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# Inventory models with reverse logistics for assets acquisition in a liquefied petroleum gas company

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## Abstract

This paper addresses a case study regarding inventory models for acquiring liquefied petroleum gas (LPG) cylinders. This is an industrial challenge that was proposed at an European Study Group with Industry, by a Portuguese energy company, for which the LPG cylinder is the main asset of its LPG business. Due to the importance of this asset, an acquisition plan must be defined in order to determine the amount of LPG cylinders to acquire, and when to acquire them, in order to optimize the investment. As cylinders are returned and refilled, the reverse logistic flows of these assets must be considered. As the classical inventory models are not suitable for this case study, three new inventory models, which account for the return of LPG cylinders, are proposed in this work. The first proposed model considers deterministic constant demand and continuous returns of LPG cylinders, with discrete replenishment from the supplier. The second model is similar, but for the case when the returned cylinders cover for the demand. A third model is also proposed considering that both the demand and the returns are stochastic in nature and the replenishment from the supplier is discrete. The three models address different scenarios that the company is either currently facing or is expecting to occur in the near future.

**MSC:** 90B05; 90B06; 90B50

**Keywords:** Inventory models; Reverse logistics; Assets acquisition; Decision making

## 1 Introduction

This work addresses an industrial challenge that consists in planning the acquisition of liquefied petroleum gas (LPG) cylinders. The challenge was proposed at an European Study Group with Industry, by a Portuguese company of the energy sector (named in this paper ALPHA for confidentiality purposes). This company started its activity in 2006 focusing in the production and distribution of biofuel. Since then, it has extended its business areas to other fuels and energy sources. The LPG cylinder business started in 2012, and since then it has experienced a continuous growth. ALPHA currently commercializes propane gas in two types of cylinders: type A with 9 kg capacity, and type B with 45 kg capacity.

In Portugal, companies selling LPG cylinders are also responsible for collecting the empty cylinders, regardless of the company from which the previous cylinders were

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bought following a direct replacement policy [13]. The empty cylinders returned to the company can be filled again with LPG and sold to the clients. As the acquisition of new cylinder bottles is expensive, reusing is a key factor for profitability. However, the cylinders are assets owned by the several companies operating in this sector, and each competitor can only refill its own cylinders. Clients frequently decide to change their LPG supplier and, when they do, they return to the new supplier a cylinder that is owned by a different company. Companies are required to return the cylinders to the company that owns them, but that process is time consuming. Therefore, companies experiencing growth, such as ALPHA, have to purchase additional cylinders to meet the demand.

The objective of this industrial challenge was to find a model to forecast the demand and return rate of each type of cylinder, and to define an assets acquisition plan, i.e., to determine *when to order* to the external supplier new LPG cylinders (Order Point) and *how many* should be bought (batch size), in order to optimize the investment.

Linear models related techniques concerning energy quantities forecasting are attracting a great attention within the energy sector. Di Persion *et al.* [8] applies an exponential smoothing model, ARMA-ARIMA and ARIMA-GARCH models for forecast of energy load in the Italian energy market. In [3] the authors model the logarithm of the spot price of electricity with a normal inverse Gaussian process. Costa e Silva *et al.* [7] presents time series data mining for forecasting energy prices of the Iberian market. Street *et al.* [14] discuss the potential for creating a flexible electricity-gas market in Brazil mainly due to the need of using natural gas for power generation in certain periods of the year. Several authors use forecasting techniques to feed information to inventory models. Lai *et al.* [11] develop an optimization model to control the inventory level of Liquid Natural Gas at a downstream facility. A stochastic evolution of the price state vector during the given time horizon is used in an inventory model that can assist in the decisions regarding regasification, sale and storage. Chebeir *et al.* [5] consider Neural Networks to forecast the demand of natural gas and use the forecasts as input for a strategic planning model that determines the economic feasibility and the development strategy for natural gas production and distribution.

The methodology used to answer the challenge addressed in the present paper can be divided into three phases:

- In the first phase, it is necessary to forecast demand, sales and the return of LPG cylinders (further details can be founded in [6]).
- Subsequently, in the second phase, this forecast is used in an inventory management model.
- Finally, in the third phase, since it is necessary to consider the return rate of LPG cylinders, reverse logistic models and closed loop supply chain models are explored.

In order to answer to what is required in the first phase, for forecasting of demand and returns, time series (TS) techniques (i.e. exponential smoothing and moving averages), Multiple Linear Regression models (MLR) and Artificial Neural Networks (ANN) were used in [6]. In order to eliminate the drawbacks of these methods and maintaining their advantages, the previous methods were combined in an ensemble method. For each method, a probability density function was defined and a Monte Carlo simulation was used. The obtained forecast values are a linear combination with weights proportional to the accuracy of each method. This methodology leads to more robust forecasts and allows to deal with nonlinearity and seasonality.

Concerning with the second phase, classical inventory models, such as the Wilson model, determine the Economic Order Quantity (EOQ) as the batch size that minimizes the total cost of stock management. A drawback of this approach is that it does not take into account reverse logistics. In the company, returned items arrive continuously and not in discrete moments. Furthermore, there are three different possible destinations for returned items: Cleaning, Requalification and Disposal. Therefore, reverse logistics (i.e. the return of cylinders) plays a crucial role in this challenge.

The focus of the current paper is in the last two phases.

The structure of this paper is the following. This Introduction section presents the problem description, the objective of the proposed industrial challenge and the adopted methodology. Section 2 briefly accounts for a literature review on the classical inventory models and the reasons why they are not applicable in this case study. Section 3 proposes an adaptation of the classical models proposed by Wilson and Teunter to the case where deterministic continuous returns are considered, along with discrete replenishment from the supplier (Sect. 3.1); the case where the deterministic continuous returns make replenishment from the supplier not needed (Sect. 3.2); and finally the case where demand and returns are stochastic (Sect. 3.3). In Sect. 4, the final conclusions are presented.

## 2 Literature review

Decision making regarding how much and when to procure the goods involved in a producing system is usually ruled by inventory management models, which intent to optimize a given cost function.

There are several methodologies proposed for inventory management. Two widely used classical inventory models are the ones presented by Wilson [10, 16] and Richter [12], which defined the two principal classes of methods used, described in following sections.

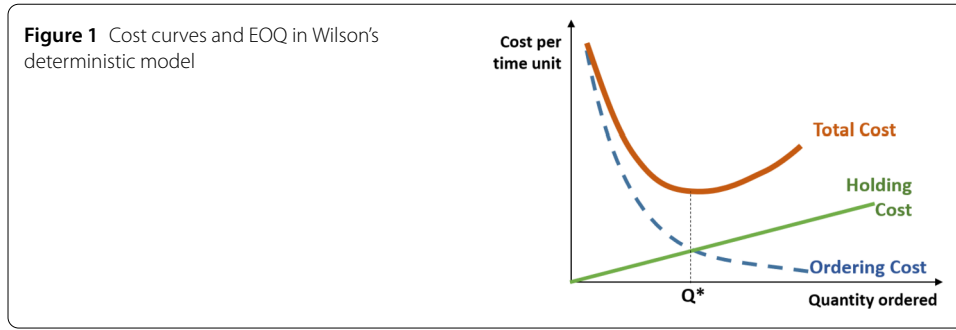
### 2.1 Economic order quantity

Classical inventory models, such as Wilson's deterministic model [10, 16], determine the EOQ as the batch size that minimizes the total cost of stock management. The total cost is the sum of three components, namely:

- Acquisition costs ( $C_A$ )—the price of acquiring the assets.
- Setup costs (or ordering costs) ( $C_S$ )—the fixed cost for every order, such as, transportation, collection, etc.
- Holding costs ( $C_H$ )—insurances, taxes, rent, electricity, salary, opportunity costs, etc.

The EOQ model is an attempt to estimate the optimal order quantity ( $Q^*$ ) by balancing the conflicting costs of holding stock and of placing replenishment orders (Fig. 1). The effect of order quantity on stock-holding costs is that, the larger the order quantity for a given item, the longer the average time in stock, and the greater will be the storage costs. On the other hand, the placing of a large number of small-quantity orders produces a low average stock, but a much higher cost in terms of the number of orders that need to be placed and the associated administrative and delivery costs. Once the forecast of the demand of LPG cylinders is determined, an EOQ model can be used for inventory management [2].

Another classical approach is the Continuous Review Policy ( $s, Q$ ), which considers probabilistic demand [2]. This is a continuous inventory checking policy, where the ordering of  $Q$  amounts is performed when the inventory level reaches the reorder point ( $s$ ).



The advantage of this policy is that it can handle scenarios where demand is high but the loss of order quantity is variant.

A drawback of the classical approaches is that they do not take into account reverse logistics, which in the industrial challenge addressed in the present paper (i.e. the return of cylinders) plays a crucial role. The plan should take in account the empty cylinders that are returned to the company, which can be either reused or disposed of. Therefore, we started by applying two inventory models with reverse flows found in the literature, using the data provided by the company. Afterwards, two deterministic models and a stochastic model tailored for this case study, were developed.

### 2.2 Inventory models with reverse flows

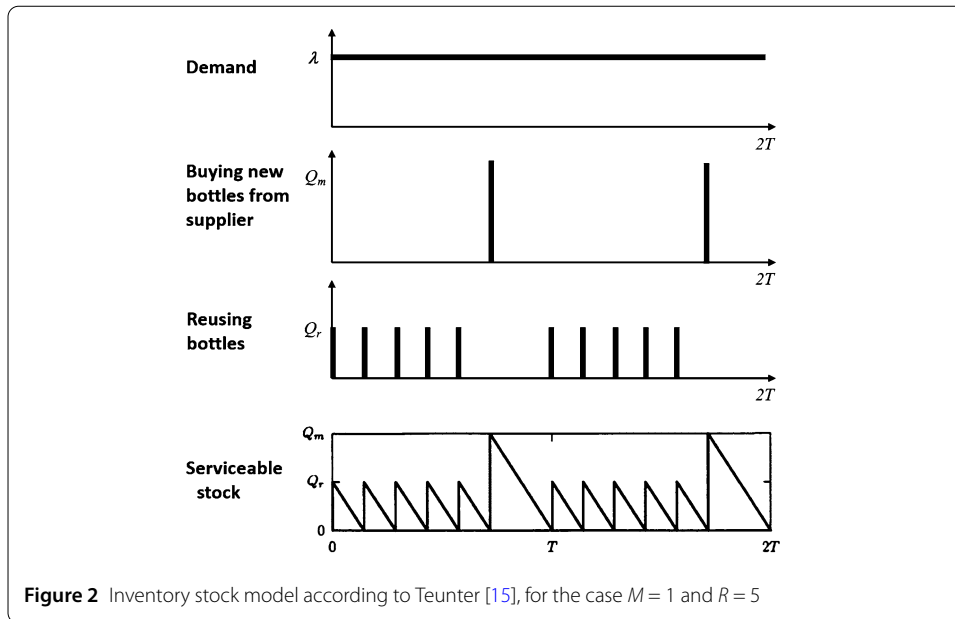
For organizations that practice reverse logistics, i.e. which reuse products and/or materials, the stock levels should account for the amount of returned items. Moreover, the inventory management plan must include the costs of the recovery operations.

Richter [12] extended the EOQ model to allow the integration of used products, which were repaired and incorporated in the production system. It assumes a stationary demand in a model with two shops. The first shop is producing new products and repairing products used by the second shop. This model considers deterministic demand and return rates, and also a constant disposal rate. Different holding costs in the first and second shops are considered in the model. In Richter's model, the returned items may either be reused or disposed of.

The inventory model developed by Teunter [15] is also based on the EOQ model, however it considers the return flow of items that can be recovered. The model proposed by Teunter also uses deterministic demand and return rates. The difference from Richter's model is that it considers a varying disposal rate instead of a constant rate. Additionally, different holding costs for manufactured, recoverable and recovered items can be considered separately. Returned items may also be either reused or disposed of. In this model,  $M$  manufacturing batches and  $R$  recovery batches succeed each other, but there can be only two cases: either  $M = 1$  or  $R = 1$ . A schematic of demand, purchases, returns, and the serviceable stock levels is shown in Fig. 2.

The formula (1) computes the Total Cost ( $TC$ ) per unit of time, for the case with  $M = 1$ , and comprehends the sum of three components: the Acquisition Costs, the Setup Costs, and the Holding Costs.

$$\begin{aligned}
 TC = & c_m \lambda (1 - \beta) + c_r \lambda \beta + c_d \lambda (g - \beta) + \frac{K_m \lambda (1 - \beta)}{Q_m} + \frac{K_r \lambda \beta}{Q_r} \\
 & + h_m \frac{1}{2} (1 - \beta) Q_m + h_r \frac{1}{2} \beta Q_r + h_n \frac{1}{2} \left( \beta Q_r + \left( \beta - \frac{g - \beta}{1 - \beta} \cdot \frac{\beta}{g} \right) Q_m \right). \tag{1}
 \end{aligned}$$



**Figure 2** Inventory stock model according to Teunter [15], for the case  $M = 1$  and  $R = 5$

The formulas for the optimal batch size for manufacturing,  $Q_m$ , and for recovery,  $Q_r$ , are, respectively:

$$Q_m = \sqrt{\frac{2K_m\lambda(1-\beta)}{h_n(1-\beta) + h_n(\beta - \frac{g-\beta}{1-\beta} \frac{\beta}{g})}} \tag{2}$$

and

$$Q_r = \sqrt{\frac{2K_r\lambda}{h_r + h_n}} \tag{3}$$

and the number of recovery batches is:

$$R = \frac{\beta}{1-\beta} \frac{Q_m}{Q_r} \tag{4}$$

where:

- $\lambda$ —demand (continuous and deterministic);
- $g$ —return percentage ( $0 < g < 1$ );
- $\beta$ —reuse percentage ( $0 < \beta < 1$ );
- $\lambda g$ —items returned;
- $\lambda\beta$ —items reused;
- $\lambda(g - \beta)$ —items disposed of;
- $t$ —continuous time variable;
- $c_m$ —cost of manufacturing an item;
- $c_r$ —cost of recovering an item;
- $c_d$ —cost for disposal of an item;
- $K_m$ —setup cost for manufacturing;
- $K_r$ —setup cost for reusing;

- $h_m$ —holding cost of a manufactured item;
- $h_r$ —holding cost of reused item;
- $h_n$ —holding cost of a recoverable item.

Other developments of the EOQ model are due to Alivoni *et al.* [1]. The authors proposed a stochastic model where the decisions between production or purchase of new items integrates product reuse, in order to identify the need of placing a production/purchasing order to avoid stock-out situations.

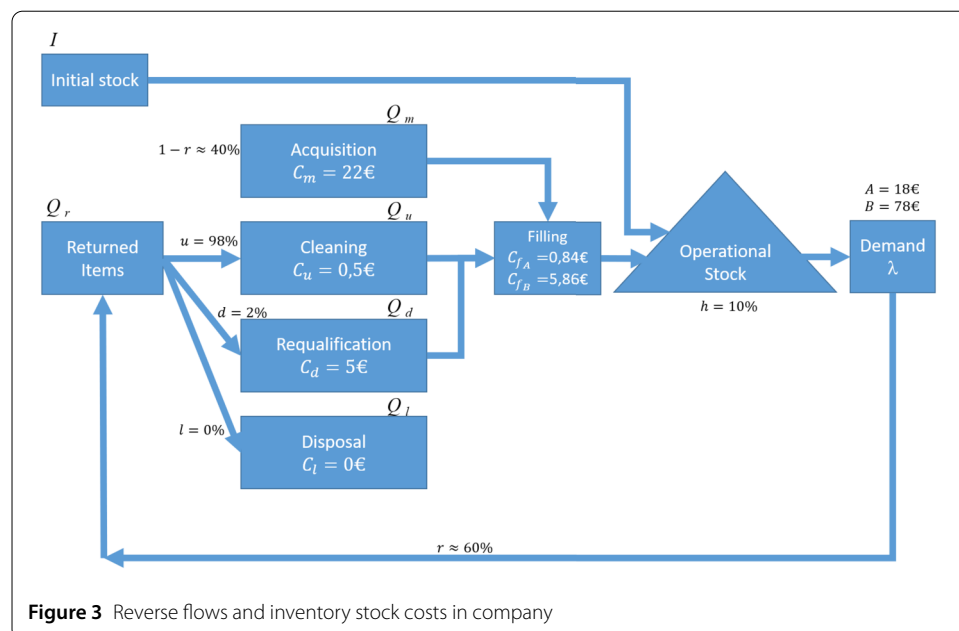
### 3 Developed inventory models

In order to solve the current industrial challenge of assets acquisition management, the formulas (1), (2), (3) and (4) from Teunter’s model were implemented in a spreadsheet for the company to compute: the total cost per unit of time (case  $M = 1$ ); the optimal batch size for manufacturing,  $Q_m$ , and for recovery,  $Q_r$ ; and the number of recovery batches,  $R$ .

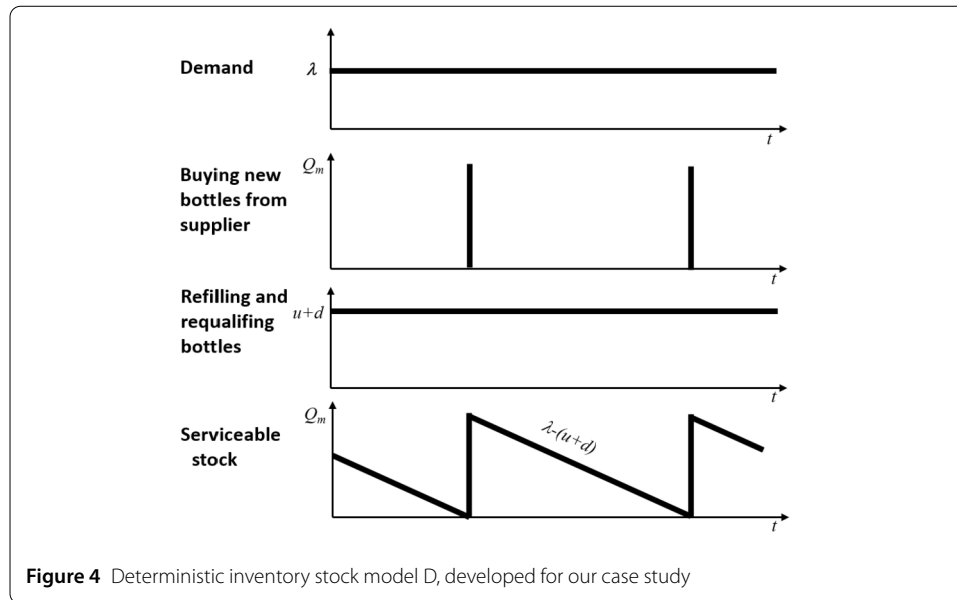
Although this is a useful tool for assisting the company manager to make decisions about assets acquisition, the models presented before do not contemplate all the specifications required in this case study.

Currently the company policy is based on continuous replenishment, i.e., the returned items ( $r$ , is approximately 60% of the total cylinders of the company) arrive continuously and not in discrete moments. There are three different possible destinations for returned items: Cleaning, Requalification, and Disposal, as depicted in Fig. 3.

The majority of the returned LPG cylinders (98%) only need cleaning, and a small percentage (about 2%) need requalification. At the moment, because this business is relatively new for the company, there are no LPG cylinders that need to be disposed of, but in the future this situation can occur. The costs and time for each of these processes are different. A cleaning unit cost of  $C_u = 0.5 \text{ €}$ , requalification cost of  $C_d = 5 \text{ €}$  and a negligible cost for disposal ( $C_l$ ) are assumed. These are example values for confidentiality reasons. The quantity of batches to clean, requalify and dispose off are denoted by  $Q_u$ ,  $Q_d$  and  $Q_l$ .



**Figure 3** Reverse flows and inventory stock costs in company



Approximately 40% of the total cylinders of the company are acquired. Returned and acquired cylinders are filled (with costs  $C_{fA}$  and  $C_{fB}$ , for A and B cylinders respectively) and go to operational stock, in order to meet the demand ( $\lambda$ ).

Teunter's model considers that both the acquired and returned cylinders arrive at discrete moments periodically in time, but actually that only happens with the acquired cylinders. In the company, the returned cylinders arrive continuously to the warehouse, and are continuously cleaned, requalified (with rate  $u + d$ ) and filled, as depicted in Fig. 4. Therefore, a continuous replenishment model could be adapted to this case study. In this setting, two cases may occur:

– *Case  $\lambda > u + d$ :*

If demand exceeds the incoming flow, it is necessary to buy new cylinders from suppliers. Thus, a Deterministic Model D, with continuous returns, is developed for this case.

– *Case  $\lambda \leq u + d$ :*

If the returned cylinders are enough to respond to the demand, buying new cylinders is unnecessary. To address this case, the Deterministic Model R, without purchases, is considered.

In both models, demand and return rate are considered deterministic constants. However, this does not occur in the company under study. For this reason, a third stochastic inventory model was developed.

On the next subsections each of the three proposed models are described.

### 3.1 Model D—deterministic continuous returns

The developed deterministic model D, based on EOQ [10, 16], considers:

- deterministic continuous constant demand,
- deterministic discrete replenishment from supplier,
- deterministic continuous constant replenishment from returned cylinders,

for the case  $\lambda > u + d$ , when returns are not enough to respond to the demand and hence the company has to buy new cylinders from the supplier (Fig. 4). In the last graph in Fig. 4,  $\lambda - (u + d)$  corresponds to the slope.

As in the classical EOQ formula, in this model, the total costs considered are the sum of the acquisition costs  $C_A$ , setup costs  $C_S$  and holding costs  $C_H$ . The acquisition costs in equation (5) consider the cases where: new cylinders are acquired from the supplier with a cost  $C_m$ , the cylinders are reused with just a cleaning cost  $C_u$ , or the case where the returned cylinders have to be requalified with a cost  $C_d$ . In these three cases, a constant filling cost is also included,  $C_{fA}$  and  $C_{fB}$ , for A and B cylinders, respectively. In the future, a disposal cost  $C_l$  could also be considered. At the moment, because this business is relatively new for the company, there are no LPG cylinders that need to be disposed of. Hence, the rate of cylinders returned and disposed of ( $l$ ) is zero. The acquisition costs are:

$$C_A = C_m(1 - r)(\lambda - I) + C_u u(\lambda - I) + C_d(r - u)(\lambda - I), \tag{5}$$

where  $\lambda$  is the constant demand rate (units/units of time),  $I$  is the initial stock,  $r$  is the return rate,  $u$  is the rate of cylinders returned and cleaned, and  $d = r - u$  is the rate of cylinders returned and requalified.

The setup costs are:

$$C_S = \frac{K_m(\lambda - I)(1 - r)}{Q_m} + \frac{K_u(\lambda - I)u}{Q_u} + \frac{K_d(\lambda - I)(r - u)}{Q_d}, \tag{6}$$

where  $K_m$  is the production fixed setup costs,  $K_u$  is the reuse fixed setup costs,  $K_d$  is the requalification fixed setup costs,  $Q_m$  is the batch size for buying new cylinders,  $Q_u$  is the batch size for reusing cylinders, and  $Q_d$  is the batch size for requalifying cylinders.

The holding costs are:

$$C_H = h_m \frac{(1 - r)Q_m}{2} + h_u \frac{uQ_u}{2} + h_d \frac{(r - u)Q_d}{2} + h_i \frac{I}{2}, \tag{7}$$

where  $h_m$  is the holding cost per new item bought per year,  $h_u$  is the holding cost per reused item per year,  $h_d$  is the holding cost per requalified item per year, and  $h_i$  is the holding cost per existent item in stock per year.

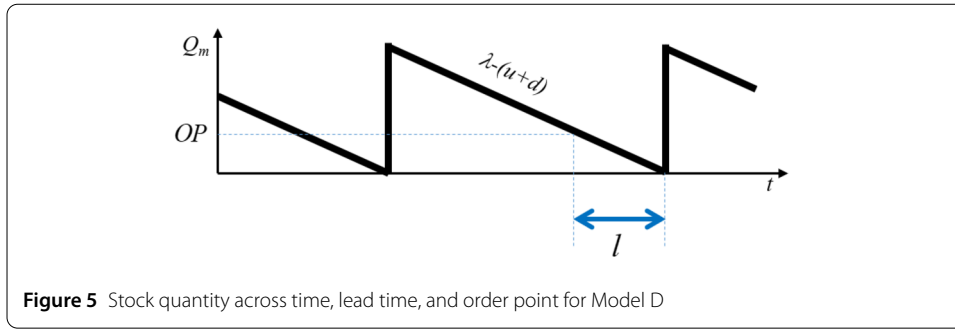
By deriving the total costs, it is possible to obtain the expression for the optimal quantities:  $Q_m^*$ —batch size for buying new cylinders;  $Q_u^*$ —batch size for reuse; and  $Q_d^*$ —batch size for requalification, that minimize the total costs, using the following expressions, respectively:

$$Q_m^* = \sqrt{\frac{2K_m(\lambda - I)}{h_m}}, \tag{8}$$

$$Q_u^* = \sqrt{\frac{2K_u(\lambda - I)}{h_u}}, \tag{9}$$

$$Q_d^* = \sqrt{\frac{2K_d(\lambda - I)}{h_d}}. \tag{10}$$





**Figure 5** Stock quantity across time, lead time, and order point for Model D

The stock levels according to this model present a “saw” shaped graph, as can be seen in Figs. 4 and 5. Given the demand  $\lambda$ , the return rates  $u + d$  and the lead time  $l$ , the quantity in stock that marks when an order should be placed to the external supplier is called the Order Point ( $OP$ ). This is the quantity that equals the exact amount of goods necessary to fulfil the demand during the lead time. The lead time is the interval of time between placing an order to the supplier and the reception of the ordered goods.

From the triangle in Fig. 5, given that the slope  $\lambda - (u + d)$  is constant, the following expression holds:

$$\frac{OP}{l} = \lambda - (u + d) \tag{11}$$

thus the  $OP$  is found as:

$$OP = (\lambda - (u + d))l. \tag{12}$$

This model was also implemented in a spreadsheet and provided to the company for being used with the company’s data.

### 3.2 Model R—deterministic without purchases

For the case  $\lambda > u + d$ , the developed deterministic model R, without purchases considers:

- deterministic continuous constant demand,
- unnecessary replenishment from supplier,
- deterministic continuous constant replenishment from returned cylinders.

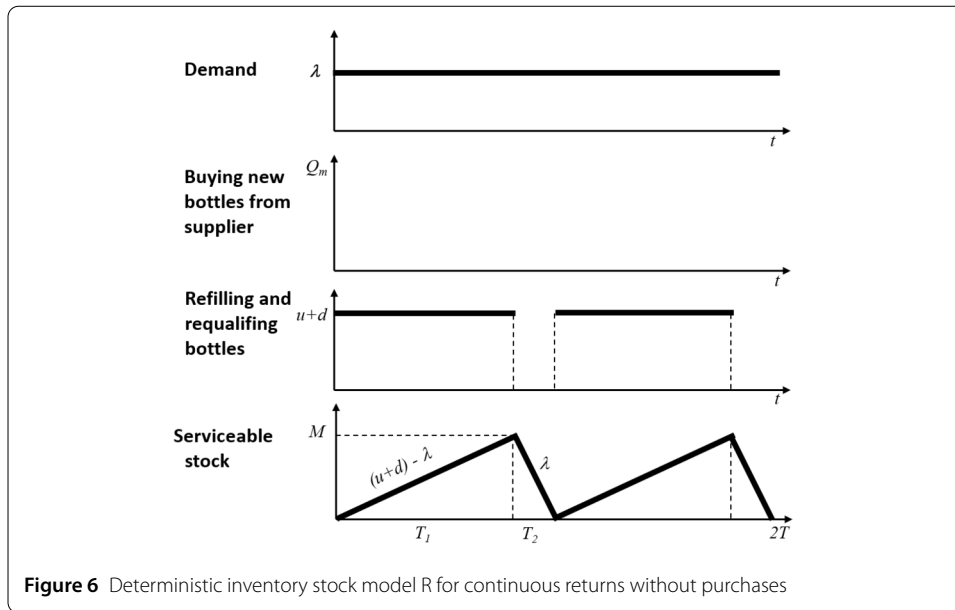
In this setting, there is a period  $T_1$  where there is simultaneously continuous replenishment of cylinders (with rate  $u + d$ ) and demand being satisfied (with rate  $\lambda$ ); and a period  $T_2$  where replenishment is interrupted and there is only demand being satisfied. The serviceable stock levels corresponding to this scenario are schematized in Fig. 6, last graph. In this Figure,  $(u + d) - \lambda$  and  $\lambda$  denote the slopes of the respective lines.

From the slopes of the main triangles in Fig. 6 last graph, we have:

$$T_1 = \frac{M}{u + d - \lambda}, \tag{13}$$

$$T_2 = \frac{M}{\lambda}, \tag{14}$$

$$M = Q - \lambda \cdot T_1 = Q \left( 1 - \frac{\lambda}{u + d} \right), \tag{15}$$



where  $M$  is the maximum stock level, and the batch size corresponds to the total production during period  $T_1$ , i.e.,  $Q = (u + d)T_1$ .

The total costs are given by:

$$TC(Q) = C_u u(D - I) + C_d d(D - I) + (K_u + K_d) \frac{D}{Q} + C_h \frac{Q}{2} \left( 1 - \frac{\lambda}{u + d} \right), \tag{16}$$

where  $D$  is the demand for the planning horizon (a year) and  $\lambda$  is the daily demand. By the differentiation of the total costs expression (16), the optimal quantity  $Q^*$  that minimizes the total cost is determine as:

$$Q^* = \sqrt{\frac{2(K_u + K_d)D}{C_h}} \sqrt{\frac{u + d}{u + d - \lambda}}. \tag{17}$$

If the lead time  $l$  is longer than the period of demand, i.e.  $l > T_2$ , then, from the slope in the blue triangle in Fig. 7:

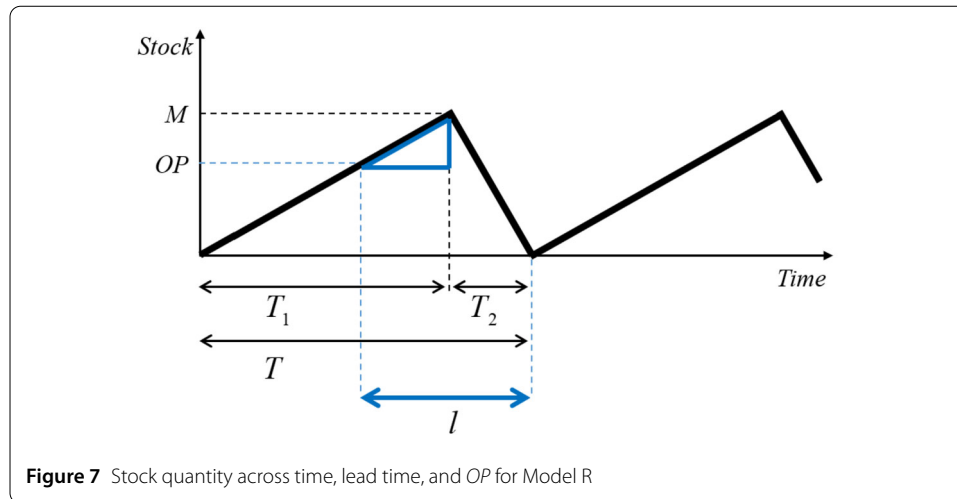
$$\frac{M - OP}{l - T_2} = u + d - \lambda \tag{18}$$

therefore the  $OP$  can be deduced as:

$$OP = M - (u + d - \lambda)(l - T_2). \tag{19}$$

Replacing  $M$  and  $T_2$  using equations (15) and (14), the order point  $OP$  is obtained as a function that depends only on the quantity of cylinders  $Q$ , the demand and reutilization rates, and lead times, as:

$$OP = Q \left( 1 + \frac{u + d}{\lambda} \right) + l(\lambda - (u + d)). \tag{20}$$



### 3.3 Model S—stochastic inventory model

In models  $D$  and  $R$ , presented in Sects. 3.1 and 3.2, it is assumed a deterministic constant demand and return rate. However, in the company, these rates are neither constant nor deterministic. In fact, both seasonality and trend are present. This is in line with a study of the German energy market by Di Persio and Perin [9], where it was found that electricity consumption is also affected by seasonality.

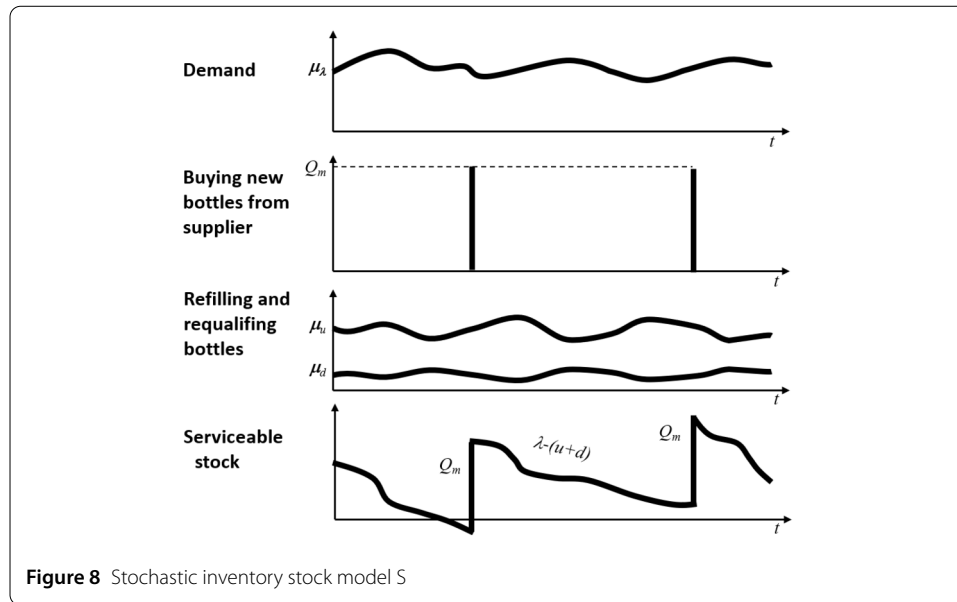
To correctly plan the acquisition of new cylinders from the supplier, it is necessary to proceed with the forecast not only for the demand, but also for the reverse logistic flows.

For this purpose, several forecasting techniques were used by Correia *et al.* in [6]. Forecasting of demand and returns was made using exponential smoothing and moving averages to compute seasonal coefficients and to forecast demand and returns. In the same work, significant relations were found between demand and Temperature, Promotional Campaigns, Sales Objectives and Expectation of Price Increase, using MLR. Similar relations were studied for returns. Furthermore, ANN were also used to forecast demand and returns. The results obtained using different forecasting techniques were compared. When ANN is used, the Mean Squared Error (MSE) is 30% of the MSE achieved using MLR, and 10% or 20% of the MSE achieved using TS, depending on the type of bottles. For further details see [6].

In order to improve the forecast, the probability density functions of the forecasts obtained by each of these individual methodologies were combined in an ensemble approach. In this approach, similar to the one proposed in [4] by Cassettari *et al.*, a weighted linear combination of the density functions of the forecasts was used for obtaining a better final forecast. This methodology leads to more robust forecasts and allows to deal with non-linearity and seasonality. The forecasted mean and Root Mean Square Error (RMSE) were used as input values for the stochastic inventory models developed in the present case study.

For this scenario, a stochastic inventory model  $S$ , based on the continuous review policy  $(s, Q)$  was presented, which considers:

- continuous stochastic demand,
- discrete replenishment from supplier,
- continuous stochastic replenishment from returned cylinders,
- constant lead times,



as depicted in Fig. 8. The graphs, from the top to the bottom, present: the mean demand,  $\mu_\lambda$ ; the acquisition of new LPG cylinders  $Q_m$ ; the mean refilling,  $\mu_u$ , and requalifying,  $\mu_d$ , number of cylinders; and the serviceable stock.

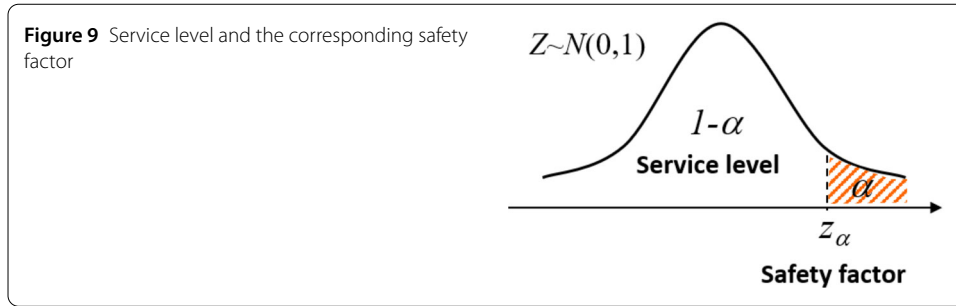
To fit a probability distribution for the demand and return rates, our data was analysed using histograms, Q-Q plots and the Shapiro Wilk test for normality. Sales of type B bottles was somewhat positively skewed, but both the sales of type A bottles, and the returns of type A and type B bottles, could be considered normally distributed, for a 1% significance level, since the Shapiro Wilk test  $p$ -values are 0.0179, 0.4317 and 0.1736, respectively. Thus, it is assumed that the demand during lead time,  $dl$ , is normally distributed with mean equal to  $\mu_{dl}$  and a standard deviation of  $\sigma_{dl}$ , i.e.  $dl \sim N(\mu_{dl}, \sigma_{dl})$ . In this case, the  $OP$  is given by:

$$OP = \mu_{dl} + z_\alpha \sigma_{dl}, \tag{21}$$

where  $z_\alpha = \Phi^{-1}(1 - \alpha)$  is the safety factor for a given service level  $1 - \alpha$  (see Fig. 9). The second term of equation (21) is called the *safety stock (SS)*. This term is intended to absorb the demand variations and to prevent out-of-stock situations. Furthermore, the parameter  $\alpha$  measures the probability of out-of-stock in a replenishment cycle. Usually,  $\alpha$  is a small value decided by the company, such as 5% or 1%. It is desirable to choose a small value for  $\alpha$  because the smaller this value is, the less is the probability of happening a situation where the company does not have enough cylinders to satisfy demand, but this causes to have a larger safety stock, which costs more money. Therefore the trade-off between costs and out-of-stock situations must be carefully planned by the company.

The demand is replaced by the random variable  $\lambda - (u + d)$ , i.e. by a linear combination of normally distributed random variables:  $\lambda \sim N(\mu_\lambda, \sigma_\lambda)$ ,  $u \sim N(\mu_u, \sigma_u)$  and  $d \sim N(\mu_d, \sigma_d)$ . For these random variables  $\mu_i$  and  $\sigma_i$  are the forecasted mean and RMSE, with  $i \in \{\lambda, u, d\}$ .

If, for simplification reasons, the independence of the random variables  $\lambda$ ,  $u$  and  $d$  is assumed, then the mean and the standard deviation of the demand is given, respectively,



by:

$$\mu_{\lambda-(u+d)} = \mu_{\lambda} - \mu_u - \mu_d \tag{22}$$

and

$$\sigma_{\lambda-(u+d)} = \sqrt{\sigma_{\lambda}^2 + \sigma_u^2 + \sigma_d^2}. \tag{23}$$

However, independence may not be a reasonable assumption in this case study. In fact, as consumers return the empty cylinders when they buy new LPG bottles, there is a significant correlation between sales and returns ( $R = 0.7$ ). Therefore, formula (23) can be adjusted to cover the case of correlated variables (Eq. (24)), where covariance can be estimated with the sample covariance observed in the data.

$$\sigma_{\lambda-(u+d)} = \sqrt{\sigma_{\lambda}^2 + \sigma_u^2 + \sigma_d^2 - 2COV(\lambda, u) - 2COV(\lambda, d) + 2COV(u, d)}. \tag{24}$$

Finally, substituting (22) and (24) in (21), the *OP* is given by:

$$OP = l(\mu_{\lambda} - \mu_u - \mu_d) + z_{\alpha}\sigma_{\lambda-(u+d)}\sqrt{l}. \tag{25}$$

This quantity, which marks when to place the order to the supplier, is the sum of the expected demand not covered by the reverse flows during the lead time, with the safety stock for the desired service level  $1 - \alpha$ .

#### 4 Conclusions

A Portuguese company in the energy sector posed a challenge that consisted in finding the best acquisition plan for LPG cylinders. To answer this industrial challenge, three inventory models with reverse flows were developed. These inventory models were implemented in spreadsheets and were given to the company’s manager to be used as decision support tools for acquisition planning.

The first model, model D, considers deterministic continuous constant replenishment from returned LPG cylinders and also discrete batches of new cylinders that are bought from the supplier. This model reflects the current situation of the company that is experiencing a considerable growth in their sales volume and whose demand is larger than the incoming flow, i.e.  $\lambda > u + d$ .

The second model, model R, considers the future situation of the company, for which the returned cylinders cover for the demand, i.e.  $\lambda \leq u + d$ . In this case replenishment from the supplier will be unnecessary.

The third model, model S, considers that both the demand and return rate are of stochastic nature. In this case non deterministic inventory models should be adapted instead. Model S was developed for a mixed scenario, regarding stochastic demand and return rate, with continuous returns and periodic discrete acquisition of new cylinders from the supplier. This last model is an approach that would better fit the particularities of the challenge proposed by this company.

As future work, results for each of the proposed inventory models, regarding real-world scenarios reflecting the company's possible situations, are to be obtained. The obtained results will assist the company's manager on planning the acquisition of LPG cylinders.

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#### Abbreviations

ANN, Artificial Neural Networks; ARMA, Autoregressive Moving Averages; ARIMA, Autoregressive Integrated Moving Averages; COV, Covariance; GARCH, Generalized Autoregressive Conditional Heteroscedasticity; EOQ, Economic Order Quantity; LPG, Liquefied Petroleum Gas; MLR, Multiple Linear Regression; MSE, Mean Square Error; OP, Order Point; RMSE, Root Mean Square Error; SS, Safety Stock; TC, Total Cost; TS, Time Series.

#### Availability of data and materials

Data sharing not applicable to this article as no datasets were analyzed during the current study.

#### Competing interests

The authors declare that they have no competing interests.

#### Authors' contributions

All authors have jointly carried out the research and worked together on the manuscript. The main idea of this paper was due to CL. All authors read and approved the final manuscript.

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