Loop Quantum Ontology: spacetime and spin-networks

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ABSTRACT

It is standardly claimed in loop quantum gravity (LQG) that spacetime both disappears, fundamentally, and emerges from spin-networks in the low energy regime. In this paper, I critically explore these claims and develop a variety of substantival and relational interpretations of LQG for which these claims are false. According to most of the interpretations I consider, including the "received interpretation", it is in fact false that spacetime emerges from spin-networks. In the process of supporting these claims, I also explain why spacetime is thought to be missing from the theory's fundamental ontology and demonstrate how this conclusion depends on our interpretation of the theory. In fact, I will argue that for a variety of interpretations spacetime survives quantization just as the electromagnetic field survives quantization. The upshot of the following analysis is a much needed clarification of the ontology of LQG and how it relates, or fails to relate, to the spacetime of general relativity.

1. Introduction

In the literature on canonical loop quantum gravity (LQG), one finds the following claims:

> The quanta of the field cannot live in spacetime: they must build "spacetime" themselves... Physical space is a quantum superposition of spin networks...a spin network is not in space it is space. (Rovelli 2004, p.9, 21)

> One influential idea based on so-called 'weave-states' proposes that the spacetime structure emerges from appropriately benign, i.e. semi-classical, spin-networks. (Huggett and Wüthrich 2013, p.279)

Such claims suggest that spacetime is not fundamental, that there are physical objects or structures outside of spacetime and that spacetime is generated by or constructed from these fundamental, non-spatiotemporal "spin-networks".[1] In this paper, I will attempt four primary tasks:

1. Explicate certain aspects of the formal structure of canonical LQG that are useful for analyzing the status of spacetime in LQG.
2. Explain why spacetime is thought to be missing in LQG.
3. Outline a variety of novel interpretations of LQG, each with different ontological commitments.
4. Critically assess the claim that spacetime emerges from spin-networks in light of these various interpretations.

I will demonstrate that there are clear and well motivated interpretations of LQG for which it is false that spacetime emerges from spin-networks either because spacetime does not disappear after all or because there are no spin-networks for spacetime to emerge from. In order to explicate the ontology of LQG, in §3 and §4 I will answer the following questions on behalf of different interpretations of the theory:

1. In providing a quantum theory of general relativity, does LQG describe spacetime as having gone missing fundamentally?
2. Are spin-networks (s-knots) included in the ontology of LQG?
3. Is spacetime emergent from or composed of spin-networks (s-knots)?

I will demonstrate that our answers to these questions depend heavily upon our interpretation of the mathematics of LQG and upon what we take spacetime to be.

The following account explores only the ontology of *canonical* LQG and does not address the ontology of covariant LQG or the closely related, group field theory approach to quantum gravity. Though this lacuna is regrettable, there are good reasons for it, namely, most of the current philosophical work on LQG takes place within the confines of the canonical approach (see the references in §3.2), and it is this literature that I hope to clarify and amend. On this topic, while there are philosophical treatises on LQG which also explicate some of the formal structure of canonical LQG, e.g. Wüthrich (2006) and Crowther (2016), what is distinct about my approach is its focus on the ontology of the theory rather than on the formalism itself. I introduce enough of the formalism as a means to end, a means for (a) understanding the putative ontology of the theory, (b) demonstrating how our understanding of the ontology has changed over the course of the theory's formal development, and (c) dispelling ontological inconsistencies which are endorsed in the philosophy of LQG literature due to a lack of care in dealing with (b).

This paper is motivated by the fact that, in general, presentations of LQG do not take appropriate care in explicating the ontology of the theory and often employ evocative language which, when taken literally and as a whole, is inconsistent. The primary reason for these inconsistencies is that our understanding and interpretation of LQG has evolved alongside

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[1] See also Smolin (2002, p.138), Wüthrich (2006, p.169) and Oriti (2014, p.15).

the development of the theory. As different pieces of the formal puzzle of LQG have been solved, our interpretation of the theory has evolved in response. And this, of course, is to be expected. Unfortunately, however, the various stages of our evolving-interpretation of LQG are all presented together in treatise on LQG, either explicitly or implicitly, rather than a single interpretation being presented throughout. Because a mixed and inconsistent ontology is presented in accounts of canonical LQG, philosophers have made mistaken or confused inferences regarding spacetime, its emergence, and its relationship to spin-networks.

In this paper, I present two families of interpretations which fall squarely across the substantival-relational divide. The relational interpretation is championed by Carlo Rovelli and has been taken up by most philosophers writing on the subject. Since the Rovellian view has become the "received account" amongst philosophers, I spend a disproportionate amount of time developing substantival interpretations to help balance the philosophy literature. To this point, it is important to note that LQG poses no new threat to substantivalism over and above that which is already present in general relativity (GR) and to some extent in Newtonian mechanics. Unfortunately, this fact is often obscured in the literature and many even suggest that manifold substantivalism comes under attack in LQG:

> Canonical loop quantum gravity ends up anyway replacing the spacetime continuum [e.g. manifold] with something more radical. (Oriti 2014, p.2, fn 3)

This view is not sequestered to academic discussions but has trickled out into the public's imagination. In a recent issue of *Scientific American*, physicist Lajos Diósi is quoted as saying "we know for sure that there will be a total scrambling of the spacetime continuity if you go down to the Planck scale" (Folger 2019, p.51). Read charitably, there is a loose sense in which these authors are right; taken literally however, their claims regarding the continuum of spacetime are false. In the transition from GR to LQG, what is scrambled is not the spacetime continuum but the geometric properties associated with it (§2.4). In short, despite suggestions to the contrary, if one were a manifold substantivalist with respect to GR, one could remain a manifold substantivalist with respect to LQG.

One final caveat, in the course of discussing the ontology of LQG – I will make use of a variety of physical and philosophical concepts: quantization, emergence, substantivalism and relationism, for example, and yet I do not provide anything like a complete account or overview of how these concepts have been used in GR and how they might be used in LQG. Historically there are two senses in which spacetime might be substantival. The first sense identifies the spacetime manifold as being the locus of the substantivalist's conviction. The idea here is that spacetime is a substantival container of physical objects and events in virtue of there being a real manifold of spacetime points. Here the emphasis is on the manifold, and it is this manifold of points which comes under attack in Earman and Norton's infamous hole argument (1987). The second sense arises in the context of general relativity and identifies the metric field as being the locus of the substantivalist's conviction. The idea here is that the substance of spacetime is encoded not in the manifold but in the dynamical *geometry* of GR. This geometry is substantival or, at least, nonrelational, since nontrivial spacetime geometry carries energy and can exist despite there being no matter in the universe to mediate its existence (Hoefer 1996, Read 2018, Martens 2019, and more critically, Rynasiewicz 1996). Now, whether or not spacetime disappears fundamentally in LQG turns on whether or not spacetime is substantival or relational and whether or not substantival spacetime is encoded in the manifold, in the geometric field or, perhaps, in both. I take no stance on this issue and will track relationalism as well as both approaches to substantivalism through the quantization of GR and into LQG.[2]

Outline:

2. Formal aspects of LQG

In this section, I explicate the theory of LQG for physics-informed nonspecialists. The primary motivation for explicating the following mathematic is so that we might better understand the ontology of LQG and in particular, the nature of spin-networks (s-knots) and their relation to spacetime. In particular, there are important interpretive issues which turn on formal features of the "Gauss" and "diffeomorphism" constraints, as we shall see in §2.2 and §2.3.

Now, since there is no interpretation-free manner of expressing the content of a physical theory, in the following, I will adopt language which is explicitly substantival. My choice in doing so is motivated by the literature itself, for, though the received interpretation of LQG, amongst philosophers, is ultimately relational, almost all formal accounts of LQG begin with language favorable to manifold substantivalism. Consider for instance:

> Nowadays, this approach is mostly pursued in a different form, based on ideas of Ashtekar. The idea of "splitting" spacetime [$\mathcal{M}$] into 3-dimensional slices [Σ], and conceiving dynamics as evolution from one slice to another, remains; but the basic dynamical variable is now, not a 3-geometry, but a 3-connection... (Isham and Butterfield 1999, p.22)

[2]There is a third possibility; Martens (2019) has noted that were spacetime emergent it might not be properly characterized as being either relational or substantival. On the one hand, emergent spacetime is not an independent, primitive substance since its existence is conditionalized on some other physical structure – namely "spin-networks". However since this structure is not matter, emergent spacetime is not technically relational either. The point is that if spacetime is emergent, then there is a third type of classification distinct from either substantivalism or relationalism.

Here and elsewhere, the bare manifold is referred to as being spacetime.[3] It is unclear how literal Chris Isham and Jeremy Butterfield intended to be interpreted when making such claims. It is usually assumed that one needs both $\mathcal{M}$ as well as some metric g in order to have a structure rich enough to represent spacetime (Einstein 1961, p.155; Maudlin 1990, p.245). Thus, I suspect that when the aforementioned physicists refer to $\mathcal{M}$ as being spacetime or the spacetime manifold, they are simply using convenient language to speak of mathematics only and are not endorsing a position on what physical spacetime is. I, on the other hand, will continue to assume (at least for the time being) that the bare manifold $\mathcal{M}$ represents a substantival spacetime.

According to this brand of substantivalism, LQG is a theory of quantum geometry and not a theory of spacetime or quantum spacetime. It might not be obvious what the difference is between these options, but this will become clear in §3.1. According to the initial interpretation to be developed, the world consists of a bare substantival spacetime manifold represented by $\mathcal{M}$, which happens to be overlaid with a geometrically charged network of relations holding between spacetime points.[4] The ontology of LQG is quantum since the geometry associated with this charged network is quantum mechanical. What geometrically charged networks are and how they are represented in the theory will be explained shortly. I will refer to this brand of substantivalism, nonpejoratively, as naïve substantivalism, though perhaps a more accurate term would have been 'naïve manifold substantivalism'.

Admittedly, we ought not be completely satisfied with naïve substantivalism as the bare manifold lacks the geometric structure which we have come to associate with spacetime. I will address this concern in §3.1 by constructing alternative interpretations of LQG and of spacetime. To reiterate then, in order to express LQG *qua* a physical theory over and above a system of mathematics, one must adopt some interpretation and, for this purpose, I have adopted naïve substantivalism. My concern at this point is to provide an introduction to some aspects of the theory's formal structure and not to debate the merits or demerits of the naïve interpretation. Soon enough, alternative interpretations will be developed and analyzed.

2.1. Spin-networks and S-knots

The theory of LQG begins with a Hamiltonian formulation of GR and proceeds to quantize the theory by quantizing the gravitational field following an approach developed by Paul Dirac. Dirac's procedure is the "canonical" route for quantizing classical theories.[5] At the end of the day, our physical system will be represented by a quantum state in some Hilbert space. However, since, as we shall see, the manifold plays an important role in defining the relevant Hilbert space, I will often refer to the ordered pair $\langle \mathcal{M}, \Psi \rangle$ as being a model of the physical system of LQG.

[3] See also Oriti (2014, p.5) and Markopolou and Smolin (2007, p.2)

[4] If there is matter or energy in the world, then the world includes these items as well.

[5] For technical details on canonical quantum gravity, see Isham (1991, 1992), Henneaux and Teitelboim (1992), Rovelli (2004), Thiemann (2007), Kiefer (2012) and Chiou (2015).

In the case of general relativity, canonical quantization entails that the physical Hilbert space associated with our system is captured by solving three "constraints" (Isham 1992, p.34–35; Baez and Muniain1994, p.445; Rovelli 2004, p.145–147, 233). In general, constraints are associated with gauge orbits through phase space. A state which evolves along one of these orbits is thought to undergo no physical evolution. Thus, in solving for these constraints, one is constructing a space of physically possible objects which have the correct set of physical degrees of freedom. The "Gauss constraint" requires that the states be invariant under SU(2) transformations, the "vector" or "diffeomorphism constraint" requires that the states be invariant under spatial differomorphisms, and the "scalar" or "Hamiltonian constraint" requires that the states be invariant under reparameterizations of time (Gambini and Pullin 2011, p.93–94; Rovelli 2004, p.146, 225). Whatever the physical world is, it will be represented by some quantum state, Ψ, which solves these constraints. Hitherto, only two of the constraints, the Gauss and diffeomorphism constraints, have been solved. I will spend a bit of time explaining the construction of the Hilbert space of states which solve these constraints, since these constraints influence our interpretation of the states and are important for understanding why spacetime is thought to be missing from the theory's fundamental ontology.

2.2. Gauss Constraint

I will explain first the Gauss constraint and then the diffeomorphsim constraint. At each stage, I will provide the naïve interpretation of the states which solve the relevant constraint(s), and will thereby unpack, in stages, the naïve ontology. In developing the theory of LQG, Rovelli implicitly endorses the naïve interpretation up through the Gauss-stage and then jettisons it when considering the diffeomorphism constraint (2004, p.238). Contrary to Rovelli, I will push the naïve interpretation through the diffeomorphism constraint-stage as a means of fleshing out the naïve interpretation. In §3, I will argue that it is this change in interpretation which Rovelli and others adopt when moving from the Gauss to the diffeomorphism constraint which has led some into fundamental confusion over the ontology of the theory. But more on this later.

In the following few paragraphs, I will provide some mathematical aspects of LQG; if the reader wishes to skip these details they are invited to do so. What is important to take away from the following discussion is that each state which passes the first test for being "physical", i.e. which solves the Gauss constraint, is built from looking at networks in $\mathcal{M}$. At the end of this subsection, I will provide the naïve interpretation for what ontology these states represent.

According to the Hamiltonian formulation of GR, one rewrites Einstein's field equations in terms of new variables, a "spin-connection", $\mathcal{A}$, and "tetrad fields", $\mathcal{E}$. It turns out that the metric field, g, can be decomposed in terms of tetrad fields and the relevant spin-connection can be defined by $\mathcal{E}$ in a manner analogous to the Levi-Civita connection (Rovelli 2004, p.34). In LQG, our quantum states are functionals of the spin-connection. A generic spin-connection is an $\mathfrak{su}(2)$-valued one-form which associates to each point in $\mathcal{M}$ an element from this Lie algebra ($\mathfrak{su}(2)$). Given that these algebraic elements will eventually be converted to concrete matrices, we can think of the spin-connection as assigning a, possibly unique, matrix to every point in space. The task is to carve this very large space of functionals (of the spin-connection) down to a physical Hilbert space of states which solve the aforementioned constraints.

In order to solve the Gauss constraint, one first identifies a graph of links (lines) and nodes (points) embedded in the mathematical manifold, $\mathcal{M}$. The links of the graph are "colored" by associating to them some representation of the SU(2) group. Every link is assigned a potentially different representation.[6] In addition to associating a color to each link, one must also color the nodes where the links meet. In essence, to color a link is to select a special vector which intertwines the vector spaces associated with the colorings of the different links. A colored network in $\mathcal{M}$ is called a "spin-network". For our purposes, the remaining details on how these networks are constructed and colored are not important. What is important is that the states, or functionals of $\mathcal{A}$, which solved the Gauss constraint are constructed with reference to networks embedded in $\mathcal{M}$.

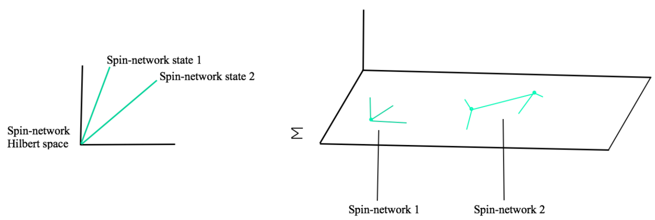

Figure 1: We "color" certain graphs in the sub-manifold Σ of $\mathcal{M}$ with quantum gravitational information. Colored graphs are called spin-networks and are used to construct the Hilbert space of spin-network states. We identify a spin-network state with each embedded network.

One can associate to each embedded spin-network a unique gauge-invariant functional of the spin-connection called a spin-network state, Ψ, (Rovelli and Upadhya 1998, p.3). These states form a basis of the Hilbert space of functionals which solve the Gauss constraint:

$$\left.\begin{array}{l}\text{topological}\\ \text{spin-network}\\ \in \mathcal{M}\end{array}\right\} \Rightarrow |\Gamma(\vec{x}), j_n, i_m\rangle \equiv \Psi. \tag{1}$$

The embedded graph $\Gamma(\vec{x})$ is a continuous series of links and nodes. The j_n keep track of which links (n) have what algebraic spin information (j) and the i_m keep track of which nodes (m) have what algebraic information (i).

I will stipulate, as part of the naïve interpretation, that structures on $\mathcal{M}$, which happen to be picked out by the physical states of LQG, are to be physically interpreted in a fairly literal way. Consequently, since spin-networks are embedded structures in $\mathcal{M}$ and are picked out by vector states of the gauge invariant Hilbert space, the ontology of LQG, according to the naïf, includes quantum gravitationally "charged" substantival graphs (Figures 1 & 2). These graphs are not mathematical objects but are composed of spacetime points which are themselves physical objects according to the naïf. These graphs are gravitationally charged, for, as we shall see, LQG represents them as having suitably quantized gravitational properties (equations (4) and (5)) encoded by the $\mathfrak{su}(2)$-field. There are times when Rovelli implicitly adopts this ontology (2004, p.147–150), even though, at the end of the day, he rejects this interpretation in favor of a relational view (§3.2). I want to note here that it is the network as a whole which represents a quantum mechanical structure of LQG since it is the entire network which is associated with a state in our Hilbert space. Since generic sub-networks in the manifold do not correspond with states in our Hilbert space, we will not interpret them as representing an item of quantum ontology in LQG.

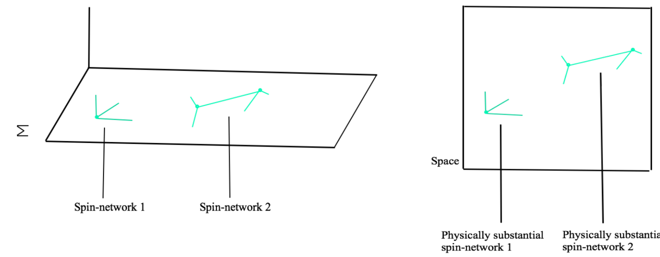

Figure 2: The naïf interprets embedded structures in $\mathcal{M}$ as literally modeling spatially-embedded graphs.

Although I have not yet discussed the observables of LQG (§2.4), on a substantival reading, these observables predicate properties of spacetime. For instance, a three-dimensional region of spacetime which includes a highly charged node is said to have a large volume and a two-dimensional surface of spacetime which is intersected by a highly charged link is said to have a large area (Figure 3). The ontology that one should have in mind at this stage in the development of the naïve interpretation is that of a charged network embedded in the fabric of spacetime which is responsible for producing the geometric properties associated with spacetime. This interpretation is cleanest at the level of the Gauss constraint. In order to solve the diffeomorphism constraint, we will have to reconsider what we take the new, more constrained set of states to represent. How to interpret states which are diffeomorphically invariant is the question where the received interpretation begins to part ways with naïve substantivalism.

2.3. Diffeomorphism Constraint

The diffeomorphism constraint requires that our physical states be invariant under smooth stretching and shifting of the corresponding network around the manifold. This constraint presents a problem if our states are defined with respect to particular network-embeddings in $\mathcal{M}$. Networks which are bolted down to defined locations in $\mathcal{M}$ are not invariant under diffeomorphisms. Therefore, in implementing the diffeomorphism constraint, our mathematical states are promoted from being tied to particular spin-networks embedded at specific places to

[6] In constructing the Hilbert space of spin-networks, one takes holonomies along the curves of the graph which convert the algebraic elements of $\mathfrak{su}(2)$ into matrices acting on some vector space. Which matrix is produced by this process depends, in part, on the representation associated with the curve.

equivalence classes, under diffeomorphisms, of such networks (Rovelli 2004, p.238–242). How this is formally achieved is not important for our purposes. What is important is that the new states, Ψ_{knt}, are no longer associated with a single network ($|\Gamma(\vec{x}), j_n, i_m\rangle$) but with all networks related to $|\Gamma(\vec{x}), j_n, i_m\rangle$ by a diffeomorphism. Effectively, we have collapsed the space of spin-network states into an equivalence class under diffeomorphisms.[7]

Once we have imposed the diffeomorphism constraint, the manner in which a graph is embedded is erased from the memory of the state, Ψ_{knt}. All that the states, Ψ_{knt}, encode about the old spin-networks is algebraic in nature: a "knot class", K, and a "coloring", c (Rovelli 2004, p.241). Now, this is not quite right as the global structure of the manifold continues to constrain the set of Ψ_{knt} states. While this is not usually emphasized in philosophical discussions, the global topological structure of the manifold constrains the initial set of embedded spin-networks with which we began (§2.2). For instance, a manifold which has a compact dimension (such as an infinitely long cylinder) will allow a different set of embedded networks from that allowed by $\mathbb{R}^4$ and therefore each choice of manifold might select out a different originating set, of spin-network states. Thus, in constructing the s-knot states and implementing the diffeomorphism constraint, the global topological structure of the manifold continues to inform the set of states we are considering. The important point, though, is that our s-knot states don't include any information about networks being embedded at *particular* places in the manifold. Thus, while global topological properties of the manifold play a background role in defining our s-knot states, local topological information of the manifold does not.

The Ψ_{knt} are both gauge and diffeomorphism invariant, and we will refer to them as s-knot states. Some authors use "spin-network state" to refer to any and all states even if they satisfy the diffeomorphism constraint in addition to the Gauss constraint. These authors allow the context to specify which mathematical structures are intended by the slightly ambiguous term. It is important to keep this in mind when reading quotes throughout this paper since, often, claims putatively about spin-networks are really claims about s-knots. In the context of the naïve interpretation, I will refer to the physical objects represented by s-knot states as *s-knots* simpliciter.

Recall, from the previous subsection, that because the physical states – the spin-network states – are associated with a mathematical network with a particular embedding in $\mathcal{M}$, the naïf interpreted these states as representing a physical network with a particular embedding in spacetime. However, the result of making our states diffeomorphically invariant is that they are no longer associated with any single graphical network in $\mathcal{M}$ but with an equivalence class of them. So, if s-knot states are not associated with a single embedded network in $\mathcal{M}$, what physical thing in spacetime do these states represent?

Here, the literature becomes a bit opaque and pushes away from the naïve interpretation. As a consequence of diffeomorphism invariance, Rovelli claims that s-knots are "abstract graphs" and no longer "in space" (Rovelli 2004, p.19–21 and 264).[8] Similarly, Wüthrich claims:

> The (abstract) [s-knot states] result after one has solved the Gauss and the spatial diffeomorphism constraints... These [s-knot states] can be represented by abstract graphs. (2006, p.92)

What these authors are committing to by calling s-knots "abstract" and failing to be in spacetime is twofold. First, the appropriate mathematical framework for talking about the physical states of LQG are algebraic graphs, not topologically embedded graphs. In other words, the graphical representation, in the topological manifold, is misleading as it requires specifying some exact location for the network. Second and more controversial, the physical objects denoted by s-knot states are not objects in a substantival manifold. Rovelli and others infer from the diffeomorphism invariance of s-knot states that the states of LQG cannot represent substantival networks in a substantival spacetime (§3.2). However, it is important to keep in mind that this inference is not required by the mathematics of LQG and is a matter of philosophical taste. Since this point is important in what follows, allow me to explain.

The diffeomorphism invariance of our states means that the mathematical description of our physical system does not refer to any particular spacetime points. Consequently, there is nothing in the theory of LQG to say whether the system is located at "these" rather than at "those" spacetime points. For those familiar with the substantival-relational debate, this is exactly the same situation that the substantivalist finds herself in both GR and in Newtonian mechanics.

In the case of GR, Einstein's field equations are invariant under diffeomorphism transformations and in the case of Newtonian mechanics, Newton's equations of motion are invariant under Galilean transformations.[9] In both cases, the empirical content of these theories is independent of the particular spacetime points at which the physical system is putatively located. With respect to the empirical opacity of particular spacetime points, there is nothing new in the move to LQG. Now, what should be our attitude towards this fact? From the fact that our theories are "blind" to the identity of the underlying spacetime points, should we conclude that there are no spacetime points? The substantivalist says "No". Let us consider Newton's attitude on absolute space:

> Absolute space, of its own nature, without relation to anything external, always remains homogeneous and immovable. Relative space is any moveable measure or dimension of this absolute space; such a measure or dimension is determined

[7] Technically, we do something stronger and group together all states which are related by any map which is continuous, invertible and such that it and its inverse are smooth except at all but a finite number of points. In other words, we allow our mappings to act non-smoothly at the vertices of our graphs (Rovelli 2004, p.232, 238–242, and 266).

[8] See also Baez and Muniain (1994, p.448) and Wüthrich (2017, p.313).

[9] And of course, Newtonian mechanics can be cast into a generally covariant form but that is not the point. The point is that with respect to the then known Galilean freedom, Newton (1687/1999) maintained the existence of an absolute manifold.

> by our senses... But since these [absolute] parts of space cannot be seen and cannot be distinguished from one another by our senses, we use sensible measures in their stead... Thus, instead of absolute places and motions we use relative ones, which is not inappropriate in ordinary human affairs, although in philosophy abstraction from the senses is required. (Newton 1687/1999, p.408–410)

According to Newton, though we can only ever measure the relative position and velocities of bodies, these bodies, nonetheless, have *absolute* positions and velocities in absolute spacetime. Likewise, despite the diffeomorphism invariance of GR many philosophers of GR have included spacetime points as part of the theory's ontology (Butterfield 1989, Maudlin 1990, Brighouse 1994, 1997, Norton 2019). For instance, in the face of the diffeomorphism invariance of GR, "sophisticated substantivalists" have tossed out the assumed primitive identity of spacetime points rather than tossing out the points themselves (Maidens 1992, Hoefer 1996). Now, I do not mean to suggest that Newton's view on substantival space is the same as that held by contemporary substantivalists, but rather I mean to draw our attention to the fact that since Newton, our physics has not depended on whether or not our system was located at "these" spacetime points rather than at "those" and despite this independence, substantivalism has continued to be a defended position. In moving to LQG, substantivalism faces no challenge that was not already present in GR and to some extent in Newtonian mechanics.

If one has reasons – presumably empiricist reasons – to interpret the diffeomorphism invariance of GR as indicating that there is no spacetime manifold in GR, then one has similar reasons for denying the manifold in the context of LQG. Rovelli and Wüthrich seem to be in this camp, though their reasoning from diffeomorphism invariance to the nonexistence of spacetime points is not given. We are told that diffeomorphism invariance demonstrates the nonexistence of the manifold but are not told why. I should offer a cautionary word against following the path of Rovelli and Wüthrich and throwing away spacetime points altogether.

While the diffeomorphism freedom inherent in GR and LQG entails that the identities of particular spacetime points are empirically opaque, the global topology of $\mathcal{M}$ constrains the metrics available to us in GR as well as the Hilbert space of s-knots in LQG. In both cases, the global properties of $\mathcal{M}$ are physically relevant. Now the question we need to ask is, can we erase from our ontology the points composing the set $\mathcal{M}$ while not also erasing the global topological properties associated with this set of points? While I suspect that this can be done, one will likely be forced to adopt an unintuitive bit of metaphysics. For instance, one way to maintain the existence of the global topological properties of the set of points $\mathcal{M}$ without also maintaining the underlying set is to insist that there can be instantiated relations without the corresponding *relata*. In this case the relations under consideration are monadic properties such as being "orientable" or "Hausdorff". In general, to allow relations or properties without *relata* is tantamount to admitting that there can be instances of redness without there being any red objects.

With respect to the manifold $\mathcal{M}$, we would have to allow "orientable" to be actualized without there being a manifold to orient. Similarly, we would have to allow the property of "Hausdorff" to be actualized without there being any spacetime points! Allow me to explain the difficulty with this suggestion. A topological space is Hausdorff if and only if every two points are included in disjoint open sets. Now, what would it mean to say that the world is Hausdorff or that the property of "Hausdorff" be actualized without there being any spacetime points to satisfy the conditions of the definition? Again, perhaps we can answer these questions and make sense of these global properties without requiring an underlying set of spacetime points, but at the very least, one would need explain how this is to be done.

In extending naïve substantivalism through the diffeormophism constraint, we will diverge from the view expressed by Rovelli and Wüthrich and in the spirit of Newton maintain that though particular spacetime points are hidden from our physics, our physical system – nonetheless – has an absolute location in the spacetime manifold. In other words, a physical s-knot is an embedded spin-network (or superposition of spin-networks) which has an absolute location in the spacetime manifold even though the s-knot *state* fails to tell us where it is embedded in the manifold. On this view and in agreement with the history of manifold substantivalism, the naïf believes that our physical system has a property, e.g. its absolute location, which is not determined by our best physical theory. Again, I am not saying that manifold substantivalists are right in thinking this but merely to outline a philosophical position which has been maintained and defended since Newton up through GR.

To be clear then, in LQG there is a mathematical manifold and how we interpret this manifold is a philosophical question. If the manifold disappears in LQG, then it disappears in GR and Newtonian mechanics as well, and it does so as a matter of interpretation, not quantization. Thus, when Daniele Oriti claims "canonical loop quantum gravity ends up anyway replacing the spacetime continuum with something more radical" (Oriti 2014, p.2, fn 3), he is merely expressing an aspect of a particular interpretation of LQG. See §3.2 for more on this interpretation.

Now, there is a related risk of confusion in LQG due to the fusing together of two disparate ideas. On the one hand, the models of LQG, $\langle \mathcal{M}, \Psi_{\text{knt}}(\mathcal{A}) \rangle$, include a smooth manifold with a smooth $\mathfrak{su}(2)$-field defined on it. On the other hand, as we shall see in the following section, the properties associated with the states of this field are discrete. So, on the one hand, there is a *continuum* manifold and, on the other hand, there is a *discrete* geometry in the form of discrete spectra. If one conflates these structures, one might mistakenly infer that the continuum itself has been decomposed into disconnected discrete blocks in accordance with our discrete geometry. This inference is not entailed by the mathematics of LQG. Let us now return to our third and final constraint, the "Hamiltonian" constraint.

Unfortunately, there is no agreed upon Hamiltonian operator with which to define the Hamiltonian constraint (Rovelli 2004, p.284–285). Nonetheless, we can still discuss some of the effects of the Hamiltonian constraint on the theory. As was the case with the previous two constraints, solving the Hamiltonian constraint will further contract the physical Hilbert space of states. Similar to the diffeomorophism constraint, the Hamiltonian constraint requires that our states be independent of our putative time coordinate. What the diffeomorphism constraint does for space the Hamiltonian constraint does for time. The resulting freedom from the time parameter seems to suggest that the physical systems of LQG are frozen in time. This frozen dynamics sits at the heart of the various "problems of time" in the LQG (Isham 1992, Kiefer 2009). Since the Hamiltonian constraint has not been solved, at the moment, s-knot states are our most reliable guide to the physical structures of canonical loop quantum gravity. In the following section, I will present the observables of LQG and explain which properties of the states they are thought to represent. Before I do this, allow me to review the interpretative transition which accompanies the move from the Gauss to the diffeomorphism constraint.

After imposing the Gauss constraint, spin-network states formally correspond with mathematically embedded networks in a mathematical manifold. The naïf interprets these mathematical structures as representing substantival networks with their particular embeddings in spacetime. In solving the diffeomorphism constraint, our mathematical states no longer correspond with particular embedded networks in our mathematical manifold. The diffeomorphism constraint erases the formal connection between these two bits of mathematics: vector states and topologically embedded mathematical networks. Despite this, the naïf interprets s-knot vector states as representing substantival networks, though, admittedly, not their particular embeddings. According to the naïf, in moving from spin-network states to s-knot states, one does not change ontology, but rather, one merely removes from the mathematics of LQG, empirically opaque aspects of substantival ontology – namely, where the embeddings occur.

2.4. Observables

Since LQG is a quantum theory aimed at replacing GR, we would like for it to include observables which approximate the geometric structure of spacetime. Area and volume observables have been defined in such a way that both spin-network states and s-knot states are eigenvectors of them (Rovelli 2004, p.248, 262 and Rovelli and De Pietri 1996, p.15). Consider the following operator:

$$\hat{\mathcal{A}}(\mathbf{S}) \equiv \lim_{n\to\infty} \sum_{k}^{n} \sqrt{-\left(\int_{\mathbf{S}_k^{(n)}} d\sigma^1 d\sigma^2 \epsilon_{abc} \frac{\partial x^a(\vec{\sigma})}{\partial \sigma^1} \frac{\partial x^b(\vec{\sigma})}{\partial \sigma^2} \frac{\delta}{\delta \mathcal{A}_c^i(\vec{\sigma})}\right)^2} \tag{2}$$

The way to interpret $\hat{\mathcal{A}}(\mathbf{S})$ is that we are measuring the value of some property, $\hat{\mathcal{A}}$, indexed by some spatial surface, $\mathbf{S}$. The $\hat{\mathcal{A}}(\mathbf{S})$ operator is the concrete "*area*" observable of LQG. The reason for italicizing *area* and *volume* is to distinguish the operators named by them from the classical pseudo-Riemannian structures we normally intend. I call the *area* observable "concrete" since it is defined in terms of embedded structures in the manifold, $\mathcal{M}$. In fact, one reason for presenting this equation is to illustrate its dependence on the manifold: the integral is defined in terms of a measure, $d\sigma^1 d\sigma^2$, over an embedded surface, $\mathbf{S}$. Moreover, the operator $\frac{\delta}{\delta \mathcal{A}_c^i(\vec{\sigma})}$ which acts on the states, Ψ, is explicitly dependent on the values of the coordinate functions $(\vec{\sigma})$ over $\mathbf{S}$.[10]

The *area* observable acts on spin-network states and has a spectrum of *area* eigenvalues:

$$\hat{\mathcal{A}}(\mathbf{S})\Psi \equiv \sum_{n\in\{\mathbf{S}\cap\Gamma(\vec{x})\}} \sqrt{j_n(j_n+1)}\Psi. \tag{3}$$

Embedded spin-networks carry charge (j_n) on their links and so contribute to the value of $\hat{\mathcal{A}}(\mathbf{S})$. An embedded network will affect the value of $\hat{\mathcal{A}}(\mathbf{S})$ for a given surface in two ways: first, the number of its links which intersect the surface $(\{\mathbf{S}\cap\Gamma(\vec{x})\})$ will change the number of things summed over in (3), and second, as we change the gravitational charge (j_n) of the links, we affect the size of each term in the sum. Thus, it is possible to increase the pseudo-Riemannian area of $\mathbf{S}$ and yet not increase $\hat{\mathcal{A}}$. For instance, consider a single embedded network with a straight link which happens to cross a flat circular surface once. If we had a metric, we could change the pseudo-Riemannian size of the circle by doubling its radius, though, since we do not change the number of times with which the surface intersects the network nor the charge of the network, we will not increase the physical area defined by $\hat{\mathcal{A}}$. Conversely, if we keep the pseudo-Riemannian size of $\mathbf{S}$ fixed but increase the charge of the link, the *area* associated with $\mathbf{S}$ will increase. These results are similar to the situation in electromagnetism: to increase the electrical charge of a surface, we must add more charge and not simply increase the pseudo-Riemannian size of the surface. In the same way, to increase the *area* of a region, we must change our network, not the bounds of our integration (Rovelli 2004, p.269–270). A similar situation holds for our volume observable: integrating over a larger region does not necessarily produce a larger *volume*.

The remarkable achievement of LQG and the reason for naming these observables *area* and *volume* is that they produce eigenvalues which approximate their Riemannian namesakes when acting on certain states. For instance, there are special spin-network states such that (Rovelli 2004, p.268):

$$\hat{\mathcal{A}}(\mathbf{S})\Psi = (\mathbb{A}(g,\mathbf{S}) + O(l_p/l))\Psi \tag{4}$$

$$\hat{\mathcal{V}}(\mathbf{R})\Psi = (\mathbb{V}(g,R) + O(l_p/l))\Psi \tag{5}$$

[10] The operators in (1) and (2) are Rovelli's, but there are other area and volume operators which one might consider. Importantly, Ashtekar and Lewandowski (1998) have defined a pair of geometry operators which disagree with Rovelli's and rely more directly on aspects of the manifold.

Here, $\mathbb{A}(g, \mathbf{S})$ is the pseudo-Riemannian area of surface, **S**, given by the metric g and $\mathbb{V}(g, R)$, the pseudo-Riemannian volume. As we pull back from the Planck scale ($l \gg l_p$), the values of our observables approach their pseudo-Riemannian counterparts. However, not all spin-network states satisfy either of these equations let alone both. The spin-network states which do satisfy these equations are called "weave-states", Ψ_w, and they form a countable basis for our spin-network Hilbert space. Consequently, a generic spin-network state can be expressed as a superposition of weave-states, a superposition of semi-classical geometries.

Throughout the remainder of this paper, I will no longer italicize 'area' and 'volume' in reference to the observables of LQG. I will refrain from doing so in order to signal that, if LQG is correct, physical areas and volumes are more accurately described by $\hat{\mathcal{A}}(\mathbf{S})$ and $\hat{\mathcal{V}}(\mathbf{R})$ than by their pseudo-Riemannian counterparts.

An important prediction of LQG is that the area of surfaces and the volume of regions come in discrete Planck-sized packages. This comes about because the graph of a network is modified by adding or subtracting whole numbered links or nodes from it. And since the j_n in equation (3) only takes on integer values, a network can only add discrete units of area to any given surface. Similar reasoning holds for the volume observable.

In order for our observables to be "Dirac" and arguably physically meaningful, we need them to be invariant along the constraint surfaces of our theory. This just means that our physical observables should enjoy the same invariances as our physical states. Therefore, the observables of LQG should be both gauge and diffeomorphism invariant. Unfortunately, our observables are explicitly dependent on particular surfaces, **S**, and regions, **R** (Rovelli 2004, p.266), and thus fail to be invariant under diffeomorphisms. Rovelli has offered some suggestions for how to construct diffeomorphism invariant observables[11] and claims that, once we have done so, the resulting observables will make no reference to particular regions and surfaces in the manifold. According to Rovelli, these observables will depend only on the algebraic information of the s-knot states (Rovelli 2004, p.262–265). However, constructing these observables is both complicated and "useless" according to Rovelli (p.266) for the spectra will be the same either way. Rovelli argues that the area and volume operators are "partial observables" and share the same spectra as their "complete" coordinate-independent counterparts. According to Rovelli, partial observables give us half of what we expect from their complete counterparts: we are given the set of available eigenvalues but are unable to calculate the exact eigenvalue of any particular s-knot state (p.263–265). This imposes a limitation on what we can say about any particular s-knot state, though it places no barrier on what we can say about s-knots in general. In general then, s-knots produce areas and volumes in discrete units (eq.(3)) through their charged links and nodes. It seems however, that there is some debate regarding the appropriateness of using equations (4) and (5), defined on spin-networks, to infer properties of s-knots. For more on this debate see, Rovelli (2007), Dittrich and Thiemann (2009) and Crowther (2016).

[11] For instance, Rovelli suggests that we might use the gauge freedom of the matter fields to make the observables diffeomorphically invariant (Rovelli and Peush 1998, p.7; Rovelli 2004, p.266).

The first half of this paper had two primary objectives. The first objective was to explain the formal structure of LQG as an introductory aide to philosophers and, the second, to explain this formal structure while highlighting the substantival interpretation of the theory. I hope to have said enough to provide a clear though admittedly bird's-eye perspective of the formal structure of LQG sufficient for beginning a philosophical analysis of the theory's ontology. In the second half of this paper, I will (1) modify the naïve interpretation in order to bring it into alignment with substantivalist attitudes in GR, (2) present two relational interpretations of LQG, and (3) answer, for various interpretations, the questions raised in §1: does spacetime exist fundamentally, do spin-networks (s-knots) exist fundamentally and does spacetime emerge from them. But before turning to these other tasks, let us remind ourselves of the naïve interpretation.

According to the naïf, s-knots live on a substantival spacetime manifold and carry gravitational charge along their links and nodes. The more charge a network has, the more volume it produces. The networks of LQG build spacetime geometry one volume at a time as geometry "radiates" from them (Figure 3). Since there is a lower bound to how much area and volume a physical network can carry, spacetime can only be geometrically parsed up to a certain scale – below which, no geometry is defined.

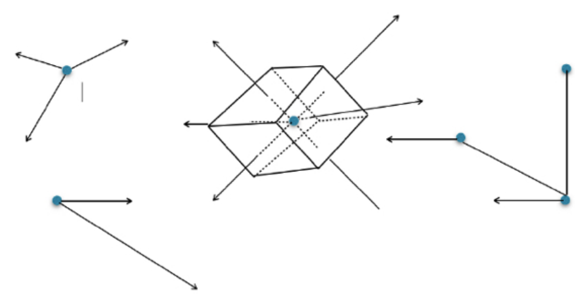

Figure 3: A series of networks: gravitationally charged links and nodes. Each node defines a volume of space and each link, an area. In this case, the network defines a cubic volume though, in the literature, one is more likely to see tetrahedra.

The ontology of this interpretation is not so different from substantival interpretations of GR. In GR, spacetime is described by $\langle \mathcal{M}, g \rangle$, which we can interpret as describing a physically substantial manifold bearing a physical geometry. In moving to naïve-LQG, we keep the physically substantial manifold but replace the physical geometry, associated with the gravitational field g, with a quantum geometry, produced by the charged networks Ψ_{knt}.

3. Interpretations and Spacetime Disappearance

In this section, I will provide six additional interpretations of LQG; four are substantival and two are relational. Each of these interpretations differs from one another and from the naïve interpretation in what they take spacetime to be.

3.1. Substantival LQG

In the following, I will amend the naïve interpretation by thickening the notion of spacetime. Rather than being a structure literally modeled by the bare manifold, $\mathcal{M}$, with no physical geometry, spacetime will be required to include some kind of geometry or quantum geometry. The motivation for thickening our notion of spacetime to include something like a metrical structure or physical geometry is that, without it, spacetime lacks a causal structure, spatial lengths and durations of time, all of which seem to be constitutive of spacetime. Consequently, perhaps spacetime is better modeled by the pair of objects $\langle \mathcal{M}, g \rangle$ or, perhaps, by $\langle \mathcal{M}, \Psi_{\text{knt}} \rangle$. In the following, I will modify the naïve interpretation in four ways in order to accommodate these two options. Since the manifold plays an essential role in modeling substantival spacetime for each of these four interpretations, these interpretations will be classified as instances of manifold substantivalism.

I do not claim that these four interpretations are the only possible elaborations of the naïve position. These interpretations have been chosen as they provide an informative vantage point from which to interpret the mathematics of LQG and evaluate the claim that spacetime emerges in LQG. Since I will be cycling through interpretations, I will thereby be cycling through what 'spacetime' means and how spacetime is formally represented in the theory. For this reason, it is important to keep in mind which interpretation is being discussed.

According to the S_1 interpretation, spacetime is the composition of a substantival manifold bearing a classical geometry represented by the ordered pair $\langle \mathcal{M}, g \rangle$. On this interpretation, both $\mathcal{M}$ and g represent physical things which jointly come together to form spacetime. If spacetime is as according to S_1, since fundamentally there is no metrical structure described by LQG, there is no spacetime fundamentally. Moreover, as part of S_1, I will stipulate that the models $\langle \mathcal{M}, \Psi_{\text{knt}} \rangle$ represent a composite "quantum spacetime".

According to the S_2 interpretation, spacetime is exactly as it is in S_1 except that the models $\langle \mathcal{M}, g \rangle$ represent a noncomposite classical spacetime while the models $\langle \mathcal{M}, \Psi_{\text{knt}} \rangle$ represent a noncomposite quantum spacetime. Here, the pieces of our models, $\mathcal{M}$, g, and Ψ_{knt}, do not represent distinct items of ontology. This interpretation, as well as S_4 below, refuses to infer from the mathematical composition of our models a corresponding composition in ontology. For these interpretations, there is only one thing represented by our models, and it cannot be ontologically parsed according to the mathematical categories of topology and geometry.

An analogy might help. Just as a person's name is decomposed into two parts, the given and the family names, the person herself cannot be ontologically decomposed along these lines. The given name does not refer to some part of a person's ontology while family name refers to what remains. So also, according to S_2, '$\langle \mathcal{M}, \Psi_{\text{knt}} \rangle$' is a name which refers to one thing which cannot be divided according to the parts of the name. Whether or not S_2 is correct in viewing spacetime this way is besides the point. S_2 is a possible interpretation whose ontology differs in important ways from the other interpretations being considered. The difference between S_1 and S_2 then is whether or not quantum spacetime is composite, composed of two things, and this difference is essential to the existence of spin-networks (s-knots) as I will explain in a moment.

There are two motivations for the following interpretations (S_3 and S_4). First, if the physical models of LQG include a four-dimensional basal manifold, then the physical structure being represented is enough like a "container" for the substantivalist to think that spacetime lives on in LQG. Second, in quantizing the electromagnetic field, we did not get rid of the field from our ontology but rather we simply gave the field a new, quantum, description. So also, according to the following interpretations, spacetime does not go missing in quantization but merely gets a new description: $\langle \mathcal{M}, \Psi_{\text{knt}} \rangle$. On this view, spacetime is like the electromagnetic field and unlike phlogiston insofar as it is a piece of ontology which survives the evolution in our understanding of the world.

According to the S_3 interpretation, spacetime is the composition of a substantival manifold and a quantum geometry represented by $\langle \mathcal{M}, \Psi_{\text{knt}} \rangle$. Since spacetime is a composite, both $\mathcal{M}$ and Ψ_{knt} represent physical things: there is a substantival manifold with a substantival network. The only significant difference between the S_1 and S_3 interpretations is whether we require spacetime to have classical or quantum properties. The difference between the S_3 and S_4 interpretations lies merely in whether or not we take spacetime to be composite.

According to the S_4 interpretation, spacetime is a noncomposite structure represented by $\langle \mathcal{M}, \Psi_{\text{knt}} \rangle$. In this case, neither $\mathcal{M}$ nor Ψ_{knt} represent anything physical on their own. In particular, the states, Ψ_{knt}, do not represent physical things called s-knots but rather these states simply provide some of the requisite mathematical structure for representing the quantum geometric properties of spacetime. As a unit, $\langle \mathcal{M}, \Psi_{\text{knt}} \rangle$ represents spacetime according to S_4, just as $\langle \mathcal{M}, g \rangle$, as a unit, represents spacetime according to S_2.

Importantly, these four less naïve interpretations are both allowed by the mathematics of LQG and disagree with each other as well as with the original naïve interpretation in what ontology they take to exist fundamentally. (For a summary see Table 1.)

- According to the naïve interpretation, the manifold $\mathcal{M}$, devoid of any metrical structure, represents substantival spacetime. Spacetime does not disappear in LQG, and there are physically substantial s-knots. On this view, whether or not geometry is classical or quantum is irrelevant for spacetime.

- According to S_1, spacetime disappears fundamentally in LQG. Spacetime, within this view, requires a classical physical geometry, and quantum spacetime requires a quantum geometry. Since the models of LQG are $\langle \mathcal{M}, \Psi_{\text{knt}} \rangle$, there is no spacetime fundamentally in LQG. Moreover, since quantum spacetime is composite, there are physically substantial s-knots embedded in the substantival manifold.

- According to S_2, both spacetime and quantum spacetime are substantival and noncomposite structures. Since the models of LQG are $\langle \mathcal{M}, \Psi_{knt} \rangle$, there is no spacetime fundamentally. Moreover, since quantum spacetime is noncomposite, there are no s-knots.

- According to S_3, substantival spacetime is a composite structure represented by $\langle \mathcal{M}, \Psi_{knt} \rangle$ and, as such, spacetime exists fundamentally. Since spacetime is composite, the states Ψ_{knt} represent physical networks embedded in the substantival manifold represented by $\mathcal{M}$.

- According to S_4, substantival spacetime is a noncomposite structure represented by $\langle \mathcal{M}, \Psi_{knt} \rangle$ and, as such, spacetime exists fundamentally, though substantival s-knots do not.

As we can see, the theory of LQG entails neither that spacetime disappears, nor that there are physical networks. Whether or not there are such things depends on our interpretation of spacetime with respect to the models $\langle \mathcal{M}, \Psi_{knt} \rangle$. Though spacetime disappears for some of these accounts, on every account we are left with a fundamental substantival structure.

Now, the difference between these interpretations is not a matter of semantics as 'spacetime' is not an empty label that we are free to attach to any structure we wish. Space, time and spacetime are not merely theoretical objects which show up in our most advanced physical theory but are objects or structures which tie together much of the way human beings conceive reality. The point is, if 'spacetime' like 'blem' is an empty label, it would not matter what we decided to tag it with – the issue would indeed be a matter of semantics. But 'spacetime' is not an empty label any more than is 'the president of the United States' – it matters to the structure of reality what these labels in fact name. It matters to the nature of reality if 'the president of the United States' names a person rather than a tree or sea creature. Why this is the case is because the president of the United States is also thought to be the commander-in-chief of the US military and to sleep in the White House. The term 'the president of the United States' is not an empty label but comes with metaphysical and ontological commitments. So too, the terms 'space', 'time' and 'spacetime' are not empty labels but come with various expectations and commitments. For instance, it is often claimed that space grounds the distinction between abstract and concrete objects, or that causes come earlier in time than their effects, or that no object can exist in more than one place in space but can exist in more than one place in time. The point is that much of the way we view reality is tied up with our notions of space and time and so it makes a difference to our understanding of reality what physical structure we label as 'space', 'time' or 'spacetime'. In particular, it makes a difference to the nature of reality if spacetime ceases to exist fundamentally, if it is emergent or if it is merely an illusion. Thus, in choosing one of the interpretations on offer one is not making a choice of semantics but a choice in how to view reality.

The final two interpretations, to which I will now turn, diverge much more radically from the naïve interpretation than the various S_i interpretations. According to the following interpretations, the manifold, $\mathcal{M}$, is a mathematical artifact and does not encode any physical information; spacetime is relational according to these interpretations and is modeled by either g or Ψ_{knt}.

3.2. Rovellian LQG

The following interpretation is largely inspired by Carlo Rovelli; however, I do not claim that the views expressed here are exactly his own. Thus, this interpretation is Rovellian, though perhaps not Rovelli's. The Rovellian interpretation has become the received interpretation among philosophers of LQG, being assumed in part, if not in whole, in almost all philosophical work on LQG: Huggett and Wüthrich (2013), Lam and Esfeld (2013), Crowther (2014, 2016, 2018), Norton (2017), Würthich (2006, 2017), Wüthrich and Lam (2018) and Le Bihan (2019).

According to the Rovellian interpretation, the diffeomorphism freedom found in both GR and LQG is evidence that $\mathcal{M}$ is something like a gauge artifact and ought not be interpreted as encoding physical structure. Since the s-knot states make no reference to any particular embedding, Rovelli concludes:

> In fact $\mathcal{M}$ (the spacetime manifold) has no physical interpretation, it is just a mathematical device, a gauge artifact... There are not spacetime points at all. The Newtonian notions of space and time have disappeared... the spacetime coordinates $\vec{x}$ and t have no physical meaning...
> (2004, p.74)

> What Newton called "space," and Minkowski called "spacetime," is unmasked: it is nothing but a dynamical object – the gravitational field...
> ...the gravitational field is the same entity as spacetime.
> (2004, p.9, 18)

However, just because $\mathcal{M}$ does not play a role in determining what physical values are observed, it is not required that we treat $\mathcal{M}$ as being a mathematical artifact. How to treat gauge orbits and related mathematical structure is a thorny philosophical issue, and it is far from settled that all such orbits ought to be taken nonrealistically. Rovelli recognizes this in "Halfway Through the Woods" (1997) when he acknowledges that, though LQG and GR are diffeomorphically invariant theories, one might still insist that there is a physical background manifold which happens to be unobservable. Though Rovelli acknowledges that the existence of an unobservable manifold is logically possible, it it not the position he endorses. According to the Rovellian interpretation presented here, there is no physical manifold represented by $\mathcal{M}$.

According to Rovelli, spacetime in GR is just the gravitational field and in quantizing it we are quantizing spacetime itself (Rovelli 2004, p.17) rather than some aspect of spacetime. Consequently, since there is no classical gravitational field in LQG, there is no classical spacetime either. Fortunately, since the weave-states produce formal expressions which approximate those of classical geometry, equations (4) and (5), there is a sense in which spacetime arises or is recovered from the quantum phenomena of LQG. When ($l \gg l_p$) and the quantum spacetime happens to be described by a weave-state, the world looks classical in the sense that the predictions of LQG are numerically similar to those of GR. In this way, we might say that spacetime is recovered or is “emergent," according to LQG in the classical regime.

However, we must not interpret claims to the effect that spacetime is recovered, in this mathematical sense, as necessitating that spacetime, as a new item of ontology, emerges in the classical regime. When ‘recovered’ is understood in the way I have used it, all that is entailed is that spacetime *qua* the formal gravitational field is a mathematically effective structure for modeling a certain class of phenomena. One might endorse the additional view according to which effective structures at low energies are genuinely emergent objects of our ontology and distinct in kind from whatever happens to be fundamental. Or contrarily, one might view effective structures as merely useful fictions. For more on effective structures and spacetime emergence in LQG, see Huggett and Wüthrich (2013), Wüthrich and Lam (2018), Le Bihan (2019), as well as (Norton, in preparation).

Much of what I have just said regarding Rovelli’s position can be modified and applied to some of the S_i interpretations. In particular, according to S_1 and S_2, classical spacetime is formally recovered when our physical system evolves to take the form of a weave-state and when our measuring devices are operating in the classical, low-energy regime. If our interpretation includes physically substantial s-knots as it does in S_1, then we might say that classical spacetime emerges in the low energy regime when our *physical* s-knots take the form of weave-states.[12]

Now, since physically embedded spin-networks and s-knots are physically substantial insofar as they are comprised of points from the physical manifold, without a substantival manifold (*a la* Rovelli), one does not have substantival networks. While Rovelli does not make this argument himself, he seems to endorse its conclusion:

> Such geometrical pictures [of topological networks] are helps for the intuition, but there is no microscopic geometry at the Planck scale and these pictures should not be taken too literally in my opinion. (Rovelli 2011, p.4)

Additionally, while Rovelli describes spacetime like a t-shirt which, when approached, reveals an underlying weave of threads, he cautions against taking these weaves “as a realistic proposal for the microstates of a given macroscopic geometry [spacetime]" (2004, p.268–269). Indeed, for Rovelli, there is no shirt since there is no basal structure but only “fields on fields" (2004, p.9). I interpret these quotes from Rovelli as cautioning us against naïvely reifying the topological networks in $\mathcal{M}$. Contrary to the naïve interpretation as well as S_1 and S_3, on the Rovellian interpretation, the world does not contain gravitationally charged networks but rather just the quantum geometric relations defined by them.

According to the Rovellian, spin-networks and s-knots are mathematical tools useful for encoding the properties of quantized relational spacetime. Both the vector states of LQG and the geometrically embedded spin-networks in $\mathcal{M}$ represent quantum geometric properties of quantum spacetime, *solely* in terms of the algebraic information they contain. Consequently, no isolated part of a topological network, not an isolated point or line, is to be taken as physically salient on its own. The network, as a whole, is physically salient but only insofar as it maps onto a vector state in the Hilbert space of LQG.[13] For the Rovellian then, there are not two physical things, quantum spacetime and physical s-knots, but only quantum spacetime represented by mathematical s-knot states. Contrary to the Rovellian, the substantivalist is committed to there being substantival spacetime points in addition to the quantum geometry represented by s-knot states. It is this additional layer of ontology, provided by the manifold, which allows the substantivalist (S_1 and S_3) to distinguish the quantum gravitational properties represented by the abstract s-knot states and physical networks composed of spacetime points.

Allow me to emphasize that for the Rovellian, what ontology there is, is given by a vector state in our Hilbert space. Since our states represent quantum geometry and not single nodes or single links, the Rovellian cannot include nodes and links as items of ontology. That being said, the mathematical information contained in the links and nodes of an embedded spin-network remain to some degree in our s-knot states in the form of degrees of freedom (K, c) (Rovelli 2004, p.241). The network ontology which the substantivalist makes essential use of is destroyed by the Rovellian leaving only a remnant in the form of internal degrees of freedom in the state. Thus, under the received, Rovellian interpretation, one cannot say that there is an ontology of links and nodes which combine in such and such a way to somehow construct classical spacetime. However, Wüthrich (2017) and Huggett and Wüthrich (2013) endorse the Rovellian interpretation, though they continue to speak of there being fundamental nodes which are associated with individualized volumes of spacetime. This is a mistake on the Rovellian interpretation as fundamental ontology is captured only by the states of the theory and not also by nodes and links. This is one aspect of the ontological inconsistency which one finds in the literature on LQG and which I hope to correct. If one wants nodes and links as items of ontology, in addition to what the states of the theory represent, one has a variety of manifold substantivalist positions to choose from.

Now, there remains conceptual space to modify the Rovellian interpretation so that spacetime does not in fact disappear; let us call this the Rovellian* interpretation. In considering the interpretations S_3 and S_4, I noted that what we discovered in

[12] Here I have been sloppy as weave-states, as I have defined them, are a kind of spin-network, not a kind of s-knot. By “weave-state", I am really referring to the diffeomorphism invariant version of a weave-state.

[13] Caveat: some sub-networks can be treated as being physically meaningful, but only because if we were to extract them from their network, they too would have a copy of themselves in the Hilbert space of LQG. Such networks are not physically meaningful as proper parts of a network but are physically meaningful as extracted networks in their own right.

quantum electrodynamics was not the disappearance of electromagnetism but its quantum nature. So too, a relationalist like Rovelli might very well claim that in LQG we discover not the disappearance of spacetime but simply its quantum nature. On this view, spacetime is just that relational structure represented by Ψ_{knt}. It is very important that we not rush past this interpretive option for it makes a difference in how we conceive of reality. There are important conceptual consequences if spacetime really ceases to exist fundamentally (Norton 2017).

In §1, I noted that there are two senses in which spacetime might be substantival. The first sense identifies the manifold as being essential for substantivalism where the second sense identifies the ontologically independent metric field as being the heir of substantival spacetime in GR. I have not yet said anything about this second sense in the transition to LQG. The primary motivations for treating the metric field as substantival, in the context of GR, is that it carries energy and can exist independent of matter fields. In other words, spacetime's existence is not mediated by relations between matter fields or degrees of freedom in those fields. Now, can we carry this reasoning over to the s-knots of LQG? In other words, can one toss out the manifold as the Rovellian does and yet maintain that substantival spacetime lives on in LQG in the form of Ψ_{knt}? I will not attempt to answer this question; though, I suppose "Yes".

In summary, according to the Rovellian interpretation, the states (Ψ_{knt}) of the theory represent a relational structure represented by the "quantum geometry" of equations (4) and (5) and do not represent physical networks. Spacetime does not exist fundamentally according to the Rovellian though it does for the Rovellian*. For ease, I have included all the relevant interpretations and their respective ontologies in Table 1.

4. Emergence

In order for spacetime to emerge from the spin-networks (s-knots) of LQG, it must be the case that spin-networks (s-knots) exist while spacetime fails to exist fundamentally. I hope that it has become clear that whether or not these conditions are satisfied depends rather heavily on one's interpretation of the theory. In the following, I assess the claim that spacetime emerges from spin-networks (s-knots), under the various interpretations considered in this paper.

- Naïve: since spacetime does not disappear fundamentally, the emergence claim is false.
- S_1: spacetime is a candidate for emergence. If spacetime emergence is accurately described by the account at the end of section §3.2, then spacetime emerges, in the low energy regime, from the non-spatiotemporal spin-networks (s-knots) so long as the physical system takes the form of a weave-state.
- S_2: since there are no s-knots, the emergence claim is false. Rather, classical spacetime emerges from quantum spacetime as we might expect of our quantum theories.

Table 1

Interpretations, Ontologies and Emergence			
Interpretation	Fundamental spacetime?	Spin-networks?	Emergence from spin-networks?
Naïve	✓	✓	✗
S1	✗	✓	✓
S2	✗	✗	✗
S3	✓	✓	✗
S4	✓	✗	✗
Rovellian	✗	✗	✗
Rovellian*	✓	✗	✗

- S_3: since spacetime does not disappear fundamentally, the emergence claim is false.
- S_4: since spacetime does not disappear fundamentally and since there are no s-knots, the emergence claim is false.
- Rovellian: since there are no s-knots *qua* an ontology of links and nodes, the emergence claim is false. Rather, classical spacetime emerges from, something like "quantum spacetime" as we might expect of any of our quantum theories.
- Rovellian*: since spacetime does not disappear fundamentally and since there are no s-knots, the emergence claim is false.

S_1 is the purest realization of the evocative claim that spacetime emerges from non-spatiotemporal networks. The ontology of this interpretation allows us to say that spacetime emerges, in the low energy limit, from the substantival networks as classical geometry radiates from its "quantum-gravitationally" charged nodes and links. There is a related, though deflated, reading of the emergence claim which is true of the S_2 and the Rovellian interpretations – namely, that classical spacetime emerges from quantum spacetime. The emergence claims which began this paper promised something extraordinary, but the only way I see to deliver on it is to adopt S_1. Outside of S_1, the emergence claim is either false or deflated; see Table 1.

5. Conclusion

In §2, I provided an exposition of LQG expressed in the language of the naïve interpretation. That interpretation describes the world as including a substantival manifold called spacetime which contains spatially embedded charged networks. These networks are responsible for the quantum geometric properties of spacetime. In §3, I provided a series of substantival and relational interpretations of LQG. These interpretations differ from one another and the original naïve interpretation in what they take spacetime to be and what they take the states of the theory to represent. Importantly, I have shown that there are clear and intuitive interpretations of LQG according to which spacetime does not disappear fundamentally. In §4, I analyzed the claim that in LQG, spacetime emerges from spin-networks and found it to be false, or deflated, for all but one interpretation. This analysis is important as most philosophical work on LQG assumes the Rovellian interpretation and does not seem to be aware that there are viable alternatives. Moreover, this same literature also endorses the claim that spacetime emerges from spin-networks – which, when taken literally, is false on the Rovellian view.

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