UDC 629.3.025.2
10.18372/2306-1472.68.10906

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ROBUST CONTROL OF INERTIALLY STABILIZED PLATFORMS FOR GROUND VEHICLES ON THE BASIS OF H_∞-SYNTHESIS

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Abstract

Purpose: *Operation of inertially stabilized platforms mounted on the ground vehicles is accompanied by influence of significant parametrical and various coordinate disturbances. To keep high operating characteristics of a system in such difficult conditions it is possible using approach to robust system design. Creation of robust inertial stabilized platforms requires further research and development in contrast to design of robust systems of motion control.* ***Methods:*** *One of the modern approaches to robust system design proposed by modern control theory is H_∞-synthesis. Problems, which are important for practical applications, it is convenient to solve using method of the mixed sensitivity as it takes into consideration conflicting design goals including robust performance and stability. The method is combined with loop-shaping that allows achieving desired amplitude-frequency characteristics of the designed system. This is achieved by choice of the appropriate weighting transfer functions, which define bounds of the designed system amplitude-frequency characteristics.* ***Results:*** *Grounded recommendations to the choice of components of inertially stabilized platforms operated on the ground vehicles are represented. The mathematical model of the system with gearless drive is developed. The optimization criterion is derived and weighting transfer functions are chosen. The structure of the robust controller in the form of quadruple of state space matrices is represented. Results of synthesised stabilization system simulation show its resistance to significant parametrical and coordinate disturbances taking place during its operation on the ground vehicle.* ***Conclusions:*** *Efficiency of the proposed design approach is proved by results of simulation in conditions of significant parametrical and coordinate disturbances. Obtained results can be widespread on inertially stabilized platforms operating on the other type of vehicles, for example, special aviation aircrafts, carrying out cartographic surveys, monitoring and other similar functions. They can be also useful for design of unmanned aerial vehicles equipment.*

Keywords: H_∞-synthesis; inertially stabilized platform; loop-shaping; method of mixed sensitivity; robust controller.

1. Introduction

Robust control is of great importance for stabilized platforms as their actual operating conditions are accompanied by parametrical and coordinate disturbances. It is also necessary to take into consideration that an actual system differs from its mathematical model used for design procedures. Uncertainties of the mathematical model are of different nature [1]

1) errors of plant parameters determination;

2) unaccounted nonlinearities and changes in operating conditions;

3) unaccounted time delays and processes of energy dissipation;

4) imperfection of measuring instruments;

5) model simplifications;

6) unaccounted dynamics at high frequencies;

7) reduction of the synthesized controller order and hardware implementation inaccuracies.

The above mentioned sources of uncertainties can be divided into structured (parametrical) and unstructured. The latter uncertainties can be caused by plant unmodeled dynamics.

One of the modern trends of the modern control theory lies in providing of the closed-loop system stability for a family of plants belonging to the given class of uncertainties. A plant is called nominal if the mathematical model uncertainties are not taken into consideration.

The specific feature of the researched system is the wide variation of some its parameters during operation and also changed operating conditions. It is expedient to solve this problem using robust control principles. This will provide requirements given to system characteristics in the above mentioned conditions.

Inertially stabilized platforms (ISP) are widely used for stabilization of sensors, cameras, and observation apparatus, which operate at vehicles of a wide class. In the general case, features of ISP are defined by its application. The main goal of ISP functioning is control by orientation of measuring axes of mounted on a vehicle devices in the inertial space [2].

Apparatus stabilized by means of ISP is widely used on ground and marine vehicles, aircrafts and spacecrafts.

Usage of ISPs allows solving the following tasks:

– stabilization of payload during angular motion of a vehicle;

– tracking of a given reference point by means of keeping constant orientation of the line-of-sight in the given direction.

It should be noted that during last years Ukrainian enterprises have been achieving significant progress in the field of design of ISPs operated on the ground vehicles. The wide area of application, and the last achievements of inertial technologies define topicality of development of new approaches to ISP design.

2. Analysis of the latest research and publications

The H_∞-synthesis approach developed in recent years and still active in research area can be the efficient instrument for design of robust systems. In general, the H_∞-synthesis approach can solve the robust stabilization and nominal performance problems [3].

Theoretical grounds and calculating procedures of the H_∞-synthesis for control systems of a wide class are represented in many textbooks [3–5]. At the same time the problem of design of robust systems for stabilization of ISPs operated on the ground vehicles just has not got the appropriate development.

It is known that optimization of actual control systems can not be based on a single cost function. For example, in some applications it makes to provide good tracking as well as to limit the control signal energy. This problem can be solved using *S* over *KS* or the mixed sensitivity approach [3]. In this case the cost function can be interpreted as a set of design objectives, for example, nominal performance, robust stabilization and minimization of signal energy.

The goal of the research is development of the procedure based on *S* over *KS* approach for design of the robust system for control of ISP operated on the ground vehicle.

3. Choice of ISP components

Nowadays fiber optic gyro rates are widely used as sensitive elements of stabilization systems. Advantages of such gyroscopes are absence of moving parts, small time of readiness to operation, high sensitivity and accuracy. But such measuring instruments are characterized by a significant mass and dimensions, and the high cost. Such situation is caused by the necessity to use the receiving-transmitting units and presence of hand operations during manufacturing. Moreover, these measuring instruments have not high resistance to shocks that is of great importance for ISPs operated on the ground vehicles.

Design of rate gyroscopes as microelectro-mechanical systems (MEMS) is one of the basic trends in the modern device-building. Such gyroscopes have a wide area of application including stabilization of platforms with payload. The process of such gyroscopes development is not completed now. It is characterized by improvement of accuracy, operating characteristics, technology, and decrease of the cost. Basic advantages of MEMS gyroscopes are simplicity of operation and low cost. At the same time their accuracy is not sufficient for some modern applications although some gyroscopes of such type are characterized by the high resistance to shocks. MEMS gyroscopes have some disadvantages including statistical dispersion caused by deviations of manufacturing conditions from the technical documentation. Moreover ageing of separate measuring instruments is implemented with the different rates. Therefore it is necessary to use correction units to improve characteristics of MEMS gyroscopes.

The digital Coriolis vibratory gyroscope (CVG) with the metallic resonator has no such disadvantages [6]. Results of comparative analysis of rate gyroscopes characteristics are given in Table 1.

Table 1

Characteristics of rate gyroscopes

№	Type of a gyro	Technology	Enterprise	Measuring range, deg/s	Bandwidth, Hz	Resistance to shocks, g
1	GT-46	Electro-mechanical	Kyiv Automatic Plant	50	40	150
2	G20-075-100	MEMS	Gladiator Technologies	75	100	500
3	SDG1000	MEMS	Systron Donner	75	>100	200
4	MAG16	MEMS	Northrop Grumman	150	400	-
5	DSP-3000	Fiber optic	KVN Industries	100	100	40
6	CVG	Vibratory	Kyiv Automatic Plant	400	100	400

The simplified mathematical model of the Coriolis vibratory gyro can be represented by the transfer function [7]

$$W_2(p) = \frac{\omega^2}{p^2 + 2\xi\omega p + \omega^2}, \qquad (1)$$

where ω is the bandwidth frequency; ξ is the damping coefficient; p is the Laplace operator.

Traditionally choice of a motor for any system including ISP is based on determination of the averaged power necessary for plant motion with the given rates and accelerations in different modes of operation. Calculation of the required power, which will provide parameters of load motion for the given inertia moment, can be carried out in the following way [8]

$$P_{tr} = 2\frac{M_{d\max} + J_n \dot{\omega}_{n\max}}{\eta}\omega_{n\max}, \qquad (2)$$

where $M_{d\max}$ is the maximum disturbing moment; J_n is the moment of inertia of the load and motor; ω_n, $\dot{\omega}_n$ are the maximum angular rate and acceleration of the load and motor, and $\eta = 0{,}8$ is the coefficient of efficiency. For practical situations can be taken $P_{dv} = kP_{tr}$. If the static moment is greater than dynamic one, $k = 0{,}5...0{,}7$. On contrary $k = 0{,}7...0{,}9$ [8].

Calculation of the nominal rotating moment can be implemented by the formula

$$M_{nom} = (J_{load} + J_{motor})(\varepsilon_1 + \varepsilon_2) + M_{dist} = \\ = (40 + 1{,}3)(1{,}75 + 1{,}1) + 11{,}5 \approx 129{,}2 \text{ Nm}, \qquad (3)$$

where J_{load} is the moment of inertia of the load; J_{motor} is the moment of inertia of the motor; ε_1 is the maximum angular acceleration of the load; ε_2 is the maximum angular acceleration caused by disturbances due to irregularities of the road or terrain; M_{dist} is the disturbing moment.

Tests of ISPs operated on the ground vehicles are implemented in conditions of the check route with the relief angular profile changing by the sinusoidal law $h(t) = 2{,}5\sin(2\pi 0{,}8t)$ in the vertical plane, and $h(t) = 2{,}0\sin(2\pi 0{,}8t)$ in the horizontal plane. Amplitudes of these expressions are measured in degrees. Respectively the maximum angular rate and acceleration will be determined by the expressions $0{,}219\cos(5{,}02t)$ and $1{,}1\sin(5{,}02t)$, where amplitudes are measured in radians. In accordance with these expressions the maximum values of the angular rate and acceleration are 0,219 rad and 1,1 rad respectively. It should be noted that random disturbances in actual operating conditions of the ground vehicle have the less amplitudes.

As to the disturbing moment M_{dist}, it includes such components as the unbalanced and friction moments. Based on the experimental data, the unbalanced moment is equal to 10,4 Nm. The friction moment may be neglected if a stabilization system uses the gearless drive based on the noncontact erection torque motor.

Choosing the motor it is necessary to take into consideration that the starting moment may exceed nominal in 3…5 times.

Analyzing values of the motor required power it is possible to define the necessity of reducer usage. It should be noted that the stabilization system with the gearless drive has significant advantages in accuracy in comparison with systems using geared drives. So, the expressions (1)–(3) provide choice of basic ISP components.

4. Mathematical model of ISP for ground vehicles

One of the basic features of ISPs applied for operation on the ground vehicles is the elastic connection between the motor shaft and the base, on which the platform with payload is mounted. Based on mathematical model for the geared drive of the stabilization system represented in [9] it is possible to develop the model for the system with the gearless drive in the following form

$$J_m\ddot{\varphi}_m = -f_{frd}\dot{\varphi}_m + \frac{c_m}{R_w}U + c_p\varphi_p - c_p\varphi_m;$$
$$J_p\ddot{\varphi}_p = -f_{fr}\dot{\varphi}_p + c_r\varphi_m - c_r\varphi_p - M_{unb}; \quad (4)$$
$$\dot{U}T_a + U = -c_e\dot{\varphi}_m + U_{PWD},$$

where J_m is the moment of inertia of the motor; φ_m is an angle of the motor turn; M_{frd} is the moment of the dry friction; c_m is constant of load at the motor shaft; R_w is the resistance of the motor armature circuit; U is the voltage of the armature circuit; J_p is the moment of inertia of the platform; φ_p is an angle of the platform turn; M_{fr} is the moment of the dry friction in platform bearings; M_{unb} is the moment of the platform imbalance; c_r is rigidity of the elastic connection between the motor and the moving platform; U_{con} is voltage at the controller output; c_e is the constant coefficient of the electromotive force.

Nonlinear moments of the dry friction can be approximated by linear relationships. Approximation coefficients are determined as ratio of the first harmonic amplitude to the angular rate amplitude [10]. In this case expressions for determination of moments $M_p = M_{frd}\text{sign}\dot{\varphi}_p$, $M_m = M_{md}\text{sign}\dot{\varphi}_m$ become $M_p = f_{frd}\dot{\varphi}_p$, $M_m = f_{frd}\dot{\varphi}_m$. Coefficients f_{frp}, f_{frm} will be determined by expressions $f_{frp} = 4M_{frp}/(\pi\Omega_p)$, $f_{frm} = 4M_{frm}/(\pi\Omega_m)$.

It should be noted that the expression for unbalanced moment determination $M_{unb}\cos\varphi_p$ can be linearized for platform angles changing in the range 5–10 degrees. In accordance with equations (4) and the above represented approaches to linearization, the integrated model of the plant and the motor looks like

$$J_m\ddot{\varphi}_m = -f_{frd}\dot{\varphi}_m + \frac{c_m}{R_w}U + c_p\varphi_p - c_p\varphi_m;$$
$$J_p\ddot{\varphi}_p = -f_{fr}\dot{\varphi}_p + c_r\varphi_m - c_r\varphi_p - M_{unb}; \quad (5)$$
$$\dot{U}T_a + U = -c_e\dot{\varphi}_m + U_{PWD},$$

where f_{frp}, f_{frm} are coefficients of linearized moments of the platform and motor friction respectively.

The model (5) can be transformed to the model in the state space after introducing new variables and reduction of an order of the differential equations (5). In this case vectors of states, controls and observations and matrices of states, controls, observations and disturbances can be represented in the following form

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \dot{\varphi}_m \\ \dot{\varphi}_p \\ \varphi_m \\ \varphi_p \\ U \end{bmatrix}; \ \mathbf{u} = \begin{bmatrix} M_{umb} \\ U_{con} \end{bmatrix}; \ \mathbf{y} = \begin{bmatrix} \dot{\varphi}_p \\ U \end{bmatrix};$$

$$\mathbf{A} = \begin{bmatrix} \frac{-f_{frm}}{J_m} & 0 & \frac{-c_r}{J_m} & \frac{c_r}{J_m} & \frac{c_m}{R_a J_m} \\ 0 & \frac{-f_{frp}}{J_p} & \frac{c_r}{J_p} & \frac{-c_r}{J_p} & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \frac{-c_e}{T_a} & 0 & 0 & 0 & \frac{-1}{T_a} \end{bmatrix};$$

$$\mathbf{B}^{\mathrm{T}} = \begin{bmatrix} 0 & \frac{-1}{J_p} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{T_a} \end{bmatrix};$$

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \ \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad (6)$$

The mathematical model (6) can be used for control system synthesis. And results of the synthesis can be checked by means of simulation based on the model taking into consideration nonlinearities of the designed system. Such model created in Simulink Toolbox is represented in Fig. 1.

5. Design of robust controller by means of H_∞-synthesis

Firstly the procedure of the H_∞-synthesis for design of control systems has been proposed by G. Zames in [11]. This approach provides the robust performance and stabilization of the designed system. In this case the design problem is formulated as a problem of the mathematical optimization directed to search of an optimal controller. Disadvantages of this approach are the mathematical complexity, heuristic choice of weighting transfer functions and deciding influence of the system model adequacy for successful solving the design problem.

The H_∞-norm characterizes the upper bound of the maximum singular value of the matrix transfer function of the closed-loop system. Usually the H_∞-norm is calculated based on representation of the system in the state space as the minimum value γ, for which the Hamiltonian matrix $\mathbf{H}$ has no eigenvalues at the imaginary axes [4, 5]

$$\mathbf{H} = \begin{bmatrix} \mathbf{A} + \mathbf{B}\mathbf{R}^{-1}\mathbf{D}^{\mathrm{T}}\mathbf{C} & \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}} \\ -\mathbf{C}^{\mathrm{T}}(\mathbf{I} + \mathbf{D}\mathbf{R}^{-1}\mathbf{D}^{\mathrm{T}})\mathbf{C} & -(\mathbf{A} + \mathbf{B}\mathbf{R}^{-1}\mathbf{D}^{\mathrm{T}}\mathbf{C})^{\mathrm{T}} \end{bmatrix}, \quad (7)$$

where $\mathbf{R} = \gamma^2\mathbf{I} - \mathbf{D}^{\mathrm{T}}\mathbf{D}$.

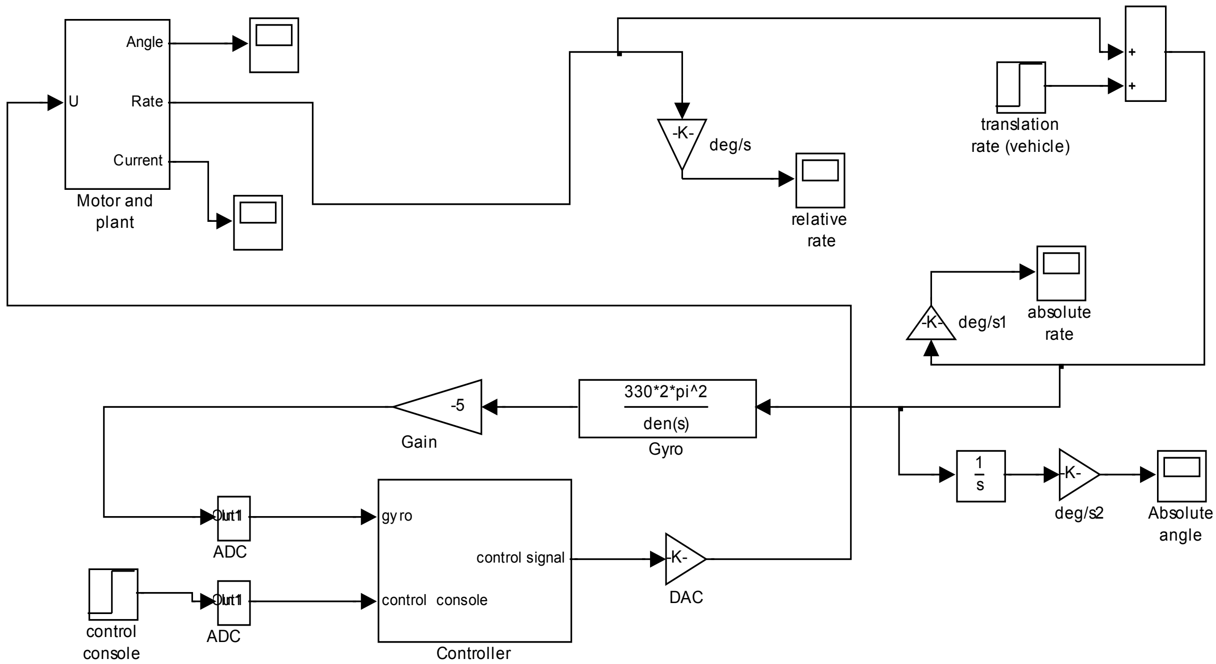


Fig. 1. The stabilization system model created in Simulink Toolbox

Design of the H_∞-synthesis controller is based on solving special Riccati equations. The mathematical description of the system in the state space must satisfy the following conditions [4, 5].

1. Pair of matrices $\mathbf{A}, \mathbf{B}_1$ must be stabilized, and pair of matrices $\mathbf{A}, \mathbf{C}_1$ – detected.

2. Pair of matrices $\mathbf{A}, \mathbf{B}_2$ must be stabilized, and pair of matrices $\mathbf{A}, \mathbf{C}_2$ – detected.

3. $\mathbf{D}_{12}^{\mathrm{T}}[\mathbf{C}_1 \quad \mathbf{D}_{12}] = [\mathbf{0} \quad \mathbf{I}]$.

4. $\begin{bmatrix} \mathbf{B}_1 \\ \mathbf{D}_{21} \end{bmatrix} \mathbf{D}_{21}^{\mathrm{T}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}$.

The above given conditions can be explained in the following way [12]. The conditions 1 and 2 guarantee absence of Hamiltonian matrix eigenvalues at the imaginary axes. These conditions correspond to Riccati equations of controller and observer and define their belonging to the definitional domain of Riccati operator dom (Ric). The condition 3 defines orthogonality of signals $\mathbf{C}_1\mathbf{x}(t)$ and $\mathbf{D}_{12}\mathbf{u}(t)$. The condition 4 corresponds to orthogonality of signals $\mathbf{B}_1\mathbf{w}(t)$ and $\mathbf{D}_{21}\mathbf{w}(t)$.

The algorithm of the suboptimal H_∞-controller is represented in [4, 5]. In accordance with this algorithm to find suboptimal H_∞-controller $\mathbf{K}(s)$ it is necessary to carry out the following steps [4, 5].

1) To find solution of the first algebraic Riccati equation $\mathbf{X}_\infty$, that is the controller equation

$$\mathbf{A}^{\mathrm{T}}\mathbf{X}_\infty + \mathbf{X}_\infty \mathbf{A} + \mathbf{C}_1^{\mathrm{T}}\mathbf{C}_1 + \mathbf{X}_\infty(\gamma^{-2}\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}} - \mathbf{B}_2\mathbf{B}_2^{\mathrm{T}})\mathbf{X}_\infty = 0$$

2) To find solution of the second algebraic Riccati equation $\mathbf{Y}_\infty$, that is the observer equation

$$\mathbf{A}\mathbf{Y}_\infty + \mathbf{Y}_\infty \mathbf{A}^{\mathrm{T}} + \mathbf{B}_1\mathbf{B}_1^{\mathrm{T}} + \mathbf{Y}_\infty(\gamma^{-2}\mathbf{C}_1^{\mathrm{T}}\mathbf{C}_1 - \mathbf{C}_2^{\mathrm{T}}\mathbf{C}_2)\mathbf{Y}_\infty = 0$$

3) To carry out checking of conditions

$$\mathrm{Re}\lambda_i[\mathbf{A} + (\gamma^{-2}\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}} - \mathbf{B}_2\mathbf{B}_2^{\mathrm{T}})\mathbf{X}_\infty] < 0, \forall i\ ;$$

$$\mathrm{Re}\lambda_i[\mathbf{A} + \mathbf{Y}_\infty(\gamma^{-2}\mathbf{C}_1^{\mathrm{T}}\mathbf{C}_1 - \mathbf{C}_2^{\mathrm{T}}\mathbf{C}_2)] < 0, \forall i\ ;$$

$$\rho(X_\infty Y_\infty) < \gamma^2 .$$

The set of permissible controllers is defined by the expression $\mathbf{K} = F(\mathbf{K}_c, Q)$,

where $\mathbf{K}_c(s) = \begin{bmatrix} \mathbf{A}_\infty & -\mathbf{Z}_\infty\mathbf{L}_\infty & \mathbf{Z}_\infty\mathbf{B}_2 \\ \mathbf{F}_\infty & 0 & \mathbf{I} \\ -\mathbf{C}_2 & \mathbf{I} & 0 \end{bmatrix}$,

$\mathbf{F}_\infty = -\mathbf{B}_2^{\mathrm{T}}\mathbf{X}_\infty$; $\mathbf{L}_\infty = -\mathbf{Y}_\infty\mathbf{C}_2^{\mathrm{T}}$;

$\mathrm{Z}_\infty = (\mathbf{I} - \gamma^{-2}\mathbf{Y}_\infty\mathbf{X}_\infty)^{-1}$,

$\mathbf{A}_\infty = \mathbf{A} + \gamma^{-2}\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}}\mathbf{X}_\infty + \mathbf{B}_2\mathbf{F}_\infty + \mathbf{Z}_\infty\mathbf{L}_\infty\mathbf{C}_2$ and Q is some stable transfer function $\| Q \|_\infty < \gamma$. For $Q(s) = 0$ the robust controller can be defined in the following way

$$\mathbf{K}(s) = \mathbf{K}_{c11}(s) = -\mathbf{Z}_\infty\mathbf{L}_\infty(s\mathbf{I} - \mathbf{A}_\infty)^{-1}\mathbf{F}_\infty .$$

This controller is called central because it has the same quantity of states as the plant $\mathbf{G}$. The obtained solution includes the observer

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}_1\gamma^{-2}\mathbf{B}_1^{\mathrm{T}}\mathbf{X}_\infty\hat{\mathbf{x}} + \mathbf{B}_2\mathbf{u} + \mathbf{Z}_\infty\mathbf{L}_\infty(\mathbf{C}_2\hat{\mathbf{x}} - \mathbf{y})$$

and feedback

$$\mathbf{u} = \mathbf{F}_\infty\hat{\mathbf{x}} .$$

6. Forming of generalized plant augmented by weighting transfer functions

The plant of the considered system represents the platform with payload, measuring system and motor. Parameters of ISPS, which are mostly changed during operation on the ground vehicles, are the moment of inertia of the platform, coefficient of the elastic connection between the motor and the moving platform and resistance of the armature circuit. The first and second parameters are changed in the range ± 50%. The resistance of the motor armature circuit varies in the range ± 70%. Variations of other parameters may be neglected as they are no more than ± 1%.

The amplitude-frequency characteristics of the closed-loop system are shown in Fig. 2. Change of the moment of inertia of the platform has been considered as a parametrical disturbance.

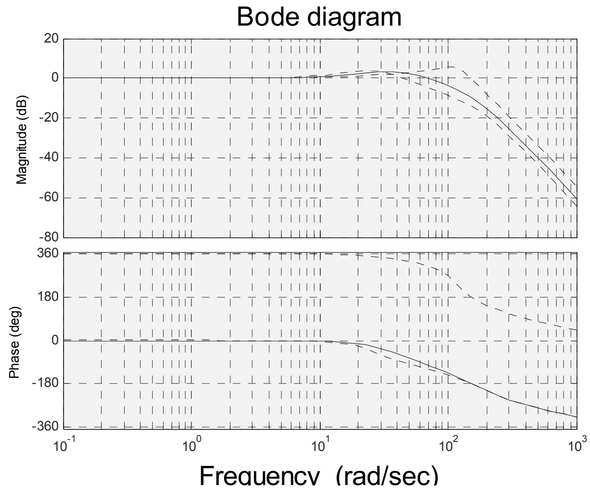


Fig. 2. Logarithmic amplitude-frequency characteristics of the nominal and disturbed systems

It is known that the standard problem of the H_∞-control can be solved by means of two Riccati equations [13]. The problem statement can be explained by the structural scheme represented in Fig. 3.

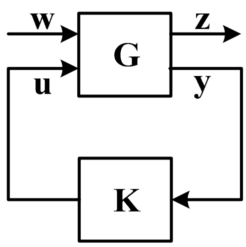


Fig. 3. The standard H_∞-configuration

Such system consists of a plant $\mathbf{G}$ and a controller $\mathbf{K}$ and can be represented by the output signals to be optimized $\mathbf{z}$, external input signals $\mathbf{w}$, control signals $\mathbf{u}$ and measured output signals $\mathbf{y}$ [4, 5]. The plant $\mathbf{G}$ and controller $\mathbf{K}$ are called generalized because they are connected with the generalized input and output signals.

As a rule, the sensitivity function $\mathbf{S} = (1 + \mathbf{GK})^{-1}$ and the complementary sensitive function $\mathbf{T} = \mathbf{GK}(1 + \mathbf{GK})^{-1}$ are used to estimate quality of robust systems designed by means of the H_∞-synthesis.

To improve quality of design process, the loop shaping can be used. In this case the plant of the designed system is augmented by weighting transfer functions as it is shown in Fig. 4. Such approach provides designing of the robust system with the desired amplitude-frequency characteristics.

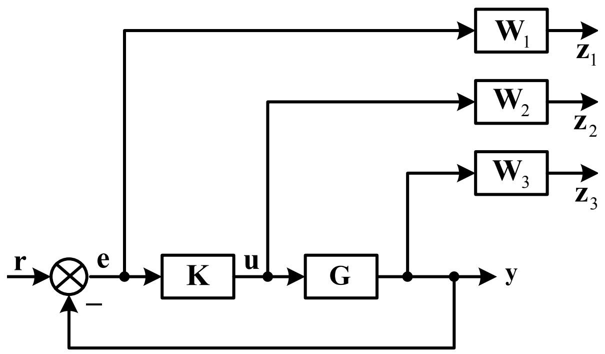


Fig. 4. The structural scheme of the system augmented by the weighting coefficients W_1, W_2, W_3

The objective function for optimization of the system represented in Fig. 4 looks like [4, 5]

$$J_{opt} = \left\| \begin{bmatrix} (\mathbf{I}+\mathbf{GK})^{-1} \\ \mathbf{K}(\mathbf{I}+\mathbf{GK})^{-1} \\ \mathbf{GK}(\mathbf{I}+\mathbf{GK})^{-1} \end{bmatrix} \right\|_{\infty} < \gamma, \qquad (7)$$

where components of the objective function are the sensitivity function, sensitivity function by control and complementary sensitivity function.

The objective function (7) is the multi-objective function. It makes it suitable for practical applications, which are always accompanied by many conflicting goals.

Choice of weighting transfer functions is based on heuristic approaches and depends on experience of a system designer. After augmentation the equations of weighting transfer functions become additional mathematical description of the system. The generalized augmented plant can be represented in the following form

$$\mathbf{P} = \left[\begin{array}{c|c} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \hline \mathbf{P}_{21} & \mathbf{P}_{22} \end{array} \right] = \left[\begin{array}{c|c} \mathbf{W}_1 & -\mathbf{W}_1\mathbf{G} \\ \mathbf{0} & \mathbf{W}_2 \\ \mathbf{0} & \mathbf{W}_3\mathbf{G} \\ \hline \mathbf{I} & -\mathbf{G} \end{array} \right]. \qquad (8)$$

Based on the expression (8) the objective function for optimization of the augmented plant becomes

$$J_{opt} = \left\| \begin{bmatrix} \mathbf{W}_1(\mathbf{I}+\mathbf{GK})^{-1} \\ \mathbf{W}_2\mathbf{K}(\mathbf{I}+\mathbf{GK})^{-1} \\ \mathbf{W}_3\mathbf{GK}(\mathbf{I}+\mathbf{GK})^{-1} \end{bmatrix} \right\|_{\infty} < \gamma. \qquad (9)$$

The method using the objective functions (7), (9) is called the method of the mixed sensitivity [4, 5].

7. Results of H_∞-synthesis

The H_∞-synthesis of the robust controller can be implemented by means of Robust Control Toolbox using functions *augtf*, *hinfopt*, which provide creation of the augmented plant and execution of the H_∞-synthesis procedure. As result of the H_∞-synthesis the discrete (0,0025 s) robust controller in the form of the quadruple matrices $\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_c, \mathbf{D}_c$ in the state space was obtained.

$$\mathbf{A}_c = \begin{bmatrix} 0{,}736 & -0{,}38 & -0{,}006 & -0{,}007 & 0{,}001 & 0{,}005 & 0{,}002 & -0{,}001 & -0{,}053 \\ -0{,}379 & 0{,}453 & 0{,}007 & 0{,}008 & 0{,}01 & 0{,}012 & 0{,}002 & 0{,}001 & -0{,}037 \\ -0{,}006 & 0{,}004 & 0{,}987 & -0{,}027 & -0{,}019 & -0{,}0004 & -0{,}004 & -0{,}0005 & -0{,}011 \\ -0{,}027 & 0{,}017 & -0{,}013 & 0{,}919 & -0{,}101 & 0{,}008 & -0{,}016 & 0{,}01 & 0{,}263 \\ 0{,}02 & -0{,}013 & 0{,}011 & 0{,}035 & 0{,}776 & -0{,}193 & 0{,}076 & 0{,}007 & 0{,}263 \\ -0{,}015 & 0{,}01 & -0{,}004 & 0{,}02 & 0{,}143 & 0{,}756 & -0{,}131 & 0{,}005 & -0{,}194 \\ 0{,}054 & -0{,}035 & 0{,}03 & 0{,}086 & 0{,}0003 & 0{,}091 & 0{,}795 & 0{,}017 & 0{,}679 \\ 0{,}00007 & -0{,}00005 & 0{,}00006 & 0{,}0002 & 0{,}0004 & 0{,}0006 & 0{,}0022 & 0{,}6 & -0{,}021 \\ 0{,}086 & -0{,}055 & 0{,}048 & 0{,}135 & -0{,}028 & 0{,}036 & -0{,}026 & 0{,}004 & 0{,}066 \end{bmatrix};$$

$$\mathbf{B}_c^T = [-5{,}225 \quad 3{,}69 \quad 8{,}026 \quad 21{,}77 \quad -17{,}02 \quad 12{,}59 \quad -45{,}26 \quad -0{,}06 \quad -71{,}66]; \ \mathbf{D}_c = 0{,}578;$$

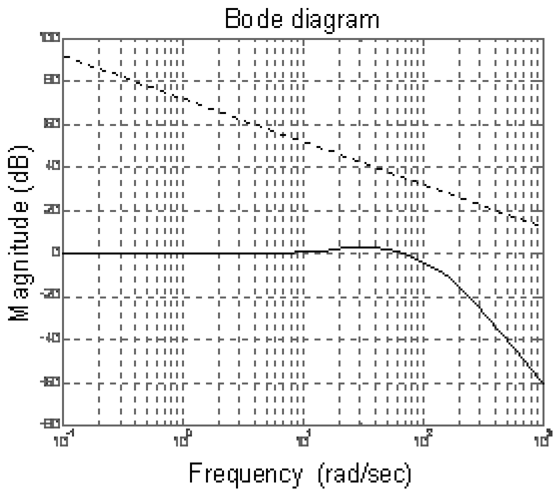
$$\mathbf{C}_c = [-0{,}001 \quad 0{,}0009 \quad 0{,}00005 \quad -0{,}0003 \quad -0{,}0001 \quad 0{,}0003 \quad 0{,}0003 \quad -0{,}0002 \quad -0{,}0086].$$

Plots of the sensitivity and complementary sensitivity functions are represented in Fig. 5. The effectiveness of the proposed approach to robust system design is proved by results of simulation represented in Fig. 6.

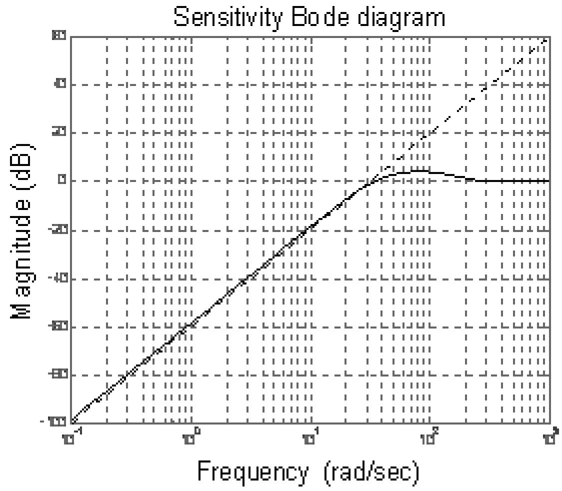
There are following recommendations for weighting transfer functions determination [5]

$$W_1 = \frac{s/M + \omega_0}{s + \omega_0 A}; \ W_2 = const; \ W_3 = \frac{s + \omega_0 M}{As + \omega_0},$$

where A is the desired maximally permissible stable error in the steady mode; ω_0 is the desired bandwidth; M is the sensitivity peak.

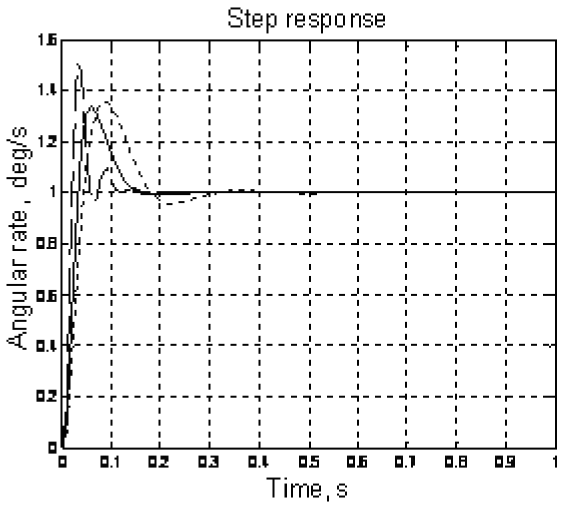


a

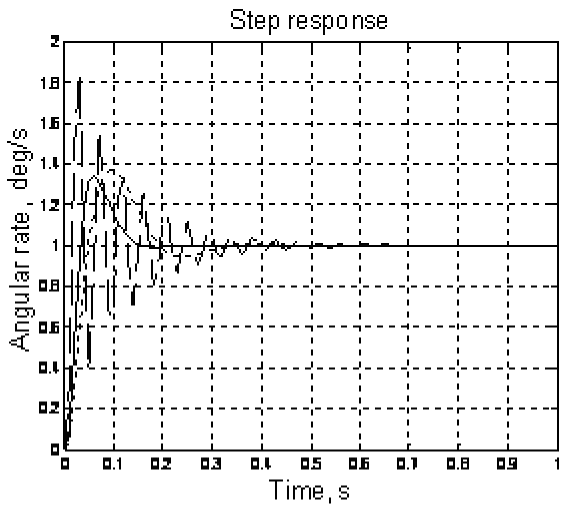


b

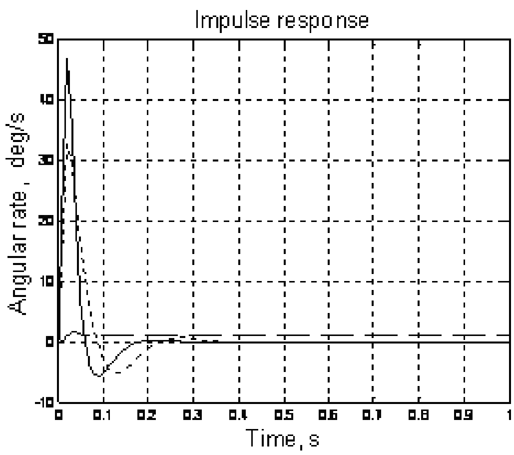
Fig. 5. Frequency characteristics of the weighting sensitivity (a) and complementary sensitivity (b) functions



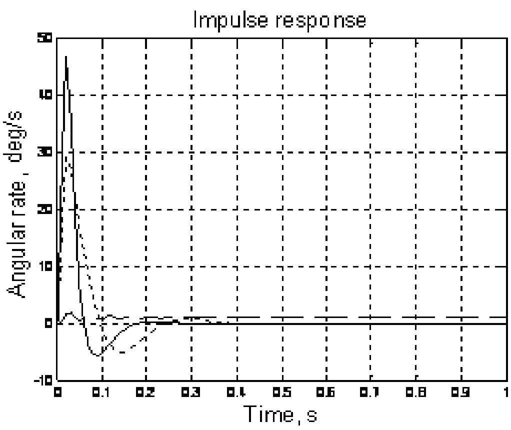
a



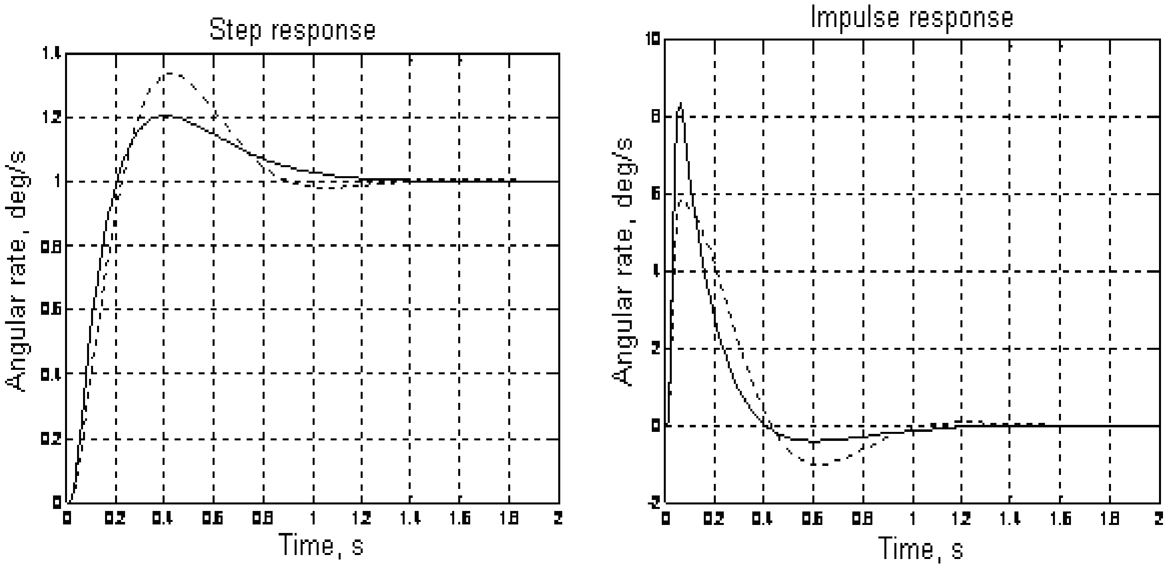
b



c



d



e f

Fig. 6. Results of robust system simulation: step response for changing inertia moment in the range ±50% and resistance of the armature circuit ± 70% (a,b); impulse response for changing inertia moment in the range ±50% and resistance of the armature circuit ± 70% (c,d); step and impulse responses for changing imbalance moment (e,f)

For the researched system the expressions for matrix weighting transfer functions were chosen in the following way

$$\mathbf{W}_1 = \begin{bmatrix} \frac{1}{s}\frac{0{,}5s+200}{s+0{,}005} & 0 & 0 \\ 0 & \frac{1}{s}\frac{0{,}5s+200}{s+0{,}005} & 0 \\ 0 & 0 & \frac{1}{s}\frac{0{,}05s+200}{s+0{,}005} \end{bmatrix};$$

$$\mathbf{W}_2 = \begin{bmatrix} 0{,}01 & 0 & 0 \\ 0 & 0{,}01 & 0 \\ 0 & 0 & 0{,}01 \end{bmatrix};$$

$$\mathbf{W}_3 = \begin{bmatrix} \frac{s}{0{,}001s+100} & 0 & 0 \\ 0 & \frac{s}{0{,}002s+200} & 0 \\ 0 & 0 & \frac{s}{0{,}005s+50} \end{bmatrix}.$$

The weighting transfer functions W_1, W_2, W_3 impose a penalty on a signal error, control signal and output signal respectively.

8. Conclusions

The H_∞-synthesis of the robust system for control of the angular motion of ISP operated on the ground vehicles is carried out. The approach for choice of components of ISP of the studied type is represented. The mathematical model of the control system with the gearless drive is obtained.

It has been shown that the synthesized controller provides the robust stability and performance for uncertainties specific to operation of ISP in conditions of significant disturbances.

Simulation of the synthesized system in conditions of the parametrical and coordinate disturbances is represented.

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Received 23 April 2016

О. А. Сущенко

Робастне управління інерціальними стабілізованими платформами для наземних рухомих об'єктів на підставі H_∞-синтезу.

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Мета: Експлуатація інерціальних стабілізованих платформ, встановлених на наземних рухомих об'єктах супроводжується впливом значних параметричних збурень. Для збереження високих експлуатаційних характеристик системи у таких складних умовах можна використовувати підхід, заснований на проектуванні робастних систем. На відміну від проектування робастних систем управління рухом, створення робастних інерціальних стабілізованих платформ потребує подальшого дослідження та розвитку. **Методи**: Одним із сучасних підходів до проектування робастних систем, що пропонується сучасною теорією управління, є H_∞-синтез. Завдання важливі для практичних застосувань зручно вирішувати за допомогою методу змішаної чутливості, оскільки він враховує суперечливі цілі проектування, у тому числі досягнення робастної якості та робастної стійкості. Цей метод поєднується з формуванням контурів управління із заданими характеристиками проектованої системи, що досягається вибором вагових передатних функцій, які визначають границі амплітудно-частотних характеристик. **Результати**: Представлено обґрунтовані рекомендації щодо вибору складових інерціальних стабілізованих платформ наземних рухомих об'єктів. Отримано критерій оптимізації та визначено вагові передатні функції. Розроблено математичну модель системи з безредукторним приводом. Визначено структуру робастного регулятора у вигляді четвірки матриць простору станів. Результати моделювання синтезованої системи стабілізації показують її стійкість до значних параметричних та координатних збурень, що мають місце в умовах експлуатації на наземному рухомому об'єкті. **Висновки:** Ефективність запропонованого підходу до проектування підтверджується результатами моделювання в умовах значних параметричних та координатних збурень. Отримані результати можуть бути поширені на інерціальні стабілізовані платформи, що експлуатуються на інших рухомих об'єктах, наприклад, літаках спеціальної авіації, що виконують функції картографічних зйомок, моніторингу та інших подібних функцій. Вони можуть також бути корисними під час проектування обладнання безпілотних літальних апаратів.

Ключові слова: інерціальні стабілізовані платформи; метод змішаної чутливості; робастний регулятор; формування контурів управління; H_∞-синтез.

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Робастное управление инерциальными стабилизированными платформами для наземных подвижных объектов на основе H_∞-синтеза

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Цель: Эксплуатация инерциальных стабилизированных платформ, установленных на наземных подвижных объектах, сопровождается действием значительных параметрических возмущений. Для сохранения высоких эксплуатационных характеристик системы в таких сложных условиях возможно использовать подход, основанный на проектировании робастных систем. В отличие от проектирования робастных систем управления движением, создание робастных инерциальных стабилизированных платформ требует дальнейшего исследования и развития. **Методы**: Одним из современных подходов к проектированию робастных систем, предлагаемых современной теорией управления является H_∞-синтез. Задачи, важные для практических применений, целесообразно решать с помощью метода, поскольку он учитывает противоречивые цели проектирования, в том числе достижение робастного качества и робастной устойчивости. Этот метод сочетается с формированием контуров управления с заданными характеристиками проектируемой системы, что обеспечивается выбором весовых передаточных функций, которые определяют границы амплитудно-частотных характеристик. **Результаты**: Представлены обоснованные рекомендации по выбору составляющих инерциальных стабилизированных платформ наземных подвижных объектов. Разработана математическая модель системы с безредукторным приводом. Получен критерий оптимизации и определены весовые передаточные функции. Определена структура робастного регулятора в виде четверки матриц пространства состояний. Результаты моделирования синтезированной системы стабилизации показывают ее устойчивость к значительным параметрическим и координатным возмущениям, которые имеют место в условиях эксплуатации на наземном подвижном объекте. **Выводы:** Эффективность предложенного подхода к проектированию подтверждается результатами моделирования в условиях значительных параметрических и координатных возмущений. Полученные результаты могут быть распространены на инерциальные стабилизированные платформы, эксплуатируемые на других подвижных объектах, например, самолетах специальной авиации, которые выполняют функции картографических съемок, мониторинги и другие аналогичные функции. Они также могут быть полезными при проектировании оборудования беспилотных летательных аппаратов.

Ключевые слова: инерциальные стабилизированные платформы; метод смешанной чувствительности; робастный регулятор; формирование контуров управления; H_∞-синтез.

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