ANALYSIS OF THE SUSPENSION BEAM IN ACCELEROMETER FOR STIFFNESS CONSTANT AND RESONANT FREQUENCY BY USING ANALYTICAL AND NUMERICAL INVESTIGATION

WONG WAI CHI

UNIVERSITI SAINS MALAYSIA

2007

ANALYSIS OF THE SUSPENSION BEAM IN ACCELEROMETER FOR STIFFNESS CONSTANT AND RESONANT FREQUENCY BY USING ANALYTICAL AND NUMERICAL INVESTIGATION

by

WONG WAI CHI

Thesis submitted in fulfillment of the
requirements for the degree
of Master of Science

June 2007

ACKNOWLEDGEMENTS

First of all, I would like to express my deepest gratitude to my family for their patience, understanding and support. Their supports allowed me to concentrate fully in my studies without hesitation. I would also like to express my sincere thanks to my supervisor, Assoc. Prof. Dr. Ishak Hj. Abd. Azid for his tremendous guidance, advice, support, assistance and encouragement throughout my postgraduate candidature period. Without him, I would not been able to complete my research in such an efficient manner. I would also like express my appreciation to my co-supervisor Prof. Dr. Burhanuddin Yeop Majlis, who allows me to study at the MEMS fabrication laboratory at IMEN, UKM and give me a chance to explore the fabrication process.

Apart from them, I would like to forward special thanks to my fellow colleagues Mr. Lee Hing Wah, Mr. Gyanaprakash, Mr. Ezral, Mr. Lim Mook Tzeng, Mr. Rosmaini, Mr. Khairul Anuar, Mr. Shahrizan, Mr. Chuah Hun Guan and Ms. Ko Ying Hao for their moral supports and help especially during the initial stage of my research. Thanks are also to all students and staff of Universiti Sains Malaysia, those who helped me either directly or indirectly in completing my research.

I am gratitude to Universiti Sains Malaysia for awarding me the GRADUTE ASSISTANT SCHEME scholarship which relieved me of financial insecurity. Last but not least, my sincere thanks, compliments and regards to anyone who had helped and supported me in one way or another.

WONG WAI CHI

June 2007

TABLE OF CONTENTS

CHAPTER TWO : LITERATURE SURVEY 9

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

Symbol	**Description**	**Unit**
a	Acceleration	$\mu m/s^2$
A	Area	μm^2
c	Cosine	-
d	Distance	μm
d_0	Sensing gap	μm
d_1		μm
E	Young’s modulus	MPa , $\mu N/\mu m^2$
f	Force	μN
fr	Resonant frequency	kHz
fr_{cal}	Calculated resonant frequency	kHz
fr_{sim}	Simulated resonant frequency	kHz
g	Gravity acceleration	$\mu m/s^2$
G	Shear modulus	MPa , $\mu N/\mu m^2$
I	Moment of inertia	μm^4
k	Stiffness constant	$\mu N/\mu m$
$k_{1/4}$	Stiffness constant of quarter model	$\mu N/\mu m$
k_{bm}	Stiffness constant due to bending moment	$\mu N/\mu m$
k_{cal}	Calculated stiffness constant	$\mu N/\mu m$
k_{eff}	Effective stiffness constant	$\mu N/\mu m$
k_{fixed}	Stiffness constant of fixed-fixed beam	$\mu N/\mu m$
k_s	Stiffness constant due to shear	$\mu N/\mu m$
k_{sim}	Simulated stiffness constant	$\mu N/\mu m$
L	Length	μm

l_{beam}	Length of suspension beam	μm
l_{finger}	Length of finger	μm
$l_{proof\ mass}$	Length of proof mass	μm
l_r	Length of second component	μm
M	Mass	Kg
m_{eff}	Effective mass	Kg
M	Moment	μN·μm
N	Number of finger	Pair
N	Shape function	-
R	Radius	μm
R	Reaction force	μN
R_a	Axial force	μN
S	Sine	-
S	shear deflection constant	-
T	Thickness	μm
u, v, w	Displacement of x, y, z direction respectively	-
U	Internal work / strain energy	pJ , μN·μm
V	Volume	μm^3
W	Width	μm
w_{beam}	Width of suspension beam	μm
w_{finger}	Width of finger	μm
w_i	weighting function	-
$w_{proof\ mass}$	Width of proof mass	μm
w_r	Width of second component	μm
W	External work	pJ ,

		μN·μm
x, y, z	Local coordinates	μm
X, Y, Z	Global coordinates	
δ	Deflection / displacement	μm
ε	Strain	μm/μm
ϕ	Angle	rad
γ	Shear strain	μm/μm
μ	Poisson's ratio	-
θ	Slope, rotation	rad
ρ	Density	kg/μm^3
σ	Stress	MPa , μN/μm^2
τ	shear stress	MPa , μN/μm^2

LIST OF APPENDICES

LIST OF PUBLICATIONS

ANALISIS KE ATAS RUSUK AMPAIAN DALAM METER PECUT BAGI PEKALI KEKUKUHAN DAN FREKUENSI RESONANSS SECARA PENYIASATAN ANALISIS DAN BERANGKA

ABSTRAK

Mikro-meterpecut yang digunakan dalam pelbagai penerapan hanya akan tercapai dengan jayanya sekiranya keperluan frekuensi resonans dan kepekaan dapat dipenuhi dan konsisten. Berdasarkan syarat-syarat tersebut, analisis struktur pada pekali kekukuhan and frekuensi resonans bagi rasuk ampaian dalam meter pecut dan seterusnya pengoptimuman kepada kepekaan haruslah dilakukan. Dengan itu, beranalisis berasal pada teori dan menggunakan kaedah unsur terhingga (FEM) telah dihasilkan dengan memberikan tumpuan ke atas ampaian rasuk meterpecut. Selain daripada itu, analisis unsur terhingga (FEA) tiga dimensi telah dihasilkan untuk menganalisis pekali kekukuhan and frekuensi resonans bagi mikro meterpecut jenis *comb finger*. Bagi FEA, terdapat dua jenis model yang digunakan iaitu model terperinci dan model rasuk. Bagi model rasuk, dua jenis jaringan telah dilakukan, iaitu jaringan terkawal and jaringan bebas. Enam jenis ampaian rasuk telah dianalisis dengan kaedah yang disebutkan. Ramalan bagi pekali kekukuhan and frekuensi resonans dengan cara beranalisis menunjukkan sedikit berbazean dengan ramalan yang diperolehi melalui simulasi FEA ANSYS® dalam julat 0-10%. Perbandingan keputusan bagi pekali kekukuhan and frekuensi resonans yang diperolehi melalui simulasi FEA ANSYS® dengan keputusan sediaada juga memuaskan, iaitu tiada berbezaan dalam ramalan frekuensi resonans dan hanya 0.59% berbezaan dalam ramalan pekali kekukuhan. Kepercayaan FEA ANSYS® dalam analisis memberikan penggalakan bagi melanjukkan analisis atas prestasi meterpecut dengan berbagai geometri and reka bentuk ampaian rasuk. Kajian ini juga telah memperkenalkan suatu reka bentuk optimum dengan menggunakan kaedah pengoptimuman Neuro-Genetic yang menggabungkan aplikasi Rangkaian Neural Tiruan (ANN) dan Algoritma Genetik (GA).

ANALYSIS OF THE SUSPENSION BEAM IN ACCELEROMETER FOR STIFFNESS CONSTANT AND RESONANT FREQUENCY BY USING ANALYTICAL AND NUMERICAL INVESTIGATION

ABSTRACT

A successful and consistent performance of micro-accelerometer which has been applied in various applications can only be achieved when the resonant frequency and the sensitivity requirement are fulfilled. In view of this, structural analysis on stiffness constant and resonant frequency for the suspension beam in accelerometer, and subsequently optimization design of accelerometer with respect to sensitivity in term of displacement against acceleration must be performed. For that reason, a two-dimensional analytical formulation derived theoretically and numerically by using finite element method (FEM) has been developed with the focus on the suspension beam of micro-accelerometer. Apart from that, a three-dimensional finite element analysis (FEA) has been simulated by using ANSYS to analyze the stiffness constant and the resonant frequency of the comb finger type capacitive micro-accelerometer. For the FEA, two types of modeling, namely the complete model and the beam model, have been used. The beam models have been discretized in two different ways, namely mapped mesh and free mesh. Six types of suspension beam have been analyzed. The prediction of stiffness constant and resonant frequency obtained using ANSYS® FEA shows slight difference compared to that obtained analytically with 0-10% difference. The stiffness constant and resonant frequency obtained using ANSYS® FEA shows a good agreement with the published results where the resonance frequency achieves zero difference and only 0.59% difference in stiffness constant. The success of the ANSYS® FEA model provides further encouragement in using the model to analyze the performance of the device for various combinations of geometry and different design in suspension beam. An optimized design of accelerometer is then obtained by using Neuro-Genetic optimization which combined the artificial neural network (ANN) and genetic algorithms (GA).

CHAPTER 1
INTRODUCTION

1.0 Overview

An introduction to the micro-accelerometer, the analysis and optimizing method, along with the problem definition, the objectives and the outline of the thesis will be presented. The main contents in this chapter are:

- Introduction to micro-accelerometer
- Structural analysis of micro-accelerometer
- Finite element method
- Neuro-genetic optimization
- Problem definition
- Thesis objective
- Thesis outline

1.1 Introduction to micro-accelerometer

1.1.1 Short history of MEMS

Micro-electromechanical systems (MEMS) can be described as a device where a mechanical function is coupled with an electrical signal and combined with other physical domains. Normally MEMS refer to a collection of microsensors and actuators which can sense their environment and have the ability to react to the changes in that environment with the use of a microcircuit control. MEMS are also referred to as micromechanics, micro machines, or Micro Systems Technology (MST). The dimension of MEMS and their components are from 100 nanometers to centimeter range. MEMS make product faster, smaller, lighter, more reliable, cheaper, and are capable of incorporating more complex functions and are usually more precise while maintaining the production cost at low level.

Richard Feynman was the first person to say or "predict" about MEMS on December 29th 1959. However, MEMS only emerged in the beginning of 1990s, with the aid of the development of integrated circuit (IC) fabrication processes, in which sensors, actuators, and control functions are co-fabricated in silicon. Since then, remarkable research progresses have been achieved in MEMS. Nowadays, MEMS devices include somehow less integrated devices, such as micro-accelerometers, inkjet printer head, and micro-mirrors for projection. The concepts and feasibility of more complex MEMS devices have been proposed and demonstrated for applications in various fields such as micro-fluidics, aerospace, biomedical, chemical analysis, wireless communications, data storage, display, and optics.

Mechanical behavior and mechanical response of those devices play major role in their performance. The behaviors of MEMS devices can be studied using numerical computing, analysis (fundamental theories) and experiments.

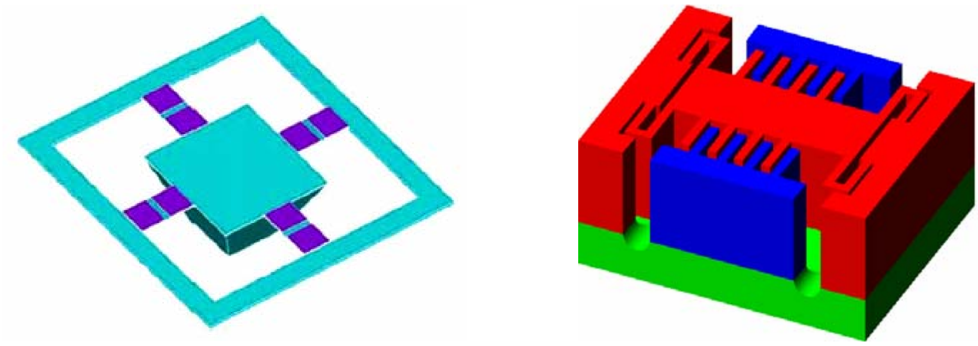
1.1.2 MEMS sensing device

Sensors are one of the main functionality of MEMS. Normally, the working principle of mechanical sensors (except a certain class of flow sensors) relies on the mechanical deformation of a construction (deflection of a membrane or a mass suspended by a beam). This deformation will then be translated into an electrical signal for presentation. The sensors need a reproducible signal which means that the structure must deform under an equal load in the same way.

The mechanical deformation due to mechanical forces can be measured in a number of ways. The measurement can be due to changes of the electrical resistance because of geometric changes of resistors or strain in the resistors (piezoelectricity), changes of electrical capacitance, changes in the resonant frequency of vibrating elements in the structure and changes of optical resonance.

1.1.3 Micro-accelerometer

Micro-accelerometers or accelerometer are one of the most important types of MEMS device, which have the second largest sales volume after pressure sensors (Yazdi et al. 1998). The large volume demand for MEMS accelerometers are due to their capability to be used in variety of applications. They can be used to measure tilt, motion, position, vibration and shock. Generally, accelerometer consists of a proof mass either seismic mass or comb finger suspended by compliant beams anchored to a fixed frame, as shown in Figure 1.1. The operation can be modeled by a second-order mass-damper-spring system. External acceleration can be measured by relative displacement (capacitance) or suspension-beam stress (piezoresistive).



(a) Seismic mass device (b) Comb finger device

Figure 1.1: Typical design of microaccelerometer

Recently, attention has been focused on the modeling and analysis on the sensitivity and the mechanical performances of comb finger devices. The comb finger type devices are one of the most important building blocks of MEMS due to their high sensitivity and high performance. Comb finger devices can be applied as either MEMS sensors or MEMS actuators. MEMS sensors normally sense the physical environment by means of mechanical sense and giving the output in electrical means. Comb finger type sensors have been used in air-bag system, chassis control, side-impact detection, antilock braking system, machinery vibration monitoring, inertial navigation, seismology and mouse. On the other hand, the MEMS actuators respond to the electronic commands and actuate the dynamic system. Comb driven actuators include RF resonators, electromechanical filters, optical MEMS, microgrippers, gyroscopes and voltmeters. They have also been used to drive vibromotors and micromechanical gears.

1.2 Structural analysis of micro-accelerometer

In general, for the design of a MEMS device with moving parts, the analysis usually starts with structural analysis and vibration analysis. Stiffness constant and subsequently resonant frequency are two important parameters which must be determined before other analyses can be carried out. Stiffness constant, which is an important intermediate parameter in the determination of natural frequency and sensitivity, is determined from a combination of physical geometrical parameters and material properties. In contrast, resonant frequency characterizes the bandwidth as well as the physical sensitivity of the accelerometer.

The sensitivity of the accelerometer is a measure of displacement with respect to acceleration. It is determined from the stiffness constant and the effective mass of the accelerometer. The resonant frequency is also determined from the effective stiffness constant and the effective mass. The relationship between sensitivity and resonant frequency with the stiffness constant and the effective mass is given in Equations 1.1 and 1.2.

$$Sensitivity = \frac{displacement}{acceration} = \frac{\delta}{a} = \frac{m_{eff}}{k} \tag{1.1}$$

$$2\pi f_r = \omega_n = \sqrt{\frac{k}{m_{eff}}} \tag{1.2}$$

where f_r is the natural frequency or resonant frequency, ω_n is the natural circular frequency, k is the stiffness constant, and m_{eff} is the effective mass.

1.3 Finite element method

The Finite Element Method (FEM) is a powerful numerical technique to solve boundary value problems. It solves a system of governing equations over the domain of a continuous physical system. This method models the structure using small interconnected element called *finite element*. Every interconnected element is linked, directly or indirectly, to every other element through common (or shared) interfaces, including nodes and/or boundary lines and/or surface. A simple approximation of the solution may be constructed, and then the local approximate solutions are put together to obtain a global approximate solution. A general solution to the governing equation can be assumed within the domain of each element. The application of the governing equations, loading and boundary conditions results in a system of equations which could be solved to find an approximate solution.

1.4 Neuro-genetic optimization

The data obtained from the finite element simulation are collected, analyzed and optimized by using Neuro-genetic method. Neuro-genetic optimization is basically the integration of genetic algorithms (GA) into the artificial neural network (ANN) platform, forming a formidable artificial intelligence tool for optimization.

1.4.1 Artificial neural network (ANN)

The working process of ANN is based on decision making process in human brain. It is categorized under artificial intelligent method and has been applied in many different fields such as control, finance, aerospace, engineering, industrial and manufacturing. Typical neural network consists of sets of input, sets of output and weighting function. Neural networks are trained to perform a particular function by adjusting the values of the connections (weights) between elements, based on a comparison of the output and the target, until a particular input leads to a specific target output. Once the network is trained, it can then be fed with any unknown input and is expected to predict the output with a high degree of accuracy.

1.4.2 Genetic algorithm (GA)

Once a well trained ANN via FEA is obtained, it is then combined with GA to form a tool for search and optimization purposes. Genetic Algorithms (GA) are adaptive search methods based on natural selection of survival of the fittest and natural genetics. They combine survival of the fittest among string structures with a structured yet randomized information exchange to form a search algorithm with some of the innovative flair of human search. GA exploits the fittest traits of old individuals to create a new generation of artificial creatures (strings). With each generation, a better population of individuals is created to replace the old population. Based on these principles, genetic algorithm is developed as a search tool that efficiently exploits historical information to speculate on new search points with expected improved performance.

1.5 Problem definition

The comb finger type is the most successful and effective type capacitive accelerometer. Resonant frequency and stiffness constant, which values depend mostly on the structural design of comb finger type accelerometer especially the suspension beam design, are two important parameters in the performance analysis of accelerometer. However, the study on the structural analysis of suspension beam in comb finger type accelerometer is not yet well established. The analytical derivation for these two parameters is only available for simple design such as straight beam. The formulae for stiffness constant and effective mass for folded beam and crab-leg beam, which are two common designs in suspension beam of MEMS devices (sensors and actuators), are either too simple or too complicated. The formula for stiffness constant and effective mass should be derived so that the performance of the accelerometer can be analyzed for common designs in suspension beam of comb finger type accelerometer. However, when the solution is obtained analytically, specific formulae need to be derived for each and every different design. Thus, this is tedious and time consuming. Therefore, a numerical solution which is applicable to all types of design for suspension beam needs to be developed.

1.6 Thesis objectives

For this project of “*Analysis of the Suspension Beam in Accelerometer for Stiffness Contant and Resonant Frequency by Using Analytical Solution and Numerical Investigation*”, there are four objectives to be achieved. They are:

- To analyze the performance of comb finger type capacitive accelerometer based on stiffness constant and resonant frequency.
- To develop analytical model to predict the stiffness constant and the effective mass of the suspension beam for comb finger type capacitive accelerometer.
- To develop a reliable finite element models to predict the stiffness constant, the effective mass and the resonant frequency of the suspension beam for comb finger type capacitive accelerometer.
- To optimize the design of the suspension beam for comb finger type capacitive accelerometer by using Neuro-Genetic methodology.

1.7 Thesis outline

The thesis is presented in five chapters which include introduction, literature survey, analytical and numerical formulations, results and discussion and finally conclusion. The first chapter gives a brief introduction on the micro-accelerometer with an overview on the comb finger type accelerometer and analyses that are required to be conducted. The objectives of the project are also presented in this chapter. In chapter two, a literature survey regarding micro-accelerometer, stiffness constant in micro-accelerometer, and the analysis using finite element simulation will be presented. This chapter deals with the past and current trends in the modeling and analysis of the micro-accelerometer. Working mechanism of comb finger type accelerometer, several design models of suspension beams, and determination of the device performance by using analytical, numerical and FEA analysis will be described in chapter three. Results from chapter three are presented and discussed in chapter four. Finally, the thesis will end with conclusions in chapter five.

CHAPTER 2
LITERATURE SURVEY

2.0 Overview

Literature reviews for four main scopes of the thesis are presented in this section. The scopes covered are shown as below:

- Micro-accelerometer
- Stiffness constant and resonant frequency in MEMS device
- Finite element simulation
- Optimization of accelerometer design

2.1 Micro-accelerometer

2.1.1 The evolvement of micro-accelerometer

According to Bernstein (2003), accelerometers are the starts of the MEMS show and currently still the leader in commercially successful MEMS technology. Yazdi et al. (1998) stated that the micro-accelerometers which have low-cost, small-size, high-performance open up new markets opportunities, and enhance the overall accuracy and performance of the systems consisting of micro-accelerometers. Throughout the innovation of micro-accelerometer, some popular suspension support designs have been suggested, including cantilever support, crab-leg/folded-beam configuration, and symmetric full-bridge supports. The physical mechanisms underlying MEMS accelerometers have moved from the traditional piezoresistive to capacitive, tunneling, resonant, thermal, and others which are not popular but feasible such as piezoelectric, optical, and electromagnetic.

2.1.2 Capacitance accelerometer as the most successful type of micro-accelerometer

Both of the review papers (Bernstein, 2003 and Yazdi et al., 1998) agreed that capacitive transduction is the most successful type. Its application varies from low-cost, large-volume automotive accelerometers, to high-precision inertial-grade microgravity devices since capacitive sensor has high sensitivity, stable dc-characteristics, low drift, low power dissipation, low temperature sensitivity, higher bandwidth, the simplicity of the sensor element itself, and no requirement for exotic materials. Yazdi et al. (1998) pointed out the disadvantage of capacitive devices which can be susceptible to electromagnetic interference (EMI), as their sense node has high impedance. However, this problem can be addressed by proper packaging and shielding the accelerometer and its interface circuit. Bernstein (2003) stated that the output of capacitance device can be analog, digital, ratiometric to the supply voltage, or any of various types of pulse modulation. Sensors with digital output are convenient when the data transmitted are without further noise degradation.

2.1.3 Micro-accelerometer in market – the important of accelerometer

Yole Développement (2006) reported that acceleration sensors have been widely used in the automotive industry for more than 30 years as motion sensing is essential for safety functions. But the real development of this market started only in 1988 when the first silicon accelerometer is used for airbag applications. The total accelerometer market value has been estimated to be $393 million in 2005 and $869 million in 2010 for a production of 161M and 502M units respectively. It is estimated that the automotive field accounts for 89% of the total volume which represents 81% of the global market value. 140 million accelerometers will be required in 2006 for the automotive field only.

Clifford (2004) forecasted that more than 100 accelerometer applications would be established soon, from PC peripherals to home networking to security, since accelerometer can measure tilt, motion, position, vibration, and shock. The major application of accelerometer is automotive applications; other applications area include biomedical applications, numerous consumers' applications, industrial applications and so on. High-sensitivity accelerometers are crucial components in self-contained navigation and guidance systems, seismometry for oil exploration and earthquake prediction, and microgravity measurements and platform stabilization in space.

2.1.4 The characteristic of micro-accelerometer

Among the specifications to be considered when choosing micro-accelerometer are output/input sensitivity, bandwidth, frequency response, resolution, noise floor, drift, linearity, dynamic range, offset, cross-axis sensitivity, shock survivability and power consumption. Yazdi et al. (1998) wrote that the challenges and future trends of micro-accelerometer is the development of low-cost, high sensitivity, low noise level, and large dynamic range, low-stress and low-drift packaging technologies, inertial-grade accelerometers with sub-μg noise levels, good long-term stability, and low temperature sensitivity. Bernstein (2003) suggested that high-g accelerometers is a niche market for accelerometers that read out very high g for crash tests, impacts, and missile and artillery shell launches.

Yazdi et al. (1998) suggested that generally, the design flow of accelerometer starting with describing stiffness constant (K) as a function of device geometry by using formulas, and then performing final design, by simulation and optimization by software. Bernstein (2003) stated that the resonant frequency is also important to determine the sensors upper useful frequency range, its sensitivity and displacement per g of acceleration.

2.2 Stiffness constant and resonant frequency in MEMS device

Stiffness constant and resonant frequency, which are the topics of this research, are two of the most important functionality that will be required in design of any MEMS devices with moving parts. Sensors and actuators, in particular, often require specific stiffness constant and resonant frequency to guarantee successful and repeatable performance.

2.2.1 Stiffness constant and resonant frequency in micro-accelerometer

The simplest design of micro-accelerometer is a proof mass suspended by a cantilever. The research on cantilever support piezoresistive accelerometer can be traced back to year 1979 (Yazdi et al., 1998) in which this design is one of the simplest designs of accelerometer. Even for this simple design, the stiffness constant and the resonant frequency are always the interest in research. Those parameters are important in both piezoresistive and capacitive accelerometer. Kovács et al. (2000) have analyzed the lowest resonant frequency, which characterized the bandwidth as well as the physical sensitivity, of cantilever- and bridge-type monolithic piezoresistive and capacitive accelerometers. The derivations have been carried out by using Rayleigh's energy principle, with the help of MATHEMATICA. The results show that the selection of geometrical parameters has opposite effects on bandwidth and device sensitivity.

The performance for different suspension beams has also been studied and compared. Yu et al. (2001) and Wang et al. (2004) presented analytical model, which is then verified with finite element model for seismic mass piezoelectric accelerometer with different suspension flexural. The suspension beams studied in those researches include cantilever, bridge type and four symmetric beams. Yu et al. (2001) studied one more design with two cantilevers and an optimal design was proposed. Wang et al.

(2004) presented the first five natural frequencies derived from stiffness constant and the resonant frequency. The dynamic analysis found that when the driving frequency is close to the fundamental resonant frequency, the output charge and the corresponding maximum stress become much larger; otherwise the output charge is relatively small. Therefore, it is reasonable that the typical range of the accelerometer frequency is chosen below one-third or one-fifth of the natural frequency. In both research, devices geometric and elastic properties have been taken as input parameters whereas resonance frequency and sensitivity are the interested output parameters. Stiffness constant has been determined as the intermediate parameter. The mechanical model of the above three researches (Kovács et al., 2000, Yu et al., 2001 and Wang et al., 2004) are based on the assumptions that the weight of the supporting beams is negligible compared to the seismic mass, the seismic mass and rim of the structure are rigid, and the seismic mass only vibrates in the z-axis. It means that, the mass of the device is only taken from the seismic mass and the stiffness constant considered in the analysis is only the stiffness of the suspension beam. Besides, the suspension beams analyzed in those works are simple straight beam.

A laterally driven accelerometer is simulated, designed and fabricated by Lüdtke, et al. (2000). The analysis is concentrated on the stiffness constant and the sensitivity which was represented by the capacitance per displacement. Chae et al. (2000), Chae et al. (2002), and Luo et al. (2002) studied on performance of comb finger type lateral capacitive sensing accelerometer. They analyzed the effect of changing device geometric and stiffness constant, and optimized the accelerometer based on device sensitivity. After the analysis by using finite element analysis, the device was fabricated and tested. Experimental result agreed well with numerical result in which the electrostatically measured sensitivity was close to the calculated value. So it was proved that the predictions by using finite element analysis are feasible.

2.2.2 Stiffness constant and resonant frequency in comb finger type device

Researches on comb finger type suspended by folded beam as used in capacitive accelerometer are seldom found although this type of design has been used as actuators for a long time. This suspension folded beam comb finger type as used in actuators in general has similar structural design and/or similar critical characteristics with comb finger type folded beam as used in capacitive accelerometer. Therefore, the research achievements of this comb finger type in actuators can be used as reference in the study of comb finger type folded beam in accelerometer.

Legtenberg et al. (1996) has presented the design, fabrication and experimental results of lateral-comb-drive actuators for large displacements at low driving voltages. The deflection behaviors of clamped–clamped beams, crab-leg beam and folded beam design had been modeled. The bending effect derived in stiffness constant and the resonance frequency was also discussed. The stiffness ratios in x- to y-direction, which is one of the important parametric for stability of large displacement device, have been found based on Hooke's law and the total potential energy. The experimentally determined stiffness constant of the folded-beam design was higher than the modeled values due to the fabrication problem.

Zhou et al. (2003) stated that electrostatic comb-drive actuators are the most widely used microactuators in MEMS. From their research, a new design called tilted folded was suggested to be used. The suspension designs were also studied numerically using the finite element method (FEM). Analytical calculations and FEM simulations were then compared. They stated that the proper design of the suspension beam is the most effective way to stabilize the actuator and achieve a large deflection. Hooke's law, the Rayleigh– Ritz method and the total potential energy were used in the derivation.

Tay et al. (2000) and Liu et al. (2002) had carried out the analysis on micro-resonator, which is the same design model with accelerometer but actuates by electrostatic forces. Tay et al. (2000) focused on the electrostatic spring effect in the study, whereas Liu et al. (2002) focused on the fabrication error in analytical derivation. Wittwer et al. (2003) carried out research on cantilever and folded beam by using Castigliano's displacement theorem to find the optimal fillet ratio for cantilever and folded suspension beam. Both bending and shear effects were analyzed. Borovic et al. (2005) studied on folded beam optical switch, with was actuated by electrostatic forces. In the research, the experimental result was verified by using MATLAB simulation.

Urey et al. (2005) studied on torsional scan mirror suspended with microbridge. By using various mirror shapes and suspension beam dimensions as parameters, a set of analytical formulae was presented to predict the natural frequency. The uniqueness of this research is that the determination of the stiffness constant and effective initial formulae for the five most important modes of common beam design in MEMS devices. Saint Venant theory was used in the determination of torsional mode, Euler-Bernoulli beam theory was used in sliding and rocking modes, and Rayleigh's method together with Timoshenko beam theory, with rotary inertia and shear deformation effects, was used in derivation of effective mass. The analytical result was partly verified by experiment. The correction factor was then introduced to reduce the error between analytical formulae and FE predictions.

Based on the available literatures, it is found that natural frequency and sensitivity of the device are popularly used to determine the performance of movable sensor and actuator. Stiffness constant are normally used as the intermitted parameter to determine natural frequency and sensitivity since stiffness constant is found based on the combination of the device geometric and material properties. Researches on stiffness constant are mostly carried out on seismic mass accelerometer and comb finger type actuator. Although comb finger type capacitive accelerometer has been agreed to be one of the type of accelerometer, but there are only a few researches on it. Since the research on comb finger MEMS actuators, which have similar basic structure with comb finger accelerometer (consist of suspension beam, proof mass and fingers), have been carried out for many years, the way of analysing the device can be used as reference in the study of accelerometer. However, those achievements can not be applied directly in micro-accelerometer, since the structure arrangement of actuators is difference form accelerometer. Besides, the critical characteristics of the accelerometer are always different with actuator. The stiffness constant and the resonant frequency are normally determined by analytical derivation, finite element simulation and experiment. The common way to determine stiffness constant of a suspension beam analytically is using Rayleigh's method, Timoshenko beam theory and Euler-Bernoulli beam theory. From the literatures, only the simplest designs of suspension beam, such as cantilever and microbridge, have been derived. Some researchers, such as Chae et al. (2000 & 2002) and Borovic et al. (2005), have derived the stiffness constant of folded beam, but it is quite simple which leads to too many assumptions and does not show accurate result compared with simulation results and experimental results. In those works, the parametric study on the design of the suspension beam or on one or various suspension designs are commonly investigated.

2.3 Finite element analysis (FEA)

2.3.1 Finite element analysis of accelerometer

Due to time consuming and high cost for fabrication of a new device for testing, finite element analysis, which assumption can provide good accuracy, is always the choice for the early state of new design. Wittwer et al. (2004) performed their research by using unmentioned software for optimal fillet ratio in cantilever and folded beam design. Tay et al. (2000) used IntelliCAD^{TM} electrostatic simulation to isolate the electrostatic spring effect and compared with the experimental observations. Dynamic electrostatic spring effect showed 20% error between experimental result compared with analytical result, whereas the difference between FE result and analytical result is only around 1%. Chae et al. (2000) and Chae et al. (2002), simulated the model of accelerometer by using both MATLAB® and ANSYS®.

2.3.2 Finite element simulation with ANSYS®

ANSYS® has been used by researchers for few years, and always achieves the requirement for a good result. In Yu et al. (2001) work, the analytical model was verified by the finite element analysis for the resonance frequency, the dynamic response of the microstructure and the sensor sensitivity by using ANSYS™ 5.3. The good results demonstrate the validity of the modeling assumptions. Wang et al. (2004) used ANSYS 5.7 to simulate the frequencies from the first to fifth mode. Since the sensitivity is proportional to the effective mass, the ignorance of the effective mass of the four transducer beam in analytical approach leads to the slightly smaller analytical sensitivity values than the results obtained by FEM. However, they still concluded that FEM results agree well with analytical result with slight difference.

In Zhou et al. (2003) work, the suspension designs were also studied numerically using the ANSYS 5.5, in which the geometric nonlinearities, such as large deflections and stress stiffening, were considered. Analytical calculations and FEM simulations were compared. The Newton–Raphson method was then used to solve, correct and resolve a nonlinear analysis. The model was then fabricated using the standard surface micromachining technology (MUMPs) and tested. The experimental results were in good agreement with the analytical predictions.

Urey et al. (2005) compared the mode frequencies between the analytical derivation results with the FE predictions using ANSYS™ for a wide range of suspension beam dimensions. They stated that the FE simulations were time consuming and engineering of the vibration mode frequencies for the optimal design was difficult without having good analytical models for fast calculations. Once a good design point was reached using the analytical calculations, FEM simulations can be used for more accurate predictions using the exact devices geometry.

From the available literatures, finite element analysis is popularly used in the natural frequency simulation since it gives reasonably accurate result. However, in most literatures, the finite element analyses are time consuming and many derivation have to be carried out as the input formulae for the finite element simulation. This step is necessary for the early version of finite element software including ANSYS®. However, for ANSYS® version 6.0 and above FE formulae are automatically program and the model can be analyzed without requiring specified formulae. In the above literature, finite element analysis using ANSYS® shows a good result compared to the analytical and experimental analysis.

2.4 Optimization of accelerometer design

Based from the literature survey, optimization in accelerometer is also investigated by several researchers. Kovács et al. (2000) and Yu et al. (2001) had optimized the seismic mass accelerometer by using analytical derivation. Chae et al. (2000) also optimized the comb finger type accelerometer by using analytical derivation. Zhou et al. (2002) proposed a general architecture for using evolutionary algorithms to achieve MEMS design synthesis. Functional MEMS devices are designed by combining parameterized basic MEMS building blocks together using Multi-objective Genetic Algorithms (MOGAs) to produce a pareto optimal set of feasible designs. The iterative design synthesis loop is implemented by combining MOGAs with the SUGAR MEMS simulation tool. Given a high-level description of the device's desired behavior, both the topology and sizing are generated. The topology or physical configuration includes the number and types of basic building blocks and their connectivity. The sizing of the designs entails assigning numerical values to parameterized building blocks. A sample from the pareto optimal set of designs is presented for a meandering resonator example, along with convergence plots.

2.5 Summary

The literature review on the analysis of micro-accelerometer has been discussed. Table 2.1 shows the summary of the literature review. From this table and from the previous discussion on literature review, it can be observed that accelerometer which is the start of MEMS is still continuing to grow, and now it is applied to various applications. Comb finger type capacitive accelerometer is the most successful type accelerometer since it gives high sensitivity in capacitance due to its large sensing area. However, the researches on comb finger type accelerometer are limited. By tracing back to the application of comb finger type device, this design has been used in actuator for a long time. The research achievements in comb finger as used in actuator can be used as the reference for comb finger type as being used in accelerometer. Regardless of being used in accelerometer or actuators, resonant frequency and stiffness constant in comb finger type are the interested parameters that are being considered in the structural analysis. These parameters have been found out analytically, numerically or experimentally.

These parameters depend mostly on the structural design of the suspension beam. However, by referring to the literature review, structural analysis of suspension beam has not been well investigated in comb finger type accelerometer. Thus, the present work attempts to fill the lack in the above study and the goal is to develop a methodology consisting of analytical and numerical investigation for structural analysis and optimization of the structural design for the suspension beam in comb finger type accelerometer.

Table 2.1 Summary of the literature review

researcher	device	sensing type	suspention beam type	device shape	research type	input parameter	output parameter	principle	tool	contribution
Borovic et al., 2005	optical switch (actuator)	electrostatic--capacitive	folded beam	comb finger	experimental, & simulation	voltage	effective mass, stiffness constant, displacement		MATLAB/ SIMULINK	compare: open loop vs closed loop control
Chae et al., 2000	accelerometer	capacitive	folded beam	comb finger	analytical, otimazation, FE, fabrication, testing	spring constant of beams, no of comb finger, length of comb finger (other devices geometry & material properties as dependent parameters)	high sensitivity, low noise, k, fn, cross axis sensitivity (FE simulation), shock resistance (FE simulation), stress(FE simulation)		MATLAB, ANSYS	micro-g performance, high-precision accelerometer, high sensitivity with low noise
Chae et al., 2002	accelerometer	capacitive	folded beam	comb finger		length of finger, no of finger, k, initial gap, thickness	resolution: sensitivity/TNEA, simulation result as above		MATLAB, ANSYS	fabrication process, system
Kovács et al. (2000)	accelerometer	piezoresistive/ capacitive	cantilever and bridge type	seismic mass	Analytical, optimazation	geometrical parameters: length, width, thickness	strain, stress, max deflection, bending moment, shear force, bandwidth & device sensitivity, physical sensitivity, eigen frequancy,	Rayleigh's energy principle	MATHEMATICA (analytical)	appropriate length ratio for minimum necessary bandwidth. Simplified model, diff beam type
Legtenberg et al., 1996	actuators	electrostatic--capacitive	clamped-clamped, crab-leg flexure,folded flexure	comb finger	design, fabrication and experimental	several suspension design	deflection behavior, axial spring constant, k ratio, resonant frequency	Hooke's law, the total potential energy		k ratio
Liu et al. 2002	resonators (actuator)	electrostatic--capacitive	folded beam	comb finger	analysis and design	proof mass area and perimeter, and the beam width	k, natural frequency, effective mass, effective E, displacement			frequency characteristics. consider fabrication error in analytical derivetion
Lüdtke, et al. 2000	accelerometer	capacitive	folded beam	comb finger	FE simulated, designed and fabricated	structure height	oscillation & frequency. K, displacement sensitivity (cap/d)		ANSYS	new fab method for high aspect acc
Luo et al., 2002	accelerometer	capacitive	multi-layer suspending springs, Corner attachment Serpentine spring. two turns each serpentine spring	comb finger	analytical, experimental		k, fn (exp), displacement sensitivity(exp), capacitive sensitivity(exp), noise, shock performance and cross axis sensitivity	Laplace transformation		multi mat, Eeff
Tay et al., 2000	resonators (actuator)	eletrostatic	folded beam	comb finger	analytical, FE, experimental	E, size parameters, structure mass	resonant frequencies, electrostatic spring effect	Rayleigh's energy principle	simulation model: IntelliCAD, FEA model: ANSYS	electrostatic spring effect. Note: unable to verify experimentally with analytical
Urey et al., 2005	torsional scan mirror (sensor)		microbridge	central mass with comb finger	analytical, FEA, experimental	various mirror shapes and flexure dimensions	stifness constant, effective inertia & natural frequency of the first five vibration modes.	Euler-Bernoulli beam theory, Rayleigh's method	ANSYS Block-Lanczos solver	correction factor
Wang et al.(2004)	accelerometer	PZT	cantilever, bridge type, four symmetric beam	seismic mass	analytical model, FEA	devices geometry (thinkness) & elestic properties (diff material)	strain, stress, bending moment, charge, displacement, sensitivity, bandwidth, resonance frequency, (FE: max acc, 1st-5th mode)	Rayleigh's energy principle	ANSYS 5.7 (FEA) Block-Lanczos solver	diff beam type, compare analytical & FE, PZT/Si thickness ratio.
Wittwer et al. 2004	general		cantilever, folded beam		FEA, analytical	devices geometry & elestic properties	displacement, compliance (c, inv of k), error	Castigliano's displacement theorem		optimal fillet ratio. analysis: combined both beanding and shear effect
Yu et al, 2001	accelerometer	Piezoelectric	cantilever, two cantilever, two symmetry (bridge), four symmetric beam	seismic mass	analytical, FEA, optimization	Material characteristics, geometry	dynamic response (fn), trade-off, sensitivity, bending moment, stress, K, noise	Hooke's law	ANSYS 5.3	dynamic response and trade-off between several design
Zhou et al. 2003	actuators	electrostatic	tilted folded beam	comb finger	analytical prediction, FEM, experimental	k ratio, beam dimension, finger gap, finger initial overlop area	negative spring constant, deflection, spring constant	Hooke's law, Rayleigh-Ritz mehtod, the total potential energy	ANSYS 5.5 (Newton-raphson method)	tilted folded beam, the stability of the comb-drive actuator is improved and the stable travel range is enhanced
Zhou et al., 2002	resonator (actuator)		Corner attachment Serpentine spring. two turns each serpentine spring	comb finger	MOGAs, SUGAR			evolutionary algorithms, Multi-objective Genetic Algorithm	combining MOGAs with the SUGAR MEMS simulation tool	methodologies for device synthesis, MOGAs in MEMS

CHAPTER 3
ANALYTICAL AND NUMERICAL FORMULATIONS

3.0 Overview

In this chapter the governing equations and the finite element formulation of stiffness constant for suspension beam in micro-accelerometer are derived and solved. The finite element simulation by using ANSYS® is also presented. The main topics discussed in this chapter are shown below:

- Working mechanism of comb finger type accelerometer
- Several design models of suspension beams
- Equivalent stiffness constant in spring mass model
- Analytical derivations
- Finite element formulations
- Finite element modeling with ANSYS

3.1 Working mechanism of comb finger type accelerometer

Comb finger type device is one of the most common designs in MEMS device, and it is normally used as sensors and actuators. It consists of two finger structures, namely fixed fingers and movable fingers. The fixed fingers are anchored to the substrate. The movable fingers are attached to a proof mass suspended by compliant/suspension beams anchored to the substrate. Those movable and fixed fingers are sometimes called sensing electrodes because they sense the acceleration and measure it by using electronic circuitry. A typical design of comb finger type accelerometer is shown in Figure 3.1.

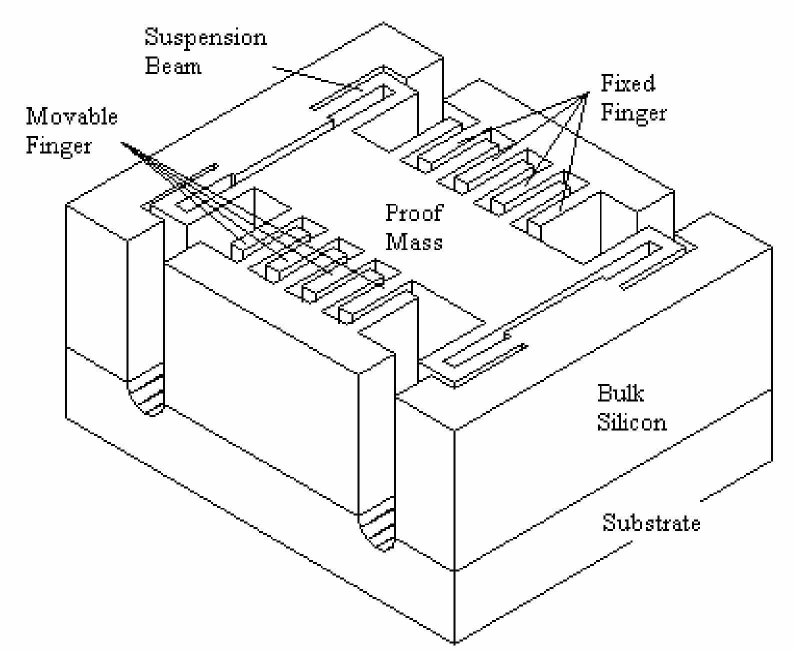
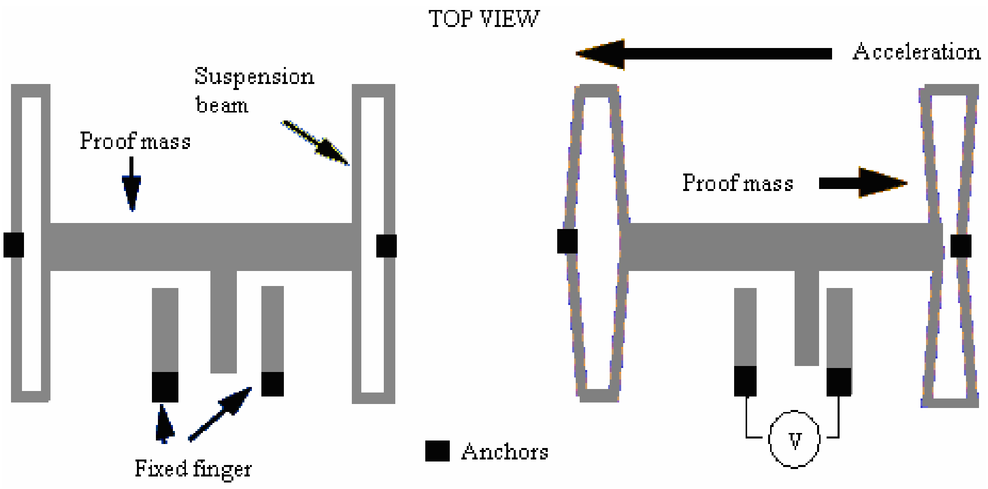


Figure 3.1: Typical design of comb finger type accelerometer (Lee et al, 2001)

In an actuator, when a voltage difference is applied between the comb structures, an electrostatic force will be generated; resulted in a deflection of the movable comb fingers, and move the movable parts to the desired position. However in a sensor, when a comb finger type accelerometer is subjected to a measured external force, in this case acceleration, the force is transferred to the proof mass through the suspension beam. The proof mass together with movable fingers move along and against the forced direction, while the fixed combs remain stationary, as shown in Figure 3.2(a) and Figure 3.2(b). This movement changes the capacitance between the fixed fingers and the movable fingers. This capacitance can be measured and calibrated with the applied external force.



(a) In equilibrium condition (b) With an acceleration toward the left

Figure 3.2 Schematic arrangement of accelerometer (Doscher 2000)

The operations and response of the comb finger type accelerometer are controlled by mass of the proof mass (M), stiffness constant (K) of suspension beam, the damping (D) of air surrounding the structure, finger overlap area (A), finger initial sensing gap (d_0), and, of cause, the initial capacitance and acceleration. Among them, the stiffness constant of the suspension beam takes significant role. The physical meaning of dimension of a comb finger type accelerometer is shown in Figure 3.3.

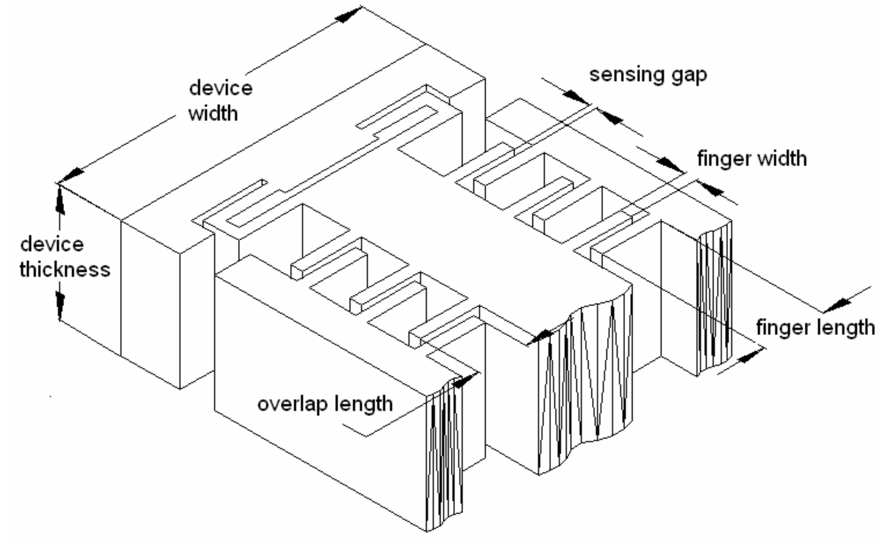
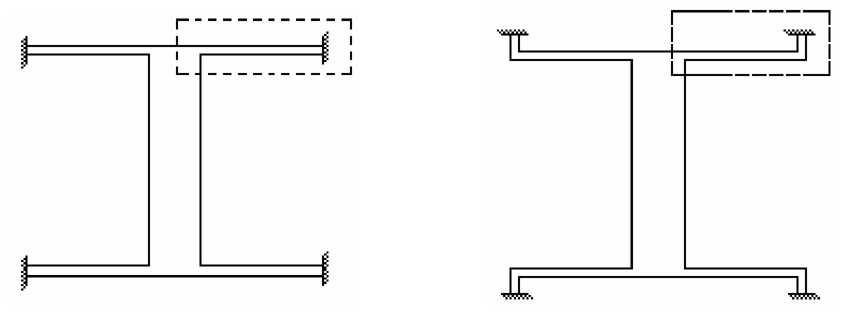


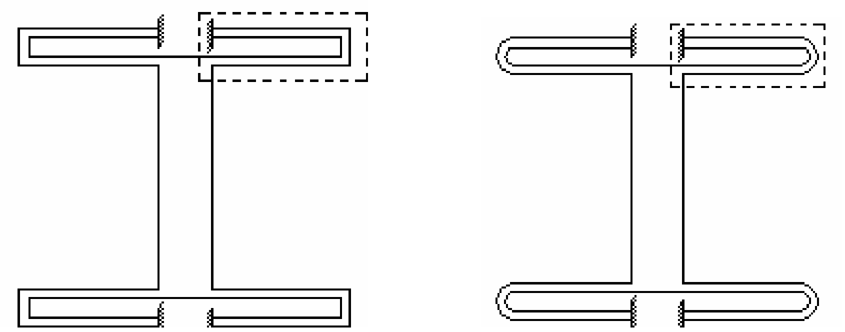
Figure 3.3: The physical meaning of dimension of a comb finger type accelerometer

3.2 Several design models of suspension beam

Several design models of commonly used suspension beam comb finger type accelerometer are shown in Figure 3.4. Only the movable parts, without fingers, are shown. The model consists of a proof mass suspended by four common beam designs namely straight beam (Fig 3.4(a)), crab-leg beams (Fig 3.4(b)), folded beam (Fig 3.4(c)), and round folded beam (Fig 3.4(d)). The simplified models have been showed here because this research focuses on the analysis of different designs of suspension beam, since the stiffness constant of the devices depends mainly on the beam design.



(a) Straight beam (b) Crab-leg beams



(c) Folded beam (d) Round folded beam

Figure 3.4: Different suspension beam design models in accelerometer for analysis

3.3 Equivalent stiffness constant in spring mass model

Most accelerometers are built on the principles of mechanical vibration. Principle component of an accelerometer is a proof mass supported by suspension beams or can be taken as springs. By referring to Figure 3.4, the proof mass is suspended by four beams in four edges. Therefore, this proof mass can be approximated by a central proof mass supported by four springs. The free body