The Kinematics and Dynamics Motion Analysis of a Spherical Robot

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***Abstract*—Mobile robot application has reach more aspect of life in industry and domestic. One of the mobile robot types is a spherical robot whose components are shielded inside a rigid cell. The spherical robot is an interesting type of robot that combined the concept of a mobile robot and inverted pendulum for inner mechanism. This combination adds to more complex controller design than the other type of mobile robots. Aside from these challenges, the application of a spherical robot is extensive, from being a simple toy, to become an industrial surveillance robot. This paper discusses the mathematical analysis of the kinematics and dynamics motion analysis of a spherical robot. The analysis combines mobile robot and pendulum modeling as the robot motion generated by a pendulum mechanism. This paper is expected to give a complete discussion of the kinematics and dynamics motion analysis of a spherical robot.**

***Index Terms*—inverted pendulum, mobile robot, path planning, pendulum, spherical robot.**

I. Introduction

The current application of mobile robot has reached more aspect of life, not only in industry but also in domestic and military. One of a mobile robot whose applications are very wide is the spherical robot. A spherical robot is the type of mobile robot where all components are shielded inside a round case including its actuators. Due to its shape, this robot is also called as ball-shaped mobile robot [1]- [4]. The outer motion looks like a rolling ball where the rolling motion needs to be controlled to ensure it can accomplish its task. The shell can be made transparent or solid depends on the application [1] [5]- [14].

The wide range application of spherical robot due to its flexible motion, the nature of rolling motion, and its shielded component giving the possibility to be a waterproof robot and deployed underwater [14] [15]. One of those applications is as an industrial surveillance robot such as piping inspection robot, in this application robot can be equipped with sensors such as camera, gas sensor, proximity sensor, etc. The robot can be set fully or semi-autonomous where the operator moves the robot and robot acts as the extended ”eye” for operator [5] [6] [7].

The driving mechanism of a spherical robot adopts the principle of an inverted pendulum, where the robot relies its motion on the actuator is moving the unbalancing inner mechanism [16]- [20]. The inner component inside the sphere equipped with actuators that attached to perpendicular axes [21] [22] [23]. During the motion, the center of mass keep on changing adjusting to the pendulum mechanism, and this unbalanced mass generates robot motion [12] [19] [24].

The kinematics and dynamics of a spherical robot are derived from the combination of a mobile robot and pendulum [18] [19] [21]- [25]. The control design of this robot is derived from dynamics to achieve path planning and tracking control and navigation [2] [8] [9] [11] [12] [15] [21]- [35].

This paper presents the literature study of the kinematics and dynamics motion analysis of a spherical robot applied as a surveillance robot. The kinematics and dynamics are derived from the possible motion of a spherical robot. The analysis combines mobile robot and pendulum modeling as the robot motion generated by the pendulum mechanism.

II. Spherical Robot Modeling

The spherical robot is another type of mobile robot whose motion is generated by the inner pendulum mechanism. The advantage of this robot is that the nature of its motion is rolling, and since all the mechanical part located inside the sphere; therefore, the application of marine or wet environment is possible. The spherical robot considered in this study is shown in Fig. 1. This robot has to actuator attached to a perpendicular axis. The robot is modeled in kinematics and dynamics.

The spherical robot in Fig. 1 configuration consists of two actuators and two counterweights to balance robot motion. The inner parts are arranged to ensure the mass of center aligned with its geometrical center. The center of mass is related to the gravity of the robot and affect the dynamics of the robot.

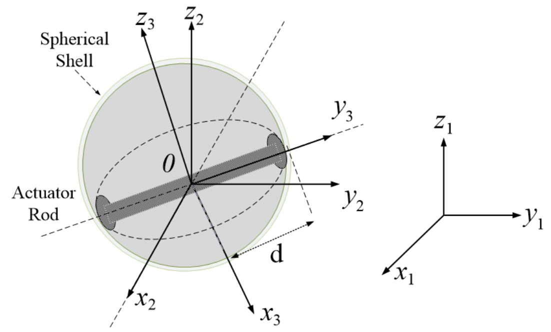

Fig. 1. A spherical robot with 2 actuators.

A. Kinematics Modeling

The spherical robot moved by two perpendicular actuators is shown in Fig. 1, and during its motion the robot rolls by its surface resulting from the ever-changing rotation and orientation. Two balance counterfeits are added to ensure the robot's stable motion and the center of gravity lays on the geometric center of the shell. Fig. 1 shows that the actuators rotational axes coincide with their corresponding symmetry axis. Frame 1 is the world coordinate frame related to the inertial reference frame. Frame 2 is the geometric center of the spherical frame, and its axis is parallel to the frame 1. Frame 3, robot body coordinate frame attached to robot geometric center. The x and y-axis are aligned with actuators rotation through the connecting rod.

If (x, y) is robot position related to the center of the spherical robot with respect to frame 1, and the position of robot in its frame is $\mathrm{q} = [x, y, \phi]^T$, then

$$ {}^3_2\mathrm{q} = {}^3_1\mathrm{q} = \begin{bmatrix} \mathrm{q}_0 \\ \vec{\mathrm{q}} \end{bmatrix} = \mathrm{q}_0 + \mathrm{q}_1 i + \mathrm{q}_2 j + \mathrm{q}_3 k, \tag{1} $$

and

$$ \left|{}^3_2\mathrm{q}\right|^2 = \mathrm{q}_0^2 + \mathrm{q}_1^2 + \mathrm{q}_2^2 + \mathrm{q}_3^2 = 1. \tag{2} $$

Based on Eq. 1 and 2, the angular velocity (ω) and acceleration (α) are given by

$$ \begin{bmatrix} 0 \\ {}^2\vec{\omega}_3 \end{bmatrix} = 2{}^2_2\dot{\mathrm{q}}{}^2_2\bar{\mathrm{q}} \tag{3} $$

$$ {}^2\vec{\alpha}_{[3]} = \frac{\mathrm{d}}{\mathrm{dt}}\left({}^2\omega_{[3]x}\right)\hat{i}_2 + \frac{\mathrm{d}}{\mathrm{dt}}\left({}^2\omega_{[3]y}\right)\hat{j}_2 + \frac{\mathrm{d}}{\mathrm{dt}}\left({}^2\omega_{[3]z}\right)\hat{k}_2 \tag{4} $$

$$ \begin{aligned} {}^2\vec{\omega}_1 &= {}^2\vec{\omega}_{[3]} \\ {}^2\vec{\alpha}_1 &= {}^2\vec{\alpha}_{[3]} \end{aligned} \tag{5} $$

where [3] indicate frame 3, ${}^2\vec{\omega}_1$ is the angular velocity of frame 1 with respect to frame 2, and ${}^2\vec{\alpha}_1$ is the angular acceleration of frame 1 with respect to frame 2.

Therefore, the angular velocity of frame 3 with respect to frame 1 is

$$ \begin{bmatrix} {}^1\omega_{3x} \\ {}^1\omega_{3y} \\ {}^1\omega_{3z} \end{bmatrix} = \begin{bmatrix} -2\mathrm{q}_1 & 2\mathrm{q}_0 & -2\mathrm{q}_3 & 2\mathrm{q}_2 \\ -2\mathrm{q}_2 & 2\mathrm{q}_3 & 2\mathrm{q}_0 & -2\mathrm{q}_1 \\ -2\mathrm{q}_3 & -2\mathrm{q}_2 & 2\mathrm{q}_1 & 2\mathrm{q}_0 \end{bmatrix} \begin{bmatrix} \dot{\mathrm{q}}_0 \\ \dot{\mathrm{q}}_1 \\ \dot{\mathrm{q}}_2 \\ \dot{\mathrm{q}}_3 \end{bmatrix}, \tag{6} $$

where ${}^1\omega_{3x}$ is the angular velocity of frame 3 with respect to frame 1.

The translational velocity of the sphere is given by

$$ \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = 2\mathrm{d} \begin{bmatrix} -\mathrm{q}_2 & \mathrm{q}_3 & \mathrm{q}_0 & -\mathrm{q}_1 \\ -\mathrm{q}_1 & -\mathrm{q}_0 & \mathrm{q}_3 & -\mathrm{q}_2 \end{bmatrix} \begin{bmatrix} \dot{\mathrm{q}}_0 \\ \dot{\mathrm{q}}_1 \\ \dot{\mathrm{q}}_2 \\ \dot{\mathrm{q}}_3 \end{bmatrix}, \tag{7} $$

where d is the radius of sphere.

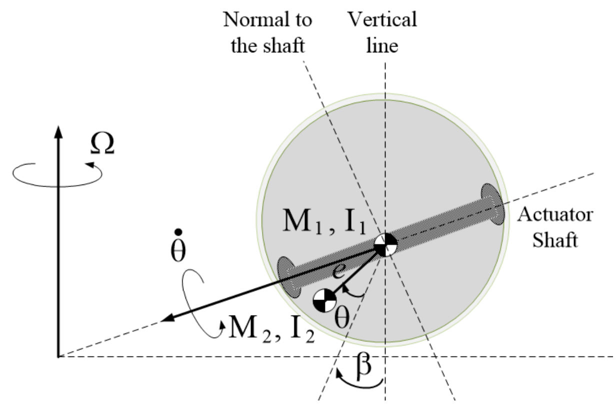

Fig. 2. The rolling motion analysis of a spherical robot.

B. Dynamic Modeling

Kinematics is the motion analysis without considering the torque applied to the robot, and dynamics considering how much torque and force applied, and effect of torque and force to robot motion and energy generated. Dynamics is required to analyze the path planning and control or the robot, and components required to derive dynamics are given in Fig. 2.

The robot acceleration is achieved by the second time derivative of Eq. 2 and 7

$$ \begin{aligned} \ddot{x} &= -2\mathrm{d}\left(\ddot{\mathrm{q}}_0\mathrm{q}_2 - \ddot{\mathrm{q}}_1\mathrm{q}_3 - \ddot{\mathrm{q}}_2\mathrm{q}_0 + \ddot{\mathrm{q}}_3\mathrm{q}_1\right) \\ \ddot{y} &= \mathrm{d}\left(\ddot{\mathrm{q}}_0\mathrm{q}_1 - \ddot{\mathrm{q}}_1\mathrm{q}_0 + \ddot{\mathrm{q}}_2\mathrm{q}_3 - \ddot{\mathrm{q}}_3\mathrm{q}_2\right) \end{aligned}, \tag{8} $$

and

$$ \left\|{}^3_2\dot{\mathrm{q}}\right\|^2 + \mathrm{q}_0\ddot{\mathrm{q}}_0 + \mathrm{q}_1\ddot{\mathrm{q}}_1 + \mathrm{q}_2\ddot{\mathrm{q}}_2 + \mathrm{q}_3\ddot{\mathrm{q}}_3 = 0. \tag{9} $$

Therefore, the spherical robot complete position is $\mathbf{q} = |x, y, \mathrm{q}_0, \mathrm{q}_1, \mathrm{q}_2, \mathrm{q}_3, \phi_x, \phi_y, \phi_z|^T$ and the applied torque is $\tau = [0, 0, 0, 0, 0, \tau_x, \tau_y, \tau_z, 0]^2$.

The dynamics of the spherical robot considered in this study can be written as follow

$$\mathrm{M}(\mathrm{q})\,\ddot{\mathrm{q}} + \mathrm{V}(\mathrm{q},\dot{\mathrm{q}}) + \mathrm{G}(\mathrm{q}) = \tau, \tag{10}$$

where $\theta_x, \theta_y, \theta_z$ are actuator angular displacement, q is state space variables, $\mathrm{M}(\mathrm{q})$ is the total mass, $\mathrm{V}(\mathrm{q},\dot{\mathrm{q}})$ is the Coriolis and Centrifugal force, $\mathrm{G}(\mathrm{q})$ is the gravitational forces, and τ is the torque.

The dynamics in Eq. 10 can be simplified and written as:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \dot{\omega} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix} + \begin{bmatrix} G_{11} \\ G_{21} \end{bmatrix} = \begin{bmatrix} \tau \\ \tau \end{bmatrix}, \tag{11}$$

where

$$\begin{aligned} &M_{11} = M_1 \mathrm{d}^2 + M_2 \mathrm{d}^2 + M_2 e^2 + I_1 + I_2 + 2M_2 \mathrm{d}e \cos(\theta+\beta), \\ &M_{12} = M_{21} = M_2 e^2 + I_1 + M_2 \mathrm{d}e \cos(\theta+\beta), \\ &M_{22} = M_2 e^2 + I_2, \\ &C_{11} = M_2 \mathrm{d}e \sin(\theta+\beta)\left(\dot{\theta}+\dot{\beta}\right) + \zeta\dot{\theta}, \\ &C_{21} = \zeta\dot{\beta}, \\ &G_{11} = G_{21} = M_2 g e \sin(\theta+\beta), \end{aligned} \tag{12}$$

and M_1 is the mass of the ball, M_2 is the mass of pendulum, θ is the rotational angle of the ball, and β is the rotation angle of the pendulum with respect to the ball.

III. Spherical Robot Motion Analysis

The spherical robot motion in this study is rolling in a horizontal motion and obstacle overcome. A spherical robot might have the ability to move uphill and downhill, and this motion can be analyzed by changing the position of gravity [10].

A. Horizontal Motion

By simplifying the robot in Fig. 2 as a rolling object, the motion analysis based on Newton Law of a spherical robot is

$$\tau = J\frac{d^2\theta}{dt^2}, \tag{13}$$

and

$$eMg\sin\theta = I_1\dot{\alpha}, \tag{14}$$

where τ is torque, J is moment of inertia, θ is the rotation angle, e is the distance between the sphere center of gravity and pendulum center of gravity, and α is the angular velocity.

In order to yield motion equation of the spherical robot for driving and steering the robot, the Lagrange equation is necessary. The Lagrange functions L of the spherical robot shown in Fig. 2 based on Eg. 14 are

$$L = K_1 + K_2 + T_1 + T_2 - U_1 - U_2, \tag{15}$$

and

$$U_1 = 0,\ U_2 = -M_2 g e \cos(\theta+\beta), \tag{16}$$

$$\begin{aligned} K_1 &= \frac{1}{2}M_1(\mathrm{d}\omega_1)^2, \\ K_2 &= \frac{1}{2}M_2\left(\mathrm{d}\omega_1 - e\cos(\theta+\beta)(\omega_1+\omega_2)\right)^2 \\ &\quad + \left(\sin(\theta+\beta)(\omega_1+\omega_2)\right)^2, \end{aligned} \tag{17}$$

$$T_1 = \frac{1}{2}J_1\omega_1^2,\ T_2 = \frac{1}{2}J_2(\omega_1+\omega_2)^2, \tag{18}$$

where K_1 is the kinetic energy of the ball, K_2 is the kinetic energy of the pendulum, T_1 is the rotational energy of the ball, T_2 is the rotational energy of the pendulum, U_1 is potential energy of the ball with respect to the height of its centroid, and U_2 is the potential energy of the pendulum with respect to the height of the ball's centroid.

The Lagrange equation of motion is derived from the second derivation of Eq. 13 and 15:

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial L}{\partial\omega_1}\right)\frac{\partial L}{\partial\theta_1} &= -\tau + \tau_f, \\ \frac{d}{dt}\left(\frac{\partial L}{\partial\omega_2}\right)\frac{\partial L}{\partial\theta_2} &= -\tau, \end{aligned} \tag{19}$$

where τ_f is the torque caused by friction between the sphere and ground.

By substituting Eq. 16-18 into Eq. 19, then

$$\begin{aligned} -\tau + \tau_f =& a_1\left(J_1 + J_2 + M_1\mathrm{d}^2 + M_2\mathrm{d}^2\right) \\ &- a_1\left(2M_2 de\cos(\theta+\beta)\right) \\ &+ a_2\left(J_2 - M_2 de\cos\left(\theta+\beta+M_2e^2\right)\right) \\ &+ M_2\mathrm{d}e\sin(\theta+\beta)(\omega_1+\omega_2)^2 \\ &+ M_2 ge\sin(\theta+\beta), \end{aligned} \tag{20}$$

$$\begin{aligned} \tau =& a_1\left(J_2 - M_2 de\cos\left(\theta+\beta+M_2e^2\right)\right) \\ &+ a_2\left(J_2 + M_2e^2\right) \\ &+ M_2 ge\sin(\theta+\beta), \end{aligned} \tag{21}$$

where $(\theta+\beta)$ is the driving angle [10].

The spherical robot has the possibility of going up or down a hill or stair. The motion equation is started from the direction of gravity (g) and the angle of the inclination (θ) that are substituted to Eg. 16:

$$\begin{aligned} U_1 &= -M_1 g\mathrm{d}\theta_1\sin\psi \\ U_2 &= -M_2 g\mathrm{d}\theta_1\sin\psi - M_2 gd\cos(\theta+\beta+\sin\psi), \end{aligned} \tag{22}$$

where ψ is the inclination of the hill/stair.

Kinetics energy and rotational energy of a spherical robot that moves up or down are the same with Eq. 17, and 18. By substituting Eq. 17, 18, and 22 into 15, the Lagrange equation for spherical robot motion to go up or down the hill are achieved as

$$
\begin{aligned}
-\tau+\tau_f =& a_1\left(J_1+J_2+M_1\mathrm{d}^2+M_2\mathrm{d}^2\right)\\
&-a_1\left(2M_2de\cos\left(\theta+\beta\right)\right)\\
&+a_2\left(J_2-M_2de\cos\left(\theta+\beta+M_2e^2\right)\right)\\
&+M_2\mathrm{d}e\sin\left(\theta+\beta\right)\left(\omega_1+\omega_2\right)^2\\
&+M_2ge\sin\left(\theta+\beta+\psi\right)\\
&-\left(M_1+M_2g\mathrm{d}\sin\psi\right)
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
\tau =& a_1\left(J_2-M_2de\cos\left(\theta+\beta+M_2e^2\right)\right)\\
&+a_2\left(J_2+M_2e^2\right)\\
&+M_2ge\sin\left(\theta+\beta+\psi\right).
\end{aligned}
\tag{24}
$$

B. Obstacle Encounter

During its assigned task, a spherical robot might encounter an obstacle, and the scenario is shown in Fig. 3. A spherical robot is moving by rolling on the floor, and this move makes it quite difficult to utilize a proximity sensor since the sensor considers the floor as the obstacle. Therefore, the spherical robot should be designed to be able to overcome obstacles without proximity sensor. One possibility is by climbing up the obstacle or jump over it. In this study, the action taken is climbing up the obstacle.

The spherical robot can overcome its obstacle is by climbing it up with the condition below

$$
\begin{aligned}
&\text{(Driving torque)} > \text{(Counter torque)}\\
&M_2g\mathrm{d}\sin\phi > \left(M_1+M_2\right)g\sqrt{d^2-\left(d-h\right)^2}\\
&\therefore h_{max} = \mathrm{d}-\sqrt{\mathrm{d}^2-\left(\frac{eM_2}{M_1+M_2}\right)}
\end{aligned}
\tag{25}
$$

where h is the height of obstacle [10].

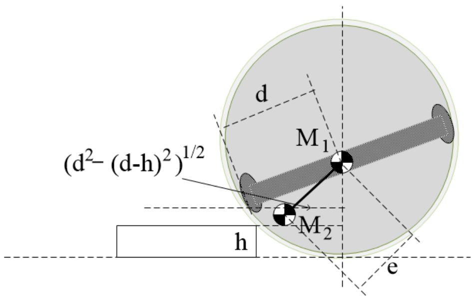

Fig. 3. A spherical robot encounter obstacle with h as its height.

IV. Path Planning and Trajectory Tracking Control Discussion

The path tracking of a spherical robot is similar to a car-like mobile robot where the steering motion is achieved by a forward motion. The difference is that in a spherical robot, the steering of angular velocity affects the angular velocity of the robot's side roll. Most of spherical robots adopt the pendulum or inverted pendulum mechanism, and from this design kinematics and dynamics are derived [4] [8] [16] [17] [20] [21] [24] [25] [26] [29] [34].

Path planning and trajectory tracking control are derived from dynamics into velocity control [9], sliding mode controller [22] [23], position control [2] [27], motion control [28], and trajectory tracking control [31] [32] [33] [35]. The path planning design of a spherical robot on current research are

Norsahperi et al. 2015 [4] investigated the possibility of bouncing mechanism in a sphere robot by using Particle Swarm Optimization (PSO) technique. By using a 3D virtual prototype, the free fall bouncing, shooting up, and the projectile scenario was presented to show the feasibility of the proposed method. Kayacan 2012 [8] designed a grey-PID-Fuzzy Controller with an adaptive step size to control the velocity of a spherical robot which is considered as a non-linear system. Fuzzy controller design was intended to tune the step size of the grey predictor. The grey PID is combined with a fuzzy controller to overcome the overshoot and settling time problem. The simulation program was presented to show effectiveness in the controller the velocity. Nagai 2008, in his master thesis [10] elaborated the control system of a spherical robot.

Azizi et al. 2013 [21] introduced dynamic modeling a spherical robot with a 3DOF inner mechanism. The kinematics and dynamics derived by Euler parameters and Kane's method. The authors add another DOF to the inner mechanism to achieve more freedom. Azizi et al. 2014 [22] proposed sliding mode controller (SMC) for position control and SMC is utilized to reduce the chattering phenomena. The control gains are determined using the Lyapunov direct method to ensure the robustness and zero convergence. The feasibility of the proposed method was verified by simulation. In 2017, Azizi et al. [23] designed point stabilization of nonholonomic spherical robot using model predictive control.

Li 2018 [24] designed a trajectory tracking control by utilizing the four-state inspired by a 2-DOF pendulum. The four states are two position and two attitude states. The controller is deduced base from the shunting model of neuro-dynamics and kinematics, and the stability was confirmed by Lyapunov's direct method. The effectiveness of the proposed method was verified using MATLAB simulation.

Furuse et al. 2015 [25] analyzed the dynamics of a spherical robot by focusing on the center of gravity. The research introduced a rigid spherical shell containing a 2DOF pendulum as the control component.

Gajbhiye 2016 [30] designed two control laws for spherical robot trajectory tracking that are the orientation tracking using a modified traced potential function and the contact position tracking using transport map. The proposed method claimed to be able to reduced orientation stabilization and demonstrated using simulation.

Wu et al. 2017, [32] introduced a passive magnetic field to detect obstacle during spherical robot deployment. The obstacle detection is used by avoidance behavior to generate the trajectory tracking method.

Cai et al. 2012 [33] discussed the practical path tracking control by focusing on the simultaneous converges of position and pose. The fuzzy controller is used along with a hierarchical sliding mode controller through a backstepping strategy. The proposed method was validated through MATLAB simulation.

A spherical robot has a wide area of application due to its natural rolling motion and can be shielded into a waterproof robot. However, due to the rolling motion and pendulum concept, it has more constraint and challenging to control. It is also more challenging to add the sensor to this robot, for example, adding proximity sensor and camera.

V. Conclusion

The spherical robot is an interesting type of robot that combined the concept of a mobile robot and pendulum. This combination leads to more complex kinematics and dynamics modeling, and based on this modeling, the control schema is derived. The nature of its motion is also adding to the nonholonomic constraint. Aside from these challenges, the application of a spherical robot is extensive, from being a simple toy, to become an industrial surveillance robot. The positioning of all mechanics part inside a shell is also making it easier to make it waterproof and applied as an underwater robot or in a wet environment such as piping inspection robot. Therefore, the study of this type of robot needs to be developed to get the best of it.

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