Decision Rules, Transparency and Central Banks

Inaugural-Dissertation
zur Erlangung der Doktorwürde
der Wirtschaftswissenschaften
der Fakultät für
Wirtschafts- and Sozialwissenschaften
der Ruprecht-Karls-Universität Heidelberg

vorgelegt von
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aus Tübingen

Heidelberg, Mai 2010

Danksagung

Diese Arbeit entstand zum größten Teil während meiner Zeit als wissenschaftlicher Assistent am Alfred-Weber-Institut der Universität Heidelberg.

Mein besonderer Dank gilt Prof. Dr. Hans Gersbach für die allzeit hervorragende Betreuung, die über seine Tätigkeit an der Universität Heidelberg hinaus reichte. So hat er mich einige Male an der ETH Zürich an seinem neuen Lehrstuhl als Gast aufgenommen. Mit seiner Unterstützung nicht nur in wissenschaftlicher Hinsicht hat er wesentlich zum letztendlichen Gelingen dieser Arbeit beigetragen. Weiterhin möchte ich mich bei Prof. Dr. Jürgen Eichberger für fruchtbare Diskussionen über spieltheoretische Themen und die Übernahme des Koreferats, bei Prof. Dr. Eva Terberger für die motivierende Unterstützung, bei Prof. Dr. Hans Haller für das Infragestellen einer Beweisidee, bis die Argumentation logisch schlüssig ist, und bei Prof. Dr. Bert Rürup für seine Ungeduld bedanken. In der Summe hat dies zusammen nicht unwesentlich dazu beigetragen, dass diese Arbeit noch einen Abschluss gefunden hat.

Weiterhin bin ich meinen Kollegen am Lehrstuhl Wirtschaftspolitik I in Heidelberg für die angenehme Atmosphäre und den regen wissenschaftlichen Austausch zu Dank verpflichtet: Dr. Volker Hahn, Dr. Markus Müller, Dr. Felix Mühe, Dr. Lars Siemers, Dr. Verena Liessem und nicht zuletzt Dr. Hans-Jörg Beilharz, mit dem ich einige Jahre das Zimmer geteilt habe und oft weitergehende Fragen über die Relevanz ökonomischer Modelle erörtern konnte.

Prof. Dr. Andreas Irmen und Prof. Dr. Timo Göschl möchte ich für die zwischenzeitliche Aufnahme an ihren Lehrstühlen danken.

Nach dem Verlassen der Universität resultierte ein weiterer anhaltender Ansporn, diese Arbeit noch fertigzustellen, aus den teils bohrenden Nachfragen meiner neuen Kollegen beim Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Entwicklung: Dr. Anna Rosinus, Dr. Wolfgang Kornprobst, Dr. Christoph Swonke und Dr. Michael Tröger.

Nicht zu vergessen sind Margot Stumm-Kadau, Gabi Rauscher und Margrit Buser, die mich bei vielen organisatorischen Dingen unterstützt haben.

Barbara Köster möchte ich meinen Dank dafür aussprechen, dass sie mich, nachdem sie mich nach meiner eigenen Aussage in der letzten Phase der Dissertation kennengelernt hat, doch nicht erst wiedersehen wollte, nachdem diese Arbeit fertiggestellt worden ist. Glücklicherweise hat sie sich zwischenzeitlich dazu entschieden, mich zu heiraten.

Außerdem war sie mir in der wirklichen Endphase beim Abzählen von Mengen von Mengen von Mengen behilflich.

Natürlich darf an dieser Stelle der Dank an meine Eltern und meinen Bruder nicht fehlen, die nicht aufgehört haben, immer wieder unangenehme Fragen nach dem Beendigen der Arbeit zu stellen.

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Chapter 1

Introduction

1.1 Overview

The trade-off between price stability and output stabilization is in the centre of monetary policy-making. This trade-off enters many macroeconomic models as the central bank is assumed to minimize some loss function consisting of inflation deviations and output deviations from some specific targets (see for example Barro and Gordon (1983), Woodford (1999)). The policy instrument to control these variables is the short-term interest rate.

Monetary policy-making is usually conducted in committees, whose members may have conflicting interests. This is evident for the Governing Council of the European Central Bank or the Board of Governors of the Federal Reserve System in the United States (Heinemann and Hüfner (2004) and Meade and Sheets (2005)). In this thesis we take a closer look at monetary policy committees. In particular, we address how decision rules and transparency requirements concerning such rules in monetary policy committees should be designed. In particular we concern ourself with the following two issues:

1. Which type of majority rule should be applied in the monetary policy committee?
2. Should the public know which decision rule the monetary policy committee applies and should the central bankers release their information about economic shocks?

To address these questions, standard monetary models with aggregate demand and supply shocks are introduced and we assume that a committee decides about the interest-rate change according to some voting rule. We develop a flexible majority rule, where the majority for interest-rate changes depends itself on the size of the interest-rate change.

Our main findings are: First, a well-designed flexible majority rule can improve welfare compared to a fixed majority rule in a simple shock structure. This insight is robust, if we apply more complex shock structures or if we introduce a simple dynamic setup. Second, transparency regarding the rule has ambiguous effects on welfare and it may not be necessary to publish the decision rule, but within our framework, we can provide a best combination of a decision rule and an information setup.

1.2 The Structure of the Thesis

The thesis is organized as follows: In chapter 2 we introduce the model and proof the first main insight in a simple setting, called the baseline model. In chapter 3 we relax the assumptions and show that our main insight is robust under many circumstances. In chapter 4 we slightly change our baseline model in order to examine the effects of transparency within our framework and chapter 5 discusses the results and concludes.

Flexible Majority Rules for Central Banks (Chapter 2)

In chapter 2, we introduce our main model, which is based on Gersbach and Pachl (2006) and on Gersbach and Pachl (2009), a shorter version of the paper. We consider a monetary policy committee, which decides about interest-rate changes in a monetary union. Aggregated social losses of the monetary union are based on a utilitarian welfare criterion and consist of the weighted sum of the loss functions of the member countries. The loss functions of the member countries are quadratic in the difference between the actual union-wide interest rate and the desired country interest rate. We consider the possibility that the monetary union is hit by a shock, which divides the union into two parts. After this, one part desires an interest-rate change, while the other wants to retain the status quo. Our main assumption, which drives the model, is the monotonic dependence of the shock size on the size of the affected region: The larger the affected part of the monetary union, the larger the shock is. Furthermore, we assume that the central bankers decide according to their home preferences. Within this framework, we compare a simple majority rule (SM), where any interest-rate change requires more than 50% of the votes and a flexible majority rule (FM). A FM-rule is characterized by the property that the larger the desired interest-rate change, the larger the required number of votes. Our main findings are, that the FM-rule is superior to the SM-rule and if the votes of the members of the committee are weighted properly, the FM-rule can even mimic the first-best solution for any aggregated shock scenario.

Extensions of the Model (Chapter 3)

In chapter 3, we relax some assumptions of the baseline model of the previous chapter. First, we drop the assumption of quadratic losses and allow for more general functional forms and show that, with some regularity condition, our main result still holds. Second, we assume that the union is hit by a weighted averaged shock, which changes aggregated social losses. We compare this with our utilitarian welfare criterion and show, that we can construct a similar FM-rule. Third, we allow for different shocks in the same direction at the same time, which includes the scenario, that we may have a large effect in one country and a small effect in another country, while another part is not affected at all. In this setting, the result for any shock scenario the FM-rule is at least as good as the SM-rule does not hold anymore. However, we can show that if a shock event divides the monetary union into three regions, the size of the desired interest-rate change linearily depends on the size of the affected region and all events occur with the same probability, the FM-rule is still superior compared to the SM-rule with regard to expected social losses of the monetary union. Fourth, we consider simultaneous shocks in different directions and can show that a well-designed FM-rule is still superior to the SM-rule. Fifth, we examine a dynamic setup with the assumptions that there exist a long-run equilibrium interest rate and fast decaying shocks. In this setup, we can also show that the FM-rule is better than the SM-rule.

Transparency (Chapter 4)

In chapter 4 we derive a loss function dependent on the actual interest-rate change, the expected interest-rate change, and the shock size, incorporating the framework of Gersbach (2003) into our model. We assume that the central bank obtains a fully informative signal about the shock. In a static framework, we examine the impact of transparency comparing three different information setups. First, the central bank does not release any information about the shock, but the public is informed about the decision rule (*Opacity 1*). Second, the central bank releases the information about the shock, but the public is not informed about the decision rule (*Opacity 2*). Third, the public is informed about both the shock and the applied decision rule (*Transparency*). It turns out that the welfare effects of the different information setups are ambiguous and the ranking depends on the shock size and the applied decision rule. But the combination of *Opacity 1* with the FM-rule is never worse than any other combination.

Discussion and Conclusion (Chapter 5)

Chapter 5 discusses and concludes.

The longer proofs and examples are given in the appendix.

Chapter 2

Flexible Majority Rules for Central Banks

2.1 Introduction

We propose a flexible majority rule for central banks. The flexible majority rule works as follows: Within a pre-specified time frame, the size of the majority necessary for adopting a change in the interest rate depends on the change in the interest rate itself. For small changes in the interest rate, only a small share of the votes is required, possibly even less than 50%. For large interest-rate changes, a larger majority is necessary, tending towards total unanimity.

We consider a model where $N \geq 2$ central bankers, representing countries, regions, or different constituencies within a country, decide on monetary policy. The central bank loss function is composed of the weighted loss functions of countries, regions, or constituencies. This is the typical case for the European Central Bank (ECB), but also applies to the Federal Reserve (Fed). In our example, we consider the ECB when the monetary union is hit by a shock dividing the union into two parts. After this, one part desires a change in monetary policy, while the other part wants to retain the status quo. For instance, some countries may be affected negatively by a negative supply or demand shock, and concern for their own country's welfare makes them want to ease monetary policy through interest-rate cuts. Other countries not affected by the shock will prefer no change in the interest rate. Under simple majority rules, a change in the interest rate will occur if, and only if, a simple majority of votes desires a change. Under flexible majority rules, small changes in the interest rate will only require a small share of supporting votes and, hence, a small number of countries to agree, whereas large changes in the interest rate require large majorities.

The key advantage of the flexible rule is that if a number of countries are hit by a shock, they can partially ease the consequences through a small small interest-rate change. Larger changes in the interest rate, however, require larger majorities, which can only be achieved if a larger number of countries are affected by the shock. The flexible majority rule aligns the severity of shocks and the socially desirable change in the interest rate. The drawbacks of simple majority rules and unanimity rules (possible exploitation by minorities, unanimity rules creating extreme veto power) can be overcome by flexible majority rules.

We distinguish between two cases. First, the vote of every central banker has the same weight; second, the vote of a central banker is weighted to the same degree as his country is weighted in the central bank loss function. Our main results are as follows: First, in both cases the flexible majority rule always leads to smaller central bank losses than the simple majority rule. Second, if every vote is weighted as described above, flexible majority rules can implement the socially optimal solution. Third, it is socially optimal for small interest-rate changes within a particular time frame to be brought about by minorities – either one large country or a set of small countries. Similarly, it is socially desirable for large interest-rate changes to require large majorities. The main intuition for our results is that flexible majority rules of the kind described above can mimic aggregate social loss minimization, which calls for small interest-rate changes when shocks are small and affect only a few countries and large interest-rate changes when shocks are larger and affect many countries.

2.2 Relation to the Literature

2.2.1 Regional Bias in Central Bank Decisions

A socially desirable procedure for making decisions in central bank committees has been the focus of a substantial body of recent literature.

Three areas have been investigated. First, the debate about optimal institutional design of the ECB has focussed on its degree of centralization. Von Hagen and Süppel (1994), Lohmann (1997), de Grauwe, Dewachter, and Aksoy (1999) and Bindseil (2001) have highlighted the advantages of a stronger role for the centrally-nominated ECB.[1] Berger (2006) suggests several ways of improving the organization of the ECB Governing Council. We suggest that flexible majority rules may partially function as a substitute

[1] The advantages of centralization have gained renewed interest in the current process of EU-enlargement (Baldwin, Berglof, Giavazzi, and Widgren (2001), and Berger, de Haan, and Inklaar (2003)).

for lack of centralization at the ECB.

Second, regional considerations appear to play a substantial role in central banks' decision-making, as has been suggested by Heinemann and Hüfner (2004) for the ECB. Meade and Sheets (2005) have highlighted the fact that the policymakers of the Federal Reserve System of the United States (especially in the Federal Open Market Committee (FOMC)) take into account developments in regional unemployment when casting votes on monetary policy.[2] From a theoretical point of view, Sophocles and Skotida (2008) suggest that there can be welfare gains if the central bank board of a monetary union includes country specific characteristics rather than only focussing on union-wide aggregated variables, and we suggest that flexible majority rules promise efficiency gains for central banks of a monetary union.

Third, there is a growing literature on the importance and effects of having monetary policy devised by a committee rather than by individuals. In her survey, Sibert (2006) suggests that an ideal monetary policy committee should not have more than five members.[3] Other recent papers provide specific arguments suggesting that monetary policy conducted by a committee is preferable to a single policy-maker. Gerlach-Kirsten (2006) derives this conclusion in a theoretical study on interest-rate-setting in monetary policy committees, and Blinder and Morgan (2005) provide support with an experimental study. We suggest that flexible majority rules might further enhance the efficiency of committee decision-making on monetary policy.[4]

2.2.2 Efficient Collective Decision-Making

On a broad conceptual level, our paper addresses the optimal design of majority rules, which has a long tradition in economic and political science.

In every collective decision problem, the question arises how a decision rule should

[2]Other sources of heterogeneity are explicitly found for the FOMC and Monetary Policy Committee (MPC) of the Bank of England (BoE). Concerning to Havrilesky and Gildea (1995) there is a difference in the voting behaviour between the subgroups of Federal Reserve Presidents and Federal Reserve board members of the FOMC. Examining the voting behaviour of the FOMC for the period from 1966 – 1996 Chappel, McGregor, and Vermilyea (2005) affirmed this view. A similar finding is described in Riboni and Ruge-Murcia (2008b), who claim that the voting behaviour of the MPC members depends on their career background (whether it is academic, private sector or BoE intern careers) or if their memberships in the MPC are internal or external. In contrast Besley, Meads, and Surico (2008) conclude that this difference in voting behaviour is much less distinctive.

[3]For a survey of the composition and voting procedures of most of the central bank committees throughout the world see Lybek and Morris (2004), Nitsch, Berger, and Lybek (2008) or Fry, Julius, Roger, Mahadeva, and Sterne (2000).

[4]Dixit and Jensen (2000) model the way in which governments could influence the central bank by offering incentive contracts.

be designed in order to achieve socially desirable outcomes. One of the most widely employed decision rules is the simple majority rule, where a proposal is accepted if it obtains more than 50% of the votes. For example, in countries with a democratic constitution, most of the processes in which politicians are elected and parliamentary decisions are taken follow the simple majority rule. An early discussion of when this rule may be optimal can be found in Rae (1969) and Taylor (1969). May (1952) has shown that the simple majority rule satisfying a number of axioms, always has a unique outcome.

Nevertheless, the simple majority rule is not optimal in all cases. The classic work by Buchanan and Tullock (1962) shows that a majority other than 50% might be optimal. Other majorities are realized, for example, in the veto or the unanimity rule in the United Nations Security Council, or the $\frac{2}{3}$ majority needed for an amendment of the constitution in the Federal Republic of Germany. As shown in many papers for well-defined frameworks simple majority rules can be improved by other voting procedures. For instance Caplin and Nalebuff (1988) show that a super majority or 64-% rule has the desirable property of the elimination of cycles.[5]

For the decision-making process in a monetary policy committee, Brückner (1998) shows that if the aggregated welfare measure is the weighted sum of preferences of the deciding members of a monetary union, then it could be welfare-enhancing to depart from the simple majority rule. This can also include leaving the principle of ”one man, one vote” with even neglecting some members in the voting process, if their weight in the welfare measure is very small. Bullard and Waller (2004) suggest that in a two generations model, where the young generation has the majority, shortcomings of the simple majority rule can be overcome if the status quo can only be changed if the younger generation convinces at least some part of the older generation. As in the previous paper in Berentsen and Strub (2009), the minority is also endowed with some veto power, which, in an extreme case, means that the majority can make the minority only a take-it-or-leave-it offer and the status quo is only changed if the minority agrees. Bó (2006) designs a more complicated voting process. In a first constitutional step, the committee determines the majority, which is needed to change the policy variable. In a second step, firstly the committee decides with simple majority about the direction of the change and secondly with the previously determined majority about the size of the change. Riboni and Ruge-Murcia (2010) consider a similar setup, but in their

[5]An early example in history for the use of the two-thirds majority rule is the voting process of the conclave, the election scheme for the pope. The two-thirds majority rule was firstly implemented by Pope Alexander III in the 3. Council of the Lateran (1179) and the detailed design followed in the 2. Council of Lyon by Pope Gregor X (1274). In Saari (1994), it is suggested, that the reason for the implementation of this rule is its stability property.

framework, the majority for the change in size is set ad hoc and is not an outcome of a pre-stage. Additionally, they propose an agenda-setting model, where the chairman of the committee makes a proposal for the change of the policy variable and the committee is only allowed to accept or reject it. In Matsen and Røisland (2005), four general scenarios are examined. First, the committee minimizes a union-wide loss function, which is entered only by aggregated union wide variables. Second, the loss function is the sum of the member loss functions. Third, every member of the committee decides according to his home preferences and the change is set due to a simple majority rule, and fourth, the change is set by the average of the desired changes of the committee members.

Fixed majorities can, however, very often lead to inefficiencies from a utilitarian perspective. Consider, for example, a collective decision problem where two groups have preferences located at two extremes. If one group is at least as big as the fixed majority needed in this decision problem,[6] it can always overrule the other group, which may lead to serious dissatisfaction on the part of the minority (see for instance Guinier (1994) and the classic work of de Tocqueville (1850)) and which is not optimal from a utilitarian perspective. As in other political decision processes, Brückner (2000) and Tarkka (1997) suggest side payments or transfers in order to compensate the minority. In our paper, we design flexible majority rules that can imitate a first-best solution in a utilitarian sense within a specified framework.

Furthermore, in the recent past there has been renewed interest in new decision rules. Casella (2005) suggests a system of storable votes, where voters can choose between the possibility of voting now or storing the vote and having an additional vote in the future. Erlenmeier and Gersbach (2001) have suggested that one might use flexible majority rules for public good provision. When a community has to decide whether to accept a new project and thus faces a simple yes/no decision, the authors show that it can be welfare-enhancing to make the required majority for the adoption of the project proposal depend on the share of individuals taxed under the proposal. In this paper, we design flexible majority rules for monetary policy, where a committee has to choose an element, i.e. a short-term interest rate, from a one-dimensional policy space, i.e. from a continuum of possible alternatives.[7] This approach incorporates various aspects of the above-quoted literature. First, we consider an aggregated loss function

[6] In this case the majority has to be greater than 50%.

[7] There are real-world examples of flexible majority rules, as has been pointed out by Amihai Glazer in personal communications. For instance, when a person buys property in Irvine in southern California, he signs a contract making him a member of a homeowner association that provides local public goods and has the right to levy annual fees. The share of votes required to implement an increase in the fees depends on the proposed fee change.

for the union.[8] Second, the votes of the committee members are weighted, and third, we abandon also the simple majority rule.

2.2.3 Decision Making in the ECB and the Fed

In order to embed our theoretical model in the actual situation of decision making in monetary policy, we briefly describe the frameworks, of the two most important central banks, the ECB and the Fed.

With the introduction of the euro on 1.1.1999, the members of the European Monetary Union (EMU) fully delegated monetary policy onto the ECB. The statute of the European System of Central Banks (ESBC) and the ECB[9] defines the Governing Council (GC) as the main authority conducting monetary policy, including decisions about the key interest rates and the supply of the reserves in the ECB (*Article 12.1*). The GC consists of the six members of the executive board[10] and the governors of the national central banks of the current 16 member states,[11] whose currency is the euro (*Article 10.1*), and each member of the GC has one vote (*Article 10.2*). Basically, the GC acts under a voting scheme of simple majority and all votes are weighted equally[12] (*Article 10.2.*). Although *Article 10.2* in combination with *Article 12.1* indicates that the GC decides about interest-rate changes with a simple majority rule, as mentioned in European Central Bank (2009) (p.92), the actual decision process is not known, since the minutes of the meetings of the GC are not published, and several statements of main figures of the GC can be read in such a way so as to indicate that interest-rate changes are made following a consensus.[13] We already mentioned that for some decisions the

[8] In an extension we show that our findings hold also for a specific union wide loss function in the sense of Matsen and Røisland (2005).

[9] If not otherwise specified, the following articles refer to European Union (2008).

[10] During the implementation of the ECB the number of executive board members need not to be six, but was restricted to a minimum of four (formerly *Article 50*), see European Central Bank (2004) (p.27) and European Union (2008) (p.250).

[11] March 2010.

[12] Exceptions are decisions concerning the eurosystem financial matters, such as increases of the capital key or the transfer of foreign reserve assets. In these cases the votes of the governors are weighted according to the national central bank's shares of the subscribed capital of the ECB (*Article 10.3*), while the votes of the executive board members are weighted zero. Additionally, there exist decisions where a larger quorum than 50% is needed, concerning mainly fundamental questions of the framework of the ECB (*Articles 14.4, 20, 47*).

[13] Duisenberg (2000a): "*First, there was no formal vote. Again, as I has hoped and as it was, it was a consensus decision.*" Duisenberg (2000b): "*You will be aware that I never comment on that. We had an intensive discussion, a prolonged discussion, which was very useful and, in the end, resulted in a consensus on what we had to do.*" Issing (2006): "*It does not matter whether this opinion is shared by everyone or by only a rough majority. Consensus certainly does not mean that there is a need for implicit unanimity...*". Trichet (2008): "*As you know, we do not vote and have never voted in the past. Today, we took a consensus decision...*".

votes of the members of the GC are weighted. But in the future, another change in the voting rights will arise from the successive enlargement of the EMU. At 1.1.1999 eleven countries started with the euro and in the year 2003, it was decided to introduce a rotation system of voting rights of the governors of the national central banks in case the number of members of the GC exceeds 21 (number of governors 15) (European Union (2003)). Although with the introduction of the euro in Slovakia this criterion was met, the rotation system was postponed until the number of governors in the GC exceeds 18 (European Union (2009)). When this new criterion is met only 15 governors obtain a voting right and voting rights will rotate every month (usually there are two meetings per month and once per month a decision is made about the interest rates). The frequency of obtaining a voting right is a monotonically increasing function of the weight[14] of the country in the EMU (European Central Bank (2009)). Officially the ECB insists that they do not abandon the principle of one-member one-vote, but since over time a country has fewer voting rights the smaller it is, there is at least some weighting in the future.[15]

In contrast to the ECB, the FOMC, the decision-making body of the Federal Reserve System, already has rotating voting rights. The FOMC consists in general of seven members of the Board of Governors[16] with permanent voting rights and five out of twelve presidents of the reserve banks. In the group of presidents, only the president of the New York Fed has a permanent voting right, while with the remaining 11 presidents, the four voting rights are rotated annually (The Federal Reserve System (2005)). But as it is also determined in the future rotation scheme of the ECB, the other presidents have the right to attend the meetings of the FOMC and thus can also influence monetary policy decisions, since the FOMC acts also under a consensus principle (The Federal Reserve System (2008)).

[14]The weight w_i of country i is calculated as follows: $w_i = \frac{5}{6}\frac{y_i^{nom}}{y_{EMU}^{nom}} + \frac{1}{6}\frac{bs_i}{bs_{EMU}}$ with y_i^{nom} Gross Domestic Product at market prices and bs_i aggregated balance sheet of Monetary Financial Institutions (MFI) of country i. According to this ranking, the governors are divided into different groups with different frequencies of voting rights. This ranking is usually done every five years or when a new country introduces the euro.

[15]Note that together with the statements in footnote 13, this weighting is not that important compared to an introduction of a rigorous voting rule, since the governors without a voting right are still members of the GC and they have the right speak in the meetings.

[16]Actually there are two vacancies (March 2010).

2.3 The Model

We consider a monetary union consisting of $N \in \mathbb{N}$ $(N \geq 2)$ countries, which make joint decisions on monetary policy in a single central bank such as the ECB. Countries are denoted by k $(k = 1, 2, ..., N)$. The monetary policy is decided in a central bank council where each country k delegates a central banker representing the interest of country k. We assume that the social loss function[17] for every single country is given by

$$L_t^k = (i_t - \tilde{i}_t^k)^2 \ . \tag{2.1}$$

The variable i_t denotes the interest rate adopted by the central bank in period t $(t \in \mathbb{Z})$, and i_t^k is the interest rate that is optimal for country k $(i_t, i_t^k \in \mathbb{R}^+_{\not{k}})$. Overall social losses, based on a weighted-utilitarian welfare criterion, are assumed to be given by

$$\mathcal{L} = \sum_{k=1}^{N} g_k L^k \ , \tag{2.2}$$

with $g_k \in (0, 1)$ and the normalization $\sum_{k=1}^{N} g_k = 1$, where g_k are the weights of the countries representing, for example, differences in GDP or in population.

We assume that in the past at $t-1$, the union has been in long-term equilibrium and the adopted interest rate i_{t-1} has been optimal. Given this status quo, we assume that the monetary union is hit by a supply or demand shock ϵ dividing the union into two parts. One part is affected by the shock, while the other part is not. We use the subset $\mathcal{K}$ to denote the countries affected by the shock, with $\mathcal{K} \subseteq \mathcal{N} = \{1, 2, ..., N\}$ and $|\mathcal{K}| = n$ the number of the affected countries. W.l.o.g., we will analyze only positive realizations of shocks and thus possible increases in the interest rate, as negative realizations would lead to corresponding declines in the interest rate.

We illustrate the working of flexible majority rules in a simple setting. First, we assume that if a shock occurs, every affected country is hit by the same aggregate shock. Second, we assume that the larger the aggregate economic weight of the countries affected by a shock, the more sizable the shock is. This idea is illustrated by a banking crisis (see Elsinger, Lehar, and Summer (2006)). Suppose that a regional banking crisis occurs, caused by a regional negative supply or demand shock and that this crisis triggers defaults of borrowers at banks. If the banking crisis is moderate, other banks in other countries might not be affected much, as only a limited number of loan defaults in the

[17] See e.g. Woodford (2003). Gersbach and Hahn (2001) show that this functional form of losses can be obtained if supply shocks are normally distributed. Riboni and Ruge-Murcia (2008a) choose a similar loss function, which is set ad hoc and the different bliss-points are motivated with a disagreement of the "correct" economic model or different information sets of the decision makers.

European interbank market connecting all banks will occur. If the regional banking crisis is more severe and many banks have to default on their interbank loans, banks in other regions may not be able to repay their outstanding debts and may default as well, so that a larger fraction of the monetary union is hit by the banking crisis.[18]

Our assumption results in a strictly monotonically increasing shock function $\epsilon(G_{\mathcal{K}})$, with $G_{\mathcal{K}}$ representing the aggregate economic weight of the affected countries, given by $G_{\mathcal{K}} = \sum_{k \in \mathcal{K}} g_k$ and $\epsilon(0) = 0$. For a country $k \in \mathcal{K}$, we assume that its desired interest-rate change $\tilde{i}_t^k$ is an increasing function of the shock size $\epsilon(G_{\mathcal{K}})$, whilst an unaffected country does not desire any interest-rate change. The desired interest rate $\tilde{i}_t^k$ of country $k \in \mathcal{N}$ can then be written as

$$\tilde{i}_t^k(\epsilon(G_{\mathcal{K}})) = \gamma_k \Delta \tilde{i}_t(\epsilon(G_{\mathcal{K}})) + i_{t-1} \tag{2.3}$$

where γ_k is a geographical indicator variable describing whether a country is affected by the shock or not and γ_k is then given by

$$\gamma_k = \begin{cases} 1 & \text{for } k \in \mathcal{K} \\ 0 & \text{otherwise} \end{cases} \tag{2.4}$$

$\Delta \tilde{i}_t(\epsilon(G_{\mathcal{K}}))$ is the desired interest-rate change (related to the long-term equilibrium i_{t-1}) if the shock ϵ has occurred, with $\Delta \tilde{i}_t(\epsilon = 0) = 0$. Summing up all subsets of $\mathcal{N}$, we obtain 2^N possible different shock scenarios in the union represented by $\mathcal{K}$.[19] We assume that these shocks are distributed according to an arbitrary probability distribution. In particular, we denote by $p_{\mathcal{K}}$ the probability that all countries in $\mathcal{K}$ are affected by the shock, with $\sum_{\text{all } \mathcal{K}} p_{\mathcal{K}} = 1$.

If we consider that i_t can be written as

$$i_t = i_{t-1} + \Delta i_t \tag{2.5}$$

where Δi_t is the actual interest-rate change from period $t-1$ to period t and with (2.3), we can write

$$L_t^k = (\Delta i_t - \gamma_k \Delta \tilde{i}_t)^2 \tag{2.6}$$

[18] In some regard we have seen such a contagion process during the recent financial crisis, although not only within a European Monetary Union, but in a global context. But, since most of this work was done before the year 2008, the current crisis is not really incorporated in this model. For example, we do not deal with the case, that the lower bound of zero interest-rate-setting is reached and the central bankers have to think about other instruments than interest-rate-setting in order to implement monetary policy decisions.

[19] Note that degeneracies are possible if the g_i's are specified. For example, $g_1 = 0.05$, $g_2 = 0.1$, $g_3 = 0.2$, $g_4 = 0.3$, $g_5 = 0.35$. Although $\epsilon(g_1 + g_5) = \epsilon(g_2 + g_4)$, these are considered to be two different shocks, because the shock does not affect the same countries.

In the following, we drop the time index t, since we are focussing on the specific period from $t-1$ to t and do not consider permanent shocks.[20] Now we can write the social loss function of the union in any specific shock scenario, denoted by $\mathcal{L}_{\mathcal{K}}$, as[21]

$$\begin{aligned} \mathcal{L}_{\mathcal{K}} &= G_{\mathcal{K}} \left(\Delta i\left(G_{\mathcal{K}}\right) - \Delta \tilde{i}\left(G_{\mathcal{K}}\right)\right)^2 + \left(1 - G_{\mathcal{K}}\right)\left(\Delta i\left(G_{\mathcal{K}}\right) - 0\right)^2 \\ &= \left(\Delta i\left(G_{\mathcal{K}}\right) - G_{\mathcal{K}} \Delta \tilde{i}\left(G_{\mathcal{K}}\right)\right)^2 + G_{\mathcal{K}}(1 - G_{\mathcal{K}}) \left(\Delta \tilde{i}\left(G_{\mathcal{K}}\right)\right)^2 \end{aligned} \tag{2.7}$$

For simplicity of exposition, we write (where suitable) in the following:

$$\Delta i\left(G_{\mathcal{K}}\right) = \Delta i_{\mathcal{K}} \quad \text{and} \quad \Delta \tilde{i}\left(G_{\mathcal{K}}\right) = \Delta \tilde{i}_{\mathcal{K}} \tag{2.8}$$

The expected social loss function is then given by

$$E[\mathcal{L}] = \sum_{\text{all}\,\mathcal{K}} p_{\mathcal{K}} \mathcal{L}_{\mathcal{K}} \tag{2.9}$$

2.4 First-Best Solution

Since $\mathcal{L}_{\mathcal{K}}$ represents the losses in every single shock scenario $\mathcal{K}$, the expected losses $E(\mathcal{L})$ are minimized if every single $\mathcal{L}_{\mathcal{K}}(\Delta i_{\mathcal{K}})$ is minimized. From equation (2.1) we see that $\mathcal{L}_{\mathcal{K}}(\Delta i_{\mathcal{K}})$ is a parabola with the minimum at $\Delta i^*_{\mathcal{K}} = G_{\mathcal{K}} \Delta \tilde{i}(G_{\mathcal{K}})$. Thus in every single shock scenario, the optimal change in the interest rate is given by

$$\Delta i_{\mathcal{K}} = \Delta i^*_{\mathcal{K}} = G_{\mathcal{K}} \Delta \tilde{i}(G_{\mathcal{K}}) \tag{2.10}$$

which results in first-best losses of

$$\mathcal{L}^*_{\mathcal{K}} = G_{\mathcal{K}}(1 - G_{\mathcal{K}}) \Delta \tilde{i}(G_{\mathcal{K}}) \tag{2.11}$$

and we end up with optimal expected losses of

$$[E(\mathcal{L})]^* = \sum_{\text{all}\,\mathcal{K}} p_{\mathcal{K}} G_{\mathcal{K}}(1 - G_{\mathcal{K}}) \Delta \tilde{i}(G_{\mathcal{K}}) \tag{2.12}$$

Note that the desired interest-rate change of a country affected by the shock is monotonically increasing in the size of the shock. The larger the shock, the larger the desired interest rate of the affected countries to stabilize the shock. In the following, we will calculate expected losses for different collective decision rules determining $\Delta i_{\mathcal{K}}$. Then we compare their expected losses among themselves and with the first-best solution.

[20] We could also motivate the dropping of the time index by assuming the central bankers to be myopic.

[21] Note that we leave out ϵ and write $\Delta\tilde{i}$ and Δi directly as functions of $G_{\mathcal{K}}$, because $\Delta\tilde{i}$ is a strictly increasing function of ϵ, and ϵ is strictly increasing in $G_{\mathcal{K}}$.

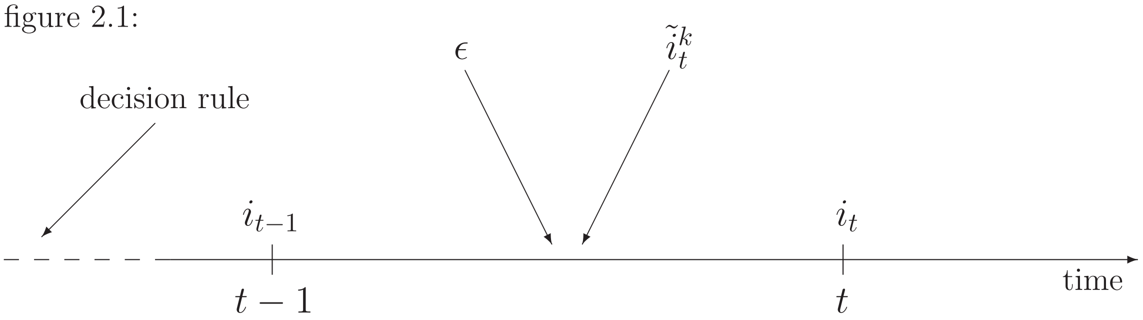
2.5 Constitution

To examine decision rules for central banks, we consider a constitutional design problem where governments of the monetary union decide which decision rule the central bank of the union will use. The selection of the decision rule is governed by the unanimity rule and occurs under a veil of ignorance, i.e. at a time when shocks are not yet known and no conflicts of interest are present. The remaining process is as follows:

Stage 1: Central bankers in the council observe whether or not their countries and other countries are affected by the shock.

Stage 2: The council decides on the change in the interest rate in accordance with the decision rule.

We will restrict rules to democratic decision processes where each central banker has one vote, which may or may not be weighted by g_k, their weight in the overall social loss function. The time-line of the events and decisions is illustrated in the following figure 2.1:



Status quo with an implemented decision rule and long-term equilibrium interest rate i_{t-1}	Shock ϵ occurs and central bankers forming their preferences about their desired interest rate $\tilde{i}_t^k$	Implementation of interest rate i_t according to the decision rule

Figure 2.1:

Pattern of the decision process

2.6 Decision Rules

We distinguish between simple majority (*SM*) and flexible majority (*FM*) decision rules.

SM: i_{t-1} will be changed in t if, and only if, a change receives a majority of more than 50% of the votes. The central bank implements the maximal interest-rate change that receives a majority of 50% of the votes when the interest rate is varied starting from i_{t-1}. Equivalently, the central bank implements the preferred interest-rate change of the median voter.[22]

FM: i_{t-1} will be changed in t if the proposed interest-rate change denoted by $\Delta\hat{i}$ ($\Delta\hat{i} \in \mathbb{R}$) obtains a share of $\alpha(\Delta\hat{i})$ votes with $\alpha(.)$ monotonically increasing and $\alpha \in [0,1]$. The central bank implements the maximum interest-rate change $\Delta\hat{i} = \Delta i_t$ that receives a share of $\alpha(\Delta\hat{i})$ votes when the interest rate is varied starting from i_{t-1}.

Practically, the FM-rule can be applied as follows. The council votes about interest changes in ascending order: $0 < \Delta\hat{i}^1 < \Delta\hat{i}^2 < ....$ As soon as an interest-rate change does not obtain the required share of votes, the last interest-rate change, which has gained the required share, will be implemented by the central bank. The important feature of flexible majority rules is that the size of the majority α depends on the proposed interest-rate change $\Delta\hat{i}$. We will see that it is optimal for small interest-rate changes to require a small share of votes, while large interest-rate changes require a large share of supporting votes. The simple majority rule represents the standard median voter outcome.

[22]When preferences are one-dimensional and single-peaked as in this paper, starting from any status quo, the median voters' most preferred outcome is the maximal change of the status quo that receives a simple majority of votes.

We now proceed as follows: We examine each decision rule separately and provide the comparison afterwards. The maximum interest-rate change for which a supporting majority exists will be chosen. We analyze both the case where every country has only one vote, and the case where the vote of every country is weighted with its importance for overall welfare g_k. In the following we describe four different decision rules:

1. a simple majority rule without weighted votes, denoted by SM_{nw}.
2. a simple majority rule with weighted votes, denoted by SM_w.
3. a flexible majority rule without weighted votes, denoted by FM_{nw}.
4. a flexible majority rule with weighted votes, denoted by FM_w.

2.6.1 Simple Majority Rules

In this part we briefly discuss the outcomes of simple majority rules, first without weighted votes, and second with weighted votes. For a given shock scenario $\mathcal{K}$, we define by $\Delta i_{\mathcal{K}}^{SM_{nw}}$ the interest-rate change under the SM_{nw}-rule and by $\Delta i_{\mathcal{K}}^{SM_w}$ the interest-rate change under the SM_w-rule.

1. <u>SM_{nw}: Simple Majority Rule without weighted votes</u>

   In this case, every country has one vote and the interest-rate is only changed, if more than 50% of the central bankers vote for a change. Hence, the change is zero, if the number of the affected countries is less than or equal to $\frac{N}{2}$. Otherwise, the affected countries implement their desired interest-rate change $\Delta\tilde{i}_{\mathcal{K}}$ since they have the majority. Then, the interest-rate change is given by

$$\Delta i_{\mathcal{K}}^{SM_{nw}} = \begin{cases} \Delta\tilde{i}_{\mathcal{K}} & \text{if} \quad |\mathcal{K}| > \frac{N}{2} \\ 0 & \text{otherwise} \end{cases} \tag{2.13}$$

   and social losses under the SM_{nw}-rule are determined by

$$\mathcal{L}_{\mathcal{K}}^{SM_{nw}} = \begin{cases} G_{\mathcal{K}}\left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 & \text{if} \quad |\mathcal{K}| \leq \frac{N}{2} \\ (1-G_{\mathcal{K}})\left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 & \text{if} \quad |\mathcal{K}| > \frac{N}{2} \end{cases} \tag{2.14}$$

   The expected social losses can be calculated as

$$E\left(\mathcal{L}_{\mathcal{K}}^{SM_{nw}}\right) = \sum_{\text{all } |\mathcal{K}| \leq \frac{N}{2}} p_{\mathcal{K}} G_{\mathcal{K}}\left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 + \sum_{\text{all } |\mathcal{K}| > \frac{N}{2}} p_{\mathcal{K}}(1-G_{\mathcal{K}})\left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 \tag{2.15}$$

The simple majority rule without weighted votes exhibits both kinds of inefficiencies associated with collective decisions. First, interest-rate changes are not implemented and are also too small, if less than half of the countries are affected by a shock. Second, adopted interest-rate changes are too large if more than half of the countries are affected by a shock but simultanously they have a weight of less than 50%.

2. SM_w: Simple Majority Rule with weighted votes

In this case every vote is weighted with the corresponding weight g_k of the overall social loss function (equation (2.2)). Compared to the SM_{nw}-rule, now the interest-rate is only changed if the weighted sum of the affected countries is larger than $\frac{1}{2}$. Then the interest-rate change under the SM_w-rule is given by

$$\Delta i_{\mathcal{K}}^{SM_w} = \begin{cases} \Delta\tilde{i}_{\mathcal{K}} & \text{if} \quad G_{\mathcal{K}} > \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{2.16}$$

and social losses under the SM_w-rule are given by

$$\mathcal{L}_{\mathcal{K}}^{SM_w} = \begin{cases} G_{\mathcal{K}} \left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 & \text{if} \quad G_{\mathcal{K}} \leq \frac{1}{2} \\ (1 - G_{\mathcal{K}}) \left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 & \text{if} \quad G_{\mathcal{K}} > \frac{1}{2} \end{cases} \tag{2.17}$$

The expected social loss are calculated as

$$E\left(\mathcal{L}_{\mathcal{K}}^{SM_w}\right) = \sum_{\text{all } G_{\mathcal{K}} \leq \frac{1}{2}} p_{\mathcal{K}} G_{\mathcal{K}} \left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 + \sum_{\text{all } G_{\mathcal{K}} > \frac{1}{2}} p_{\mathcal{K}} (1 - G_{\mathcal{K}}) \left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 \tag{2.18}$$

The simple majority rule with weighted votes slightly improves the outcome of the SM_{nw}-rule, since now a large number of small countries cannot overrule anymore a small number of large countries, if for example the aggregated weight of the large countries is larger than 50% and they are the only countries affected by a shock.

2.6.2 Flexible Majority Rules

In this part we modify the rule that the interest rate can only be changed with the fixed majority of more than 50% of the votes. We change to flexible majority rules, where the required share of votes depends on the size of the proposed interest-rate change. Again, we discuss flexible majority rules without (FM_{nw}) and with (FM_w) weighted votes and for a given shock scenario $\mathcal{K}$, we define by $\Delta i_{\mathcal{K}}^{FM_{nw}}$ the interest-rate change under the FM_{nw}-rule and by $\Delta i_{\mathcal{K}}^{FM_w}$ the interest-rate change under the FM_w-rule.

1. $\underline{FM_{nw}\text{: Flexible Majority Rule without weighted votes}}$

We introduce a flexible majority rule (FM_{nw}) that will prove to be welfare-enhancing compared to the SM_{nw}-rule. In particular, we define the FM_{nw}-rule as follows:

$$\alpha^{FM_{nw}}(\Delta\hat{i}) = \begin{cases} 0 & \text{if} & & \Delta\hat{i} & = & 0 \\ \frac{1}{N} & \text{if} & 0 & < & \Delta\hat{i} & \leq & \Delta I(1) \\ \frac{2}{N} & \text{if} & \Delta I(1) & < & \Delta\hat{i} & \leq & \Delta I(2) \\ \vdots & & & & & & \\ \frac{N-1}{N} & \text{if} & \Delta I(n-2) & < & \Delta\hat{i} & \leq & \Delta I(n-1) \\ 1 & \text{if} & \Delta I(n-1) & < & \Delta\hat{i} & & \end{cases} \tag{2.19}$$

where $\Delta I(n)$ is given by

$$\Delta I(n) = \begin{cases} \min\limits_{\mathcal{K}}\{G_{\mathcal{K}}\Delta i_{\mathcal{K}}\} \quad \text{s.t.}\ |\mathcal{K}| = n \quad \text{if} \quad n \leq \frac{N}{2} \\ \\ \max\limits_{\mathcal{K}}\{G_{\mathcal{K}}\Delta i_{\mathcal{K}}\} \quad \text{s.t.}\ |\mathcal{K}| = n \quad \text{if} \quad n > \frac{N}{2} \end{cases} \tag{2.20}$$

with n the number of countries affected. Note that $\alpha^{FM_{nw}}(\Delta\hat{i})$ is monotonically increasing in $\Delta\hat{i}$, since $\Delta I(n)$ is monotonically increasing in n. The flexible majority rule has the following property: Small interest-rate changes require a small share of votes, while large interest-rate changes require a large share of supporting votes.

In the following proposition, we show which interest-rate changes result in any shock scenario $\mathcal{K}$ by applying the FM_{nw}-rule defined in equation (2.19).

Proposition 1

For any shock scenarios $\mathcal{K}$ the FM_{nw}-rule defined by $\alpha^{FM_{nw}}$, the monetary policy committee implements an interest-rate change of

$$\Delta i_{\mathcal{K}}^{FM_{nw}} = \begin{cases} \Delta I(n) & \textit{if} \quad |\mathcal{K}| = n \leq \frac{N}{2} \\ \\ \Delta\tilde{i}_{\mathcal{K}} & \textit{if} \quad |\mathcal{K}| = n > \frac{N}{2} \quad \textit{and} \quad \Delta I(n) \geq \Delta\tilde{i}_{\mathcal{K}} \\ \\ \Delta I(n) & \textit{if} \quad |\mathcal{K}| = n > \frac{N}{2} \quad \textit{and} \quad \Delta I(n) < \Delta\tilde{i}_{\mathcal{K}} \end{cases} \tag{2.21}$$

Proof of proposition 1:
The reasoning for this is as follows:

(i) Suppose $\mathcal{K}$ is fixed and $|\mathcal{K}| = n \leq \frac{N}{2}$. For the FM_{nw}-rule we obtain $\Delta i_{\mathcal{K}}^{FM_{nw}} = \Delta I(n) = \{\min_{\mathcal{K}'}\{G_{\mathcal{K}'}\Delta i_{\mathcal{K}'}\}\,\text{s.t.}\,|\mathcal{K}'| = n\}$ (where $\mathcal{K}' \subseteq \mathcal{N}$ denotes, similar to the definition of $\mathcal{K}$, the countries affected by the shock), since the affected countries desire an interest-rate change of $\Delta\tilde{i}_{\mathcal{K}}$ but they can only change the interest rate up to $\Delta I(n) < \Delta\tilde{i}_{\mathcal{K}}$.

(ii) Suppose $\mathcal{K}$ is fixed, $|\mathcal{K}| = n > \frac{N}{2}$ and $\Delta I(n) \geq \Delta\tilde{i}_{\mathcal{K}}$. For the FM_{nw}-rule we obtain $\Delta i_{\mathcal{K}}^{FM_{nw}} = \Delta\tilde{i}_{\mathcal{K}}$, since the desired interest-rate change $\Delta\tilde{i}_{\mathcal{K}}$ of the affected countries is less than their possible interest-rate change $\Delta I(n) = \{\max_{\mathcal{K}'}\{G_{\mathcal{K}'}\Delta i_{\mathcal{K}'}\}\,\text{s.t.}\,|\mathcal{K}'| = n\}$.

(iii) Suppose $\mathcal{K}$ is fixed, $|\mathcal{K}| = n > \frac{N}{2}$ and $\Delta I(n) < \Delta\tilde{i}_{\mathcal{K}}$. For the FM_{nw}-rule we obtain $\Delta i_{\mathcal{K}}^{FM_{nw}} = \Delta I(n) = \{\max_{\mathcal{K}'}\{G_{\mathcal{K}'}\Delta i_{\mathcal{K}'}\}\,\text{s.t.}\,|\mathcal{K}'| = n\}$, since the affected countries desire an interest-rate change of $\Delta\tilde{i}_{\mathcal{K}}$ but they can only change the interest rate up to $\Delta I(n) < \Delta\tilde{i}_{\mathcal{K}}$.

■

The social losses under the FM_{nw}-rule are then given by

$$\mathcal{L}_{\mathcal{K}}^{FM_{nw}} = \begin{cases} \left(\Delta I(n) - G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}\right)^2 + G_{\mathcal{K}}(1-G_{\mathcal{K}})\left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 & \text{if} \quad |\mathcal{K}| = n \leq \frac{N}{2} \\ (1-G_{\mathcal{K}})\left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 & \text{if} \quad |\mathcal{K}| = n > \frac{N}{2} \text{ and } \Delta I(n) \geq \Delta\tilde{i}_{\mathcal{K}} \\ \left(\Delta I(n) - G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}\right)^2 + G_{\mathcal{K}}(1-G_{\mathcal{K}})\left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 & \text{if} \quad |\mathcal{K}| = n > \frac{N}{2} \text{ and } \Delta I(n) < \Delta\tilde{i}_{\mathcal{K}} \end{cases} \tag{2.22}$$

and expected social losses are calculated as

$$E\left(\mathcal{L}_{\mathcal{K}}^{FM_{nw}}\right) = \sum_{\substack{\text{all } |\mathcal{K}|\leq \frac{N}{2} \text{ and} \\ \text{all } |\mathcal{K}|=n>\frac{N}{2} \text{ with } \Delta I(n)<\Delta\tilde{i}_{\mathcal{K}}}} p_{\mathcal{K}}\left(\left(\Delta I(n) - G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}\right)^2 + G_{\mathcal{K}}(1-G_{\mathcal{K}})\left(\Delta\tilde{i}_{\mathcal{K}}\right)^2\right)$$
$$+ \sum_{\text{all } |\mathcal{K}|>\frac{N}{2} \text{ with } \Delta I(n)\geq\Delta\tilde{i}_{\mathcal{K}}} p_{\mathcal{K}}(1-G_{\mathcal{K}})\left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 \tag{2.23}$$

This also improves the SM_{nw}-rule. The intuition for this is as follows. With the FM_{nw}-rule, an interest rate from the interval $[\Delta i_{\mathcal{K}}^{SM_{nw}}, G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}]$ or $[G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}, \Delta i_{\mathcal{K}}^{SM_{nw}}]$ is adopted. Hence, the FM_{nw}-rule represents a compromise between countries affected by the shock and the other countries, which is welfare-enhancing compared to the SM_{nw}-rule.

2. FM_w: Flexible Majority Rule with Weighted Votes

Now, we flexible majority rule (FM_w), where both the required votes for an interest-rate change depend on the size of the desired interest-rate change and the votes are weighted with their corresponding weight of the overall social loss function. It turns out that we can construct a FM_w-rule that minimizes $E[\mathcal{L}]$. According to the first-best solution (equation (2.10)), for every shock scenario $\mathcal{K}$ we know that we should aim to implement

$$\Delta i_{\mathcal{K}}^{FM_w} = \Delta i_{\mathcal{K}}^{*} = G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}} \tag{2.24}$$

via the FM_w-rule. In proposition 2 we establish the existence of an optimal flexible majority rule.

Proposition 2

There exists a function $\alpha^{FM_w}(\Delta\hat{i})$, which determines for every interest-rate change the necessary share of votes such that under this flexible majority rule $\alpha^{FM_w}(\Delta\hat{i})$, the central bank committee will always implement an interest-rate change that minimizes the social loss function (2.7) *for every given shock scenario $\mathcal{K}$ and $\alpha^{FM_w}(\Delta\hat{i})$ is given by*

$$\alpha^{FM_w}(\Delta\hat{i}) = \begin{cases} G_{(1)} = 0 & \text{if} \quad \Delta\hat{i} = 0 \\ G_{(2)} & \text{if} \quad 0 < \Delta\hat{i} \leq G_{(1)}\Delta\tilde{i}_{(1)} \\ G_{(3)} & \text{if} \quad G_{(1)}\Delta\tilde{i}_{(1)} < \Delta\hat{i} \leq G_{(2)}\Delta\tilde{i}_{(2)} \\ \vdots & \\ G_{(2^N-1)} & \text{if} \quad G_{(2^N-2)}\Delta\tilde{i}_{(2^N-2)} < \Delta\hat{i} \leq G_{(2^N-1)}\Delta\tilde{i}_{(2^N-1)} \\ G_{(2^N)} = 1 & \text{if} \quad G_{(2^N-1)}\Delta\tilde{i}_{(2^N-1)} < \Delta\hat{i} \end{cases} \tag{2.25}$$

where $G_{(k)}$ $(k = 1...2^N)$ is the ordered ascending row of all $G_\mathcal{K}$ with

$$0 = G_{(1)} < G_{(2)} \leq G_{(3)} \leq ... \leq G_{(2^N-1)} < G_{(2^N)} = 1 \tag{2.26}$$

and $\Delta\tilde{i}_{(k)} = \Delta\tilde{i}(G_{(k)})$ (in cases of indifferences with $G_\mathcal{K} = G_{\mathcal{K}'}$ $(\mathcal{K} \neq \mathcal{K}')$ the ordering is randomly chosen, since in this case the numbering is not crucial).

Proof of Proposition 2:
First, observe that $\alpha^{FM_w}(\Delta\hat{i})$ is monotonically increasing in the interest-rate change $\Delta\hat{i}$. Second, for every shock scenario $G_{(k)}$ one has $\Delta\tilde{i}_{(k)} \geq G_{(k)}\Delta\tilde{i}_{(k)}$. But since $\Delta\tilde{i}_{(k)}$ is the desired interest-rate change of the affected countries and due to the construction of α^{FM_w} the interest-rate change $G_{(k)}\Delta\tilde{i}_{(k)}$ is the maximum interest-rate change a group of countries with economic weight $G_{(k)}$ can implement, in a pairwise ascending ballot the interest rate is changed up to $G_{(k)}\Delta\tilde{i}_{(k)}$. With the unique relation of $G_{(k)} \longrightarrow G_\mathcal{K}$, this directly implies, that under the FM_w-rule the optimal interest-rate change $\Delta i_\mathcal{K}^{FM_w} = \Delta i_\mathcal{K}^*$ is implemented in a given shock scenario $\mathcal{K}$. Altogether, α^{FM_w} minimizes expected overall social losses, because expected overall social losses $E[\mathcal{L}]$ are the weighted sum of all $\mathcal{L}_\mathcal{K}$, with the probability weights $p_\mathcal{K}$.

■

Under the FM_w-rule this leads in every shock scenario $\mathcal{K}$ to an interest-rate change of

$$\Delta i_\mathcal{K}^{FM_w} = G_\mathcal{K}\Delta\tilde{i}_\mathcal{K} \tag{2.27}$$

by inserting equation (2.27) in (2.7) social losses are given by

$$\mathcal{L}_\mathcal{K}^{FM_w} = G_\mathcal{K}(1 - G_\mathcal{K})\left(\Delta\tilde{i}_\mathcal{K}\right)^2 \tag{2.28}$$

and the expected losses are calculated as

$$E\left(\mathcal{L}_{\mathcal{K}}^{FM_w}\right) = \sum_{\text{all}\,\mathcal{K}} p_{\mathcal{K}} G_{\mathcal{K}}(1 - G_{\mathcal{K}}) \left(\Delta\tilde{i}_{\mathcal{K}}\right)^2 \tag{2.29}$$

An immediate consequence of proposition 2 is the following corollary:

Corollary 1
Applying α^{FM_w} leads to the first-best solution.

Corollary 1 follows from the observation that a first-best solution means implementing the interest-rate change that minimizes $\mathcal{L}_{\mathcal{K}}$ for every single shock scenario. Therefore, corollary 1 follows directly from (2.27).

As a very simple example, we consider a case with two countries $g_1 = 0.3$, $g_2 = 0.7$ and $\Delta\tilde{i}_{\{\emptyset\}} = 0$, $\Delta\tilde{i}_{\{1\}} = 1$, $\Delta i_{\{2\}} = 2$ and $\Delta\tilde{i}_{\{1,2\}} = 3$. The optimal function $\alpha^{FM_w}(\Delta\hat{i}_{\mathcal{K}})$ is then given by

$$\alpha^{FM_w}(\Delta\hat{i}) = \begin{cases} 0 & \text{if} & & & \Delta\hat{i} & = & 0 \\ 0.3 & \text{if} & 0 & < & \Delta\hat{i} & \leq & 0.3 \\ 0.7 & \text{if} & 0.3 & < & \Delta\hat{i} & \leq & 1.4 \\ 1 & \text{if} & 1.4 & < & \Delta\hat{i} & & \end{cases} \tag{2.30}$$

α^{FM_w} is calculated by determining the optimal interest-rate change $\Delta i^*_{\mathcal{K}}$ and subsequent inversion. We obtain $\Delta i^*_{\{1\}} = 0.3 \cdot 1 = 0.3$ and $\Delta i^*_{\{2\}} = 0.7 \cdot 2 = 1.4$. Applying this flexible majority rule, we see that no interest-rate change occurs when no country is affected, whereas a change of 3 occurs when all countries are affected, because in this case every country wants exactly a change of 3. If only the smaller country is affected, it will desire a change of 1. But with its share of 30% of the total votes, it can only implement a change up to 0.3. Since its private losses are decreasing in $[0, 0.3]$, the central bank will adopt a change of 0.3 in the interest rate. By the same argument, the change in interest rate will be 1.4 when only the large country is affected, which has a share of 70% of the votes.

This flexible majority rule further improves the outcome, since it aligns the required votes for an interest-rate change with the size of the optimal interest-rate change in any shock scenario. Note that the functionality of the flexible majority rule highly depends on the assumption that the larger the affected region, the larger the desired interest-rate change of the affected countries. Later on, we show that our result still holds for a broader class of loss functions and another aggregation process.

2.7 Comparison of the Different Decision Rules

In this section we formally compare the different decision rules defined in the previous chapter and their outcomes. This is summarized in the following proposition

Proposition 3

In any given shock scenario $\mathcal{K}$ we have

$$\begin{array}{llllll} (i) & \mathcal{L}_{\mathcal{K}}^{FM_w} & \leq & \mathcal{L}_{\mathcal{K}}^{SM_w} & \leq & \mathcal{L}_{\mathcal{K}}^{SM_{nw}} \\ (ii) & \mathcal{L}_{\mathcal{K}}^{FM_w} & \leq & \mathcal{L}_{\mathcal{K}}^{FM_{nw}} & \leq & \mathcal{L}_{\mathcal{K}}^{SM_{nw}} \end{array} \tag{2.31}$$

The proof is given in Appendix 6.1. The intuition for this finding is as follows.

Since the FM_w-rule guarantees the first-best outcome, the first part of the inequalities (2.31) from proposition 3 follows directly from corollary 1.

In order to see, that the SM_w-rule is improving compared to the SM_{nw}-rule, consider the case where the interest-rate changes of the SM_w and SM_{nw}-rule fall apart for a specific shock scenario. First, if $G_{\mathcal{K}} > \frac{1}{2}$ and the number of the affected countries $n \leq \frac{N}{2}$ the interest-rate change under the SM_w-rule and the SM_{nw}-rule deviate in different directions from the optimal interest-rate change $\Delta i_{\mathcal{K}}^* = G_{\mathcal{K}} \Delta \tilde{i}_{\mathcal{K}}$, but the deviation from the optimal interest-rate change is less for the SM_w-rule and thus, because of the symmetry of aggregated social losses, SM_w outperforms SM_{nw}. In the opposite case ($G_{\mathcal{K}} \leq \frac{1}{2}$ and $n > \frac{N}{2}$), the same argumentation leads to the superiority of SM_w over SM_{nw}. The improvement of the FM_{nw}-rule compared to the SM_{nw}-rule follows from a similar consideration. Again the distance to the optimal interest-rate change is diminished compared to SM_{nw}-rule. But in this case both decision rules (FM_{nw} and SM_{nw}) deviate $\Delta i_{\mathcal{K}}^*$ in the same direction. Explicitly with the FM_{nw}-rule, an interest rate from the interval $[\Delta i_{\mathcal{K}}^{SM_{nw}}, G_{\mathcal{K}} \Delta i_{\mathcal{K}}]$ or $[G_{\mathcal{K}} \Delta i_{\mathcal{K}}, \Delta i_{\mathcal{K}}^{SM_{nw}}]$ is adopted. Hence, as already earlier stated, the FM_{nw}-rule represents a compromise between countries affected by the shock and the other countries. Thus, the FM_{nw}-rule is welfare-enhancing compared to the SM_{nw}-rule.

From proposition 3 follows directly the following corollary:

Corollary 2

$$\begin{array}{llllll} (i) & E\left(\mathcal{L}_{\mathcal{K}}^{FM_w}\right) & < & E\left(\mathcal{L}_{\mathcal{K}}^{SM_w}\right) & \leq & E\left(\mathcal{L}_{\mathcal{K}}^{SM_{nw}}\right) \\ (ii) & E\left(\mathcal{L}_{\mathcal{K}}^{FM_w}\right) & \leq & E\left(\mathcal{L}_{\mathcal{K}}^{FM_{nw}}\right) & < & E\left(\mathcal{L}_{\mathcal{K}}^{SM_{nw}}\right) \end{array} \tag{2.32}$$

This follows from the property, that $E[\mathcal{L}_{\mathcal{K}}]$ is the weighted sum of all $\mathcal{L}_{\mathcal{K}}$ and the possibility of equality of first SM_w and SM_{nw} and second FM_w and FM_{nw} for example is fulfilled if every country has the same weight (i.e $g_k = \frac{1}{N}$ for all k). The comparison of SM_w and FM_{nw} is ambiguous. In the following table we list in general cases the relation between SM_w and FM_{nw} for different shock scenarios $\mathcal{K}$. For simplicity we exclude $G_{\mathcal{K}} = 0, 1$.

	$G_{\mathcal{K}}$	n	$\Delta I(n)$	Δi^*	$\Delta i_{\mathcal{K}}^{FM_{nw}}$	$\Delta i_{\mathcal{K}}^{SM_w}$	$\mathcal{L}_{\mathcal{K}}^{FM_{nw}} - \mathcal{L}_{\mathcal{K}}^{SM_w}$
1.	$< \frac{1}{2}$	$\leq \frac{N}{2}$		$G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}$	$\Delta I(n)$	0	< 0
2.	$< \frac{1}{2}$	$> \frac{N}{2}$	$\geq \Delta\tilde{i}_{\mathcal{K}}$	$G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}$	$\Delta\tilde{i}_{\mathcal{K}}$	0	> 0
3.	$= \frac{1}{2}$	$> \frac{N}{2}$	$\geq \Delta\tilde{i}_{\mathcal{K}}$	$G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}$	$\Delta\tilde{i}_{\mathcal{K}}$	0	$= 0$
4.	$= \frac{1}{2}$	$> \frac{N}{2}$	$< \Delta\tilde{i}_{\mathcal{K}}$	$G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}$	$\Delta I(n)$	0	< 0
5.	$> \frac{1}{2}$	$\leq \frac{N}{2}$		$G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}$	$\Delta I(n)$	$\Delta\tilde{i}_{\mathcal{K}}$	$\lesseqgtr 0$
6.	$> \frac{1}{2}$	$> \frac{N}{2}$	$< \Delta\tilde{i}_{\mathcal{K}}$	$G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}$	$\Delta I(n)$	$\Delta\tilde{i}_{\mathcal{K}}$	< 0
7.	$> \frac{1}{2}$	$> \frac{N}{2}$	$\geq \Delta\tilde{i}_{\mathcal{K}}$	$G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}$	$\Delta\tilde{i}_{\mathcal{K}}$	$\Delta\tilde{i}_{\mathcal{K}}$	$= 0$

Table 2.1:
Comparison of FM_{nw} and SM_w.

The entries in the last column of the table can directly be verified by comparing the absolute differences $|\Delta i^* - \Delta i_{\mathcal{K}}^{FM_{nw}}|$ and $|\Delta i^* - \Delta i_{\mathcal{K}}^{SM_w}|$. For the rows, we obtain

1. $\Delta i^* \geq \Delta I(n) > 0 \Longrightarrow \mathcal{L}_{\mathcal{K}}^{FM_{nw}} < \mathcal{L}_{\mathcal{K}}^{SM_w}$.
2. $\Delta\tilde{i}_{\mathcal{K}} - \Delta i^* = (1 - G_{\mathcal{K}})\Delta\tilde{i}_{\mathcal{K}} > G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}$, since $G_{\mathcal{K}} < \frac{1}{2} \Longrightarrow \mathcal{L}_{\mathcal{K}}^{FM_{nw}} > \mathcal{L}_{\mathcal{K}}^{SM_w}$.
3. $\Delta\tilde{i}_{\mathcal{K}} - \Delta i^* = (1 - G_{\mathcal{K}})\Delta\tilde{i}_{\mathcal{K}} = G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}$, since $G_{\mathcal{K}} = \frac{1}{2} \Longrightarrow \mathcal{L}_{\mathcal{K}}^{FM_{nw}} = \mathcal{L}_{\mathcal{K}}^{SM_w}$.
4. $\Delta I(n) - \Delta i^* < (1 - G_{\mathcal{K}})\Delta\tilde{i}_{\mathcal{K}} = G_{\mathcal{K}}$, since $G_{\mathcal{K}} = \frac{1}{2} \Longrightarrow \mathcal{L}_{\mathcal{K}}^{FM_{nw}} < \mathcal{L}_{\mathcal{K}}^{SM_w}$.
5. $\Delta i^* - \Delta I(n) \lesseqgtr (1 - G_{\mathcal{K}})\Delta\tilde{i}_{\mathcal{K}}$, since $G_{\mathcal{K}} > \frac{1}{2} \Longrightarrow \mathcal{L}_{\mathcal{K}}^{FM_{nw}} \lesseqgtr \mathcal{L}_{\mathcal{K}}^{SM_w}$.
6. $\Delta i^* \leq \Delta I(n) < \Delta\tilde{i}_{\mathcal{K}} \Longrightarrow \mathcal{L}_{\mathcal{K}}^{FM_{nw}} < \mathcal{L}_{\mathcal{K}}^{SM_w}$.
7. $\Delta i_{\mathcal{K}}^{FM_{nw}} = \Delta i_{\mathcal{K}}^{SM_w} \Longrightarrow \mathcal{L}_{\mathcal{K}}^{FM_{nw}} = \mathcal{L}_{\mathcal{K}}^{SM_w}$.

If we revisit the construction of the FM_{nw}-rule in equations (2.19) and (2.20), we observe that this rule is somehow ad hoc defined, in order to improve SM_{nw}. Therefore, we introduce two further construction principles for a flexible majority rule without weighted votes. First a rule denoted by $FM_{nw'}$, which is oriented at the already defined

FM_{nw}-rule. This means, we do not violate the property to be better than the SM_{nw}-rule in every shock scenario $\mathcal{K}$, but we try to improve the social outcome compared to the FM_{nw}-rule. Second, we abandon the property to be better than the SM_{nw}-rule in every shock scenario $\mathcal{K}$ and mainly consider to minimize expected overall social losses with a rule denoted by $FM_{nw''}$.

1. $FM_{nw'}$:

   We define $\alpha^{FM_{nw'}}$ as follows,

$$\alpha^{FM_{nw'}}(\Delta\hat{i}) = \begin{cases} 0 & \text{if} & & \Delta\hat{i} & = & 0 \\ \frac{1}{N} & \text{if} & 0 & < & \Delta\hat{i} & \leq & \Delta I'(1) \\ \frac{2}{N} & \text{if} & \Delta I'(1) & < & \Delta\hat{i} & \leq & \Delta I'(2) \\ \vdots & & & & & & \\ \frac{N-1}{N} & \text{if} & \Delta I'(n-2) & < & \Delta\hat{i} & \leq & \Delta I'(n-1) \\ 1 & \text{if} & \Delta I'(n-1) & < & \Delta\hat{i} & & \end{cases}, \tag{2.33}$$

   where $\Delta I'(n)$ is obtained from the following minimization problem:

$$\min_{\Delta I'(1)...\Delta I'(N-1)} \left\{ \sum_{\text{all}\,\mathcal{K}} p_{\mathcal{K}} \mathcal{L}_{\mathcal{K}} \right\} \quad \begin{array}{ll} \text{s.t.} & \Delta I'(n) \leq \Delta I'(n+1) \\ & \text{for all}\, n = 1, 2...N-2 \\ \text{s.t.} & \mathcal{L}_{\mathcal{K}}^{FM_{nw'}} \leq \mathcal{L}_{\mathcal{K}}^{SM_{nw}} \end{array} \tag{2.34}$$

   The existence of $\alpha^{FM_{nw'}}$ follows directly from the existence of $\alpha^{FM_{nw}}$, since $\alpha^{FM_{nw}}$ satisfies the constraints in the minimization problem of equation (2.34). This decision rule is by definition ex ante and ex post not worse than the SM_{nw}-rule.

2. $FM_{nw''}$:

   In this case, we can abandon the assumption that for all $\mathcal{K}$ the flexible majority rule without weighted votes should not be worse than the SM_{nw}-rule and focus only on the expected losses. But in this case, it is possible that after the realization of a specific shock $\mathcal{K}$ and the implementation of an interest rate change according to $FM_{nw''}$, the outcome is worse than that of SM_{nw} and thus there exists a $\mathcal{K}$, such that ex post SM_{nw} is better than $FM_{nw''}$. Similarly to the definition of $\alpha^{FM_{nw'}}$ we define $\alpha^{FM_{nw''}}$ as,

$$\alpha^{FM_{nw''}}(\Delta\hat{i}) = \begin{cases} 0 & \text{if} & & \Delta\hat{i} & = & 0 \\ \frac{1}{N} & \text{if} & 0 & < & \Delta\hat{i} & \leq & \Delta I''(1) \\ \frac{2}{N} & \text{if} & \Delta I''(1) & < & \Delta\hat{i} & \leq & \Delta I''(2) \\ \vdots & & & & & & \\ \frac{N-1}{N} & \text{if} & \Delta I''(n-2) & < & \Delta\hat{i} & \leq & \Delta I''(n-1) \\ 1 & \text{if} & \Delta I''(n-1) & < & \Delta\hat{i} \end{cases}, \qquad (2.35)$$

where $\Delta I''(n)$ is obtained from the following minimization problem:

$$\min_{\Delta I''(1)...\Delta I''(N-1)} \left\{ \sum_{\text{all}\,\mathcal{K}} p_{\mathcal{K}} \mathcal{L}_{\mathcal{K}} \right\} \quad \begin{array}{l} \text{s.t. } \Delta I''(n) \leq \Delta I''(n+1) \\ \text{for all}\, n = 1, 2...N-2 \end{array} \qquad (2.36)$$

Since this rule has another degree of freedom compared to $FM_{nw'}$ it follows directly that $E[\mathcal{L}_{\mathcal{K}}^{FM_{nw''}}] \leq E[\mathcal{L}_{\mathcal{K}}^{FM_{nw'}}]$, but the general statement, that for all $\mathcal{K}$ we obtain $\mathcal{L}_{\mathcal{K}}^{FM_{nw''}} \leq \mathcal{L}_{\mathcal{K}}^{SM_{nw}}$, is not true anymore.

In order to illustrate the different decision rules, in appendix 6.2.1 (B Examples) we provide a detailed example, where in particular we calculate the different flexible majority rules. The intuition underlying the advantages of flexible majority rules runs as follows: It is socially desirable for small interest-rate changes to be possible if only a small part of the union is affected by a shock. This is not possible under simple majority rules, because the 50% majority always fully determines the monetary policy. By contrast, applying flexible majority rules means that minorities can also change the interest rate to a small degree. Additionally, for the social optimum large interest-rate changes should only be possible if a large part of the union is really affected by a shock. But again, simple majority rules already provide the possibility for large interest-rate changes if only less than 50% of the union is affected. Under flexible majority rules, the larger the interest-rate change, the larger the required share of votes. This means that large interest-rate changes can require a share of votes larger than 50%.

Chapter 3

Extensions of the Model

In this chapter, we extend our model in different directions. At first, we vary the social loss function, in which one the one hand we leave the quadratic form and assume only a convex, single-peaked, form and on the other hand we assume that the social target is not evaluated from a utilitarian aggregation, but it is set ad hoc focussing on an averaged shock to the union. Second, we leave the strict separation of the union in only two parts (affected by the shock and not affected) and allow for heterogeneous shocks differing in size and direction, and third we introduce a dynamic setup, in which shocks can occur in every period.

3.1 Different Loss Functions

3.1.1 General Convexity

In the previous chapter, we used the standard approach with a quadratic loss function, which is widely applied in the context of monetary policy (see i.e Svensson (2003) and Woodford (2003)). Following Duarte (2009) this approach entered monetary policy the first time in the work of Poole (1970) and Kareken (1970). But within their approach, only a quadratic loss in deviation from an output target entered the objective function, while in our approach, although it is one-dimensional in interest-rate change, the loss function is derived from a standard loss function, which is both quadratic in deviations from an output target and an inflation target. Such an approach can be found in Sargent and Wallace (1975), who consider an ad hoc loss function which is quadratic in output and prices. This model is followed by the seminal paper of Kydland and Prescott (1977), who start with a general loss function that depends on a policy variable and an agent's decision variable. Throughout their paper they later focus for computational reasons on quadratic losses, which are motivated by a second-order approximation. But this approximation is not further explained in detail. Most of the current models, as well as ours, are based on a framework developed in Barro and Gordon (1983), followed by Rogoff (1985). Regarding Duarte (2009), the "quadratic approach" attained monetary policy from optimal control theory and two theoretical papers by Theil (1957) and Simon (1956), who showed the certainty equivalence under quadratic losses. But afterwards this approach was widely adopted without further motivation, until Rotemberg and Woodford (1997), followed by Erceg, Henderson, and Levin (2000), gave a rigorous treatment of deriving a quadratic loss function based on a Taylor expansion of a general welfare function. But in recent years, this approach has been increasingly criticized. Kim and Kim (2003) generally discuss the limitations of log linearization and second-order approximations of a general welfare measure in different model setups. Others, often referring to Cukierman and Meltzer (1986), generally attack the quadratic approximation asking for asymmetric or skewed loss functions. This view is represented by Goodhart (2001), who qualitatively argues for a skewed loss function, or Chadha and Schellekens (1999) and al Nowaihi and Stracca (2003), who explicitly apply more general and asymmetric functional forms. For example the increasing risk-averse type $L_1 = |x - \tilde{x}|^\alpha$ $(\alpha > 0)$, which includes the quadratic form at $\alpha = 2$ or the constant absolute risk averse typ[1] $L_2 = \Theta(\tilde{x} - x)e^{\beta_1(\tilde{x}-x)} + \Theta(x - \tilde{x})e^{\beta_2(x-\tilde{x})}$ (with the Heavyside[2]) function $\Theta(.)$ and $\beta_1, \beta_2 > 0$). In this context, there has been also developed a literature which models preferences with the so-called linex-function $L_3 = \frac{1}{\gamma^2}\left(e^{\gamma(x-\tilde{x})} - \gamma(x - \tilde{x})\right)$ introduced in economics by Varian (1974), which also incorporates the quadratic case in the limit of $\gamma \to 0$. For example, this functional form is applied in Ruge-Murcia (2003), Nobay and Peel (2003), and Surico (2008). Generally, we denote with $x \in \mathbb{R}$ the decision variable and with $\tilde{x}$ the target, in which we assume w.l.o.g. $\tilde{x} > 0$.

In order to integrate our model in this body of literature, we replace our loss function with a more general functional form and show that our results hold for a much broader class of preferences. Generally, social losses for any single central banker k are just a function depending on the interest-rate change Δi and his target $\gamma_k \Delta\tilde{i}$, which are represented by

$$L = L(\Delta i, \gamma_k \Delta\tilde{i}(G_{\mathcal{K}})) \tag{3.1}$$

[1] Note that in case $\beta_1 \neq \beta_2$, losses are asymmetric with respect to deviations below and above target $\tilde{x}$.

[2]

$$\Theta(y) = \begin{cases} 0 & \text{if} \quad y \leq 0 \\ 1 & \text{if} \quad y > 0 \end{cases}, \quad y \in \mathbb{R}\,.$$

In contrast to the previous assumption, we abstract from the quadratic shape of social losses and we only claim for $L(.,.)$ to be twice continuously differentiable and strictly convex in the first variable (interest-rate change). Since the following discussion applies for the general case of aggregated losses of a union, in which the target of the affected part is an increasing function in the size of the affected part, we replace Δi with x, $\Delta\tilde{i}(G_{\mathcal{K}})$ with $\tilde{x}(G)$, in which G corresponds to the weight $G_{\mathcal{K}}$ of the affected set $\mathcal{K}$ of the union. At this point, we repeat that $G \in [0,1]$, $\tilde{x}(G=0)=0$ and $\frac{d}{dG}\tilde{x}(G) \geq 0$, while we omit $G=0,1$ if this would lead to unallowable mathematical operations. In this general case of social losses, we show in proposition 4 that aggregated social losses given by the convex combination of the affected and unaffected part

$$\mathcal{L} = GL(x, \tilde{x}(G)) + (1-G)L(x,0) \tag{3.2}$$

have, in combination with single-peakedness at the target $\tilde{x}$, a unique optimum and with some further assumption (see proposition 5) it is generally possible to construct a flexible majority rule, which leads to the first-best solution.

Proposition 4
Suppose social losses are strictly convex in x and single-peaked at $\tilde{x}(G)$, then aggregate social losses have a unique optimum.

Proof of Proposition 4:
Single-peakedness, convexity, and twice continuously differentiability imply

(i) $\frac{\partial L(x,\tilde{x}(G))}{\partial x} \gtrless 0 \quad \text{if} \quad x \gtrless \tilde{x}(G)$

(ii) $\frac{\partial^2 L}{\partial x^2} > 0$

The FOC[3] for the overall social optimum is given by

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \iff \frac{\partial L(x,\tilde{x}(G))}{\partial x} = \frac{G-1}{G} \cdot \frac{\partial L(x,0)}{\partial x} \tag{3.3}$$

Condition (3.3) has a unique solution $x^* \in [0, \tilde{x}(G)]$, since

(i) $\frac{\partial L(x,\tilde{x}(G))}{\partial x}$ and $\frac{\partial L(x,0)}{\partial x}$ are increasing in x. (convexity)

(ii) $\frac{\partial L(x,\tilde{x}(G))}{\partial x} < 0$ and $\frac{\partial L(x,0)}{\partial x} > 0$ for $x \in (0, \tilde{x}(G))$. (single-peakedness)

(iii) $\left.\frac{\partial L(x,\tilde{x}(G)))}{\partial x}\right|_{x=\tilde{x}(G)} = \left.\frac{L(x,0)}{\partial x}\right|_{x=0} = 0$ and $\frac{G-1}{G} < 0$ for $G \in (0,1)$. (TDC[4])

[3] First-order condition.
[4] Twice continuously differentiable.

Hence the solution $x^* \in [0, \tilde{x}(G)]$ of equation (3.3) is a local optimum. The uniqueness of x^* as a global minimum is obtained by the SOC,[5] which is given by

$$\frac{\partial^2 \mathcal{L}_{\mathcal{K}}}{\partial x^2} = G\frac{\partial^2 L(x, \tilde{x}(G))}{\partial x^2} + (1-G)\frac{\partial^2 L(x, 0)}{\partial x^2} > 0 \tag{3.4}$$

Condition (3.4) holds, because of the convexity of $L(x, \tilde{x}(G))$ in x. This implies that $\mathcal{L}$ is globally convex and therefore the local minimum Δx^* is also a global minimum.

■

In proposition 4 we derived the first-best solution but in order to ensure that it is possible to implement this outcome via a flexible majority rule according to proposition 2 in section 2.6.2, we need x^* to be increasing in G. The necessary condition for this property is calculated in the following proposition:

Proposition 5
Suppose $\frac{1}{G^2} \cdot \frac{\partial L(x^*,0)}{\partial x^*} > \frac{\partial^2 L(x,\tilde{x}(G))}{\partial x^* \, \partial \tilde{x}} \cdot \frac{\partial \tilde{x}}{\partial G}$ *then* $x^*(G)$ *is monotonically increasing in* G.

Proof of Proposition 5:
If we insert the optimum $x^*(G)$ into the FOC in equation (3.3) and differentiate both sides with respect to G, we obtain

$$\begin{aligned} \frac{\partial^2 L(x^*(G),\tilde{x}(G))}{\partial (x^*)^2}\frac{\partial x^*}{\partial G} + \frac{\partial^2 L(x^*(G),\tilde{x}(G))}{\partial x^* \, \partial \tilde{x}}\frac{\partial \tilde{x}}{\partial G} &= \frac{1}{G^2}\frac{\partial L(x^*(G),0)}{\partial x^*} + \frac{G-1}{G}\frac{\partial^2 L(x^*(G),0)}{\partial (x^*)^2}\frac{\partial x^*}{\partial G} \\ &\Longleftrightarrow \\ \frac{\partial x^*}{\partial G}\underbrace{\left[\frac{\partial^2 L(x^*(G),\tilde{x}(G))}{\partial (x^*)^2} + \frac{1-G}{G}\frac{\partial^2 L(x^*(G),0)}{\partial (x^*)^2}\right]}_{>0} &= \frac{1}{G^2}\frac{\partial L(x^*(G),0)}{\partial x^*} - \frac{\partial^2 L(x^*(G),\tilde{x}(G))}{\partial x^* \, \partial \tilde{x}}\frac{\partial \tilde{x}}{\partial G} \end{aligned} \tag{3.5}$$

which implies the condition given in proposition 5.

■

Altogether we have shown, that convexity and single-peakedness guarantees a unique optimizer of aggregated social losses. Together with the condition

$$\frac{1}{G^2}\frac{\partial L(x^*,0)}{\partial x^*} > \frac{\partial^2 L(x,\tilde{x}(G))}{\partial x^* \, \partial \tilde{x}}\frac{\partial \tilde{x}}{\partial G}$$

from proposition 5, we know from the proof of proposition 2 in section 2.6.2, that we can construct a flexible majority rule with weighted votes, which leads to the optimal outcome. Furthermore, all other results hold under the same conditions, as the construction of the flexible majority rule without weighted votes and the use of simple

[5]Second order condition.

majority rules do not depend on the quadratic specification of preferences. In the following, we examine the previously addressed loss functions L_1, L_2 and L_3, questioning, if they meet the criterions for the existence of a flexible majority rule. In order to examine a broader family of functions, we relax the assumption of continuously differentiability of functions to the case of distributions and regard two functions which are not differentiable in their extremum.

(i) For $L_1(x, \tilde{x}) = |x - \tilde{x}(G)|^\alpha$ with $(\alpha > 1)^6$ we obtain:[7]

$$\frac{d^2 L_1}{dx^2} = \quad \alpha(\alpha - 1)(x - \tilde{x}(G))(\Theta(x - \tilde{x}(G)) - \Theta(\tilde{x}(G) - x)) \tag{3.6}$$

$$x^*(G) = \quad \underset{x}{\operatorname{argmin}}\{GL_1(x, \tilde{x}(G)) + (1 - G)L_1(x, 0)\} = \frac{\tilde{x}(G)}{1 + A_1(G)} \tag{3.7}$$

with $A_1(G) = \left(\frac{1-G}{G}\right)^{\frac{1}{\alpha-1}}$. From equations (3.6) and (3.7) we directly obtain, that L_1 is strictly convex in x, if $\alpha > 1$, which also implies that $x^*(G)$ is monotonically increasing in G, since we obtain

$$\frac{dx^*(G)}{dG} = \underbrace{x^*(G)}_{>0}\Big(\underbrace{\frac{d\tilde{x}(G)}{dG}}_{>0} + \underbrace{\frac{A_1(G)}{(1 + A_1(G))(\alpha - 1)(1 - G)G}}_{>0}\Big) > 0 \tag{3.8}$$

(ii) For $L_2(x, \tilde{x}) = \Theta(\tilde{x}(G) - x)e^{\beta_1(\tilde{x}-x)} + \Theta(x - \tilde{x})e^{\beta_2(x-\tilde{x}(G))}$ $(\beta_1, \beta_1 > 0)$ we obtain:

$$\frac{d^2 L_2}{dx^2} = \quad \beta_1^2 \Theta(\tilde{x}(G) - x)e^{\beta_1(\tilde{x}-x)} + \beta_2^2 \Theta(x - \tilde{x})e^{\beta_2(x-\tilde{x}(G))} > 0 \tag{3.9}$$

$$\begin{aligned} x^*(G) = &\quad \underset{x}{\operatorname{argmin}}\{GL_2(x, \tilde{x}(G)) + (1 - G)L_2(x, 0)\} \\ = &\quad \begin{cases} 0 & \text{if} \quad \tilde{x}(G) \leq \frac{A_2(G)}{\beta_1} \\ \frac{1}{\beta_1+\beta_2}\left(\beta_1 \tilde{x}(G) - A_2(G)\right) & \text{if} \quad \frac{A_2(G)}{\beta_1} < \tilde{x}(G) < -\frac{A_2(G)}{\beta_2} \\ \tilde{x}(G) & \text{if} \quad \tilde{x}(G) \geq -\frac{A_2(G)}{\beta_2} \end{cases} \end{aligned} \tag{3.10}$$

with $A_2(G) = \ln\left(\frac{\beta_2}{\beta_1}\frac{1-G}{G}\right)$. We can directly see that L_2 is convex and $x^*(G)$ is increasing in G, since $A_2(G)$ is monotonically decreasing in G. Note that there exist parameter constellations, in which firstly $x^*(G)$ is constant with $x^*(G) = 0$ and secondly $x^*(G)$ coincides with the regional optimum of the affected part of the union with $x^*(G) = \tilde{x}(G)$. But this does not affect the functionality of the flexible majority rule.

[6] Note, that for $\alpha \leq 1$ L_1 is not strictly convex in x.

[7] We use $\frac{d|y|}{dy} = -\Theta(y) + \Theta(y)$ and $\frac{d^2|y|}{dy^2} = \delta(y)$, with $\delta(y)$ represents the Dirac delta distribution and $y \in \mathbb{R}$.

(iii) For $L_3 = \frac{1}{\gamma^2}\left(e^{\gamma(x-\tilde{x}(G))} - \gamma(x-\tilde{x}(G))\right)$ with $\gamma > 0$, we obtain

$$\frac{d^2 L_3}{dx^2} = e^{\gamma(x-\tilde{x}(G))} \tag{3.11}$$

$$x^*(G) = \underset{x}{\operatorname{argmin}}\{GL_3(x,\tilde{x}(G)) + (1-G)L_3(x,0)\} = \tilde{x}(G) - A_3(G) \tag{3.12}$$

with $A_3(G) = \frac{1}{\gamma}\ln(G + (1-G)e^{\gamma\tilde{x}(G)}) \in (0, \tilde{x}(G))$. We see that convexity is directly fulfilled and $x^*(G)$ is increasing in G since we have $\frac{dx^*(G)}{dG} = \frac{\gamma G \frac{d\tilde{x}(G)}{dG} + e^{\gamma\tilde{x}(G)} - 1}{\gamma(G+(1-G)e^{\gamma\tilde{x}(G)})} > 0$

Altogether we could show, that our setup of flexible majority rules applies to a broad class of loss functions, which include some concrete loss functions applied in monetary policy beyond the quadratic approach.[8]

3.1.2 Weighted Averaged Shock

In this section we compare our weighted utilitarian welfare criterion of the previous sections with another approach, in which the central bank focusses on weighted economic shocks. In our model, this can be incorporated by defining the aggregated social losses of the union by[9]

$$\bar{\mathcal{L}}_{\mathcal{K}}(\Delta i_{\mathcal{K}}) = \left(\Delta i_{\mathcal{K}} - \Delta\tilde{i}\left(\bar{\epsilon}\left(G_{\mathcal{K}}\right)\right)\right)^2 \tag{3.13}$$

where $\bar{\epsilon}\left(G_{\mathcal{K}}\right)$ is the weighted average shock of the whole union given by

$$\bar{\epsilon}\left(G_{\mathcal{K}}\right) = G_{\mathcal{K}}\epsilon\left(G_{\mathcal{K}}\right) + (1-G_{\mathcal{K}})\epsilon(0) = G_{\mathcal{K}}\epsilon\left(G_{\mathcal{K}}\right) \tag{3.14}$$

$\bar{\mathcal{L}}_{\mathcal{K}}$ is minimized by

$$\Delta i_{\mathcal{K}} = \Delta\bar{i}_{\mathcal{K}} = \Delta\tilde{i}\left(G_{\mathcal{K}}\epsilon\left(G_{\mathcal{K}}\right)\right) \tag{3.15}$$

If we compare $\Delta\bar{i}_{\mathcal{K}}$ with the minimizer $\Delta i^*_{\mathcal{K}}$ of the weighted utilitarian loss function introduced, we observe that $\Delta\bar{i}_{\mathcal{K}}$ has the same properties as $\Delta i^*_{\mathcal{K}}$ (i.e both are increasing in $G_{\mathcal{K}}$ and $\Delta\bar{i}_{\mathcal{K}}, \Delta i^*_{\mathcal{K}} < \Delta\tilde{i}_{\mathcal{K}}$ for $0 < G_{\mathcal{K}} < 1$). Hence, we can again apply our construction principle and flexible majority rules will also be welfare-enhancing under such circumstances.

[8] Note that another widely applied type of loss function, the constant relative risk-averse type $L_4 = \frac{(x-\tilde{x}(G)+1)^{(1-\rho)}}{1-\rho}$, only meets the convexity criterion for $\rho < 0$, but this case is similar to L_1.

[9] We are grateful to a referee for this suggestion.

3.2 Heterogeneous Shocks

So far our model induces a division of the union into two parts. One part is affected by the shock, the other part is not affected. In this section, we extend our model to the case, whereby the union can be hit by different shocks at the same time. In particular, we distinguish between two different cases.

First, we assume, that shocks can differ in size, but not in their direction. For example, suppose that one part is hit by a large shock, another part by a smaller one and the remaining part is not affected. We continue to assume that the shock increases in the size of the affected region. This means, we allow that some of the 2^N possibilities of forming a region $\mathcal{K}$ and representing shocks in the baseline-model may occur simultaneously.

Second, we assume, that shocks can differ also in direction (i.e positive and negative), but with the restriction, that only *one positive* and *one negative* shock can occur within one period. This means, that in this setup, the union is divided into three regions. One part desires an increase in interest rates, another part a decrease, and the remaining part wants no change at all.

Since we have shown in the baseline-model, that decision rules with weighted votes are better than without, in the following we compare only the FM_w-rule and SM_w-rule and abandon the FM_{nw}-rule and SM_{nw}-rule.

3.2.1 Extended Shock Scenario

In our baseline-model, the monetary union is separated into two parts by a shock whereby one part desires to remain at the status quo and the other part desires one unique interest-rate change. This means, that the countries of the affected part are assumed to have all the same preferences about an interest-rate change. Now we introduce an extended shock scenario denoted by $\mathcal{E}$, which allows us to divide the union into more than two subgroups after $\mathcal{E}$ has occurred. In an extended shock scenario $\mathcal{E}$, we still assume that there is one part of the union that is not affected by the shock and all countries forming this group do not desire an interest-rate change. But the other part affected by "some" shock is not treated as a homogeneous group anymore. Now, it is possible, that different countries of the latter group desire different interest-rate changes. We define this in a similar way as in the baseline shock scenario described in chapter 2.3. We assume, that affected countries represented by $\mathcal{K}$ are forming subgroups $\mathcal{K}_i$ with $\mathcal{K}_i \cap K_j = \emptyset$ ($i \neq j$ and $i, j \in \{1, 2, \dots, 2^N\}$), since a country cannot simultaneously be an element of two subgroups and $\bigcup_i \mathcal{K}_i = \mathcal{K}$. The countries forming a subset $\mathcal{K}_i$ are assumed to be affected by the same shock, which is defined by the same function $\epsilon(.)$ introduced in equation 2.3. Thus, since the subset $\mathcal{K}_i$ has the aggregated weight $G_{\mathcal{K}_i}$, this group is affected by a shock of size $\epsilon(G_{\mathcal{K}_i})$. An extended shock scenario is formally defined by

$$\begin{aligned} &\mathcal{E} =: \{\epsilon(G_{\mathcal{K}_1}), \epsilon(G_{\mathcal{K}_2}), ..., \epsilon(G_{\mathcal{K}_m})\}\,, \\ &\textstyle\bigcup_{i=1}^m \mathcal{K}_i = \mathcal{K} \quad \mathcal{K}_i \cap K_j = \emptyset,\ m \in \{1, 2, \dots, 2^N\} \quad \text{and} \quad \mathcal{K}_i \neq \emptyset \end{aligned} \tag{3.16}$$

where we assumed w.l.o.g $\mathcal{K}_i \neq \emptyset$, because this excludes the trivial case $\mathcal{K} = \emptyset$ and degeneracies, if we would allow an extended shock scenario $\mathcal{E}$ with $0 \in \mathcal{E}$.

In an extended shock scenario $\mathcal{E}$, we have $m+1$ groups, which have in general different preferences concerning the interest-rate change. The countries in the set $\mathcal{N} \setminus \bigcup_{i=1}^m \mathcal{K}_i$, who prefer the status-quo and the countries forming the sets $\mathcal{K}_i$ $(i = 1, 2, \dots m)$, who desire interest-rate changes[10] of $\Delta\tilde{i}(\epsilon(G_{\mathcal{K}_i}))$. This is again similarly defined as in equation 2.3 and we use in the following

$$\Delta\tilde{i}_{\mathcal{K}_i} = \Delta\tilde{i}(\epsilon(G_{\mathcal{K}_i})) \quad \text{and} \quad \Delta i_{\mathcal{E}} = \Delta i(\mathcal{E}) \tag{3.17}$$

where $\Delta i(\mathcal{E})$ is the interest-rate change in extended shock scenario $\mathcal{E}$ under some decision rule, while $\Delta i_{\mathcal{E}}^{FM_w}$ and $\Delta i_{\mathcal{E}}^{SM_w}$ are the explicit interest-rate changes under FM_w and SM_w respectively.

For example consider a union consisting of four countries with the weights g_1, g_2, g_3 and g_4. A possible extended shock scenario is then given by $\mathcal{E} = \{\epsilon(g_1 + g_4), \epsilon(g_2)\}$. This implies that countries 1 and 4 desire together an interest-rate change of $\Delta\tilde{i}(\epsilon(g_1 + g_4))$, country 2 desires a change of $\Delta\tilde{i}(\epsilon(g_2))$, while country 3 desires to remain at the status quo.

With this extension, we obtain with the utilitarian aggregation for union-wide social losses in an extended shock scenario $\mathcal{E}$

$$\begin{aligned} \mathcal{L}_{\mathcal{E}} &= \textstyle\sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i} \left(\Delta i_{\mathcal{E}} - \Delta\tilde{i}_{\mathcal{K}_i}\right)^2 + (1 - \sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i}) \left(\Delta i_{\mathcal{E}}\right)^2 \\ &= \left(\Delta i_{\mathcal{E}} - \textstyle\sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i} \Delta i_{\mathcal{K}_i}\right)^2 + C_{\mathcal{E}} \end{aligned} \tag{3.18}$$

and $C_{\mathcal{E}} := \sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i} \left(\Delta\tilde{i}_{\mathcal{K}_i}\right)^2 - \left(\sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i} \Delta\tilde{i}_{\mathcal{K}_i}\right)^2$ a term not depending on the

[10] Note that although $\mathcal{K}_i \cap K_j = \emptyset$, it is possible that $G_{\mathcal{K}_i} = G_{\mathcal{K}_j}$ and $\epsilon(G_{\mathcal{K}_i}), \epsilon(G_{\mathcal{K}_j}) \in \mathcal{E}$ $(i \neq j)$. But although both groups desire the same interest-rate change, this differs from the case, where these sets form a common subgroup, since then, they desire an interest-rate change $\Delta\tilde{i}(\epsilon(G_{\mathcal{K}_i}) + \epsilon(G_{\mathcal{K}_j})) > \Delta\tilde{i}(\epsilon(G_{\mathcal{K}_i}))$.

implemented interest-rate change. Furthermore, the number of possible extended shock scenarios $N_{\mathcal{E}} := \sum_{l=0}^{N} \binom{N}{l} \left[\sum_{k=1}^{l} \binom{l}{k} \right]$, including the shock scenarios of the baseline model.[11]

We observe, that in an extended shock scenario $\mathcal{E}$ the optimal interest-rate change is given by

$$\Delta i_{\mathcal{E}}^{*} = \sum_{\text{all}\, \mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i} \Delta \tilde{i}_{\mathcal{K}_i} \tag{3.19}$$

the weighted sum of all desired interest-rate changes $\Delta \tilde{i}_{\mathcal{K}_i}$ in the affected subsets $\mathcal{K}_i$.

The time-line with an extended shock scenario is similar as in figure 2.1. We have only to replace the single shock ϵ with the extended shock scenario $\mathcal{E}$. Anything else remains the same, but note that the desired interest rates (i.e. interest-rate changes) include now all subgroups $\mathcal{K}_i$ with desired interest-rate changes of $\Delta \tilde{i}_{\mathcal{K}_i}$.

Again, we want to compare the FM_w-rule, formerly derived in equation 2.25 with the SM_w-rule. We repeat, that this implies, regarding equations 2.27, that with a share of G votes one can implement an interest-rate change of

$$\Delta i^{FM_w}(G) = G \Delta \tilde{i}(G) \tag{3.20}$$

where we dropped the index $\mathcal{K}$ and we assume that $G \in \{G_{(1)}, G_{(2)}, ..., G_{(2^N)}\}$ (for the ordering see equation 2.26). In the following, we show, that in general the FM_w-rule is not first-best anymore and that even in the simple case, in which the union is only divided into three parts by an extended shock scenario, there exist extended shock scenarios, such that SM_w outperforms FM_w. In order to more deeply understand the relation between the FM_w-rule and the SM_w-rule, we examine the case, that the monetary union is divided in three parts by an extended shock scenario in detail. It turns out, that although not under all circumstances, but in most cases, the FM_w-rule is still better than the SM_w-rule. At least, we can show, that if we assume that $\Delta \tilde{i}(G)$ is a linear function of G and all extended shock scenarios are uniformly distributed (i.e. every extended shock scenario $\mathcal{E}$ occurs with the same probability, denoted by $p_{\mathcal{E}}$), that expected losses under the FM_w-rule are smaller than under the SM_w-rule.

[11] $\binom{0}{1} := 1$.

3.2.1.1 FM_w-rule and First-Best

In order to characterize the effect of the concept of extended shock scenarios onto the FM_w-rule, we provide a simple example, which shows that the FM_w does not always lead anymore to the first best outcome. The intuition is as follows: Suppose an extended shock scenario $\mathcal{E}$ has occurred, then the optimal interest-rate change for the union would be $\Delta i^*_{\mathcal{E}} = \sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i} \Delta\tilde{i}_{\mathcal{K}_i}$. But if all affected regions $\mathcal{K}_i$ form a coalition, they can change interest rate up to

$$\Delta i_{\mathcal{E}} = \left(\sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i} \right) \Delta\tilde{i}(\sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i}) > \sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i} \Delta\tilde{i}_{\mathcal{K}_i}$$

In general, it is possible that there exists a desired interest-rate change $\Delta\tilde{i}_{\mathcal{K}_i}$ of an affected region, which is larger than the optimal interest-rate change $\Delta i^*_{\mathcal{E}}$ and simultaneously this region joins a coalition with other (larger) affected regions $\mathcal{K}_j$ in order to implement an interest-rate change of at least $\Delta\tilde{i}_{\mathcal{K}_i}$. This can be seen in the following example:

Suppose

(i) the union consists of three countries with $(g_1, g_2, g_3) = (0.25, 0.35, 0.4)$.

(ii) $\Delta\tilde{i}(\epsilon(G))$ is given by $\Delta\tilde{i}(\epsilon(G)) = G$.

(iii) Country 2 and 3 are separately affected by a shock, which means $\mathcal{E} = \{\epsilon(g_2), \epsilon(g_3)\} = \{\epsilon(0.35), \epsilon(0.4)\}$.

The optimal interest-rate change is then $\Delta i^*_{\mathcal{E}} = 0.35^2 + 0.4^2 = 0.2825$. But since countries 2 and 3 can change together the interest rate up to $\Delta i^{FM_w}(0.35 + 0.4) = 0.75^2 = 0.5625$, the FM_w-rule implements in this extended shock scenario the desired interest-rate change of $\Delta\tilde{i}(g_2) = 0.35$ of the smaller affected country, an interest-rate change which is larger than the optimal interest-rate change. The reason for this is, that both affected countries are allowed to change the interest rate even by a larger amount than the desired change of 0.4 of the larger country. But since the smaller country could not gain anything by exceeding its desired interest-rate change, country 2 will leave the coalition after having voted for its desired interest-rate change of 0.35.

Following the reasoning of the previous example, we prove proposition 6. We claim that the property of a possible violation of the first-best condition in the baseline-model by the FM_w-rule in an extended shock scenario $\mathcal{E}$ depends on a specific shape of the function $\Delta\tilde{i}(G)$.

Proposition 6
Suppose $\Delta\tilde{i}(G)$ is strictly concave or convex, or concave and convex parts of $\Delta\tilde{i}(G)$ alternate. Then, there exist a shock function $\Delta\tilde{i}(G)$ and a specific extended shock scenario $\mathcal{E}$, such that the implemented interest-rate change, applying the FM_w-rule, is not optimal: $\Delta i_{\mathcal{E}}^{FM_w} \neq \Delta i_{\mathcal{E}}^{*}$.

Proof of proposition 6:
In the example with $(g_1, g_2, g_3) = (0.25, 0.35, 0.4)$, $\Delta\tilde{i}(\epsilon(G)) = G$ and extended shock scenario $\mathcal{E}_j = \{\epsilon(g_2), \epsilon(g_3)\}$, we have seen that $\Delta i_{\mathcal{E}}^{*} \neq \Delta i_{\mathcal{E}}^{FM_w}$. But since $\Delta\tilde{i}(\epsilon(G)) = G$ is linear in G, there exists always a slight perturbation $\delta(G)$ $(0 < |\delta(G)| \ll G)$ of $\Delta\tilde{i}(G)$=G, such that the perturbed shock function $\Delta\tilde{i}_{\delta}(G) := \Delta\tilde{i}(G) + \delta(G)$ has a non-zero curvature and $\Delta i_{\mathcal{E}}^{FM_w} \neq \Delta i_{\mathcal{E}}^{*}$ still holds.

■

Proposition 6 shows, that in general FM_w is not optimal anymore under the circumstances of extended shock scenarios. The outcome depends highly on the shape of the functional forms and the size of the affected regions.

3.2.1.2 FM_w-rule versus SM_w-rule

We have already seen, that FM_w is not first-best anymore. Thus, we turn to the comparison of FM_w and SM_w. Since it is very plausible, that with no further restrictions, we always can find an example (i.e. an extended shock scenario $\mathcal{E}$) in which SM_w is ex post better than FM_w. Therefore, we restrict ourselves to the case, where the union is only divided into three parts: The first part is not affected and the affected part is divided into two subgroups. We assume that the non-affected part has weight G_0 and w.l.o.g. $G_1 \leq G_2$ are the corresponding weights of the two affected regions, with $G_0, G_1, G_2 \in \{G_{(1)}, G_{(2)}, ..., G_{(2^N)}\}$, with $G_0 + G_1 + G_2 = 1$. In the following, we show that even in this simple setup, the relation between FM_w and SM_w is not unique. In table 3.1 we provide all outcomes comparing FM_w and SM_w for all possible relations[12] of G_1 and G_2.

The verification for the entries in table 3.1 are given in appendix 6.1 (A Proofs) in argumentation 1.

In table 3.1 we see, that in most cases the FM_w-rule is better or least as good as the SM_w-rule. But there are also three cases in which, in specific parameter settings, the

[12]Note, that this is sufficient, since G_1 and G_2 fully determine G_0 by $G_0 = 1 - G_1 - G_2$.

		$G_2 > \frac{1}{2}$	$G_2 \leq \frac{1}{2} \wedge G_1 + G_2 > \frac{1}{2}$	$G_1 + G_2 \leq \frac{1}{2}$
$\Delta i_{\mathcal{E}}^{FM_w} = (G_1 + G_2)\Delta\tilde{i}(G_1 + G_2)$		$+$	$+$	$\pm$
$\Delta i_{\mathcal{E}}^{FM_w} = \Delta\tilde{i}(G_1)$	$\geq \Delta i_{\mathcal{E}}^{*}$	$+$	$=$	$\pm$
	$< \Delta i_{\mathcal{E}}^{*}$	$\pm$	$=$	$+$
$\Delta i_{\mathcal{E}}^{FM_w} = G_2\Delta\tilde{i}(G_2)$		$+$	$+$	$+$

Table 3.1:
This table shows the comparison of the FM_w-rule and SM_w-rule, when the union is divided into three regions by an extended shock scenario for the different possible cases. G_1 and G_2 are the weights of the affected regions (w.l.o.g. $0 < G_1 \leq G_2$).

"$+$" means FM_w is better than SM_w.

"$=$" means FM_w is as good as SM_w.

"$\pm$" means that the comparison of FM_w and SM_w is ambiguous, and the decision, which decision rule is the better one, depends simultaneously on the size of the affected regions and the curvature of the shock function $\Delta i(G)$.

SM_w-rule might be better than the FM_w-rule. This implies that ex ante, we cannot generally conclude that the FM_w-rule is better than the SM_w-rule, because if the cases where SM_w is better than FW_w have a very large probability of occurrence, they can totally outweigh the cases where FM_w is better than SM_w, regarding the expected value of the overall loss function. But nevertheless, if the probability distribution does not put that much weight on the possibilities whereby the SM_w-rule is better than the FM_w-rule, then, ex ante FM_w will be better. This is also supported by the fact, that there exists no case, in which SM_w is definitely better than FM_w, independent of the shape of the shock function.

As a consequence of table 3.1, we obtain the following corollary

Corollary 3
Suppose the weights g_k of the countries are given and there exists an extended shock scenario $\mathcal{E}$ such that $\mathcal{L}_{\mathcal{E}}^{FM_w} > \mathcal{L}_{\mathcal{E}}^{SM_w}$. Then, there exists a probability distribution $P(X = \mathcal{E})$ with the probability $p_{\mathcal{E}}$ for an extended shock scenario $\mathcal{E}$, such that

$$E(\mathcal{L}^{FM_w}) > E(\mathcal{L}^{SM_w}) \tag{3.21}$$

where $E(\mathcal{L}^{FM_w}) = \sum_{\text{all}\,\mathcal{E}} p_{\mathcal{E}} \mathcal{L}_{\mathcal{E}}^{FM_w}$ and $E(\mathcal{L}_{\mathcal{E}}^{SM_w})$ is defined accordingly.

Proof of corollary 3:
Since all extended shock scenarios include also the shock scenarios that the union is only separated into two regions, there exists a $\mathcal{E}' \neq \mathcal{E}$ with $\mathcal{L}_{\mathcal{E}'}^{FM_w} < \mathcal{L}_{\mathcal{E}'}^{SM_w}$ ($\mathcal{E}'$ another extended shock scenario). Then choose $p_{\mathcal{E}}$ such that $p_{\mathcal{E}}(\mathcal{L}_{\mathcal{E}}^{FM_w} - \mathcal{L}_{\mathcal{E}}^{SM_w}) > \sum_{\text{all}\,\mathcal{E}'} p_{\mathcal{E}'}(\mathcal{L}_{\mathcal{E}'}^{FM_w} - \mathcal{L}_{\mathcal{E}'}^{SM_w})$ which is always possible for $p_{\mathcal{E}}$ almost 1 and all $p_{\mathcal{E}'}$ very small.

■

Corollary 3 shows, that it is also possible that ex ante the FM_w-rule is worse than the SM_w-rule. Of course, this not very surprisingly, since a finite probability distribution is just a weighting of the events and can therefore be chosen properly, but nevertheless it is still interesting, that there exist both, parameter constellations in which SM_w is better than FM_w in a single extended shock scenario and also ex ante expected losses of SM_w are smaller than the losses of FM_w.

3.2.1.3 Ex Ante Comparison of FM_w and SM_w

Since we have seen in chapter 3.2.1.2, that the relation between the FM_w-rule and the SM_w-rule highly depends on the functional form $\Delta\tilde{i}(.)$, the extended shock scenario $\mathcal{E}$, and the probability distribution represented by $p_{\mathcal{E}}$, we conclude with an investigation of a very simple setup. First, we assume a linear dependency between desired interest-rate changes and the economic weight G of affected regions. As we have already seen, then FM_w is not always first-best anymore and there exists extended shock scenarios $\mathcal{E} = \{G_1, G_2\}$, such that SM_w is better than FM_w (see table 3.1). Additionally, we assume, that all possible extended shock scenarios occur with the same probability. With this assumptions, we can show the following proposition 7.

Proposition 7
Suppose the shock function, including the desired interest-rate changes, is given by $\Delta\tilde{i}(G) = A \cdot G$ $(A > 0)$, $p_{\mathcal{E}'} = p_{\mathcal{E}}$ $(\forall\, \mathcal{E}, \mathcal{E}')$ *and the union consists of three countries, represented by their weights* g_1, g_2 *and* g_3. *Then*

$$E(\mathcal{L}^{FM_w}) < E(\mathcal{L}^{SM_w}) \tag{3.22}$$

In order to proof proposition 7, we show, that if the weight of the largest member of the union is not greater than $\frac{1}{2}$, the FM_w-rule is ex post not worse than the SM_w-rule.

Lemma 1
Suppose the union consists of three countries with weights $g_i \leq \frac{1}{2}$ $(i, = 1, 2, 3)$ *and* $\Delta\tilde{i}(G) = G \Longrightarrow \mathcal{L}_{\mathcal{E}}^{FM_w} \leq \mathcal{L}_{\mathcal{E}}^{SM_w}$ *for all* $\mathcal{E}$.

The proofs of lemma 1 and proposition 7 are given in appendix 6.1 (A Proofs).

The setup of a linear shock function and the uniform probability distribution can be interpreted as a benchmark case, since in practice, it is not easy to determine the curvature of the shock function and the true probability distribution. Therefore, we choose the linear form, as a compromise between concavity and convexity, and the uniform probability distribution is motivated by the fact, that if we do not know anything about the probability distribution, the uniform distribution can be interpreted as the distribution over all possible distributions. Altogether we have shown, that with further restrictions on parameters and functions, FM_w tends still to be “better” than SM_w.

3.2.2 Separating Shock Scenario

So far, we consider only shock scenarios in which affected countries desire only positive interest-rate changes. In the previous chapter, we already extended our baseline-model with the possibility, that the monetary union is divided by a shock into more than two regions. But nevertheless the desired interest rates of affected countries, although now they can differ, are still only positive. In some cases, this does not seem very plausible in practice, especially with regard to the ECB, where we often have the discussion between a “dovish” or “hawkish” monetary policy within the ECB governing council. For example, it is almost certain, that in July 2008 in the prelude of the interest rate rise of the ECB, there was a discussion in the central bank board meeting whether to raise or to cut interest rates. In order to incorporate preferences differing in direction in our model, we introduce another simple shock scenario, denoted by separating shock scenario within our framework. A separating shock scenario is derived from an extended shock scenario with $|\mathcal{E}| = 2$ (i.e. now we allow for the division of the affected region $\mathcal{K}$ into two subsets, denoted by $\mathcal{K}^+$ and $\mathcal{K}^-$). In more detail a separating shock scenario $\mathcal{E}^\pm$ is defined as:

$$\mathcal{E}^\pm = \{\epsilon(G_{\mathcal{K}^+}), \epsilon(-G_{\mathcal{K}^-})\} \tag{3.23}$$

with $G_{\mathcal{K}^+}, G_{\mathcal{K}^-} \in \{G_{(1)}, G_{(2)}, ..., G_{(2^N)}\}$, $\mathcal{K}^+ \cap \mathcal{K}^- = \emptyset$ and $\mathcal{K}^+ \cup \mathcal{K}^- = \mathcal{K}$ in which in this case $\mathcal{K}$ denotes only the property that a country, element of $\mathcal{K}$, is affected. Furthermore, we have to extend $\epsilon(.)$ to the negative part of the number line. This is done by point reflection of ϵ at the origin. This implies that $\epsilon(-G_{\mathcal{K}^-}) = -\epsilon(G_{\mathcal{K}^-})$. Consider for example a union that consists of three countries with weights g_1, g_2 and g_3. Then, we obtain 16 additional separating shock scenarios complementing the one-direction shock scenarios of the baseline model.

$$\begin{array}{lll}
\{\epsilon(g_1), \epsilon(-g_2)\}, & \{\epsilon(g_1), \epsilon(-g_3)\}, & \{\epsilon(g_2), \epsilon(-g_3)\}, \\
\{\epsilon(g_1 + g_2), \epsilon(-g_3)\}, & \{\epsilon(g_1 + g_3), \epsilon(-g_3)\}, & \{\epsilon(g_2 + g_3), \epsilon(-g_1)\} \\
\{\epsilon(-g_1), \epsilon(+g_2)\}, & \{\epsilon(-g_1), \epsilon(+g_3)\}, & \{\epsilon(-g_2), \epsilon(+g_3)\}, \\
\{\epsilon(-g_1 - g_2), \epsilon(+g_3)\}, & \{\epsilon(-g_1 - g_3), \epsilon(+g_3)\}, & \{\epsilon(-g_2 - g_3), \epsilon(+g_1)\}
\end{array}$$

For completeness, we have to define the desired interest-rate changes if a country is an element of negatively hit set $\mathcal{K}^-$. For simplicity, we also assume, $\Delta\tilde{i}(.)$ to be point symmetric to the origin. This implies:

$$\Delta\tilde{i}(\epsilon(-G_{\mathcal{K}^-})) = -\Delta\tilde{i}(\epsilon(G_{\mathcal{K}^-})) = -\Delta\tilde{i}(G_{\mathcal{K}^-}) \tag{3.24}$$

Again, we introduce the short-cuts

$$\begin{aligned}
\Delta\tilde{i}_{\mathcal{K}^+} &:= \Delta\tilde{i}(\epsilon(G_{\mathcal{K}^+})) \\
\Delta\tilde{i}_{\mathcal{K}^-} &:= \Delta\tilde{i}(\epsilon(-G_{\mathcal{K}^-})) = -\Delta i(\epsilon(G_{\mathcal{K}^-})) \\
\Delta i_{\mathcal{E}^\pm} &:= \Delta i(\mathcal{E}^\pm)
\end{aligned} \tag{3.25}$$

With this notation we obtain for the aggregated social loss function:[13]

[13] Note that $\Delta\tilde{i}_{\mathcal{K}^-}$ is negative.

$$
\begin{aligned}
\mathcal{L}_{\mathcal{E}^{\pm}} &= G_{\mathcal{K}^{+}}\left(\Delta i_{\mathcal{E}^{\pm}} - \Delta\tilde{i}_{\mathcal{K}^{+}}\right)^{2} + G_{\mathcal{K}^{-}}\left(\Delta i_{\mathcal{E}^{\pm}} - \Delta\tilde{i}_{\mathcal{K}^{-}}\right)^{2} + \left(1 - G_{\mathcal{K}^{+}} - G_{\mathcal{K}^{-}}\right)\left(\Delta i_{\mathcal{E}^{\pm}}\right)^{2} \\
&= \left(\Delta i_{\mathcal{E}^{\pm}} - \left(\overbrace{G_{\mathcal{K}^{+}}\Delta\tilde{i}_{\mathcal{K}^{+}}}^{>0} + \underbrace{G_{\mathcal{K}^{-}}\Delta\tilde{i}_{\mathcal{K}^{-}}}_{<0}\right)\right)^{2} + C^{\pm}
\end{aligned}
\tag{3.26}
$$

with $C^{\pm} := G_{\mathcal{K}^{+}}\left(\Delta\tilde{i}_{\mathcal{K}^{+}}\right)^{2} + G_{\mathcal{K}^{-}}\left(\Delta\tilde{i}_{\mathcal{K}^{-}}\right)^{2} - \left(G_{\mathcal{K}^{+}}\Delta\tilde{i}_{\mathcal{K}^{+}} + G_{\mathcal{K}^{-}}\Delta\tilde{i}_{\mathcal{K}_{j}^{-}}\right)^{2}$ independent from $\Delta i_{\mathcal{E}^{\pm}}$. Hence, the optimal interest-rate change is given by

$$
\Delta i = \Delta i^{*}_{\mathcal{E}^{\pm}} = \overbrace{G_{\mathcal{K}^{+}}\Delta\tilde{i}_{\mathcal{K}^{+}}}^{>0} + \underbrace{G_{\mathcal{K}^{-}}\Delta i_{\mathcal{K}^{-}}}_{<0} \tag{3.27}
$$

Observe, that it is not possible to apply directly the FM_w-rule defined in the baseline-model, since there is no instruction on how to deal with the fact, that we have two regions differing in their desired interest-rate changes. In order to address this question, we introduce a two-step voting process for a flexible majority rule. In a first step, central bankers decide secretly using the SM_w-rule whether the interest rate will be lowered or raised,[14] but the votes are collected personalized in the sense, that after the voting, the aggregated weight of the votes for a raising, a lowering, and abstention are known[15] (similar pre-stages in a voting process about monetary policy are discussed in Bó (2006) and Riboni and Ruge-Murcia (2010)). Second, the losing weights without abstentions are aggregated and the decision about the amount of the interest-rate change follows a modified functional form of required majorities compared to the FM_w-rule. This flexible majority rule is denoted by $FM_w^{\mathcal{E}^{\pm}}$ and defined as

<u>$FM_w^{\mathcal{E}^{\pm}}$-rule:</u>

(i) The weights of the losing votes in the SM_w-voting stage about the direction of an interest-rate change are aggregated without the votes of abstention and denoted by $\bar{G} \in \{G_{(1)}, G_{(2)}, ..., G_{(2^N)}\}$.

(ii) The monetary policy committee decides about the amount of the interest-rate change in which a group with a share of $G \in \{G_{(1)}, G_{(2)}, ..., G_{(2^N)}\}$ can change the interest rate according to

[14]Note, that a set $\mathcal{K}^{+}$ or $\mathcal{K}^{-}$ does not necessarily need a weight of $G_{\mathcal{K}^{+}}$ or $G_{\mathcal{K}^{-}}$ larger than 50% , since we never excluded abstention and, in contrast to the baseline-model, for a central banker it is not a priori obvious to fail with abstention.

[15]We disregard possible problems in a dynamic setup arising from the fact, that since the set of members of the monetary union is finite and generally $g_k \neq g_l$ ($l \neq k$ and $k, l \in \{1, 2, ...N\}$), ex post it could be partially possible to assign the voting to the decisions of, “raise, lower, and abstention”.

$$\Delta i^{FM_w^{\mathcal{E}\pm}}(\nu, G, \bar{G}) := (-1)^\nu (G\Delta i(G) - \bar{G}\Delta i(\bar{G})) \tag{3.28}$$

with $\nu = 0$ if the outcome of the SM_w-voting stage has been an increase and $\nu = 1$ if the outcome has been a decrease.

Now, we fully specify the $FM_w^{\mathcal{E}\pm}$-two-stage-rule:

$\underline{FM_w^{\mathcal{E}\pm}\text{-two-stage-rule}}$

(i) Central bankers decide with the SM_w-rule whether the interest is lowered or raised.

(ii) Central bankers decide according to the $FM_w^{\mathcal{E}\pm}$-rule about the ultimate interest-rate change.

In the following we compare the $FM_w^{\mathcal{E}\pm}$-two-stage-rule and the SM_w-rule and show the following proposition:

Proposition 8

(i) The $FM_w^{\mathcal{E}\pm}$-two-stage-rule is not first-best in every separated shock scenario.

(ii) The $FM_w^{\mathcal{E}\pm}$-two-stage-rule is never worse than the SM_w-rule in every separated shock scenario.

The proof is given in appendix 6.1 (A Proofs).

The intuition for this result is as follows. In the first step of the $FM_w^{\mathcal{E}\pm}$-two-stage-rule we apply the SM_w-rule to determine only the direction of an interest-rate change. Since in the second step, this information about the direction can already be incorporated into the following decision about the size of the interest-rate change, the outcome can be improved compared to a decsion process in which we apply only the SM_w-rule.

3.3 Dynamic Setting

We already mentioned, that so far, we do not consider a dynamic setting within our framework (i.e. a sequential interest-rate change from period $t = 0$ to period $t = 1$ to period $t = 2$ etc.). This addresses the problem of how to change interest rates from period to period. Consider for example the following sequence of events.

1. $t = 0$, the union is in a long-run equilibrium with $i_0 = i^*$.
2. At the end of $t = 0$ a shock occurs, dividing the union into two regions, one is affected by a shock and desires an interest-rate change, while the other region prefers the status quo.
3. At the beginning of period $t = 1$, the central bankers change, due to their applied decision rule, the interest rate and we obtain $i_1 = i_0 + \Delta i_1 = i^* + \Delta i_1$.
4. Assume the shock decays within one period, no other shock occurs and thus at the end of period $t = 1$, the union returns to an economic environment close to the long-run equilibrium. In this case, it is plausible, that the central bank committee prefers to return to an interest rate of i^*. This means, in the voting process, a retraction of the interest rate increase Δi_1 of the previous period should receive a majority. But since our framework defined the loss function of a single central banker as $L_t^k = (i_t - \tilde{i}_t^k)^2$ with $\tilde{i}_t^k = \gamma_k \Delta \tilde{i}_t(\epsilon(G_{\mathcal{K}})) + i_{t-1}$ (see equations 2.1 and 2.3), this implies that, from period $t = 1$ to period $t = 2$, when no new shock occurs, the preferred interest-rate change of central banker k would be $\tilde{i}_2^k = i_1 + 0 = i^* + \Delta i_1$. Hence, an interest rate cut back to i^* would fail to receive a majority.

Therefore, we assume that central banker's calculate their losses concerning a long-run equilibrium interest rate i^* as bliss point,[16] which changes their losses to

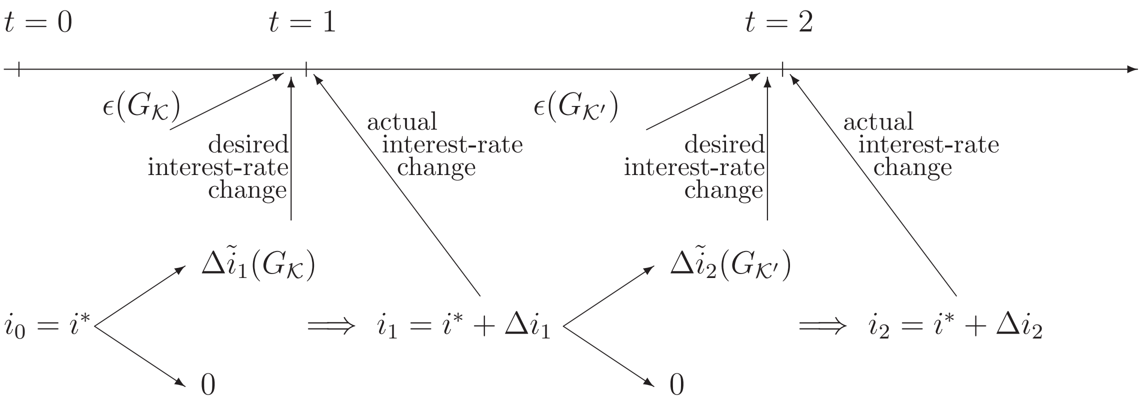
$$L_t^k = (i^* - \tilde{i}_t^k)^2 \tag{3.29}$$

Similarly, we calculate the desired interest rate $\tilde{i}_t^k$ of the k-th central banker and the actual interest-rate change i_t in period t with i^* as reference:

$$\begin{aligned} \tilde{i}_t^k &= \gamma_k \Delta \tilde{i}_t(\epsilon(G_{\mathcal{K}})) + i^* \\ i_t &= i^* + \Delta i_t \end{aligned} \tag{3.30}$$

[16]This long-run interest rate i^* can also depend on time, for example if the conditions of the general framework of the union are changing, but we will skip this in the following, because we assume that this change is small compared to a desired change, if a member is hit by a shock. Another possibility is to say, that central bankers can only unanimously change the long-run bliss point i^*.

Furthermore, in order to keep our dynamic setup as simple as possible, we assume, that shocks decay, within one period and from period to period we apply our baseline-model. This means, every shock in every period divides the union only in two regions. The dynamic sequence of events is illustrated in figure 3.1.



$$L_0 = 0 \qquad \begin{array}{ll} L_1^k = (\Delta i_1 - \Delta\tilde{i}_1(G_{\mathcal{K}}))^2,\ k \in \mathcal{K} & L_2^k = (\Delta i_2 - \Delta\tilde{i}_2(G_{\mathcal{K}'}))^2,\ k \in \mathcal{K}' \\ L_1^k = (\Delta i_1)^2,\ k \notin \mathcal{K} & L_2^k = (\Delta i_2)^2,\ k \notin \mathcal{K}' \quad (\mathcal{K}' \subseteq \mathcal{N}) \end{array}$$

Figure 3.1:

Sequence of events in a dynamic setting.

In $t = 0$ the union starts in the long-run equilibrium i^*. At the end of period $t = 0$ the shock $\epsilon(G_{\mathcal{K}})$ occurs in which we also allow for negative shocks $\epsilon(-G_{\mathcal{K}})$, similarly defined as in chapter 3.3. At the beginning of period $t = 1$, an interest-rate change of Δi_1 with respect to i^* is implemented according to the applied decision rule in the union. At the end of period $t = 1$, $\epsilon(G_{\mathcal{K}})$ has vanished and another shock $\epsilon(G_{\mathcal{K}'})$ occurs. At this point, only the central bankers of region $\mathcal{K}'$ desire an interest-rate change with respect to i^*. Observe, that if we choose i_1 as reference in case of $\Delta i_1 < \Delta\tilde{i}_2(G_{\mathcal{K}'})$ it is possible, that central bankers in region $\mathcal{K}'$ desire an increase of interest rates, while central bankers of the region $\mathcal{N} \setminus \mathcal{K}'$ desire an interest-rate cut with respect to i_1. But since our reference point is i^*, only the region $\mathcal{K}'$ desires an interest-rate change, while the other central bankers prefer the status quo.

With this setting and the change in loss functions, we still end up in the social optimum by applying the flexible majority rule:

Proposition 9
Suppose an occurring shock ϵ dividing the union into two regions lasts only for one period and central bankers try to minimize their loss functions

$$L_t^k = (i^* - \tilde{i}_t^k)^2 \tag{3.31}$$

from period to period. Then the FM_w-rule ensures the minimization of overall social losses in every shock scenario at time t

$$\mathcal{L}_t = \sum_{k=1}^{N} g_k L_t^k \tag{3.32}$$

Proof of Proposition 9:
Since the bliss point in the central bankers loss functions is now the long-run interest rate i^* of the no-shock scenario and interest-rate changes are calculated with respect to i^*, L_t^k can still be rewritten to $L_t^k = (\Delta i_t - \Delta \tilde{i}_t)^2$. This implies that the proof follows the same argumentation as the corresponding proof of proposition 2 in the baseline model.

■

The reason for the working of the FM_w-rule is that we only changed the frame of reference. For that reason from period to period, we maintain the property of our baseline-model, that the union is always only divided into two differing regions by a shock. This would fail, if we allow for longer decaying shock, because in this case a country could be hit by a new shock, while an old one has not already decayed. This would create an additional shock situation compared to a region which was previously not hit. Something similar would happen, if the reference is not independent of time, since then, as we have already mentioned, a situation similar to the framework with positive and negative shocks can occur.

Chapter 4

Transparency

4.1 Introduction

Central bank transparency is a widely discussed topic in the literature of the last decades. Most empirical analyses not only favour transparency but also shows that since the 1990s central banks have started to transform from more or less opaque institutions into more transparent regimes. But nevertheless from the theoretical side the question of an optimal degree of transparency has still not answered in a conclusive manner. For a recent overview, see Cruijsen and Eijffinger (2007) and Dincer and Eichengreen (2009). Especially the effect of releasing information about the applied economic models, that the central banks apply (Cukierman (2002)) is far from being clear-cut. For instance Cukierman (2009) and Mishkin (2004) argue that there is a limit of transparency, because even within central banks, there is both uncertainty about the right economic model or the right output target. Furthermore, it is pointed out that the communication process with the public about the right economic modelling could be too complicated. In contrast Geraats (2008) argues that especially in the case of the ECB, welfare can be improved by being more transparent about the central bank's objectives and macroeconomic forecasts as well as being more transparent in the decision-making process.

Within this context, in our model, we can ask if it is welfare-improving or harmful if, on the one hand, the central bank shares its economic expertise contained in the knowledge about the shock, and on the other hand, communicates their decision process whilst revealing its explicit decision rule.[1]

[1] Especially the latter case is interesting since there is not much known about the decision process in the central bank board of the ECB. Although in the constitution of the ESCB (European System of Central Banks), it is defined that "Each member of the Government Council shall have one vote" (*Article 10*), "the Governing Council shall act by simple majority" (*Article 10*) and the "Governing

For that purpose we explicitly construct different information setups, in which we define how information about the shock and the decision rule is released to public. In these different information setups, we determine the welfare implications. Until now, we have dealt with the information setup, in which the public is not informed about the shock size, but they are informed about the decision rule. Now we extend this setup with two cases. First, the public is not informed about the decision rule but they are informed about the shock size and second the public is fully informed, both about the decision rule and the shock size. We do not consider the case, in which the public is neither informed about the decision rule nor the shock size, because the outcome does not differ from the case of the initial information setup. The reason is that the public does not expect a change in interest rate independent from the decision rule, if they are not informed about the shock. Hence, the information about the decision rule does not alter the reaction of the public. In order to implement the different information setups within our model, we have to define how public expectations enter our framework. This is done within a standard Barro-Gordon model (see Barro and Gordon (1983) or Rogoff (1985)), where we mainly follow the approach of Gersbach (2003).

4.2 The Model and Expectations

The previous chapters are based on social loss functions for every member k of the monetary union

$$L^k = (\Delta i - \gamma_k \Delta\tilde{i}(\epsilon(G_{\mathcal{K}})))^2 \tag{4.1}$$

This loss function is derived from the assumption that the information about the shock ϵ is not released to the public, which implies that the public expects $\mathbf{E}(\epsilon) = 0$, where we assume a symmetrically distributed shock with zero mean and $\mathbf{E}(.)$ denoting the expected value of a variable (Gersbach and Hahn (2001)). As a consequence this implies that the public does not expect an interest-rate change. Within this setup, we derived the best FM_w-rule, which implements an interest-rate change of

$$\Delta i^{FM_w} = G\Delta\tilde{i}(\epsilon(G)) \tag{4.2}$$

if the union is divided into two parts by a shock $\epsilon(G)$.[2] Since we have shown in the previous chapters that, in general, the decision rules with weighted votes outperform

Council formulate the monetary policy ... including ... key interest rates" (*Article 12*). We know from several statements of board members, that there is no explicit rule which the committee follows in their meetings (see also footnote 13).

[2]We dropped the index $\mathcal{K}$, since we assume that the shock depends only on the size of the affected region and it does not matter which countries have an aggregated weight of $G_{\mathcal{K}} = G$ representing the set $\mathcal{K}$.

the rules without weighted votes, we consider only decision rules with weighted votes. In the following, we compare the setup, which we call OP_1 (the central bank is still opaque about its knowledge of the shock size, but the decision rule is known), with a fully transparent information setup TP (the public is informed about both, the shock size and the decision rule) and another opaque setup OP_2 (the public is informed about the shock size, but not about the decision rule). Therefore, we need to know how the public expectations about an interest-rate change to enter the loss function, since in information setups, where the public is informed about the size of the shock, their expectations will not generally equal zero as in the information setup OP_1.

For that purpose we introduce expectations within a standard log-linearized Barro-Gordon model (see Barro and Gordon (1983)) similar to Gersbach (2003).[3]

Social losses of a central banker are defined as[4]

$$l = \pi^2 + \alpha y^2 \quad \alpha > 0 \tag{4.3}$$

with the Philipps-Curve

$$y = \beta(\pi - \pi^e) - \epsilon \quad \beta > 0 \tag{4.4}$$

where y denotes output, π inflation, π^e expected inflation of the public and ϵ is zero for the unaffected region and $\epsilon(G)$ for the affected part. We assume that ϵ is a shock symmetrically distributed around zero and the absolute value of ϵ is an increasing function of the size G of the affected region. The log of natural output is normalized to zero and therefore we abandon a central bank output target above natural output. This is done, because we do not want to incorporate the time inconsistency problem[5] in our analysis (Barro and Gordon (1983) and Binder (1997)). For simplicity the inflation target is also normalized to zero. In this setup, we can focus on the pure transparency effect, induced by the information of the public about the shock or the decision rule.

Inserting the Philipps-Curve into the loss function (4.3), we obtain

$$l = \pi^2 + \alpha(\beta(\pi - \pi^e) - \epsilon)^2 \tag{4.5}$$

which yields by quadratic completion

$$l = (1 + \alpha\beta^2)\left[(\pi^2 - \frac{a\beta}{1 + a\beta^2}(\beta\pi^e + \epsilon))^2 + \frac{a}{(1 + a\beta^2)^2}(\beta\pi^e + \epsilon)^2\right] \tag{4.6}$$

[3]This model is somewhat simpler than the aggregate supply and demand model used in Gersbach and Hahn (2001), but social losses for OP_1 can similarly derived.

[4]As in Gersbach (2003) social loss functions do not differ between central bankers and the public.

[5]Note that this is a special property of the quadratic loss function, while with more general loss functions, especially with an asymmetric shape (see also chapter 3.1.1), the problem does not vanish (i.e. see Surico (2008)).

Suppose in the past, we start from a long-run optimal equilibrium with no shocks (or already long decayed shocks) with zero inflation and an interest rate which is matching the natural real interest rate r. A change in nominal interest rate i can then be represented by $\Delta i = i - r$ and an expected change by $\Delta i^e = i^e - r$ respectively. Together with the Fisher equation $i = \pi + r$ and $i^e = \pi^e + r$ (i.e. Woodford (2003) and Wicksell (1898)), we can substitute inflation and expected inflation in equation (4.6) by the interest-rate change Δi and expected interest-rate change Δi^e. We obtain

$$l(\Delta i, \Delta i^e, \epsilon) = (1 + \alpha\beta^2)\left[(\Delta i - \frac{a\beta}{1 + a\beta^2}(\beta\Delta i^e + \epsilon))^2 + \frac{a}{(1 + a\beta^2)^2}(\beta\Delta i + \epsilon)^2\right] \quad (4.7)$$

For further calculations we use the following short-cuts:

$$A := \frac{a\beta}{1 + a\beta^2}\beta \quad B := \frac{a\beta}{1 + a\beta^2} \quad C := \frac{a}{(1 + a\beta^2)^2} \quad (4.8)$$

and redefine social losses as

$$l(\Delta i, \Delta i^e, \epsilon) = (\Delta i - (A\Delta i^e + B\epsilon))^2 + C(\beta\Delta i^e + \epsilon)^2 \quad (4.9)$$

where we left out the factor $(1+\alpha\beta^2) > 0$, since this is only a monotonic transformation which does not change the preferences.

The time-line is set as follows:

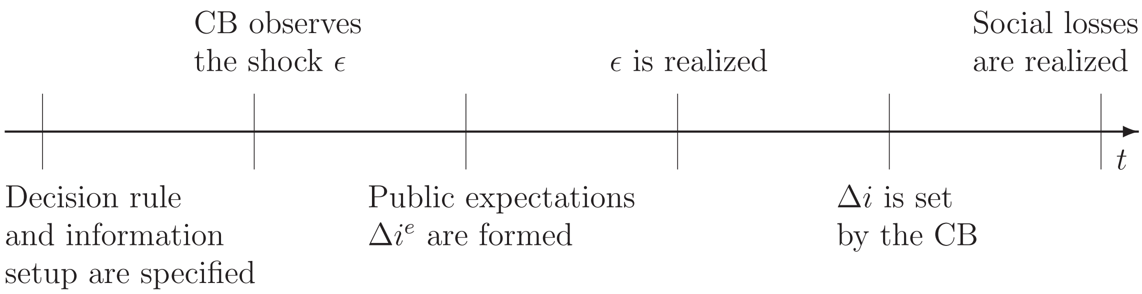


Figure 4.1:
Sequence of events (CB: Central Bank).

This means, that the central bank cannot change the information setup or the decision rule during the decision process about the interest-rate change. In the following we explicitly define the three different information setups:

1. *Opacity* 1 (OP_1): The information about the shock is not released and public is informed about the decision rule (this is our baseline scenario used in the previous chapters).

2. *Opacity* 2 (OP_2): The information about the shock is released, but the public does not know whether the SM_w-rule or the FM_w-rule is applied. The public assigns probability p for SM_w and probability $(1-p)$ for FM_w with $p \in [0,1]$.

3. $Transparency$ (TP): The Public is fully informed by the central bankers. The information about the shock is released and the public knows the decision rule of the central bank committee.

Generally, we obtain the reaction function of a central banker by minimizing equation (4.9) with respect to Δi and Δi^e is fixed. This yields

$$\Delta i^R(\Delta i^e, \epsilon) = (A\Delta i^e + B\epsilon) \tag{4.10}$$

where either the shock size $\epsilon(G)$ or zero is inserted, depending on whether the central banker represents a country of the affected region or not.

Note that $\Delta i^R(\Delta i^e, \epsilon)$ is both increasing in shock size ϵ and expected interest-rate change Δi^e. Furthermore, the derivative of Δi^R with respect to expected interest-rate change Δi^e is less than one, since $A, B > 0$ and $A < 1$. Inserting this in social losses of every single central banker of the union and similarly aggregating social losses as in the previous chapters where we defined overall social losses as the weighted sum of the affected and the unaffected region (see equation 2.7), we obtain

$$\mathcal{L} := l^G + l^{1-G} \tag{4.11}$$

with

$$l^G := Gl(\Delta i, \Delta i^e, \epsilon(G)) = G\left((\Delta i - (A\Delta i^e + B\epsilon(G)))^2 + C(\beta\Delta i^e + \epsilon(G))^2\right) \tag{4.12}$$

for the affected region and

$$l^{1-G} := (1-G)l(\Delta i, \Delta i^e, 0) = (1-G)\left((\Delta i - A\Delta i^e)^2 + C(\beta\Delta i^e)^2\right) \tag{4.13}$$

for the unaffected region.

This yields

$$
\begin{aligned}
\mathcal{L} &= Gl(\Delta i, \Delta i^e, \epsilon(G)) + (1-G)l(\Delta i, \Delta i^e, 0) \\
&= (\Delta i - A\Delta i^e - GB\epsilon(G))^2 + C(\beta\Delta i^e + G\epsilon(G))^2 + \\
& \quad G(1-G)(B^2+C)[\epsilon(G)]^2
\end{aligned}
\tag{4.14}
$$

For the comparison of transparency and opacity, we need to specify a FM_w-rule, that is applied in all different information frameworks. Following the keep it as simple as possible principle (Mishkin (2007)), we use the information setup (OP_1) of the previous chapters as a benchmark. Since the public is not informed about the shock, the expected interest-rate change is $\Delta i^e = 0$. This yields a simple linear dependency between the desired interest-rate change of the affected region and the shock size $\epsilon(G)$ with $\Delta i = B\epsilon(G)$. The optimal FM_w-rule is therefore given by

$$\Delta i^{FM_w}(G) = GB\epsilon(G) \tag{4.15}$$

which means that for an interest-rate change of $\Delta i = GB\epsilon(G)$, we need a share of G of the votes.[6]

In the following, we calculate for every triple

(decision rule, shock size, information setup)

the resulting triple of

(expected public interest-rate change, implemented interest-rate change, overall social losses)

4.3 *Opacity* 1

We briefly reproduce the results of the previous chapters for our specific loss function. With rational expectations of the public, we obtain for all possible shock scenarios

$$\Delta i^e = A\Delta i^e + B\overbrace{\mathbf{E}(\epsilon)}^{=0} \quad \Longrightarrow \Delta i^e = 0 \tag{4.16}$$

[6]Note that although we apply the FM_w-rule defined in equation (4.15), in every information setup one can think in different information setups about other flexible majority rules, since (4.15) is only optimally derived for OP_1. But then non zero public expectations must enter the new rule, which highly complicates a derivation.

because the public has no information about the shock size and $A \in (0,1)$. Furthermore, the implemented interest-rate change Δi and therefore overall social losses depend on the applied decision rule and the shock size $\epsilon(G)$. In the following we calculate the aggregated social losses applying the SM_w-rule and the FM_w-rule.

4.3.1 Simple Majority Rule

If the decision rule is SM_w we have to distinguish the cases $G > \frac{1}{2}$ and $G \leq \frac{1}{2}$, since in the first case the affected region can change the interest rate, while in the second case it cannot change the interest rate.

1. $G > \frac{1}{2}$: Since the central bankers of the effected part know the shock size $\epsilon(G)$ and public expectations are $\Delta i^e = 0$, they change the interest rate up to their bliss point
$$\Delta i = \Delta i^R(0, \epsilon(G)) = B\epsilon(G) \tag{4.17}$$
Inserting this into the social loss functions of the affected region l^G and the unaffected region l^{1-G}, we obtain aggregated social losses of
$$\begin{aligned} \mathcal{L}^{OP_1} &= Gl(B\epsilon(G), 0, \epsilon(G)) + (1-G)l(B\epsilon(G), 0, 0) \\ &= [\epsilon(G)]^2 \left(B^2(1-G) + GC\right) \end{aligned} \tag{4.18}$$

2. $G \leq \frac{1}{2}$: In this case the interest rate is not changed since the bliss point of the not affected region is given by
$$\Delta i = \Delta i^R(0, 0) = 0 \tag{4.19}$$
and aggregated social losses are calculated to
$$\begin{aligned} \mathcal{L}^{OP_1} &= Gl(0, 0, \epsilon(G)) + (1-G)l(0, 0, 0) \\ &= [\epsilon(G)]^2 \left(G(B^2 + C)\right) \end{aligned} \tag{4.20}$$

4.3.2 Flexible Majority Rule

In this case the bliss point of central bankers affected by the shock is again given by
$$\Delta i^R(0, \epsilon(G)) = B\epsilon(G) \tag{4.21}$$
But now, due to the FM_w-rule, they can only change the interest rate up to
$$\Delta i^{FM_w} = GB\epsilon(G) = G\Delta i^R(0, \epsilon(G)) \tag{4.22}$$

Since $G \leq 1$, the possible interest-rate change of the affected region is lower than their desired change of $\Delta i = B\epsilon(G)$. This implies that they exhaust the possible change, because their losses are decreasing in $\Delta i \in [0, B\epsilon(G)]$ and we obtain an implemented interest rate of $\Delta i = GB\epsilon(G)$. Inserting this into aggregated social losses, we obtain

$$\begin{aligned} \mathcal{L}^{OP_1} &= Gl(GB\epsilon(G), 0, \epsilon(G)) + (1-G)l(GB\epsilon(G), 0, 0) \\ &= [\epsilon(G)]^2 \, G \, (B^2(1-G) + C) \end{aligned} \tag{4.23}$$

4.4 *Transparency*

In the transparent regime, where the public is instantaneously informed about the shock size $\epsilon(G)$ by the central bankers, the public expectations about interest-rate change depend also on the applied decision rule. Therefore, we cannot generally calculate the public expectations as we have done in the case of the information setup OP_1, but we have to immediately incorporate the decision rule. Again, we start with the SM_w-rule for the cases $G > \frac{1}{2}$ and $G \leq \frac{1}{2}$ and finish with the FM_w-rule.

4.4.1 Simple Majority Rule

For rational expectations of the public we obtain again

$$\Delta i^e = A\Delta i^e + B\mathbf{E}(\epsilon) \tag{4.24}$$

But now, we have to insert $\mathbf{E}(\epsilon) = \epsilon(G)$ if $G > \frac{1}{2}$, because the public is informed about the shock size and they know that the members of the affected region can change the interest rate as much as they desire. In the other case ($G \leq \frac{1}{2}$) we obtain $\mathbf{E}(\epsilon) = 0$, because in this case the public knows that the unaffected region with $\epsilon = 0$ is the decisive part.

1. $G > \frac{1}{2}$: In this case we obtain[7]

$$\Delta i^e = A\Delta i^e + B\epsilon \quad \Longrightarrow \quad \Delta i^e = \frac{B}{1-A}\epsilon(G) \quad (A \in (0,1)) \tag{4.25}$$

[7] Technically, the upshifting of the interest change Δi by the factor $\frac{1}{1-A}$ follows from the same considerations as in the time inconsistency problem of the Barro and Gordon (1983) model. But since we are not assuming an output target above natural output, this problem arises only in a regime, when the public is informed about the shock size, because otherwise the reaction function runs through the origin and therefore the time inconsistency problem is absent. This effect is also debated in Blanchard and Fischer (1989). Altogether this repeats the result of Gersbach (2003) if the signal is fully informative and we set in our model $G = 1$.

Note that $A \in (0,1)$ is a crucial condition in this model, because otherwise the public would have negative expectations (or even Δi^e is undetermined if $A = 1$) about the interest-rate change although the shock ϵ is positive. Since the central bankers of the affected region can change the interest rate according to their preferences, we obtain:

$$\Delta i = \Delta i^R(\Delta i^e, \epsilon(G)) = A\frac{B}{1-A}\epsilon(G) + B\epsilon(G) = \frac{B}{1-A}\epsilon(G) \tag{4.26}$$

and this yields overall social losses of

$$\begin{aligned} \mathcal{L}^{TP} &= Gl(\tfrac{B}{1-A}\epsilon(G), \tfrac{B}{1-A}\epsilon(G), \epsilon(G)) + (1-G)l(\tfrac{B}{1-A}\epsilon(G), \tfrac{B}{1-A}\epsilon(G), 0) \\ &= [\epsilon(G)]^2 \left(B^2(1-G) + CG(1 + G\tfrac{\beta B/G}{1-A}(\tfrac{\beta B/G}{1-A} + 2))\right) \end{aligned} \tag{4.27}$$

2. $G \leq \frac{1}{2}$: In this case public knows that the central bankers of the unaffected region are the relevant group with bliss point $\Delta i^R(\Delta i^e, 0)$, which implies

$$\Delta i^e = A\Delta i^e \quad \Longrightarrow \quad \Delta i^e = 0 \tag{4.28}$$

which again yields

$$\Delta i = \Delta i^R(0,0) = 0 \tag{4.29}$$

Then, overall losses are given by

$$\begin{aligned} \mathcal{L}^{TP} &= Gl(0,0,\epsilon(G)) + (1-G)l(0,0,0) \\ &= [\epsilon(G)]^2 G(B^2 + C) \end{aligned} \tag{4.30}$$

4.4.2 Flexible Majority Rule

In this regime the public knows the shock size $\epsilon(G)$ and they additionally know, that the central bankers of the affected region can change the interest rate up to

$$\Delta i^{FM_w} = GB\epsilon(G) \tag{4.31}$$

In the following proposition, we show that this possible interest-rate change is simultaneously the expected interest-rate change Δi^e of the public and the implemented interest rate Δi of the central bank committee.

Proposition 10
Suppose the public is informed about the shock size $\epsilon(G)$ and additionally they know that the central bank board decides about the interest-rate change according to the FM_w-rule, then

(i) Public expectations are $\Delta i^e = GB\epsilon(G)$

(ii) the central bank board implements an interest-rate change of $\Delta i = GB\epsilon(G)$

Proof of proposition 10:
The bliss point of the central bankers of the affected region $\Delta i^R(\Delta i^e, \epsilon(G))$ is both increasing in the shock size $\epsilon(G)$ (and thus in G, the size of the affected region) and the expected interest-rate change Δi^e because

$$\frac{\partial \Delta i^R(\Delta i^e, \epsilon)}{\partial G} = B\frac{\partial \epsilon(G)}{\partial G} > 0 \quad \text{and} \quad \frac{\partial \Delta i^R(\Delta i^e, \epsilon)}{\partial \Delta i^e} = A > 0 \tag{4.32}$$

This implies that, if we assume $\Delta i^e > 0$, the bliss point $\Delta i^R(\Delta i^e, \epsilon(G))$ of the central bankers in the affected region is not smaller than Δi^{FM_w}, because

$$\begin{aligned} (G-1)\tfrac{B\epsilon(G)}{\Delta i^e} < 0 < A \Longleftrightarrow \\ \Delta i^R((\Delta i^e, \epsilon(G)) = A\Delta i^e + B\epsilon(G) > \Delta i^{FM_w}(\epsilon(G)) = GB\epsilon(G) \end{aligned} \tag{4.33}$$

Additionally, we know that the expectations Δi^e must have the same sign as the shock, since the slope of the reaction function $\frac{\partial \Delta i^R(\Delta i^e,\epsilon)}{\partial \Delta i^e} = A$ is positive and smaller than one (see figure 4.2). Then we can conclude that, firstly, the public expects an interest-rate change of $\Delta i^e = GB\epsilon(G)$ because they know that the bliss point of the affected region is not less than $GB\epsilon(G)$ and this is simultaneously the maximum possible interest-rate change. Second, central bankers of the affected region have due to their reaction function an incentive to exceed this interest-rate change, but they are also restricted to an interest-rate change of $\Delta i = GB\epsilon(G)$ because of the FM_w-rule.

■

Inserting

$$\Delta i = \Delta i^e = GB\epsilon(G) \tag{4.34}$$

in the overall social losses we obtain

$$\begin{aligned} \mathcal{L}^{TP} &= Gl(GB\epsilon(G), GB\epsilon(G), \epsilon(G)) + (1-G)l(GB\epsilon(G), GB\epsilon(G), 0) \\ &= [\epsilon(G)]^2 G\left(B^2(1-G(1-A^2)) + C(1+G\beta B(\beta B+2))\right) \end{aligned} \tag{4.35}$$

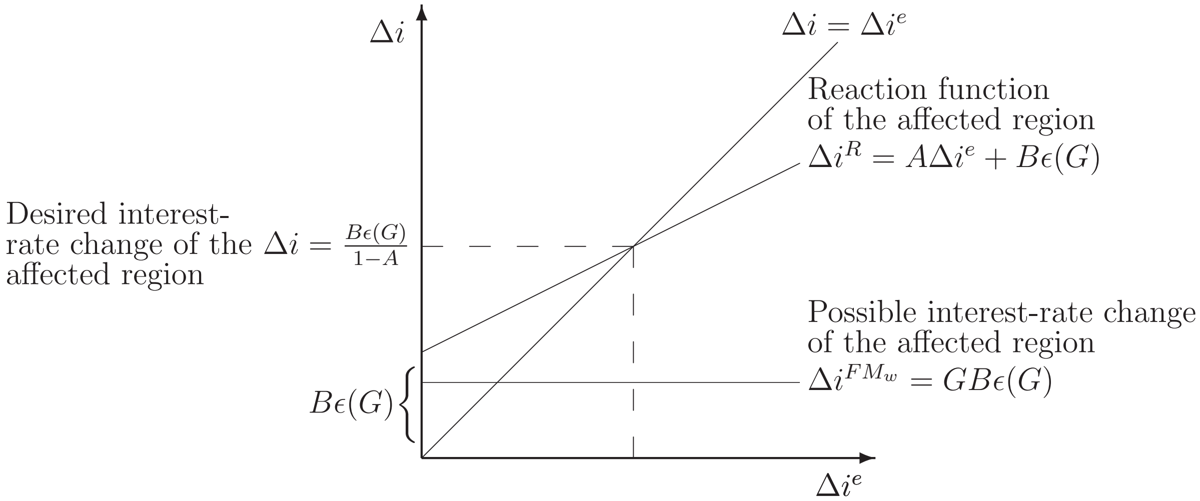


Figure 4.2:
interest-rate change under FM_w and TP

4.5 ***Opacity*** **2**

In this regime, we assume that the public is informed about the shock size, but they do not know which decision rule the central bank board is applying. We assume that the public weights the SM_w-rule with probability $p \in [0, 1]$ and the FM_w-rule with probability $1 - p$, where p is commonly known. The expected interest-rate change is then the convex combination of the expected interest-rate changes of decision rules SM_w and FM_w and can be written as

$$\Delta i^e = p\Delta i^e_{SM_w} + (1-p)\Delta i^e_{FM_w} \tag{4.36}$$

with $\Delta i^e_{FM_w} = GB\epsilon(G)$ and $\Delta i^e_{SM_w} = A\Delta i^e + \Theta(G - \frac{1}{2})B\epsilon(G)$ (with $\Theta(x)$ the Heavyside-function. Note that following proposition 10, we can already insert $GB\epsilon(G)$ for the interest-rate change, assuming the decision rule is FM_w, because we have still $\Delta i^e \geq 0$ and thus $\Delta i^R(\Delta i^e, \epsilon) + B\epsilon(G) \geq \Delta i^{FM_w} = GB\epsilon(G)$. But we cannot insert $\Theta(G - \frac{1}{2})\frac{B}{1-A}\epsilon(G)$ for the interest-rate change assuming the decision rule is SM_w following equations (4.25) and (4.28) from the transparent information setup (TP), because $\Delta i^e_{SM_w}$ depends itself on Δi^e.

With these considerations we obtain for every shock scenario $\epsilon(G)$ an expected interest-rate change of

$$\Delta i^e = p(A\Delta i^e + \Theta(G - \tfrac{1}{2})B\epsilon(G)) + (1-p)GB\epsilon(G) \tag{4.37}$$

Again, we have to distinguish between the cases $G \leq \frac{1}{2}$ and $G > \frac{1}{2}$.

1. $G > \frac{1}{2} \Longrightarrow \Theta(G - \frac{1}{2}) = 1 \Longrightarrow \Delta i^e = p(A\Delta i^e + B\epsilon(G)) + (1-p)GB\epsilon(G)$ and we obtain
$$\Delta i^e = \frac{p/G + (1-p)}{1-pA} GB\epsilon(G) = \overline{p}GB\epsilon(G) \tag{4.38}$$
2. $G \leq \frac{1}{2} \Longrightarrow (G - \frac{1}{2}) = 0 \Longrightarrow \Delta i^e = pA\Delta i^e + (1-p)GB\epsilon(G)$ and we obtain
$$\Delta i^e = \frac{(1-p)}{1-pA} GB\epsilon(G) = \underline{p}GB\epsilon(G) \tag{4.39}$$

where we have introduced $\overline{p} := \frac{p/G+(1-p)}{1-pA}$ and $\underline{p} := \frac{(1-p)}{1-pA}$. After the calculation of the expected interest-rate changes, the central bank board implements the interest-rate change according to the decision rule.

4.5.1 Simple Majority Rule

As in the previous cases we distinguish between $G > \frac{1}{2}$ and $G \leq \frac{1}{2}$.

1. $G > \frac{1}{2}$: In this case the central bankers of the affected region can change the interest rate according to their bliss point, which yields
$$\Delta i = \Delta i^R(\overline{p}GB\epsilon(G), \epsilon(G)) = \frac{1/G + A(1-p)}{1-pA} GB\epsilon(G) = \widetilde{p}GB\epsilon(G) \tag{4.40}$$
where we have introduced $\widetilde{p} := \frac{1/G+A(1-p)}{1-pA} = A\overline{p} + \frac{1}{G}$. Inserting this into the overall social losses, we obtain
$$\begin{aligned} \mathcal{L}^{OP_2} &= (\widetilde{p}GB\epsilon(G), \overline{p}G\epsilon(G), \epsilon(G)) + (1-G)l(\widetilde{p}GB\epsilon(G), \overline{p}G\epsilon(G), 0) \\ &= [\epsilon(G)]^2 \left(B^2(1-G) + CG(1 + G\overline{p}\beta B(\overline{p}\beta B + 2))\right) \end{aligned} \tag{4.41}$$
2. $G \leq \frac{1}{2}$: In this case, the unaffected region is the decisive group for the interest-rate change and since their bliss point is $\Delta i^R(\underline{p}GB\epsilon(G), 0)$, they implement an interest-rate change of
$$\Delta i = \Delta i^R(\underline{p}GB\epsilon(G), 0) = A\underline{p}GB\epsilon \tag{4.42}$$
Inserting this into overall social losses, we obtain
$$\begin{aligned} \mathcal{L}^{OP_2} &= Gl(A\underline{p}GB\epsilon(G), \underline{p}GB\epsilon(G), \epsilon(G)) + (1-G)l(A\underline{p}GB\epsilon(G), \underline{p}GB\epsilon(G), 0) \\ &= [\epsilon(G)]^2 G \left(B^2 + C(1 + G\underline{p}\beta B(\underline{p}\beta B + 2))\right) \end{aligned} \tag{4.43}$$

4.5.2 Flexible Majority Rule

For general G we obtain $\Delta i = GB\epsilon$, since the bliss point is increasing in the expected interest-rate change, which is also positive and thus the affected region will exhaust their possible interest-rate change following the same argumentation as in proposition 10. But now we have to distinguish between the cases $G \leq \frac{1}{2}$ and $G > \frac{1}{2}$ since the expected interest-rate changes differ for these cases.

1. $G > \frac{1}{2}$: Inserting $\Delta i = GB\epsilon$ and $\Delta i^e = \overline{p}GB\epsilon$ into the loss function, we obtain

$$\begin{aligned} \mathcal{L}^{OP_2} &= Gl(GB\epsilon(G), \overline{p}G\epsilon(G), \epsilon(G)) + (1-G)l(GB\epsilon(G), \overline{p}G\epsilon(G), 0) \\ &= [\epsilon(G)]^2 G\left(B^2(1 - G(1 - \overline{p}^2 A^2)) + C(1 + G\overline{p}\beta B(\overline{p}\beta B + 2))\right) \end{aligned} \tag{4.44}$$

2. $G \leq \frac{1}{2}$: Inserting $\Delta i = GB\epsilon$ and $\Delta i^e = \underline{p}GB\epsilon$ into the loss function, we obtain

$$\begin{aligned} \mathcal{L}^{OP_2} &= Gl(GB\epsilon(G), \underline{p}G\epsilon(G), \epsilon(G)) + (1-G)l(GB\epsilon(G), \underline{p}G\epsilon(G), 0) \\ &= [\epsilon(G)]^2 G\left(B^2(1 - G(1 - \underline{p}^2 A^2)) + C(1 + G\underline{p}\beta B(\underline{p}\beta B + 2))\right) \end{aligned} \tag{4.45}$$

4.6 Transparency versus Opacity

Before we start with a detailed welfare comparison, we generally examine the difference between two information setups in order to show that the relation of the expectations in two different information setups play the crucial role. Suppose in two different information setups I and J ($I \neq J$ and $I, J \in \{OP_1, OP_2, TP\}$), the public has interest rate expectations of Δi_I^e and Δi_J^e, while central bankers change the interest rate to Δi_I and Δi_J respectively. Then the difference of overall social losses $\mathcal{L}_I - \mathcal{L}_J$ in these two setups is given by

$$\begin{aligned} \mathcal{L}_I - \mathcal{L}_J &= G\left(l(\Delta i_I, \Delta i_I^e, \epsilon(G)) - l(\Delta i_J, \Delta i_J^e, \epsilon(G))\right) \\ &\quad + (1-G)\left(l(\Delta i_I, \Delta i_I^e, \epsilon(G)) - l(\Delta i_J, \Delta i_J^e, \epsilon(G))\right) \\ &= [(\Delta i_I - A\Delta i_I^e) - (\Delta i_J - A\Delta i_J^e)] \\ &\quad \cdot [(\Delta i_I - A\Delta i_I^e) + (\Delta i_J - A\Delta i_J^e) - 2GB\epsilon(G)] \\ &\quad + C\beta\,[\Delta i_I^e - \Delta i_J^e]\,[\beta(\Delta i_I^e + \Delta i_J^e) + 2G\epsilon(G)] \end{aligned} \tag{4.46}$$

For the SM_w-rule in every information setup I we obtain $(\Delta i_I - A\Delta i_I^e) = 0$ for $G \leq \frac{1}{2}$ and $(\Delta i_I - A\Delta i_I^e) = B\epsilon(G)$ for $G > \frac{1}{2}$, because for $G \leq \frac{1}{2}$ central bankers of the unaffected region with bliss point $A\Delta i_I^e$ determine the interest-rate change and in the other case interest-rate change is given by $\Delta i_I = A\Delta i_I^e + B\epsilon$. Hence, in this case the difference of overall social losses between two information setups I and J is given by

$$\mathcal{L}_I - \mathcal{L}_J = C\beta\,[\Delta i_I^e - \Delta i_J^e]\,[\beta(\Delta i_I^e + \Delta i_J^e) + 2G\epsilon(G)] \tag{4.47}$$

In the above case with the FM-rule, the interest rate is always changed to the maximum allowed level in the realized shock scenario, which implies that in different information setups, we obtain $\Delta i_I = \Delta i_J = \Delta i$. This yields for the difference of overall social losses between two information setups I and J

$$\mathcal{L}_I - \mathcal{L}_J = [\Delta i_I^e - \Delta i_J^e]\left[(\Delta i_I^e - \Delta i_J^e)(A^2 + C\beta^2) + 2G\epsilon(G)(AB + C\beta) - 2A\Delta i\right] \tag{4.48}$$

Since the interest-rate change is explicitly given by $\Delta i = GB\epsilon(G)$, we obtain

$$\mathcal{L}_I - \mathcal{L}_J = [\Delta i_I^e - \Delta i_J^e]\left[(\Delta i_I^e - \Delta i_J^e)(A^2 + C\beta^2) + 2G\epsilon(G)C\beta\right] \tag{4.49}$$

Altogether, we obtain the general result that OP_1 cannot be worse than the other information setups, because within OP_2 and TP, public expectations about interest-rate changes are non-negative and almost always positive (the exception is TP and $G \leq \frac{1}{2}$), whilst public interest rate expectations in OP_1 are vanishing, which implies $\Delta i_I^e - \Delta i_{OP_1}^e \geq 0$.

Now we turn to the detailed discussion of the relation between the information setups. The comparison[8] runs as follows: We pairwise calculate the difference of the overall social losses between the different information setups given a shock scenario $\epsilon(G)$ and the decision rule. Although we already know from the previous section that OP_1 is at least as good as OP_2 or TP, for completeness we also present the explicit outcome of the differences in overall social losses.

4.6.1 *Transparency* versus *Opacity* 2

1. SM_w-rule and $G > \frac{1}{2}$:
   In this case we obtain from equations (4.27) and (4.41)

$$\mathcal{L}^{OP_2} - \mathcal{L}^{TP} = [\epsilon(G)]^2 CG^2\beta B\left(\beta B(\overline{p} + \frac{1/G}{1-A}) + 2\right)\left(\overline{p} - \frac{1/G}{1-A}\right) \leq 0 \tag{4.50}$$

[8] We exclude in this analysis the trivial cases $G = 0, 1$.

This follows from $\overline{p}(p=0)=1$, $\overline{p}(p=1)=\frac{1/G}{1-A}$ and $\frac{\partial \overline{p}(p)}{\partial p}=\frac{1-G(1-A)}{G(1-pA)^2}>0$, which implies that $\overline{p}\leq\frac{1/G}{1-A}$ for $p\in[0,1]$, because $\overline{p}(p=0)=1<\frac{1/G}{1-A}$ since $A,G\in(0,1)$ and $\overline{p}(p)$ is monotonically increasing in $p\in[0,1]$.

2. SM_w-rule and $G\leq\frac{1}{2}$:
From equations (4.30) and (4.43) we obtain

$$\mathcal{L}^{OP_2}-\mathcal{L}^{TP}=[\epsilon(G)]^2CG^2\underline{p}\beta B\left(\underline{p}\beta B+2\right)\geq 0 \tag{4.51}$$

since $\underline{p}\geq 0$.

3. FM_w-rule and $G>\frac{1}{2}$:
From equations (4.35) and (4.44) we obtain

$$\mathcal{L}^{OP_2}-\mathcal{L}^{TP}=[\epsilon(G)]^2G^2\left(B^2A^2+\beta B(\beta B(1+\overline{p})+2)\right)(\overline{p}-1)\geq 0 \tag{4.52}$$

This follows from the comparison of TP and OP_2 with decision rule SM_w and $G>\frac{1}{2}$, because in this case we have shown that $\overline{p}\geq 1$ for $p\in[0,1]$.

4. FM_w-rule and $G\leq\frac{1}{2}$:
Similar to the case of $G>\frac{1}{2}$ from equations (4.35) and (4.45) we obtain

$$\mathcal{L}^{OP_2}-\mathcal{L}^{TP}=[\epsilon(G)]^2G^2\left(B^2A^2+\beta B(\beta B(1+\underline{p})+2)\right)(\underline{p}-1)\leq 0 \tag{4.53}$$

This follows from $\underline{p}(p=0)=1$, $\underline{p}(p=1)=0$ and $\frac{\partial \underline{p}(p)}{\partial p}=-\frac{1-A}{(1-pA)^2}<0$, which implies that $\underline{p}\leq 1$ for $p\in[0,1]$, because $\underline{p}(p=0)=1$ and $\underline{p}(p)$ is monotonically decreasing in $p\in[0,1]$.

Altogether we have shown, that for a shock scenario with $G>\frac{1}{2}$ and the SM_w-rule the opaque regime OP_2 is better than transparency TP and in the case of $G\leq\frac{1}{2}$ transparency TP outperforms OP_2 while for the FM_w-rule the reverse relations are true.

4.6.2 *Transparency* versus *Opacity* 1

1. SM_w-rule:

   (a) $G>\frac{1}{2}$:
   In this case we obtain from equations (4.27) and (4.18)

$$\mathcal{L}^{OP_1}-\mathcal{L}^{TP}=-[\epsilon(G)]^2CG^2\beta B\frac{1/G}{1-A}(\beta B\frac{1/G}{1-A}+2)<0 \tag{4.54}$$

(b) $G \leq \frac{1}{2}$:
In this case we obtain from equations (4.30) and (4.20)

$$\mathcal{L}^{OP_1} - \mathcal{L}^{TP} = 0 \tag{4.55}$$

2. FM_w-rule:
In this case we obtain from equations (4.23) and (4.35)

$$\mathcal{L}^{OP_1} - \mathcal{L}^{TP} = -[\epsilon(G)]^2 G^2 (A^2 B^2 + C\beta B(\beta B + 2)) < 0 \tag{4.56}$$

This shows that for every shock scenario, the opaque regime OP_1 is at least as good as transparency. It is interesting to note, that both welfare outcomes coincide, in the case of the SM_w-rule and $G < \frac{1}{2}$, because in this case central bankers cannot profit from output gains, since public expectations vanish also for TP.

4.6.3 *Opacity* 2 versus *Opacity* 1

1. SM_w-rule:

(a) $G > \frac{1}{2}$:
In this case we obtain from equations (4.41) and (4.18)

$$\mathcal{L}^{OP_1} - \mathcal{L}^{OP_2} = -[\epsilon(G)]^2 C G^2 \overline{p}\beta B(\overline{p}\beta B + 2) < 0 \tag{4.57}$$

(b) $G \leq \frac{1}{2}$:
In this case we obtain from equations (4.43) and (4.20)

$$\mathcal{L}^{OP_1} - \mathcal{L}^{OP_2} = [\epsilon(G)]^2 C G^2 \underline{p}\beta B(\underline{p}\beta B + 2) \leq 0 \tag{4.58}$$

2. FM_w-rule:

(a) $G > \frac{1}{2}$:
In this case we obtain from equations (4.44) and (4.23)

$$\mathcal{L}^{OP_1} - \mathcal{L}^{OP_2} = -[\epsilon(G)]^2 G^2 (\overline{p}A^2 B^2 + C\overline{p}\beta B(\overline{p}\beta B + 2)) < 0 \tag{4.59}$$

(b) $G \leq \frac{1}{2}$:
In this case we obtain from equations (4.45) and (4.23)

$$\mathcal{L}^{OP_1} - \mathcal{L}^{OP_2} = -[\epsilon(G)]^2 G^2 (\underline{p}A^2 B^2 + C\underline{p}\beta B(\underline{p}\beta B + 2)) \leq 0 \tag{4.60}$$

This shows also that for every shock scenario, the opaque regime OP_1 is at least as good as the half-transparent information setup OP_2, where the public is only informed about the shock, but not informed about the decision rule. Note the interesting case, in which the public fully misspecifies their expectations about the decision rule in the information setup OP_2. This is the case, in which the implemented decision rule is FM_w, the weight of the affected region is less than $\frac{1}{2}$ and public weights SM_w with $p = 1$. This is the only case, in which OP_2 is as good as OP_1, because in this scenario OP_2 totally mimics the expectations about an interest-rate change of OP_1, since the public erroneously assume that the central bankers decide according to the SM_w-rule and therefore, the public expects no interest-rate change as is the case in the OP_1 information setup.

4.7 Summary and Overall Comparison

In this section, we summarize our results and in table 4.1 we present the expectations about interest-rate changes and the finally implemented interest-rate changes in the different frameworks regarding the information setup and the decision rule. Table 4.2 ranks the different information setups based on their welfare implications, where we exclude the polar cases $p = 0, 1$ in the information setup OP_2, in order to obtain unique relations between the different information setups for the remaining parameter constellations. The rankings for $p = 0, 1$ are separately listed in table 4.3.

			OP_1	OP_2	TP
SM_w	$G > \frac{1}{2}$	Δi^e	0	$\frac{p/G+(1-p)}{1-pA} GB\epsilon(G)$	$\frac{1}{1-A} B\epsilon(G)$
		Δi	$B\epsilon(G)$	$\frac{1/G+A(1-p)}{1-pA} GB\epsilon(G)$	$\frac{1}{1-A} B\epsilon(G)$
	$G \leq \frac{1}{2}$	Δi^e	0	$\frac{(1-p)}{1-pA} GB\epsilon(G)$	0
		Δi	0	$A\frac{(1-p)}{1-pA} GB\epsilon(G)$	0
FM_w	$G > \frac{1}{2}$	Δi^e	0	$\frac{p/G+(1-p)}{1-pA} GB\epsilon(G)$	$GB\epsilon(G)$
		Δi	$GB\epsilon(G)$	$GB\epsilon(G)$	$GB\epsilon(G)$
	$G \leq \frac{1}{2}$	Δi^e	0	$\frac{(1-p)}{1-pA} GB\epsilon(G)$	$GB\epsilon(G)$
		Δi	$GB\epsilon(G)$	$GB\epsilon(G)$	$GB\epsilon(G)$

Table 4.1: interest-rate changes

	$G > \frac{1}{2}$	$G \leq \frac{1}{2}$
SM_w	$\boldsymbol{OP_1} \succ OP_2 \succ TP$	$\boldsymbol{OP_1} \sim \boldsymbol{TP} \succ OP_2$
FM_w	$\boldsymbol{OP_1} \succ TP \succ OP_2$	$\boldsymbol{OP_1} \succ OP_2 \succ TP$

Table 4.2: Ranking of the different information setups for $p \notin \{0, 1\}$

The intuition for ranking is as follows. If the decision rule is SM_w and $G > \frac{1}{2}$ in information setup OP_2 for $p < 1$ public expects a lower interest-rate change than in the transparent case (TP) and therefore the incentive to upshift the interest rates of the central bankers of the affected region is lower, which results in lower aggregated welfare losses. In the case of $G \leq \frac{1}{2}$ we can argue in the opposite way, that public has non-zero expectations in the information setup OP_2, because the FM_w-rule would allow a slight interest-rate change, which induces the unaffected region to raise the interest rate, which increases overall welfare losses compared to the transparent case. For the FM_w-rule the argumentation runs the other way around. For $G > \frac{1}{2}$ the assumption, that the decision rule could also be SM_w pushes for OP_2 the public expectations of interest-rate changes further than in the transparent case TP, because in this case the expectations are limited by the maximum allowed interest-rate change of FM_w. For $G \leq \frac{1}{2}$ the spurious anticipation of SM_w in OP_2 lowers the public expectations of interest-rate changes compared to TP, leading to lower social losses, although finally implemented interest-rate changes coincide. As we have seen in chapter 4.6 in our model, the differences in overall social losses of different information setups are mainly driven by the various expectations about interest-rate changes and, therefore, OP_1 is always at least as good as the other two information setups, because in OP_1 the public interest-rate change expectations vanish, since the public is not informed about the shock size. This result is not really surprising, since FM_w is optimally designed for OP_1 and the other information setups are incorporating this decision rule, although this result is not totally trivial.

Take for example the case, whereby OP_2 is as good as OP_1 if public totally misspecifies their expectations about the decision rule in the case whereby $G \leq \frac{1}{2}$ and FM_w is the decision rule (see table 4.3 and equation (4.60)), because of the misleading expectations of the public, we obtain in both information setups the same expectations about interest-rate changes and, induced by the FM_w-rule, also the same final interest-rate change.

After this discussion, we can ask if there exists a framework of an information setup and/or decision rule, which can outperform OP_1 combined with the FM_w-rule. In order

$p = 1$	$G > \frac{1}{2}$	$G \leq \frac{1}{2}$
SM_w	$\boldsymbol{OP_1} \succ OP_2 \sim TP$	$\boldsymbol{OP_1 \sim TP \sim OP_2}$
FM_w	$\boldsymbol{OP_1} \succ TP \succ OP_2$	$\boldsymbol{OP_1 \sim OP_2} \succ TP$

$p = 0$	$G > \frac{1}{2}$	$G \leq \frac{1}{2}$
SM_w	$\boldsymbol{OP_1} \succ OP_2 \succ TP$	$\boldsymbol{OP_1 \sim TP} \succ OP_2$
FM_w	$\boldsymbol{OP_1} \succ TP \sim OP_2$	$\boldsymbol{OP_1} \succ OP_2 \sim TP$

Table 4.3: Ranking of the different Information setups in the polar case $p = 0, 1$.

to answer this question, we globally minimize the overall loss function $\mathcal{L}(\Delta i, \Delta i^e, \epsilon)$ with respect to Δi and Δi^e in a given shock scenario $\epsilon(G)$. This is done in proposition 11.

Proposition 11
In any fixed shock scenario $\epsilon(G)$, the overall social loss minimum is given by

$$\underset{\Delta i, \Delta i^e}{\operatorname{argmin}} \left\{\mathcal{L}(\Delta i, \Delta i^e, \epsilon(G))\right\} = \begin{pmatrix} \Delta i^\dagger \\ \Delta i^{e\dagger} \end{pmatrix} = G\epsilon \left[\begin{pmatrix} B \\ 0 \end{pmatrix} - \frac{1}{\beta} \begin{pmatrix} A \\ 1 \end{pmatrix} \right] \tag{4.61}$$

with

$$\mathcal{L}(\Delta i^\dagger, \Delta i^{e\dagger}, \epsilon(G)) = [\epsilon(G)]^2 \, G(1-G) \left(B^2 + C\right) \tag{4.62}$$

Proof of proposition 11:
The necessary condition for a minimum is $\vec{\nabla}(\mathcal{L}) = \begin{pmatrix} \frac{\partial}{\partial \Delta i} \\ \frac{\partial}{\partial \Delta i^e} \end{pmatrix} \mathcal{L} = \binom{0}{0}$, which leads to $2 \begin{pmatrix} \Delta i - A\Delta i^e - GB\epsilon(G) \\ -A\Delta i + (A^2 + C\beta^2)\Delta i^e + \epsilon(G)(AGB+1) \end{pmatrix} = \binom{0}{0} \Longleftrightarrow \binom{\Delta i}{\Delta i^e} = \begin{pmatrix} \Delta i^\dagger \\ \Delta i^{e\dagger} \end{pmatrix} = G\epsilon \left[\binom{B}{0} - \frac{1}{\beta}\binom{A}{1}\right]$.

The sufficient condition for a minimum is that the Hessian of $\mathcal{L}$ ($Hess(\mathcal{L})$) is positive definit for $\begin{pmatrix} \Delta i^\dagger \\ \Delta i^{e\dagger} \end{pmatrix}$. Calculating $Hess(\mathcal{L})$ we obtain

$$Hess(\mathcal{L}) = \begin{bmatrix} \frac{\partial^2}{\partial \Delta i^2} & \frac{\partial^2}{\partial \Delta i \Delta i^e} \\ \frac{\partial^2}{\partial \Delta i \partial \Delta i^e} & \frac{\partial^2}{\partial (\Delta i^e)^2} \end{bmatrix} \mathcal{L} = 2 \begin{bmatrix} 1 & -A \\ -A & A^2 + c\beta^2 \end{bmatrix} \tag{4.63}$$

Since the main minors are both positive, because $M_1 = \frac{\partial^2}{\partial \Delta i^2}\mathcal{L} = 1 > 0$ and
$M_2 = Det(Hess(\mathcal{L})) = C\beta^2 > 0$, we can conclude that $Hess(\mathcal{L})$ is positive definit[9] for

[9] Note that $Hess(\mathcal{L})$ is positive definite for all $\begin{pmatrix} \Delta i^\dagger \\ \Delta i^{e\dagger} \end{pmatrix}$.

$\binom{\Delta i^\dagger}{\Delta i^{e\dagger}}$. Inserting $\Delta i^\dagger$ and $\Delta i^{e\dagger}$ in $\mathcal{L}(\Delta i, \Delta i^e, \epsilon(G))$ implies equation 4.62.

■

From proposition 11, we obtain that indeed there exists a vector $(\Delta i^\dagger, \Delta i^{e\dagger})^T$ such that $\mathcal{L}(\Delta i^\dagger, \Delta i^{e\dagger}, \epsilon(G)) < \mathcal{L}(GB\epsilon(G), 0, \epsilon(G))$, the outcome of OP_1 combined with the FM_w-rule, because[10]

$$\mathcal{L}(\Delta i^\dagger, \Delta i^{e\dagger}, \epsilon(G)) - \mathcal{L}(GB\epsilon(G), 0, \epsilon(G)) = -\left[\epsilon(G)\right]^2 GC < 0$$

(see equations (4.23) and (4.62)). But from proposition 11 we also obtain that generally $\Delta i^{e\dagger} < 0$ and $\Delta i^\dagger < 0$ if $B < A/\beta$. This implies that in order to globally minimize $\mathcal{L}$, we would have to look for a framework, in which the public expectations should aim in the opposite direction than the shock $\epsilon(G) > 0$. Additionally there exists a parameter constellations where even the central bankers should implement an interest-rate change Δi with an opposite sign than the shock. Since this is very counter-intuitive, in the following we restrict the domain of $(\Delta i, \Delta i^e)$ to the non-negative quadrant $\mathbb{R}_0^{2+}$. With this restriction we show the following proposition:

Proposition 12
Suppose $\Delta i, \Delta i^e \geq 0$, $\epsilon(G)$ is fixed and overall losses of a monetary union are given by

$$\begin{aligned}\mathcal{L}(\Delta i, \Delta i^e, \epsilon(G)) &= (\Delta i - A\Delta i^e - GB\epsilon(G))^2 + C(\beta\Delta i^e + G\epsilon(G)) \\ &\quad + G(1-G)\epsilon(G)(B^2 + C)\end{aligned} \tag{4.64}$$

Then, there exists no better framework for the decision of interest-rate change than OP_1 in combination with FM_w.

Proof of proposition 12:
Since $(\Delta i^\dagger, \Delta i^{e\dagger})^T \notin \mathbb{R}_0^{2+}$ and $Hess(\mathcal{L})$ is positive definite (see footnote 9) for all $(\Delta i, \Delta i^e)^T \in \mathbb{R}^2$, we can conclude that

$$\underset{\Delta i, \Delta i^e \geq 0}{\operatorname{argmin}} \{\mathcal{L}\} \in \{(x, 0); (0, y)\} \quad \text{with} \quad x, y \in \mathbb{R}_0^+ \tag{4.65}$$

because the graph of $\mathcal{L}$ represents an upward-opened elliptical paraboloid with the minimum at $(\Delta i^\dagger, \Delta i^{e\dagger})^T$. Hence, it is sufficient to show that

$$\min_{\Delta i^e \geq 0} \{\mathcal{L}(0, \Delta i^e, \epsilon(G))\} > \min_{\Delta i \geq 0} \{\mathcal{L}(\Delta i, 0, \epsilon(G))\} \tag{4.66}$$

[10]With the trivial exception of no shock $G = 0$.

because we already know that $\mathcal{L}(\Delta i, 0, \epsilon(G))$ is minimized at $\Delta i = GB\epsilon(G)$ and $\mathcal{L}(GB\epsilon(G), 0, \epsilon(G))$ is the welfare outcome of OP_1 in combination with FM_w. Furthermore, we have $\frac{\partial \mathcal{L}(0,\Delta i^e,\epsilon(G))}{\partial \Delta i^e} = 0 \iff \Delta i^e = -G\epsilon(G)\frac{AB+C\beta}{A^2+C\beta^2} < 0$ and therefore $\operatorname{argmin}_{\Delta i^e \geq 0} \{\mathcal{L}(0, \Delta i^e, \epsilon(G))\} = 0$ since $\mathcal{L}(0, \Delta i^e, \epsilon(G))$ is an upward-opened parabola in Δi^e. But $\mathcal{L}(0, 0, \epsilon(G)) - \mathcal{L}(0, GB\epsilon(G), \epsilon(G)) = -GB\epsilon(G) < 0$ which concludes.

$\blacksquare$

Altogether we have the interesting result that within our framework the setup OP_1 in combination with FM_w can only be improved, if public is misled about the direction of the shock and for specific parameter constellations even the central bank should change the interest rate in another direction than the shock would intuitively demand. Anecdotally we want to point out that the ECB has increased the key interest rate in July 2008 although the signals of the upcoming crisis could not have been overlooked and also the public was not really surprised by this change, since inflation and also inflation expectations (i.e. break-even inflation) were still rising at that point.

Chapter 5

Discussion and Conclusion

Our investigation suggests that flexible majority rules may be a useful tool for central banks. But nevertheless there are a variety of conceptual and practical issues which need to be dealt with.

First, allowing minorities to initiate a change in the interest rates may invite cycling in a dynamic setting, since interest-rate changes might be revised immediately. We showed only in a very simple case, where shocks decay within one period, how this shortcoming of our rule could be overcome. Such undesirable cycling can be avoided by restricting flexible majority rules to genuine majorities or a revision rule. A revision rule stipulates that interest-rate change reversals within a particular time frame, say a year, require a share of supporting votes larger than the share of opposing votes for the initial interest-rate change. It is still necessary to eliminate strategic voting under such reversal rules.

Second, most central bankers would deny, that they mainly focussing on their home country. Instead they often argue that their focus is on the issues of the whole monetary union, but we think, that it is useful to examine a framework like ours as a polar case, since there is at least some heterogeneity in preferences among central bankers.

Third, in some kind our flexible majority rule can be seen as a tool of preference aggregation, since this framework has the potential to form a broader range of coalitions within the decision process. In this sense flexible majority rules can be seen as a kind of a compromize between a consensus decision rule, where a small minority has a strong veto power, which can inhibit the decision process and the widely applied simple majority rule, whereby conversely a larger minority can easily be overruled. As we have already mentioned, there are many real world examples, where larger majorities than 50% are needed for a change of the status quo. But of course, it is difficult to quantize the correct majority depending on the quality of some decision. However, in the case of monetary policy, with the interest rate, we have an explicit quantifiable decision variable. Therefore, we think that in this field a decision rule, in which the required share of votes for a change depends on the size of the change itself, can improve the yes/no character of the simple majority rule.

Fourth, with regard to the practicability of a decision rule aspect, the simplicity of our concept of a flexible majority rule seems important. Although, we have to admit the shortfall of our framework that in real world, we do not know the quantitative implication of a shock ex ante, we think that introducing for example some ad hoc step function like an interest rate change of 25 basispoints require more than 50% of the votes 50 basispoints require 75% and more than 75 basispoints require unanimity, could improve the decision process in a monetary policy committee.

Chapter 6

Appendix

6.1 A Proofs

Proof of Proposition 3:
The first part of the inequalities, $\mathcal{L}_{\mathcal{K}}^{FM_w} \leq \mathcal{L}_{\mathcal{K}}^{SM_w}$ and $\mathcal{L}_{\mathcal{K}}^{FM_w} \leq \mathcal{L}_{\mathcal{K}}^{FM_{nw}}$ follows directly from proposition 2 and corollary 1, since there it is shown, that the FM_w-rule implements for every shock scenario $\mathcal{K}$ the first-best solution and hence, the FM_w-rule can never be worse than any other decision rule.

In order to show the other inequalities, first note that for $G_{\mathcal{K}} = 0, 1$ every defined decision rule implements the same interest-rate change. Therefore we exclude these trivial cases in the following.

- $\mathcal{L}_{\mathcal{K}}^{SM_w}$ versus $\mathcal{L}_{\mathcal{K}}^{SM_{nw}}$

  From equations (2.14) and (2.17), we can conclude

  (i) Suppose $G_{\mathcal{K}} \leq \frac{1}{2}$ and $|\mathcal{K}| \leq \frac{N}{2}$, then under both decision rules the interest is not changed and we obtain $\mathcal{L}_{\mathcal{K}}^{SM_w} = \mathcal{L}_{\mathcal{K}}^{SM_{nw}}$.

  (ii) Suppose $G_{\mathcal{K}} > \frac{1}{2}$ and $|\mathcal{K}| > \frac{N}{2}$, then both decision rules implement the same interest changed $\Delta i_{\mathcal{K}}^{SM_{nw}} = \Delta i_{\mathcal{K}}^{SM_w} = \Delta\tilde{i}_{\mathcal{K}}$ and we obtain $\mathcal{L}_{\mathcal{K}}^{SM_w} = \mathcal{L}_{\mathcal{K}}^{SM_{nw}}$.

  (iii) Suppose $G_{\mathcal{K}} \leq \frac{1}{2}$ and $|\mathcal{K}| > \frac{N}{2}$, then we obtain
  $\mathcal{L}_{\mathcal{K}}^{SM_w} - \mathcal{L}_{\mathcal{K}}^{SM_{nw}} = (2G_{\mathcal{K}} - 1)[\Delta\tilde{i}_{\mathcal{K}}]^2 \leq 0$.

  (iv) Suppose $G_{\mathcal{K}} > \frac{1}{2}$ and $|\mathcal{K}| \leq \frac{N}{2}$, then we obtain
  $\mathcal{L}_{\mathcal{K}}^{SM_w} - \mathcal{L}_{\mathcal{K}}^{SM_{nw}} = (1 - 2G_{\mathcal{K}})[\Delta\tilde{i}_{\mathcal{K}}]^2 \leq 0$.

  and $(i) - (iv)$ leads to $\mathcal{L}_{\mathcal{K}}^{SM_w} \leq \mathcal{L}_{\mathcal{K}}^{SM_{nw}}$ for all $\mathcal{K}$

- $\mathcal{L}_{\mathcal{K}}^{FM_{nw}}$ versus $\mathcal{L}_{\mathcal{K}}^{SM_{nw}}$

  From equation 2.14 and proposition 1 together with the definition of $\Delta I(n)$ in equation 2.20 we can conclude

  (i) Suppose $\mathcal{K} = n \leq \frac{N}{2}$, then

  $$\begin{aligned}\mathcal{L}_{\mathcal{K}}^{FM_{nw}} - \mathcal{L}_{\mathcal{K}}^{SM_{nw}} &= (\Delta I(n) - G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}})^2 - (G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}})^2 \\ &= \Delta I(n)(\Delta I(n) - 2G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}) \qquad < 0\end{aligned}$$

  since $\Delta I(n) \leq G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}$.

  (ii) Suppose $\mathcal{K} = n > \frac{N}{2}$ and $\Delta I(n) \geq \Delta\tilde{i}_{\mathcal{K}}$, then both decision rules implement the same interest-rate change $\Delta i_{\mathcal{K}}^{FM_{nw}} = \Delta i_{\mathcal{K}}^{SM_{nw}} = \Delta\tilde{i}_{\mathcal{K}}$ and we obtain $\mathcal{L}_{\mathcal{K}}^{FM_{nw}} = \mathcal{L}_{\mathcal{K}}^{SM_{nw}}$.

  (iii) Suppose $\mathcal{K} = n > \frac{N}{2}$ and $\Delta I(n) < \Delta\tilde{i}_{\mathcal{K}}$, then

  $$\begin{aligned}\mathcal{L}_{\mathcal{K}}^{FM_{nw}} - \mathcal{L}_{\mathcal{K}}^{SM_{nw}} &= (\Delta I(n) - G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}})^2 - (\Delta\tilde{i}_{\mathcal{K}} - G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}})^2 \\ &= (\Delta I(n) - \Delta\tilde{i}_{\mathcal{K}})(\Delta I(n) + \Delta\tilde{i}_{\mathcal{K}} - 2G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}}) \quad < 0\end{aligned}$$

  since $G_{\mathcal{K}}\Delta\tilde{i}_{\mathcal{K}} \leq \Delta I(n) < \Delta\tilde{i}_{\mathcal{K}}$.

  and $(i) - (iii)$ leads to $\mathcal{L}_{\mathcal{K}}^{FM_{nw}} \leq \mathcal{L}_{\mathcal{K}}^{SM_{nw}}$ for all $\mathcal{K}$.

■

Argumentation for the entries of table 3.1

Argumentation 1
We divide the union into three regions. This means, that two regions are affected by separate shocks and they desire different interest-rate changes and the third region is not affected by a shock and desires therefore no interest-rate change. The weights of the two affected regions are denoted by G_1 and G_2 (w.l.o.g. $0 < G_1 \leq G_2 < 1$), which implies that the weight of the non-affected region is given by $G_0 = 1 - G_1 - G_2$. Thus, the desired interest-rate changes for the three regions are formally given by:

Weight of the region		*Desired interest-rate change*
G_0	$\longrightarrow$	*0*
G_1	$\longrightarrow$	$\Delta\tilde{i}(G_1)$
G_2	$\longrightarrow$	$\Delta\tilde{i}(G_2)$

By the monotonicity of $\Delta\tilde{i}(.)$ we have $\Delta\tilde{i}(G_2) \geq \Delta\tilde{i}(G_1)$ and the optimal interest-rate change for this extended shock scenario $\mathcal{E} = \{\epsilon(G_1), \epsilon(G_2)\}$ is given by[1]

$$\Delta i^* = G_1 \Delta\tilde{i}(G_1) + G_2 \Delta\tilde{i}(G_2) \tag{6.1}$$

In order to compare the FM_w-rule and the SM_w-rule, we distinguish between three cases of possible relations between G_1 and G_2:

1. $G_2 > \frac{1}{2}$*: This implies, that the larger affected region can change the interest rate alone, applying the SM_w-rule.*
2. $G_1 + G_2 > \frac{1}{2}$ and $G_2 \leq \frac{1}{2}$*: This implies, that two affected regions can change the interest rate together, applying the SM_w-rule.*
3. $G_1 + G_2 \leq \frac{1}{2}$*: This implies, that two affected regions cannot change the interest rate together, applying the SM_w-rule.*

1. $G_2 > \frac{1}{2}$:

SM_w: $G_2 > \frac{1}{2} \Longrightarrow \Delta i^{SM_w} = \Delta\tilde{i}(G_2)$

FM_w: *The interest-rate change will be at least $G_2\Delta\tilde{i}(G_2)$, since the larger region can implement this interest-rate change alone. But which interest-rate change is actually implemented depends on the relation between $\Delta\tilde{i}(G_1)$, the desired interest-rate change of the smaller affected region, $G_2\Delta\tilde{i}(G_2)$, the possible interest-rate change that the larger affected region can implement alone, and $(G_1 + G_2)\Delta\tilde{i}(G_1 + G_2)$, the interest-rate change, both affected regions can implement together.*

(a) $(G_1 + G_2)\Delta\tilde{i}(G_1 + G_2) < \Delta\tilde{i}(G_1) \Longrightarrow$
both affected regions vote for $\Delta i^{FM_w} = (G_1 + G_2)\Delta\tilde{i}(G_1 + G_2)$, since monotonicity of $\Delta\tilde{i}(.)$ implies $\Delta\tilde{i}(G_1) \leq \Delta\tilde{i}(G_2)$. Furthermore, since

$$\Delta i^{FM_w} = (G_1 + G_2)\Delta\tilde{i}(G_1 + G_2) =$$

$$G_1\Delta\tilde{i}(G_1 + G_2) + G_2\Delta\tilde{i}(G_1 + G_2) >$$

$$G_1\Delta\tilde{i}(G_1) + G_2\Delta\tilde{i}(G_2) = \Delta i^*$$

we obtain $\Delta i^* \leq \Delta i^{FM_w} < \Delta i^{SM_w}$, which implies $\mathcal{L}^{FM_w} < \mathcal{L}^{SM_w}$. This is illustrated in figure 6.1

[1]We dropped the index $\mathcal{E}$ for optimal interest-rate change, since within this proof, we examine only one specific $\mathcal{E}$.

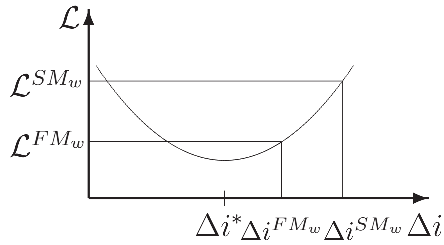


Figure 6.1:
Overall social losses with the FM_w-rule and the SM_w-rule under the conditions $G_2 > \frac{1}{2}$ and $(G_1 + G_2)\Delta\tilde{i}(G_1 + G_2) < \Delta\tilde{i}(G_1)$

(b) $G_2\Delta\tilde{i}(G_2) < \Delta\tilde{i}(G_1) \leq (G_1 + G_2)\Delta\tilde{i}(G_1 + G_2) \Longrightarrow$
both affected regions vote for $\Delta i^{FM_w} = \Delta\tilde{i}(G_1)$, *because although they could vote together for* $(G_1+G_2)\Delta\tilde{i}(G_1+G_2)$, *the smaller affected region will leave the coalition at* $\Delta\tilde{i}(G_1)$, *since this is the regional optimum.*
For the comparison of the FM_w*-rule and the* SM_w*-rule, we have to distinguish between two more cases, whether the implemented interest-rate change is larger or lower than the first-best interest-rate change.*

i. $\Delta\tilde{i}(G_1) \geq \Delta i^*$
In this case $\mathcal{L}^{FM_w} < \mathcal{L}^{SM_w}$, *which is shown by the same argumentation as in (a), because we have*

$$\Delta i^{SM_w} = \Delta\tilde{i}(G_2) > \Delta\tilde{i}(G_1) = \Delta i^{FM_w} \geq \Delta i^*$$

ii. $\Delta\tilde{i}(G_1) < \Delta i^*$
In this case, the comparison of the FM_w*-rule and the* SM_w*-rule is ambiguous, which is shown by the following examples:*

A. *Suppose* $\Delta\tilde{i}(G) = G$, $G_1 = 0.37$ *and* $G_2 = 0.6 \Longrightarrow$

$$\begin{array}{lclcl} \Delta i^* & = & G_1\Delta\tilde{i}(G_1) + G_2\Delta\tilde{i}(G_2) & = & 0.4969 \\ \Delta i^{SM_w} & = & \Delta\tilde{i}(G_2) & = & 0.6 \\ \Delta i^{FM_w} & = & \Delta\tilde{i}(G_1) & = & 0.37 \\ & & \Longrightarrow & & \end{array}$$

$|\Delta i^{SM_w} - \Delta i^*| = 0.1031 < 0.1269 = |\Delta i^{FM_w} - \Delta i^*|$
This implies because of the symmetry of the loss function with respect to the optimum Δi^* *that* $\mathcal{L}^{FM_w} > \mathcal{L}^{SM_w}$. *This is illustrated in figure 6.2 (A).*

B. *Suppose* $\Delta\tilde{i}(G) = G$, $G_1 = 0.27$ *and* $G_2 = 0.51 \Longrightarrow$

$$\begin{array}{lclcl}\Delta i^* & = & G_1\Delta\tilde{i}(G_1)+G_2\Delta\tilde{i}(G_2) & = & \mathit{0.333}\\ \Delta i^{SM_w} & = & \Delta\tilde{i}(G_2) & = & \mathit{0.51}\\ \Delta i^{FM_w} & = & \Delta\tilde{i}(G_1) & = & \mathit{0.27}\end{array}$$

$$\Longrightarrow$$

$|\Delta i^{SM_w} - \Delta i^*| = 0.177 < 0.063 = |\Delta i^{FM_w} - \Delta i^*|$

This implies because of the symmetry of the loss function with respect to the optimum Δi^* that $\mathcal{L}^{FM_w} < \mathcal{L}^{SM_w}$. This is illustrated in figure 6.2 (B).

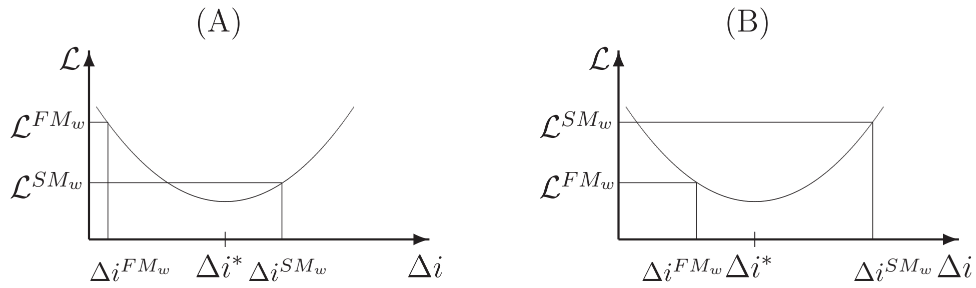


Figure 6.2:
Overall social losses with the FM_w-rule and the SM_w-rule under the conditions $G_2 > \frac{1}{2}$ and $G_2\Delta i(G_2) < \Delta\tilde{i}(G_1) \leq (G_1+G_2)\Delta\tilde{i}(G_1+G_2)$.
(A) $\Delta\tilde{i}(G) = G$, $G_1 = 0.37$ and $G_2 = 0.6$
(B) $\Delta\tilde{i}(G) = G$, $G_1 = 0.27$ and $G_2 = 0.51$

(c) $\Delta\tilde{i}(G_1) \leq G_2\Delta\tilde{i}(G_2) \Longrightarrow$
the smaller affected region leaves the coalition also at $\Delta\tilde{i}(G_1)$, but the larger affected region can implement the higher interest-rate change $\Delta i^{FM_w} = G_2\Delta\tilde{i}(G_2)$ alone. Comparing the FM_w-rule and the SM_w-rule, we obtain $\mathcal{L}^{FM_w} < \mathcal{L}^{SM_w}$. This follows from

$$\begin{aligned}&2G_1\Delta\tilde{i}(G_1) \overset{\Delta\tilde{i}(G_1)\leq G_2\Delta\tilde{i}(G_2)}{\leq} 2G_1G_2\Delta\tilde{i}(G_2) \overset{1>G_2>\frac{1}{2}}{<}\\ &G_1\Delta\tilde{i}(G_2) \overset{G_0>0}{<} (1-G_2)\Delta\tilde{i}(G_2) \Longrightarrow\end{aligned} \tag{6.2}$$

$$G_1(\Delta\tilde{i}(G_2) - \Delta\tilde{i}(G_1)) < \Delta\tilde{i}(G_2) - G_1\Delta i(G_1) - G_2\Delta\tilde{i}(G_2)$$

$$\overset{2\Delta\tilde{i}(G_1)\leq\Delta\tilde{i}(G_2)}{\Longrightarrow}$$

$$\underbrace{G_1\Delta\tilde{i}(G_1)}_{|\Delta i^{FM_w}-\Delta i^*|} < \underbrace{\Delta\tilde{i}(G_2) - G_1\Delta\tilde{i}(G_1) - G_2\Delta\tilde{i}(G_2)}_{|\Delta i^{SM_w}-\Delta i^*|} \tag{6.3}$$

2. - *SM_w:*
     $G_2 \leq \frac{1}{2}$ *and* $G_1 + G_2 > \frac{1}{2} \Longrightarrow \Delta i^{SM_w} = \Delta\tilde{i}(G_1)$
   - *FM_w:*
     Similarly to (1.) we distinguish between three cases:

     (a) $(G_1 + G_2)\Delta\tilde{i}(G_1 + G_2) < \Delta\tilde{i}(G_1) \Longrightarrow$
     $\Delta i^{FM_w} = (G_1 + G_2)\Delta\tilde{i}(G_1 + G_2)$ *(see (1.a)) and* $\mathcal{L}^{FM_w} < \mathcal{L}^{SM_w}$ *since* $\Delta i^* < \Delta i^{FM_w} < \Delta i^{SM_w}$.

     (b) $G_2\Delta\tilde{i}(G_2) < \Delta\tilde{i}(G_1) \leq (G_1 + G_2)\Delta\tilde{i}(G_1 + G_2) \Longrightarrow$
     $\Delta i^{FM_w} = \Delta\tilde{i}(G_1)$ *(see (1.b)) and* $\mathcal{L}^{FM_w} = \mathcal{L}^{SM_w}$ *since* $\Delta i^{FM_w} = \Delta i^{SM_w}$.

     (c) $\Delta\tilde{i}(G_1) \leq G_2\Delta\tilde{i}(G_2) \Longrightarrow$
     $\Delta i^{FM_w} = G_2\Delta i(G_2)$ *(see (1.c)) and* $\mathcal{L}^{FM_w} \leq \mathcal{L}^{SM_w}$ *since* $\Delta i^{SM_w} \leq \Delta i^{FM_w} < \Delta i^*$.

3. - *SM_w:*
     $G_1 + G_2 \leq \frac{1}{2} \Longrightarrow \Delta i^{SM_w} = 0$
   - *FM_w:*
     Similarly to (1.and 2.) we distinguish between three cases:

     (a) $(G_1 + G_2)\Delta\tilde{i}(G_1 + G_2) < \Delta\tilde{i}(G_1) \Longrightarrow$
     $\Delta i^{FM_w} = (G_1 + G_2)\Delta\tilde{i}(G_1 + G_2)$ *(see (1.a)) and the comparison of the FM_w-rule and the SM_w-rule is ambiguous.*

     A. *Suppose* $\Delta\tilde{i}(G) = G^2$, $G_1 = \frac{9}{144}$ *and* $G_2 = \frac{16}{144} \Longrightarrow$
     $$|\Delta i^{SM_w} - \Delta i^*| = \frac{4825}{144^3}$$
     $$|\Delta i^{FM_w} - \Delta i^*| = \frac{10800}{144^3}$$
     $$\Longrightarrow$$
     $\mathcal{L}^{FM_w} > \mathcal{L}^{SM_w}$.

     B. *Suppose* $\Delta\tilde{i}(G) = \sqrt{G}$, $G_1 = \frac{9}{144}$ *and* $G_2 = \frac{16}{144} \Longrightarrow$
     $$|\Delta i^{SM_w} - \Delta i^*| = \frac{91}{12^3}$$
     $$|\Delta i^{FM_w} - \Delta i^*| = \frac{34}{12^3}$$
     $$\Longrightarrow$$
     $\mathcal{L}^{FM_w} < \mathcal{L}^{SM_w}$.

     (b) $G_2\Delta\tilde{i}(G_2) < \Delta\tilde{i}(G_1) \leq (G_1 + G_2)\Delta\tilde{i}(G_1 + G_2) \Longrightarrow$
     $\Delta i^{FM_w} = \Delta\tilde{i}(G_1)$ *(see (1.b))*

For the comparison of the FM_w-rule and the SM_w-rule, we have to distinguish between two more cases:

i. $\Delta\tilde{i}(G_1) \geq \Delta i^*$

In this case, the comparison of the FM_w-rule and the SM_w-rule is ambiguous, which is shown by the following examples:

A. *Suppose* $\Delta\tilde{i}(G) = G^2$, $G_1 = 0.10$ *and* $G_2 = 0.15 \Longrightarrow$

$$\begin{aligned} |\Delta i^{SM_w} - \Delta i^*| &= \mathit{0.0043} \\ |\Delta i^{FM_w} - \Delta i^*| &= \mathit{0.0056} \\ &\Longrightarrow \end{aligned}$$

$\mathcal{L}^{FM_w} > \mathcal{L}^{SM_w}$.

B. *Suppose* $\Delta\tilde{i}(G) = G^3$, $G_1 = 0.10$ *and* $G_2 = 0.15 \Longrightarrow$

$$\begin{aligned} |\Delta i^{SM_w} - \Delta i^*| &= \mathit{0.00060625} \\ |\Delta i^{FM_w} - \Delta i^*| &= \mathit{0.00039375} \\ &\Longrightarrow \end{aligned}$$

$\mathcal{L}^{FM_w} < \mathcal{L}^{SM_w}$.

ii. $\Delta\tilde{i}(G_1) < \Delta i^* \Longrightarrow$

$\mathcal{L}^{FM_w} < \mathcal{L}^{SM_w}$ because $0 = \Delta i^{SM_w} < \Delta i^{FM_w} \leq \Delta i^*$.

(c) $\Delta\tilde{i}(G_1) \leq G_2\Delta\tilde{i}(G_2) \Longrightarrow$

$\Delta i^{FM_w} = G_2\Delta\tilde{i}(G_2)$ *(see (1.c)) and* $\mathcal{L}^{FM_w} \leq \mathcal{L}^{SM_w}$ *since* $0 = \Delta i^{SM_w} \leq \Delta i^{FM_w} < \Delta i^*$.

Proof of lemma 1:

Suppose w.l.o.g. $g_1 \leq g_2 \leq g_3$. If the region is divided into two regions by the extended shock scenario (i.e. $\mathcal{E} = \{\epsilon(g_\alpha)\}$ or $\mathcal{E} = \{\epsilon(g_\alpha + g_\beta)\}$ $(\alpha \neq \beta,\ \alpha, \beta = 1, 2, 3)$) $\mathcal{L}_{\mathcal{E}}^{FM_w} \leq \mathcal{L}_{\mathcal{E}}^{SM_w}$ follows directly from corollary 1.[2] Suppose that $g_3 < \frac{1}{2}$ and the union is divided into three regions ($\mathcal{E}_j = \{\epsilon(g_\alpha), \epsilon(g_\beta)\}$ $(\alpha \neq \beta,\ \alpha, \beta = 1, 2, 3)$). Suppose $g_3 < \frac{1}{2}$, this implies, we have always the case, that $G_1 + G_2 > \frac{1}{2}$ and $G_2 < \frac{1}{2}$. This is generally discussed in argumentation 1 of table 3.1 at point (2.) and we can conclude that $\mathcal{L}_{\mathcal{E}}^{FM_w} \leq \mathcal{L}_{\mathcal{E}}^{SM_w}$ in this case. Now, only the case $g_3 = \frac{1}{2}$ is left, whereas the cases $\mathcal{E} = \{\epsilon(g_1), \epsilon(g_3)\}$ and $\mathcal{E} = \{\epsilon(g_2), \epsilon(g_3)\}$ are also obtained by argumentation 1 of table 3.1 at point (2.). Thus $\mathcal{E} = \{\epsilon(g_1), \epsilon(g_2)\}$ with $g_1 + g_2 = g_3 = \frac{1}{2}$ is the last case, which is left over. This case satisfies the conditions of point (3.) in the argumentation 1 of table 3.1. Here only for (a) and (b), it is possible to violate the inequality $\mathcal{L}_{\mathcal{E}_j}^{FM_w} \leq \mathcal{L}_{\mathcal{E}_j}^{SM_w}$. But for (a), we have $\Delta i^{FM_w} = (g_1 + g_2)^2$, $\Delta i^{SM_w} = 0$ and $\Delta i^* = g_1^2 + g_2^2$, which implies $|\Delta i^{SM_w} - \Delta i^*| - |\Delta i^{FM_w} - \Delta i^*| = (g_1 - g_2)^2 \geq 0$ and for (b) $\Delta i^{FM_w} = g_1$, $\Delta i^{SM_w} = 0$ and $\Delta i^* = g_1^2 + g_2^2$, which implies $|\Delta i^{SM_w} - \Delta i^*| - |\Delta i^{FM_w} - \Delta i^*| = \underbrace{2(g_1^2 + g_2^2)}_{\geq \frac{1}{4}} - \underbrace{g_1}_{\leq \frac{1}{4}} \geq 0$

■

Proof of Proposition 7:

W.l.o.g. we assume that $g_1 \leq g_2 \leq g_3$. For the ex ante comparison of the FM_w-rule and the SM_w-rule, we have to determine the sign of $E(\Delta\mathcal{L}_{\mathcal{E}}) = E(\mathcal{L}_{\mathcal{E}}^{SM_w} - \mathcal{L}_{\mathcal{E}}^{FM_w})$. With the uniform distribution and $\Delta\tilde{i}(G) = A \cdot G$, we obtain from equation (3.18)

$$
\begin{aligned}
E(\Delta\mathcal{L}_{\mathcal{E}}) &= \textstyle\sum_{\text{all}\,\mathcal{E}} p_{\mathcal{E}} \left[\left(\Delta i_{\mathcal{E}}^{SM_w} - \sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i} \Delta\tilde{i}_{\mathcal{K}_i}\right)^2 - \left(\Delta i_{\mathcal{E}}^{FM_w} - \sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i} \Delta\tilde{i}_{\mathcal{K}_i}\right)^2 \right] \\
&= A^2 p \textstyle\sum_{\text{all}\,\mathcal{E}} \left[\left(\frac{\Delta i_{\mathcal{E}}^{SM_w}}{A^2} - \sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i}^2\right)^2 - \left(\frac{\Delta i_{\mathcal{E}}^{FM_w}}{A^2} - \sum_{\text{all}\,\mathcal{K}_i \in \mathcal{E}} G_{\mathcal{K}_i}^2\right)^2 \right]
\end{aligned} \tag{6.4}
$$

with $p = p_{\mathcal{E}} = \frac{1}{\#\mathcal{E}}$ ($\#\mathcal{E}$:= number of possible extended shock scenarios). We observe that it is sufficient to show the proposition only for $\Delta i(G) = G$ since both decision variables, $\Delta i_{\mathcal{E}}^{SM_w}$ and $\Delta i_{\mathcal{E}}^{SM_w}$ are both scaling with the linear parameter $\frac{1}{A}$.

We know, that $\mathcal{L}_{\mathcal{E}}^{SM_w} - \mathcal{L}_{\mathcal{E}}^{FM_w} \geq 0$ if the union is only divided into two regions by a shock (see corollary 1 and table 3.1). Additionally from lemma 1, it is sufficient to proof this only for the case that $g_3 > \frac{1}{2}$. This means, that we only have to deal with points (1.) and (3.) in the argumentation 1 of table 3.1.

[2]We omitted the trivial cases of $\mathcal{E} = \{\epsilon(0)\}$ and $\mathcal{E} = \{\epsilon(1)\}$.

- The only candidates of $\mathcal{E}$ for 1.(b) with negative $\Delta\mathcal{L}_{\mathcal{E}}$ are $\mathcal{E} = \{\epsilon(g_1), \epsilon(g_3)\}$ and $\mathcal{E} = \{\epsilon(g_2), \epsilon(g_3)\}$. But $\mathcal{E} = \{\epsilon(g_1), \epsilon(g_3)\}$ cannot satisfy the constraints of 1.(b), because $g_1 \leq g_2 \leq g_3$ and $g_3 > \frac{1}{2} \Longrightarrow g_1 < \frac{1}{4}$ and $g_3^2 > \frac{1}{4}$, which violates the condition $g_3^2 = G_2\Delta\tilde{i}(G_2) < \Delta\tilde{i}(G_1) = g_1$. Thus, only $\mathcal{E} = \{\epsilon(g_2), \epsilon(g_3)\}$ satisfying the conditions of 1.(b) remains for a negative $\Delta\mathcal{L}_{\mathcal{E}}$.

- The only candidate of $\mathcal{E}$ for 3.(a) and 3.(b) with negative $\Delta\mathcal{L}_{\mathcal{E}}$ is $\mathcal{E} = \{\epsilon(g_1), \epsilon(g_2)\}$. But 3.(a) with $\Delta\mathcal{L}_{\mathcal{E}} < 0$ is not possible, because we would obtain

$$\begin{array}{lcl} \Delta i_{\mathcal{E}}^{FM_w} & = & (G_1 + G_2)\Delta\tilde{i}(G_1 + G_2) = (g_1 + g_2)^2 \\ \Delta i_{\mathcal{E}}^{SM_w} & = & 0 \\ \Delta i_{\mathcal{E}}^{*} & = & g_1^2 + g_2^2 \end{array} \tag{6.5}$$

which implies

$$|\Delta i_{\mathcal{E}}^{SM_w} - \Delta i_{\mathcal{E}}^{*}| - |\Delta i_{\mathcal{E}}^{FM_w} - \Delta i_{\mathcal{E}}^{*}| = g_1^2 + g_2^2 - [(g_1 + g_2)^2 - (g_1^2 + g_2^2)] = (g_1 - g_2)^2 \geq 0 \tag{6.6}$$

Thus only $\mathcal{E} = \{\epsilon(g_1), \epsilon(g_2)\}$ satisfying the conditions of 3.(b) remains for a negative $\Delta\mathcal{L}_{\mathcal{E}}$.

In the following table, we summarize all extended shock scenarios $\mathcal{E}$, the interest-rate changes, applying the different decision rules, the optimal interest-rate change, and the difference $\Delta\mathcal{L}_{\mathcal{E}}$ between the losses of the FM_w-rule and the SM_w-rule. In order to show, that $E(\Delta\mathcal{L}_{\mathcal{E}})$ is positive, we must distinguish three cases:

1. Only $\Delta\mathcal{L}_{\mathcal{E}_9}$ is negative.

2. Only $\Delta\mathcal{L}_{\mathcal{E}_{11}}$ is negative.

3. $\Delta\mathcal{L}_{\mathcal{E}_9}$ and $\Delta\mathcal{L}_{\mathcal{E}_{11}}$ are simultaneously negative.

This distinction is necessary, because if $\Delta\mathcal{L}_{\mathcal{E}_9}$ and $\Delta\mathcal{L}_{\mathcal{E}_{11}}$ are simultaneously negative, we have more constraints on g_1, g_2, and g_3, than if they are separately negative.

1. If only $\Delta\mathcal{L}_{\mathcal{E}_9} < 0 \Longrightarrow E(\Delta\mathcal{L}_{\mathcal{E}}) > 0$.

   This follows directly from the fact, that $\Delta\mathcal{L}_{\mathcal{E}_9}$ is already outweighed by $\Delta\mathcal{L}_{\mathcal{E}_4}$ and $\Delta\mathcal{L}_{\mathcal{E}_8}$:

   $\Delta\mathcal{L}_{\mathcal{E}_4} + \Delta\mathcal{L}_{\mathcal{E}_8} + \Delta\mathcal{L}_{\mathcal{E}_9} = (g_1 + g_2)^2 + 2g_1(1 - g_1)g_2^2 > 0$

2. If only $\Delta\mathcal{L}_{\mathcal{E}_{11}} < 0 \Longrightarrow E(\Delta\mathcal{L}_{\mathcal{E}}) > 0$.

$\mathcal{E}$	$\Delta i_{\mathcal{E}}^{FM_w}$	$\Delta i_{\mathcal{E}}^{SM_w}$	$\Delta \hat{i}_{\mathcal{E}}^{*}$	$\Delta \mathcal{L}_{\mathcal{E}}$	sign
$\mathcal{E}_1 = \{\epsilon(0)\}$	0	0	0	0	$=$
$\mathcal{E}_2 = \{\epsilon(1)\}$	0	0	0	0	$=$
$\mathcal{E}_3 = \{\epsilon(g_1)\}$	g_1^2	0	g_1^2	g_1^4	$+$
$\mathcal{E}_4 = \{\epsilon(g_2)\}$	g_2^2	0	g_2^2	g_2^4	$+$
$\mathcal{E}_5 = \{\epsilon(g_3)\}$	g_3^2	g_3	g_3^2	$[(1-g_3)g_3]^2$	$+$
$\mathcal{E}_6 = \{\epsilon(g_1+g_2)\}$	$(g_1+g_2)^2$	0	$(g_1+g_2)^2$	$(g_1+g_2)^4$	$+$
$\mathcal{E}_7 = \{\epsilon(g_1+g_3)\}$	$(g_1+g_3)^2$	(g_1+g_3)	$(g_1+g_3)^2$	$[(1-g_2)g_2]^2$	$+$
$\mathcal{E}_8 = \{\epsilon(g_2+g_3)\}$	$(g_2+g_3)^2$	(g_2+g_3)	$(g_2+g_3)^2$	$[(1-g_1)g_1]^2$	$+$
$\mathcal{E}_9 = \{\epsilon(g_1)\epsilon(g_2)\}$	g_1	0	$g_1^2+g_2^2$	$[(g_1^2+g_2^2)]^2-$ $[g_1-(g_1^2+g_2^2)]^2$	$\pm$
$\mathcal{E}_{10} = \{\epsilon(g_1)\epsilon(g_3)\}$	g_3^2	g_3	$g_1^2+g_3^2$	$[g_3-(g_1^2+g_3^2)]^2-$ $[g_3^2-(g_1^2+g_3^2)]^2$	$+$
$\mathcal{E}_{11} = \{\epsilon(g_2)\epsilon(g_3)\}$	g_2	g_3	$g_2^2+g_3^2$	$[g_3-(g_2^2+g_3^2)]^2$ $-[g_2-(g_2^2+g_3^2)]^2$	$\pm$

Table 6.1:
"=" $\rightarrow \Delta\mathcal{L}_{\mathcal{E}} = 0 \rightarrow SM_w$ is as good as FM_w, "+" $\rightarrow \Delta\mathcal{L}_{\mathcal{E}} > 0 \rightarrow SM_w$ is worse than FM_w, "$\pm$" $\rightarrow \Delta\mathcal{L}_{\mathcal{E}} \gtrless 0 \rightarrow SM_w$ is ambiguous compared to FM_w.

In order to proof this, we omit $\Delta\mathcal{L}_{\mathcal{E}_9}$ and $\Delta\mathcal{L}_{\mathcal{E}_{10}}$, because they are both non-negative in this case and therefore it is sufficient to show that[3]
$\sum_{j=1}^{8} \Delta\mathcal{L}_{\mathcal{E}_j} + \Delta\mathcal{L}_{\mathcal{E}_{11}} > 0$.

We define $x := g_1$, $y := g_2 \Longrightarrow g_3 = 1 - x - y$

$$\begin{aligned} \mathcal{V}(x,y) &:= \textstyle\sum_{j=1}^{8} \Delta\mathcal{L}_{\mathcal{E}} + \Delta\mathcal{L}_{\mathcal{E}_{11}} \\ &= 4y^4 + 4x^4 + 8xy^3 + 8x^3y + 12x^2y^2 + 4y^3 - 2x^3 + 6xy^2 + 2x^2y \\ &\quad -10y^2 - 3x^2 - 12xy + 4x + 6y - 1 \end{aligned} \tag{6.7}$$

From the conditions of 1.(b), with $G_2 = g_3 = 1 - x - y$ and $G_1 = g_2 = y$ we obtain

$$\begin{array}{lll} G_2 > \frac{1}{2} & \Longrightarrow & \frac{1}{2} > x + y \\ G_2^2 < G_1 & \Longrightarrow & x > 1 - y - \sqrt{y} \\ G_1 \leq (G_1 + G_2)^2 & \Longrightarrow & y \leq (1-x)^2 \end{array} \tag{6.8}$$

[3]To omit 9 and 10 in this case and 10 in case (3.) has no further reason, except that I had already done many calculations with the sum of 1 to 8 and therefore realized at some point, that I can prove my conjecture omitting 9 and 10 or 10.

and thus the domain $\mathcal{D}$ of $\mathcal{V}(x,y)$ is given by

$$\mathcal{D} := \left\{ (x,y) \,\middle|\, \frac{1}{2} > x+y,\; x > 1-y-\sqrt{y},\; y \le (1-x)^2,\; x \ge 0,\; y \ge 0 \right\} \tag{6.9}$$

In order to show, that $\mathcal{V}(x,y) > 0$ for $(x,y) \in \mathcal{D}$, we start with the proof, that

$$\mathcal{V}_x := \tfrac{\partial \mathcal{V}}{\partial x} = \underbrace{8y^3 + 6y^2 - 12y + 4}_{a_{\mathcal{V}}(y)} + \underbrace{16x^3 - 6x^2 - 6x}_{b_{\mathcal{V}}(x)} + \underbrace{24xy(x+y) + 4xy}_{c_{\mathcal{V}}(x,y)} > 0$$

$$\text{for} \quad (x,y) \in \bar{\mathcal{D}} := \left\{ (x,y) \,\middle|\, \tfrac{1}{4} \ge x \ge 0\,,\; \tfrac{1}{2} \ge y \ge \tfrac{1}{4} \right\} \tag{6.10}$$

with $\mathcal{D} \subseteq \bar{\mathcal{D}}$ (from (6.8) we have $g_3 > \frac{1}{2} \wedge g_2 > g_3^3$, which leads together with $g_2 \ge g_1 \ge 0$ to $\frac{1}{2} \ge y \ge \frac{1}{4}$ and $\frac{1}{4} \ge x \ge 0$).

With

$$\begin{aligned} \tilde{a}_{\mathcal{V}} &= 16y^2 - 16y + \tfrac{9}{2} && \Longrightarrow \quad a_{\mathcal{V}} - \tilde{a}_{\mathcal{V}} = 8(y - \tfrac{1}{4}(y - \tfrac{1}{2})) && \ge 0 \\ \tilde{b}_{\mathcal{V}} &= 2x^2 - 7x && \Longrightarrow \quad b_{\mathcal{V}} - \tilde{b}_{\mathcal{V}} = 16x(x - \tfrac{1}{4})^2 && \ge 0 \\ \tilde{c}_{\mathcal{V}} &= 10xy && \Longrightarrow \quad c_{\mathcal{V}} - \tilde{c}_{\mathcal{V}} = (4(x+y)+1)xy && \ge 0 \end{aligned} \tag{6.11}$$

we obtain $\mathcal{V}_x \ge \tilde{\mathcal{V}}_x := \tilde{a}_{\mathcal{V}} + \tilde{b}_{\mathcal{V}} + \tilde{c}_{\mathcal{V}}$ for $(x,y) \in \bar{\mathcal{D}}$.

Observe that $\tilde{\mathcal{V}}_x(x,y) = 16y^2 - 16y + 2x^2 - 7x + 10xy + \frac{9}{2}$ $((x,y) \in \mathbb{R}^2)$ is a paraboloid with a global minimum at $(x^*,y^*) \notin \bar{\mathcal{D}}$, since

$$\vec{\nabla} \tilde{\mathcal{V}}_x = 0 \Longleftrightarrow (x^*,y^*) = \left(\frac{16}{7}, -\frac{3}{14} \right) \quad \text{with} \quad \mathcal{V}_x(x^*,y^*) = -\frac{25}{14} \tag{6.12}$$

and

$$Hess(\tilde{\mathcal{V}}_x(x^*,y^*)) = \begin{pmatrix} 4 & 10 \\ 10 & 32 \end{pmatrix} \tag{6.13}$$

is positive definite, because $Hess(\tilde{\mathcal{V}}_x(x^*,y^*))$ has two positive eigenvalues[4] λ_1, λ_2.

Since $(x^*,y^*) \notin \bar{\mathcal{D}}$, we know that $\min_{(x,y)\in\bar{\mathcal{D}}}\{\tilde{\mathcal{V}}_x\}$ lies on the boundary of $\bar{\mathcal{D}}$. Since $\bar{\mathcal{D}}$ is a closed rectangle in $\mathbb{R}^2$, we have to compare the four minima lying on the sides of $\bar{\mathcal{D}}$. From this, we obtain $\min_{(x,y)\in\bar{\mathcal{D}}}\{\tilde{\mathcal{V}}_x\} = \frac{7}{256} > 0 \Longrightarrow \mathcal{V}_x > 0$ for $(x,y) \in \bar{\mathcal{D}}$.

This proves, that $\mathcal{V}$ is increasing in x for $(x,y) \in \bar{\mathcal{D}}$.

Generally, we can show that $\mathcal{V}(0,y) > 0$ for $y \ge \frac{3}{10}$.

[4] $\mathrm{Det}\left[Hess(\tilde{\mathcal{V}}_x(x^*,y^*)) - \lambda\,\mathbb{E} \right] = \lambda^2 - 36\lambda + 28 \Longrightarrow \lambda_1, \lambda_2 > 0$, because $\left(\frac{36}{2}\right)^2 = 324 > 28$.

$$\mathcal{V}(0,y) = 4y^4 + 4y^3 - 10y^2 + 6y - 1 \tag{6.14}$$

is a Polynom of degree 4 ($P(\mathcal{O}^4)$) in y with the following properties:

(a) $\lim_{y\to\pm\infty} \mathcal{V}(0,y) = \infty$.

(b) $V(0,-3) = 107 > 0$, $V(0,0) = -1 < 0$, $V(0,\frac{13}{44}) = \frac{31333}{937024} > 0$

(c) $\text{cub}^4(\mathcal{V}(0,y)) = \frac{16399}{13824} > 0$

where $\text{cub}^4(P(\mathcal{O}^4))$ is the discriminant of the associated cubic equation[5] of $\mathcal{V}(0,y) = 0$.

From (c) follows that $\mathcal{V}(0,y)$ has only two real roots y_1 and y_2 (Cardano (1545) and Ferrari (1545)) and from (b), we obtain $-3 < y_1 < 0$ and $0 < y_2 < \frac{13}{44}$. Together with (a), this implies that $\mathcal{V}(0,y) > 0$ for $y \geq \frac{3}{10} > \frac{13}{44}$ and with $\mathcal{V}_x > 0$ for $(x,y) \in \bar{\mathcal{D}}$ we end up with

$$\mathcal{V}(x,y) > 0 \quad \text{for} \quad (x,y) \in \left\{ (x,y) \,\middle|\, \frac{1}{4} \geq x \geq 0\,,\ \frac{1}{2} \geq y \geq \frac{3}{10} \right\} \tag{6.15}$$

For $y \leq \frac{3}{10}$, we incorporate to $\bar{\mathcal{D}}$ the constraint $x > 1 - y - \sqrt{y}$ from the domain $\mathcal{D}$. Observe that $1 - y - \sqrt{y}$ is decreasing in y, $1 - \frac{3}{10} - \sqrt{\frac{3}{10}} > \frac{1}{10}$ and

$$\mathcal{V}\left(\frac{1}{10}, y\right) = 4y^4 + \frac{24}{5}y^3 - \frac{232}{25}y^2 + \frac{1207}{250}y - \frac{1579}{2500} > 0 \quad \text{for} \quad y \geq \frac{1}{4} \tag{6.16}$$

[5]Suppose $P^w(w) = w^4 + aw^3 + bw^2 + cw + d$ ($a,b,c,d,w \in \mathbb{R}$). Within three steps, this polynom is transformed into a polynom of degree 3 without a quadratic term:

Step 1 P^w is transformed into a polynom of degree 4 without a cubic term by $w = x - \frac{a}{4} \Longrightarrow$ $P^x(x) = x^4 + px^2 + qx + r$ with $p = b - 3a^2/8 \quad q = a^3/8 - ab/2 + c$ $r = -(3a^4 + 16a^2b + 64ac - 256d)/256$

Step 2 The cubic resolvent $P^y(y)$ of P^x is given by $P^y(y) = y^3 + \alpha y^2 + \beta y + \gamma$ with $\alpha = -2p$ $\beta = p^2 - 4r \quad \gamma = q^2$

Step 3 The cubic resolvent P^y is transformed into a polynom of degree 3 without a quadratic term by $y = z - \frac{\alpha}{3} \Longrightarrow P^z(z) = z^3 + \tilde{p}z + \tilde{q}$ with $\tilde{p} = \beta - \alpha^2/3 \quad \tilde{q} = 2\alpha^3/27 - \alpha\beta/3 + \gamma$

$cub^4(P^w)$ is then given by $cub^4(P^w) = \left(\frac{\tilde{p}}{2}\right)^3 + \left(\frac{\tilde{q}}{2}\right)^2$.

because similar to equation (6.14), $\mathcal{V}\left(\frac{1}{10}, y\right)$ has the following properties:

(a) $\lim_{y\to\pm\infty} \mathcal{V}(\frac{1}{10}, y) = \infty$.

(b) $V(0,-3) = \frac{239411}{2500} > 0$, $V(\frac{1}{10}, 0) = -\frac{1579}{2500} < 0$, $V(\frac{1}{10}, \frac{1}{5}) = \frac{19}{2500} > 0$

(c) $\mathrm{cub}^4(\mathcal{V}(0,y)) = \frac{73408722673}{1440000000000} > 0$

((a)-(c) directly imply, that $\mathcal{V}\left(\frac{1}{10}, y\right) > 0$ for $y \geq \frac{1}{4}$). Together with $\mathcal{V}_x > 0$ for $(x,y) \in \bar{\mathcal{D}}$ we obtain

$$\mathcal{V}(x,y) > 0 \quad \text{for} \quad (x,y) \in \left\{(x,y) \middle| \frac{1}{4} \geq x \geq 0,\ \frac{3}{10} \geq y \geq \frac{1}{4},\ x > 1 - y - \sqrt{y}\right\} \tag{6.17}$$

and (6.15) together with (6.17) imply

$$\mathcal{V}(x,y) > 0 \quad \text{for} \quad (x,y) \in \mathcal{D} \tag{6.18}$$

3. If $\Delta\mathcal{L}_{\mathcal{E}_9} < 0$ and $\Delta\mathcal{L}_{\mathcal{E}_{11}} < 0 \Longrightarrow E(\Delta\mathcal{L}_{\mathcal{E}}) > 0$.

   Again, we omit $\Delta\mathcal{L}_{\mathcal{E}_{10}} > 0$ and show that $\sum_{j=1}^{9} \Delta\mathcal{L}_{\mathcal{E}_j} + \Delta\mathcal{L}_{\mathcal{E}_{11}} > 0$.

   With $x := g_1$, $y := g_2$ we obtain

$$\begin{aligned} V(x,y) &:= \textstyle\sum_{j=1}^{9} \Delta\mathcal{L}_{\mathcal{E}_j} + \Delta\mathcal{L}_{\mathcal{E}_{11}} \\ &= 4y^4 + 4x^4 + 8xy^3 + 8x^3y + 12x^2y^2 + 4y^3 + 8xy^2 + 2x^2y \\ &\quad -10y^2 - 4x^2 - 12xy + 4x + 6y - 1 \end{aligned} \tag{6.19}$$

From the conditions of 1.(b), with $G_2 = g_3 = 1 - x - y$ and $G_1 = g_2 = y$ we obtain

$$\begin{array}{lcl} G_2 > \frac{1}{2} & \Longrightarrow & \frac{1}{2} > x + y \\ G_2^2 < G_1 & \Longrightarrow & x > 1 - y - \sqrt{y} \\ G_1 \leq (G_1 + G_2)^2 & \Longrightarrow & y \leq (1-x)^2 \end{array} \tag{6.20}$$

and from the conditions of 3.(b) with $G_1 = g_1 = x$ and $G_2 = g_2 = y$ we obtain

$$\begin{array}{lcl} G_1 + G_2 \leq \frac{1}{2} & \Longrightarrow & \frac{1}{2} \geq x + y \\ G_2^2 < G_1 & \Longrightarrow & y^2 < x \\ G_1 \leq (G_1 + G_2)^2 & \Longrightarrow & y \geq \sqrt{x} - x \end{array} \tag{6.21}$$

and thus the domain D of $V(x,y)$ is given by

$$D := \left\{(x,y)\,\middle|\,\frac{1}{2} > x+y,\ x > 1-y-\sqrt{y},\ y \leq (1-x)^2,\ y^2 < x,\ y \geq \sqrt{x}-x,\ x \geq 0,\ y \geq 0\right\} \tag{6.22}$$

In order to show, that $V(x,y) > 0$ for $(x,y) \in \mathcal{D}$, we start with the proof, that

$$V_x := \tfrac{\partial V}{\partial x} = \underbrace{4(1-3y+2y^2+2y^3)}_{a_V(y)} + \underbrace{16x^3-8x}_{b_V(x)} + \underbrace{4xy+24xy^2+24x^2y}_{c_V(x,y)} > 0$$
$$\text{for}\quad (x,y) \in \bar{D} := \left\{(x,y)\,\middle|\,\tfrac{1}{4} \geq x \geq \tfrac{1}{16}\,,\ \tfrac{1}{2} \geq y \geq \tfrac{1}{4}\right\} \tag{6.23}$$

with $D \subseteq \bar{D}$

(from (6.20) we have $g_3 > \frac{1}{2} \wedge g_2 > g_3^3$, which leads together with $g_2 \geq g_1 \geq 0$ and $g_2^2 < g_1$ (see (6.21)) to $\frac{1}{2} \geq y \geq \frac{1}{4}$ and $\frac{1}{4} \geq x \geq \frac{1}{16}$)

With

$$\begin{aligned}
\tilde{a}_V &= 17y^2 - \tfrac{152}{10}y + \tfrac{43}{10} &\Longrightarrow\quad a_V - \tilde{a}_V &= 8y^3 - 9y^2 + \tfrac{16}{5}y - \tfrac{3}{10} &\geq 0\\
\tilde{b}_V &= 2x^2 - 7x &\Longrightarrow\quad b_V - \tilde{b}_V &= 2x(4x-1)(2x-1) &\geq 0\\
\tilde{c}_V &= 10xy &\Longrightarrow\quad c_V - \tilde{c}_V &= (24(x+y) - \tfrac{15}{2})xy &\geq 0
\end{aligned} \tag{6.24}$$

we obtain $V_x \geq \tilde{V}_x := \tilde{a}_V + \tilde{b}_V + \tilde{c}_V$ for $(x,y) \in \bar{D}$.

- $a_V - \tilde{a}_V \geq 0$ because $\Delta a_V := a_V - \tilde{a}_V$ is a polynom of degree 3 ($P(\mathcal{O}^3)$) in y with the following properties:
  (a) $\lim_{y\to\pm\infty} \Delta a_v = \pm\infty$.
  (b) $\Delta a_v(0) = -\frac{3}{10} < 0$, $\Delta a_v(\frac{2}{5}) = \frac{13}{250} > 0$
  (c) $\mathrm{cub}^3(\Delta a_v) = \frac{331}{27648000} > 0$

  where $\mathrm{cub}^3(P(\mathcal{O}^3))$ is the discriminant of the cubic equation[6] $\Delta a_v = 0$. From (c) follows that Δa_v has exactly one real root (Cardano (1545)) y_1 with $0 < y_1 < \frac{2}{5}$ (see (b)) and together with (a) this implies that $\Delta a_v \geq 0$ for $y \geq \frac{1}{4} > \frac{2}{5}$.
- $b_V - \tilde{b}_V \geq 0$ because $\Delta b_V := b_V - \tilde{b}_V$ is a polynom of degree 3 with three real roots at $x_1 = 0$, $x_2 = \frac{1}{4}$ and $x_3 = \frac{1}{2}$ and since $\lim_{x\to\pm\infty} = \pm\infty$, Δb_V must be non-negative for $x \in [x_1, x_2] = [0, \frac{1}{4}]$.

[6]Suppose $P(w) = w^3 + \alpha w^2 + \beta w^2 + \gamma$ ($\alpha, \beta, \gamma, w \in \mathbb{R}$), then $\mathrm{cub}^3(P) = \left(\frac{\tilde{p}}{2}\right)^3 + \left(\frac{\tilde{q}}{2}\right)^2$ with $\tilde{p} = \beta - \alpha^2/3$ $\quad\tilde{q} = 2\alpha^3/27 - \alpha\beta/3 + \gamma$ (see also footnote 5).

- $c_V - \tilde{c}_V \geq 0$ because $\min_{(x,y)\in D}\{24(x+y)\} = 24(\frac{1}{4}+\frac{1}{16}) = \frac{15}{2}$.

Observe that $\tilde{V}_x(x,y) = 17y^2 - \frac{76}{5}y + 12x^2 - 10x + \frac{23}{2}xy + \frac{43}{10}$ $((x,y) \in \mathbb{R}^2)$ is a paraboloid with a global minimum at $(x^*,y^*) \in \bar{D}$ with $\tilde{V}_x(x^*,y^*) > 0$, since

$$\vec{\nabla}\tilde{V}_x = 0 \Longleftrightarrow (x^*,y^*) = \left(\frac{4996}{12675}, \frac{3304}{13675}\right) \quad \text{with} \quad \tilde{V}_x(x^*,y^*) = \frac{43129}{136750} \tag{6.25}$$

and

$$Hess(\tilde{V}_x(x^*,y^*)) = \begin{pmatrix} 34 & \frac{23}{2} \\ \frac{23}{2} & 24 \end{pmatrix} \tag{6.26}$$

is positive definite, because $Hess(\tilde{V}_x(x^*,y^*))$ has two positive eigenvalues[7] λ_1, λ_2. Since the global minimum is positive and lies within $\bar{\mathcal{D}}$, we obtain directly V is increasing in x for $x \in \bar{\mathcal{D}}$.

Additionally, $V(\frac{1}{16}, y) => 0$ is a polynom of degree 4 with the following properties:

(a) $\lim_{y\to\pm\infty} \mathcal{V}(0,y) = \infty$.

(b) $V(\frac{1}{16}, -3) => \frac{1634393}{16384}$, $V(\frac{1}{16}, 0) = -\frac{12487}{16384} < 0$, $V(0, \frac{23}{100}) = \frac{43919521}{6400000000} > 0$

(c) $\text{cub}^4(\mathcal{V}(0,y)) = \frac{40110380749}{824633720832} > 0$

From (c) follows that $V(\frac{1}{16}, y)$ has only two real roots y_1 and y_2 and from (b), we obtain $-3 < y_1 < 0$ and $0 < y_2 < \frac{23}{100}$. Together with (a), this implies that $V(\frac{1}{16}, y) > 0$ for $y \geq \frac{1}{4} > \frac{23}{100}$ and with $V_x > 0$ for $(x,y) \in \bar{D}$ it is shown that $V(x,y) > 0$ for $(x,y) \in D$.

$\blacksquare$

[7] $\text{Det}\left[Hess(\tilde{V}_x(x^*,y^*)) - \lambda\,\mathbb{E}\right] = \lambda^2 - 58\lambda + \frac{2735}{4} \Longrightarrow \lambda_1, \lambda_2 > 0$, because $58^2 = 3364 > 2735$.

Proof of proposition 8:
We start with the proof of (i) and w.o.l.g. $G_{\mathcal{K}^+} \geq G_{\mathcal{K}^-}$ throughout the whole proof. Observe in voting stage 1, the regions $\mathcal{K}^+$ and $\mathcal{K}^-$ vote always for their preferred direction of interest-rate changes, because these regions cannot gain anything by deviating to another direction or by abstention, taking into account the decision of the other regions. This implies, that the region, which prefers to retain the status quo, in the following denoted by $\mathcal{K}^= = \mathcal{N} \setminus \{\mathcal{K}^+, \mathcal{K}^-\}$ takes into account, that the affected regions vote for their preferred direction of interest-rate changes. This implies, that if $\mathcal{K}^=$ chooses abstention $\Delta i = G_{\mathcal{K}^+}\Delta\tilde{i}(G_{\mathcal{K}^+}) - G_{\mathcal{K}^-}\Delta\tilde{i}(G_{\mathcal{K}^-})$ is the implemented interest-rate change after voting stage 2, because $\mathcal{K}^+$ is winning voting stage 1 and since the preferred interest-rate change $\Delta\tilde{i}(G_{\mathcal{K}^+})$ of region $\mathcal{K}^+$ is larger than

$$\Delta i^{FM_w^{\mathcal{E}^\pm}} = G_{\mathcal{K}^+}\Delta\tilde{i}(G_{\mathcal{K}^+}) - G_{\mathcal{K}^-}\Delta\tilde{i}(G_{\mathcal{K}^-})$$

the maximum possible interest-rate change, then $\Delta i^{FM_w^{\mathcal{E}^\pm}}$ is implemented, since $\bar{G} = G_{\mathcal{K}}^-$. In order to calculate the best response of $\mathcal{K}^=$, we distinguish between three cases:

1. $G_{\mathcal{K}^=} > \frac{1}{2}$:

   Suppose $\mathcal{K}^=$ votes for raising interest rates, then raising wins voting stage 1 and $\Delta i = G_{\mathcal{K}^+}\Delta\tilde{i}(G_{\mathcal{K}^+}) - G_{\mathcal{K}^-}\Delta\tilde{i}(G_{\mathcal{K}^-})$ is the implemented interest-rate change, which equals the outcome of abstention.

   Suppose $\mathcal{K}^=$ votes for lowering interest rates, then lowering wins voting stage 1 and interest rates are not changed in voting stage 2, because region $\mathcal{K}^-$ cannot lower interest rates, because $G_{\mathcal{K}^-}\Delta\tilde{i}(G_{\mathcal{K}^-}) - \bar{G}\Delta\tilde{i}(\bar{G}) > 0$ and $G_{\mathcal{K}^+} = \bar{G}$. But no interest-rate change is the preferred interest rate of $\mathcal{K}^=$ and therefore improves the outcome of $\mathcal{K}^=$ compared to abstention. This implies, that region $\mathcal{K}^=$ has an incentive to vote together with the smaller region in voting stage 1. But since voting in voting stage 1 is secret, $\mathcal{K}^=$ does not know, which is the right direction. But additionally voting together with the larger affected region does not worsen the outcome of $\mathcal{K}^=$. This implies that $\mathcal{K}^=$ randomly chooses to raise or to lower interest rate rates.

2. $G_{\mathcal{K}^+} > \frac{1}{2}$: Suppose $\mathcal{K}^=$ votes for raising interest rates, then again $\Delta i = G_{\mathcal{K}^+}\Delta\tilde{i}(G_{\mathcal{K}^+}) - G_{\mathcal{K}^-}\Delta\tilde{i}(G_{\mathcal{K}^-})$ is implemented in voting stage 2.

   Suppose $\mathcal{K}^=$ votes for lowering interest rates, then still raising wins voting stage 1. But interest rates are changed only up to $\Delta i = G_{\mathcal{K}^+}\Delta\tilde{i}(G_{\mathcal{K}^+}) - (G_{\mathcal{K}^=} + G_{\mathcal{K}^-})\Delta\tilde{i}(G_{\mathcal{K}^=} + G_{\mathcal{K}^-})$ since now $\bar{G} = G_{\mathcal{K}^-} + G_{\mathcal{K}^=}$. This also improves the outcome for $\mathcal{K}^=$ compared to abstention. But again $\mathcal{K}^=$

cannot be sure about the right direction and therefore randomly chooses to raise or to lower interest rates.

3. $G_{\mathcal{K}^+}, G_{\mathcal{K}^-}, G_{\mathcal{K}^=} \leq \frac{1}{2}$:

   By the same argumentation as in (1.) $\mathcal{K}^=$ randomly chooses to raise or to lower interest rates.

Altogether, we obtain as possible outcomes which are not first-best (see equation 3.27), $\Delta i = 0$ in cases of $G_{\mathcal{K}^+} \neq G_{\mathcal{K}^-}$ (see (1.) and (3.)) and
$\Delta i = G_{\mathcal{K}^+}\Delta\tilde{i}(G_{\mathcal{K}^+}) - (G_{\mathcal{K}^=} + G_{\mathcal{K}^-})\Delta\tilde{i}(G_{\mathcal{K}^-} + G_{\mathcal{K}^=})$ (see (2.)). This proofs (i) of proposition 8.

Now we compare the possible outcomes calculated in (1.)-(3.) with the outcomes of the SM_w-rule in order to proof (ii). First, we directly observe that if the $FM_w^{\mathcal{E}^\pm}$-two-stage-rule implements first-best, this can never be worse than the outcome of SM_w. Therefore, we have only to consider the three cases in which the outcome of $FM_w^{\mathcal{E}^\pm}$-two-stage-rule can differ from first-best.

1. $G_{\mathcal{K}^=} > \frac{1}{2}$ and the $FM_w^{\mathcal{E}^\pm}$-two-stage-rule implements $\Delta i = 0$, then this coincides with the outcome of SM_w, since the region $\mathcal{K}^=$ wins the SM_w vote.

2. $G_{\mathcal{K}^+} > \frac{1}{2}$ and the $FM_w^{\mathcal{E}^\pm}$-two-stage-rule implements
   $\Delta i = G_{\mathcal{K}^+}\Delta\tilde{i}(G_{\mathcal{K}^+}) - (G_{\mathcal{K}^=} + G_{\mathcal{K}^-})\Delta\tilde{i}(G_{\mathcal{K}^-} + G_{\mathcal{K}^=})$. In this case SM_w implements $\Delta i = \Delta\tilde{i}(G_{\mathcal{K}^+})$ the preferred interest-rate change of region $\mathcal{K}^+$ since $G_{\mathcal{K}^+} > \frac{1}{2}$. Considering equation 3.26 it is sufficient to compare the absolute deviation of the outcome of SM_w (i.e. $\Delta\tilde{i}(G_{\mathcal{K}^+})$) from the optimal interest-rate change $\Delta i^*_{\mathcal{E}^\pm}$ with the deviation of the outcome of the $FM_w^{\mathcal{E}^\pm}$-two-stage-rule
   (i.e $G_{\mathcal{K}^+}\Delta\tilde{i}(G_{\mathcal{K}^+}) - (G_{\mathcal{K}^=} + G_{\mathcal{K}^-})\Delta\tilde{i}(G_{\mathcal{K}^-} + G_{\mathcal{K}^=})$) from the optimal interest-rate change $\Delta i^*_{\mathcal{E}^\pm}$. This directly implies

   $$
   \begin{aligned}
   &|\Delta\tilde{i}(G_{\mathcal{K}^+}) - (G_{\mathcal{K}^+}\Delta\tilde{i}(G_{\mathcal{K}^+}) + G_{\mathcal{K}^-}\Delta\tilde{i}(G_{\mathcal{K}^-}))| - \\
   &|G_{\mathcal{K}^+}\Delta\tilde{i}(G_{\mathcal{K}^+}) + (G_{\mathcal{K}^=} + G_{\mathcal{K}^-})\Delta\tilde{i}(G_{\mathcal{K}^=} + G_{\mathcal{K}^-}) - \\
   &(G_{\mathcal{K}^+}\Delta\tilde{i}(G_{\mathcal{K}^+}) + G_{\mathcal{K}^-}\Delta\tilde{i}(G_{\mathcal{K}^-}))| > 0 \\
   &\qquad\qquad\Longleftrightarrow \\
   &(1 - G_{\mathcal{K}^+})(\Delta\tilde{i}(G_{\mathcal{K}^+}) - \Delta\tilde{i}(1 - G_{\mathcal{K}^+})) > 0
   \end{aligned}
   $$

   which is true since $G_{\mathcal{K}^+} > \frac{1}{2}$ and $\Delta\tilde{i}(x)$ strictly monotonically decreasing for $x \leq 0$ and strictly monotonically increasing for $x \geq 0$.

3. $G_{\mathcal{K}^+}, G_{\mathcal{K}^-}, G_{\mathcal{K}^=} \leq \frac{1}{2}$ no region has a majority and therefore interest rates are not changed under the SM_w, which coincides with the possible non-first-best outcome of the $FM_w^{\mathcal{E}^\pm}$-two-stage-rule.

Altogether we have shown that $FM_w^{\mathcal{E}^\pm}$-two-stage-rule is never worse than the SM_w.

■

6.2 B Examples

6.2.1 Baseline-Model

We consider a monetary union consisting of three countries and economic weights of $g_1 = 0.1$, $g_2 = 0.2$ and $g_3 = 0.7$. Applying the ordering according to (2.26), we obtain $G_{(1)} = 0$, $G_{(2)} = 0.1$, $G_{(3)} = 0.2$, $G_{(4)} = 0.3$, $G_{(5)} = 0.7$, $G_{(6)} = 0.8$, $G_{(7)} = 0.9$, and $G_{(8)} = 1$. Furthermore, we assume that $\Delta i\big(G_{\mathcal{K}}\big) = 10G_{\mathcal{K}}$ and that all shocks are uniformly distributed, which implies that $p_{\mathcal{K}} = \frac{1}{8}$ for all $\mathcal{K}$. The social losses in every shock scenario of the simple majority rules SM_{nw} and SM_w can directly be calculated from equations (2.14) and (2.17). The calculation for the flexible majority rules follows equation (2.19) for FM_{nw}, equation (2.33) for $FM_{nw'}$, equation (2.35) for $FM_{nw''}$, and equation (2.25) for FM_w. Explicitly we obtain

$$\alpha^{FM_{nw}}(\Delta\hat{i}) = \begin{cases} 0 & \text{if} & & & \Delta\hat{i} & = & 0 \\ \frac{1}{3} & \text{if} & 0 & < & \Delta\hat{i} & \leq & 0.1 \\ \frac{2}{3} & \text{if} & 0.1 & < & \Delta\hat{i} & \leq & 8.1 \\ 1 & \text{if} & 8.1 & < & \Delta\hat{i} & & \end{cases} \tag{6.27}$$

$$\alpha^{FM_{nw'}}(\Delta\hat{i}) = \begin{cases} 0 & \text{if} & & & \Delta\hat{i} & = & 0 \\ \frac{1}{3} & \text{if} & 0 & < & \Delta\hat{i} & \leq & 0.2 \\ \frac{2}{3} & \text{if} & 0.2 & < & \Delta\hat{i} & \leq & 7.2 \\ 1 & \text{if} & 7.2 & < & \Delta\hat{i} & & \end{cases} \tag{6.28}$$

$$\alpha^{FM_{nw''}}(\Delta\hat{i}) = \begin{cases} 0 & \text{if} & & & \Delta\hat{i} & = & 0 \\ \frac{1}{3} & \text{if} & 0 & < & \Delta\hat{i} & \leq & 2.65 \\ \frac{2}{3} & \text{if} & 2.65 & < & \Delta\hat{i} & \leq & 7.25 \\ 1 & \text{if} & 7.25 & < & \Delta\hat{i} & & \end{cases} \tag{6.29}$$

$$\alpha^{FM_{nw''}}(\Delta\hat{i}) = \begin{cases} 0 & \text{if} & & & \Delta\hat{i} & = & 0 \\ 0.1 & \text{if} & 0 & < & \Delta\hat{i} & \leq & 0.1 \\ 0.2 & \text{if} & 0.1 & < & \Delta\hat{i} & \leq & 0.4 \\ 0.3 & \text{if} & 0.4 & < & \Delta\hat{i} & \leq & 0.9 \\ 0.7 & \text{if} & 0.9 & < & \Delta\hat{i} & \leq & 4.9 \\ 0.8 & \text{if} & 4.9 & < & \Delta\hat{i} & \leq & 6.4 \\ 0.9 & \text{if} & 6.4 & < & \Delta\hat{i} & \leq & 8.1 \\ 1 & \text{if} & 8.1 & < & \Delta\hat{i} & & \end{cases} \tag{6.30}$$

In the following table, we compare all six different decision by means of their outcome in all shock scenarios $\mathcal{K}$ and in the last row we calculate the expected social losses.

$G_{\mathcal{K}_j}$	$\Delta i\,(G_{\mathcal{K}})$	n	$\mathcal{L}_{\mathcal{K}}^{SM_{nw}}$	$\mathcal{L}_{\mathcal{K}}^{SM_{w}}$	$\mathcal{L}_{\mathcal{K}}^{FM_{nw}}$	$\mathcal{L}_{\mathcal{K}}^{FM_{nw'}}$	$\mathcal{L}_{\mathcal{K}}^{FM_{nw''}}$	$\mathcal{L}_{\mathcal{K}}^{FM_{w}}$
0	0	0	0	0	0	0	0	0
0.1	1	1	0.1	0.1	0.09	0.1	0.09	0.09
0.2	2	1	0.8	0.8	0.73	0.68	3.2	0.64
0.3	3	2	6.3	2.7	6.3	6.3	6.3	1.89
0.7	7	1	34.3	14.7	33.33	32.38	15.3525	10.29
0.8	8	2	12.8	12.8	12.8	10.88	10.9625	10.24
0.9	9	2	8.1	8.1	7.29	8.1	8.0125	7.29
1	10	3	0	0	0	0	0	0
$E[\mathcal{L}_{\mathcal{K}}]$			7.8	4.9	7.5675	7.305	5.4897	3.805

Table 6.2: Comparison of the outcomes of the different decision rules.

6.2.2 Transparency

In order to illustrate our results, we list the different welfare outcomes for parameter values for a large (figure 6.3) and a small shock (figure 6.4). There we can see, that indeed the outcome of OP_1 in combination with FM_w has the lowest overall social losses, if we restrict Δi and Δi^e to non-negative values. In the tables 6.3 and 6.4, we provide the corresponding values for Δi and Δi^e and $\mathcal{L}$ in the parameter setups with $A := 1/3$ $B := 1$ $\beta := 1$ $C := 1$ $p := 1/3$, where we set $G := 1/3$ and $\epsilon(1/3) := 1$ for the small shock and $G := 2/3$ and $\epsilon(2/3) := 2$ for the large shock.

Large shock ($G = \frac{2}{3}$): $\mathcal{L}(\Delta i, \Delta i^e, 2) = (\Delta i - \frac{1}{3}\Delta i^e - \frac{4}{3})^2 + (\Delta i^e + \frac{4}{3})^2 + \frac{16}{9}$					
		OP_1	TP	OP_2	$First\,Best$
SM_w	Δi	2	3	$\frac{58}{21}$	$\frac{8}{9}$
	Δi^e	0	3	$\frac{16}{7}$	-$\frac{4}{3}$
	$\mathcal{L}$	4	21	$\frac{2252}{147}$	$\frac{16}{9}$
FM_w	Δi	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{8}{9}$
	Δi^e	0	$\frac{4}{3}$	$\frac{16}{7}$	-$\frac{4}{3}$
	$\mathcal{L}$	$\frac{32}{9}$	$\frac{736}{81}$	$\frac{2272}{147}$	$\frac{16}{9}$

Table 6.3:
Social losses, expected interest-rate changes, and finally implemented interest-rate changes in the case of a large shock.

Small shock ($G = \frac{1}{3}$): $\mathcal{L}(\Delta i, \Delta i^e, 1) = (\Delta i - \frac{1}{3}(\Delta i^e + 1))^2 + (\Delta i^e + \frac{1}{3})^2 + \frac{4}{9}$					
		OP_1	TP	OP_2	$First\,Best$
SM_w	Δi	0	0	$\frac{1}{12}$	$\frac{2}{9}$
	Δi^e	0	0	$\frac{1}{4}$	$-\frac{1}{3}$
	$\mathcal{L}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{43}{48}$	$\frac{4}{9}$
FM_w	Δi	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{9}$
	Δi^e	0	$\frac{1}{3}$	$\frac{1}{4}$	$-\frac{1}{3}$
	$\mathcal{L}$	$\frac{5}{9}$	$\frac{73}{81}$	$\frac{19}{24}$	$\frac{4}{9}$

Table 6.4:
Social losses, expected interest-rate changes, and finally implemented interest-rate changes in the case of a small shock.

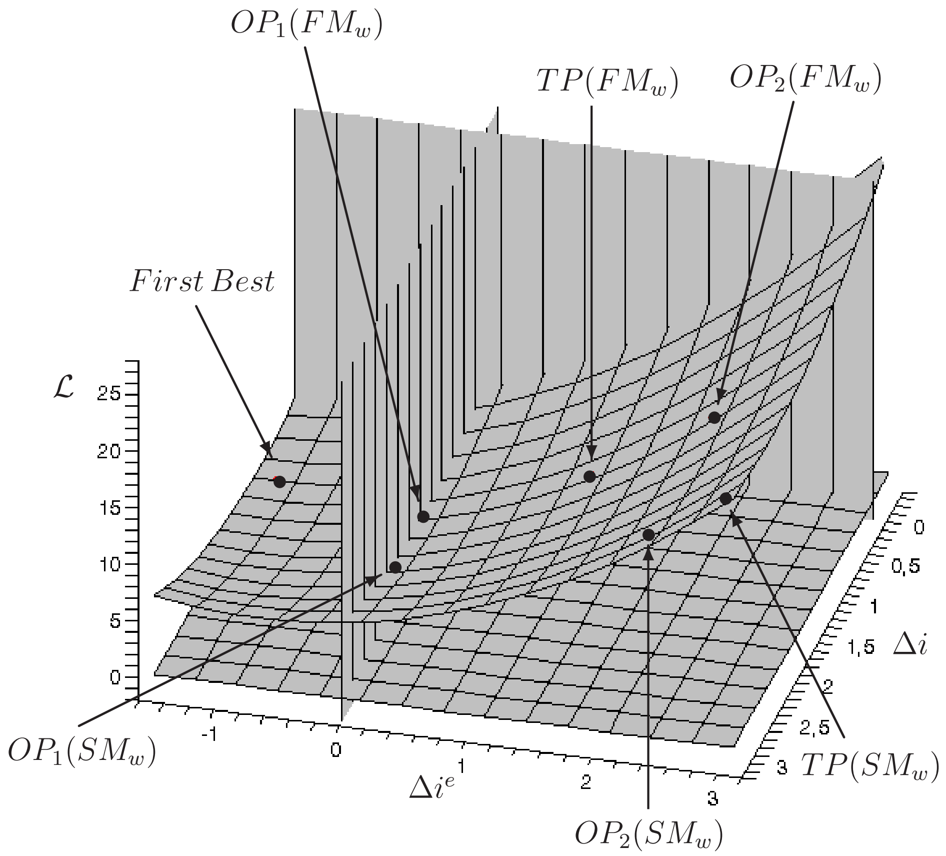


Figure 6.3:
Overall social losses, in the case of a large shock.

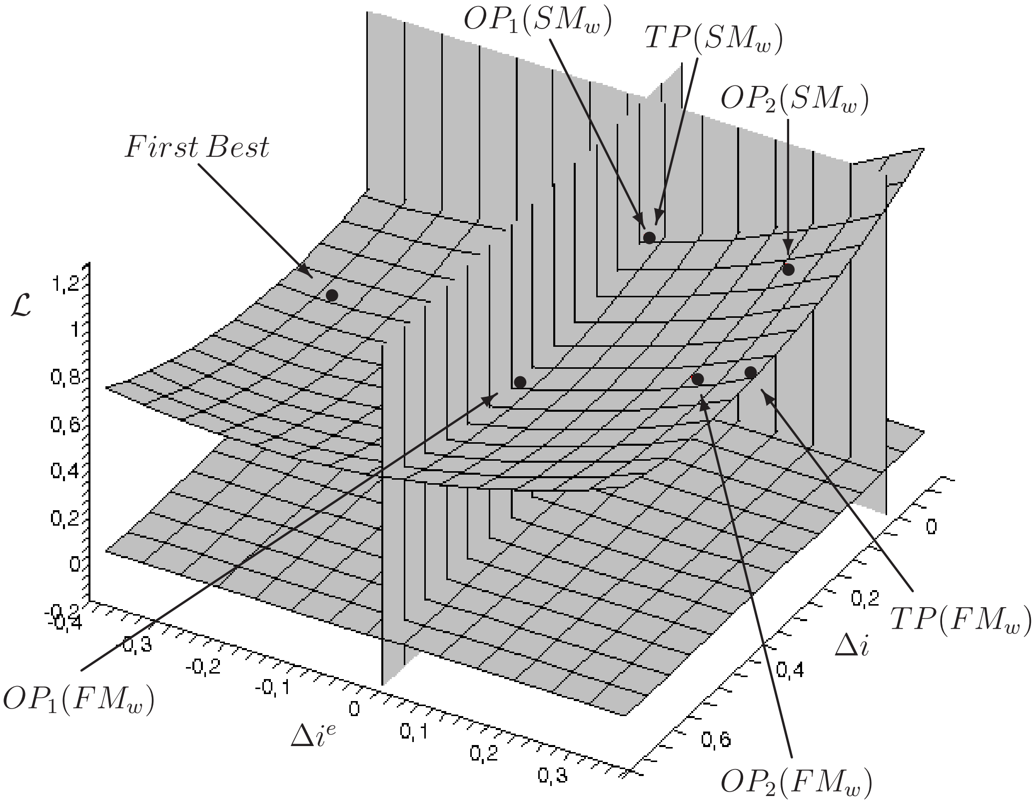


Figure 6.4:
Overall social losses, in the case of a small shock.

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