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Fast S-parameter Convolution for Eye Diagram Simulations of High-speed Interconnects

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Abstract

With the increase in signal frequency and the complexity of high-speed interconnects, signal integrity has become a prominent issue in modern electronic devices. In this aspect, accurate and fast simulation methodologies to analyze the performance of high-speed channels are of paramount importance in the pre-layout and post-layout design stages. One of the main tools used by designers in this regard is the eye diagram, which provides a statistical representation of the signal quality and system performance, and allows design engineers to quickly assess channel parameters such as jitter and signal-to-noise ratio. However, simulations of eye diagrams are slow as it relies on the simulation of a long series of a pseudo-random bit sequence (PRBS). In this paper, we show that the simulations of eye diagrams can be sped-up by using a fast S-parameter convolution method. We show that when S-parameters with properly chosen reference impedances are used as the representation of the channel, the impulse response decays very rapidly and thus only a small amount of points need to be retained to obtain an accurate solution. The channel can then be represented as a series of delayed delta functions, and this reduced set of points is used in the convolution process. This results in large computational savings. We apply the method to a real channel and show the improvement over regular convolution on eye diagram simulations.

Keyword: Signal integrity, eye diagram, interconnect simulations

1. Introduction

The simulations of electrical interconnects characterized by a multiport data is an integral part of the signal integrity analysis of modern devices. This need arises as the operating frequencies and signal speeds continue to increase which has led to issues such as ringing, crosstalk and jitter. While previously minimal and negligible, these issues are now much more prominent and can result in operational errors in digital systems if not accounted for correctly. Thus, research on transient simulations of interconnect networks have received a considerable amount of interest in recent years [1] – [4]. The two main approaches to interconnect simulations from tabulated data are model-order reduction (MOR) based techniques and convolution based techniques.

In the MOR method, the system is first approximated by a reduced order model, often in the form of a rational function, which can then be used to obtain the transient port responses of the system [1], [2]. The drawback of this method lies in the complexity of the model identification step, in which a curve fitting routine is often used to approximate the system response to that of a rational function. In addition, physical properties such as stability and passivity are not guaranteed and has to be checked and enforced [3], [4].

On the other hand, in the convolution based method [5], the frequency response of the system is used directly to obtain the port responses of the system through a direct convolution with the input. However, this process can be time consuming as the complexity depends on the number of data points in the impulse response of the system and the input. In this work, we apply a fast S-parameter convolution approach to speed up the simulation. The method is used to perform a long simulation of a random bit sequence to generate the eye diagram of a channel and comparisons are made with the traditional approach.

2. S-parameter Convolution

For a multiport structure characterized by its scattering parameters or S-parameters, the response, *B* to an input *A* can be written in the frequency domain as

$$B(\omega) = S(\omega) \cdot A(\omega)\,. \tag{1}$$

In the time domain, this becomes

$$b(t) = s(t) * a(t) \tag{2}$$

where $*$ indicates the convolution operator. For discrete data points, (2) can be written as

$$b(t) = s(t) * a(t) = s(0)a(M)\Delta t + \sum_{\tau=1}^{M} s(\tau)a(M-\tau)\Delta t \tag{3}$$

where Δt is the time step and *M* is the index of the current time step. The calculation of (3) is numerically expensive as it involves the calculation of the history function *h(t)* at every time step where *h(t)* is given as

$$h(t) = \sum_{\tau=1}^{M} s(\tau)a(M-\tau)\Delta t\,. \tag{4}$$

3. Fast S-parameter Convolution

The burden in the calculation of (3) can be alleviated by noticing that the impulse response of an S-parameter decays rapidly with time especially when the reference impedance is properly chosen. For example, the impulse response of S_{11} of a two port interconnect structure is given in Fig. 1. Out of the 1601 points, only 90 had magnitudes greater than 0.01% of the maximum magnitude value. In other words, a majority of the points in *s(t)* will have small magnitudes and can be neglected. As a reformulation, the discrete frequency domain S-parameters can be represented as

$$S(q) = \sum_{k=1}^{L} c_k e^{j2\pi qk} \tag{5}$$

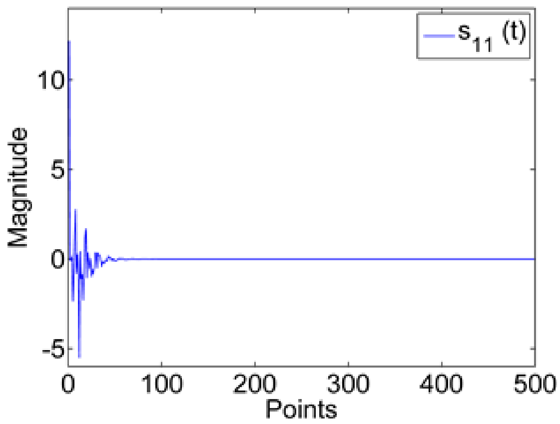

Fig. 1. Impulse response of S_{11} of an interconnect showing the rapid decay of the function. Only the first 500 points are shown.

where c_k and *k* are parameters to be determined and *L* is the order of the approximation that satisfies $L << N$ where *N* is the total number of simulation points. Using this representation, the time domain function of (5) can be represented as a train of impulses whose weights are given by c_k

$$s(p)=\sum_{k=1}^{L}c_k\delta(p-k) \tag{6}$$

and convolution with an input *a(p)* can then be represented as:

$$h(p)=\left[\sum_{k=1}^{L}c_k\delta(p-k)\right]*a(p)=\sum_{k=1}^{L}c_k a(p-k)\,. \tag{7}$$

where the c_k can be obtained obtained by taking the inverse discrete Fourier transform or IFFT of the frequency-domain transfer function. The reader is referred to [6] for the full formulation.

4. Eye Diagram Simulations using Fast S-parameter Convolution

In this work, we apply the method presented in the previous section for eye diagram simulations of interconnects characterized by their tabulated data. The S-parameters of a 2-port interconnect structure is generated using a commercial numerical tool for the frequency range of 10 MHz to 10 GHz. Port 1 is then driven by a 50 Ω source while port 2 is terminated with a 50 Ω load. In addition, both ports are assumed to have a 0.1 nF parasitic capacitance. A random bit sequence of 1000 ones and zeroes with rise and fall times of 1 ns and a pulse width of 3 ns is then sent at port 1 and the output at port 2 is obtained using both the regular convolution and fast convolution approaches. The result for the first 500 ns is shown in Fig. 2.

From this result, the eye diagram at port 2 is generated by overlapping the responses over the period of one bit. The results obtained from both regular convolution and fast convolution are shown in Fig. 3.

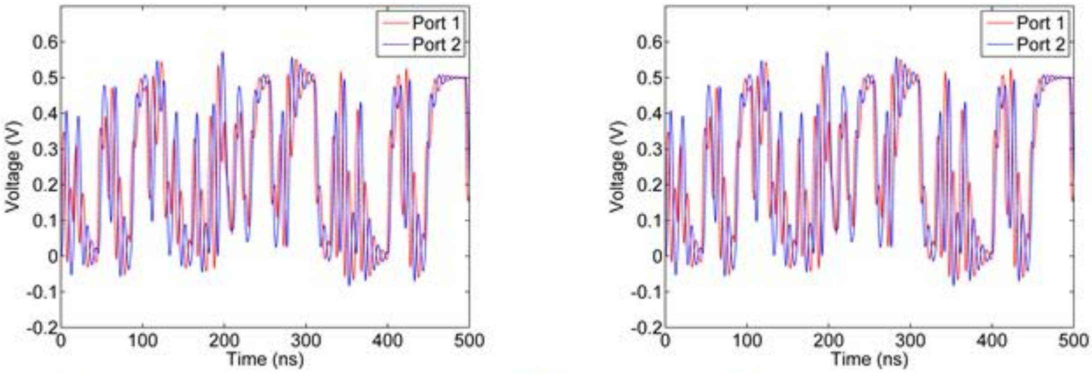

Fig. 2. Transient simulation of a random bit pattern. Left: Using full convolution. Right: Using fast convolution.

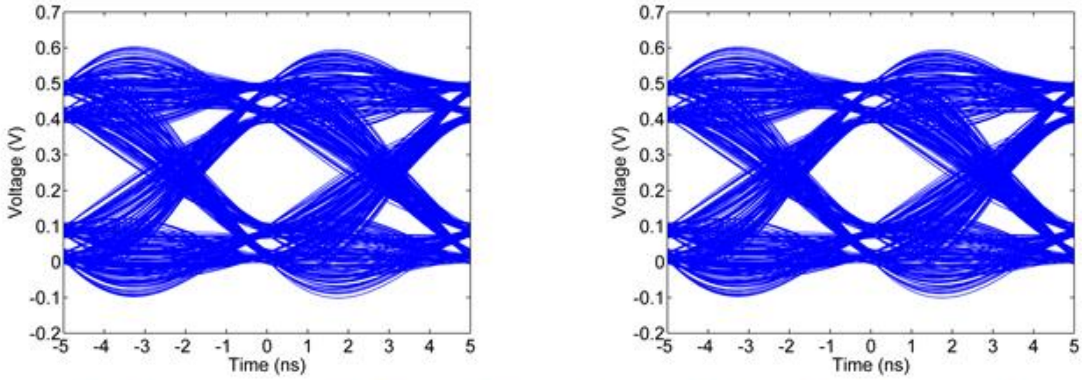

Fig. 3. Eye diagram. Left: Using full convolution. Right: Using fast convolution.

We see that both methods produce comparable results in terms of accuracy. In terms of runtime, the regular convolution method required 55.0 s to finish the simulation, in which 48.1 s was used to calculate the solution to (4). Using the fast convolution method, the runtime of the convolution step could be reduced to only 4.3 s by using (7) and retaining only [90, 189, 189, 85] points out of the original [1601, 1601, 1601, 1601] points of the original S-parameters [S_{11}, S_{12}, S_{21}, S_{22}]. The final simulation time of the fast S-parameter method was 12.4 s. We note that a total of 100,000 points were simulated to generate this eye diagram. All simulations were performed in Matlab R2009a on a desktop computer with a dual core Intel Xeon processor at 2.4 GHz and 4 GB of RAM.

5. Conclusion

In this paper, we have presented a method to speed-up the simulations of eye diagrams using a fast S-parameter convolution approach. The example tested showed a 11.2× improvement in speed for the solution of the discrete convolution part and a 4.4× speed-up for the overall simulation. We note that this computational savings is expected to be even more significant for large networks with a high number of ports.

6. Acknowledgements

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7. References

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