STRUCTURAL MODELLING AND ANALYSIS OF THE BEHAVIOURAL DYNAMICS OF FOREIGN EXCHANGE RATE

By

YIP CHEE YIN

June 2006

Thesis submitted in fulfilment of the
Requirements for the degree of
Doctor of Philosophy

ACKNOWLEDGMENTS

I wish to thank my dissertation supervisor, Professor Quah Soon Hoe, to whom I am in great debt for his general guidance in this research and in all my graduate studies. His advice and support have been invaluable to me. I would also like to extend a big thank you to Professor James Hamilton of the University of California, San Diego, for supplying me with materials, data and advice.

Last but not least, I would like to thank the School of Mathematical Sciences, and also all the staff, lecturers and professors alike for their help one way or another.

I also would like to express my deep gratitude to my family for their unconditional love and support. Their encouragement was undoubtedly priceless. Lastly, I would like to thank my wife, Mee Yen, for her endless support at all times and in all situations. This dissertation is dedicated to her and my two children, Seng Jon and Seng Joe.

CONTENTS

List of Figures

List of Tables

List of Abbreviations and Symbols

With respect to notations and abbreviations, we have the following rules:

1 Britpus denotes the daily exchange rates series for British Pound.

2 LBritpus denotes the log daily exchange rates series.

3 DLBritpus denotes the daily exchange rates returns series

1 WeBritpus denotes the weekly exchange rates series

2 LWeBritpus denotes the log weekly exchange rates series

3 DLWeBritpus denotes the weekly exchange rates returns series

4 QBritpus denotes the quarterly exchange rates series.

5 LQBritpus denotes log the quarterly exchange rates series

6 Lconsumption denotes the log quarterly consumption series.

7 Bold capital letter stands for matrix and small letter for vector.

8 Without bold means that the letter represents scalar variable.

9 Small letter t stands for one particular period, capital N stands for total number of periods or observations.

10 Lcons stands for Lconsumption.

11 Cy or C stands for cycle.

12 RMSE denotes root mean square error

13 MAPE denotes mean absolute prediction error

14 AR denotes autoregression

15 AR_1 denotes autoregressive model of order 1

16 MA denotes moving average

17 MA_1 denotes moving average model of order 1

18 ARMA denotes autoregressive moving average

19 ARIMA denotes autoregressive integrated moving average

20 ARFIMA denotes autoregressive fractionally integrated moving average

21 D denotes the first difference and DD denotes the second difference

22 Dm denotes Denmark

23 Sn denotes Sweden

24 Sz denotes Switzerland

25 Sin denotes Singapore

26 HK denotes Hong Kong

27 Frnfrus denotes French Francs

28 Dtchgus denotes Dutch Guider

29 Austrus denotes Australian Dollar

30 Euro denotes European Dollar

31 Jappynus denotes Japanese Yen

32 Germdus denotes German Deutschmark

33 Swisfus denotes Switzerland Francs

34 Sgd denotes Singapore Dollar

35 Twd denotes Taiwan Dollar

36 Php denotes Philippine Pesos

37 Sdr denotes Currency based on the country’s reserve.

PEMODELAN STRUKTUR DAN ANALISIS TINGKAHLAKU DINAMIK BAGI KADAR PERTUKARAN WANG ASING

ABSTRAK

Tesis ini berkaitan dengan Kadar Wang Pertukaran Asing (KWPA) yang dihasilkan oleh satu regime urusniaga bebas. Pada amnya, kita mengkaji Pemodelan Struktur dan Analisis Tingkahlaku Dinamik Kadar Pertukaran Wang Asing. Kita bermula dengan menganggap KWPA sebagai satu siri kewangan. Kita memodelkan KWPA dengan menggunakan dua jenis metadologi: kaedah ARFIMA dan kaedah Penapis Kalman (KFBM) dengan harapan ia boleh menghasilkan satu model KWPA yang terbaik. Objektif kita ialah untuk menggunakan model yang terbaik ini sebagai satu alat bagi menunjukkan keadaan stabil bagi kadar pertukaran wang asing dalam jangka masa yang panjang. Dalam proses membuat analisis, kita juga hendak mengkaji tingkahlaku dinamik KWPA.

Kita telah membina satu model YQ- ARFIMA yang dinamik and sesuai untuk pemodelan siri ingatan yang panjang dan pendek. Dinamik bermaksud bahawa parameters YQ-AFRIMA boleh diubahkan dengan mengikut ciri-ciri siri masa yang berkenaan. Perubahan yang mudah ini boleh dilakukan dengan menggunakan ujian *t* secara berturut-turut. Model YQ-ARFIMA ini didapati adalah lebih baik daripada model KFBM. Kita juga dapati YQ-ARFIMA menunjukkan prestasi 12 kali lebih baik daripada KFBM dalam pemodelan struktur siri masa dalam jangkamasa panjang. Dalam jangkamasa pendek, YQ-AFRIMA menunjukkan prestasi yang lebih baik lagi dengan suatu nilai RMSE, lebih kurang 40 kali dan nilai MAPE, lebih kurang 36 kali lebih tepat daripada kaeah KFBM. Sungguh pun begitu, model KFBM didapati lebih baik dalam analisis perubahan (*breaks*) struktur. Satu

sebab ialah ia lebih sesuai untuk memodelkan siri yang mempunyai komponen berkitaran (*cyclical components*). Saiz sampel yang paling baik untuk pemodelan jangka masa pendek ialah di antara 200 dan 1300 pemerhatian sementara bagi jangka masa panjang saiz sampel yang paling sesuai ialah antara 1500 and 2000 pemerhatian.

Kita juga dapati model kita adalah stabil merentasi 22 siri KWPA antara bangsa dan juga stabil merentasi perubahan saiz sampel. Kita dapati YQ-ARFIMA adalah lebih baik daripada random walk dalam ramalan yang diukur dari segi fungsi hilang RMSE dan MAPE. Dengan ketepatan ini, kita boleh menggunakan YQ-ARFIMA untuk menunjukkan tingkahlaku reversi min (*mean reversion*) bagi KWPA.

Kita menyiasat pengaruh perubahan pada model kita. Kita dapati keputusan adalah lebih teruk jika kita mengabaikan perubahan dalam analisis ramalan. Tambahan pula, kita telah membina satu kaedah pembahagian (*dissection*) untuk mengkaji cerapan luar biasa (*outliers*) apabila bilangannya adalah kurang. Kita dapati dalam pemodelan, cerapn luar biasa (*outliers*) yang teruk, lebih baik di buang untuk menambah kejituan ramalan.

Kita dapati komponen berkitaran (*cyclical components*) KWPA berubah dengan secara positif mengikut turun-naik komponen berkitaran bagi nilai keupayaan para pengguna. Ini menunjukkan terdapat satu hubungan dalam jangka masa panjang di antara KWPA dan nilai keupayaan para pengguna. Dalam lain perkataan, kedua-dua siri bergerak bersama. Dengan keputusan ini, kita sekurang-kurangnya boleh mengetahui tanda perubahan KWPA dengan meneliti nilai kupayaan para pengguna.

Dalam pemodenan satu persamaan tunggul, kita dapati dengan menambah satu komponen berkitaran sahaja daripada keupayaan para pengguna boleh memburukkan keupayaan ramalan YQ-ARFIMA. Walaupun begitu, kita dapati juga dengan menambah komponen berkitaran daripada LQBritpus bersama-sama dengan komponen yang sama daripada Lconsumption kepada model yang bukan terbaik, boleh menambahkan keupayaan ramalan. Tetapi bagi model yang terbaik, tambahan komponen berkitaran tidak langsung mengaruh keupayaan ramalannya.

STRUCTURAL MODELLING AND ANALYSIS OF THE BEHAVIOURAL DYNAMICS OF FOREIGN EXCHANGE RATE

ABSTRACT

This thesis deals specifically with the foreign exchange rates that resulted from free float regimes. In general, we study the structural modelling and analysis of the behavioural dynamics of foreign exchange rates. We start with the recognition that foreign exchange rate is a financial time series. We model the foreign exchange rate by using two popular methodologies: ARFIMA and a Kalman filter based method (KFBM) with the hope that it can produce the best exchange rate model. Our objective is to use this best model to show the mean reverting behaviour of the exchange rates. In the course of carrying out the experiments and analysis, we would like to study the behavioural dynamics of the exchange rate.

We have developed a dynamic YQ-specified ARFIMA model for the long and short term memory modelling. Here 'dynamic' simply means that the model parameters can be altered according to the characteristics of the particular time series. This seemingly easy alteration is made possible by using sequential *t* tests. This YQ-specified ARFIMA model performs very well and much better than the Kalman filter based method. We have shown without any doubts that YQ-ARFIMA is about 12 times better in term of the RMSE values and 10.6 times better in term of the MAPE values, than the Kalman filter based method in long-term memory modelling. Moreover, for short-term memory, the YQ-ARFIMA performs even better, giving a RMSE value of 40 times and a MAPE value of 36 times

better than the Kalman filter based method. However, KFBM seems to do better in analysing structural breaks. The possible reason is that KFBM is more suitable for series with cyclical components. In addition, output results show that for short-term memory modelling, the ideal sample size is about 200 to 1300 observations while for long-memory modelling the ideal sample size is between 1500 and 2000 observations.

The output of experimental results shows that this YQ-specified ARFIMA model is robust across 22 foreign exchange markets and across sample sizes. We have found that this YQ-ARFIMA beats the random walk model soundly in out of sample forecasting in terms of the loss functions RMSE and MAPE. With this positive result, we use this YQ-ARFIMA model as a tool to show the mean reverting behaviour of the exchange rates.

We investigated the influence of the breaks on our predictive models. We found that it is costly to ignore structural breaks in forecasting. We have devised a practical method, which we refer to as the dissection method for the correction of outliers when there are not many of them in the series. However, we have found that discarding the section of the data that contains extreme outliers can improve the predictive power of the model tremendously.

We have shown that the cyclical components (stationary) of the exchange rate series are positively related with the corresponding cyclical components of the consumption series. This implies exchange rate series have a long run relationship with consumption. To put it differently, they move together. With this result, we can at least keep track of the sign of the exchange rate by examining the consumption rate.

In the single equation dynamic specification modelling, we have shown that adding only one cyclical component of consumption into the model can deteriorate the predictive ability of the YQ-ARFIMA model. However, we have found that adding the appropriate cyclical component of the LQBritpus series together with that of Lconsumption to models other than the best-fitted model can improve its predictive ability substantially. For the best fitted-model, no combination of the cyclical components can improve its predictive ability at all.

CHAPTER 1

INTRODUCTION

1.1 Outline

The currency crisis in Asia in 1997 has created havoc and disorder in the financial and economic systems of countries like Malaysia, South Korea, Indonesia and Thailand. To prevent such currency crises from recurring, it is imperative for us to study the foreign exchange rate (forex) time series. However, we have inherited an “international monetary system” that has no international standardized measurement of exchange rate. This happened since the collapse of The Gold Standard (1880-1914) system, The Gold Exchange Standard (1925 – 1936) system and The Bretton Woods System (1947 – 1970) collapsed. Over the thirty years, since the breakdown of the Bretton Woods system, countries have adopted a wide variety of regimes, ranging from dollarization and currency boards to simple pegs, basket pegs, crawling pegs, and target zones to clean and dirty floats. Figure 1.1 shows clearly that, the forex for United Kingdom during the period before the breakdown of the Bretton Woods System, 1972, is either constant or decrease sharply due to devaluation of the currency.

As a whole, all these regimes can be classified into two main types: pegged or float. In this thesis, we make mainly empirical research on floating regimes and in our case, free floats. Unlike stocks and derivatives, forex time series are not pure series generated by economic agents. Moreover, the forex trading volume is rather small compared to the volume of the stock traded in the stock exchange.

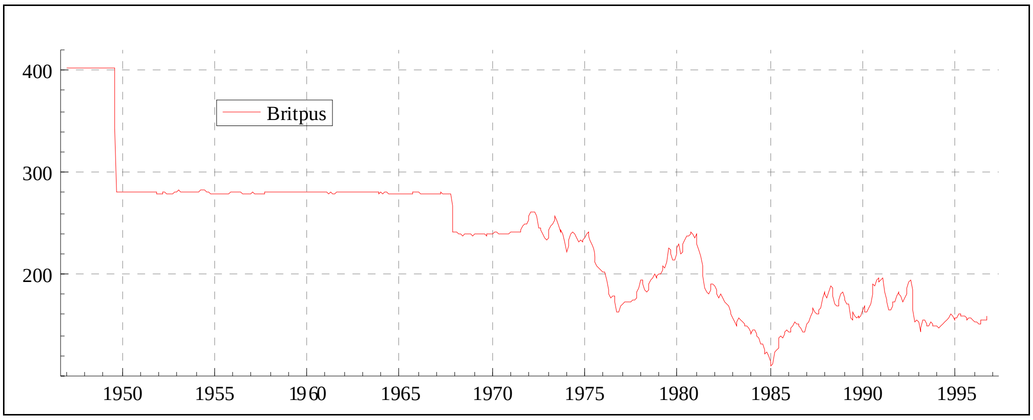

Figure 1.1 – Graph of British pound per US dollar exchange rate
(Note: This exchange rate Britpus is obtained from the Federal Reserve Board US)

On top of this, the time span of the exchange rate series is rather short if we consider only the period after the breakdown of the Bretton Woods System for empirical research. Perhaps, it is because of this small trading volume, that the central bank is able to intervene in its trading activity in times of needs.

Relatively speaking, there are not many research papers available on this subject. The reason could be that this intervention of the central bank makes the forex time series artificial, and artificial time series is very difficult to model or analyse. Another reason is that the time span for the exchange rate series after the recent floats is rather short for statistical inference and tests. The next question is “What are the uses of studying exchange rate?”

Exchange rate behaviour used to be described by a simple proposition, viz., the long run purchasing power parity (PPP), which states that the national price levels should be the same when expressed in a common currency. The long run PPP hypothesis has created two types of exchange rates: the nominal exchange rate, which is defined as the price of one currency in terms of another, and the real exchange rate, which is defined as the nominal exchange rate adjusted for the differences in the relative national price level. Whether the long run PPP holds or the real exchange rate is stationary have important economic implications on a number of fronts. Among these fronts, the easier to understand is that it is used to determine the degree of misalignment of the nominal exchange rate and the appropriate policy response, the setting of exchange rate parities, and the international comparison of national income levels. Our primary interest in the long run PPP is to verify this mean reversion behaviour by using some existing tools, proposing a new tool, and constructing an effective exchange rate model for accurate forecasting.

It is by now an accepted fact that financial time series exhibit three important properties. These three important properties form the basic ingredients for further research and analysis.

1 Large absolute returns occur more frequently than one would expect if the data follows a normal distribution.
2 These large absolute returns tend to occur in clusters.
3 Large negative returns tend to appear more often than large positive ones in stock markets, while it may be true the other way around for foreign exchange rates (see Franses, 1998; Cochrane, 1999).

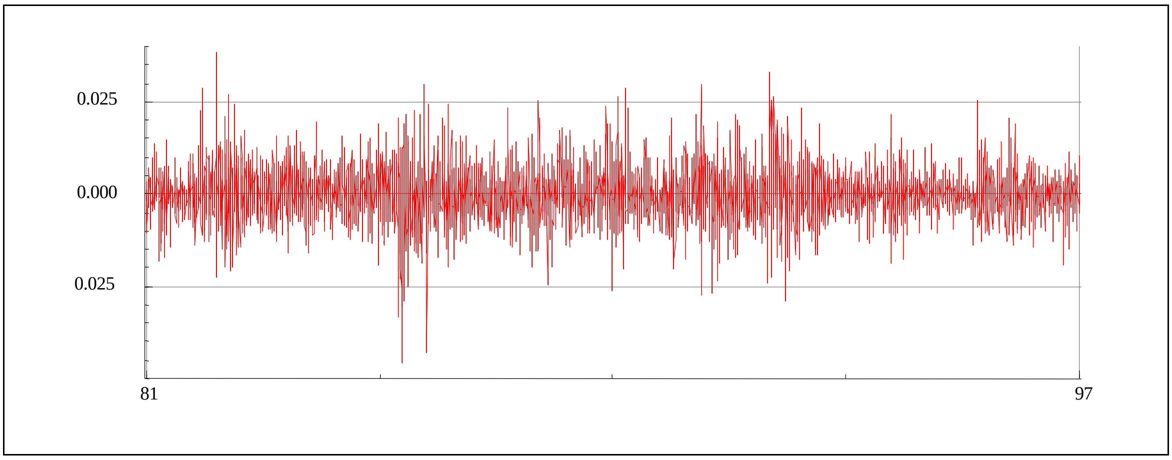

Figure 1.2 – Exchange rate returns occur in clusters
(Note: DLBritpus denotes the returns of British pound exchange rate per US dollar)

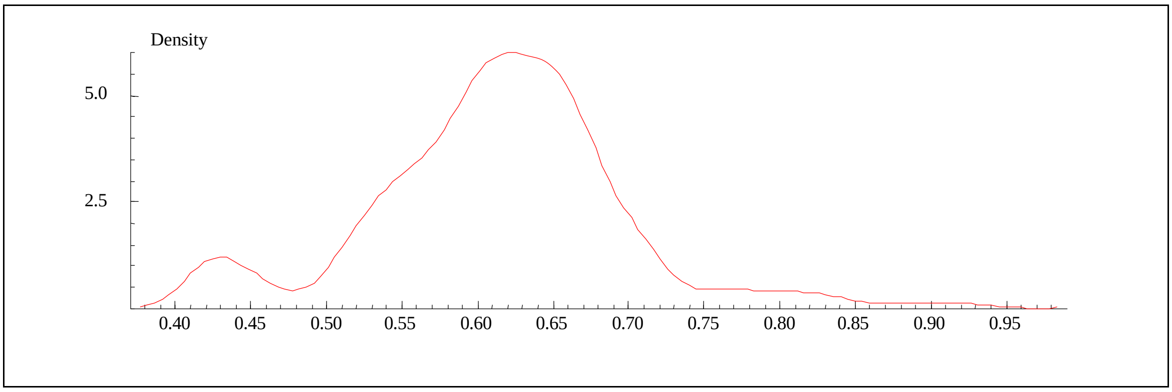

Figure 1.3 – Fat tails and shape peak of the density graph of Britpus
(Note: Britpus denotes British pound exchange rate per US dollar)

Large absolute returns occur in clusters as in Figure 1.2, and Figure 1.3 shows the fat tails and shape peak of Britpus, which are the general characteristics of financial time series.

Forex is an example of a financial time series, and thus, is expected to exhibit the above characteristics as widely documented for the past decade. In statistical terms, fact number one is equivalent to saying that the actual kurtosis is very much larger than 3 for the actual distribution. In graphical terms, it implies that the actual density graph has a sharp peak and fat tails. Figure 1.3 shows clearly these two characteristics of Britpus. We are going to work on these fat tails. In volatility research, the contributing factor for these fat tails is that there exist additive outliers (AO), innovation outliers (IO), level change outliers (LC) and variance change outlier (VC). The existence of these outliers (aberrant observations), it can be said as the main cause for nonstationarity in the financial time series. It is accepted that these outliers cannot be modelled by the standard GARCH (see Baillie and Bollerslev (1989)) model. One possible cause for this outcome is that the GARCH model (generalized autoregressive conditional heteroskedastic) model is not able to capture these so-called outliers on the returns. We assume that in a structural time series model for foreign exchange rate, the model cannot capture such outliers. Thus, we have to devise methods to deal with these outliers, which cause not only structural changes but also affect the forecasting performance of the relevant models. Generally, there are two methodologies to analyse a structural time series.

1 The Box and Jenkins methodology popularised by Box and Jenkins in 1970, and improved and refined to advanced forms such as the Autoregressive Integrated Moving Average (ARIMA) and the Autoregressive Fractional Moving Average (ARFIMA).

2 The Kalman filter and state space representation system used to extract unobservable variables from the observed series (hereafter referred to as KFBM).

This thesis basically tries to find an effective exchange rate structural model, which can outperform the random walk model in out of sample forecasting. If this structural model has this property, we can use it as a tool to ascertain the mean reversion behaviour of the exchange rate for any sample size. To obtain this structural model, first we have to address the question on which methodology should be preferred in structural modelling: the ARFIMA method or the Kalman filter based method (KFBM). In the course of answering this question, we will come up with the best structural model for use as a tool to determine whether an exchange rate series exhibits mean reverting behaviour. In studying exchange rate behaviour, we also propose a procedure for modelling the additive outliers in a structural time series model, analyse the structural breaks and their influence on forecasting, investigate the relationship between consumption rate and exchange rate, and, lastly, determine whether the cyclical components of the consumption rate or the exchange rate can in any way improve the predictive ability of our best structural model, the Yip-Quah specified ARFIMA model (hereafter referred to as YQ-ARFIMA).

The layout for the rest of this chapter is as follows: In Section 1.2, we discuss the objectives of this dissertation. In Section 1.3, we present the constraints of the research and then, we discuss the methodologies used in the research in Section 1.4. In Section 1.5, we discuss our data set and the software used in the research. Then, we discuss the three common forms of a financial time series in Section 1.6. In section 1.7, we give an outline for each subsequent chapter.

1.2 Objectives of this Research

Our main objective in this thesis is to find an effective exchange rate structural model, which can outperform the simple random walk model in out of sample forecasting. We can use this structural model as a new tool to verify the parity reversion and thereafter, make accurate prediction of exchange rate. We proceed as follows: first, we are going to look for a variant of the ARFIMA model that is able to produce good forecasting results. We can achieve this by making a comparative study between this variant of the ARFIMA model and the Kalman filter based method for structural modelling. We would then determine whether this variant of the ARFIMA model can be used as a tool to ascertain the mean reverting behaviour of the exchange rate, by checking whether this model can beat the random walk model or not. In addition, we will use the fractional integration technique and the impulse response function to verify the existence of parity reversion for any sample size. Only then, we will study the behaviour of exchange rate with respect to the influence of outliers, structural breaks, consumption rate and whether the stationary components of consumption rate can in any way help to improve the predictive ability of our structural model.

1.3 Constraints of the Research

There are constraints in any research. Very often, these constraints are unavoidable and cannot be got rid of completely. For research on exchange rates, our first constraint is that if the condition prevailing in the exchange rate market is not favourable to the country concerned, the central bank will, very often than not, intervene in the trading. This creates an artificial element in the market, making its series extremely difficult to model.

Secondly since there are no standardized regimes, not many free float exchange rate data sets are available for research. Many of these free float exchange rate data series have very short time span because quite a number of them switched to currency pegging regimes about a decade ago. Thirdly, we have great difficulty in obtaining the real time data especially, the most up-to-date data. This creates a difficulty when we perform out of sample forecasting. Thus, very often, we can only perform pseudo out of sample forecasting. Here “pseudo” means: pick a date near the end of the sample, estimate our forecasting model using data up to that date, and then use the estimated model to make a forecast of the data left out of the sample.

1.4 Methodology

There are two main methodologies used in the research. One of them is the state space representation system of a Kalman filter. We use the state space model to generate the residuals and the state variables for the observable foreign exchange rate data when the data is fitted with a structural dynamic time series model. Residuals and state variables in this case are unobservable and we use a Kalman filter to extract it. We use a state space representation system with a Kalman filter basically because of two reasons.

1. Many researchers have shown the applicability of the Kalman filter. For example, Fama and Gibbons (1982) modelled real interest rate by using a Kalman filter. The result obtained was exceptionally good. There is also the research by Halsey (2000) on stationary components of earnings and stock prices by using a Kalman filter.
2. A Kalman filter can extract the unobservable with excellent accuracy and moreover, it can give very good maximum likelihood estimates. With this property, it can be used to do the job of smoothing the series and to do point wise forecasting. It is a popular technique now in macroeconomic and financial research.

Besides the state space representation system using a Kalman filter, we also use the Box and Jenkins methodology (ARFIMA) for long-term memory modelling. We construct the most suitable dynamic specification for the ARFIMA modelling of the foreign exchange rates. We test the stability of the model by using 22 foreign exchange rates series, which are specially chosen from different parts of the world: Canada, Australia, Mexico, South America, Europe, Africa and Asia. This geographical variation will definitely enhance the robustness of the model after stability testing. To compare and contrast the effectiveness of each of the methodologies used in structural modelling, we perform the actual modelling on long-term memory and short-term memory series.

Next, we use a different setting and a different population, that is, Ringgit per foreign currency to repeat the experiment. This will verify the external validity of our empirical study. We also compare the robustness of each of the methodologies with regard to sample size. For comparing, we use the root mean square error (RMSE) and the mean absolute prediction error (MAPE) values.

To confirm the output, we use a sign test, which is usually called the S statistics to test whether there is any difference between the two models. Next, we use a new forecast criterion to determine whether the forecasts are consistent or not. If the forecast is consistent, the forecasted values and the actual values should be cointegrated. We use a 8-step ahead forecasting for our comparison study. Many research papers such as the one by Diebold and Mariano (1997) show that short horizon forecasting is more reliable than long horizon forecasting. Moreover a short duration forecasting is also in line with the concept of Martingale theory, used widely in financial research. However, since we are dealing with exchange rate, which most probably has the mean reverting behaviour, we also consider long horizon, 100 steps ahead forecasts.

1.5 The Data and Software

There are two types of daily exchange rate data sets used in the analysis. First, we have twenty-two daily exchange rate series for our analysis. These twenty-two daily exchange rate data sets were obtained from the Federal Reserve Board of the United States of America (US). All of them are shown in Table 1.1. Among these exchange rate series, there are three series, which had been terminated in 1998 because of the adoption of Euro as a common currency. These three exchange rate series are the French Franc, the German Deutschmark and the Dutch Guilder. Many of these data sets have quite a number of missing observations. We use the cubic spline technique to select the best values to fill in these missing observations. Bandwidths are chosen after repeated testing on whether the cubic spline fit the curves or not. Most of these data sets started from 1st January 1981 to 31st December 2004. However, quite a number of them started from 1st January 1990 to 31st December 2004, and some started from 1st January 2000 to 31st December 2004.

The second set of exchange rates is obtained from Bank Negara Malaysia. All the values of this second data set are in the number of ringgit per unit foreign currency. As we use this second set for the purpose of external validity test, only 6 series from this second set of data are used for the analysis. However, a main weakness in this second set of exchange rate series is that they all cover the period from 1st January 2000 to 31st December 2004. Thus, the time span is short even though the frequency of the observations is within reasonable limits. Table 1.2 shows these six series, which are divided into two main zones, viz., Europe and Asia.

We include Dutch guilder, German Deutschmark and French franc in experiment for the sake of obtaining the best structural exchange rate model. These exchange rates were used because they are considered as some of the best free float data sets.

In Chapters 11 through to Chapter 13, we mainly use the British pound per US dollar exchange rate data, Britpus, for our experiment. Britpus is transformed into two forms: weekly data, WeBritpus and quarterly data, QBritpus. This is because we find that weekly data is more suitable for structural breaks analysis and quarterly data is more meaningful for co-movement (move in parallel) analysis. Beside the exchange rates, we also use the UK real consumption, taken from Harvey and Scott (1994). These twenty-two forex data series and the consumption data can be found in the CD enclosed in this thesis

Table 1.1 – Names of the 22 exchange rates used in the experiment.

America	Europe	Africa	Asia
Canada Dollar (Can)	European Dollar (Eur)	South Africa Rand (Ran)	Singapore Dollar (Sid)
Mexico Pesos (Pes)	British Pound (Britpus)		Thailand Baht (Bah)
Brazil Real (Rea)	Denmark Kroner (Kne)		Malaysia Ringgit (Rin)
Venezuela Bolivar (Bol)	Sweden Kronor (Kno)		South Korea Won (Won)
Australian Dollar (Austrus)	Switzerland Francs (Swisfus)		Chinese Yuan (Yua)
	French Francs (Frnfrus)		Indian rupees (Rup)
	German Deutschmark (Germdus)		Japanese Yen (Japynus)
	Dutch Guilder (Dtchgus)		Hong Kong Dollar (Hkd)

Table 1.2 – Names of the 6 exchange rates (Ringgit per foreign currency) used for the external validity experiment

Europe	Asia
British Pound (Britpus)	Singapore Dollar (Sid)
European Dollar (Eur)	Thailand Baht (Bah)
	Hong Kong Dollar (Hkd)
	Japanese Yen (Japynus)

In Chapters 11 through to Chapter 13, we mainly use the British pound per US dollar exchange rate data, Britpus, for our experiment. Britpus is transformed into two forms: weekly data, WeBritpus and quarterly data, QBritpus. This is because we find that weekly data is more suitable for structural breaks analysis and quarterly data is more meaningful for co-movement (move in parallel) analysis. Beside the exchange rates, we also use the UK real consumption, taken from Harvey and Scott (1994). These twenty-two forex data series and the consumption data can be found in the CD enclosed in this thesis.

With respect to software, we mainly use Eviews 4.1, PcGive10.3, Stamp 6.20 and, occasionally, we use Gauss 5.0 for certain situations only.

1.6 Three Different Forms of the Data

There are three different forms of a data series, which can be used in any research. Which form is more suitable depends to a large extent on the intended outcome of the research. In this section, we discuss all the three forms of a data series: the original data series, the log data series and the returns series. We use all these three forms of data in our research.

1.6.1 The original data

The raw data series can provide a better intrinsic measure of the central tendency of the series. However for cases where the distribution of the raw data series is highly skewed, the conditional mean $E(\boldsymbol{y}_t|x_t)$ where y_t denotes the dependent variable and x_t denotes the explanatory variable, may not be a useful measure of the central tendency, and estimates will be undesirably influenced by extreme observations, which we refer to as outliers. In Chapter 1 through to Chapter 8, we use the raw data series for the experiments. This is largely because our objective is to compare and contrast the two structural modelling methodologies.

1.6.2 The log data

This form of the data series is widely used in the research. Basically there are four reasons for its widespread use.

1. Many nonlinear functions can be transformed into linear functions by using the logarithm. Thus, we have $E[\log(\boldsymbol{y_t})|x_t]$ is roughly linear in x_t over the range of x_t, while $E[y_t|x_t]$ is nonlinear. y_t and x_t as before denote the dependent and explanatory variables respectively. This is an advantage because linear models are easier to report and interpret. For example, we interpret the regression coefficients as percentage changes when the log series is used.

2. The errors in $\varepsilon_t = \log(y_t) - E[\log(y_t)|x_t]$ may be less heteroskedastic than the errors from the linear specification when raw data is used. We can show this by using Taylor's theorem, and then apply the delta method. Let $\mu_t = E(\boldsymbol{x_t})$ and $Var(\boldsymbol{x_t}) = k\mu_t^2$. Then the variance of the log is approximately constant as shown below. We use Taylor's theorem which is given by:

$$
\begin{aligned}
f(x) &\approx f(\mu) + f'(\mu)(x-\mu) + \ldots. \\
Var(f(x)) &\approx Var(f(\mu)) + Var(f'(\mu)(x-\mu)) + \ldots. \\
Var(f(x)) &\approx [f'(\mu)]^2 Var(x)
\end{aligned}
$$

By setting $f(x_t) = \log x_t$, we have:

$$Var(\log x_t) \approx \left(\frac{1}{\mu_t}\right)^2 Var(x_t) = k$$

Thus:

$$Var(f(\mathbf{x})) \approx [f'(\mu)]^2 Var(\mathbf{x}) \Rightarrow Var(\log \mathbf{x}_t) \approx \left(\frac{1}{\mu_t}\right)^2 Var(\mathbf{x}_t) = k \qquad (1.1)$$

However this is not consistent and the reverse may be true also.

3 If the distribution of y_t is highly skewed, the conditional mean $E[y_t|x_t]$ may not be a useful measure of central tendency, and outliers will influence the estimates. In this case, the conditional mean $E[\log(\mathbf{y}_t)|\mathbf{x}_t]$ may be a better measure of central tendency.

4 A good approximation of a returns series can be easily computed by taking the difference between two consecutive observations in the log form.

1.6.3 The returns series

The returns is defined as the quotient obtained by dividing the difference between the two consecutive observations by the first observation of the two consecutive observations. This form of the returns series is widely used in finance for the simple reason that we are interested in the returns for our investment. The second reason is that by using the returns series, we may not need to difference the original series again to obtain a stationary time series if the raw series is nonstationary.

1.7 Outlines of Subsequent Chapters

In Chapter 2, we give an overview of the theoretical background involved. This is to prepare the readers on the sort of probability and statistical theory used in the research.

In presenting the time series concepts, we stress on its difference from cross-sectional regression and how the difference is reconciled. We also present the *F* tests, *t* tests and, most important of all the so-called Hall preferential t test. We are of the opinion that, with this, we can easily guide the readers as to what contributions we have made.

We present and discuss two methodologies, ARFIMA and KFBM, which we are going to compare and contrast so as to obtain the best structural time series model for exchange rate in Chapter 3.

In Chapter 4, we present a concise literature review. From the literature review, we state categorically, the objective of each experiment, which we are going to conduct after critical analyse of the results of the relevant research papers.

In Chapter 5, we discuss the mean reverting and random walk behaviour of the exchange rate. First, we present the latest development in the study of these two behaviours with respect to the controversies or rather puzzle encountered. With that, we present our experiment. We use fractional dynamics, and impulse response function in addition to unit root tests, to ascertain mean reverting behaviour.

Subsequently, in Chapter 6, we first make an exploratory investigation of eight foreign exchange rate series by using graphical analysis and hypothesis testing. We develop the most suitable dynamic specification for the ARFIMA modelling of long-term memory. We test the stability of the specification by fitting model to the 8 foreign exchange rates series. We also test its robustness with regard to variations in sample size. To test the external validity of the model, we fit the model to 6 more exchange rates specially chosen from different parts of the globe.

In Chapter 7, we use the series Britpus for the experiment. We generate the component series for Britpus. Next, we regress Britpus on all its three components in a single equation dynamic modelling. We perform pseudo out of sample forecasting by assuming the case of uncertain variances. We assume that its effectiveness in forecasting depends only on the values of RMSE and MAPE. We test its robustness by fitting it to the eight-forex series, and then across different sample sizes.

We state the conclusions obtained from the results obtained from Chapter 6 and Chapter 7 in Chapter 8 . Next, we use each of the two structural models for modelling short-term memory series. We vary the sample size and for each variation, we perform the modelling. We compare and contrast each of the output of the experiments. In this way, we can make decision on which model or, rather, methodology is the best, and under what conditions, it is the best.

In Chapter 9, we repeat the experiment in Chapter 6 in a new setting, that is, foreign exchange rate in Ringgit per unit of foreign currency. We perform this additional experiment in order to verify whether our best structural model can still be workable in a new setting and population.

We perform empirical analysis in Chapter 10, where we devise and execute experiments to compare and contrast the forecasting ability of three models, the standard ARFIMA, the YQ-ARFIMA and the random walk model in the context of exchange rate. We use two loss functions, RMSE and MAPE, for the comparison. To further confirm the results, we perform a forecast consistent procedure for the experiment.

In Chapter 11, we present a theoretical framework for a structural break model based on the partial sum concept. We use recursive least squares and recursive residual sums of squares to investigate the structural breaks and outliers in the DLWeBritpus series. Next, we investigate the influence of the structural breaks on the predictive performance of our KFBM model. With respect to outliers, we use the simple ARFIMA model to model the mild outliers, and for shape outliers, we use two approaches: we scale them down to normal size, and then, we exclude them altogether. We then perform structural modelling again on DLWeBritpus, but this time with the corrections of the outliers. The output from this last experiment is intended to serve as a check on the validity of the forecasting ability of the two models developed earlier, and also to determine the usefulness of outlier correction in general.

In Chapter 12, we construct a structural model for Lconsumption. We use the Kalman filter to extract the cyclic components of the consumption (Lconsumption) and LQBritpus series. We regress LQBritpus on Lconsumption, and, after this, we regress Lconsumption on its three important components. Next, we regress the trend cyclic components of LQBritpus on the trend cyclic components of Lconsumption in order to obtain their relationship.

In Chapter 13, we perform a bivariate structural time series modelling of LQBritpus and Lconsumption with the intention of obtaining more robust and precise estimates of their cyclic components. Next, we perform a single equation dynamic modelling of the cyclic components of LQBritpus on Lconsumption. We want to determine and confirm whether the exchange rate and the consumption move in parallel. We also want to investigate whether the inclusion of the cyclic components as regressors can improve significantly the predictive power of our YQ-specified ARFIMA model. Finally in Chapter 14, we conclude the thesis. We present the empirical results and make inferences based on the experiments and discuss their implications. We list our contributions in the research. Last, but not least, we suggest future areas of research.

CHAPTER 2

THEORETICAL BACKGROUND

2.1 Introduction

The objective of this chapter is to give a brief overview of all the basic financial econometric theory as well as statistical theory involved in the analysis. As for the methodologies used, we will introduce them briefly in this chapter and discuss them in more details in Chapter 3. The financial econometric theory and statistical theory used in the research are: independent and serially correlated data, long memory, short memory, white noise, linear and nonlinear regression, autocorrelation function, partial autocorrelation function, Akaike, Schwarz and Hannan-Quinn information criteria, theory of ARMA model, *t*-tests, Wald tests, *F* tests, R^2 tests, unit roots and its use in testing, Granger causality tests, structural modelling, stochastic trends, forecasting and selection criterion for the best forecasting model, structural breaks, and real time data versus transformed data.

2.2 Independent and Serially Correlated Data

In our research, we deal mainly with time series data. In general, there are two types of data available for research; time series data and cross-sectional data. Regression techniques and OLS estimators are developed specifically for cross-sectional data. Can we apply these techniques without alterations to time series data? This section deals mainly with the answer to this question. In the course of answering this question, we will point out the important role played by the characteristics “independent and serially correlated" of the data. We will provide a formal definition for independent and serially correlated data after the explanation.

There is a basic difference between time series data and cross-sectional data. The former can only have a single realization generated by the data generating process (DGP), and the latter can be generated by multiple realizations of the DGP. This major difference eliminates altogether the possibility of using random sampling distributional techniques and sampling statistical inferences for time series analysis. However, if we impose the restriction of ergodic stationarity on the time series, the time average over the elements of the time series will be consistent for the ensemble mean of the cross-sectional data series. Normally, in practice, any time series we encounter would have more than 100 data. With this length of data, we would be able to use large sample theory (asymptotic theory) for making statistical inferences (see Stock and Watson, 2003). Thus, we can use asymptotic properties of the time series data to construct confidence levels for hypothesis testing and make statistical inferences.

The major tool used to derive statistics and construct confidence levels for studying cross-sectional data is the Central Limit Theorem (CLT) initiated by Lindeberg-Levy. However, this CLT is valid only for independent and identical distributional observations (i.i.d) as is usually assumed in regression analysis. As most of the time series data are serially correlated, we cannot apply the CLT wholesale without generalization and alteration. This generalization and alteration of the CLT were done by Gordin (1969) and subsequently restated by White (1984). We call this final version of the CLT as Gordin's CLT for zero-mean ergodic stationary process (see Hayashi, 2000). Thus, we have to use Gordin's version of CLT to derive statistics and construct confidence intervals for time series analysis. We shall now explain the term independent, serially correlated, stationarity and ergodicity.

The observations in most financial time series are serially correlated and seldom are independent. Serially correlated observations and how to test its existence play an important role in structural analysis because the output results will be very different if we take serially correlated observations as independent observations. It is an accepted fact, that independent will imply no serial correlation but the reverse is not true in general. However, for normal random variables, no serial correlation can imply independent. In time series analysis, one important assumption is that the time interval between two consecutive observations is uniformly the same, and we refer to such property of the time series as stationarity. Stationarity is a restriction imposed on the time series so that the number of parameters needs to be estimated is minimum. To estimate the parameters of a time series, the time series must exhibit stationarity behaviour and ergodicity as well. Stationarity simply means that the joint probability distribution of any set of k observations in the sequence of observations is the same regardless of the origin, *t*, in the time scale. Ergodicity means that events separated far enough in time are "asymptotically independent". Put differently, the observations are not too persistent, and every observation contains some information, which are not available in other observations. However, recent research found and confirmed that most time series are nonstationary and very often cointegrated. We shall discuss this new development in Section 2.8.

Definition: If $\{y_t\}_{t=1}^{t=N}$ represents a time series of data for the sample period from t = 1 to $t = N$, then the time interval between any two conservative time must be the same. If the observations are independent, we have the following relations:

$$Cov[y_{t-k}y_t] = 0 \text{ and } E[y_{t-k}y_t] = 0 \tag{2.1}$$

for any t and k with $k < t$

However, if the observations were serially correlated instead, we would have the following relations:

$$Cov[y_{t-k}y_t] \neq 0 \text{ and } E[y_{t-k}y_t] \neq 0 \tag{2.2}$$

for any t and k with $k < t$

Definition: A first order serial correlated linear regression model is defined as:

$$y_t = a + bx_t + u_t \qquad \text{where } u_t = \rho u_{t-1} + \varepsilon_t, \text{ and } |\rho| \leq 1 \tag{2.3}$$

with the x's being non random:

$$E(u_t) = 0,\ Var(u_t) = C, \text{ and } \forall\, t,\ Cov(u_t u_k) \neq 0 \tag{2.4}$$

where C denotes a constant.

The above model can be represented in a more compact way as shown below:

$$y_t = \mu + (1 - \rho L)^{-1}\varepsilon_t \tag{2.5}$$

where $\mu = E(y_t) = a + bx_t$.

Notice that Equation (2.5) is in the moving average form. A moving average process will generate serially correlated data. When Ordinary Least Squares (OLS) or Generalized Least Squares (GLS) is used to estimate the parameters in the above-reduced equation, we would find that the OLS or GLS is no longer unbiased or consistent. We refer to such problem as simultaneous equation bias and we use instrumental variable regression to overcome the problem. Thus, very often, we avoid any lagged values in the dependent variables when doing OLS regression. However, we can also overcome this problem by testing the validity of each parameter of the output equation after doing the regression by using the *t* tests, which is robust, consistent and pivotal. This can be done easily by using econometric software package, for examples, Eviews and PcGive.

As for the analysis, we can use a graphical method or hypothesis testing to determine the correlation of a set of data. Figure 2.1 shows graphically what serially correlated data set means. Notice how the variation within one period can be used to indicate the likely variation in the next period. The data set of GDP in US is obtained from " Econometric Analysis" by William H. Greene (2003).

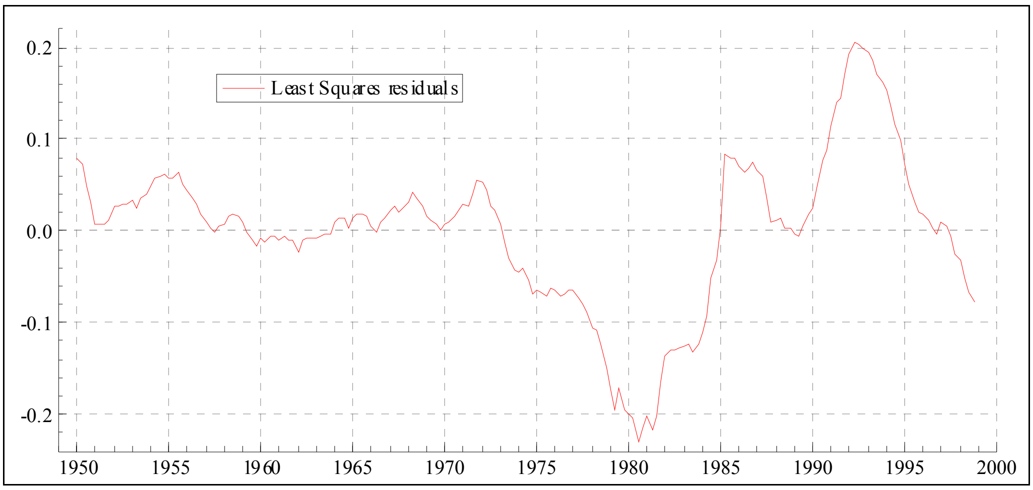

Figure 2.1 – Variation pattern of serially correlated data- GDP in US

As for additional graphical analysis, we shall discuss it under the heading of the autocorrelation function ACF and the partial autocorrelation function PACF in Section 2.5. In this aspect, scatter plots can also be used to detect the correlation of the data. In practice, we divide the scatter plot into four quadrants. Equal number of points in each quadrant simply means lack of correlation. Excess points in any quadrant indicate correlation. There are two common tests for the null hypothesis testing of no serial correlation. They are the Durbin Watson Test and the Ljung-Box Test.

The Durbin-Watson Test

Durbin Watson test is a test for the existence of serially correlated random disturbances of the dependent variable. It is a classical test. The model has to have an intercept and the regressor has to be non-random. The test statistic is given by:

$$d = \frac{\sum_{t=2}^{N}(\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}(\hat{u}_t)^2} \tag{2.6}$$

For testing the null hypothesis of no serial correlation against serial correlation, we have the following decision rule:

Reject if: $d < d_{\text{lower}}$

Accept if: $d > d_{\text{upper}}$

No conclusion if $d \in (d_{\text{lower}}, d_{\text{upper}})$