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Premium Auctions and Risk Preferences: An Experimental Study

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Abstract

In premium auctions, the highest losing bidder receives a reward from the seller. This paper studies the private value English premium auction (EPA) for different risk attitudes of bidders. We explicitly derive the symmetric equilibrium for bidders with CARA utilities and conduct an experimental study to test the theoretical predictions. In our experiment, subjects are sorted into risk-averse and risk loving groups. We find that revenues in the EPA are significantly higher when bidders are risk loving rather than risk averse. These results are partly consistent with theory and confirm the general view that bidders' risk preferences constitute an important factor that affects bidding behavior and consequently also the seller's expected revenue. However, individual subjects rarely follow the equilibrium strategy and as a result, revenue in our experiment is lower than in the symmetric equilibrium.

JEL-classification numbers: D44, C92.

Key words: premium auction, risk preference, Holt-Laury method, experimental economics.

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1 Introduction

Premium auctions have been used across Europe to sell houses, land, boats, machinery, or inventory of insolvent businesses for centuries (Goeree and Offerman, 2004). As the name already suggests, the distinguishing feature of premium auctions is a commitment on the part of the seller to pay a cash reward (premium) to a set of highest losing bidders. The amount of that premium is determined according to some pre-specified rule. This kind of tactics is commonly believed to be able to enhance competition and to induce higher bids. Therefore, even though it appears that the seller is giving away some of the profits, the final equilibrium payoff to the seller could still be higher with a premium rather than without one.

Theoretical justifications for such premium tactics remain inconclusive, however. Existing studies have demonstrated the potential value of premium auctions in the special case of an asymmetric setting, e.g., when there is one strong bidder who competes with several weak bidders (e.g., Ayres and Cramton, 1996; Goeree and Offerman, 2004; Milgrom, 2004; Hu, Offerman, and Onderstal, 2010).

In a recent paper, Hu, Offerman, and Zou (2011) consider a *symmetric* English premium auction (EPA) model. Just as in an English auction (EA), the auctioneer raises the price in the EPA until all but one bidder have withdrawn. The remaining bidder wins the object and pays the price at which the auction ends. Unlike in the EA, the last two remaining bidders in the EPA receive a premium from the seller that is determined by a pre-specified function of the difference between the prices at which the second- and third-to-last bidders withdraw. Hu et al. (2011) show that the seller's expected revenue increases in bidders' risk tolerance: the more risk loving they are, the higher revenue.

In this paper, we derive a closed-form solution of the equilibrium bid function in a classical symmetric, private values setting assuming that bidders exhibit constant absolute risk aversion (Proposition 1). This solution allows us to quantify the qualitative results of Hu et al. (2011) regarding the revenue comparison between the EPA and the standard English auction

(EA). The solution reveals that equilibrium bids for risk loving subjects are stunningly high for bidders with low values. In fact, if values are distributed uniformly on $[0, 100]$, for the specific parameters used in our experiment, risk loving bidders should bid more than 80 even if their value is close to 0.

In an EA, bidders should bid their value regardless of risk preference. By the revenue equivalence theorem, revenues for risk neutral bidders should be the same for EPA and EA. Hence, revenue for the EPA with risk loving bidders should be higher then for an EA and vice versa for risk averse bidders. In fact, we show that revenue in a EPA should be about 50% higher than the corresponding EA for realistic parameters of risk lovingness and about 25% lower for realistic values of risk aversion in our specific setting.

This prediction raises two important empirical issues. The first issue is the empirical validity of the prediction: Is it true that in practice the more risk averse (loving) bidders are, the less (more) expected revenue will be generated through an EPA? The second issue is the extent to which the premium practice would make a difference: Is risk preference a sufficient reason for a seller to adopt a premium policy in practice (rather than a standard English auction, for example)?

In this paper, we investigate these issues through a series of experiments. A novel aspect of our experiments is that we first sort prospective subjects into risk averse and risk loving groups in order to ensure that our experiment is more in line with the model assumption of ex ante symmetric bidders. This is done using the method proposed by Holt and Laury (2002) in an online experiment, which takes place several weeks before the main auction experiment. We then invite each group to separate EPA sessions.

Our results are partly consistent with theory and confirm the general view that bidders' risk preferences constitute an important factor that affects bidding behavior and consequently also the seller's expected revenue. However, individual subjects rarely follow the equilibrium strategy and as a result, revenue in our experiment is lower than in the symmetric equilibrium.

The rest of the paper is organized as follows. Section 2 presents the model and theoretical results. Section 4 introduces the experimental design. In Section 5 we present the experimental results. In Section 6 we conclude.

Some proofs and the instructions for the experiment are collected in the appendix.

2 Auction rules and theoretical predictions

In this section we introduce the auction rules and derive theoretical predictions. Suppose there is a single indivisible object for sale via an English premium auction (EPA).[1] Just like in a standard ascending English auction, the price in an EPA increases continuously. The last remaining bidder receives the good and pays the price at which the second-to-last bidder quits (the sales price). There is one important difference between the EPA and an English auction: in an EPA, the last and the second-to-last remaining bidders both receive a premium from the seller. This premium is a percentage of the difference between the sales price and the price at which the third-to-last bidder quits. The according percentage is announced before the auction starts. Ties are resolved randomly.

Suppose there are n (> 2) bidders with private values v that are independently distributed ex ante according to a cumulative distribution function F, which has a continuously differentiable density function $f = F'$ that is strictly positive on its support $[L, H] \subset \mathbb{R}_+, H > L$.

For analytical convenience, we perceive the EPA w.l.o.g. as a two-stage auction. In the first stage, the price rises continuously from zero and each bidder stays in the auction until he makes an irrevocable decision to exit. This stage ends as soon as only two bidders, called *finalists*, remain. The price level p_3 at which the third-to-last bidder quits is called the bottom price. This price can be viewed as an endogenously generated reserve price for the second stage.

In the second stage, the price level rises from p_3 until one of the finalists quits at some p_2. The last bidder who stays wins the object and pays the sales price p_2. In addition, both finalists receive a cash premium from the seller that is equal to $(p_2 - p_3)/m$, for $m \geq 2$. For instance, the seller may announce to the bidders prior to the auction that he will give back the

[1] See Goeree and Offerman (2004) and Hu, Offerman, and Zou (2011) for details.

amount $p_2 - p_3$ to the two finalists and let them have an equal share of it, in which case $m = 2$. Another example is where the seller pre-announces that he will "go Dutch" with the two finalists and equally share with them the amount $p_2 - p_3$, in which case $m = 3$.

In order to obtain tractable equilibrium solutions, we assume that the bidders have constant absolute risk aversion (CARA) and that they have the same utility function parameterized by λ,

$$u(x) = \frac{1 - \exp(-\lambda x)}{\lambda}, \qquad \lambda \in \mathbb{R}, \tag{1}$$

where λ can be positive (risk averse) or negative (risk loving). If a bidder quits before entering the second stage, his utility equals $u(0) = 0$. We also normalize the seller's reservation value for the object to be zero, so that the sale will take place with certainty.

We shall focus on symmetric equilibria in which all bidders adopt the same bidding strategies b^I and b^{II} for stages I and II, respectively.[2] By backward induction, the strategy pair (b^I, b^{II}) is an *EPA symmetric equilibrium* if (i) given any bottom price p_3 and updated information, $b^{II}(v, p_3)$ maximizes the expected utility of a finalist with value v provided the other finalist adopts the same strategy b^{II}, and (ii) given the common knowledge about b^{II}, all bidders will adopt b^I in the first stage—that is, a bidder with value v will quit at the price $p = b^I(v)$. Hu et al. (2011, Theorems 1 and 2) show in a more general setting that such a symmetric equilibrium pair exists and is unique, and that the bid functions (provided they are bounded from above) are necessarily differentiable and strictly increasing in the private signal (or value as in the present context) of the bidders. The analysis of Hu et al. (2011) implies that in our setting the bid function b^I is defined implicitly by $b^I(v) = b^{II}(v, b^I(v))$ and b^{II} is characterized by a differential equation with the boundary condition $b^{II}(H, p_3) \equiv H$.

Given that we assume CARA utility functions, we can solve explicitly for the EPA equilibrium bid function. In particular, we show that in this

[2] It is known that even in ex ante symmetric settings there may exist multiple *asymmetric* equilibria (e.g., Maskin and Riley, 2003). We focus on symmetric equilibria in this study.

case, the bid function $b^{II}(v, p_3)$ is independent of the bottom price p_3. We also explicitly derive a number of comparative statics results concerning how the premium and the bidder risk attitudes affect bidding behavior as well as the seller's expected revenue.

Given any value vector $(v_1, ..., v_n)$, let $v_{(1)}, v_{(2)}$, and $v_{(3)}$ denote the largest, second largest, and third largest value from this vector. Let (b^I, b^{II}) be the symmetric EPA equilibrium. Because b^I is increasing, the first stage will end with $p_3 = b^I(v_{(3)})$ and both finalists will have an updated cumulative probability distribution function $\left[F(y) - F(v_{(3)})\right] / \left[1 - F(v_{(3)})\right]$ about the other finalist's value v.

Now fix any bottom price $p_3 \in [L, H)$. The expected utility of the finalist who has value $v \in [v_{(3)}, H]$ and who bids as though his value were $z \in [v_{(3)}, H]$ equals

$$\begin{aligned} U(v,z) &= \frac{1}{1-F(v_{(3)})}\int_{v_{(3)}}^{z} u\left(v - b^{II}(y,p_3) + \frac{1}{m}(b^{II}(y,p_3) - p_3)\right) dF(y) \\ &\quad + \frac{1-F(z)}{1-F(v_{(3)})} u\left(\frac{1}{m}(b^{II}(z,p_3) - p_3)\right). \end{aligned} \tag{2}$$

where the first term on the right-hand side results from winning and the second term from losing (with premium collected).

Proposition 1 *Assume that u is given by (1). Then, for arbitrary $m \geq 2$ and $\lambda \in \mathbb{R}$, the EPA symmetric equilibrium bid functions satisfy $b^I(v) = b^{II}(v, p_3) = b(v)$, which is given by*

$$b(v) = -\frac{1}{\lambda}\ln\left(\int_v^H e^{-\lambda x} dM(x|m)\right), \text{ where } M(x|m) = 1 - \left(\frac{1-F(x)}{1-F(v)}\right)^m. \tag{3}$$

Proof. See the Appendix. ■

For the special case with risk neutral bidders, it can be checked by taking the limit as $\lambda \to 0$ that the equilibrium bid function converges to

$$b(v) = v + \int_v^H \left(\frac{1-F(x)}{1-F(v)}\right)^m dx \tag{4}$$

as shown in Hu et al. (2011). The second term on the right-hand side of this expression indicates the over-bidding above the true value due to the premium.

Now let $b_\lambda(v|m)$ denote the equilibrium bid function associated with the CARA parameter λ and the premium rule m. The comparative statics results of the next proposition follow straightforwardly.

Proposition 2 *The EPA equilibrium $b_\lambda(v|m)$ has the following properties:*

(i) $b_\lambda(v|m)$ decreases in λ for all $m \geq 2$.

(ii) $b_\lambda(v|m)$ decreases in m for all $\lambda \in \mathbb{R}$.

Proof. See the Appendix. ■

3 Parameters used in the experiment

We focus on the special case in which the bidders' private values are uniformly distributed on $[0, 100]$, i.e., $F(v) = v/100$ with density $f(v) = 1/100$. The result of Proposition 2 (ii) shows that a higher premium results in a larger expected differences in average revenues between groups of bidders with different risk preferences. To make this difference as pronounced as possible, we use the largest premium rate, i.e., $m = 2$.

Substituting the assumed parameters into (3) we get

$$\begin{aligned}
& \int_v^H e^{-\lambda x} dM(x|2) \\
= & -\frac{2}{\lambda}\left(\frac{1}{100-v}\right)^2 \left(-e^{-\lambda v}(100-v) + \int_v^{100} e^{-\lambda x} dx\right) \\
= & -\frac{2}{\lambda}\left(\frac{1}{100-v}\right)^2 \left(-e^{-\lambda v}(100-v) - \frac{1}{\lambda}\left(e^{-\lambda 100} - e^{-\lambda v}\right)\right) \\
= & -\frac{2}{\lambda}\frac{e^{-v\lambda}(v-100) - \frac{1}{\lambda}\left(e^{-100\lambda} - e^{-v\lambda}\right)}{(v-100)^2}.
\end{aligned}$$

Consequently, the equilibrium bid function is given by

$$b_\lambda(v) = -\frac{1}{\lambda}\ln\left(-\frac{2}{\lambda}\frac{e^{-v\lambda}(v-100) - \frac{1}{\lambda}\left(e^{-100\lambda} - e^{-v\lambda}\right)}{(v-100)^2}\right). \tag{5}$$

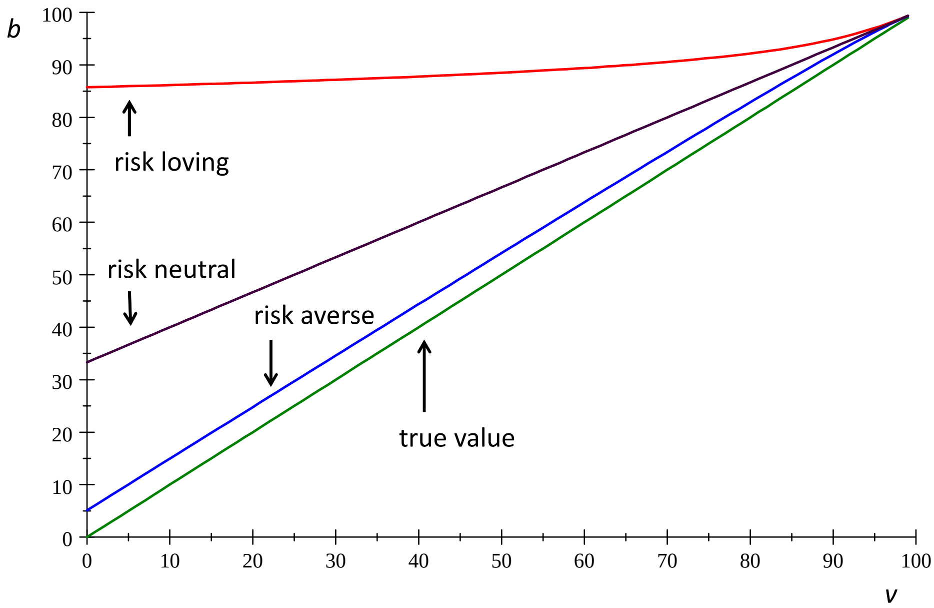

Figure 1: Euilibrium bid functions for various risk preferences. Here, F is assumed to be uniform on $[0, 100]$ and $m = 2$. The risk averse bid function is plotted for $\lambda = 0.7$ and the risk loving bid function for $\lambda = -0.5$.

The risk neutral case can be derived from (4) for $m = 2$ and $F(v) = v/100$, giving

$$b_0(v) = \frac{2}{3}v + \frac{100}{3}.$$

To illustrate, we depict in Figure 1 the equilibrium bid functions of risk averse, risk neutral, and risk loving bidders. The figure confirms that the premium, in general, induces the bidders to bid higher than their true values. It also shows that the bids are uniformly higher (lower) if the bidders are more risk tolerant (averse). Note that the differences are particularly pronounced for small values of v.

We now calculate the expected revenues and compare them to that of an English auction. Let $f^n_{(2)}(y)$ denote the density of the second-highest value among the n bidders, and let $f^n_{(2),(3)}(y, z)$ denote the joint density of the second-highest and third-highest values. These functions are given by

$$\begin{aligned}
f_{(2)}^{n}(y) &= 10^{-4} n(n-1)\left(\frac{y}{100}\right)^{n-2}(100-y) \\
f_{(2),(3)}^{n}(y,z) &= n(n-1)(n-2)F(z)^{n-3}(100-F(y))f(z)f(y) \\
&= 10^{-6} n(n-1)(n-2)\left(\frac{z}{100}\right)^{n-3}(100-y) \text{ for } z<y,\ n\geq 3.
\end{aligned}$$

In an English auction (EA), bidders should bid up to their true values regardless of risk preferences so that the EA revenue is

$$\begin{aligned}
R(n) &= \int_0^{100} y f_{(2)}^{n}(y)dy \\
&= 10^{-4}\int_0^{100} yn(n-1)\left(\frac{y}{100}\right)^{n-2}(100-y)dy.
\end{aligned}$$

We focus on the case with four bidders. Therefore

$$R(4)=10^{-4}\int_0^{100} 12y\left(\frac{y}{100}\right)^{2}(100-y)dy=60.$$

The EPA expected revenue is

$$\begin{aligned}
& R_m(n,\lambda) \\
= & \int_0^{100}\int_z^{100}\left(b_\lambda(y)-\frac{2}{m}(b_\lambda(y)-b_\lambda(z))\right) f_{(2),(3)}^{n}(y,z)dydz \\
= & \int_0^{100}\left(1-\frac{2}{m}\right) b_\lambda(y)(100-y)n(n-1)\left(\frac{1}{100}\right)^{n} y^{n-2}\,dy \\
& +\frac{2}{m}\int_0^{100} b_\lambda(z)10^{-6}n(n-1)(n-2)\left(\frac{z}{100}\right)^{n-3}\frac{1}{2}(z-100)^2\,dz.
\end{aligned}$$

For $m=2$ and $n=4$, the EPA revenue function reduces to the expected bid of the bidder with the third highest value,

$$R_2(4,\lambda)=12*10^{-8}\int_0^{100} b_\lambda(z)z\,(z-100)^2\,dz.$$

The Holt-Laury tests allows to observe λ-values that range roughly from -1 to 1.2. Figure 2 shows the expected revenues of the sellers in the EPA as a function of λ compared to those of the EA (which are constant of course).

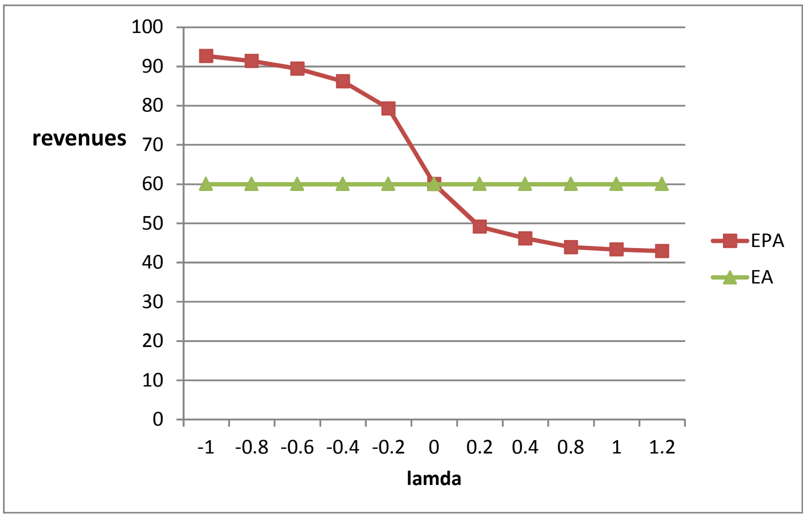

Figure 2: Seller's expected revenues in the symmetric equilibrium of the EPA and the EA for the range of λ-values observed in the experiment.

4 Experimental design

The experiment was conducted in two parts. Part I was an internet experiment designed to elicit subjects' risk preferences with a Holt-Laury (2002) questionnaire. Part II was the actual auction experiment conducted in the laboratory of the University of Heidelberg.

Several weeks before the auction experiment, subjects from the subject pool of the University of Heidelberg were recruited via the ORSEE online recruiting system (Greiner, 2004) to participate in an online questionnaire with monetary incentives. In total, 368 subjects participated. All participants were directed to Charlie Holt's Vecon website[3] and filled in the standard Holt-Laury questionnaire online (see Appendix for the questionnaire). The Holt-Laury questionnaire consists of 10 choices between two binary lotteries. By observing a subject's switching point from the less risky to the more risky lottery, an interval for the subject's degree of constant absolute risk aversion as measured by λ can be determined.

[3]See `http://people.virginia.edu/~cah2k/programs.html`

The riskier lottery yielded payoffs of either 4.85 or 1.1 euro, payoffs in the safer lottery were 3 or 2.6 euro. We added 1 euro to the standard payoffs to compensate subjects for the effort of picking up their payment in person.

We discarded 48 observations which were either incomplete, inconsistent (because subjects switched back and forth between columns or because they did not choose a strictly dominating safe option), or where subjects were trying to participate multiple times. This left us with 320 subjects.

A few weeks later we invited subjects to the lab grouped by their risk aversion, i.e. we formed sessions of risk loving and risk averse subjects.[4] Thus, the treatment variable was the risk attitude of subjects. In each auction, four bidders had the opportunity to bid for an object. The number of auction periods was 15 in all sessions and this was commonly known. Bidders interacted in fixed groups of four for these 15 periods. Bidders' valuations were chosen independently (across periods and subjects) from a uniform distribution on $[0, 1, 2, ..., 100]$. To improve the comparison across treatments, these value realizations were drawn ahead of time and the same set of realizations was used for both our treatments.

The auction experiment was computerized using the z–Tree software package provided by Fischbacher (2007). Each auction was conducted as follows. On a first computer screen, subjects were shown their bidder ID, which was randomly and independently determined anew each round (which made it impossible to identify other subjects in their group), and their value for the object. On the second screen (see Figure 3) there were three areas. On the left area, a rising red bar indicated the current price. The current price was also shown numerically on the right side in the center. The price increased with a speed of 2 points per second for prices below 34 and with a speed of 1 point per second for higher prices. On the upper right side of the screen, subjects were reminded of their ID and value. On the lower right side, there was the "exit" button which allowed subject to exit the auction. The exit prices of other bidders were indicated on the price bar by a black

[4] In particular, we classified subjects that chose the safer lottery 7, 8, or 9 times on the Holt-Laury questionnaire as risk averse and subjects that chose the safe lottery 0-3 times as risk loving.

line together with the ID of the bidder.

There was one interesting design issue that requires some elaboration and which may also be relevant for other experiments. After we had collected data from three groups for each treatment, we noted that subjects sometimes seemed to exit the auction as soon as they heard mouse clicks originating from other subjects. Although the information whether, and at what price, a bidder had left the auction was also provided visually, we were concerned that subjects would react to clicks by subjects in a different group. To address this concern, we changed the design in two ways. First, we adjusted the increments in which prices were increasing from 1 point to 0.1 points making ties much less likely (keeping the overall speed of the price movement the same). Second, we employed a new z-Tree feature that triggered the exit from the auction by moving the mouse pointer into the exit area (hovering) without clicking a mouse button. However, our results indicate that neither change appears to affect the exit behavior in a substantial way.[5]

After each auction period, bidders were informed about the selling price, whether they obtained the object, the value of the object to them, the amount of the premium if any, and their profit from this auction.

Instructions (see Appendix B) were written on paper and distributed in the beginning of each session. When subjects were familiar with the rules,

[5] To test whether these changes have an effect on how often subjects leave immediately after some other subject has left the auction, we count the number of auction periods in which at least two subjects leave within an interval of 0.5 seconds. When the price increases at 1 point per second, subjects have on average 0.5 seconds to leave the auction at the same price as a predecessor assuming that the predecessor's exact time of exit is randomly determined. Therefore, an interval of length 0.5 seconds allows us to best compare sessions in which the price increases at increments of 1 to sessions in which the increment is 0.1.

Out of 30 periods in which mouse clicks could be heard and where the increment was 1, there were 8 periods in which at least two subjects left the auction within an interval of 0.5 seconds (27%). There were 60 periods in which mouse clicks could also be heard but the increment was reduced to 0.1 and at least two subjects left within 0.5 seconds of each other in 25 of these periods (42%). Out of 60 periods in which the increment was also 0.1 and where it was not necessary to click to exit the auction, 25 (42%) satisfy the criterion that at least two subjects left the auction within 0.5 seconds. Therefore, it does not appear that reducing the increment leads to a reduction in simultaneous exits. Moreover, whether subjects hear others click or not does not appear to affect the frequency of simultaneous exits.

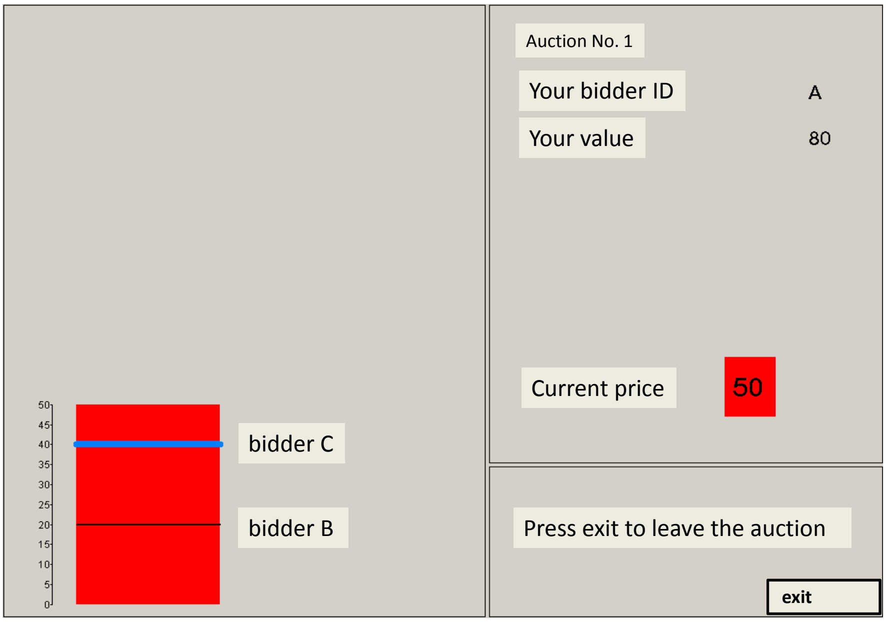

Figure 3: Screen shot of the bidding stage

they had to answer an extensive set of online test questions. We recorded the time a subject took to fill in the test questions. As soon as all subjects had answered all questions correctly, we started the first round.

For both treatments (i.e. risk attitudes) we had five independent groups of subjects, which yielded a total of 40 ($= 2 \times 5 \times 4$) subjects who participated in the auction experiment.[6] Subjects were paid the sum of their earnings from 5 randomly selected periods. Points were converted to euro at a rate of 3:1. Additionally, subjects started with an endowment of 5 euro. The average payoff was 13.5 euro including the show-up fee.[7] Experiments typically lasted about 45 - 60 minutes including instruction time.

[6] The number of observations was limited by the difficulty of recruiting a sufficient number of risk loving subjects to the lab.

[7] Except for the first two sessions, we also paid a show-up fee of 3 euro to all subjects in addition to their other earnings. In one sessions, we increased this show-up fee to 8 euro since we had to restart the software, which lead to some delay.

Table 1: **Mean Revenues**

	experimental	theoretical predictions	
treatment	EPA	EPA	EA
risk loving	60.70 (21.52)	84.15 (7.89)	58.03 (21.50)
risk averse	49.88 (21.54)	54.52 (26.72)	58.03 (21.50)

Note: Standard deviations based on 75 observations (5 times 15 periods) in parentheses. All values are calculated based on the same draws of valuations.

5 Experimental results

As a first step we look at the revenues obtained by the seller in the auctions. Table 1 lists the mean revenues for the experimental EPA separately for the treatment with risk loving subjects and for risk averse subjects. Average revenues with risk loving subjects (60.70) are much higher than those for risk averse subjects (49.88). This difference is significant based on a MWU-test ($p = 0.032$, two-sided) based on the average revenues of each group as one independent observation.

A similar picture emerges when we control for the value of bidders (recall that the same value realizations were used in both treatments). Table 2 displays an OLS regression (with clustered standard errors) of bids on values, a dummy for the risk loving treatment, and periods. In addition we included the time (in seconds) it took subjects to correctly answer the test questions given the possibility that subjects who took longer would bid differently from the rest.

As expected, value is highly significant. There also seems to be a significant positive time trend in bids although in absolute numbers it is fairly small (average bids increase by about 8 points over the experiment). The time it took a subject to answer the test questions does not have any influence on his bid. Most relevant for our question is the fact that the dummy for risk loving subjects is highly significant, which supports the evidence

Table 2: **How are bids determined?**

Dependent variable: exit prices (bids)	coefficient	std. error
value	0.728^{***}	0.05
dummy for risk loving	7.043^{***}	2.04
seconds to answers test questions	-0.004	0.01
period	0.583^{**}	0.23
constant	14.416^{***}	3.04
R^2	0.54	

Note: Standard errors are clustered by groups. The number of clusters is 10. The number of observations is 461, which results because we cannot observe exits prices for subjects that - as winners - did not exit. In some cases the last bidders exited simultaneously, in which case we can observe all exit prices. ***,** ,* significant at 1%, 5%, 10% level.

from the MWU-tests.

Result 1 *As predicted by theory, revenues in the EPA are significantly higher with risk loving bidders than with risk averse bidders.*

As theoretical benchmarks, Table 1 also lists the revenues as derived from the equilibrium bid function (5) above and based on the realizations of bidders' values that were actually used in the experiment. As one can see, the revenues in the experiment fall short of the theoretical prediction for both treatments but particularly for risk loving bidders. The differences for both treatments are significant according to a (paired) Wilcoxon test (both p-values are 0.043, two-sided).

Result 2 *The revenues from the EPA are substantially and significantly lower than predicted by the theoretical bidding function (5).*

Finally, Table 1 also lists the revenues that are predicted for a regular English auction based on the realizations of bidders' values that were actually used in the experiment assuming that bidders follow their weakly dominant strategy.[8] Comparing these to the experimental data from the EPA, we find that revenues are slightly but not significantly higher ($p = 0.50$, two-sided, Wilcoxon test) in the EPA for risk loving subjects and they are significantly lower ($p = 0.043$, two-sided, Wilcoxon) for risk averse subjects.

Result 3 *As predicted by theory, revenues of the EPA for risk averse bidders are significantly lower than (predicted) revenues of an EA. In contrast to the theoretical prediction, revenues of the EPA for risk loving bidders are not significantly higher than those predicted for an EA.*

We will now turn to individual bidding behavior to find out where and why the theoretical predictions fail. Figure 4 shows the exit prices of subjects for the different values. Also shown are the 45 degree line, which would correspond to subjects bidding exactly their value, and three different equilibrium bidding lines, which result from applying the equilibrium bid function (5) to the various λ-values corresponding to the choices from the Holt-Laury questionnaire.[9] In particular, the top panel of Figure 4 shows exit prices for risk averse bidders, i.e. subjects that chose the safe lottery 7, 8, or 9 times on the Holt-Laury questionnaire. The bottom panel of Figure 4 shows the same for risk loving subjects, i.e. subjects who chose the safe lottery 0-3 times.

In both treatments, there are some subjects who make the weakly dominated choice of exiting the auction at prices below their value. A large number of exit prices seems to be fairly close to the value. For risk averse subjects, this implies that they are also close to the equilibrium predictions.

[8] In experiments, bids and revenues for an EA are usually very close to the theoretical prediction, see e.g. Coppinger et al. (1980); Kagel et al. (1987); and Goeree and Offerman (2004).

[9] Recall that an agent's utility function is given by (1). We define a value for λ for each subject using the following procedure. We observe the subject's choices in the Holt-Laury questionnaire. These choices are typically consistent with an entire interval of values for λ. We simply use the midpoint of these intervals except when the according interval is not defined. That is the case when subjects are very risk loving (they always chose the risky lottery) or when they are very risk averse (they choose the safe lottery 9 times). In these two cases, we use the least risk loving respectively the least risk averse value for λ that is still consistent with the subject's choices.

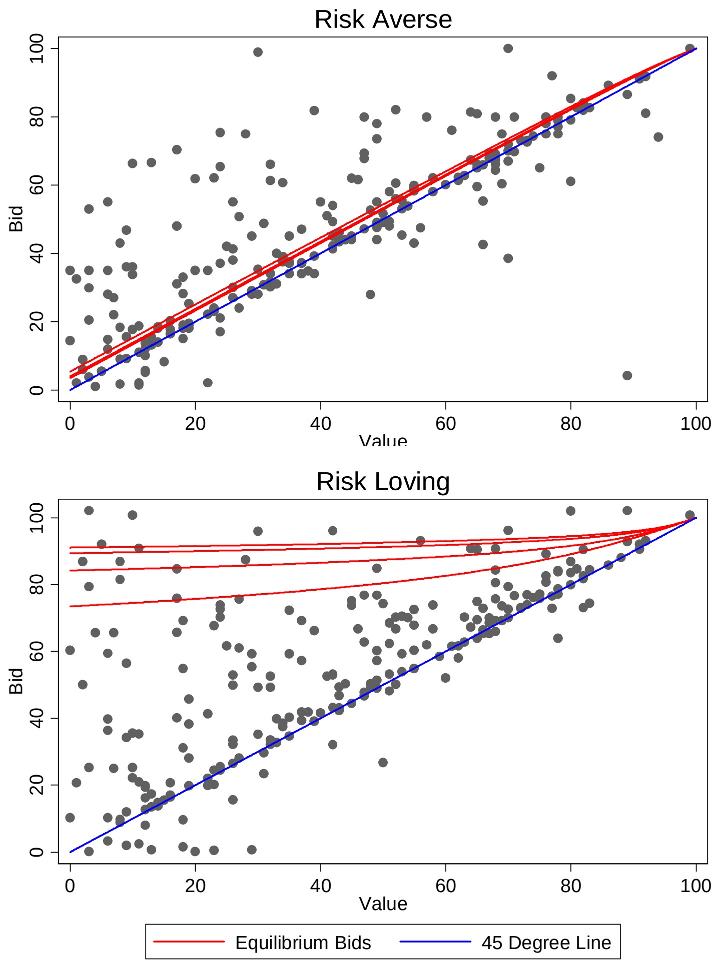

Figure 4: Scatter plot of bids (i.e. exit prices) of subjects depending on their values. The different equilibrium bidding functions are calculated for midpoints of the cutoff values for λ corresponding to choosing the safe lottery $7, 8$, or 9 times (top panel) and $0, 1$, or 2 (bottom panel).

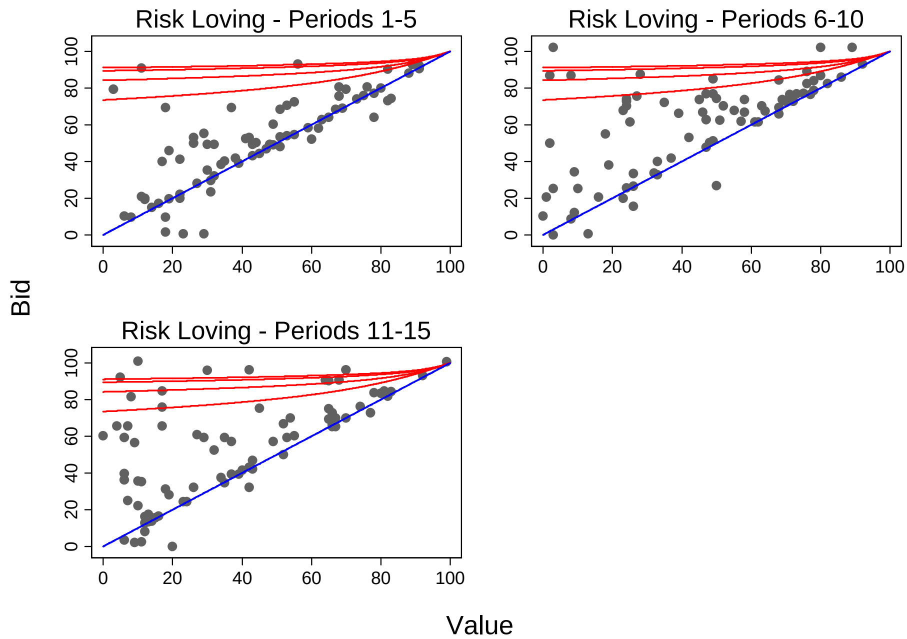

Figure 5: Scatter plot of exit prices of risk loving subjects, separately for the first 5 periods, the middle 5 periods, and the final 5 periods.

For risk loving bidders, only few exit prices are close to the equilibrium prediction, which holds in particular for low values. Figure 5 shows the same as the bottom panel of Figure 4 but split into the first 5 periods, the middle 5 periods, and the final 5 periods. Although there is a slight increase in the frequency of high bids when values are low, there does not seem to be a convergence to the equilibrium bid function.

If subjects do not bid according to the equilibrium bidding function, what do they do instead? Several alternative strategies come to mind. Subjects could simply bid up to their value like in an English auction (we call this strategy: at_value). A slightly more sophisticated strategy would be to stay in the auction until the price reaches the value plus the current premium, where the current premium is simply the difference between the current price and the bottom price (value_plus_premium). Although it is

Table 3: **Classifications of bidding behaviors**

Strategy	# of cases	average profit
all	600	4.34
eq_bid	212	12.42
at_value	301	10.20
below_value	347	9.16
value_plus_premium	266	10.24
after_second	244	2.67
after_first	170	1.27

unclear how such a strategy could be profitable, we also think it possible that subjects stay in the auction until the second bidder (or the first bidder) exited (after_second resp. after_first).

Table 3 classifies all bids according to whether they are within 3 points of the predicted exit price for the above strategies. For example, if value_plus_premium predicts that a bidder should exit at 90, then all bids that are between 87 and 93 are counted as success. The exception is the strategy below_value which counts all bids that are less than value plus 3 (and therefore subsumes all cases of at_value). While only 212 of 600 cases can be classified as being close to equilibrium bids, 301 respectively 347 cases are classified as at_value and below_value.

Table 3 also lists the average profit of all bids that are consistent with a particular strategy. While the after_second and after_first strategies clearly do much worse than the rest, the remaining strategies are fairly close together. While the equilibrium strategy is the best strategy empirically, the pressure to abandon a simple strategy like at_value is minor, which could explain why subjects did not converge to equilibrium behavior.

We can also ask how often the same subject can be classified into one of the categories of Table 3 out of all his 15 decisions. Figure 6 shows for each classification a frequency distribution of the number of subjects whose behaviors are compatible with this particular theory in t out of 15 rounds.

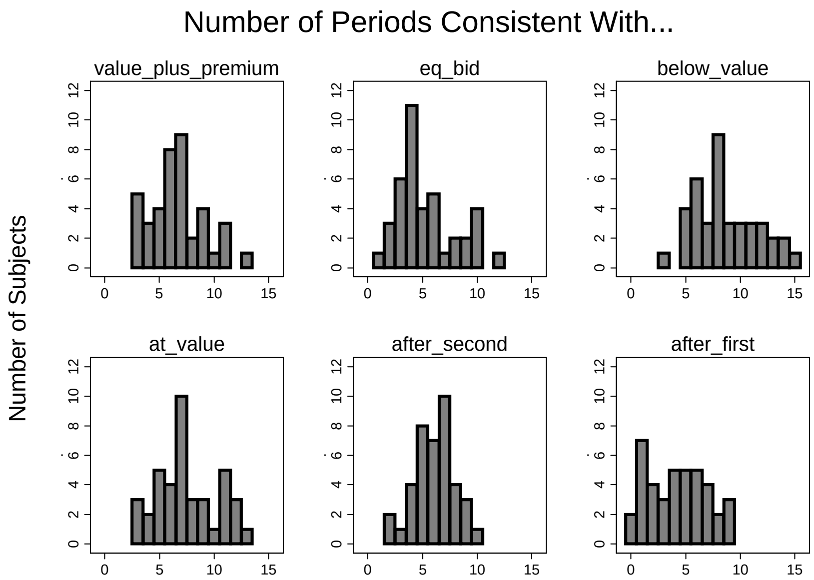

Figure 6: Frequency distribution of the number of periods for which the behavior of subjects is compatible with a particular classification.

For example, there are 9 subjects whose behavior is compatible with the strategy value_plus_premium in 7 of the 15 rounds.

To shed further light on whether bidders follow the equilibrium strategy, we test whether bids are independent of the bottom price, which they should be in the symmetric equilibrium with CARA bidders. For that purpose, we consider bids that are submitted after the bottom price has been established (i.e., after the third to last bidder dropped out). We run a regression (see Table 4) in which we explain how much subjects bid using two explanatory variables and an intercept: The first explanatory variable is simply the theoretical prediction given a subjects' private value and risk preference as revealed in the first stage of our experiment. The second explanatory variable is the bottom price. We find that the bottom price has a significant positive effect on bids, both when subjects are risk loving and when they are risk averse (clustering standard errors by group and running two separate

Table 4: **How are bids determined?**

Dependent variable: exit prices (bids)	risk averse	risk loving
equilibrium bid	.365***	.431*
	(.064)	(.173)
bottom price	.520***	.703***
	(.082)	(.092)
constant	14.88**	−10.37
	(3.32)	(17.44)
n	74	80
R^2	0.83	0.74

Note: Standard errors (in parentheses) are clustered by groups. The number of clusters is 5. We only use bids that were submitted after the bottom price had been established. The number of observations is not the same for the two treatments because subjects sometimes quit simultaneously. ***,** ,* significant at 1%, 5%, 10% level.

regressions for risk loving and risk averse subjects). One possible explanation for this observation is that a number of subjects follow the strategy we labeled after_second: They keep bidding until the third-to-last bidder left and then quit shortly afterwards.

Result 4 *Individual bidding behavior in the EPA is diverse. Many subjects seem to simply bid their value, possibly adjusted for the premium. Others seem to bid in accordance with the equilibrium bidding strategy. Some bidders seem to follow heuristics like leaving the auction as soon as one or two other bidders have left the auction.*

Given that most subjects' behavior is not consistent with any of the strategies discussed above for all 15 periods (see Figure 6), it would be natural to consider a dynamic model to explain the data. One possible approach is learning direction theory (Selten and Stöcker, 1986). The idea is simply that subjects reflect on whether their expected payoff would have been higher or lower had they submitted a higher or lower bid. If submitting a higher bid in the last period would have resulted in a higher payoff, they are likely to bid more aggressively in the following period and vice versa. In our experiment, it is often impossible for a bidder to establish the effect of a higher bid on his payoff in the previous round. However, there are situations where at least the sign if not the magnitude of the effect of a higher bid on a bidder's payoff can be established.

Consider a bidder who wins the auction and whose value is strictly lower than the sales price. Such a bidder can only gain by decreasing his bid by one increment. In the worst case, his profit remains unchanged but it might also increase by an increment.[10] On the other hand, bidding even more aggressively in this situation would not change such a bidder's profits for sure. Therefore, we would expect winning bidders whose values are strictly lower than the sales price to bid less aggressively in the subsequent period.

When a losing bidder's value is strictly higher than the sales price, submitting a slightly higher bid can only lead to a higher or equal profit while decreasing the bid by one increment would lead to lower or equal profits.[11] Therefore, we would expect such a bidder to bid more aggressively in the subsequent period.

When a winning bidder's value is higher than the sales price, the bidder cannot possibly obtain a higher profit by adjusting his bid. When a bidder's value is lower than the sales price and the bidder is not winning the auction,

[10] We can distinguish 2 possible consequences of decreasing one's bid by one increment in that situation: (i) The bidder still wins the auction at the same price. As a result, his profit remains unchanged. (ii) The bidder no longer wins the auction. In that case, his profit strictly increases: He saves one increment because he no longer has to purchase the good at a price that exceeds his value. On the other hand, he might lose up to half an increment because he might now receive a lower premium or no premium at all. The premium can at most decrease by half an increment because the sales price can at most decrease by one increment. If the bidder no longer receives the premium at all, the premium must have been either zero or one increment. Otherwise, it is not possible that a previously winning bidder loses the premium due to reducing his bid by only one increment.

[11] If such a bidder increases his bid by one increment, his profit increases if he receives a strictly positive premium (irrespective of whether he already received it before increasing his bid). It increases even more strongly if in addition, the bidder wins the auction. On the other hand, decreasing his bid by an increment can lead to a strictly lower profit if the bidder received a strictly positive premium and he either loses that premium due to submitting a lower bid or the premium decreases because his bid determined the sales price. If the bidder did not receive a strictly positive premium, decreasing his bid will lead to the same zero profit.

it is unclear whether a higher bid would have resulted in higher or lower profits: If the higher bid had been sufficient for the bidder to win the auction, profits would have been lower. If, on the other hand, increasing the bid had resulted in the bidder receiving the premium but not winning the auction, it would have been a good idea to bid more aggressively.

We therefore only obtain a clear prediction for the case when a bidder whose value is lower than the sales price wins the auction (bid less aggressively) and for the case when a bidder whose value is higher than the sales price does not win the auction (bid more aggressively). To test whether these predictions are borne out in the data, we need to define what we mean by "bidding more aggressively": We use the model described in Table 2 to generate predicted bids and then simply compare actual bids to these predictions. When the difference between the prediction and the actual bid increases in the next period, we say that the according subject is bidding more aggressively. Table 5 summarizes these results. When a winning bidders' value is lower than the sales price, the according bidder indeed bids less aggressively in the subsequent period in 6 out of 7 instances (Table 5, row 1). When a losing bidder's value is higher than the sales price, the bidder bids more aggressively as predicted by learning direction theory in 24 out of 43 cases (Table 5, row 4). Table 5 also indicates the average of the change of the difference between prediction and actual bids (Dresidual). These results suggest that some subjects might indeed try to adjust their bids in accordance with learning direction theory.

The fact that some subjects simply bid according to their value while others bid according to the equilibrium bidding function has some further consequences. In particular in the risk loving treatments, it happened that bidders with low values bid much higher than bidders with higher values. As a result, ex-post efficiency may suffer in the EPA. Table 6 lists three measures of efficiency for both treatments. (1) The share of auctions with a Pareto optimal outcome, i.e. in which the bidder with the highest value wins. (2) A measure for the absolute surplus created, the value of the winner divided by the highest value. And (3), a measure for the relative surplus created, the difference between the winner's value and the lowest value divided by

Table 5: **Learning Direction Theory**

	obs.	Dresidual	% more aggressive
Wins auction & value $<$ sales price	7	-6.48	14.29%
Wins auction & value $\geq$ sales price	3	-4.03	33.33%
Doesn't win auction & value $\leq$ sales price	368	-0.09	50.00%
Doesn't win auction & value $>$ sales price	43	3.67	55.81%

Note: To obtain the variable *Dresidual*, we first compute the difference between the predicted bid using the model described in Table 2 and actual bids (residuals). We then subtract the residual of the subsequent period from the residual of the current period and take the mean over all observations that satisfy the conditions given in column 1 to obtain *Dresidual*. If the residual increases, the bidder bids less aggressively and vice versa. The column *% more aggressive* indicates the percentage of observations where residuals decrease.

the difference of the highest and the lowest value. In an English auction, all three measures are usually well above 0.95, often above 0.99 (see e.g. Coppinger et al. 1980 or Goeree and Offerman, 2004). It should be noted, however, that the EPA may perform well with respect to ex-ante efficiency even though we observe relatively low levels of ex-post efficiency (Hu et al. 2012). With respect to ex-post efficiency, we can summarize:

Result 5 *Compared to the English auction, the EPA yields relatively low ex-post efficiency values in particular for the case of risk loving bidders.*

6 Conclusion

This paper reports the results of an experimental study on symmetric private-value premium auctions. As in most of the auctions literature, we limit our attention to the case in which the bidders have homogeneous risk preferences. Assuming constant absolute risk aversion, we derive the symmetric equilibrium bidding function for an English Premium Auction (EPA) and show that the seller's revenue is monotonic in bidders' degree of risk tolerance: the more risk tolerant the bidders are, the higher the seller's expected

Table 6: **Efficiency measures**

treatment	share of auctions won by highest value bidder	$\frac{v_{(winner)}}{v_{(1)}}$	$\frac{v_{(winner)} - v_{(4)}}{v_{(1)} - v_{(4)}}$
risk loving	0.71	0.89 (0.24)	0.85 (0.32)
risk averse	0.83	0.96 (0.14)	0.94 (0.18)

Note: Standard deviations based on 75 observations in parentheses. All values are calculated based on the same draws of valuations.

revenue.

In order to bring our experimental tests closer to the homogeneous assumption of the model, we first measure the subjects' risk preferences using the method developed by Holt and Laury (2002). We then separate the subjects according to their revealed risk preferences into risk averse and risk loving groups. A series of experiments are conducted for each group of subjects separately, and the results confirm that risk loving subjects bid more aggressively than risk averse subjects. Therefore, our experimental evidence provides support for the theory that the seller's revenue is higher when bidders are risk loving rather than risk averse.

Even though those subjects who follow the equilibrium strategy earn high payoffs in our experiment, most bids are not consistent with the equilibrium bidding function. We also observe no evidence for convergence towards the equilibrium. In particular, risk loving subjects bid less than their equilibrium bids. As a result, the seller's revenue in the EPA we conducted is lower than predicted.

When comparing the seller's revenue in the EPA to the revenue a seller would presumably have earned in a standard ascending English Auction (EA), we find that revenue is lower in the EPA than those predicteded for an EA when bidders are risk-averse, which corresponds to the theoretical prediction. However, contrary to theory, expected revenue in the EPA does not exceed revenue in an EA when bidders are risk loving. Moreover, efficiency in the EPA tends to be lower than in an EA.

Compared to the commonly made assumption that all bidders are risk neutral, our model is more general in that it allows for various degrees of risk aversion. Since we find that bidders with different risk preferences indeed bid differently in the EPA, this generalization can lead to an improvement of the predictive power of the model. However, we still assume that all bidders exhibit the same degree of risk aversion. An even more general model would allow different bidders to have different degrees of risk aversion.[12] Such a heterogeneous model would significantly extend the scope of possible applications but we have to leave this for future work.

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Appendix A: Proofs

Proof of Proposition 1. Differentiating $U(v,z)$ in (2) with respect to z and setting the first order condition to 0 at $z=v$ yield

$$\begin{aligned} 0 &= \left[u\left(v - b^{II}(v,p_3) + \frac{1}{m}(b^{II}(v,p_3) - p_3)\right) - u\left(\frac{1}{m}(b^{II}(z,p_3) - p_3)\right)\right] f(v) \\ &\quad + (1-F(v))\, u'\left(\frac{1}{m}(b^{II}(v,p_3) - p_3)\right) \frac{1}{m} \frac{\partial b^{II}(v,p_3)}{\partial v}. \end{aligned}$$

Substituting the functional form (1) of u, the above equation simplifies to

$$\frac{\partial b^{II}}{\partial v} = m\frac{\exp(\lambda(b^{II}-v))-1}{\lambda}\frac{f(v)}{1-F(v)}. \tag{6}$$

Since p_3 does not appear explicitly in this differential equation, $b^{II}(v,p_3)$ is independent of p_3 and it therefore can be written as $b(v)$. Rearranging the terms in (6), and multiplying both sides by $\exp(-\lambda b(v))$, we obtain

$$\left[\frac{\lambda\,(1-F(v))}{m}b'(v)+f(v)\right]\exp(-\lambda b(v)) = f(v)\exp(-\lambda v). \tag{7}$$

Notice that

$$\begin{aligned}
&\frac{\partial}{\partial v}\left(\frac{1}{m}(1-F(v))^m\exp(-\lambda b(v))\right)\\
= & -\left[(1-F(v))^{m-1}f(v)+\frac{1}{m}(1-F(v))^m\lambda b'(v)\right]\exp(-\lambda b(v)).
\end{aligned}$$

Hence, (multiplying both sides of (7) by $(1-F(v))^{m-1}$), (6) is equivalent to

$$-\frac{\partial}{\partial v}\left(\frac{1}{m}(1-F(v))^m\exp(-\lambda b(v))\right) = (1-F(v))^{m-1}f(v)\exp(-\lambda v).$$

Let v be replaced by x. Then, integrating the above expression over $[v,H]$ gives

$$\begin{aligned}
\frac{1}{m}(1-F(v))^m\exp(-\lambda b(v)) &= \int_v^H (1-F(x))^{m-1}\exp(-\lambda x)dF(x)\\
\text{or equivalently, }\exp(-\lambda b) &= -\int_v^H \exp(-\lambda x)d\left(\frac{1-F(x)}{1-F(v)}\right)^m. \quad (8)
\end{aligned}$$

It is now easily checked that equation (8) is equivalent to (3), with $b(v)\equiv b^{II}(v,p_3)$ that is independent of p_3.

Finally, since b^{II} does not depend on p_3, we have $b^I(v)=b(v)$ as well.□

Proof of Proposition 2. (i) Let $\lambda_1<\lambda_2$. For $v=H$, we have $b_{\lambda_1}(H|m) = b_{\lambda_2}(H|m) = H$. To verify this, by L'Hospital's rule, taking limit of $b_\lambda(v|m)$ in (3) as $v\uparrow H$ yields

$$\begin{aligned}
\lim_{v\uparrow H} b_\lambda(v|m) &= -\frac{1}{\lambda}\ln\left(\lim_{v\uparrow H}\frac{\left(m\int_v^H e^{-\lambda x}(1-F(x))^{m-1}dF(x)\right)}{(1-F(v))^m}\right)\\
&= -\frac{1}{\lambda}\ln\left[\lim_{v\uparrow H}e^{-\lambda v}\right] = H.
\end{aligned}$$

Let $\Delta b(v) := b_{\lambda_1}(v|m) - b_{\lambda_2}(v|m)$ and suppose hypothetically that there exists a $v \in [L, H)$ such that $\Delta b(v) \leq 0$. From (6) we have

$$\begin{aligned}
\frac{\partial \Delta b}{\partial v} &= m\frac{f(v)}{1-F(v)}\left(\frac{\exp(\lambda_1(b_{\lambda_1}-v))-1}{\lambda_1} - \frac{\exp(\lambda_2(b_{\lambda_2}-v))-1}{\lambda_2}\right) \\
&\leq m\frac{f(v)}{1-F(v)}\left(\frac{\exp(\lambda_1(b_{\lambda_2}-v))-1}{\lambda_1} - \frac{\exp(\lambda_2(b_{\lambda_2}-v))-1}{\lambda_2}\right) \\
&< m\frac{f(v)}{1-F(v)}\left(\frac{\exp(\lambda_2(b_{\lambda_2}-v))-1}{\lambda_2} - \frac{\exp(\lambda_2(b_{\lambda_2}-v))-1}{\lambda_2}\right) \qquad (9) \\
&= 0.
\end{aligned}$$

where the weak inequality follows from the assumption $\Delta(b) \leq 0$ and the strict inequality follows from the general property that

$$\frac{\partial}{\partial x}\frac{\exp(x)-1}{x} = \frac{e^x}{x^2}\left(e^{-x} - (1-x)\right) > 0, \quad \forall x \neq 0.$$

Since $\Delta b(H) = 0$ and Δb is differentiable, the inequality in (9) contradicts $\Delta b(v) \leq 0$. So we must have $\Delta b(v) > 0$ or that $b_{\lambda_1}(v|m) > b_{\lambda_2}(v|m)$ for all $v < H$.

(ii) Let $m_1 < m_2$. The cumulative distribution function $M(x|m)$ defined in (3) is an increasing function of m. Therefore, $M(x|m_1)$ dominates $M(x|m_2)$ in the sense of first-order stochastic dominance. If $\lambda > 0$, the integrand $e^{-\lambda x}$ is a decreasing function of x and therefore the integral in (3) increases in m. Because $-\frac{1}{\lambda} < 0$, this implies $b_\lambda(v|m_1) > b_\lambda(v|m_2)$. If $\lambda < 0$, the integral in (3) decreases in m. But then by $-\frac{1}{\lambda} > 0$ again we have $b_\lambda(v|m_1) > b_\lambda(v|m_2)$. For the case with $\lambda = 0$, the property can be verified directly from (4).

Appendix B: Instructions

These are the translations of the original instructions, which were in German. The originals are available upon request from the authors.

Introduction

Welcome to the AWI Lab. In today's experiment, you will participate in 15 auctions. The experiment will take about one hour, and at the end of the experiment, you will be paid in cash. The payment you receive for the experiment depends on your own decisions, the decisions of the other participants, and on chance.

You can make all your choices at your computer. Please do not talk to other participants. If you have any questions, please raise your hand, and someone will come over.

Before we start the experiment, you are required to answer a few test questions correctly to make sure that you have understood the instructions. Now please read the instructions carefully. You may use paper and pencil and take notes any time.

Instructions

Groups

Before the experiment starts, all participants are divided into groups of four. These groups remain unchanged throughout the experiment. You will interact only with the members of your group, but not with the members of other groups.

What is the value of a good for an individual bidder?

The experiment consists of 15 auctions. In each group at each auction a good is sold to one of the 4 bidders. Prior to each one of the 15 auctions, a value for the good is determined randomly for each bidder. This value is an integer between 0 and 100. The value is randomly generated by the computer, and each integer is selected with the same probability.

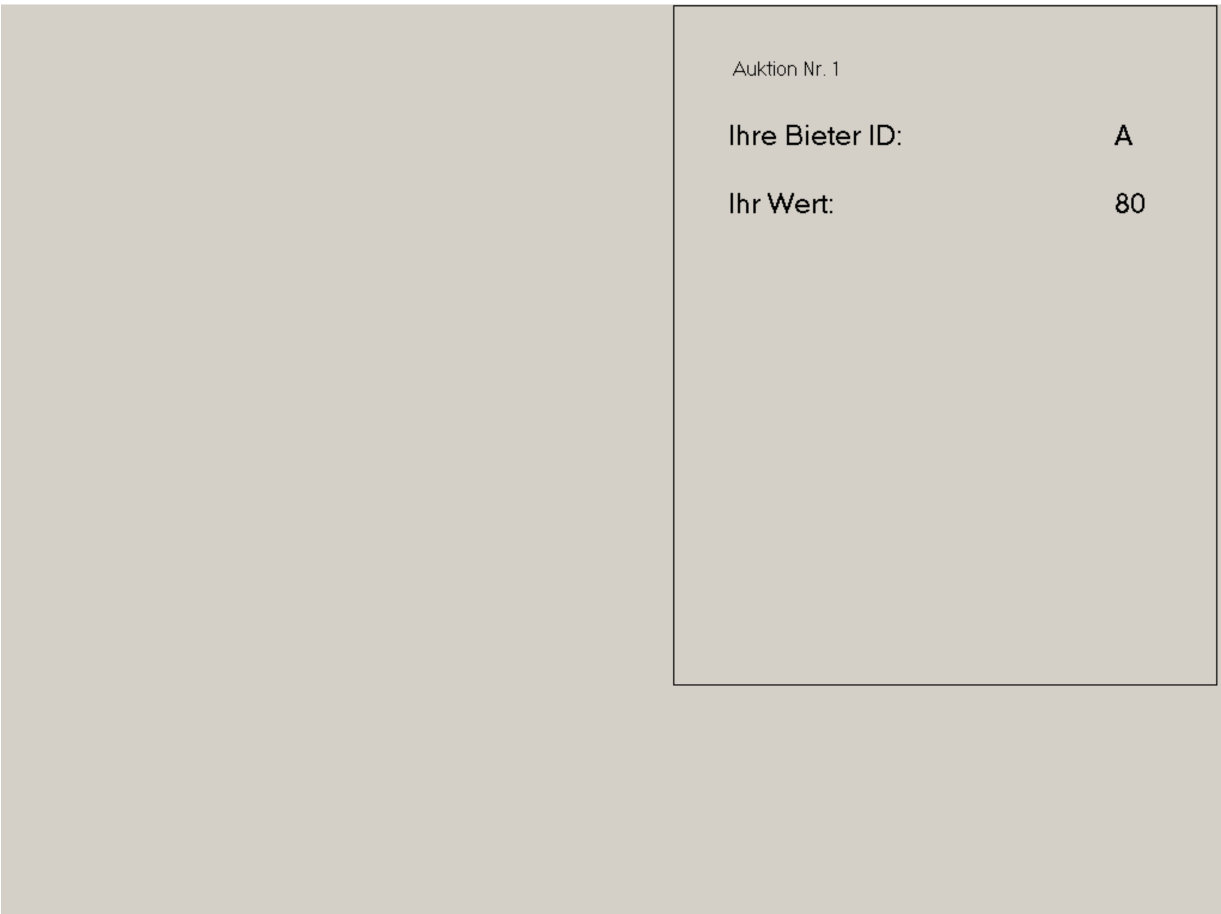
The value is determined independently for each bidder. The values of other bidders have no effect on the value determined for you. The value for each of the 15 auctions is also determined independently. The values in other auctions have no effect on the values determined in the current auction. Therefore your values for the good are most likely different from the values of other bidders in your group. Additionally, you most likely do not have the same value for all 15 auctions.

Each bidder knows his/her own value, but not the values of other bidders.

Screen 1: Value and Bidder ID

Prior to each of the 15 auctions, you will be shown your value for 5 seconds. You will see the screen shown in Figure 1. This screen also shows your Bidder ID. This ID serves as an identification of the different bidders within a group. Your Bidder ID is randomly determined for each of the 15 auctions. Therefore you and any other participant are unlikely to be assigned the same Bidder ID each time.

Figure 1: Screen 1:



{Translation: Auction No. 1

Your bidder ID: A

Your value: 80}

Auction procedure

At the start of each auction, the price of a good is 0. Every 0.5 seconds, the price is raised by one point. Once the price has reached 34 points, the price will be raised only every second by one point.

You can decide to drop out of the auction any time by moving the cursor onto the blue rectangle at the right bottom on your screen (see Figure 2). Clicking the mouse is not required. Once you have moved the cursor onto the blue rectangle, you drop out of the auction.

Once 3 or 4 bidders of your group have dropped out, the auction is finished. The bidder who did not drop out purchases the good at the current price. If several bidders drop out simultaneously as last bidders in an auction, the buyer will be determined randomly: all of the bidders dropped out last have the same chance to buy the good at the current price.

Please look at Figure 2. On the left part of your screen, the red bar shows the current price. You can also see when another bidder of your group drops out of the auction. In Figure 2, for example, bidder B dropped out at price 20. The right part of the screen shows your Bidder ID, your value, and the current price (in a red field).

Figure 2: Screen 2

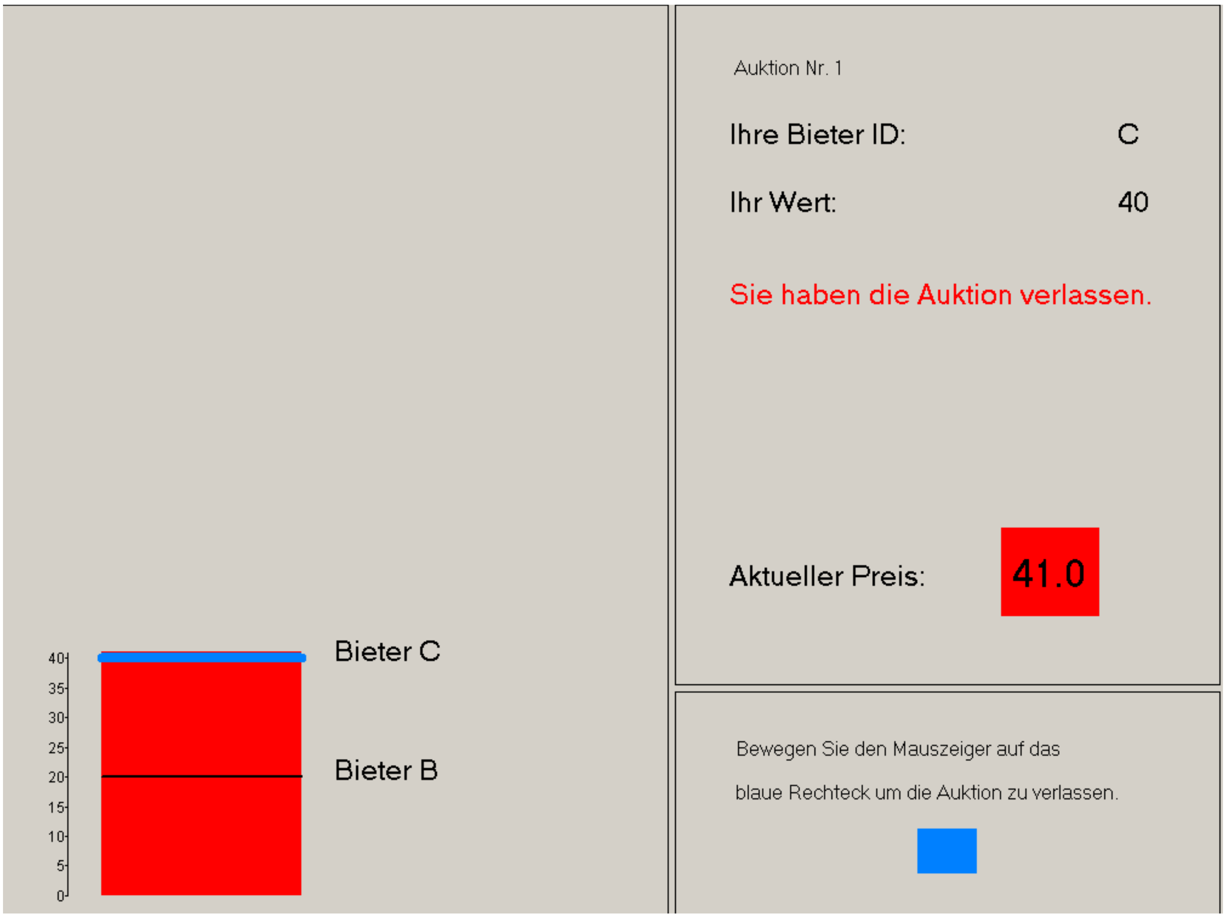
{Translation: Auction No. 1, Your Bidder ID: A, Your Value: 80, Current price: 30.0
Bidder B, “Please move the mouse pointer to the blue rectangle to leave the auction”}

End of Auction

When you move the cursor onto the blue rectangle, you will be notified that you have dropped out of the auction. Figure 3, for example, shows that bidder C dropped out at price 40. When 3 or more bidders of your group have dropped out, the auction is finished.

When the current price has reached 150 points, the auction is finished, even if fewer than 3 bidders have dropped out of the auction. The good will be sold at a price of 150 points to one of the remaining bidders, with the same probability for each bidder.

Figure 3: Screen 3



{**Translation**: Auction No. 1, Your Bidder ID: C, Your Value: 40, "You have left the auction." Current Price: 41.0. Bidder B, Bidder C, "Please move the mouse pointer to the blue rectangle to leave the auction"}

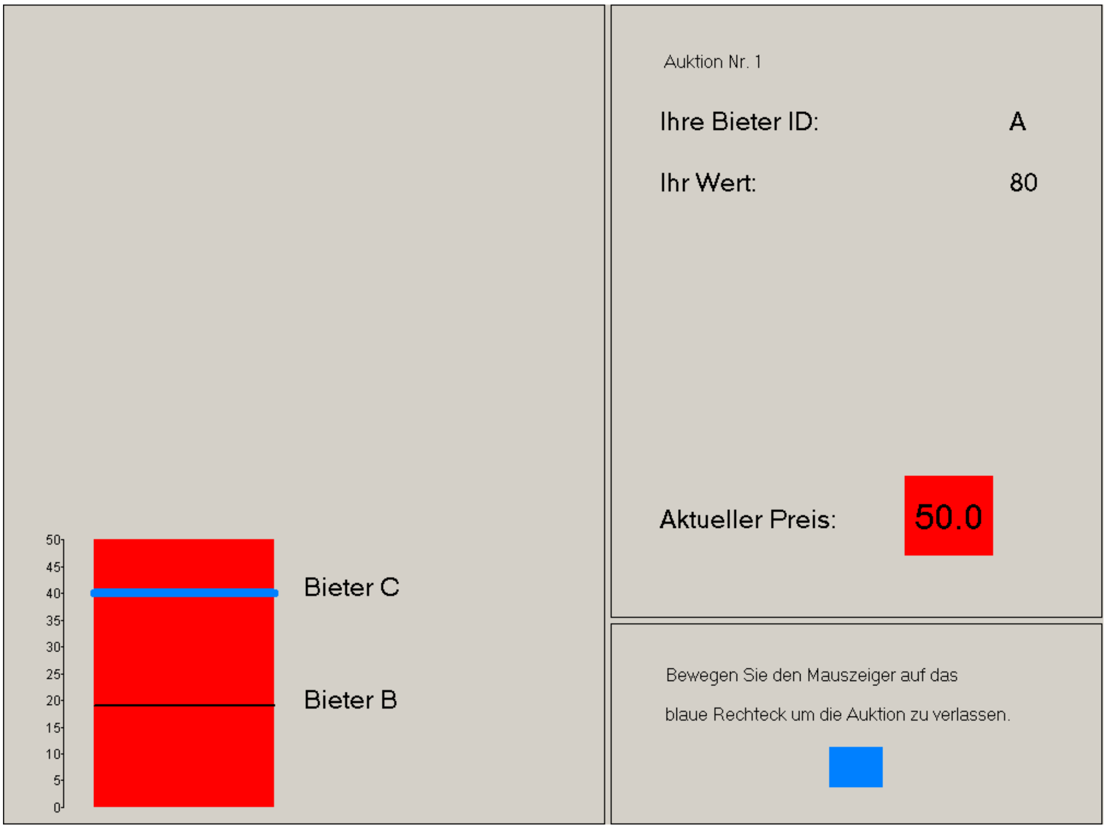
Calculating your profit

The bidder who buys the good receives a profit equal to the difference between the value of the good and the sales price. Additionally the buyer and the bidder who dropped out third (that is, the next to last remaining bidder) receive a bonus. This means you can make a profit in this auction even if you do not buy the good.

For calculating this bonus, two prices are relevant: the sales prices and the so-called basis price. The basis price is the price at which the second bidder drops out. Assume, for example, that bidder B drops out at price 20. Then bidder C drops out at price 40. So in this example the basis price is 40. The basis price is indicated by a thick blue line (see Figure 4).

The bonus equals half the difference between sales price and basis price, and is therefore always positive. Example: Bidder B drops out at price 20. Bidder C drops out at price 40. The basis price is therefore 40. Bidder D drops out at price 50. So the sales price is 50. In this case the bonus is 5: (sales price 50 – basis price 40)/2. Bidders B and C receive a profit of 0. Bidder D dropped out third and receives a profit of 5 (the bonus). Bidder A purchases the good. His/her profit equals the difference between the sales price of 50 and the value of the good plus the bonus of 5.

Figure 4: Screen 4



Your profit for each auction is calculated as follows:

- If you drop out first or second: 0
- If you drop out third: bonus = (sales price – basis price)/2
- If you purchase the good: your value – sales price + bonus

We consider another example: Assume that bidder A drops out at price 20. Bidders B and C drop out simultaneously at price 30. Since only one bidder stays, the auction ends, and bidder D buys the good at price 30. As the second bidder dropped out at price 30, the basis price is 30. So in this case the bonus is 0. Bidders A, B, and C gain a profit of 0, and bidder D gains a profit equaling the difference between his value of the good and the sales price 30.

We consider a final example to illustrate the calculation of profits. Assume that the values of bidders A – D are allocated in the following manner:

- Bidder A: 10
- Bidder B: 30
- Bidder C: 50
- Bidder D: 70

We assume that bidders A - D will drop out at the following prices:

- Bidder A: 15
- Bidder B: 50
- Bidder C: 80
- Bidder D: 80

The good is purchased at sales price 80 by bidder 3 or bidder 4, with equal probabilities. Since the second bidder dropped out at price 50, the basis price is 50. Therefore the bonus is 15 (sales price 80 – basis price 50)/2. The two last remaining bidders, that is, bidders C and D, receive a bonus of 15 each.

Now we look at the profit of bidder C. If bidder C buys the good, he/she pays the sales price of 80 for the good. He/she gets a value of 50 plus the bonus of 15, thus suffering a loss of 15 (value 50 – sales price 80 + bonus 15 = profit -15). As bidders C and D dropped out at price 80, each of them buys the good with a probability of 0.5. If bidder D buys the good, the profit of bidder C is 15: He/she only gets the bonus.

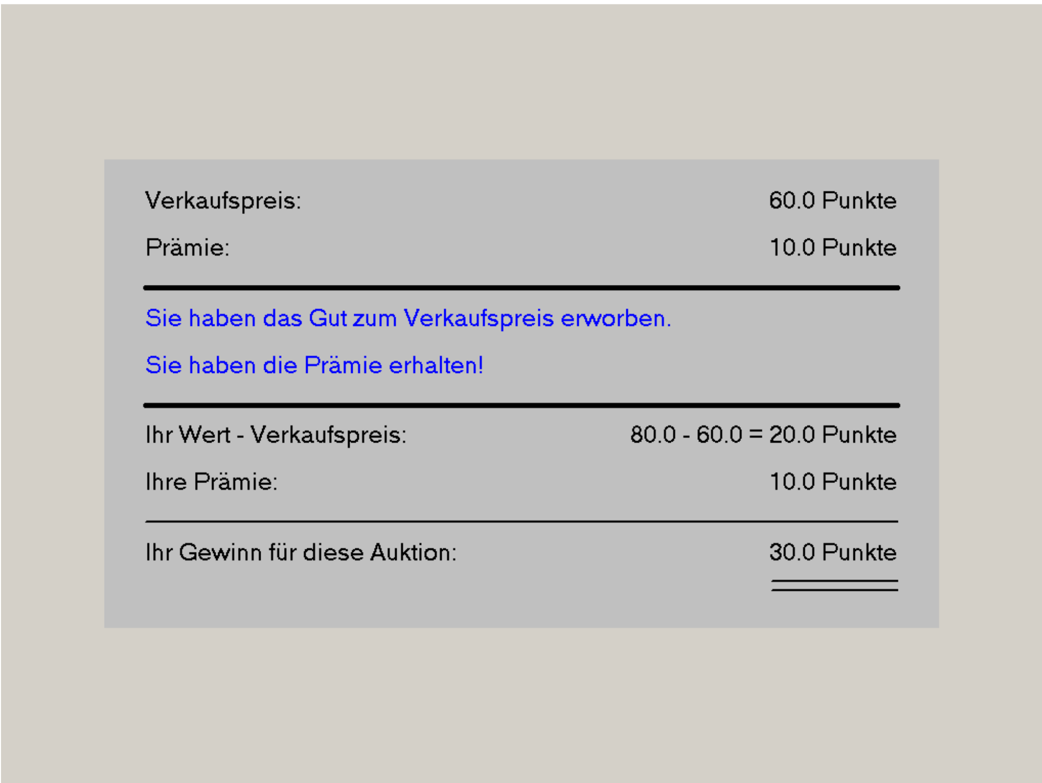
Display of Profits

At the end of the auction, the results of the auction are displayed for 15 seconds (see Figure 5). At the top you can see the sales price and the bonus. Next, in the middle of the screen, you learn whether you have purchased the good and whether you received the bonus. If you have purchased the good or received the bonus, this information is printed blue. It is printed *black* if you have *not* purchased the good or *not* received the bonus.

In the bottom part of the screen your profits are presented in detail. If you have purchased the good, the difference between your value and the sales price is displayed. If you did not purchase the good, the entry is left blank. Next, your bonus is displayed. If you have not received a bonus, the value is 0. Your profit of the auction equals the sum of the two positions (value – sales price + your bonus).

Now look at the example in Figure 5. In this auction the sales price is 60, and the bonus is 10. The bidder has bought the good, and he/she gets the bonus. So his/her profit is 30.

Figure5: Screen 5



{Translation:

Sales Price:	60.0 points
Bonus:	10.0 points

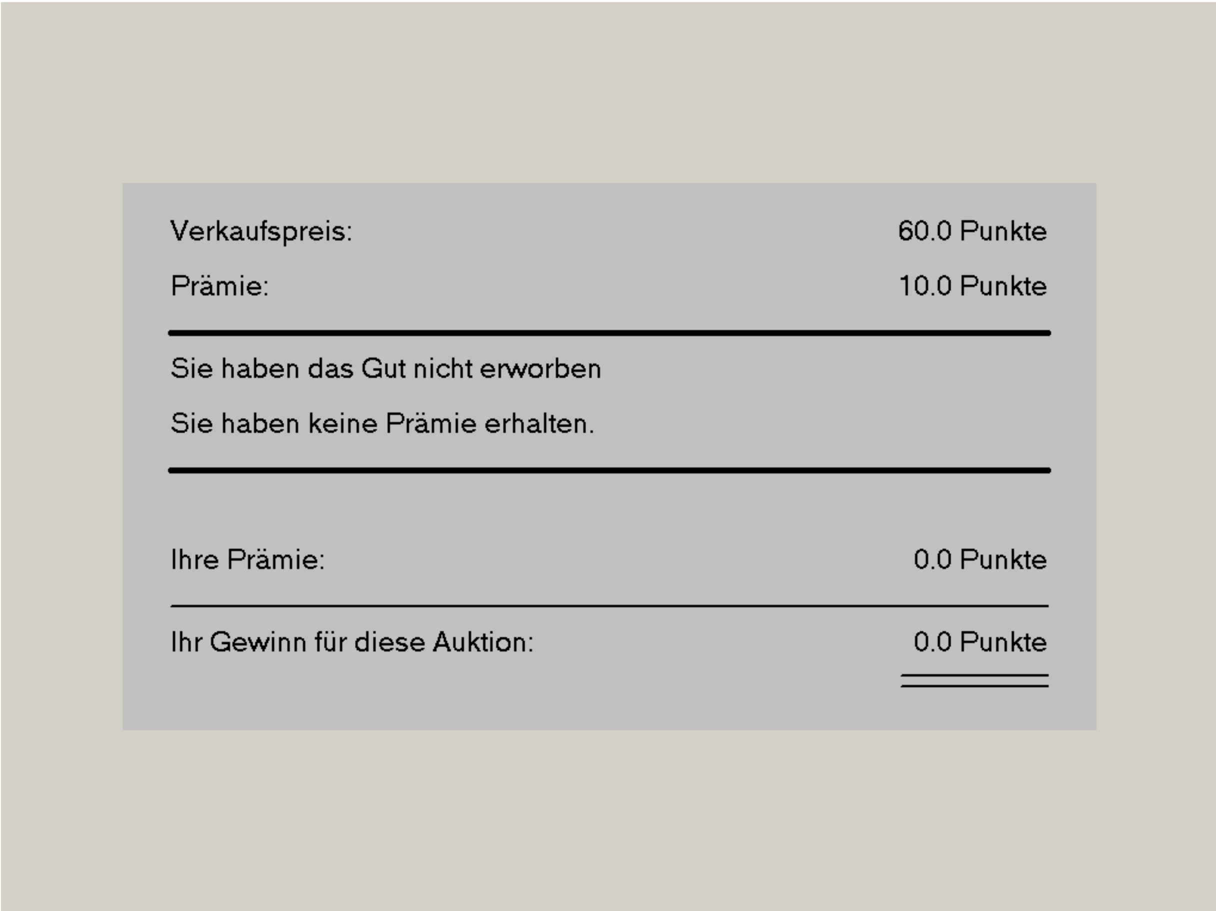
You have purchased the good at the sales price.
You have received the bonus!

Your Value – Sales Price: 80.0 – 60.0 = 20.0 points
Your Bonus 10.0 points

Your profit in this auction: 30.0 points}

In the example in Figure 6 the bidder has not purchased the good, nor received a bonus. So his/her profit is 0.

Figure 6: Screen 6



{Translation:

Sales Price: 60.0 points

Bonus: 10.0 points

You have not purchased the good.
You have not received the bonus.
Your Bonus 0.0 points

Your profit in this auction: 0.0 points}

Payment

At the end of the experiment 5 out of 15 auctions will be randomly selected. This is done by one of the participants randomly drawing without replacement 5 out of 15 balls, each marked with a number. Only the profits you gained in the 5 selected auctions will be paid. For 3 points you receive a payment of 1 euro. Additionally, you receive 5 euros. Your payment is calculated as follows:

Payment = Your profits gained in the 5 selected auctions / 3

+ 5 euros

You will receive your payment in cash at the end of the experiment. Each participant will only learn his/her own payment.

Test questions

Now you should be able to answer the test questions. Please raise your hand if you need help.

Summary

- The experiment consists of 15 auctions
- In each auction one good is sold to 1 of 4 bidders of your group
- You stay in the same group throughout the 15 auctions
- For each bidder and each of the 15 auctions, a value between 0 and 100 is selected randomly
- The price of a good is 0 at the beginning of the auction, and it is raised at regular intervals
- You can decide to drop out anytime by moving the cursor onto the blue rectangle
- When 3 or more of the 4 bidders have dropped out of your group, the auction is finished
- The bidder who did not drop out buys the good at the current price
- The basis price is the price at which the second bidder drops out
- Your profit in an auction is:
    - If you drop out first or second: 0
    - If you drop out third: bonus =(sales price – basis price)/2
    - If you buy the good: Your personal value – sales price + bonus
- At the end of the experiment, 5 auctions will be randomly selected
- Your payment is:

  Your profits gained in the 5 selected auctions / 3

  + 5 euros