n$. Yet, if f is differentiable at x_0, then both $D^+f(x_0)$ and $D^-f(x_0)$ are nonempty and $D^+f(x_0) = D^-f(x_0) = \{Df(x_0)\}$. Reciprocally, if for a function f, both $D^+f(x_0)$ and $D^-f(x_0)$ are nonempty, then f is differentiable at x_0 and $D^+f(x_0) = D^-f(x_0) = \{Df(x_0)\}$, where Df denotes the derivative of f. Given two continuous functions f_1 and f_2, two nonnegative numbers λ_1 and λ_2 and $p_i \in D^+f_i(x)$, for $i = 1, 2$, $\lambda_1 p_1 + \lambda_2 p_2 \in D^+(\lambda_1 f_1 + \lambda_2 f_2)(a)$. A similar proposition holds for D^-. Another property that we will use is that, whenever x_0 is a local maximum of f in Ω, $0 \in D^+f(x_0)$. Finally, $D^+f(x_0) \neq \emptyset$ if the function f is concave. See, for instance, Bardi and Capuzzo-Dolcetta (1997) for these and for other properties of the F-super– and subdifferentials of a function.

The next theorem characterizes the F-differentials of the value function $f(x) = \max_{y \in \Gamma(x)} F(x, y)$, where $F : X \times Y \to \mathbf{R}$ is continuous, with $X, Y \subseteq \mathbf{R}^n$, and where Γ is a correspondence from X to Y is nonempty, compact valued and continuous. The result is well known in the case in which the correspondence Γ is constant (i.e., when $\Gamma(x) = Y$ for all $x \in X$), but for the general case it is a generalization of the Benveniste-Scheinkman envelope argument. We will apply the theorem to show the validity of the Euler equations in our model, which is a non-trivial issue due to the lack of concavity.

Theorem 3. *Consider the problem described above, $f(x) = \max_{y\in\Gamma(x)} F(x,y)$. Let x_0 be an interior point of X and $y_0 \in \Gamma(x_0)$ satisfying: (i) $f(x_0) = F(x_0, y_0)$, and (ii) there is a ball $B(x_0, \varepsilon)$ in X with center x_0 and radius $\varepsilon > 0$, such that for all $x \in B(x_0, \varepsilon)$, $y_0 \in \Gamma(x)$. Then $D^-_x F(x_0, y_0) \subseteq D^- f(x_0)$ and $D^+ f(x_0) \subseteq D^+_x F(x_0, y_0)$, where $D^\pm_x F(x_0, y_0)$ denotes the F–upper/lower differential of the function $x \mapsto F(x, y_0)$.*

Proof. By Bergé's Theorem, f is continuous and the optimal policy correspondence is nonempty. Assumptions (i) and (ii) ensure that the function $x \mapsto f(x) - F(x, y_0)$ is well defined on the ball $B(x_0, \varepsilon)$ and attains a local minimum at x_0. If $D^-_x F(x_0, y_0)$ is empty, then there is nothing to prove. Suppose that it is nonempty. Let φ be continuous in $B(x_0, \varepsilon)$ and differentiable at x_0 such that $F(x, y_0) - \varphi(x)$ has a local minimum at x_0 and $F(x_0, y_0) = \varphi(x_0)$. Then $f(x) - \varphi(x) \geq F(x, y_0) - \varphi(x) \geq 0$ and $f(x_0) - \varphi(x_0) = F(x_0, y_0) - \varphi(x_0) = 0$ by (i). Thus x_0 is a local minimum of $f - \varphi$, and so $D\varphi(x_0) \in D^- f(x_0)$. Now, if $D^+ f(x_0) = \emptyset$ then $D^+ f(x_0) \subseteq D^+_x F(x_0, y_0)$, trivially. If $D^+ f(x_0) \neq \emptyset$, let φ be continuous in $B(x_0, \varepsilon)$ such that $D\varphi(x_0) \in D^+ f(x_0)$ and $f - \varphi$ has a local maximum at x_0, with $(f - \varphi)(x_0) = 0$. Then $F(x, y_0) - \varphi(x) \leq f(x) - \varphi(x) \leq 0 = F(x_0, y_0) - \varphi(x_0)$, for all $x \in B(x_0, \varepsilon)$. Hence, x_0 is a maximum of $x \mapsto F(x, y_0) - \varphi(x)$, and so $D\varphi(x_0) \in D^+_x F(x_0, y_0)$. □

Remark 4. *Note that (ii) is satisfied when (x_0, y_0) is an interior point of the graph of Γ, although it may be fulfilled more generally, as we will show in our housing model. On the other hand, $D^-_x F(x_0, y_0) \neq \emptyset$ implies $D^-_x f(x_0) \neq \emptyset$. Hence, if f is concave then f is differentiable. This is the classical envelope theorem of dynamic programming.*

After this preliminary exposition, we turn to our specific problem, given by (B.1)–(B.3). In the results that follow, we will assume that there are selections of g^a_o, g^a_r and g^θ_b such that g^a_o and g^a_r are interior, and

$$0 \leq g^\theta(a) < p^{-1}\left(\frac{a}{1+\tau_b}\right), \tag{C.1}$$

for all $a \in A$. Hence, we do not assume uniqueness of the optimal policies.

Define $a_{min} = (1+\tau_b)\, p_{min} - (1-\delta)\, \overline{p}$. This is the threshold value of a above which $D(a)$, as defined in (B.4), is nonempty. Denote by $a_{part} > a_{min}$ the maximum $a > a_{min}$ such that $g^\theta(a) = \theta_0$ for $a_{min} < a \leq a_{part}$ (if it exists).

Our strategy for proving that the value functions are differentiable at the optimal policies consists on showing that both the F-subdifferential and the F-superdifferential are nonempty along the optimal paths. This is the content of the results that follow. As a byproduct, we prove that the Euler equations hold. We use this result in our computation (see Section E.3).

Lemma 5. *Let $a_0 > \underline{a}$. Then (i) $u_c(g^c_o(a_0), \hbar) \in D^- W_o(a_0)$, and (ii) $u_c(g^c_r(a_0), h_r) \in D^- W_r(a_0)$.*

Proof. We only prove (i), since the proof of (ii) is similar. W_o satisfies the Bellman equation (B.1). Since $g^a_o(a_0)$ is interior, given that the feasible correspondence is a closed interval, there is an open interval I centered at a_0, such that $g^a_o(a) \in (-(1-\delta)\overline{p}, R(w+a))$ for all $a \in I$. Thus (i) and (ii) in Theorem 3 hold. Moreover, taking $\alpha = (1-\pi_{\xi_o})\,(1-\pi_\mu)$, the function

$$F(a, g^a_o(a_0)) = u\left(w + a - g^a_o(a_0)/R, \hbar\right) + \beta\, \alpha\, W_o\left(g^a_o(a_0)\right) + \beta\,(1-\alpha)\; W_b\left(g^a_o(a_0) + \overline{p}\right),$$

is differentiable with respect to a, with derivative $u_c(g_o^c(a_0), \hbar)$ at $a = a_0$, as the second and third summands in the definition of F are constant. Theorem 3 then implies $u_c(g_o^c(a_0), \hbar) \in D^- W_o(a_0)$. □

To explore whether $D^- W_b$ is nonempty, we rewrite the problem of a potential buyer in an equivalent form. Let

$$W(a, m_b) = \begin{cases} W_r(a), & \text{if } a \leq a_{\min},\ m_b \in [0,1], \\ m_b \left(W_o(a - (1+\tau_b)\hat{p}(m_b)) - W_r(a)\right) + W_r(a), & \text{if } a > a_{\min},\ m_b \in \overline{D}(a), \end{cases} \tag{C.2}$$

where $\overline{D}(a)$ is defined in (B.7). Let $\widetilde{D}(a) = \{0\}$ for $a \leq a_{\min}$, and $\widetilde{D}(a) = \overline{D}(a)$ for $a > a_{\min}$. The correspondence $\widetilde{D}$ is nonempty, compact valued and continuous. Formally, we are identifying the choice θ_0 in the original problem with $m_b = 0$. Given this, it is clear that the original problem is equivalent to the new formulation: $\max W(a, m_b)$ subject to $m_b \in \widetilde{D}(a)$. Note that W is piecewise continuous and, when restricted to the graph of $\widetilde{D}$, it is continuous. For, if $(a_n, (m_b)_n)$ is a pair of sequences converging to $(a_{\min}, m_b)$ along the graph of $\widetilde{D}$, where $m_b \in [0,1]$, then for $a_n > a_{\min}$, $(m_b)_n = \widehat{p}^{-1}(a_n) \to \widehat{p}^{-1}(a_{\min}) = 0$, and for $a_n < a_{\min}$, $(m_b)_n = 0$. Hence,

$$W(a_n, (m_b)_n) \to 0(W_o(0) - W_r(a_{\min})) + W_r(a_{\min}) = W_r(a_{\min}) = W(a_{\min}, 0), \tag{C.3}$$

as $n \to \infty$. Since $m_b = 0$ is feasible for any a and $g_b^\theta(a) = 0$ in the region $a \leq a_{part}$ (if a_{part} exists), $W_b(a) = W_r(a)$ in this region.

Lemma 6. *Let $a_0 > \underline{a}$. Then $D^- W_b(a_0) = D^- W_r(a_0)$, for $a_0 \leq a_{part}$, and*

$$m_b\left(g_b^\theta(a_0)\right) p_o + \left(1 - m_b\left(g_b^\theta(a_0)\right)\right) p_r \in D^- W_b(a_0), \tag{C.4}$$

for $a_0 > a_{part}$, where $p_o = u_c\left(g_o^c(a_0 - (1+\tau_b)\, p(g_b^\theta(a_0))), \hbar\right)$ and $p_r = u_c(g_r^c(a_0), h_r)$.

Proof. For $\underline{a} < a < a_{part}$, $W_b(a) = W_r(a)$, so (i) is trivial. At $a = a_{part}$, $m_b = 0$ is the optimal choice (it is the only feasible choice, given our reformulation of the problem). Although not interior to the graph of $\widetilde{D}(a)$, this choice satisfies condition (ii) in Lemma 3, that is, $0 \in \widetilde{D}(a)$ in a neighborhood of a_{part} (for all a, actually). Hence, $D^- W_b(a_{part}) \neq \emptyset$. Let $a_0 > a_{part}$. Since g_b^θ is interior, the optimal $g^{m_b}(a_0)$ is interior. Thus the function of a

$$F(a, g^{m_b}(a_0)) = g^{m_b}(a_0) W_o(a - (1+\tau_b)\hat{p}(g^{m_b}(a_0))) + (1 - g^{m_b}(a_0)) W_r(a)) \tag{C.5}$$

is well defined in a suitable interval centered at a_0. Moreover, $D_a^- F(a_0, g^{m_b}(a_0)) \neq \emptyset$. To see this, take $p_o \in D^- W_o\left(a_0 - (1+\tau_b)\,\hat{p}(g^{m_b}(a_0))\right)$ and $p_r \in D^- W_r(a_0)$, which exist by Lemma 5. By one of the properties mentioned just above Theorem 3, $g^{m_b}(a_0)\, p_o + (1 - g^{m_b}(a_0))\, p_r \in D_a^- F(a_0, g^{m_b}(a_0))$, or, equivalently,

$$m_b\left(g_b^\theta(a_0)\right) p_o + \left(1 - m_b\left(g_b^\theta(a_0)\right)\right) p_r \in D_a^- F(a_0, g_b^\theta(a_0)), \tag{C.6}$$

with p_o and p_r as described in the statement of the lemma. Since $D_a^- F(a_0, g_b^\theta(a_0)) \subseteq D^- W_b(a_0)$ by Theorem 3, the result in the lemma holds. □

The fact that the F-subdifferential of the value function is nonempty is not enough to get differentiability, since the value functions need not be concave. Below we follow the path initiated in Rincón-Zapatero (2020) to prove differentiability in the absence of concavity, which uses the optimality condition and the special structure of the Bellman equation. This will provide us with conditions for the nonemptiness of the F-superdifferential of the value functions at the optimal policies.

Lemma 7. *Let $a_0 > \underline{a}$. Then $u_c(g_r^c(a_0), h_r) \in R\beta\, D^+ W_b(g_r^a(a_0))$.*

Proof. Consider the Bellman equation (B.2) and the function of a' given by

$$F(a_0, a') := u(w - r_h + a_0 - a'/R, h_r) + \beta W_b(a'). \tag{C.7}$$

Since $g_r^a(a_0)$ is an interior optimum to the Bellman equation (B.2), $0 \in D^+_{a'} F(a_0, g_r^a(a_0))$. But, since u is of class C^1, $D^+_{a'} F = \{-u_c/R\} + \beta D^+ W_b$. Hence, $-u_c(g_r^c(a_0), h_r) \in R\beta D^+ W_b(g_r^a(a_0))$. □

Our next result shows that W_b is differentiable at the renter's optimal policy, and establishes the validity of the renter's Euler equation.

Proposition 8. *Let $a > \underline{a}$. Then*
(i) W_b is differentiable at $g_r^a(a)$;
(ii) the Euler equation

$$\begin{aligned} &- u_c(g_r^c(a), h_r) \\ &\quad + R\beta\Big[m_b\left(g_b^\theta(a')\right) u_c\left(g_o^c(a' - (1+\tau_b)\, p(g_b^\theta(a'))), \hbar\right) + \left(1 - m_b\left(g_b^\theta(a')\right)\right) u_c(g_r^c(a'), h_r)\Big] = 0 \end{aligned}$$

holds, where $a' = g_r^a(a)$.

Proof. By Lemma 6 and Lemma 7, both the F-super– and the F-subdifferential of W_b are nonempty at $g_r^a(a)$. Hence, W_b is differentiable at $g_r^a(a)$. The derivative is, on the one hand, the unique element in $D^- W_b(g_r^a(a))$, that is, $W_b'(g_r^a(a)) = \frac{1}{R\beta} u_c(g_r^c(a), h_r)$ and, on the other hand, the unique element in $D^+ W_b(g_r^a(a))$, that is

$$W_b'(g_r^a(a)) = u_c(g_r^c(a'), h_r), \tag{C.8}$$

for $g_r^a(a) \le a_{part}$, and

$$W_b'(g_r^a(a)) = m_b\left(g_b^\theta(a')\right)\Big[u_c\left(g_o^c(a' - (1+\tau_b)\, p(g_b^\theta(a'))), \hbar\right) - u_c(g_r^c(a'), h_r)\Big] + u_c(g_r^c(a'), h_r), \tag{C.9}$$

for $g_r^a(a) > a_{part}$, where $a' = g_r^a(a)$ in both (C.8) and (C.9). Since $m_b\left(g_b^\theta(a_{part})\right) = 0$, (C.9) encompasses (C.8). Equating $\frac{1}{R\beta} u_c(g_r^c(a), h_r)$ to (C.9), we obtain the renter's Euler equation. □

Proposition 9. *Let $a > a_{part}$. Then*
(i) W_o is differentiable at $g_o^a(a)$;
(ii) the Euler equation

$$\begin{aligned} -u_c(g_o^c(a), \hbar) + R\beta\alpha\, u_c(g_r^c(a'), \hbar) + R\beta(1-\alpha)\, m_b\left(g_b^\theta(a')\right) u_c\left(g_o^c(a' - (1+\tau_b)\, p(g_b^\theta(a'))), \hbar\right) \\ + R\beta(1-\alpha)\,(1 - m_b\left(g_b^\theta(a')\right)) u_c(g_r^c(a'), h_r) = 0, \end{aligned}$$

holds, where $a' = g_o^a(a) + \overline{p}$ *and* $\alpha = (1 - \pi_{\xi_o})(1 - \pi_\mu)$.

Proof. From (B.1), the function of a'

$$F(a, a') = u(w + a - a'/R, \hbar) + \beta\,\alpha\, W_o\,(a') + \beta(1 - \alpha)\, W_b\,(a' + \overline{p}) \tag{C.10}$$

satisfies $0 \in D^+_{a'} F(a, g_o^a(a))$. Since both u and W_b are differentiable,

$$-u_c(g_o^c(a), \hbar) + R\beta\,(1 - \alpha)\, W_b'(g_o^a(a) + \overline{p}) \in -R\beta\,\alpha D^+ W_o(g_o^a(a)),$$

so $D^+ W_o$ is nonempty at $g_o^a(a)$. This, combined with Lemma 5, implies that W_o is differentiable at $g_o^a(a_0)$. Also, its derivative at this point is given, on the one hand, by $u_c(g_o^c(a), \hbar)$, and, on the other hand, by $\frac{1}{R\beta\alpha} u_c(g_o^c(g_o^a(a)), \hbar) - \frac{(1-\alpha)}{\alpha} W_b'(g_o^a(a) + \overline{p})$. Equating both expressions, and replacing $W_b'(g_r^a(a))$ by its value in (C.9), we obtain the Euler equation in (ii). □

Now we study concavity. Concavity of the value functions is proved in intervals where the optimal consumption policy of the renters is nondecreasing.

Proposition 10. *W_b is concave in intervals I of the image of g_r^a if and only if g_r^c is nondecreasing in the preimage of this subset, $(g_r^a)^{-1}(I)$.*

Proof. Note that W_b is differentiable in I by Proposition 9. Also, if $a' \in I$, there is $a > \underline{a}$ such that $a' = g_r^a(a)$ and $W_b'(a') = u_c(g_r^c(\theta), h_r)/R$ by Lemma 5. Let $a_i' \in I$ and let $a_i > \underline{a}$ such that $a_i' = g_r^a(a_i)$, for $i = 1, 2$. Without loss of generality, suppose that $a_1' > a_2'$. By the Mean Value Theorem,

$$W_b(a_1') - W_b(a_2') = W_b'(\theta')(a_1' - a_2') = \frac{1}{R} u_c(g_r^c(\theta), h_r)(a_1' - a_2'), \tag{C.11}$$

where $a_2' < \theta' < a_1'$ and where $\theta' = g_r^a(\theta)$. Since g_r^a is nondecreasing, $a_2 < \theta < a_1$, and since g_r^c is non decreasing, $g_r^c(a_2) \leq g_r^c(\theta) \leq g_r^c(a_1)$. Now, $u(\cdot, h_r)$ is concave, so $u_c(g_r^c(\theta), h_r) \leq u_c(g_r^c(a_2), h_r) = RW_b'(a_2')$. Hence,

$$W_b(a_1') - W_b(a_2') \leq W_b'(a_2')(a_1' - a_2'), \tag{C.12}$$

and so W_b is concave in C. Obviously, the reasoning above is reversible. □

Proposition 11. *Let I be an interval of A such that $g_r^a(I)$ is an interval. Then W_r is strictly concave in I if and only if g_r^c is nondecreasing in I.*

Proof. Let $a_1, a_2 \in I$ and let $\lambda_1, \lambda_2 \in [0, 1]$. Since $g_r^a(I)$ is convex, $\lambda_1 a_1 + \lambda_2 a_2 \in I$, $\lambda_1 g_r^a(a_1) + \lambda_2 g_r^a(a_2) \in g_r^a(I)$. Also, $(\lambda_1 a_1 + \lambda_2 a_2, \lambda_1 g_r^a(a_1) + \lambda_2 g_r^a(a_2))$ belongs to the graph of the buyer's feasible correspondence, since it is convex. Moreover, W_b is concave in $g_r^a(I)$ by Proposition 10. Then

$$\begin{aligned} W_r(\lambda_1 a_1 + \lambda_2 a_2) &\leq u(\lambda_1 a_1 + \lambda_2 a_2, \lambda_1 g_r^a(a_1) + \lambda_2 g_r^a(a_2), h_r) + \beta W_b(\lambda_1 g_r^a(a_1) + \lambda_2 g_r^a(a_2)) \\ &\leq \lambda_1 u(a_1, g_r^a(a_1), h_r) + \lambda_2 u(a_2, g_r^a(a_2), h_r) + \beta\lambda_1 W_b(g_r^a(a_1)) + \beta\lambda_1 W_b(g_r^a(a_1)) \\ &= \lambda_1 W_r(a_1) + \lambda_2 W_r(a_2), \end{aligned}$$

where we have used the fact that u is concave and W_b is concave in the image of g_r^a. Hence, W_r is concave in I. Strict concavity of W_r follows from strict concavity of u. □

Proposition 12. *Let I be an interval of A such that both $g_o^a(I)$ and $g_r^a(I)$ are intervals and $\{\overline{p}\} + g_o^a(I) \subseteq g_r^a(I)$. Then W_o is strictly concave in I*

Proof. We use the fact that the restriction of the operator T_o to the set $\mathcal{F}$ is a contraction. This restricted operator is defined in the obvious way. First, fix the buyer's value function W_b which, given the hypotheses of the proposition and Proposition 10, is concave in $g_r^a(I)$. The restricted operator is then

$$T_o^b(f_o)(a) = \max_{c,a'} \left\{ U^b(c, \hbar) + \beta\alpha f_o(a') \right\}, \tag{C.13}$$

where $U^b(c, a') = u(c, \hbar) + \beta(1-\alpha)W_b(a' + \overline{p})$ is strictly concave, and $\alpha = (1 - \pi_{\xi_o})(1 - \pi_\mu)$. Hence, if f_o is concave, $T_o^b f_o$ is concave. By Stokey-Lucas-Prescott, the limit of the iterating sequence $(T_o^b)^n$ is concave and thus W_o is concave. Once this is proved, the dynamic programming equation in (C.13) implies that W_o is in fact strictly concave, since U_b strictly concave. □

D Proofs of Propositions 1 to 3

The characterization results in Section 3.3 follow from the properties of the value functions established in Sections B and C. Potential buyers solve problem (B.8), or, equivalently, the problem described right after Lemma 5. Under the conditions of Theorem 1, an optimal solution to this problem exists, by the Theorem of the Maximum. Since the price function $\hat{p}$ in (B.6) is strictly increasing and strictly convex, the concavity result in Proposition 12 implies that, conditional on participating in the afternoon market, the optimal solution is unique under the assumptions in Proposition 1. Hence, by the Theorem of the Maximum, the associated policy function is continuous. This proves Proposition 1. Proposition 2 then follows from the differentiability W_o and the concavity result in Proposition 12.

Proof. Since W_o is differentiable (Proposition 9), the optimal solution of buyers who find it optimal to participate in the afternoon market is characterized by the first-order condition:

$$\begin{aligned} &[W_o\,(a - (1+\tau)\hat{p}(m_b)) - W_r(a)] - m_b\,(1+\tau_b)\,\hat{p}'(m_b)\,W_o'\,(a - (1+\tau_b)\hat{p}(m_b)) \\ &= \hat{\lambda}(a)\,(1+\tau_b)\,\hat{p}'(m_b), \end{aligned} \tag{D.1}$$

where $\hat{\lambda}(a)$ is the Lagrange multiplier of the borrowing constraint in (B.7). The result is trivial if $\lambda(a) > 0$. If $\lambda(a) = 0$, (D.1) can be written as:

$$\left(\frac{1}{1+\tau_b}\right)\left(\frac{W_o\,(a - (1+\tau_b)\,p) - W_r(a)}{m_b W_o'\,(a - (1+\tau_b)\,p)}\right) = \hat{p}'(m_b). \tag{D.2}$$

This equation has a unique solution (Proposition 1). The term in the left-hand side is the buyer's marginal rate of substitution of p for m_b. Buyers prefer high values of m_b and low values of p. Given the assumption on $g_r^c(a)$, W_o is strictly concave, by Proposition 12. If $(W_o\,(a - (1+\tau_b)\,p) - W_r(a))$

increases with a for a given p, this implies that the buyer's marginal rate of substitution is strictly increasing in a and, hence, so is the optimal value of m_b. □

The proof of Proposition 3 is based on the original problem where potential buyers choose θ. The result follows from the continuity and differentiability of W_b and W_r, and Proposition 1.

Proof. Let $\tilde{W}_b(a)$ denote the value of a potential buyer conditional on participating in the afternoon market, that is, the value of problem (3.2). Let $\tilde{g}_b^\theta(a)$ be the associated policy function. Then

$$W_b(a) = \max\{\tilde{W}_b(a), W_r(a)\}, \tag{D.3}$$

and $\tilde{g}_b^\theta(a) = g_b^\theta(a)$ if $W_b(a) = \tilde{W}_b(a) > W_r(a)$. Since θ_0 is only feasible choice for a potential buyer when $a \leq a_{min} = (1+\tau_b)\, p_{min} - (1-\delta)\, \overline{p}$, $W_b(a) = W_r(a)$ on this range. Suppose $a > a_{min}$, so the constraint set of problem (3.2) is nonempty. Applying the Envelope theorem to the Lagrangian of this problem yields

$$\tilde{W}_b'(a) - W_r'(a) = m_b\left(\tilde{g}_b^\theta(a)\right)\left(W_o'\left(a - (1+\tau_b)\, p(\tilde{g}_b^\theta(a))\right) - W_r'(a)\right) + \lambda(a). \tag{D.4}$$

The righthand side of (D.4) is strictly positive because $m_b(\theta) > 0$ for all $\theta \in \mathbf{R}_+$, the term in brackets is strictly positive by assumption, and $\lambda(a) \geq 0$. Thus $\tilde{W}_b(a) - W_r(a)$ is strictly increasing for $a > a_{min}$. By assumption, $W_b(a) = \tilde{W}_b(a) > W_r(a)$ for some a. Since $\tilde{W}_b$ and W_r are continuous, there then exists a_{part} such that $W_b(a) = \tilde{W}_b(a) > W_r(a)$ for all $a > a_{part}$ and $W_b(a_{part}) = \tilde{W}_b(a_{part}) = W_r(a_{part})$. Since $p(g_b^\theta(a)) > p_{min}$ for $a > a_{part}$, $p(\theta)$ is continuous, and so is $g_b^\theta(a)$ on this range (by Proposition 1), $p(\lim_{a \to a_{part}^+} g_b^\theta(a_{part})) > p_{min}$. Thus $a_{part} > a_{min}$ and, by continuity, this inequality also holds for any $a < a_0$ sufficiently close to a_{part}. Since $\tilde{W}_b(a) - W_r(a)$ is strictly increasing on this range, $W_b(a) = W_r(a) > \tilde{W}_b(a)$ and so $g_b^\theta(a) = \{\theta_0\}$ for any $a < a_{part}$. □

Finally, when the borrowing constraint holds for some buyers and is slack for others, the existence of the threshold a_1 follows directly from the following result, which uses the differentiability of W_o and W_r and the strict monotonicity of W_r.

Lemma 13. *If $a < a'$ and $\hat{\lambda}(a), \hat{\lambda}(a') > 0$ then $\hat{\lambda}(a') < \hat{\lambda}(a)$.*

Proof. If $\hat{\lambda}(a) > 0$, the price paid by a buyer with assets a is $\frac{a+(1-\delta)\overline{p}}{(1+\tau_b)}$. Thus (D.1) implies

$$\begin{aligned} \hat{\lambda}(a) &= \frac{W_o\left(-(1-\delta)\,\overline{p}\right) - W_r(a)}{(1+\tau_b)\hat{p}'(m_b)} - m_b W_o'\left(-(1-\delta)\overline{p}\right) \\ &= \frac{W_o\left(-(1-\delta)\,\overline{p}\right) - W_r(a)}{(1+\tau_b)\hat{p}'(m_b)} - m_b\, u_c\left(g_o^c\left(-(1-\delta)\,\overline{p}\right), \hbar\right), \end{aligned} \tag{D.5}$$

where the last equality follows from the Envelope theorem. Also, since $\hat{p}(m_b)$ is given by (B.6), m_b satisfies

$$\frac{\kappa_s + (1-\beta)\overline{p}}{\hat{m}_s(m_b)} + \beta\overline{p} = \frac{a + (1-\delta)\overline{p}}{(1+\tau_b)}. \tag{D.6}$$

If assets increase from a to a' then m_b increases, since $\hat{m}_s$ is strictly decreasing, and so does $\hat{p}'(m_b)$, since $\hat{p}$ is strictly increasing and strictly convex. Since W_r is strictly increasing by Theorem 1, (D.5) then implies $\hat{\lambda}(a') < \hat{\lambda}(a)$. □

E Computation

In order to compute a stationary equilibrium it is best to rewrite the problems of potential buyers and intermediaries so that, instead of choosing m_b taking $\hat{p}(m_b)$ as given, they choose p taking as given the inverse of the increasing function $\hat{p}(m_b)$, which we denote by $m_b(p)$. For this, it is crucial that $m_b(\theta)$ is a function instead of a correspondence. In particular, we cannot use the standard "truncated" Cobb-Douglas matching function. In our calibration, we use the class of matching functions in Chaumont and Shi (2018) (though the urn-ball matching function could also be used).

E.1 The matching function and the equilibrium price schedule

Given the Walrasian price $\overline{p}$, equation (B.6) determines m_s as a function of p:

$$m_s(p) = \frac{\kappa_s + (1-\beta)\,\overline{p}}{p - \beta\,\overline{p}}. \tag{E.1}$$

This function is strictly decreasing and strictly convex with $m_s(p_{min}) = 1$ and $\lim_{p\to\infty} m_s(p) = 0$, and does not depend on the choice of the matching technology.

We take $m_s(\theta) = (1+\theta^{-\gamma})^{\frac{-1}{\gamma}}$ with $\gamma > 0$, and $m_b(\theta) = m_s(\theta)/\theta$. Thus $\hat{m}_s(m_b) = \left(1 - m_b^{\gamma}\right)^{1/\gamma}$, and we can write

$$m_b(p) = \left(1 - m_s(p)^{\gamma}\right)^{1/\gamma}, \tag{E.2}$$

$$\theta(p) = \frac{m_s(p)}{\left(1 - m_s(p)^{\gamma}\right)^{1/\gamma}}. \tag{E.3}$$

Here, $\theta(p)$ is the inverse of $p(\theta)$, so it is strictly decreasing and strictly convex with $\lim_{p\to\infty}\theta(p) = 0$ and $\lim_{p\to p_{min}}\theta(p) = \infty$. Also, $m_b(p)$ is strictly increasing with $m_b(p_{min}) = 0$ and $\lim_{p\to\infty} m_b(p) = 1$. As shown in Appendix B, $m_b(p)$ is strictly concave provided $-\hat{m}_s{}'(m_b)/\hat{m}_s(m_b)$ is non decreasing. This last assumption can be further relaxed. For instance, for the value of γ used in our calibration to match the value of median TTB in the data (and, in fact, for any $\gamma < 1$), the assumption only holds for values of m_b above some threshold. Yet we only require that it holds for the range of values of m_b which correspond the submarkets that are active in equilibrium (since eliminating inactive submarkets does not change the problem of a potential buyer). One can easily verify that it suffices to check that the slope of $-\hat{m}_s{}'(m_b)/\hat{m}_s(m_b)$ is positive for the lowest value of m_b observed in equilibrium (which corresponds to the optimal choice of a marginal buyer). If so, $m_b(p)$ is strictly concave on the range of prices which correspond to the set of active submarkets, and the results in Propositions 1 to 3 again hold.

E.2 The optimal choice of potential buyers

In order to extend the method in Fella (2014) to our framework, we proceed in two steps. The problem of those potential buyers who participate in the afternoon market in equilibrium can be written as

$$\begin{array}{ll} W_b(a) & \max_p \{W_r(a) + m_b(p)\,[W_o\,(a-(1+\tau_b)\,p) - W_r(a)]\,\} \\ \text{s. t.} & p_{min} \leq p \leq \frac{a+(1-\delta)\,\overline{p}}{(1+\tau_b)}, \end{array} \tag{E.4}$$

with associated policy function $g^p(a)$. By Proposition 3, the constraint $p \geq p_{min}$ does not bind. The buyer's gains from trading at price $p > p_{min}$ are $S(a,p) = W_o\,(a-(1+\tau)p) - W_r(a)$.

By Theorem 1, $S(a,p)$ is strictly decreasing in p. Hence, if $S(a,p_{min}) \leq 0$ then $S(a,p) < 0$ for all $p > p_{min}$, and non-participation is optimal in this case. Suppose that $S(a,p_{min}) > 0$, so the gains from participation are positive. It is direct to check from the first-order condition of problem (E.4) that the Lagrange multiplier of the borrowing constraint is given by $\lambda(a) = m_b'(p)[S(a,p) - \tilde{S}(a,p)]$, where $\tilde{S}(a,p) = \frac{m_b(p)}{m_b'(p)} u_c\,(g_o^c\,(a-(1+\tau_b)\,p)\,, \hbar)\,(1+\tau_b)$. Hence, at an optimal solution, $S(a,p) \geq \tilde{S}(a,p)$, with equality if the constraint does not bind. By the Envelope Theorem, $W_o' = u'\,(g_o^c\,(a-(1+\tau_b)\,p))\,(a-(1+\tau_b)\,p)$, so $g_o^c(a)$ is non-decreasing if W_o is concave, since u is strictly concave. Since m_b is strictly increasing and strictly concave, this implies that $\tilde{S}(a,p)$ is strictly increasing in p and non-increasing in a. Also, $\tilde{S}\,(a,p_{min}) = 0$ regardless of the value of a, since $m_b\,(p_{min})\,/m_b'\,(p_{min}) = 0$. There is then a unique value p which solves $S(a,p) = \tilde{S}(a,p)$ (in line with Proposition 1), and for this value $S(a,p) > 0$. There are then two cases: (i) if $p \leq (a+(1-\delta)\,\overline{p})\,/(1+\tau_b)$ then $g_p(a) = p$, and (ii) otherwise, $g^p(a) = (a+(1-\delta)\,\overline{p})\,/(1+\tau_b)$.

We use the following algorithm to find $g^p(a)$. Given the value functions W_o, W_r and the policy function g_o^c:

1. Check that $S\,(a,p_{min}) > 0$, so the agent's gains from participation are positive. (Otherwise, $g^\theta(a) = \theta_0$).
2. Find the maximum price the agent is willing to pay. This is equal to $p_r = \widetilde{p}$ where $S\,(a,\widetilde{p}) = 0$ if $\widetilde{p} \leq (a+(1-\delta)\,\overline{p})\,/(1+\tau_b)$. Otherwise, $p_r = (a+(1-\delta)\,\overline{p})\,/(1+\tau_b)$.
3. If $\tilde{S}\,(a,p_r) > S\,(a,p_r)$ use any solver to find a price $p \in (p_{min}, p_r)$ for which $\tilde{S}\,(a,p) = S\,(a,p)$.
4. If $\tilde{S}\,(a,p_r) \leq S\,(a,p_r)$, set $p = p_r$.

If $S(a,p)$ is increasing in a, as in our quantitative model, the above arguments imply that both p_r and $g^p(a)$ increase with a (in line with Proposition 2). Agents with low assets are constrained and choose $p = (a+(1-\delta)\,\overline{p})\,/(1+\tau_b)$. Wealthier agents are unconstrained.

E.3 The choice of financial assets

Let us focus on the problem solved by the renter at night. The expression for the Euler equation of the problem depends on whether the agent can participate in the competitive search market in the next afternoon. Thus there are two cases. If $g_r^a(a) + (1-\delta)\,\overline{p} < (1+\tau_b)p_{min}$, the Euler equation is:

$$-u_c\,(g_r^c(a), h_r) + R\,\beta\,u_c\,(g_r^c(a'), h_r) \leq 0, \tag{E.5}$$

with equality if $a' = g_r^a(a) > 0$. If $g_r^a(a) + (1-\delta)\,\overline{p} \geq (1+\tau_b)p_{min}$, the Euler equation becomes

$$-u_c\left(g_r^c(a), h_r\right) + R\,\beta\,\left[m_b\left(g^p(a')\right)\,u_c\left(g_o^c\left(a' - (1+\tau_b)\,g^p(a')\right), \hbar\right) + \left(1 - m_b\left(g^p(a')\right)\right)\,u_c\left(g_r^c\left(a'\right), h_r\right)\right] + R\,\beta\,\frac{m_b'\left(g^p(a')\right)}{1+\tau_b}\,\left[S\left(a', g^p(a')\right) - \tilde{S}\left(a', g^p(a')\right)\right] \leq 0, \quad \text{(E.6)}$$

with equality if $a' = g_r^a(a) > 0$. The problem solved by owners is similar, except for the fact that they can borrow up to $(1-\delta)\,\overline{p}$. We build on Fella (2014) and solve for the optimal consumption rule using a modified version of his generalized endogenous grid method. The algorithm is as follows:

1. Choose an initial guess for $(W_o^j, W_r^j, g_o^{c,j}, g_r^{c,j})$. For the owner's value function, we use the value function of an owner that is never hit by any shock as an initial guess. For the renter, we use that of a renter who never participates in the afternoon market. The consumption policy function of the renter will have a discontinuity point. We choose $\underline{a}^j = (1+\tau_b)\,p_{min}$ as the first guess for this point.

2. Solve the afternoon problem as outlined in Section E.2 to find $g^p(a)$ and $W_b(a)$.

3. For a given grid for next period's assets, ga, we use the Euler equation to find consumption today. We know that, if $ga < \underline{a}^j$, the Euler equation is (E.5); otherwise it is (E.6). We need to interpolate to obtain the consumption policy function as a function of the grid of assets today. We also need to be aware that there is a discontinuity at $\underline{a}^j$. This is key to use interpolation to find the policy function of consumption (as a function of assets today). To find the maximum in the region of assets that correspond to participation and non-participation, respectively, we follow Fella (2014). There is a cutoff point below which the renter knows that she will not participate in the afternoon market in next period. Save the node as $\underline{a}^{j+1}$. Save W_o^{j+1}, W_r^{j+1}, $g_o^{c,j+1}$, $g_r^{c,j+1}$.

4. Go to step 2. Iterate until convergence.

A grid of 400 points in financial assets gives very high accuracy and is very fast.

E.4 The stationary distribution

We cannot use Monte Carlo simulations in this setup because of the curse of dimensionality. Monte Carlo simulations are a good approximation of the invariant distribution when we are certain that the law of large numbers holds across simulations. This is not the case here, however, because the taste and moving idiosyncratic shocks only occur when the agent is an owner, and tenure depends on a choice. In our economy, any change in the distribution of financial assets implies a change in the number of submarkets which are active in equilibrium. Using Monte Carlo simulations would require using a sample so large that the law of large numbers holds in each possible submarket, which is computationally unfeasible.

We thus solve for the stationary distribution as in Huggett (1993) and as explained in Ríos-Rull (1997). We use a much finer grid than the one used to solve the household's problem (800 points in our case) and guess the distribution of owners and renters at night. Then we use the policy functions for financial assets to integrate numerically and find the distribution of non-traders and potential buyers in the afternoon as shown in equations (A.1)–(A.4). We iterate until convergence.

E.5 The algorithm to find the stationary equilibrium

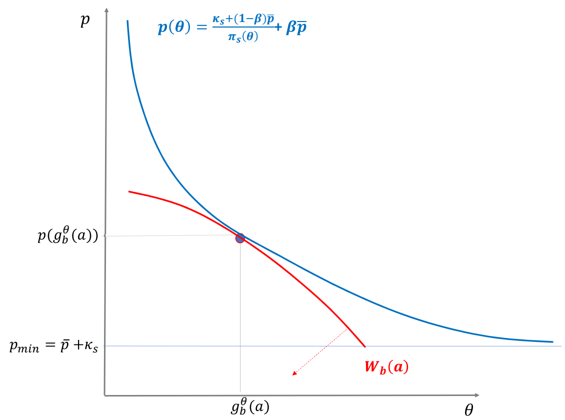
1. Choose an initial guess for the Walrasian price $\overline{p}$ and obtain the price function in (E.2).
2. Solve the household's afternoon problem as stated in Subsection E.3.
3. Find the invariant distribution. Calculate the mass N of non traders in the afternoon.
4. Given the stationary distribution of buyers, use (2.11) to calculate the density of buyers for each level of financial assets, $b(a)$. For the buyers who participate, use $g^p(a)$ to calculate the probabilities of selling and buying, and thus

$$\widetilde{\theta}(a) = \frac{m_s\left(g^p(a)\right)}{m_b\left(g^p(a)\right)}. \tag{E.7}$$

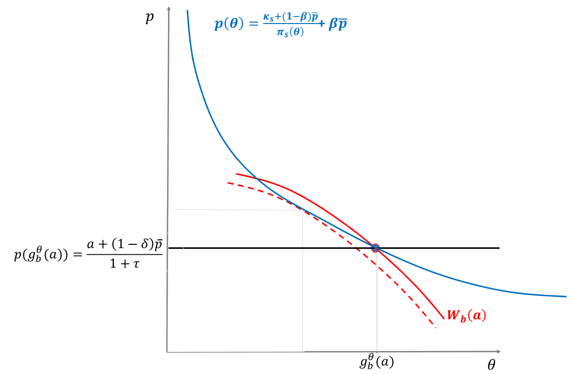
5. Find the amount of vacant homes needed to satisfy the rational expectations condition at the guessed prices:

$$S' = \int_A \frac{b(a)}{\widetilde{\theta}(a)}\, da. \tag{E.8}$$

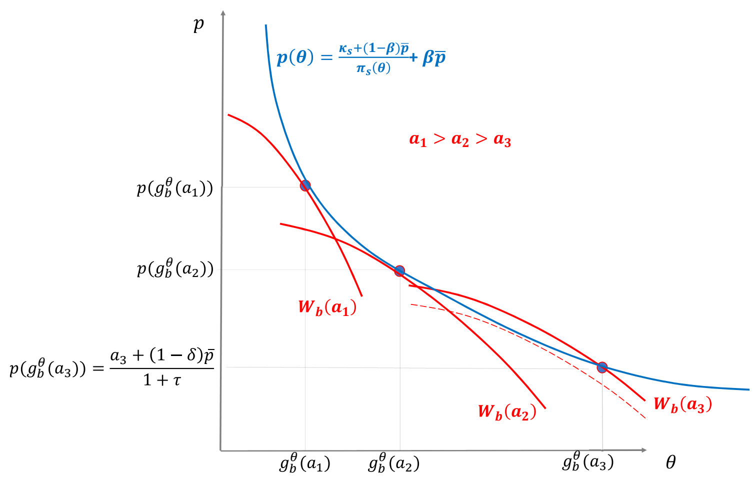
6. Compare S' with the actual number of vacant homes, $H - N$. If $S' > H - N$ (the price $\overline{p}$ is too low), update $\overline{p}$ upwards. If $S' < H - N$, update $\overline{p}$ downwards. Go back to step 1.



(a) The optimal choice of an unconstrained buyer.

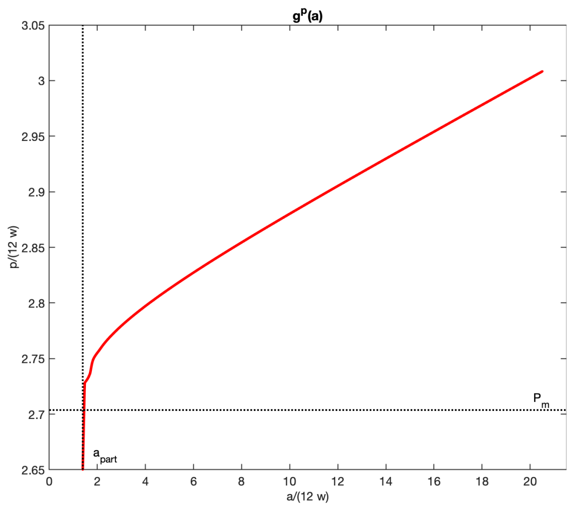


(b) The optimal choice of a borrowing-constrained buyer.

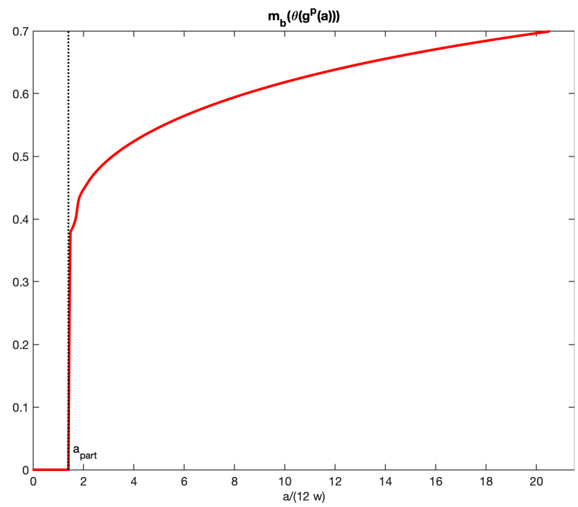


(c) Positive sorting.

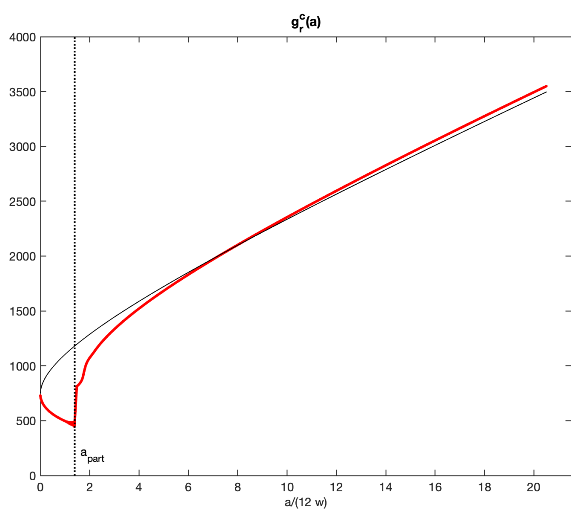
Figure 1: The choice of submarket.



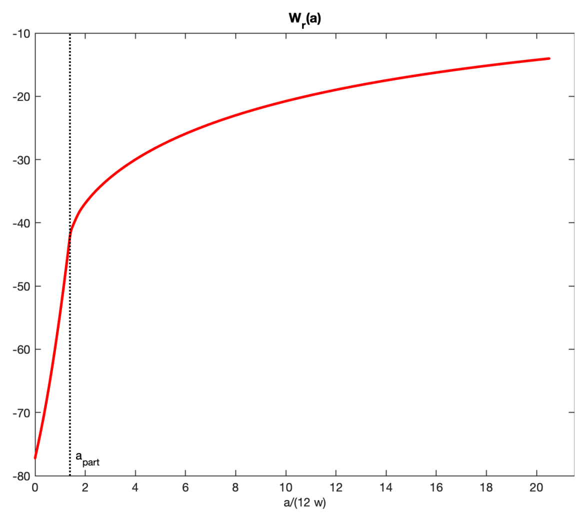
(a) The price policy function $g^p(a)$



(b) Probability of buying as a function of financial assets $m_b\left(g^p(a)\right)$

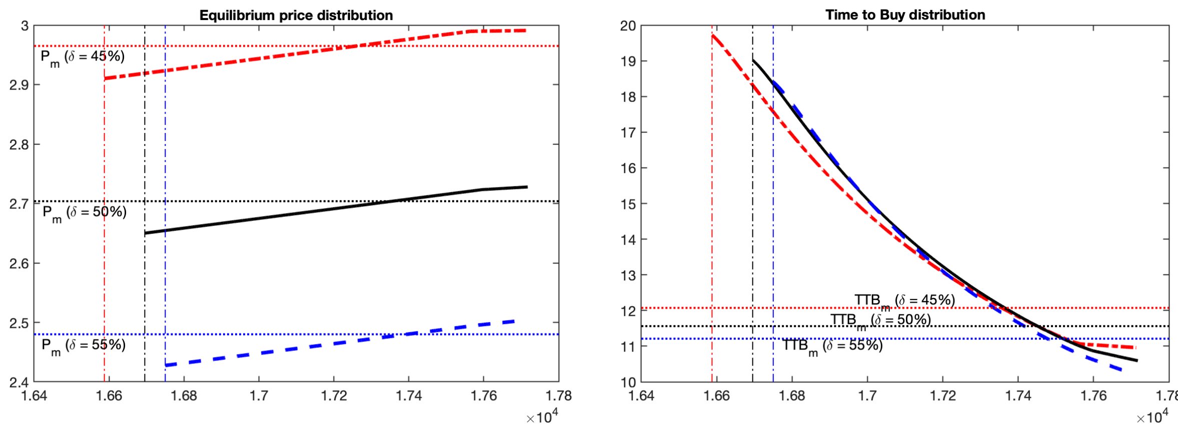


(c) The renter's consumption policy function $g_r^c(a)$



(d) The renter's value function $W_r(a)$

Figure 2: Policy functions



(a) (b)

Figure 3: Prices and TTB as a function of financial assets for different values of δ.

Table 1: Calibration

Param.	Observation	Value
w	Monthly wage	1000.0000
r (annually)	Díaz & Luengo-Prado (IER 2010)	0.0391
r_h	AHS, median housing costs renters 28% of income	$0.25\,w$
τ_b	Indirect taxes on buyers	0.0250
τ_s	Indirect taxes on owners	0.0600
κ_s	Cost of posting a vacancy	0.0000
δ	SCF, median LTV ratio = 41.04%	0.5000
π_μ	NAR: Median tenure of 10 years	0.0059
π_{ξ_o}	Annual mobility of owners = 3.2 %	0.0025
π_{ξ_r}	Annual mobility of renters = 12 %	0.0100
σ	Risk aversion parameter	2.0000
$h_r/\hbar$	Homeownership rate = 69.43%	0.9992
β	Median W/E for renters = 0.3450	0.8480
γ	Median TTB (NAR) [10 12]	0.6552
H/N (%)	Median H/E for owners = 2.7223	70.6447

Notes: The model period is a month. Annualized values. The monthly wage w is the numeraire and has been set $w = 1000$.

Table 2: The benchmark steady state

Target	Source	Data	Bench.	Walras
$\overline{p}/(12\,w)$	-	-	2.6500	1.0070
Homeownership rate	SCF mean1989-2007	69.4258	69.6940	70.6447
Median H/E owners	SCF median 1989-2007	2.7223	2.7039	1.0070
Median LTV ratio (%)	"	41.0448	38.6170	42.3390
Median W/E renters	"	0.3450	0.2933	0.5274
Median TTB	NAR 2017	[10-12]	11.5470	168.8001
Rent-to-Price ratio (%)	Sommer and Sullivan (2018)	[8-15]	9.4338	24.8261
Median TOM	NAR 2017	[4-17]	10.1940	0.0000
Months of Supply	NAR 2017	5.4737	2.6204	1.0000
Vacancy rate (%)	AHS, mean 2011-2015	2.1766	2.1763	0.8418
Mean error (%)	Lisi and Iacobini (2013)	2.3500	0.4123	0.0000
Coeff. of Variation (%)	Lisi and Iacobini (2013)	2.2500	0.5152	0.0000
% of sales error > 1%	Zillow 5% interval	16.3000	5.6968	0.0000
Gini of renters wealth G_r	-	-	0.5351	0.1956
Gini of wealth (all) G	-	-	0.2183	0.2016
Part. rate	-	-	7.2963	62.6471

Notes: Median TTB refers to median time to buy, whereas median TOM refers to time to sell. Both statistics are reported in weeks. The rest of the statistics are reported in annual terms.

Table 3: Long run changes in the down payment

Target	$\delta = 0.45$		$\delta = 0.5$	$\delta = 0.55$	
	Fixed H	Const.	Benchmark	Fixed H	Const.
$\overline{p}/\overline{p}_{bench}$	1.0981	1.0000	1.0000	0.9160	1.0000
H/H_{bench}	1.0000	1.0916	1.0000	1.0000	0.0000
Homeownership rate	69.7520	76.0860	69.6940	69.6450	0.0004
Median H/E owners	2.9648	2.7034	2.7039	2.4799	2.7116
Median LTV ratio	43.7330	43.5330	38.6170	33.5140	33.6090
Median W/E renters	0.3030	0.4011	0.2933	0.2839	0.0081
Rent-to-Price ratio (%)	8.5908	9.4339	9.4338	10.2980	9.4339
Median TTB	12.0520	11.6010	11.5470	11.2040	11.4860
Median TOM	9.8128	10.1310	10.1940	10.5120	10.6840
Months of Supply	2.5207	2.6057	2.6204	2.7045	2.7128
Vacancy rate (%)	2.0953	2.3626	2.1763	2.2445	0.0000
% of sales error > 1%	5.1526	5.9006	5.6968	6.1728	2.3768
Price range	1.0294	1.0311	1.0313	1.0332	1.0314
Gini of renters wealth	0.5298	0.4592	0.5351	0.5380	0.0000
Gini of wealth (all)	0.2181	0.1633	0.2183	0.2184	0.0149
Participation rate	7.6732	10.1360	7.2963	6.9999	0.0000
Marg. buyer $a/(12\,w)$	1.3823	1.2588	1.3913	1.3958	1.5238
% buyers that trade	24.8180	25.7360	26.0300	27.0700	30.2710

Table 4: Steady state comparisons

Target	Bench.	$\Delta w =$ 5%		$\tau_b = 0$		$\tau_s = 0$		$\gamma = 1.5$	
		Fixed H	Const.	Fixed H	Constr.	Fixed H	Constr.	Fixed H	Constr.
$\overline{p}/\overline{p}_{bench}$	1.0000	1.0943	1.0000	1.0637	1.0000	1.0438	1.0000	1.1190	1.0000
H/H_{bench}	1.0000	1.0000	1.0912	1.0000	1.1481	1.0000	1.0827	1.0000	1.2271
Homeownership rate	69.6940	69.7010	73.4720	69.7030	73.5720	69.6850	73.8510	70.1399	86.2179
Median H/E owners	2.7039	2.6826	2.5662	2.6076	2.6020	2.8223	2.5600	2.9844	2.6656
Median LTV ratio	38.6170	38.7090	36.7770	41.1750	38.8040	44.5080	43.7340	40.6805	39.9121
Median W/E renters	0.2933	0.2969	0.6782	0.2667	0.7048	0.3062	0.6888	0.0159	0.4623
Rent-to-Price ratio (%)	9.4338	8.6206	9.4338	8.8690	9.4338	9.0381	9.4338	8.3745	9.4338
Median TTB	11.5470	11.5780	11.6270	11.6470	9.8817	11.5640	12.1600	7.1998	7.0023
Median TOM	10.1940	10.1380	9.8859	10.1410	12.8680	10.2830	9.5428	6.9882	6.5586
Months of Supply	2.6204	2.6082	2.6567	2.6046	3.2473	2.6364	2.6181	1.4257	1.4462
Vacancy rate (%)	2.1763	2.1664	5.4920	2.1634	10.0550	2.1893	4.2637	1.4761	1.3784
% of sales error > 1%	5.6968	5.7387	17.1840	5.2955	2.7778	4.7314	0.0000	0.0000	0.0000
Price range	1.0313	1.0310	1.0327	1.0318	1.0358	1.0311	1.0364	1.0046	1.0051
Gini of renters wealth	0.5351	0.5281	0.3021	0.5338	0.2833	0.5324	0.2996	0.7392	0.5437
Gini of wealth (all)	0.2183	0.2169	0.1368	0.2169	0.1244	0.2060	0.1159	0.2555	0.1020
Participation rate	7.2963	7.3732	22.8580	7.3025	24.8170	7.0645	22.6370	4.3324	10.1399
Marg. buyer $a/(12\,w)$	1.3913	1.3810	1.3250	1.2784	1.2619	1.4522	1.3250	1.5582	1.3913
% buyers that trade	26.0300	25.7670	25.6810	26.0190	37.4150	26.8730	20.5550	55.0571	56.1626

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