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Network Formation and International Trade

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1 Introduction

During the last 60 years the WTO (World Trade Organization) and its General Agreement on Tariffs and Trade (GATT) have played an important role in liberalizing trade worldwide and increase economic outcome, job prospects and business opportunities. Since the foundation of the GATT in 1947 member countries have attempted to reduce protectionism within nine trading rounds and globally liberalize world trade. Membership of the GATT has risen from 23 to currently 153 members. In total more than 90 % of the world trade volume is produced by members of the WTO. The results of the world trading rounds are apparent: tariffs on industrial commodities have sunk from more than 40 % to less than 4 % within the last 40 years. Furthermore, total world export increased from 5,000 billion dollars to more than 8,000 billion dollars between 1998 and 2004[1].
Although the multilateral trading system succeeded in liberalizing trade barriers in the world global free trade has not been achieved. During the last two negotiation rounds trade liberalization seemed to stagnate and less tariff liberalization could be achieved.

Due to the economic impact trade liberalization has on countries of the WTO, one may well enquire as to why global free trade could not be achieved so far and why countries stick to protectionism. In many cases countries adopt trade policy to increase welfare and protect certain interest groups. The following example is borrowed from Krugman and Obstfeld (2000, p. 218):
In 1981 the U.S. government asked Japan to limit its exports of cars to the U.S.. This raised the prices for foreign cars in the U.S. and forced consumers to buy domestic cars that they did not like that much. On the other hand, Japan's government pursued a policy in which Japanese consumers were forced to buy incredibly expensive domestic beef and citrus products instead of cheap imports from the U.S..
This example demonstrates that governments often pursue a trade policy that can be detrimental to national welfare and especially to consumer surplus. The most popular instrument of trade policy are tariffs. *Specific tariffs* charge a fixed amount on each unit sold and do not depend on the price of a good, whereas an *ad valorem*

[1] See e.g. United Nations (2006).

	Country B	
	Cooperate	Defect
Country A Cooperate	400,400	50,500
Defect	500,50	100,100

Figure 1: The Prisoner's Dilemma in Trade Policy

tariff levies a certain percentage of the value of the imported good such that it changes with the price of the good. Tariffs have the purpose of raising the price of a good in the importing country and lowering it in the foreign market. Countries impose tariffs on other countries strategically to protect their own industry and to establish a competitive advantage. These strategies increase domestic welfare but influence outside countries. As a consequence countries impose positive tariffs on other countries such that countries face a prisoner's dilemma in trade policy. This was first analyzed by Brander (1986), who illustrates the tariff policy dilemma by means of a prisoners' dilemma game to address the question of why two countries end up with protectionism, even though they would both be better off under free trade. When we use numbers to represent a country's payoff we can illustrate this conflict by means of a 2×2 strategic form game between country A and B.[2] Both countries obtain a payoff of 400 when they cooperate and eliminate tariffs whereas they both obtain a payoff of 100 when they impose high tariffs against each other. When country A imposes zero tariffs on market B whereas market B imposes non-cooperative tariffs on market A, Country A will obtain a payoff of 50 whereas country B receives a payoff of 500. Figure 1 illustrates the dilemma. In this game, the dominant strategy is to defect, since this yields each country the highest payoff, irrespective of what the other country chooses, whereas both can reach the highest payoff when they cooperate.

Instead countries form regional trade agreements (RTAs). The current wave of RTAs has created a debate between different groups of economists, those who see them as harmful for multilateral trade liberalization (multilateralists) and others who see

[2]The following example is based on Baldwin (1989).

them as enhancing global free trade (regionalists). The effect of regional trade agreements and the process of multilateral liberalization plays a central role in the current discussion of regional integration. The webside of the WTO provides us with the following information[3]:

"Regional Trade Agreements (RTAs) have become in recent years a very prominent feature of the Multilateral Trading System (MTS).
The surge in RTAs has continued unabated since the early 1990s. Some 421 RTAs have been notified to the GATT/WTO up to December 2008. Of these, 324 RTAs were notified under Article XXIV of the GATT 1947 or GATT 1994; 29 under the Enabling Clause; and 68 under Article V of the GATS. At that same date, 230 agreements were in force."

In spite of or even because of the trend towards regional integration from 1948 to 2007 the trade volume increased from 59 billion dollars to 13,570 billion dollars and the WTO partners were expecting an increase in trade volume of an additional 100 billion dollars.

The WTO is based on its most favoured nation (MFN) clause (Article I), which requires that the conditions applied to the most favoured trading nation (i.e. the one with the least restrictions) apply to all trading nations in the GATT. Precisely when a country offers a low tariff to its most favoured nation, it has to offer each other member of the GATT the same tariff. The MFN clause ensures equal opportunities, especially concerning import duties. This principle relates to the non-discrimination principle among GATT countries and implies that member countries may not discriminate against other countries within the WTO in their tariff policy. The only exception to the MFN principle is Article XXIV of the GATT that permits the formation of RTAs among member countries with the condition to eliminate within union trade barriers on "substantially all trade " (GATT Article XXIV). This article is in conflict with Article I as it allows preferential access within member countries under the condition that the preferential access has to be granted on sub-

[3]See http://www.wto.org/english/tratop_e/region_e/region_e.htm, last accessed on August 26, 2009.

stantially all goods.

In the following we will introduce some key definitions that all relate to arrangements of trade in goods and that persistently appear in the literature of regionalism: preferential trade agreement (PTA), free trade area (FTA) and customs union (CU).

A preferential trade agreement (PTA) is a regional trade agreement between two or more countries with the aim of offering advanced access to its member markets, in which member countries reduce tariffs among each other but maintain tariffs with countries outside the PTA.

GATT Article XXIV states that "a free trade area (FTA) shall be understood to mean a group of two or more customs territories in which the duties and other restrictive regulations of commerce [...] are eliminated on substantially all the trade between the constituent territories in products originating in such territories."
A FTA is a PTA in which member countries agree on common tariff elimination with members of the FTA but choose individual tariffs to countries outside the FTA. One example for a FTA is the North American Free Trade Area (NAFTA) in which the U.S , Canada and Mexico agreed upon tariff elimination but the U.S. may impose different external tariffs to countries outside the FTA than Canada or Mexico.

In contrast to a FTA, in a customs union (CU) member countries eliminate their tariffs but maintain common external tariffs on countries outside the union. One example of such a CU is the European Union, in which all countries within the union offer the same tariffs to an external countries like the U.S. for the import of goods. CU and FTAs are included in the term PTA, which is a widerly used term.

The proliferation of PTAs has lead many people to fear that they undermine the process of multilateral liberalization. It has been discussed whether regional trade agreements influence trade relations of WTO members. Some trade theorists argue that regionalism is compatible with multilateralism whereas others fear that the increasing number of PTAs serves as a substitute for multilateral tariff reduction[4]. On its website the WTO states the following[5]:

> "The proliferation of RTAs, especially as their scope broadens to include policy areas not regulated multilaterally, increases the risks of inconsistencies in the rules and procedures among RTAs themselves, and between RTAs and the multilateral framework."

The formation of PTAs demands that the reduction of tariffs among member countries does not lead to an increase on countries outside the PTA. Still multilateralists argue that the process of regionalism does hinder the process of multilateral liberalization and the formation of PTAs (both CU and FTA) influences tariffs on members outside of the PTA. They argue that a PTA lowers incentives for tariff reduction within the world trading system and PTAs are a substitute for multilateral tariff reduction. In fact, Germany's share of exports to fellow members of the European Union as a percentage of total exports is 62 % [6]. Between 1992 und 2003, 2.5 billion additional jobs were created within the European Union. Due to the elimination of Europe-wide tariffs, the European Union Gross Domestic Product turned out to be 164.5 billion higher due to the Single European Market in 2002[7]. Sometimes bilateral agreements are a precursor of multilateral liberalization. A large body of theoretic literature with the pioneering work of Viner (1950) and his analysis of trade diversion and trade creation concludes that a CU is not necessarily welfare enhancing for all countries.

In spite of its relevance in the world trading system the GATT has been the source of much controversy in the last decades. The regionalism debate has received a great

[4]See Bhagwati and Panagariya (1996) and Panagariya (2000), two of the leading multilateralists, for overviews on this topic that are in the view of those who see regional integration as harmful. See Winters (1996) for a more neutral overview of theoretic research on the impact of regional trade agreements on multilateral liberalization.

[5]See http://www.wto.org/english/tratop_e/region_e/scope_rta_e.htm, last accessed on August 26,2009.

[6]See Statistisches Bundesamt (2008).

[7]See Commission of the European Communities (2003).

deal of attention during the last three decades. Researchers have argued that the new wave of regionalism hinders the process of multilateral liberalization which may be the cause of stagnating tariff reduction within the current world trade negotiating round. The first wave of regionalism took place in the 1950s after the formation of the European Union and did not spread beyond western Europe. Viner (1950) in his prominent book *The Customs Union Issue* addressed with his static concepts of *trade diversion* and *trade creation* the question whether regional trade agreements are welfare enhancing or welfare diminishing for members. Viner's argumentation is based on the formation of the EU. Newer literature, however, reexamines the problem as the new wave of regionalism started during the 1960s. The European Union has moved aggressively to draw its eastern and central European neighbours into the customs union and the U.S. has continued to promote a FTA among American countries. Japan and many Asian countries have formed a trading bloc, the Association of South East Asian Nations (ASEAN). This observation has raised a lot of discussion concerning the question of whether the trend will lead to a world split into two or three large trading blocs. As the process of regionalism and the formation of trading blocs in the world effect all countries, Bhagwati (1991) asks whether the formation of trading blocs serves as a *building bloc* or a *stumbling bloc* towards global free trade. Section 2 of the introduction provides an overview of the relevant literature in this field.

In section 1, we review the historical evolution of preferential trade agreements in the world trading system and its importance for the multilateral trading rounds.

Section 3 of the introduction provides an overview of the thesis.

1.1 Historical Evolution of Preferential Trade Agreements

The constitution of the world trading system is embodied in the General Agreement on Tariffs and Trade (GATT). Since its creation in 1947 its membership has grown from initially 23 members to more than 150 members in 2009. Through the first seven rounds of trade negotiations the average ad valorem tariff has fallen from 40 % to less than 4 %. Given the significant impact that the GATT has on the world trading system it is therefore important to enhance further tariff reduction and promote the process of multilateral liberalization.

Since its creation in 1947, there have been nine rounds of negotiations (the ninth (Doha) round is still in progress). The first five were characterized by parallel bilateral negotiations[8]; when Germany wanted to offer a tariff reduction with members of the WTO, it could ask each country separately for reciprocal tariff reduction. This way of negotiating substantially reduced tariffs. The sixth round of negotiations (Kennedy Round) resulted in a across-the-board tariff reduction of an average of 35 %. By the mid-1980s tariffs on almost all goods had been considerably reduced. However, for agricultural goods the liberalization process has moved slowly.
The eighth round of negotiation (Uruguay Round) lasted from 1986 to 1994 and was the most exhausting round, with the aim to reduce agricultural subsidies. It provided a tariff reduction of additional 40 % and most importantly, the Uruguay Round made a contribution to the liberalization of the agricultural and clothing sectors. Many attempts were necessary to conclude the talks and in 1992 an agreement was finally reached. This agreement was the predecessor of the World Trade Organization (WTO) that was founded in 1995 and which replaced the GATT. It expanded the scope of the GATT from traded goods to trade within the service sector and towards intellectual property rights[9].

Meanwhile the number of regional trade agreements has increased dramatically. In a regional trade agreement a country offers another country preferential access to its own market and vice versa. GATT Article XXIV allows countries to grant preferential access to other countries when they form a PTA. The first wave of regionalism started in 1950 when the European Union was founded and was limited to regional trade agreements within western Europe and among developing countries. For instance the Central American Common Market (CACM) in 1961 constituted a CU among countries of central America. During the first wave, while the European Union widened its CU across Europe, the U.S. maintained exclusively tariff nego-

[8] See Krugman and Obstfeld (2000, p. 236).

[9] For theoretic literature on optimal patent policy and protection of intellectual property rights see Nordhaus (1969), Grossman and Lai (2004) and Müller-Langer (2009).

tiations within the multilateral framework of the GATT. During the 1960s initial proposals were made by the Canadians for a North-American free trade agreement that was not undertaken by the U.S. government. Whereas in the first seven rounds of multilateral tariff negotiations tariffs were globally reduced, during the Uruguay round, the eighth round of negotiations, trade negotiations proceeded tough between the EU and the U.S. and the U.S. agreed to enter into PTAs with Israel in 1985 and with Mexico and Canada in 1994. In addition South-American countries formed a customs union (MERCOSUR[10]) in 1991 and within four years the trade volume among South American countries had tripled.[11] Meanwhile the European Union expanded its territories with the inclusion of Greece, Portugal, Spain, Austria, Finland and Sweden, which all took place from 1981 to 1995. While the European Union continued to expand its CU with eastern European countries the U.S continued its efforts to form an American free trade zone. In 1994 the Uruguay round was finally completed and signed by all 123 members. The persistency of the Uruguay round and the new wave of regional integration started the new debate as to whether the world would develop into a number of competing trading blocs that hinder the process of multilateral liberalization.

Lastly, the failure of the multilateral trading system was apparent in view of the eight-year-long Doha negotiation round which, in the end, failed due to insuperable conflict between the U.S., China and India. In 2001, the Doha round of negotiations started and has not yet reached a conclusion. In July 2008 the last Doha discussions broke down as the member countries failed to reach a compromise on the agricultural sector. China and India insisted on protectionist tariffs for the agricultural sector in order to increase their influence within the WTO. They were the major representatives of the interests of the group of the developing countries and insisted on protective tariffs for the poor farmers to safeguard the domestic agricultural sector. They would rather stick to their regional trade agreements (primarily with Asian countries) which are of lesser economic importance but of great political importance

[10]MERCOSUR stands for *Mercado Común del Sur*.

[11]But inspite of its huge effect on regional trade volume, the trade diversion effect of this customs union to the world market was remarkably high and trade reports from the World Bank say that the formation of the CU might have had negative welfare effects on the member countries.

than cooperate multilaterally.
Whereas the U.S. does not want to support the "protectionism" of India and China the globalization process seems to abut on a limit. Instead countries form bilateral agreements, which leads to a decreasing interest in multilateral tariff reduction as different interest groups want to maintain a certain tariff structure within the regional trade agreement. Whether regionalism serves as a substitute for multilateralism will be a main topic of the present thesis.
Meanwhile, countries like Brazil have called for Doha negotiations to recommence. Luiz Inácio Lula da Silva, president of Brazil, called several country leaders to urge them to renew negotiations and to complete the Doha round but currently no one really knows whether the trade negotiations can be completed successfully.

The current stagnation of WTO negotiations is characteristic for the development of the world trading system in the past few decades. The increasing number of PTAs is seen by many economists as the main reason for the deceleration of the liberalization process. Does regionalism constitute a threat to multilateralism? And if so, what is the nature of the threat? And what can be done to countervail the threat? The following section reviews the literature on regionalism and highlights its main contributions and views. Regionalists argue that the process of regional integration facilitates global free trade whereas others, viewing themselves as multilateralists, argue that they detract from multilateral liberalization.

1.2 The Regionalism Debate

PTAs and their impact on the multilateral trading system have long been object of discussion for many researchers[12]. The question of whether the process of regional trade formation hinders tariff reduction within the WTO has been addressed by many as what is referred to as the *regionalism debate*. The purpose of this section is to bring together the theoretical and empirical literature addressing the regionalism debate.

Ricardo (1817) demonstrates the effect of two countries moving from a policy of

[12]See Panagariya (2000) for an overview on this issue.

not trading with each other to trading with each other. He showed by means of a two good, one factor general equilibrium model, that welfare from trade is higher in both countries from the opening of the markets. His model suggests that each country should specializes in the production of this good in which it has a comparative advantage and exports the good to the foreign market. Conversely, it should imports the other good from the foreign market and one ends up in a situation in which both countries can gain from trade with each other.
Shortly after the implementation of GATT Article XXIV, Viner (1950) opened the discussion on regional integration in *The Customs Union Issue* where he analyzed by means of an example the effect of regional integration on third countries. He showed that a trade-creating customs union can be detrimental to countries outside this union.

He introduced the concepts of *trade diversion* and *trade creation* which we explain in the following:
Suppose the U.S. and Canada form a PTA under GATT Article XXIV such that they eliminate tariffs among each other and maintain their external tariffs on foreign countries. Let's assume the U.S. has three different sources of wheat: It can either produce it by itself for a price of 9, import it from Canada for a price of 7 or import it from Australia for a price of 6.

When tariffs are 0, the U.S. will import its wheat from Australia, since this is the low-cost production source. If tariffs are 2 it is cheapest to import from Australia, whereas if $t = 4$, the U.S. will produce its own wheat at a cost of 9.
Consider now the case of a CU between the U.S. and Canada such that both countries eliminate tariffs on each other which allows the U.S. to import its wheat from Canada for a price of 7. If the initial tariff was $t = 4$ the customs union shifts the expensive production costs from the U.S. to the source of lower production costs Canada. In this case the CU is beneficial for both countries, the U.S. and Canada. If the initial tariff was $t = 2$ the formation of a CU shifts the imports from the low production cost country Australia to Canada. This effect is called the trade diversion effect and is illustrated in Table 1.

Cost of wheat in the U.S.:	$t = 0$	$t = 2$	$t = 4$
Source: U.S.	9	9	9
Source: Canada	7	9	11
Source: Australia	6	8	10
Source: Canada, after CU	7	7	7

Table 1: The Trade Diversion Effect

In this case the trade diversion effect simply diverts trade from countries outside the CU to trade inside the CU and can therefore be detrimental to world welfare. Instead the trade creation effect substitutes the high-cost production in the domestic market with imports from the trade union partner.

It is obvious that the trade diversion effect can be seen as negative for world welfare whereas the trade creation effect can be seen as something positive.[13]

Viner's (1950) seminal work was the first to show that free trade does not in general improve welfare for everyone. The original idea by Ricardo[14], that free trade improves everybody's welfare, was therefore disproved and the discussion of the formation of regional trade agreements was launched.

Kemp and Wan (1976) were the first to show that when a group of countries forms a customs union such that external countries are not affected by the formation of

[13]Whereas in Viner's opinion trade creation could be understood as a "good thing" and trade diversion as a "bad thing", Lipsey (1957) and Meade (1955) showed that a trade diverting customs union can also increase world welfare. In his analysis, Viner (1950) assumed that commodities are consumed in fixed proportions, independent of the relative prices, and ruled out substitution possibilities between different commodities. Meade (1955) and Lipsey (1957) both concluded that when consumption effects are present the general assertion that trade creation can be seen as something positive and trade diversion as something negative no longer applies. For a survey on the issue of CUs see Lipsey (1960).

[14]Ricardo (1817) assumed that two countries have different cost of production such that each country specializes in the production of its low costs products and imports the other goods from the foreign market. Moreover, he did not consider the effect of a FTA between two countries on third markets.

this CU by freezing external tariffs on outside countries, a common tariff vector exists and a certain transfer among the member countries such that no country is worse off under the CU. This can be reached by taking the external tariff vector as a constraint and equalizing the marginal rate of substitution and marginal rate of transformation for each good across countries in the CU. Moreover, they showed that this result is valid regardless of the number of countries or the state of development and income of countries. They concluded that there are finite steps where at each step customs unions are created such that no country is worse off. After a finite number of steps in which the process of CU formation continues the world reaches a free trade equilibrium in which no country is worse off.

The result of Kemp and Wan could not be extended towards free trade areas for many years. The problem was that whereas countries within a CU chose a common external tariff and face equal prices for all goods within each member country such that the marginal rate of substitution in all countries was equal across all goods, under individual external tariffs as in a FTA prices in each member country differ. In 2002, Krishna and Panagariya suggested a solution for the existence of welfare-enhancing free trade areas in which countries within the free trade area freeze individual external tariffs on non-member countries. They showed that a group of countries can form a free trade agreement and maintain individual external tariffs on non members and still all countries can be better off. Krishna and Panagariya (2002) show that as long as goods that are produced within the free trade area are traded free across member countries a welfare enhancing free trade area exists in which prices across member countries differ. In the proposed welfare-enhancing FTA the quantities that each country imports individually from outside remain equal under the pre-FTA and post-FTA equilibrium. Since external prices are fixed the welfare for outside countries does not change. The welfare for FTA countries increases in goods that are produced inside. As the price in member countries differs all goods that are produced inside are supplied in the high-price market which is the country with the higher external tariff, such that the supply is zero in the low-price market. To clear markets the low-price market price is increased. At the new price the total demand is satisfied by the imports from outside and its total supply is sold in the high price market. As the markets in both countries have to be cleared the prices in the high-price country have to be reduced such that demand equals supply. Krishna and Panagariya (2002) verify graphically that the total welfare in the FTA is higher than the pre-FTA welfare and, as the welfare for outside countries did not change, the total welfare effect is positive. An important aspect of the analysis is that due to the rules of origins countries inside the free trade agreement are not allowed to import products from outside to sell them in the high-price market such that prices differ among countries of the FTA.[15]

Regionalism is not just apparent in the formation of new FTAs and CUs but also in the enlargement of existing PTAs. In 1994 the NAFTA trading zone was founded by its three member states (Canada, Mexico and the U.S.). In recent years, the U.S. has entered into negotiations with other South and Central American countries to form a trading zone of the Americans. The EU has increased its member countries from 15 to 27 within the last six years and in particular extended its territories with countries from eastern Europe. Negotiations are taking place with Turkey, Croatia, Albania and others. In 1992 the Asian Free Trade Area (AFTA) was founded by countries of the south-east Asian area and has grown to ten countries.

Many economists during the last two decades have begun to consider the possibility that the GATT could give rise to a world of three main trading blocs. The worry of most multilateralists is whether countries that have joined a trading bloc will be more protectionists towards countries outside the bloc. They are facing the realization that the GATT as something that was originally supposed to aid liberalize now helps to hinder trade liberalization[16].

[15]As Krishna and Panagariya (2002) provide only an existence result, this does not imply that a PTA necessarily enhances welfare for all countries and a path towards global free trade will be adopted. The question of whether a PTA indeed increases welfare such that it enhances countries' incentives for multilateral liberalization and multilateral free trade will be adopted is investigated later in the section by Krishna (1998) and Levy (1997).

[16]The worry that regional free trade such as trading blocs may undermine the effort to obtain multilateral trade liberalization is also discussed by Krugman (1991b).

This question has led some researchers to study the impact of increasing bloc formation and bloc size and the division of the world into trading blocs on world welfare. First, Krugman (1991a) asks how welfare varies with the number of trading blocs. He sets up a model in which the world consists of a large number of symmetrical "provinces" in which he shows that world welfare is maximized when the world consists of one large trading bloc (free trade), reaches a minimum when the number of "provinces" is separated into three symmetrical trading blocs and increases when the number of symmetrical trading blocs increases. Krugman (1993) provides a simulation of his model in which he sets different ad valorem tariff rates and shows that the number of trading blocs that minimizes world welfare is either two or three. This result is really surprising as we can observe that the currrent trend goes towards two or three trading blocs. The intuition is the following: When the set of provinces is partitioned into two trading blocs, trade diversion takes place. Each province trades more with the provinces in the same trading bloc. As no additional trade creation takes place the total effect is negative. When the number of trading blocs is three trade creation and trade diversion take place at the same time. Trade diversion takes place due to the increasing number of trading blocs as countries that originally traded with half of the provinces now trade with less countries as the size of each trading bloc decreases. On the other hand, trade creation results from additional trade from the formation of a new trading bloc with countries that were already outside. Whenever the number of symmetrical trading blocs grows, the trade creation effect dominates the trade diversion effect. With an increase in the number of trading blocs the size of a single trading bloc decreases and more trade takes place with countries from outside the trading bloc and this trade creation effect is larger than the trade diversion effect due to the formation of the new trading zone.

When we neglect the question of whether the formation of PTAs has a positive or a negative impact on world welfare, one can ask the question of whether the dynamic process of regional integration will continue until all trade barriers are reduced worldwide. Bhagwati (1991) introduced a dynamic time path approach to investigate the question of whether PTAs act as *stumbling blocs* or as *building blocs* towards global free trade. To analyze this issue he addressed the following two questions:

- Assuming that the formation of PTAs and multilateral liberalization do not interact with each other, will membership of PTAs increase until a worldwide membership is reached and global free trade is achieved?
- Through permanent expansion of PTAs will the process of this dynamic time path hinder or facilitate the process of multilateral liberalization when the formation of PTAs and multilateralism interact?

The time path of liberalization can be divided into two alternatives. First, Bhagwati (1991) assumes that the time-path of world trade liberalization through PTAs and multilateral trade negotiations are independent. He illustrates the process by means of a dynamic time path diagram and shows that PTAs can be a building bloc towards global free trade if their time path of PTAs continues until each country has preferential access to every other country such that global free trade can be achieved due to continued expansion of PTAs. Second, he assumes that regionalism and multilateral liberalization interact and asks whether PTAs enhance or hinder multilateral tariff liberalization.

He did not answer the questions directly but first issued the warning in 1991 and challenged trade economists to tackle this problem. His work has inspired a voluminous, theoretical body of literature as well as recent evidence that suggests that there is indeed a clash between preferential and multilateral liberalization (Bhagwati (1993)). He argues that, in contrast to the first wave of regional integration in the 1960s, the new wave will endure. He calls the question of whether PTAs are *stumbling blocs* or *building blocs* to global free trade the *dynamic time-path* issue.

This first question was then addressed by Baldwin (1995) and Andriamananjara (2002).
Baldwin (1995) investigates, assuming that multilateralism and regionalism do not interact, whether a trading bloc will continue to expand until free trade is achieved. In this context, Baldwin (1995) proposed a model that concentrates on the incentives of non-members to join a PTA. His conclusion is that the PTA creates a so-called "domino" effect in which non-member countries want to join the PTA. The argument is that a trading bloc is detrimental to non-member countries' firms

as they decrease profits in the bloc market in an imperfect competition model due to lower tariffs for firms inside. As the number of bloc members increase, firms face a tariff disadvantage in an even greater number of countries. In his political economy framework firms then lobby for entry into the PTA and countries that were previously happy to be non-members of the trading bloc demand membership. In this model, given the dynamic time path, the domino effect results in worldwide free trade in the way that each country has PTAs with countries from the rest of the world.

In a more recent paper, Andriamananjara (2002) addresses the question as to whether the increasing number of trading blocs will expand more and more such that they encompass the whole world. He assumes that the outside tariff is fixed and investigates the decision for outsiders to seek entry into the trading bloc, whereby the decision to seek entry depends on the domestic firm's profit. He shows that as soon as the trading bloc reaches a certain size, insiders will block entry for outsiders and the process of regionalism will fail to expand into global free trade. His model is based on a Cournot competition model with symmetrical countries. He proposes a political economy framework in which producers play the decisive role and continue expansion of a preferential trading area as long as producers find it profitable. He finds that equilibrium bloc size depends on the entry conditions of the PTA. When there is "open membership" the trading bloc contains all countries such that global free trade is reached. When membership is selective in the way that firms inside the PTA can choose whether or not to accept new members, expansion stops before global free trade is achieved. As an increase of the trading bloc increases total profit but as further expansion of the bloc size will decrease the member firm's profit this has to be shared among a larger number of firms. In this context regionalism can be seen as a stumbling bloc towards global free trade.[17]

The second question Bhagwati (1991) addresses has been analyzed by Krishna (1998)

[17] Andriamananjara's analysis is strongly based on the assumption that producers play a decisive role and does not consider consumer surplus effects of the increasing size of a trading zone. The effects of the political economy framework on the economic outcome and the trading regime that results will be addressed later in the thesis.

and Levy (1997) who model the impact of regionalism on multilateral tariff reduction. The question these papers address is whether the process of regionalism hinders or facilitates multilateral tariff cooperation when regionalism interacts with multilateralism.

Using a political-economy model in which decisions are made by majority voting, Levy (1997) shows that a bilateral FTA can undermine political support for multilateral liberalization. In his model he shows that bilateral PTAs can never enhance multilateral liberalization. After countries have signed a bilateral trade agreement they lose the desire for multilateral cooperation. In his model agents in a country first have to decide whether a proposed bilateral trade agreement is examined and later whether multilateral tariff cooperation is supported. It is assumed that a majority of voters have to support the proposal and the option to form a FTA is exercised only if it increases the median voter's utility. In the first part of his paper he presents a Heckscher-Ohlin model[18] in which he shows that a bilateral trade agreement cannot preclude the feasibility of multilateral liberalization such that in a standard Heckscher-Ohlin model a PTA neither helps nor hinders free trade. In the second part he presents a differentiated product market and shows that due to the introduction of product variety, in which countries are compensated by increased variety gains, a PTA serves as a stumbling bloc. This makes a bilateral trade agreement feasible that was first rendered infeasible in part I of his paper.

Krishna (1998) receives a similar result. In a three-country oligopoly model he addresses the question as to whether an initially multilateral liberalization remains feasible after two countries have formed a FTA. His result is that the formation of

[18] Heckscher and Ohlin (see Feenstra (2004, p. 4ff)) showed in a general equilibrium two good, two factor economy, that a country produces and exports relatively more of the good which is relatively abundant locally. Relative endowments of the factors of production determine a country's comparative advantage. Countries have comparative advantages in those goods for which the required factors of production are relatively abundant locally as the prices of goods are determined by the prices of their inputs. Goods that require inputs that are locally abundant will be cheaper to produce than those goods that require inputs that are locally scarce. While Ricardo (1817) showed that technological differences in one factor economies matter, Heckscher and Ohlin emphasized the factor endowment as the basis for trade.

a FTA lowers countries' incentives for multilateral liberalization. Moreover, he finds that a trade diverting PTA is more likely to be supported politically by both partner countries.
More specifically, he presents a three-country model with a single good produced in each country. Firms compete in each market and the political economy framework is one in which governments base their decision to form a FTA on the domestic firm's profit. Krishna (1998) investigates the effect of a FTA between two countries on the tariffs set on the outside market. Since GATT Article XXIV permits FTAs when tariffs on virtually all products are eliminated tariffs between the two trading partners will be set to zero.
The effect of a FTA on welfare is that each country obtains higher access to the foreign market such that a firm's profit in the foreign market increases but decreases in its domestic market. The balance between loss and gain in the two trading partner markets seems to be like a zero sum game. But both partners additionally gain due to the trade diversion effect as trade from the outside market is diverted to the inside FTA market.
He shows that FTAs reduce domestic incentives to strive for multilateral tariff reduction and moreover that this effect is larger, the larger the trade diversion effect induced by the FTA.[19]

Whereas Krishna (1998) and Levy (1997) concentrate more on the political economy side to investigate whether the process of regionalism enhances incentives for multilateral liberalization, Bagwell and Staiger (1997a) and (1997b) investigate whether PTAs affect the outcome of a multilateral liberalization process during a temporary *transition phase*. They differ between customs unions and free trade areas and show that the formation of both can imply totally reversed conclusions concerning multilateral cooperation.
In Bagwell and Staiger (1997a) they find that the formation of a free trade area influences multilateral cooperation and can undermine multilateral low tariffs in the short run, during the period in which mutilateral tariffs are negotiated. Their

[19]In chapter 4 of the thesis we will investigate whether the results of Krishna (1998) hold, when we consider endogenous tariff formation and strategic formation of trade agreements.

conclusion is rather negative but also points out that in the long run multilateral liberalization will continue after the regional trade agreements are etablished.
In a companion paper, Bagwell and Staiger (1997b) show that the formation of a customs union in the short run does enhace the multilateral liberalization process but in the long run this conclusion has to be reversed when the customs union is already well established.
To understand the results Bagwell and Staiger emphasize two different effects: the *market power* effect and the *trade diversion* effect. Whereas the trade diversion effect occurs under both, a CU and a FTA, the market power effect only appears under a CU. The market power effect appears when two countries in a customs union impose their common external tariffs on the outside countries and this implies an increase of the tariffs for non-member countries. As in a FTA countries choose their individual external tariffs, this effect does not occur under a FTA. When the market power effect of a customs union is sufficiently important, in the short run a customs union might enhance multilateral tariff cooperation. Non-member countries, who anticipate the market power effect in the long run, are more willing to cooperate with member countries during the transition phase to avoid a trade war, as confrontations with member countries can be more detrimental when the market power effect is strong. On the contrary, they argue that the trade diversion effect of a FTA leads to a higher multilateral tariff in the short run as countries anticipate during the negotiations that trade will be diverted away due to the free trade agreement. Thus countries recognize that trade will be diverted away from non-member countries to member countries. In the short run the incentive to deviate unilaterally is large as compared with the now smaller discounted future value of maintaining a cooperative relationship as the trade volume between member and non-member countries will be lower. When a FTA is formed, in the short run countries increase multilateral tariffs.

There are some papers that concentrate directly on countries' incentives to form PTAs and investigate in a political economy framework under which conditions countries will take advantage of the opportunity to form a PTA. One may ask whether countries will exercise the option to form a PTA and how the decision to form a PTA effects the choice of external tariffs. In this context, Grossman and Helpman

(1995) explicitly model the decision-making process. They investigate in a political economy framework conditions under which a FTA will emerge and the strategic incentives for countries to form FTAs. In their model they emphasize the role of different interest groups in the decision to form a FTA. They introduce a sequential game in which at stage one political competition between different interest groups determines a country's preferences and in the second stage countries choose, based on their political preferences, the optimal international policy that describes whether a FTA is enforced. A FTA between two countries may have different effects on their welfare which depend on the size of the industry in the trading partner's market. When the exporting market is small compared to the world market such that it has no influence on the prices of the importing country, exporting firms can benefit from the preferred access to the FTA-partner market and increase the exporting firms' profits. This tariff effect is called *enhanced protection*, whereas with *reduced protection*, an exporting firm sees falling prices as the large share of the exporting industry on the total world market is large such that it can influence prices in the importing market. As a FTA requires the consent of both countries to be formed both countries have to gain from the FTA. Whenever the fraction of all industries that have enhanced protection due to a FTA is high in both countries, a FTA is more likely to be enforced as a sufficient number of exporters exist in both countries who lobby for the agreement resulting in increased tariff protection.

Some authors have also investigated the opposing question of whether the process of multilateral liberalization increases countries' incentives to form PTAs. Ethier (1998) shows that regionalism may be seen as a direct consequence of multilateral liberalization and a guarantee for its survival. He builds up a model in which the world is divided into several developed and several less-developed countries in which developed countries are symmetrical. Each developed country produces a differentiated, traded good that is produced in two stages. The first stage, intermediate product, uses skilled labor and can be produced either in a developed or a less-developed country whereas the second stage product, which requires human capital, can only be produced in the developed country. Each less-developed country can trade its produced intermediate good in exchange of the final good. Without multilateral liberalization countries choose autarky and produce a low-quality product instead on their own. Ethier (1998) shows that multilateral liberalization among developed countries increases the motivation of less-developed countries to reform and to open their markets as the employment of skilled labor for the intermediate good expands and the value for the intermediate input rises. Some less-developed countries are now willing to liberalize trade. They form regional trade agreements with developed countries which has the effect that as the less-developed country becomes the developed market's sole supplier, the other less-developed markets lose market access. This induces other less-developed countries to seek regional liberalization with other developed market countries. This process continues until all developed markets have formed PTAs with all less-developed markets such that free trade is reached. Ethier (1998) finds that in spite of the low trade advantages that less-developed countries obtain from a PTA with a developed market country, they seek to form trade agreements with developed market countries. The reason is that they obtain more access to the developed market than other less-developed countries that do not participate in a PTA with the developed market country. This result is in line with the observation that a country like Mexico seeks PTAs with a developed market like the U.S..[20]

Freund (2000) observes in a symmetrical, three country oligopoly model, based on the model of Krishna (1998), that as initial multilateral tariffs are high, the welfare gains from a PTA are lower than the welfare gains from free trade whereas when initial multilateral tariffs are low, welfare gains from a PTA are higher than from free trade. This result comes from the welfare effect of lowering tariffs. Lower tariffs increase competition, which leads to lower firm profits and tariff revenue reduction. When initial multilateral tariffs are low the gains from joining a PTA outweigh the gains from eliminating all tariffs. PTAs result in higher welfare gains than multilateral free trade as the decrease in firms' profits and the loss in tariff revenue are lower from a PTA. She concludes that the process of regionalism can be seen as a result of the multilateral liberalization process. A preferential trade agreement that was initially unfeasible under high multilateral tariffs can be made feasible under

[20]In chapter 5 we show that this increases the low-income market's welfare.

multilateral cooperation.

Considerable attention has been given on the welfare effects of regional trade agreements and its implications for the multilateral trading system. The importance of these issues motivates an examination of country's incentives to form those trading agreements and especially of its strategic stability of different trading structures. The question of whether we will reach free trade by continued expansion of regional trade agreements also raises the question of whether we get stuck in some intermediate stages. And if so, can we characterize these stable trading structures. The following thesis addresses this question and considers strategic formation of trade agreements. It analyzes stable and efficient trading structures and investigates the effects of additional PTAs on multilateral tariff choice. To analyze stability and investigate a country's incentives to form and sever trade agreements and its optimal tariff policy under different trading regimes, one has to define a country's outcome as a function that depends on the network of international trade agreements. Chapter 2 of the thesis illustrates how network structures can be integrated into economic modelling and how it is a country's payoff, dependent on the network structure, that provides a basis for countries' strategic decision making.

1.3 Overview of the Thesis

This introduction has reviewed the empirical and theoretical findings of the increasing formation of regional trade agreements and its impact on the multilateral trading system. We have argued that the network of international trade agreements is important to determine the economic outcome of a country. This observation motivates an analysis of the incentives of countries to form trade agreements and of the strategic stability of trading systems. To model the strategic formation of trade agreements one needs a game theoretic tool that directly analyzes the strategic stability of trading systems. We will argue that network formation models as introduced by Jackson and Wolinsky (1996) can be seen as a new and very useful tool to analyze certain problems in international trade. The network formation approach offers an alternative method to analyzing countries' behaviour with respect to the choice of optimal tariffs and trading agreements. This thesis applies the network formation theory to models in international trade and analyzes the effects of bilateral trade agreements on multilateralism by means of game-theoretic equilibrium concepts.

The first aim of the thesis is to provide a framework which allows us to analyze the strategic formation of bilateral and multilateral links. Furthermore, we provide an equilibrium concept to investigate the strategic stability of trading networks. The equilibrium notion extends the well-known concepts from bilateral network games introduced in Jackson and Wolinsky (1996) to a more general setting that allows not just links between two players but also between any number of players. When it comes to an analysis of strategic network formation in international trade models where countries are those to form links between one another, we shall investigate the following questions:

First, we shall investigate why countries form additional preferential trade agreements but still maintain a multilateral GATT agreement. Furthermore, we want to find good reasons why we can observe that the number of GATT members is even increasing.
A lot of literature has tried to address the question of what the incentives for countries to form trade agreements are but still the question remains as to why they continue keeping their existing multilateral agreements. Is the GATT still beneficial or why don't countries simply return to a purely bilateral world?

A lot of research has been done regarding the question of why countries in a multilateralized world form additional preferential trade agreements. Existing literature investigates, while starting from a multilateralized world, what incentives countries have to form additional bilateral links and tries to answer the question as to whether additional bilateral links increase individual and global welfare. The time path approach of Bhagwati (1993) investigates whether regionalism will lead to discriminatory multilateral free trade for all through continued expansion of the regional blocs until universal free trade is reached, or will it fragment the world economy; will we evolve into a world of stumbling blocs or building blocs motivated models to introduce endogenous preferential trade formation. But Baldwin (2006) observed, and as was earlier stated in Deardorff and Stern (1994, p. 27), that regional and bilateral tariff reduction go hand-in-hand with multilateral liberalization and preferential trade agreements have coexisted with multilateralism from the start. When we introduce the process of strategic linkformation, will we observe that the process of multilateral liberalization is replaced by the formation of PTAs or can free trade be reached as a stable state? What structures will emerge when countries choose bilateral and multilateral links simultaneously? This thesis also analyzes the effect of asymmetry among countries regarding the incentives to form trade agreements where we concentrate on two types of asymmetry: income level and market size.

A large body of literature investigates the effect of PTAs on multilateral tariffs and vice versa. Freund (2000) observes that multilateral tariff reduction can affect the formation of PTAs. The conclusion is that many of the current PTAs may be a result of the success of the GATT lowering tariffs. Krishna (1998) shows that the formation of a PTA between two countries lowers their incentives for multilateral liberalization as they increase tariffs on third countries. We shall address the question of whether the formation of PTAs lead to a more open multilateral trading system, when we consider strategic link formation of countries. We also investigate the effects of PTA formation on multilateral tariff choice.

The thesis is structured as follows. In chapter 2 we will present a simple trade model that demonstrates how the network of international trade agreements determines a country's outcome. The model is based on the three-country competing exporter model by Bagwell and Staiger (1999b). Afterwards we will argue that this formulation of a country's outcome motivates an examination of countries' incentives to form trade agreements and of the strategic stability of different trade agreements. We will argue that the investigation of multilateral and bilateral trade agreement formation requires a new tool to analyze the stability which has not been used in network formation games so far. Instead of bilateral graphs we will analyze the formation of hypergraphs that allow the formation of trade agreements between more than just two players.

In chapter 3, we will first present the basic concepts that introduce the theory of hypergraphs. We will present graphic illustrations to explain the basic definitions and notations. Afterwards, we will redefine the basic concepts of network formation games as introduced by Jackson and Wolinsky (1996) to allow for links between players that include more than just two players.
In the remaining part of chapter 3 we will review the development of the network formation literature. We start with the cooperative game theory approach of Myerson (1977) in which he assumes that players are connected in an underlying graph structure that describes players' cooperation possibilities where the payoff each player receives is allocated corresponding to a cooperative game. We continue with the framework of Jackson and Wolinsky (1996) in which each player's payoff depends on the exact network structure. We further provide an overview of recent applications of network formation in economic models and provide equilibrium and efficiency concepts in the context of social and economic networks. We will also highlight the main examples from network theory and define a new equilibrium concept for hypergraph games that we call *multilateral stability*. This definition extends the pairwise stability notion of Jackson and Wolinsky (1996) towards multilateral link formation and requires that no subset of the player set has an incentive to add an additional link and no player has an incentive to sever any of its existing links.
Further, we will highlight the importance of multilateral link formation, define basic definitions and concept and investigate whether the results of the connection model by Jackson and Wolinsky (1996) will change when we allow multilateral link formation. We will give a corresponding definition on the efficiency of networks that characterizes network structures that maximize the total value which is calculated as the sum of the payoffs of the players. Later we will investigate the connection between the efficiency and stability of networks.

As carried out in section 2 of the introduction a growing body of theoretic literature has examined the impact of PTAs on multilateral liberalization and vice versa in recent decades. To understand countries' incentives to form PTAs and to explain the current trend towards PTAs in chapter 4 we consider strategic link formation between countries where countries can form bilateral free trade agreements (FTAs) as well as a multilateral agreement (e.g. GATT). We use a network formation approach to make bilateral as well as multilateral link formation endogenous. To examine the stability of different trading structures we use the notion of multilateral stability which we defined in chapter 3. We consider a three-country setting with a firm in each country that produces a homogenous good and competes as a Cournot oligopolist in each market. We observe that multilateral liberalization coexists with FTAs and that a multilateral trade agreement exists in every equilibrium. If we allow for heterogeneity of countries with respect to market size we find that a bilateral free trade agreement between small countries may exist in equilibrium. We further provide intuition for the results of the model.

In chapter 5, we present an international trade model in the style of Bagwell and Staiger (1999b) to analyze the impact of FTAs on multilateral cooperation in a three country framework where countries are asymmmetrical with respect to income. We use a network formation approach in which countries can sign bilateral and multilateral trade agreements and consider multilateral stability to analyze the shape of network structures that emerge in equilibrium. Our equilibrium concept suggests that global free trade is a stable state whenever countries' income levels are relatively similar. We also model multilateral bargaining under MFN and find that when all countries have the same number of FTAs, free trade is an optimal bargaining solution. Whenever the network structure is asymmetrical in the way that some countries belong to very few FTAs and others are strongly networked, free trade cannot be achieved. We also show that when multilateral tariffs are sufficiently low a country's welfare gains are higher when it has few FTAs.

In chapter 6, we provide axiomatic characterizations for two prominent allocation rules in the context of graph restricted games, the Myerson value and the position value. We assume that players are connected by links in a hypergraph[21] where the hypergraph describes communication possibilities. In contrast to a cooperative ga-

[21] The hypergraph that describes the linking structure between players will later be referred to as a *conference structure*.

me, the value function in network games is defined on the conference structure, since we allow different hypergraph structures that connect the same individuals to lead to different values. We show that the Myerson value of a corresponding hypergraph game is the unique allocation rule that satisfies fairness and efficiency. We argue that these axioms are desirable in the context of international trade networks.

Chapter 7 summarizes the main results of the thesis and outlines directions for further research.

2 Network Formation and the Regionalism Debate

We have argued that the structure of international trade agreements is significant for the economic outcome of each country and countries' optimal tariff policy. In the following we will present a model that demonstrates how a trading system influences a country's outcome. Furthermore, it shows how optimal tariff choice depends on the trading structure and how this influences each country's payoff.
Furthermore, we we will argue why the network formation approach is a helpful tool to analyze strategic formation of trade agreements. We will argue that a mathematical tool is required to model the simultaneous formation of bilateral and multilateral trade agreements and the interaction of both that has not been applied in economics so far.

2.1 A Simple Model

In the following we introduce an international trade setting of Bagwell and Staiger (1999b) with three countries and three different goods in which each country is served by competing exporters from the other countries. Each country is endowed with two of the goods but has a positive demand for each of the goods such that each country imports one good that is supplied by the competing exporters. The goods are subject to tariffs that the importing countries impose on the exports of the other two markets. We demonstrate how network structures can be integrated into an economic model, in particular an international trade model, in order to determine each country's payoff subject to the underlying network of trade agreements. Therefore we view a country as a node in the network and a trade agreement as a link in the network. A link allows entry into the foreign market and the tariffs to enter depend on the kind of trade agreeement. We allow countries to be involved in bilateral and multilateral trade agreements and determine optimal specific tariffs on the imports of the goods with respect to the trading regime a country faces. We develop a model in which any structure of trading systems is allowed between a fixed number of countries.

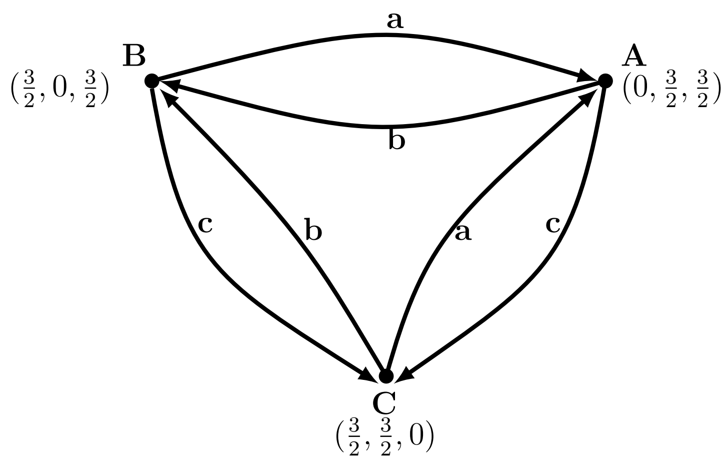


Figure 2: Pattern of trade between countries A, B and C.

We will denote the set of countries by $N = \{A, B, C\}$ and the set of goods will be denoted by $M = \{a, b, c\}$. We will assume that country $j \in N$ is endowed with zero units of good $J \in M$, $\frac{3}{2}$ units of good I and $\frac{3}{2}$ units of good K, whereas I, J, $K \in M$. Countries $i \in N$ and $k \in N$ are endowed with zero units of good I and K, respectively, and $\frac{3}{2}$ of the other two goods. Hence, in general we can say that all countries demand each of the three goods, such that all countries $j \in N$ have to import good J from country i and k whereas j exports good I to country i and good K to country k.

Figure 2 illustrates the pattern of trade. Note that in this setup it is not possible for one country to import one good from a country and export it to another country. As Bagwell and Staiger (1999b) argue this assumption can be justified given that fixed costs usually accrue when serving a new market which implies that a firm has to supply a very high amount to compensate the costs. Furthermore, this framework simplifies the analysis in such a way that one can analyze and isolate important effects of PTAs on multilateral tariffs.

Each country i's demand for good J is given by:

$$D(P_i^J) = \alpha - \beta \cdot P_i^J.$$

In this case the price of one good does not depend on the price of the other goods[22].

The following no-arbitrage conditions for good J relates the price of good J in the importing country to the price in the exporting countries

$$P_j^J = P_i^J + t_i^J = P_k^J + t_k^J, \tag{1}$$

where t_i^J and t_k^J are the tariffs that country j imposes on the imports of good J from country i and country k, respectively, and P_j^J denotes the price of good J in country j.
The import function of good J in country j is given by:

$$IM_j^J(P_j^J) = D(P_j^J), \tag{2}$$

whereas country j's exports of good I are given by:

$$X_j^I(P_j^I) = \frac{3}{2} - [\alpha - \beta \cdot P_j^I]. \tag{3}$$

The market clearing condition for good J can now be written as:

$$IM_j^J(P_j^J) = X_i^J(P_i^J) + X_k^J(P_k^J), \tag{4}$$

such that the total export of good J from market k and i has to equal the total import of good J in market j.

To define a country's outcome as a function of the network of trading agreements we will in the following assume that equilibrium prices and tariffs imposed on the other markets as well as the goods traded between the countries depend on the underlying trading system. We assume that countries only trade when there is a trade agreement between them. When a trade agreement does not exist between the countries, no trade takes place and each country consumes its endowment. As soon

[22] Our analysis in chapter 4 is based on this model where we allow for different endowment levels. Here, we merely adopt the model of Bagwell and Staiger (1999b) to demonstrate that one can define a country's outcome as a function on the network of trading agreements between all countries. The linear demand function can be derived from a quasilinear utility function which will be discussed in detail in chapter 4 of the thesis and we will not address this issue at this stage of the thesis.

as a trade agreements between two countries exists each country exports its good to the foreign market. A trade agreement between country i and j enables country i to export good J into market j.

The quantity that country i supplies in the foreign markets depends not only on its own trade agreements to the other countries but also on the trade agreements of its trading partners and therefore on the trading system as a whole. Equilibrium prices and tariffs imposed on the exports are determined by the optimization problem of the countries.
In the following we will describe the set of possible trading systems that may exist between countries A, B and C.

Let $N = \{A, B, C\}$ be the set of countries. With $\mathcal{L} \subseteq 2^N$, $\mathcal{L}$ is called a *trading system* on N.
With this notation a subset $L \in \mathcal{L}$, with $L \subseteq N$, represents a trade agreement between the countries in L.[23] We assume $|L| \geq 2 \; \forall L \in \mathcal{L}$ as a trade agreement exists between at least two countries.

We assume that countries choose optimal tariffs with respect to maximizing domestic welfare in a given trading system $\mathcal{L}$. Optimal tariffs in a trading system $\mathcal{L}$ are determined as follows: A global (multilateral) link ($L = N$) between all countries represents the GATT as a multilateral trading agreement which represents a trading system under MFN. Under the GATT regime we assume that each country chooses its welfare maximizing tariffs with respect to the MFN clause. Each country levies the same tariffs on each of the countries with which they are linked multilaterally, due to the non-discrimination requirement of the MFN clause. Whenever the trading system contains only one multilateral trade agreement between all countries with $\mathcal{L} = \{N\}$, the trading structure is called global and is denoted with $\mathcal{L}^G$. This trading system is represented in Figure 3a). In chapter 4 and chapter 5 of the thesis

[23] Formally, trading systems can be more generally described as hypergraphs, which allows us to model multilateral agreements between countries. In chapter 3 of the thesis we present a more formal representation of hypergraphs. Two papers that introduce games played on a fixed hypergraph structure are Durieu et al. (2005) and van den Nouweland et al. (1992).

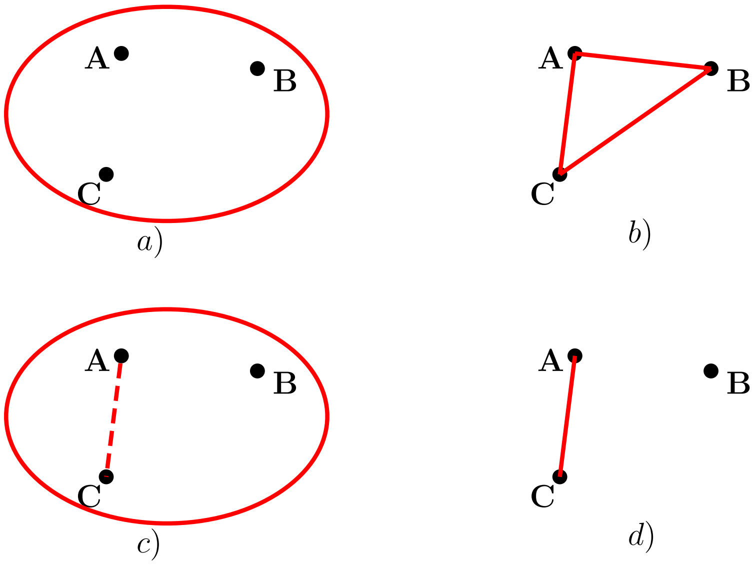


Figure 3: *a*) Multilateral GATT between countries A, B and C. $\mathcal{L} = \{\{A, B, C\}\}$. *b*) Bilateral trade agreements with $\mathcal{L} = \{\{A, B\}, \{B, C\}, \{A, C\}\}$. *c*) GATT with a PTA between A and C, $\mathcal{L} = \{\{A, B, C\}, \{A, C\}\}$. *d*) A bilateral trade agreement between A and C with $\mathcal{L} = \{\{A, C\}\}$.

we will see how a multilateral link, and therefore the fact that countries trade under MFN, effects countries' optimal tariff decision.

Countries can also form bilateral links. When two countries have a bilateral trade agreement under MFN, both trading partners eliminate tariffs due to Article XXIV of the GATT. In this case a trade agreement represents a PTA (here FTA)[24] in which we assume that countries eliminate tariffs between each other. In Figure 3*c*) a multilateral trade agreement (GATT) exists between all countries and in the scope of the GATT countries A and C have formed a bilateral free trade agreement (dashed line). In this trading system, tariffs between A and C are eliminated and countries A and C merely choose their welfare maximizing MFN tariffs on country B.

In Figure 3*b*) each pair of countries is linked bilaterally but no multilateral agreement (GATT) exists. Without the MFN clause each country imposes its non-cooperative

[24]During the whole thesis a bilateral PTA represents a FTA between two countries such that tariffs are eliminated and no trade barriers between these two countries exist.

welfare maximizing Nash tariffs on its trading partner. In this case a country's optimal tariff on one trading partner might differ from the optimal tariff on another trading partner. In this situation tariff discrimination may take place and depends crucially on the linking structure of the other countries. The complete trading system $\mathcal{L}^N$ is the family of subsets of N with $\mathcal{L}^N = \{L \in 2^N || L| = 2\}$, in which each pair of countries is linked bilaterally and countries choose non-cooperative, discriminatory tariffs on the other markets.

Figure 3d) represents a trading system without MFN in which a bilateral trade agreement between A and C exists. In this case tariffs are not eliminated as this trading system does not represent a trading situation under MFN (GATT) such that A and C merely levy their optimal non-cooperative tariffs against each other. As no trade agreement between A and B (C and B, respectively) exists, no trade takes place between them and country B lives in autarky.

When there is no trade agreement between two countries, neither bilateral nor multilateral, we assume that no trade takes place. A trading system without any trade agreements represents the case of autarky. Each country consumes its own endowment and no trade takes place. This is represented by an empty trading system and is denoted by $\mathcal{L}^e$.

With this notation we are now able to determine for each trading system $\mathcal{L}$ each country i's optimal tariffs on the imports from foreign markets on good I, $t^I(\mathcal{L}) = (t^I_j(\mathcal{L}), t^I_k(\mathcal{L}))$. Next, given each trading system and the optimal tariffs one can calculate each country's outcome that is given by its welfare as a function of the trading system $\mathcal{L}$, $Y_i(t(\mathcal{L}), \mathcal{L})$. This is calculated as the sum of producer surplus, consumer surplus and tariff revenue over all goods which gives us a representation of each country's payoff function that depends on the underlying network structure.

$$Y_i(t(\mathcal{L}), \mathcal{L}) = CS_i(t(\mathcal{L}), \mathcal{L}) + TR_i(t(\mathcal{L}), \mathcal{L}) + \Pi_i(t(\mathcal{L}), \mathcal{L}) \tag{5}$$

Where country i's consumer surplus equals:

$$CS_i(t(\mathcal{L}), \mathcal{L}) = \frac{1}{2\beta}[(\alpha - \beta P_i^J(t(\mathcal{L}), \mathcal{L}))^2 + (\alpha - \beta P_i^K(t(\mathcal{L}), \mathcal{L}))^2 + (IM_i^I(\mathcal{L}))^2]$$

its tariff revenue equals

$$TR_i(t(\mathcal{L}),\mathcal{L}) = \sum_{j\in N\setminus\{i\}} X_j^I(\mathcal{L})\cdot t_j^I(\mathcal{L})$$

and its producer surplus over domestic and foreign sales is given by:

$$\begin{aligned}\Pi_i(t(\mathcal{L}),\mathcal{L}) &= X_i^J(\mathcal{L})\cdot P_i^J(t(\mathcal{L}),\mathcal{L}) + X_i^K(\mathcal{L})\cdot P_i^K(t(\mathcal{L}),\mathcal{L})\\ &+ (\frac{3}{2} - X_i^J(\mathcal{L}))\cdot P_i^J(t(\mathcal{L}),\mathcal{L}) + (\frac{3}{2} - X_i^K(\mathcal{L}))\cdot P_i^K(t(\mathcal{L}),\mathcal{L})\end{aligned}$$

The expressions for consumer surplus, firm profit and tariff revenue can be derived from the linear demand function and the market clearing condition in equation (4).

The next two example demonstrate how the equilibrium prices and tariffs are calculated under a given trading regime.

Example 2.1. *Autarky, in which no trade takes place, is represented by an empty trading system $\mathcal{L}^e$. All countries consume their endowments and the price of good J in market i is given by:*

$$\frac{3}{2} = \alpha - \beta\cdot P_i^J(t(\mathcal{L}^e),\mathcal{L}^e).$$

Since $P_i^J(t(\mathcal{L}^e),\mathcal{L}^e) = \frac{\alpha-\frac{3}{2}}{\beta}$ and tariff revenues are zero social welfare is given by:

$$Y_i(t(\mathcal{L}^e),\mathcal{L}^e) = \frac{1}{2\beta}[(\frac{3}{2})^2 + (\frac{3}{2})^2] + \frac{3}{2}(\frac{\alpha-\frac{3}{2}}{\beta}) + \frac{3}{2}(\frac{\alpha-\frac{3}{2}}{\beta}) = \frac{12\alpha-9}{4\beta}.$$

Example 2.2. *In the global trading system all countries are members of the GATT and impose the same tariff on each trading partner. From (1)-(4) we can calculate:*

$$P_i^J(t(\mathcal{L}^G),\mathcal{L}^G) = \frac{\alpha}{\beta} - \frac{3}{3\beta} - \frac{t^J(\mathcal{L}^G)}{3}.$$

Substituting in (5) and maximizing social welfare in country j with respect to $t^J(\mathcal{L}^G)$ yields:

$$t^J(\mathcal{L}^G) = \frac{3}{8\beta},$$

and prices in market i for good J are given by:

$$P_i^J(t(\mathcal{L}^G),\mathcal{L}^G) = \frac{\alpha}{\beta} - \frac{1}{\beta} - \frac{1}{8\beta}.$$

Social welfare is given by:

$$Y_i(t(\mathcal{L}^G), \mathcal{L}^G) = \frac{1}{2\beta}[(\frac{6}{8\beta})^2 + (\frac{9}{8\beta})^2 + (\frac{9}{8\beta})^2] + \frac{3}{2}(\frac{2\alpha}{\beta} - \frac{18}{8\beta}) + 2 \cdot \frac{3}{8\beta}(\frac{3}{2} - \frac{1}{8} - 1),$$

where the first term is consumer surplus, the second term is firm i's profit and the last term reflects tariff revenues.

Equation (5) reflects the connection between a country's outcome and the overall trading structure between all countries and shows that the network structure directly influences each country's optimal tariff choice as well as the total welfare given by the sum of tariff revenue gained from its trading partners, consumer surplus and firm profit. We are now able to investigate the strategic formation of such trade agreements and, based on an equilibrium notion, predict the outcome of stable trading networks. One questions that naturally arises when modelling the economic outcome as a function of the underlying trading system is what countries incentives are to maintain and sever trading agreements and which trading systems are stable in the sense that no country has an incentive to alter the structure of the trading network. One may also ask for conditions under which stable trading systems turn out to be efficient when we interpret efficiency as an overall societal criterion of a trading system. In order to discuss efficiency and stability of trading systems in the next chapter we will define the two concepts. We will provide a more general framework to describe countries' payoff as a function of the trading system, by means of a so-called *allocation rule*. The overall outcome of all countries in a trading system can be more generally described as a *value function*.

2.2 Reasoning of Network Formation in International Trade Models

The above example relates a country's economic outcome to the underlying network of international trade agreements. Given that network structure matters in international trade it is important to understand which trading networks will form and to understand the strategic incentives of countries to form trade agreements. As Bhagwati (1993) already addressed the question of whether the formation of PTAs serves as a building bloc or a stumbling bloc towards global free trade, one can address the question of whether the strategic formation of PTAs continues until free trade is reached. This requires us to apply a strategic network formation approach and to formulate equilibrium concepts to investigate whether global free trade can be achieved as a stable state. The literature on regionalism reviewed in chapter 1 considers the trading regime as fixed and does not investigate strategic formation of trade agreements where the trading structure evolves endogenously. Neither do they examine the incentives of countries to form trade agreements and the strategic stability of different trading structures.

The idea that a market can be viewed as a node and a trade agreement as a link between nodes is a main contribution of the thesis. To model strategic link formation of trade agreements the thesis is based on network formation games and strategic stability concepts first introduced by Jackson and Wolinsky (1996). This allows us to explicitly model countries' incentives to strategically form trade agreements and investigate the stability of different trading systems. The network formation approach is a helpful tool to explain the current evolution of the world trading system. It may provide an explanation for the failure of the multilateral trading round and whether the world trading system might not necessarily reach free trade. It enables us to analyze whether the current trading system may be stable and why we get stuck at an intermediate tariff level in the multilateral trading system.

Hoping to gain an insight into how far the movement towards free trade agreements continues and whether global free trade can be achieved as a stable state, Furusawa and Konishi (2007) use a network formation approach where countries can sign bilateral FTAs. Countries can trade differentiated industrial commodities and countries may differ in their industrialization level. The payoff of each country depends on the network structure as well as on the total value generated by all countries. One can now study the incentives for countries to form trade agreements and the strategic stability of different trading structures.
In contrast to Furusawa and Konishi (2007), Goyal and Joshi (2006a) use a Cournot competition model in which a homogenous good is produced in each market and

in which a link between two players represents a FTA. The objective function of each country is given by its welfare function that depends on the network of trading agreements.[25]

The network formation approach allows a very flexible analysis where countries like the U.S. can have a bilateral FTA with Canada and can have an additional FTA with Australia. This does not require that Canada and Australia have a FTA as well. There is another strand of literature that includes the strategic formation of customs unions into the regionalism debate. As already mentioned, members of a customs union have to build up the same PTAs with countries outside the customs union and each country can only be member of one CU such that this literature uses a coalition formation approach rather than a network formation approach to analyze strategic stability of endogenously evolving trading zones. Some authors that model the formation of trading zones as a coalitional game are Bond et al. (2004), Bond and Syropoulos (1996) and Yi ((1996),(2000)). Bond et al. (2004) shed light on the question of whether the formation of customs unions serves as a building bloc or a stumbling bloc towards global free trade and show that the formation of a customs union may undermine multilateral liberalization. Bond and Syropoulos (1996) investigate the size of trading zones of welfare and world tariffs. They show that world welfare may be minimized when there are four or more trading zones in the world.

Yi (2000) shows in a model of intra-industry trade that the formation of a free trade agreement between two countries is Pareto-improving for all countries. Furthermore he shows that global free trade may not be achieved as an equilibrium outcome. Whereas in another paper (Yi (1996)), he shows that the formation of a customs union increases member countries' welfare but simultaneously decreases non-members' welfare and global free trade is a stable outcome. This contrast is due to the tariff externalities. Since in a customs union member countries set their

[25] Belleflamme and Bloch (2004) model the formation of market sharing agreements where a link between two firms represents an agreement that one firm does not offer its product in the other firm's market and vice versa. A firm's profit function now depends on the structure of market sharing agreements as well as on the total payoff generated from all firms.

external tariffs jointly against non-member countries external tariffs in a customs union are higher than individual welfare-maximizing tariffs on third parties in a FTA. Countries have a strong incentive to become members of the customs union and global free trade will be achieved. They show that a free trade agreement between two countries has positive effects on third countries since tariffs on third parties decrease as two countries sign into a FTA and free trade may not be achieved.

Whereas the coalition formation literature considers the evolution of the world into trading blocs it is not able to analyze the formation of bilateral and multilateral links at the same time. The literature has so far only been able to investigate the formation of either bilateral trade agreements or customs unions. There is, as yet, no approach to model the formation of bilateral and multilateral trade agreements. This is of course a strong restriction since many countries are engaged in both types of agreements simultaneously. We can see for example that the United States and Australia have a bilateral FTA and both countries are members of the GATT, a multilateral trading agreement that encompasses more than 150 other members. We therefore want to investigate the question as to what the incentives of countries are to form bilateral and multilateral trade agreements.

While existing literature on strategic formation of trade agreements concentrates on the question whether the formation of PTAs alone achieves global free trade, it neglects the question of whether the process of PTA formation facilitates or hinders multilateral trade liberalization. But as Furusawa and Konishi (2007) have already pointed out, multilateral trade liberalization and PTA formation interact with each other and coevolve over time.

This thesis addresses this issue and provides a theoretic framework which allows countries to form multilateral and bilateral trade agreements simultaneously. Furthermore it enables interpretation of various kinds of trade agreements differently with respect to the imposed tariff structure. Literature on PTAs has either analyzed the effect of a PTA on multilateral tariff reduction or whether the process of regionalism converges into free trade. The introduction of multilateral and bilateral network formation allows the analysis of both topics at the same time and how they interact. The simultaneous development of multilateral and bilateral trade agreements requires an analytical tool which, to date, has not been used in network formation games. Bilateral graphs that are used in the standard literature of network formation are only capable of modeling bilateral link formation of players. For multilateral and bilateral link formation we introduce a generalization of graphs, called hypergraphs.

Section 1 of chapter 3, first, provides an overview of the main contributions to the literature of network games based on the seminal paper by Jackson and Wolinsky (1996), where networks are represented by means of bilateral graphs. In the second part of chapter 3 we provide a theoretic framework that allows us to analyze the strategic formation of bilateral and multilateral trade agreements.

3 Network Games

Network structures play an important role for the economic outcome. A large body of empirical studies has highlighted the importance of network structure in many social and economic fields[26].
Empirical studies in international economics have shown that, as the network of international trade agreements gets more and more complex, the world trade volume increased with an increase in the density of trade alliances between countries in the world trading system. The World Trade Organization enlarged its membership up to more than 150 countries and at the same time the number of regional and bilateral trading arrangements increased during the last twenty years, such that almost each country is involved in a RTA. In the course of the development of the world trading system the total world trade volume increases with the lowering of world trade tariffs and the formation of regional trade agreements. This observation shows, that there is a connection between a country's payoff and the network of trading arrangements that encompasses him.
Another field of interest in economic and social networks are social contact networks: When one wants to apply for a job he has to get an overview on all possible jobs available. One has to gather information by searching the internet, reading newspaper announcements and talking to friends. The likelihood for someone to find a suitable job depends on his own information and on the information of his personal contacts. When his friends are well informed about vacant job opportunities, he will find a job with less effort. The effort and time someone invests in gathering information depends on his friends' level of information and the level of their information depends on the level of information of their friend and so on. This suggests that individual outcome is influenced by the pattern of their social connections.

Given the importance of social networks in determining economic and social outcome, it is important to have theories of how such networks form and how they

[26]For empirical works on networks of firm alliances see Baker, Gibbons and Murphy (2008). For networks of co-authors see Goyal, van der Leij and Moraga-Gonzalez (2006) and for networks in labor markets Rees (1966) and Granovetter (1973) and Montgomery (1991). A survey of empirical works on networks is given by Newman (2003).

influence economic and social outcome. In what follows we will survey the recent literature on network formation. We will proceed by presenting two application that have been extensively studied in the network literature: the connection model and the trading example. We will then describe the related main theoretical findings. The network formation theory has already been applied to many economic fields. We will therefore provide a survey of the main contributions in the field of job-market networks, firm-cooperation networks and buyer-seller networks.
In the second part of this chapter, we will motivate the introduction of multilateral linking structures in the literature of network formation games, redefine the basic concepts of network formation and revise the main results in the theory of bilateral network games.

3.1 Contributions to the Theory of Economic Networks

In the strategic network formation literature, nodes are viewed as players and links between nodes represent bilateral relationships between players. We can for example view a node as a country and a link as a trade agreement between countries, or we can view nodes as firms and links represent collaboration agreements. In social contact networks a node can be viewed as an individual and the link as a social contact or a friendship to other individuals. Players obtain a payoff that depends on the network structure and different linking structures may lead to different economic or social outcome for the players.

Myerson (1977) was the first who modelled economic payoffs dependent on network structure. Since the basic models of cooperative game theory as introduced by von Neumann and Morgenstern (1944) disregard restrictions in communication among players, in some cases communication restrictions exist. In a transferable utility cooperative game Myerson (1977) assumes that players' communication possibilities were restricted by an underlying undirected communication graph. The nodes of the graph represent the players and the links represent conferences in which players can communicate and negotiate plans such that the conference structure describes communication possibilities between players. He defined a *value function* that assigns a value to each coalition of players. It can be interpreted as the total worth the

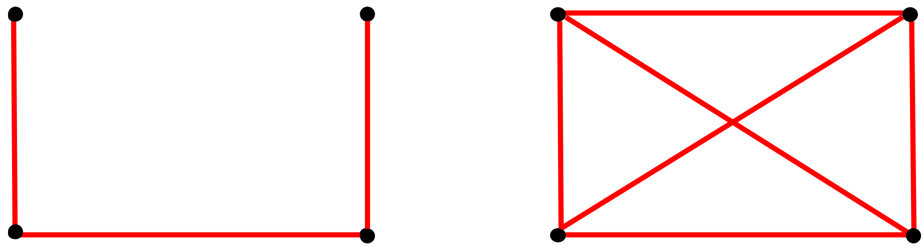


Figure 4: In Myerson's framework the set of players generates the same value in both graph structures.

coalition can achieve by coordinating their actions. A set of players can only generate a value when its members are connected in the network. The value function, as in cooperative game theory, depends on a group of players that are connected, independent of how exactly they are connected. A network structure that connects the same groups of individuals could generate the same value as a network structure that connects the same groups of individuals but with a lower density of links. In Figure 4 nodes represent players and the links represent communication possibilities. In Myerson's framework the total value generated by all players is the same in both graph structures.

Next Myerson (1977) defines an *allocation rule* on the class of communication situations as a function that assigns to each communication graph a payoff vector that describes the payoff each player receives from a given communication graph. He defines an allocation rule, the so-called Myerson value (a variation of the Shapley value) as a solution concept for a cooperative game where cooperation possibilities among players are restricted by an underlying graph structure. He then characterizes the Myerson value as the unique allocation rule that satisfies fairness and component efficiency. Fairness means that two players should gain equally from forming the link between them. Component efficiency means that the total value generated by a connected component in a given graph structure should be allocated among the members of the component.

Although Myerson (1977) defines a class of cooperative games based on a graph structure he does not directly analyze the endogenous formation of the underlying linking structure. Aumann and Myerson (1988) were the first who modelled a game where the graph structure is endogenously evolving. They introduced an extensive form game where pairs of players are listed in a certain order and one pair after the other has the opportunity to form a link, knowing the decisions of all pairs of players that came before them and predicting the steps that come after them. After a link was formed it cannot be deleted anymore. The game stops when each pair of players had another opportunity to form the link after the last link was formed. Consider the following example from Myerson (1991, p. 448):

Example 3.1. *Consider a three player game with* $N = \{1, 2, 3\}$ *in which a total value of* 300 *is allocated among the three players such that no player's payoff is negative and the sum of all the players' payoffs does not exceed* 300*. Moreover, the value is allocated with respect to the Myerson value allocation rule that satisfies fairness and efficiency.*
Pairs of players are listed in a certain order in which they can decide of whether to form a link or not. Two players can only negotiate the allocation of the value when they are linked. The linking structure describes which coalitions can negotiate or coordinate effectively in a coalitional game. For each linking structure the Myerson value allocation rule is shown in Table 2*.*

Let's assume the order of pairs which can choose whether to form a link is given by $\{1, 2\}, \{1, 3\}, \{2, 3\}$*. After the link* $\{1, 2\}$ *is in place players* 1 *and* 2 *cooperate such that the Myerson value provides the payoff vector* $(150, 150, 0)$*. Player* 1 *would predict that if he formed a link with player* 3 *in the next round such that the allocation is given by* $(200, 50, 50)$*, in the subsequent round players* 2 *and* 3 *will form a link which implies an allocation of* $(100, 100, 100)$ *and worsens his situation in the cooperation structure* $\{\{1, 2\}\}$*. Therefore the cooperation structure* $\{\{1, 2\}\}$ *will result as a subgame perfect equilibrium.*

Due to its extensive form the game is very hard to analyze and the network structure that results depends tremendously on the order of pairs of players. As the

linking structure	Myerson value allocation rule
$\emptyset$	(0,0,0)
$\{\{1,2\}\}$	(150,150,0)
$\{\{1,3\}\}$	(150,0,150)
$\{\{2,3\}\}$	(0,150,150)
$\{\{1,2\},\{1,3\}\}$	(200,50,50)
$\{\{1,2\},\{2,3\}\}$	(50,200,50)
$\{\{1,3\},\{2,3\}\}$	(50,50,200)
$\{\{1,2\},\{1,3\},\{2,3\}\}$	(100,100,100)

Table 2: The Myerson value allocation rule (see Myerson (1991, p.448)).

above example shows a complete network structure where each player is linked with every other player does not necessarily evolve. The Myerson value of player 1 and 2 is larger when the network consists only of a link between player 1 and 2 than in the complete network.

Much easier to analyze, Myerson (1991, p.448) mentions a strategic form game that described a linking formation game that results in a network and constitutes the simplest form of a linkformation game one can think of.[27] The idea is the following: Each player's strategy set is given by the player set such that a player's strategy of player i, s_i, describes with whom player i wants to be linked. A link between two players i and j is formed, whenever i belongs to the strategy of player j and vice versa. Therefore each strategy vector s over the whole player set explicitly describes a cooperation structure. Due to its simplicity it has the undesirable feature that there exists a multiplicity of Nash equilibria. Moreover it can be shown that any cooperation structure can be sustained in a Nash-equilibrium. In order to make use of this game one really needs refinements of the equilibrium concept to narrow the set of equilibrium networks and to exclude the inefficient ones.[28]

[27] For a more formal discussion of the linking game see Dutta et al. (1998). Slikker et al. (2000) extend the linking game of Myerson (1991) toward hypergraphs and called it the conference formation game.

[28] For a survey on the cooperative part of network formation see van den Nouweland (2005).

Needless to say, the value that is generated by the interaction of a group of players may be different for different linking structures and densities of the network structure. The network of social interactions influences economic outcome. We could see that in chapter 2 the total outcome produced under free trade differs to the outcome of another trading regime. In contrast to the cooperative game theory literature that models the value that a coalition of players can achieve as a function of the player set connected in an underlying graph structure, Jackson and Wolinsky (1996) were the first to define a value function directly on the network structure. The allocation rule assigns to each player the payoff it obtains from a certain linking structure and its direct and indirect linking environment. This payoff can be interpreted as the utility or production that an individual obtains from the social interaction that occurs through the network. In section 1 of chapter 2 the payoff function was given by a country's welfare in a given trading system. They assume that links are strategically formed by the players who have the opportunity to form bilateral links to other players that shape the network structure and hence the players' payoffs.
To model strategic network formation from a game theoretic perspective one needs to define stability concepts to compare different trading structures. Jackson and Wolinsky (1996) define a stability concept that serves as a direct requirement for networks to be resistant against individual deviations. The concept they introduce is called *pairwise stability.* A network is pairwise stable, whenever (i) no player is better off when he severs one of his existing links, and (ii) no pair of players would benefit from adding an additional link to the network. The first condition is based on the idea that one player is needed to severe any of his existing links without the consent of the other player. The second condition requires that both players benefit from the additional link (whereas one of the two players has to be at least strictly better off such that the existing network is not stable anymore).

The stability concept mentioned above is static in the sense that it merely characterizes networks that are stable with respect to individual incentives. In contrast to the static model of network formation one can also consider the network formation process as a dynamic process. Watts (2001) modelled the first approach of a dynamic network formation process where players can add and severe links one after the other. The idea is to start with an empty network and at times $t = \{1, 2, ...\}$ a pair of players is randomly selected where the players myopically decide whether to delete the link if it already exists, or add the link to the existing network, whenever both players want to add the link. Furthermore, Watts (2001) introduces the concept of an *improving path*[29] which denotes a sequence of networks where each network followed by its predecessor either defeats the network in the way that an additional link was added or that an existing link was severed. Watts (2001) calls a network to be stable, when there is some time t in which the improving path stops and no additional link is added and no existing link is severed. Therefore a stable state is a pairwise stable network that can be reached from an improving path starting from the empty network. This formulation requires agents to be myopic since a change in one player's links can leave other players with less payoff which might induce them to change their links in the next round. We can therefore conclude that a network is pairwise stable if and only if it has no improving path emanating from it.

Another dynamic network formation approach was made by Jackson and Watts (2002). They model network formation in a way that individuals form and sever links based on the improvement that the resulting network offers them relative to the current network. As in the pairwise equilibrium, a link between two players can be formed only if both players agree to form the link, while a single player can sever an existing link. Each individual receives a payoff based on the underlying network structure. The authors use the concept of an improving path to analyze dynamic network formation which has the properties that (i) each network in the sequence of networks differs from the previous network by the addition or deletion of a single link, and (ii) the addition or deletion of the link is beneficial for the players who form or sever the link. The notion of improving path is myopic in the sense that players form and sever links through myopic considerations and do not consider how their decision to form or sever a link might influence the future evolution of the network.

[29]The concept of an improving path will be explored in more detail in section 2 of this chapter, where we redefine the concept when not just pairs of players are randomly selected but larger groups of players can reconsider their links.

Furthermore Jackson and Watts (2002) introduce the notation of a *cycle*, which is a set of networks in which each member of the cycle has an improving path to any other member of the cycle. The improving path will end up either in a pairwise stable network or in a cycle. There are cases in which a pairwise stable network always exists and no cycles. However, there are also cases in which no pairwise stable network exists but a cycle exists. The trading example in section 1.3 of this chapter shows that it is possible that no pairwise stable network exists but that the improving path will result in a cycle.

While pairwise stability is a relatively weak equilibrium concept and easy to analyze, it does not consider deviations where players can delete more than just one link at a time or add and delete links at the same time. A lot of refinements have been considered to narrow the set of stable networks which turn out to be a subset of the pairwise stable networks[30]. One prominent refinement has been made by Jackson and van den Nouweland (2005) which they call the *strong stability*. They formed the term *obtainable*, which regards to a network that is obtainable from another network via deviations of a coalition S with $S \subseteq N$ which requires that (i) any new link that is added can only be between players of S, and (ii) at least one player of any severed link must be a member of S. This condition already implies that a link can be severed unilaterally, whereas the first condition implies that the consent of both players is needed to form a link. The equilibrium concept of strong stability now requires that a network is called strongly stable, whenever there exists no subset of players $S \subseteq N$ such that another network is obtainable via deviations of S where no player of S is worse off under the new network and at least one player is better off. Therefore no coalition of players can jointly deviate and form a new linking structure in which they all improve[31].

[30]Refinements of the pairwise equilibrium concepts were first considered by Dutta and Mutuswami (1997).

[31]This stability notion has already applied in the context of networks of oligopolies (Goyal and Joshi (2006b)) where firms are relatively well informed about the market structure and can coordinate their action more flexible. It might, however, not be very applicable in the case of international trade networks where trade negotiations take place over years and legal conditions as the GATT system restrict countries' options for action.

3.1.1 Applications of Network Games

Although the literature in network games is relatively new, the equilibrium concept as suggested by Jackson and Wolinsky (1996) has already been applied in many economic and social fields. Some applications can be found in the context of buyer and seller networks, Kranton and Minehart (2001) and Corominas-Bosch (2004), in social contact and labor market networks, Calvo-Armengol (2004) and Calvo-Armengol and Jackson (2004), and in industrial organization as for example in R&D networks, Goyal and Joshi (2003), and firm collaboration networks, Goyal and Moraga-Gonzalez (2001)[32]. In the following we will consider how models of network have been applied to these fields to investigate incentives of players to strategically form and sever links.

Kranton and Minehart (2001) analyze a model with a fixed number of buyers and sellers and an exogenously given network structure. To exchange goods a buyer and a seller have to be linked such that trading possibilities are represented by the linking structure. They elaborate on a player's influence that depends on its position in a given network and investigate the equilibrium prices under a given network structure. Competition in networks takes place by means of the English auction and assigns the unit of good from a seller to the buyer with the highest valuation. A player's valuation is random and a buyer knows its own valuation for the unit of good but not the valuation of the others. Sellers announce prices simultaneously and buyers drop out of the auction as soon as the price exceeds their valuation. They obtain that buyers tend to elaborate a large number of links with different sellers to pool demand uncertainties. The allocation rule is given by a player's payoff that is determined by the overall network of buyers and sellers. Corominas-Bosch (2004) investigates a model similar to the one in Kranton and Minehart (2001) where the prices for goods are determined by an alternation offers bargaining process rather than through an auction. A link is necessary for a buyer and seller to bargain over the unit of good hold by the seller. They model the bargaining process as a variation of the Rubinstein bargaining model where the expected payoff of a buyer and a seller can be calculated as a function of the network structure where each players bargai-

[32]See a survey of this literature in Goyal and Moraga-Gonzalez (2003).

ning power depends on its position in the network. First sellers announce prices and each buyer receives offers from those sellers to whom he is linked. Buyers can choose whether to accept one of the offers or not. Afterwards the pairs of sellers and buyers that have successfully traded the goods are cleared from the market and in the next round the buyers are to call out prices. Each period the role of the proposer switches as long as there are no linked pairs of buyers and sellers left in the market, whereas each period a buyer's and a seller's valuation of the good is discounted by a common discount factor $\delta \in (0, 1)$. She defines a player's expected payoff as a function of the link pattern between all buyers and sellers.

Calvo-Armengol (2004) analyzes a model in which initially all workers are employed and may randomly lose their jobs with a certain probability. They hear about job opportunities through their links to other workers. He investigates the relation between the density of the personal contact network and the success in finding new jobs. He shows that each worker's information flow of new job opportunities is determined by the shape of direct and two-links-away contacts given by the social network structure in the way that direct links are always beneficial for the information flow whereas two-links-away contacts are detrimental for the information flow. When one of a worker's direct contacts becomes better connected the direct contact might pass his information on to one of his other direct contacts which decreases the probability for a player to receive the information. Other indirect contacts have no influence on the probability of a worker to obtain a new job. He defines a worker's payoff as a function of the direct and indirect contacts in the network and the total value of a network is given by the sum of all workers' payoffs.
Calvo-Armengol und Jackson (2004) introduced a model in which workers are linked through a network of social contacts and obtain information about job opportunities through their linking structure. Over time workers randomly lose their jobs and new job opportunities randomly arrive. They investigate two different aspects. First, they investigate the correlation between the employment status of workers in the same network. They show that there is a positive correlation between the employment status of connected workers. Furthermore they investigate how the duration of unemployment affects future employment in a given network structure. They find

that with a higher density of the network a longer duration of unemployment lowers the probability of getting a job. The intuition of the result is the following: The longer the duration of unemployment, the more likely it is that the neighbours of an unemployed are also unemployed and therefore do not pass on information about job opportunities. A workers probability of finding a new job is determined by the current employment situation of the other workers and the overall network of social contacts.

Goyal and Joshi (2003) analyze firms' incentives to form collaborative links to reduce their cost of production. They find that under different market competitions firms have an incentive to form collaborative alliances. For a given network structure they can calculate equilibrium quantities in the Cournot competition case supplied by each firm and equilibrium prices in the Bertrand competition case, respectively. Given the quantities of all firms one can calculate each firm's payoff as a function of the network of collaboration alliances and the quantities supplied by each firm in the network. One ends up with a well defined allocation rule and value function, which is given by the overall payoff of all firms. Further, they can be adopted to analyze incentives for firms to form such collaboration alliances and to investigate stability and efficiency of network structures.
Goyal and Moraga-Gonzalez (2001) study incentives of competing firms to form collaboration alliances to invest in R&D. Firms can form pairwise collaboration links to share R&D knowledge about a cost-reducing technology such that the set of pairwise collaboration links defines a network. Given the network of collaboration links each firm chooses a level of R&D effort (costs to invest in R&D). After firms chose their effort level, each firm chooses a quantity to supply in the market. They investigate different network structures with respect to effort level of the firms. They define a firm's payoff as its profit that derives from the network of collaboration agreements and the effort level of all firms in the network. Goyal and Moraga-Gonzales (2001) are now able to investigate firms' incentives to strategically form R&D collaboration links.

These and many other applications to network games in economic and social en-

vironments have been applied that show the potential of network games to describe economic situations. Still relatively few attempts have been made to model strategic link formation in international trade by means of network formation games.[33]
In the following we will discuss the connection model as an introducing example in network formation which has often been discussed in the literature. In the connection model players obtain a payoff from direct and indirect connections, whereas the payoff from an indirect connection decreases with the distance between the two players. Players have to weight the costs of forming new links with the returns from direct and indirect links.

3.1.2 The Symmetric Connection Model

Jackson and Wolinsky (1996) introduced the connection model in which links between players can be interpreted as social relationships that provide benefits to players in the social network. Forming a link with another individual requires to make some costly effort but in turn it allows access to the benefits available to the latter via his own links. In the following model we can think of a network of social relations and social contacts in which players benefit from indirect connections as from the "friends of a friend".
This idea of players forming costly links but benefit by indirect links is known in the literature under the name *symmetric connection model*. There exist two versions of the connection model. The first one was introduced in Bala and Goyal (2000) where individuals can form links unilaterally[34]. The other version was first mentioned in Jackson and Wolinsky (1996) and considers non-directed formation of links, where players form links on the basis of bilateral agreements. Both versions have been extensively studied in the literature.[35] As the present thesis concentrates on the formation of trade agreements which require the consent of all parties involved

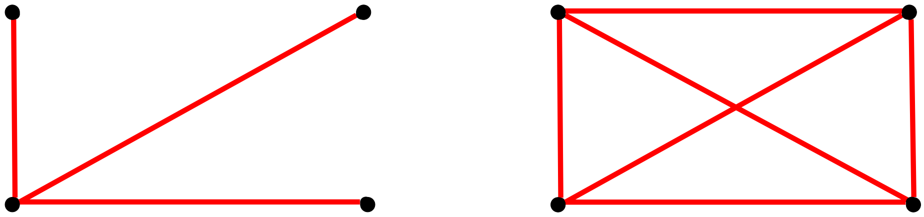
[33]Goyal and Joshi (2006a) as well as Furusawa and Konishi (2007) model the formation of trade agreements as a network formation game. The models consider exclusively the formation of bilateral links.

[34]In section 1.4 of this chapter we will briefly mention the literature on unilaterally link formation where networks of social contacts are modelled by means of directed graphs. It is based on the pioneering paper by Bala and Goyal (2000).

[35]See for example Johnson and Gilles (2000) and Jackson and Rogers (2005).

and are therefore described by means of non-directed networks, we will in the following report the main findings of the symmetric connection model of Jackson and Wolinsky (1996).

In the symmetric connection model each player pays cost c for the formation of a link and obtains from each direct and indirect link a payoff, where the payoff from a link decreases with the distance two (indirectly) linked players have. Formally, the payoff player i obtains from a direct link to another player is given by δ, where $\delta \in (0, 1)$, and this payoff decreases with the distance that two players have in a network. δ denotes the decay factor from indirect links such that the payoff from an indirect link decreases with the distance between the two players in the social network.
Consider for example the network with 4 players where player 1 is connected to player 2 and player 3 but not directly connected with player 4. Player 4 however is directly connected with player 3. Player 1 gets the benefit δ from his direct links with player 2 and 3 and the benefit of δ^2 from the indirect link with player 4. As it is assumed that $\delta \in (0, 1)$ this network leads to a smaller payoff for player 1 from the connection with player 4 than from the connection with player 2 and 3.
Jackson and Wolinsky (1996) defined efficiency of networks in the way that a network structure is efficient whenever it maximizes the total payoff of all players, where the payoff of a network is calculated as the sum of the payoffs of all players. In the symmetric connection model we find that when costs are very low, the complete network is the unique efficient network structure as the formation of a direct link is always more profitable than the gains from an indirect link. For very high linking costs the empty network is efficient, whereas for moderate costs a star network, which describes a situation in which all players are directly linked to one center player and there are no other links such that each player is indirectly linked to each other player, will be efficient. In Figure 5 we illustrate the star network and the complete network for the case of 4 players. Jackson and Wolinsky (1996) adopt the notion of pairwise stability as a necessary requirement for networks to be stable which implies that the formation of a new link requires both players to be formed whereas deletion can be done unilaterally.



a) Star Network b) Complete Network

Figure 5

The results on stability can be summarized as follows: When the linking costs are very high, the unique stable network is empty since no player will ever have an incentive to form a link. When linking costs are very low, the complete network will be formed in which each player is linked bilaterally to any other player since direct links are more beneficial than indirect links. When linking costs are of a moderate level, players will form a star network, in which one player has bilateral links to all the other players and there are no additional links.

One of the central questions Jackson and Wolinsky (1996) address is whether efficient networks will form when self-interested players can choose to form and severe links. They point out that the set of efficient networks and equilibrium networks does not always coincide. In the symmetric connection model, in spite of its simplicity of the payoff structure, there exists a trade-off between individual and social incentives. Consider the case of moderate costs when the costs of a direct link exceed the benefits from a direct link with $\delta < c$. Then relationships are only beneficial whenever a player is involved in some direct and some indirect links. It is clear that a star network cannot be an equilibrium as the center player will always severe its direct links. It can however be shown that in this cost range a star network might be efficient for some values of $c > \delta$, in the case when gains from indirect connections (δ^2) are high enough such that the total value of a star network exceeds the total value of an empty network (which is zero). This model of social connections between players depicts the conflict that can occur between social and individual incentives.[36]

It is clear that this stability concept is not very strong as it only considers the deviation of single links. In our discussion in section 2.6 of chapter 3 we introduce the notion of a stronger stability concept in which players are allowed to delete more than just one link at a time.

The connection model concentrates on bilateral link formation, where networks are modelled by means of non-directed bilateral graphs. In section 2.3 of this chapter we will report the main findings of Jackson and Wolinsky (1996) on stability and efficiency when we extend the framework of Jackson and Wolinsky (1996) toward multilateral link formation, where networks are modelled by means of non-directed hypergraphs. In a hypergraph links can include more than just two players. We define efficiency and stability notions for hypergraph networks and compare the results of the connection model with the ones in Jackson and Wolinsky (1996). Afterwards we will discuss the conflict between the two concepts.

The following example illustrates that there are cases in which there are no pairwise stable networks and only cycles exist.

3.1.3 Trading Example (Non-existence of Pairwise Stable Networks)

The following example was first discussed in Jackson and Watts (2002) and is called the *trading example* in which players benefit from trade with other players and trade can only flow along links.

Consider a society that consists of at least 4 players. There are two goods x and y that can be traded among the players and each player has a Cobb-Douglas utility function $u(x, y) = x \cdot y$. With probability $\frac{1}{2}$ each player is endowed either with

[36]There have been a number of experiments done on network formation which examine whether the network structures predicted by the theoretic models arise or whether the formation of equilibrium networks is unlikely. For references see Kosfeld (2004), Berninghaus et al. (2006) and Goeree et al. (2008).

$(1,0)$ or $(0,1)$, and the endowment is randomly disributed with an independent and identical distribution. Players obtain gains from trade with other players when they are in the same connected component in the way that a Walras-equilibrium appears between all players in a connected component, independent of the density of links in the connected component.

Consider for instance network $g = \{\{1,2\}\}$ that consists of a single bilateral link between player 1 and 2. There is a $\frac{1}{2}$ probability that both player have a different endowment such that the utility for each player is $\frac{1}{4}$ and a player's expected utility is given by $\frac{1}{8}$.

Players have to pay costs c to maintain a link. Let the cost to each player in a link be given by $c = \frac{5}{96}$. What can be shown is that there does not exist a pairwise stable network. A player obtains a payoff of 0 when he has no links to any other player. The expected utility of being connected to one player is $\frac{1}{8}$. The expected utility of being connected to two players is $\frac{1}{6}$ and of being connected to three players is $\frac{3}{16}$. It is obvious that the expected utility of a player is increasing with the number of players that a player is directly or indirectly connected to, ignoring the cost of links, but with a diminishing return to the number of players added to the network. It is clear that each connected component with k players has $k-1$ links since each additional link can be deleted without reducing the expected utility of each player but lowering the costs of links.

With $n = 4$ each connected component that contains a player who has just one link cannot be stable. A player who is connected to a player that is not connected to anyone else loses at most $\frac{1}{6} - \frac{1}{8} = \frac{1}{24}$ of the expected utility by severing the link to the end player but the additional payoff he gains is $\frac{5}{96}$. Therefore the network will result in a network with two pairs of players. But this cannot be stable either since two players will form an additional link with $\frac{1}{8} - \frac{5}{96} < \frac{3}{16} - \frac{10}{96}$. Therefore this results in a cycle.

In section 2.5 of this chapter we will see that a stable network exists when players can form multilateral links.

3.1.4 Closing Remarks

So far we have focussed on the formation of non-directed networks where the link between two players requires the consent of both players. Non-directed links are a more appropriate tool to model networks of friendship, buyer seller networks and firm network and especially international trade agreements, where the consent of both parties is required for the formation of a link between them. More specific we concentrated on link formation that contained both players involved. There is a large strand of literature that concentrates on the formation of directed networks where players unilaterally form links. Directed networks may play an important role in the context of telephone networks, the world wide web or the network of citations in refereed journals, where players can unilaterally build up new links. For instance, when we consider a network of researchers that reports who cites who, this network is directed and, as citations can be done without the other researcher's permission, links can be formed unilaterally.

One pioneering paper by Bala and Goyal (2000) investigate stability in a way that players can form links without the consent of the other player and where the costs of a link are beared by the player that formed the link. They consider directed link formation and differ between models of *one-way flow* in which the payoff from a link only flows to the player that formed the link whereas in a *two-way flow* model the payoff from a link goes to both players involved in that link. The equilirium concept they introduce allows to adopt the Nash equilibrium since players can non-cooperatively form bilateral links. They consider a direct version of the connection model and find that in the one-way flow model, when the costs of forming a link are low as compared to the payoff received from a direct link, the complete network, in which each player has formed a link with every other player, is stable. When linking-costs are very high the unique Nash-network is the empty network. As in the model of Jackson and Wolinsky (1996) they obtain an incompatibility between stable and efficient networks for moderate linking costs. The efficiency concept they propose is the same as in Jackson and Wolinsky (1996) and characterizes the network that maximizes total outcome. It is clear that with moderate linking costs, whenever the star network might be efficient, it is not necessarily stable. For instance, the center player that formed a link to each of the other players will sever its direct links as soon as the direct linking costs exceed the payoff from a direct link whereas the externality that the star network structure provides for the other players through indirect connections might be very large.

Another paper by Haller and Sarangi (2001) investigates non-cooperative network formation and allows heterogeneity among players with respect to link failure or link success, respectively. They investigate the robustness of the results of Bala and Goyal (2000) and find that neither an empty nor a connected network may arise in equilibrium.[37]

The literature on network formation has not introduced multilateral link formation so far. The pioneering works by Jackson and Wolinsky (1996) and Bala and Goyal (2000) and the literature based on their models considers bilateral links between pairs of players. However, this strand of literature does not consider links between a larger group of players, as in multilateral trade agreements and multilateral organizations. In the present thesis we provide a theoretic framework to extend the original concepts and results towards a more general framework in which we allow the strategic formation of multilateral and bilateral trade agreements.

3.2 Network Formation Games and Hypergraphs

The organization of individual agents into networks plays an important role in the determination of the outcome of many social and economic interactions. Given the numerous occurrence of social and economic networks in real life situations and the importance in determining the outcome of the interaction among linked players, it is essential to have theories about how such network structures matter and how they form. One way to investigate network formation was introduced by Jackson and Wolinsky (1996), where self-interested individuals can form bilateral links to other individuals to improve their payoffs. Their approach considers non-directed networks, where a link needs the consent of both players to be formed, but can be

[37]For more details on the case of non-cooperative network formation see e.g. the chapter by Goyal (2003).

deleted unilaterally. In another paper Bala and Goyal (2000) consider one-sided link formation, where players can form bilateral agreements. Both papers consider the formation of pairwise links.

Often players are involved in multilateral relationships such as multilateral trade agreements, in which players in a multilateral trade agreement have exclusive trade conditions with members of the multilateral trade agreement but not with members outside, or social organizations, in which players only share information with a group of players. Some players may even be members of many multilateral organizations or trade agreements. We are interested in understanding how multilateral linking structures are going to form, because multilateralism can lead to different economic outcomes than bilateralism. Bagwell and Staiger (1999a) investigate optimal tariff policy in the GATT system. They find out, that in a bilateral negotiation game players choose an optimal tariff level different from that under the multilateral GATT regime, since the non-discrimination principle as claimed in Article I of the GATT demands the same tariffs among all members of the GATT system, whereas in bilateral linking situations player choose non-cooperative external tariffs.

To allow multilateral link formation we introduce the structure of hypergraphs. A hypergraph is a generalization of a graph. While in a graph a link connects two players, in a hypergraph a link can connect any number of players. While the formation of a link in a graph requires the consent of both players involved, the formation of a link in a hypergraph requires the consent of all players involved in that link. One advantage of hypergraphs is that they allow players to be involved in many multilateral links. Many examples appear in international trade, where most countries are member of the GATT and some smaller groups of players have regional free trade agreements such as the NAFTA (North American Free Trade Agreement), in a way that all members of the NAFTA are also GATT members.

One characteristic of the GATT is that due to the MFN clause a member country has to offer each country of the GATT the same non-discriminatory tariffs and does not have the opportunity to sever the connection to a single member country

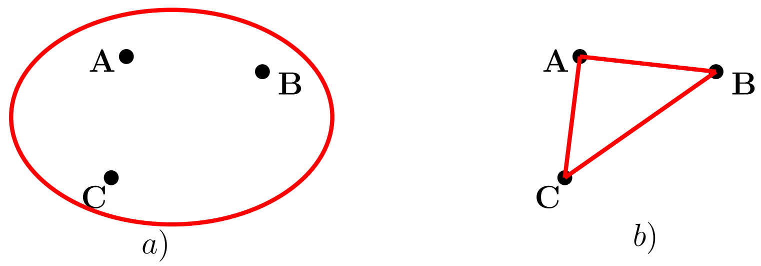


Figure 6: *a*) Hyperlink between A, B and C. *b*) Bilateral link formation.

within the GATT. An example for this characteristic is shown in Figure 6*a*). Country A, B and C are members of the WTO and have agreed upon common tariff reduction. In the bilateral linking case (Figure 6*b*)) as in Jackson and Wolinsky (1996) player C has the opportunity to sever his existing link with player B but keep his bilateral link with player A. In contrast, in a hyperlink players are connected in a way that does not allow one member to terminate trading arrangements with single members within the GATT. Therefore hyperlinks are used to model multilateral trade agreements.

Although the literature on network economics is well established, the literature on hypergraphs in economics is still in its infancy. Durieu et al. (2005) investigate network games that are played on a fixed hypergraph structure. Van den Nouweland et al. (1992) investigate the Myerson value and the position value for hypergraph communication situations. In both papers the value function depends on a set of players whereas we allow for different values that depend on the exact structure of a network. We let the hypergraph structure be endogenous and allow players to choose their linking partners.

The first aim of this chapter is to present the basic concepts of networks that will be used in this thesis. The study of networks has a long tradition in mathematics in the theory of graphs. In the following we provide the basic concepts in the theory of hypergraphs. Hypergraphs generalize pairwise graphs and consist of a set of nodes and a collection of non-singleton groups of nodes. Most of the definitions listed below can be found in Berge (1989). Section 1 presents the basic graph theoretic concept used in the present thesis and provides graphic illustrations to clarify and distinguish different terms and definitions.

The second aim is to provide equilibrium and efficiency concepts based on the graph theoretic structure presented in section 1 and motivate the introduction of hypergraph into models of network formation. We proceed by revisiting the connection model when network structures are described by hypergraphs. The next section analyzes the compatibility between efficiency in the context of hypergraphs and concludes with some general results.
Stronger efficiency and stability concepts are discussed at the end of chapter 3.

The remainder of this chapter is organized as follows. In section 2.1 of this chapter we introduce the basic concepts and properties of hypergraphs and present some graphic illustrations. In section 2.2 we define the notion of multilateral stability and define efficiency of hypergraphs. In section 2.3 we introduce the connection model to illustrate the new stability concept and continue in section 2.4 with an analysis of the compatibility between efficiency and stability of hypergraphs. For each of these models we describe the efficient and the stable hypergraphs. We also prove an existence result of multilaterally stable networks in section 2.5 of this chapter and present allocation rules for which stable hypergraphs always exist. In section 2.6 we provide a discussion of the stability notion and in section 2.7 a discussion of the efficiency notion. Section 2.8 concludes.

3.2.1 Concepts

In the following we will consider a set of nodes $N = \{1, ..., n\}$ where n is finite. Relations between nodes will be expressed by subsets of N that we denote with L, $L \subseteq N$ and a *hypergraph* $\mathcal{L}$ on N is a family of subsets of N with $\mathcal{L} = \{L_1, ..., L_m\}$. The set of all possible hypergraphs on N is denoted by $\mathcal{H}$.

Neighbours and Degree
Let $N_i(\mathcal{L}) = \{j \in N \mid \exists L \in \mathcal{L} : \{i, j\} \subseteq L\}$ denote the set of nodes with which

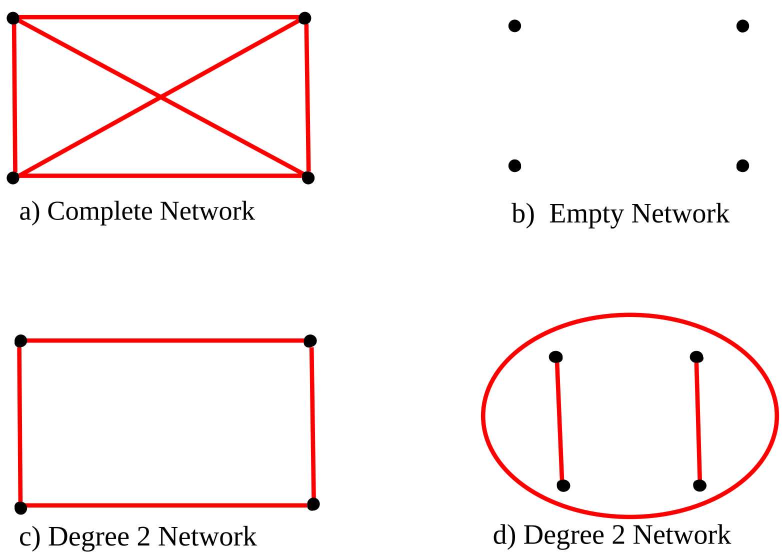


Figure 7: Regular networks for $n = 4$

i has a link and will be referred to as the set of *neighbours* of node i in $\mathcal{L}$. With $\eta_i(\mathcal{L}) =\mid N_i(\mathcal{L}) \mid$ we denote the number of neighbours of node i in $\mathcal{L}$.
We define the *degree* $d_{\mathcal{L}}(i)$ of i as the number of links in that node i is included such that $d_{\mathcal{L}}(i) =\mid \mathcal{L}_i \mid$ where $\mathcal{L}_i = \{L \in \mathcal{L} | i \in L\}$.
A hypergraph $\mathcal{L}$ is said to be *regular* when all nodes have the same degree, i.e. $d_{\mathcal{L}}(i) = d_{\mathcal{L}}$, $\forall i \in N$. Figure 7 represents regular hypergraphs for $n = 4$. The *complete* hypergraph $\mathcal{L}^N$ is a regular network in which $\mathcal{L}^N = \{L \in 2^N \mid |L| = 2\}$ and $d_{\mathcal{L}}(i) = n-1$ $\forall i \in N$. The *empty* network $\mathcal{L}^e$ is a regular network in which $d_{\mathcal{L}}(i) = 0$ $\forall i \in N$.

The average degree in hypergraph $\mathcal{L}$ is defined as $\sum_{i\in N} \frac{d_{\mathcal{L}}(i)}{n}$. It can easily be verified that the average degree in Figure 7c) and d) is equal to 2. The average degree in the complete network in Figure 7a) is 3.

A hypergraph is *linear* if each pair of links in the hypergraph intersects in at most one node. For example, the hypergraph of Figure 8 is linear.

Paths and Components

A central tool to analyze whether one node can be reached from another is the notion of a *path*. We say that there is a path between two nodes i and j in $\mathcal{L}$ either when there exists a set $L \in \mathcal{L}$ such that $\{i,j\} \subseteq L$ or there exist distinct nodes $i_1, i_2, ...i_n$ such that $\{i, i_1\} \subseteq L_1$, ..., $\{i_n, j\} \subseteq L_n$ for all $\{L_1, ..., L_n\} \subseteq \mathcal{L}$. Figure 8 describes a hypergraph with $n = 5$. A possible path in this hypergraph is 3, 2, 4, 5.

We say that two nodes belong to the same *component* if and only if there exists a path between them. Formally, a nonempty network $\mathcal{L}' \subseteq \mathcal{L}$ is a component of $\mathcal{L}$ if $\forall i \in N(\mathcal{L}')$ and $j \in N(\mathcal{L}')$, there exists a path in $\mathcal{L}'$ connecting i and j, and for any $i \in N(\mathcal{L}')$ and $j \notin N(\mathcal{L}')$ then there does not exist a path in $\mathcal{L}$ between i and j. We want to emphasize that a path consists of a sequence of links within a network. Later when we introduce an improving path we rather speak of a sequence of different networks. We will denote the set of components in $\mathcal{L}$ with $C(\mathcal{L})$. Furthermore, a hypergraph is called *connected* if there exists a path between any pair of nodes. Formally, $\mathcal{L}$ is said to be connected if each pair of players i and j, $\forall i, j \in N$, is connected in $\mathcal{L}$. Then the network $\mathcal{L}$ consists of one component with $C(\mathcal{L}) = \{\mathcal{L}\}$. In this case the hypergraph consists of one single connected component and is called *connected*. In Figure 8 we can see that node 1 and node 5 belong to the same connected component. Moreover the hypergraph consists of one single component and is therefore connected.

The distance between two nodes i and j in $\mathcal{L}$ is the length of the shortest path between i and j and will be denoted by $t_{i,j}(\mathcal{L})$. If there exists no path between player i and j in network $\mathcal{L}$ then by convention we set $t_{i,j}(\mathcal{L}) = \infty$. When $\mathcal{L}$ is connected, the average distance between nodes of hypergraph $\mathcal{L}$ is given by:

$$t(\mathcal{L}) = \frac{\sum_{i \in N} \sum_{j \in N} t_{i,j}(\mathcal{L})}{n(n-1)}.$$

In the complete network the average distance is given by $\frac{n(n-1)}{n(n-1)} = 1$ (see Figure 7).

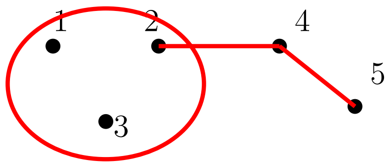


Figure 8: Linear Hypergraph

Cliques

A *clique* is a maximal subset of nodes with the property that every pair of nodes has a link. Formally, a set of nodes $N' = \{i_1, i_2, ...i_k\} \subset N$, where $k \geq 3$ is a clique if, for every pair i and $j \in N'$, there exists an $L \in \mathcal{L}$ for that $\{i, j\} \subseteq L$ and there exists no subset $N'' \subset N'$ with this property. In the complete network of Figure 7a) there exists one clique that contains all nodes and each pair of nodes has a bilateral link.

3.2.2 A Model of Network Formation

The importance of network structure on economic outcome motivates an examination of the incentives of players to form links and of the strategic stability of different structures when linking decisions depend on players' payoffs. A simple way to analyze stable network structures is to examine the requirement that individuals do not benefit from altering the structure by single deviations.
In this chapter we adopt the basic concept of pairwise stability but allow players to form and sever multilateral links.

First we investigate the stable and efficient structures for the connection model. We show that the results of Jackson and Wolinsky (1996) are a special case of ours, whereas in our framework we obtain a larger range of possible stable networks but therefore also a larger range of possible efficient networks. Then we give a simple existence proof of multilaterally stable networks by introducing the concept of an improving path. We also provide examples in which multilaterally stable networks exist and an example in which we obtain that each improving path results in a cycle. We obtain existence of a multilaterally stable network in the trading example and

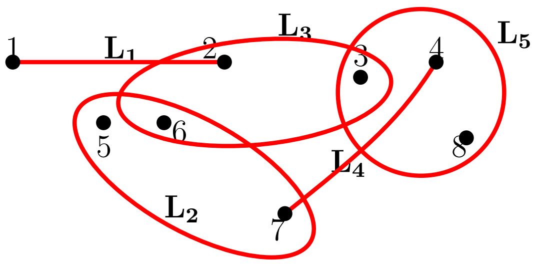


Figure 9: $n = 8$, $\mathcal{L} = \{L_1, L_2, L_3, L_4, L_5\}$ where $L_1 = \{1, 2\}$, $L_2 = \{5, 6, 7\}$, $L_3 = \{2, 3, 6\}$, $L_4 = \{4, 7\}$ and $L_5 = \{3, 4, 8\}$.

demonstrate with an example that multilaterally stable networks not always exist.

Consider a finite number of identical players $N = \{1, ..., n\}$ and assume $n \geq 3$.
In our setting we concentrate on undirected links as in Jackson and Wolinsky (1996) which means that a link between players needs the consent of all players involved in that link.[38]
With the aid of hypergraphs we are able to model multilateral agreements between players.

Definition 3.1. *Let $N = \{1, ..., n\}$ be a finite set of nodes. A family of subsets of N, $\mathcal{L}$, where $\mathcal{L} = \{L_1, ..., L_m\}$ is a set of links, $\mathcal{L} \subseteq 2^N$, is called a hypergraph on N.*

In the following the set of nodes will represent the set of players and the term network will be used as a synonym for the word hypergraph. Since each player is linked with himself we restrict our attention to hypergraphs $\mathcal{L}$ with $\mathcal{L} \subseteq \{L \in 2^N || L| \geq 2\}$. The set of all possible hypergraphs that satisfy this definition is denoted with $\mathcal{H}$. A hypergraph is shown in Figure 9.

If $L \in \mathcal{L}$, we say that all player $i \in L$ have a direct link. This could for example mean that a set of countries has a multilateral free trade agreement. In industrial

[38]In this thesis we will consider non-directed links such that a link between a group of players requires the consent of all players involved. For directed networks see Bala and Goyal (2000).

organization this could mean that a group of firms forms an alliance. In social contact networks it means that a group of people shares and exchanges information.
A *global* hypergraph is denoted by $\mathcal{L}^G$ and consists of a single link that contains all players in N with $\mathcal{L}^G = \{N\}$.

The complete network $\mathcal{L}^N$ is the family of subsets of N with $\mathcal{L}^N = \{L \in 2^N || L| = 2\}$. The *star* with center i, which we denote by $\mathcal{L}_i^S$, has only bilateral links from the central player i to each of the other players with $\mathcal{L}_i^S = \{L \in 2^N || L| = 2 \text{ and } i \in L\}$. We denote the empty network by $\mathcal{L}^e$.
$N(\mathcal{L})$ denotes the set of players $i \in N$ that have at least one direct link in $\mathcal{L}$, $N(\mathcal{L}) = \{i \in N \mid \exists L \in \mathcal{L} : i \in L\}$ and, as already mentioned above, $N_i(\mathcal{L})$ denotes the set of players that are directly linked with player i in a multilateral agreement in network $\mathcal{L}$.

Value Function
The value of a hypergraph is represented by a real valued function $v : \mathcal{H} \to \mathbb{R}$, which specifies for each hypergraph $\mathcal{L} \in \mathcal{H}$ the total value $v(\mathcal{L})$ generated by $\mathcal{L}$. In most applications it will be the aggregate of individual payoffs or productions of a hypergraph, with $v(\emptyset) = 0$. The set of all possible value functions is denoted by $\mathcal{V}$. In chapter 2 it can be understood as the aggregate of all countries' payoffs in a trading system $\mathcal{L}$.[39]

Allocation Rule
An allocation rule is a function $Y : \mathcal{H} \times \mathcal{V} \to \mathbb{R}^n$ that describes how the value is distributed among the players and assigns a payoff $Y_i(\mathcal{L}, v)$ to each player $i \in N$ in the network $\mathcal{L} \in \mathcal{H}$ for the value function v. An allocation rule for example assigns to each firm in a collaboration network its total profit. In an international trade network an allocation rule represents a country's welfare in a network of international trade agreements. In our model of chapter 2 this was represented by equation (5).

[39] Of course, in chapter 2 and in the following chapters 4 and 5 the value of an empty network can be positive. For simplification, we will maintain the normalization $v(\emptyset) = 0$ for the rest of the chapter.

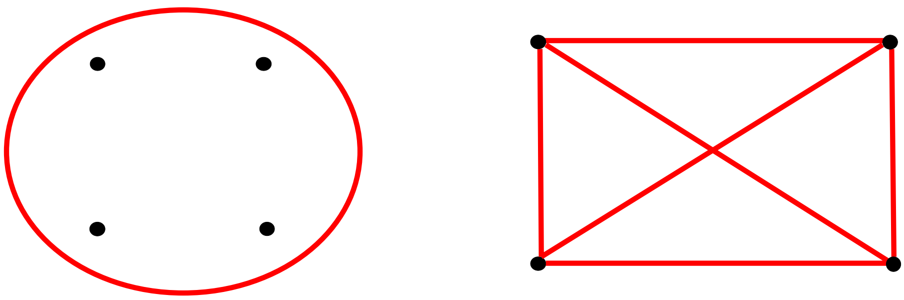


Figure 10: Complete graph and the global hypergraph for $n = 4$

When v is fixed, we will write $Y_i(\mathcal{L})$.[40]

Stability

We introduce the following notations:

- For $L \notin \mathcal{L}$, $\mathcal{L} \cup \{L\}$ is the network we obtain from $\mathcal{L}$ when we form the link L.
- For $L \in \mathcal{L}$, $\mathcal{L} \backslash \{L\}$ is the network we obtain from $\mathcal{L}$ when we sever the link L, if $L \in \mathcal{L}$.

Furthermore for any set of links $\mathcal{L}' \subseteq \mathcal{L}$ we define

- $\mathcal{L} \backslash \mathcal{L}'$ is the network we obtain from $\mathcal{L}$ by severing all links in $\mathcal{L}'$.

The formation of a link requires the consent of all players involved, but severance can be done unilaterally.
With $\tilde{\mathcal{L}} = \mathcal{L} \cup \{L\}$ or $\tilde{\mathcal{L}} = \mathcal{L} \backslash \{L\}$ the networks $\tilde{\mathcal{L}}$ and $\mathcal{L}$ are called *adjacent*.
We introduce the following stability concept:

Definition 3.2. *A hypergraph $\mathcal{L} \in \mathcal{H}$ on N with $\mathcal{L} = \{L_1, ..., L_m\}$ is called multila-*

[40]In the connection model of section 2.3 we will define a player's payoff as a function that only depends on the network structure and not on the value function as the value of a network is fixed and defined as the aggregate payoff over all players.

terally stable with respect to Y and v, if

$$(i)\;\; Y_i(\mathcal{L}, v) \geq Y_i(\mathcal{L}\backslash\{L\}, v)\; \forall L \in \mathcal{L}, \quad \forall i \in L \; and$$
$$(ii)\; Y_i(\mathcal{L} \cup \{L\}, v) > Y_i(\mathcal{L}, v) \Rightarrow \exists j \in L,$$
$$such\ that\ Y_j(\mathcal{L} \cup \{L\}, v) < Y_j(\mathcal{L}, v) \quad \forall L \notin \mathcal{L}$$

The above definition describes a situation in which no country has an incentive to sever any of its existing links and no subset of countries wants to form an additional agreement.

Since in the above definition the formation of a new multilateral link needs the consent of all players included in the link, this definition differs from the noncooperative Nash equilibrium concept.

A network $\mathcal{L}$ that is not multilaterally stable is said to be *defeated* by either network $\tilde{\mathcal{L}} = \mathcal{L} \cup \{L\}$ if condition (ii) is violated for $L \notin \mathcal{L}$, or it is defeated by network $\tilde{\mathcal{L}} = \mathcal{L}\backslash\{L\}$ if condition (i) is violated for $L \in \mathcal{L}$.

Efficiency

In order to study efficient hypergraphs, we consider the aggregate payoff of all players.

Definition 3.3. *A hypergraph $\mathcal{L}^* \in \mathcal{H}$ on N is said to be strongly efficient relative to v, if $v(\mathcal{L}) = \sum_{i \in N} Y_i(\mathcal{L}, v) \leq v(\mathcal{L}^*) = \sum_{i \in N} Y_i(\mathcal{L}^*, v),\ \forall \mathcal{L} \in \mathcal{H}$.*

The term strong efficiency indicates maximal total value and not Pareto efficiency but one can easily verify that the set of efficient networks is a subset of the Pareto efficient structures. In this case, v is fixed and defined as the total aggregate payoff. In the following we will simply refer to the set of strongly efficient networks as the set of efficient networks.

In the context of chapter 2 one can interpret an efficient network as a trading system that maximizes the aggregate payoff over all countries and can be understood as a trading system that maximizes world welfare.

3.2.3 The Connections Model for Multilateral Stability

We consider a modification of the symmetric connection model introduced by Jackson and Wolinsky (1996) in which linking costs incur for the formation of a new link. Linking costs can be interpreted as negotiation costs that occur when installing the link. This applies in the context of international trade when costs for multilateral trade negotiations occur and the costs of negotiations increase with the number of countries involved in the multilateral trading agreement. In bilateral link formation models linking costs are paid for each single link that a player forms. Introducing multilateral agreements we can allow costs to be shared between players involved in this agreement. Costs may differ in either to form a multilateral or a bilateral agreement. One argument could be that in some applications like international trade and liberalization of markets bilateral link costs from i to j are more expensive than costs for each player in a multilateral agreement. Since in a multilateral agreement costs are negotiated and shared between the number of players involved in this agreement, we introduce different cost functions for multilateral links.

In the linear costs model it is assumed that each player bears the same costs for each member of a multilateral link and his costs increase linear with the number of players involved in the link.

Linear costs

With a linear cost function $C_i(\mathcal{L}) = \sum_{L \in \mathcal{L}_i(\mathcal{L})}(|L| - 1) \cdot c$, where $c > 0$, we obtain the payoff function:

$$Y_i(\mathcal{L}) = \sum_{j \neq i} \delta^{t_{ij}(\mathcal{L})} - \sum_{L \in \mathcal{L}_i(\mathcal{L})} (|L| - 1) \cdot c, \tag{6}$$

where $\mathcal{L}_i(\mathcal{L}) = \{L \in \mathcal{L} | i \in L\}$ denote the family of groups player i belongs to. $\delta \in (0, 1)$, $t_{i,j}(\mathcal{L})$ is the number of links in the shortest path between i and j in $\mathcal{L}$. In Figure 11 the payoff for player i in a given network structure is calculated.

If $L_1 \subset L_2$ for some L_1, $L_2 \in \mathcal{L}$, we call L_1 a sublink of L_2.
First we obtain a lemma to narrow the set of possible stable networks.

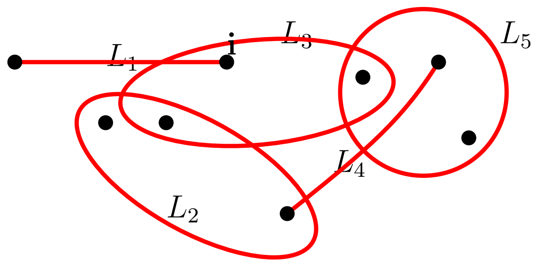


Figure 11: Player i's payoff is given by:

$$Y_i(\mathcal{L}) = 3 \cdot \delta + 4 \cdot \delta^2 - c - 2 \cdot c.$$

Lemma 3.1. *A stable network $\mathcal{L}$ cannot contain any two links L_1, $L_2 \in \mathcal{L}$ such that L_1 is a sublink of L_2.*

Proof. Assume that L_1, $L_2 \in \mathcal{L}$ such that $L_1 \subset L_2$. We set $\mathcal{L}' = \mathcal{L} \backslash \{L_1\}$. We have that for any $i \in L_1$: $Y_i(\mathcal{L}') > Y_i(\mathcal{L})$ such that player i will sever link L_1. □

The emerging stable graph structures are:

Proposition 3.1. (i) *If $\delta - c > \delta^2 > 0$, each network $\mathcal{L}$ is stable where each player is precisely once directly linked with each other player such that $\mathcal{L}$ does not contain subsets L_1, L_2 such that L_1 is a sublink of L_2.*

(ii) *For $c > \delta > 0$ the empty network is stable.*

(iii) *If $0 < c < \delta$ but $0 < \delta(1 - \delta) < c$ a star network is stable. But these are not necessarily the only stable networks.*[41]

(iv) *$(\delta - c) > 0$ is sufficient for stability of the global trading system.*

Our results correspond with the results in Proposition 2 in Jackson and Wolinsky (1996) only when linking costs are very high. In the other cases we obtain a larger range of possible stable structures. In Jackson and Wolinsky (1996), Proposition 2(i),

[41] Jun and Kim (2008) consider a modification of the symmetric connection model in which additional fixed costs for installing a multilateral link occur. The variation of the cost function in the connection model implies that only direct multilateral link is beneficial and a star network cannot be stable.

the unique stable network is the complete network whenever $\delta - c > \delta^2 > 0$. But in our framework this is not the unique stable network. The global network can also be stable since each player's payoff is given by $Y_i(\mathcal{L}^G) = (n-1) \cdot \delta - (n-1) \cdot c = Y_i(\mathcal{L}^N)$. Each player is directly linked with every other player and no player has an incentive to sever the global link. Moreover, the global network in which each player receives the same payoff as in the complete network is stable as long as $\delta - c > 0$. The reason is that a player is forced to sever the global connection to each other player and all players will end up in the empty network. The star network cannot be reached.

The next example demonstrates that for very low linking costs stable networks exist in which a player is more than just once directly connected to another player.

Example 3.2. *Consider* $n = 5$, $\mathcal{L} = \{\{1, 2, 3, 4\}, \{3, 4, 5\}, \{1, 5\}, \{2, 5\}\}$ *and parameter values* $\delta - 2 \cdot c - \delta^2 > 0$. *Each player is directly linked to each player and* $\{1, 2, 3, 4\} \cap \{3, 4, 5\} = \{3, 4\}$. *This hypergraph is stable since no player has an incentive to add an additional link and player* 3 *and* 4 *will not have an incentive to sever one of their existing links as* $Y_4(\mathcal{L}) = Y_3(\mathcal{L}) = 4 \cdot \delta - 5 \cdot c > 3 \cdot \delta + \delta^2 - 3 \cdot c = Y_4(\mathcal{L} \backslash \{\{3, 4, 5\}\}) = Y_3(\mathcal{L} \backslash \{\{3, 4, 5\}\})$ *and* $Y_4(\mathcal{L}) = Y_3(\mathcal{L}) > 2 \cdot \delta + 2 \cdot \delta^2 - 2 \cdot c = Y_4(\mathcal{L} \backslash \{\{1, 2, 3, 4\}\}) = Y_3(\mathcal{L} \backslash \{\{1, 2, 3, 4\}\})$.

To characterize the efficient networks we start with the following lemma.

Lemma 3.2. *An efficient network* $\mathcal{L}$ *cannot contain any two links* L_1, $L_2 \in \mathcal{L}$ *such that* L_1 *is a sublink of* L_2.

Proof. Assume that L_1, $L_2 \in \mathcal{L}$ such that $L_1 \subset L_2$. We set $\mathcal{L}' = \mathcal{L} \backslash \{L_1\}$. We have that for any $i \in L_1$: $Y_i(\mathcal{L}') > Y_i(\mathcal{L})$. We further obtain that $Y_i(\mathcal{L}') = Y_i(\mathcal{L})$ for all $i \notin L_1$. Therefore we can conclude that $v(\mathcal{L}') > v(\mathcal{L})$. □

When we consider different values for c and δ we can characterize the efficient network structure:

Proposition 3.2. (i) *If* $\delta - c > \delta^2 > 0$, *the complete graph* g^N *and the global hypergraph are efficient, but they are not the only efficient networks.*

(ii) *If* $\delta^2 > \delta - c$ *and* $c < \delta + \frac{(n-2)}{2} \cdot \delta^2$ *a central sponsored star is the efficient network structure.*

(iii) *If* $\delta^2 > \delta - c$ *and* $c > \delta + \frac{(n-2)}{2} \cdot \delta^2$*, the only efficient structure will be the empty network.*

In contrast to Proposition 1 in Jackson and Wolinsky (1996) we obtain a larger variety of efficient network structures in Proposition 3.2. (i). The complete network is still efficient in our framework but we can also obtain a global network as an efficient structure.
When we analyze the tension between efficiency and stability we can observe that the set of stable networks is only efficient when linking costs are very high. When costs are very low with $0 < \delta^2 < \delta - c$ the complete network is efficient as well as stable whereas for $\delta < c$ and $c < \delta + \frac{n-2}{2} \cdot \delta > 0$ the star network is not stable but efficient. However, as soon as $c > \delta + \frac{n-2}{2} \cdot \delta^2 > \delta$ a star network is no longer efficient. In this cost range the only efficient and stable network is the empty network.
Furthermore, whenever $\delta - c > 0$ the global network is stable but not efficient for values $\delta^2 > \delta - c > 0$. In this costs range the star network is the unique efficient network.
In Jackson and Wolinsky (1996), the set of stable and efficient networks coincides when linking costs are low, whereas we obtain in contrast to them that for low linking costs, stable networks are not in general efficient. Consider for example for $n = 4$ the network $\mathcal{L} = \{\{1,2,3\},\{2,3,4\},\{1,4\}\}$. With $\delta - 2c > \delta^2 > 0$ $\mathcal{L}$ is stable but not efficient. When we choose $\mathcal{L}' = \{\{1,2,3\},\{2,4\},\{1,4\},\{3,4\}\}$ we can see that $v(\mathcal{L}') = 12 \cdot \delta - 12 \cdot c > v(\mathcal{L}) = 12 \cdot \delta - 14 \cdot c$.

Decreasing costs

In the following we shall demonstrate in what way the set of efficient networks changes, when we change the allocation rule.
We assume that the cost each player pays within a multilateral agreement is "concave" with respect to the number of players included in this agreement. This may apply in social contact networks when the costs to build up further links to players within the same community are lower than linking costs to players outside of the

community[42]. The cost function is given by:

$$C_i(\mathcal{L}) = \sum_{L \in \mathcal{L}_i(\mathcal{L})} \sqrt{|L| - 1} \cdot c, \qquad c > 0. \tag{7}$$

We shall now investigate efficient hypergraphs when the cost function is decreasing. It can easily be verified that Lemma 3.2 still holds. For a decreasing average cost function as given in (7), we obtain the following efficient structures:

Proposition 3.3. (i) *If $\delta > c$ and $\delta - (\frac{2(n-1)-n\sqrt{n-1}}{(n-1)^2-(n-1)}) \cdot c > \delta^2$, then the global hypergraph structure is the unique efficient network.*

(ii) *If $\delta - (\frac{2(n-1)-n\sqrt{n-1}}{(n-1)^2-(n-1)}) \cdot c < \delta^2$ and $c < \delta + \frac{(n-2)}{2} \cdot \delta^2$ a central sponsored star is the unique efficient network structure.*

(iii) *If $c > \delta + \frac{(n-2)}{2} \cdot \delta^2$, the unique efficient structure is the empty network.*

A comparison with Jackson and Wolinsky (1996) shows that the complete graph is no longer efficient. For relatively low linking costs we obtain that only the global network can be efficient since the average linking costs to each player in a link decrease with the number of players involved in that link. When direct links are beneficial it is always more efficient when all players are connected by means of a global link than by means of bilateral links between each pair of players.

Another comparison with Jackson and Wolinsky (1996) shows that when we only allow for bilateral links such that $| L |= 2 \; \forall L \in \mathcal{L}$, the cost function of equation (7) becomes linear with $C_i(\mathcal{L}) = \sum_{L \in \mathcal{L}_i(\mathcal{L})} \sqrt{|L| - 1} c = \sum_{L \in \mathcal{L}_i(\mathcal{L})} c$. This corresponds to the symmetric connection model of Jackson and Wolinsky (1996).

3.2.4 The Compatibility between Efficiency and Stability

We noticed in section 2.3 as we calculated multilaterally stable networks that efficiency and stability of networks are not always compatible. Furthermore, we showed

[42] Here a community is represented by a multilateral link between a group of players.

by means of the connection model that for certain parameter ranges stable networks are inefficient. The next section deals with the question whether it is always possible to find a multilaterally stable network that is efficient when we are free to define the allocation rule.

We first define further characteristics of value functions and allocation rules and introduce a permutation as a bijective function $\pi : N \rightarrow N$ of the player set. For each $\mathcal{L} \in \mathcal{H}$ and $L \in \mathcal{L}$ we can define a network $\mathcal{L}^\pi$ such that $i \in L \longleftrightarrow \pi(i) \in L^\pi$ such that $\mathcal{L}^\pi$ has the same structure as network $\mathcal{L}$, with all that has changed is the label of the players. We can now define:

Definition 3.4. *An allocation rule $Y : \mathcal{H} \times \mathcal{V} \rightarrow \mathbb{R}^n$ is anonymous if, for any permutation π, $Y_{\pi(i)}(\mathcal{L}^\pi, v^\pi) = Y_i(\mathcal{L}, v)$.*

Thus with anonymity of an allocation rule the payoff allocated to an individual does not depend on the label of an individual but on the network structure $\mathcal{L}$ and the corresponding value function v. The allocation only changes according to the relabelling.

Definition 3.5. *The value function v is called anonymous if $v(\mathcal{L}) = v(\mathcal{L}^\pi)$ for all permutations π and networks $\mathcal{L}$.*

An anonymous value function allocates the same value to networks that have the same architecture independent of the labels.

Definition 3.6. *An allocation rule $Y : \mathcal{H} \times \mathcal{V} \rightarrow \mathbb{R}^n$ is efficient if $\sum_i Y_i(\mathcal{L}, v) = v(\mathcal{L})$ $\forall$ v and $\mathcal{L}$.*

Under efficiency no value is wasted and the total value generated from the network should be allocated among the players.

Definition 3.7. *A value function $v : \mathcal{H} \rightarrow \mathbb{R}$ is component additive if $v(\mathcal{L}) = \sum_{\mathcal{C} \in C(\mathcal{L})} v(\mathcal{C})$.*

For component additive value functions the value of a network is simply the sum of the value of its components such that the value of one component does not depend on the value of the other components. This condition on value functions rules out

externalities across components and is satisfied in the connection model as well as in many other economic situations.
The next definition states that when a value function is component additive, the value that is generated by a component will be allocated among the members of the component.

Definition 3.8. *An allocation rule $Y : \mathcal{H} \times \mathcal{V} \to \mathbb{R}^n$ is called component efficient if for any component additive v, $\mathcal{L}$ and all components $\mathcal{C} \in C(\mathcal{L})$*

$$\sum_{i \in N(\mathcal{C})} Y_i(\mathcal{L}, v) = v(\mathcal{C}), \tag{8}$$

where $N(\mathcal{C})$ denotes the set of players in component $\mathcal{C}$.

Thus component efficiency applies when the members of a component have no incentive to allocate value to members outside of the component whenever there are no externalities across components such that the value function is component additive. It is important to note that component efficiency is merely required from value functions that are component additive. Otherwise component efficiency of an allocation rule when the value function does not satisfy component additivity would directly violate efficiency of the allocation rule.
The following result can be shown by means of an extension of the proof in Jackson and Wolinsky (1996).

Theorem 3.1. *There does not exist an anonymous and component efficient allocation rule Y such that for every v there exists an efficient network that is multilaterally stable.*

As in Jackson and Wolinsky (1996) the theorem does not state that there does not exist an allocation rule that satisfies component efficiency and anonymity for which a multilaterally stable network always exists. In section 2.5 we will see that for certain allocation rules that satisfy component additivity and anonymity there always exists a multilaterally stable network.
Theorem 3.1 states that one cannot always design an allocation which is anonymous and component efficient such that at least one efficient network is multilaterally stable.

The next two allocation rules are of particular interest: the *egalitarian* allocation rule and the *component-wise egalitarian* allocation rule.
The *egalitarian* allocation rule Y^e is defined by:

$$Y_i^e(\mathcal{L}, v) = \frac{v(\mathcal{L})}{n}, \tag{9}$$

for all i and $\mathcal{L}$. The egalitarian allocation rule splits the value of the network equally among the players regardless of which role they play in the network. It is obvious that this allocation rule satisfies anonymity and efficiency but not component efficiency, as all player always obtain an equal share of the total value of the network.
The *component-wise egalitarian* allocation rule for a component additive v satisfies component efficiency and is given by:

$$Y_i^{ce}(\mathcal{L}, v) = \begin{cases} \frac{v(\mathcal{C})}{|N(\mathcal{C})|} & \text{if there exists a } \mathcal{C} \in C(\mathcal{L}) \text{ such that } i \in N(\mathcal{C}), \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

This allocation rule allocates the total value generated by a component to the member of the component in the way that each member of a component receives the same payoff. As component additivity implies that disconnected players generate no value we have that $Y_i^{ce}(\mathcal{L}, v) = 0$ if there exists no component $\mathcal{C} \in C(\mathcal{L})$ such that $i \in N(\mathcal{C})$.
We start the analysis by introducing certain characteristics for links of a given network $\mathcal{L}$. We introduce the term of a critical link. A link $L \in \mathcal{L}$ is called critical if it is contained in component $\mathcal{C}$ of network $\mathcal{L}$ and its deletion splits $\mathcal{C}$ into components $\mathcal{C}_1$, $\mathcal{C}_2$, ...,$\mathcal{C}_k$.

Definition 3.9. *A link L is called critical to network $\mathcal{L}$ if $\mathcal{L}\backslash\{L\}$ has more components than $\mathcal{L}$ or if at least one player in L is only included in link L.*

The last condition states that one of the players in L will become disconnected when L is severed.

Example 3.3. *Let $n = 6$ and $\mathcal{L} = \{\{1, 2, 3, 4, 5, 6\}, \{1, 2\}, \{3, 4\}, \{5, 6\}\}$ as in Figure 12. Now link $L = \{1, 2, 3, 4, 5, 6\}$ is critical to $\mathcal{L}$ and its deletion splits the network into three components $\mathcal{C}_1 = \{\{1, 2\}\}$, $\mathcal{C}_2 = \{\{3, 4\}\}$ and $\mathcal{C}_3 = \{\{5, 6\}\}$.*

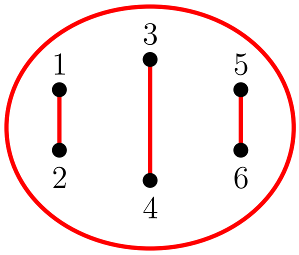


Figure 12. Network with a critical link $L = \{1, 2, 3, 4, 5, 6\}$.

Consider the next property on networks and value functions that is needed for the next result.

Definition 3.10. *The pair $(\mathcal{L}, v)$ satisfies critical link monotonicity if for any critical link in $\mathcal{L}$ and its associated components $\mathcal{C}$, $\mathcal{C}_1$, $\mathcal{C}_2$, ...,$\mathcal{C}_k$ we have that $v(\mathcal{C}) \geq v(\mathcal{C}_1) + v(\mathcal{C}_2) + ... + v(\mathcal{C}_k)$ implies that $\frac{v(\mathcal{C})}{|N(\mathcal{C})|} \geq max[\frac{v(\mathcal{C}_1)}{|N(\mathcal{C}_1)|}, \frac{v(\mathcal{C}_2)}{|N(\mathcal{C}_2)|}, ..., \frac{v(\mathcal{C}_k)}{|N(\mathcal{C}_k)|}]$.*

Note that in bilateral graphs the deletion of a bilateral link can split a component into at most two components. As Figure 12 demonstrates the severance of a critical link in a hypergraph can split a component into more than just two components.

Lemma 3.3. *If $\mathcal{L}$ is efficient relative to a component additive v, then $\mathcal{L}$ is multilaterally stable for Y^{ce} relative to v if and only if $(\mathcal{L}, v)$ satisfies critical link monotonicity.*

For a given allocation rule Lemma 3.3 describes for which class of value functions the set of multilaterally stable networks and efficient networks coincide.

We shall now consider the characteristics of the egalitarian allocation rule.

Definition 3.11. *The allocation rule Y is independent of potential links if $Y(\mathcal{L}, v) = Y(\mathcal{L}, w)$ for all networks $\mathcal{L}$ and value functions v and w such that there exists a link L such that v and w agree on every graph except $\mathcal{L} \cup \{L\}$.*

Definition 3.12. *An allocation rule is pairwise monotonic if $\mathcal{L}'$ defeats $\mathcal{L}$ implies that the value $v(\mathcal{L}') > v(\mathcal{L})$.*

Pairwise monotonicity implies that each efficient network is multilaterally stable since an efficient network which maximizes total value cannot be defeated under pairwise monotonicity. This property of allocation rules together with the independence of potential links uniquely characterizes the allocation rule.
The next result is a slight extension of Theorem 3 of Jackson and Wolinsky (1996).

Theorem 3.2. *If Y is anonymous, pairwise monotonic, efficient and independent of potential links, then $Y_i(\mathcal{L}, v) = \frac{v(\mathcal{L})}{n}$, $\forall i$, $\mathcal{L}$ and anonymous v.*

3.2.5 The Existence of Multilaterally Stable States

In this section we want to analyze existence results on multilaterally stable networks by means of the trading example presented in section 1.3 of chapter 3.
Jackson and Watts (2002) show that for the case of three player the only pairwise stable structure consists of a bilateral link between two players and one player in isolation whereas for $n \geq 4$ no pairwise stable network exists. Instead we obtain a closed cycle of networks emanating from an improving path of networks.
The definition of an improving path and of a closed cycle of networks as formulated by Jackson and Watts (2002) will be restated for hypergraphs in the following.

Definition 3.13. *An improving path from $\mathcal{L}$ to another network $\mathcal{L}'$ is a finite sequence of adjacent networks $\mathcal{L}^1, ..., \mathcal{L}^K$ with $\mathcal{L}^1 = \mathcal{L}$ and $\mathcal{L}^K = \mathcal{L}'$ such that for any $k \in \{1, .., K-1\}$ either:*

(i) $\mathcal{L}^{k+1} = \mathcal{L}^k \backslash \{L\}$ for some L such that $Y_i(\mathcal{L}^k \backslash \{L\}) > Y_i(\mathcal{L}^k)$ for some $i \in L$, or

(ii) $\mathcal{L}^{k+1} = \mathcal{L}^k \cup \{L\}$ for some L such that $Y_i(\mathcal{L}^k \cup \{L\}) \geq Y_i(\mathcal{L}^k)$ for all $i \in L$ and $Y_j(\mathcal{L}^k \cup \{L\}) > Y_j(\mathcal{L}^k)$ for some $j \in L$.

An improving path is a sequence of networks in which the transition from one network to the next is either caused by the severance of a single link or by the formation of a new link, where severance can be done unilaterally but the formation of a new link requires all linking partners' consent. It requires agents to be myopic since a change in one player's links can leave other players with less payoff which might induce them to change their links in the next round.

Note that a network is multilaterally stable if and only if it has no improving path emanating from it. To see this consider that when we have a multilaterally stable network it means that no player has an incentive to deviate from the structure by single links. This means that it is not possible for any group of players to improve by forming an additional link (condition (ii) of Definition 3.2.) and that no player can improve by severing an existing link (condition (i)). Any improving path will stop at a multilaterally stable network which has no improving path emanating from it. An improving path does not necessarily lead to a multilaterally stable network. There are also situations in which an improving path will result in a cycle. Furthermore Jackson and Watts (2002) formally introduce the notation of a cycle which is a path of networks in which each member of the cycle has an improving path to any other member of the cycle. We define:

Definition 3.14. *(i) A set of networks Z forms a cycle when for all $\mathcal{L}, \mathcal{L}' \in Z$ there exists an improving path that connects $\mathcal{L}$ with $\mathcal{L}'$.*
(ii) A cycle Z is called maximal if it is not a proper subset of a cycle.
(iii) A cycle Z is called closed if there is no network $\mathcal{L} \in Z$ that belongs to an improving path of $\mathcal{L}'$ whereas $\mathcal{L}' \notin Z$. A closed cycle is always maximal.

There are three possibilities for the existence of improving paths and cycles. First it is possible that there are no improving paths emanating from any network such that each network is multilaterally stable. Second, every improving path results in at least one multilaterally stable network. And third, there exists at least one improving path that does not end such that a cycle exists as the number of possible network structures is finite.
With the notion of an improving path we can easily extend the result of Jackson and Watts (2002, Lemma 1):

Lemma 3.4. *For any v and Y there exists at least one closed cycle or one multilaterally stable network.*

Proof. We start with an arbitrary network. If it does not lie on an improving path to any other network then it is multilaterally stable by definition. If it lies on an improving path then it can either stop in finite steps since there is only a finite number of networks. Then this network is multilaterally stable or the improving path will not stop. However, in this case it will either lead back to itself and form a cycle, or it lies on an improving path to a closed cycle. The maximum of a closed cycle will be the set of all networks which is finite as the set of players is finite. Hence there always exists at least one multilaterally stable network or one closed cycle. □

The trading example of Jackson and Watts (2002) demonstrates the non-existence of a pairwise stable network. The next (extended) version of the trading example shows that although we can exclude pairwise stable networks for $n \geq 4$ multilaterally stable network may exist when we allow multilateral link formation.

Example 3.4. (The Trading Example revised)

Consider the trading example with $N = \{1, 2, 3, 4\}$ *where costs of* $\frac{5}{96}$ *occur to each player for each link, regardless of whether the link contains two or more than two players. Therefore the total costs for a multilateral link with four players are* $\frac{20}{96}$ *whereas the costs of a bilateral link are* $\frac{10}{96}$*. Starting with an empty network it is possible that first the global link between all players is formed since all players are better off with* $\frac{3}{16} - \frac{5}{96} > 0$*. We can show that* $\mathcal{L} = \{N\}$ *is multilaterally stable since no subset of players has an incentive to form an additional link and no player has an incentive to sever the global link. Hence the improving path starting from the empty network stops at the global network which is multilaterally stable by definition.*

Another stable network consists of a single multilateral link between three players, e.g. $\mathcal{L} = \{\{1, 2, 3\}\}$*, since starting from an empty network all players have an incentive to form the link. This network is stable since the addition of an additional link with player* 4 *leads to a payoff of* $\frac{3}{16} - \frac{10}{96} < \frac{1}{6} - \frac{5}{96}$ *for at least one player who is in the intersection of both links such that no additional link is formed.*

On the other hand consider that starting from an empty network a pair of players (player 1 *and* 2*) forms a bilateral link with* $\frac{1}{8} - \frac{5}{96} > 0$*. Player* 3 *and* 4 *will also form a link. From network* $\{\{1, 2\}, \{3, 4\}\}$ *there are two improving paths. One leads to the formation of a link between* 2 *and* 3 *which will result in a cycle as shown by Jackson and Watts (2002), and the other improving path leads to the formation of a multilateral link between all four players since* $\frac{3}{16} - \frac{10}{96} > \frac{1}{8} - \frac{5}{96}$*. This network, in turn, lies on an improving path to the global network. We can show that each cycle* *leads to an improving path to a multilaterally stable network and no closed cycle exists.*

This example can be generalized to more than four players by considering that the payoff of a single player from the addition of a multilateral link increases as the number of players in a multilateral link increases. Therefore for $n \geq 4$ there always exists a multilaterally stable network.
This example demonstrates that for certain allocation rules multilaterally stable networks can exist but no pairwise stable networks exist.

The following example shows that it is possible to have only closed cycles and no multilaterally stable network exists.

Example 3.5. *Players in a social network benefit from direct and indirect connections. Not considering the costs from each link they obtain a payoff of* $Y_i(\mathcal{L}) = \sqrt{\sum_{L \in \tilde{\mathcal{L}}_i(\mathcal{L})} 2 \cdot (|L| - 1)}$ *such that the payoff function is concave in the number of links. With* $\tilde{L}_i(\mathcal{L})$ *we denote the links with which player* i *is either directly or indirectly connected.*
Furthermore we obtain for the costs of player i *in network* $\mathcal{L}$: $C_i(\mathcal{L}) = \sum_{L \in \mathcal{L}_i(\mathcal{L})} (|L| - 1)^2$ *such that the cost function is convex in the number of players included in a link. For* $n = 4$ *we can show that no multilaterally stable network exists: Starting with the empty network we observe that a set of three players or more never has an incentive to form a multilateral link. The additional costs of at least* 4 *will always be higher than the additional payoff received (which will be* 2 *for* 3 *players).*
A bilateral link between a pair of players i *and* j *is always beneficial as starting from an empty network* $Y_i(\{\{i,j\}\}) = Y_j(\{\{i,j\}\}) = \sqrt{2 \cdot 1} - 1 > 0$. *A network with two pairs of players is formed* ($\{\{i,j\},\{k,l\}\}$, $i \neq j \neq k \neq l$)*. Note that two players that are not connected have an incentive to form a bilateral link since the new payoff will be* $Y_i(\{\{i,j\},\{k,l\},\{i,k\}\}) = Y_k(\{\{i,j\},\{k,l\},\{i,k\}\}) = \sqrt{2 \cdot 3} - 2 > \sqrt{2 \cdot 1} - 1$ *and a line network is formed. For the two terminal nodes we have:* $Y_j(\{\{i,j\},\{k,l\},\{i,k\}\}) = Y_l(\{\{i,j\},\{k,l\},\{i,k\}\}) = \sqrt{2 \cdot 3} - 1 > \sqrt{2 \cdot 1} - 1$. *Now each player* i *has an incentive to sever its link with player* j *since* $\sqrt{2 \cdot 2} - 1 > \sqrt{2 \cdot 3} - 2$. *Now player* k *is also going to sever one of his links since*

$\sqrt{2 \cdot 2} - 2 = 0 < \sqrt{2 \cdot 1} - 1$ *and we are back in the situation with one single bilateral link. One example of a cycle is:* $\{\{1,2\},\{3,4\}\}$ *to* $\{\{1,2\},\{2,3\},\{3,4\}\}$ *to* $\{\{1,2\},\{2,3\}\}$ *to* $\{\{12\}\}$ *to* $\{\{1,2\},\{3,4\}\}$*. However this cycle is not closed, since we could also have* $\{\{1,2\},\{3,4\}\}$ *to* $\{\{1,2\},\{1,4\},\{3,4\}\}$*. Therefore there are a number of alternative improving paths from each of these networks in the cycle and a closed cycle will consist of a larger number of networks.*
Since no link with at least three players is ever going to be formed, independent of the former network structure, we have that there does not exist a multilaterally stable network.

While the above example demonstrates that multilaterally stable networks do not necessarily exist, we can proof that for certain allocation rules a multilaterally stable network always exists and that under these allocation rules cycles never occur. For the egalitarian allocation rule $Y_i^e(\mathcal{L}, v) = \frac{v(\mathcal{L})}{n}$ it is obvious that a stable network always exists:[43] The total value generated by network $\mathcal{L}$ is shared equally among all players, independent of their position in the network. To see that there always exists a stable structure consider network $\tilde{\mathcal{L}}$ which is efficient and thus maximizes the total value. Since any change in the structure will reduce the total value, the value allocated to each player is decreasing too. Hence no player has an incentive to deviate and the network that maximizes the value function is stable. Since there always exists at least one efficient network there always exists at least one multilaterally stable network.

It was mentioned earlier that the egalitarian allocation rule fails to satisfy component efficiency. Another allocation rule which allocates to each player of a component the same value generated by a component independent of his position is given by the *component-wise egalitarian* allocation rule. It can be shown that one can always find a multilaterally stable network under the component-wise egalitarian allocation rule:

$$Y_i^{ce}(\mathcal{L}, v) = \begin{cases} \frac{v(\mathcal{C})}{|N(\mathcal{C})|} & \text{if there exists a } \mathcal{C} \in C(\mathcal{L}) \text{ such that } i \in N(\mathcal{C}), \\ 0 & \text{otherwise,} \end{cases}$$

[43] See Jackson and Wolinsky (1996) for a corresponding definition on bilateral linking structures.

for each v that is component additive and any network $\mathcal{L}$. Component additivity demands that the value is only allocated to the members of the component and not to members outside.

To see that there always exists a multilaterally stable network consider a component $\mathcal{C}$ that maximizes for each player $i \in N(\mathcal{C})$ the payoff $Y_i^{ce}(\mathcal{L}, v)$ such that no player in this component is going to deviate and change the network structure. For the remaining $n- \mid N(\mathcal{C}) \mid$ players we can also find a component $\tilde{\mathcal{C}}$ that maximizes the payoff for all players $i \in \tilde{\mathcal{C}}$ and so on. The resulting network is multilaterally stable. It can easily be verified that Y^{ce} is component efficient and anonymous. It thus provides an example for an allocation rule that shows that Theorem 3.1 is not a non-existence result. For some allocation rules there always exists a multilaterally stable network.

To see why the assumption of component additivity of the component-wise allocation rule is important, note that the value of two disconnected players always has to equal zero. Otherwise we cannot use the argument above to demonstrate that there always exists a multilaterally stable network. To show existence and to rule out cycles we shall introduce another concept of the allocation rule and the value function.

Definition 3.15. *Y and v exhibit no indifference if for any two networks $\mathcal{L}$ and $\tilde{\mathcal{L}}$ which are adjacent we have that either $\mathcal{L}$ defeats $\tilde{\mathcal{L}}$ or $\tilde{\mathcal{L}}$ defeats $\mathcal{L}$.*

This definition says that when altering the network structure from $\mathcal{L}$ to $\tilde{\mathcal{L}}$ or vice versa by a single deviation, at least one of the players that altered the network structure has to be strictly better off.

Another allocation rule for which the existence proof is less obvious is the Myerson value allocation rule[44]. The next result shows that under the Myerson value allocation rule, which is defined as the Shapley value of a characteristic function game $Ch_{\mathcal{L},v}$, defined on hypergraph networks, a multilaterally stable network always exists.

[44]The Myerson value allocation rule was first suggested by Myerson (1977) as a solution concept for cooperative games where communication possibilities of players were described by an underlying graph. He characterized the Myerson value as the unique allocation rule that satisfies efficiency and fairness. In chapter 6 of the thesis we provide a corresponding characterization in the context of network games.

The Shapley value $SV(v)$ of v is defined by:

$$SV_i(v) = \sum_{S \subseteq N} \frac{(n-|S|)!(|S|-1)!}{n!}(v(S) - v(S\backslash\{i\})),$$

for all $i \in N$.

We define a characteristic function game $Ch_{\mathcal{L},v}$ with

$$Ch_{\mathcal{L},v}(S) = \sum_{\mathcal{C} \in C(S/\mathcal{L})} v(\mathcal{C}),$$

where $S/\mathcal{L}$ is the set of links in $\mathcal{L}$ that contain only the players in S and the function $Ch_{\mathcal{L},v}$ assigns to each coalition S the value the coalition can achieve in the network $\mathcal{L}$.

Theorem 3.3. *There always exists a multilaterally stable network for the Myerson value allocation rule $MV_i(\mathcal{L}, v) = SV_i(Ch_{\mathcal{L},v})$. Moreover all improving paths emanating from any network $\mathcal{L}$ lead to a multilaterally stable network. Thus there do not exist any cycles under the Myerson value allocation rule.*

Proof. First we introduce a Potential function[45] $P(\mathcal{L}) = \sum_{S \subseteq N} \frac{(n-|S|-1)!|S|!}{n!}(Ch_{\mathcal{L},v}(S))$.
With
$SV_i(Ch_{\mathcal{L},v}(S)) = \sum_{S \subseteq N} \frac{(n-|S|-1)!|S|!}{n!}(Ch_{\mathcal{L},v}(S) - Ch_{\mathcal{L},v}(S\backslash\{i\}))$ we can calculate:

$$\begin{aligned}
&SV_i(Ch_{\mathcal{L},v}(S)) - SV_i(Ch_{\mathcal{L}\backslash\{L\},v}(S)) \\
&= \sum_{S \subseteq N} \frac{(n-|S|-1)!|S|!}{n!}[(Ch_{\mathcal{L},v}(S) - Ch_{\mathcal{L},v}(S \setminus \{i\})) - (Ch_{\mathcal{L}\backslash\{L\},v}(S) - Ch_{\mathcal{L}\backslash\{L\},v}(S \setminus \{i\}))] \\
&= \sum_{S \subseteq N} \frac{(n-|S|-1)!|S|!}{n!}[(Ch_{\mathcal{L},v}(S) - (Ch_{\mathcal{L}\backslash\{L\},v}(S)] = P(\mathcal{L}) - P(\mathcal{L}\backslash\{L\}), \qquad (11)
\end{aligned}$$

for any i, $\mathcal{L} \in \mathcal{H}$ and $L \in \mathcal{L}$, since one can easily verify that for all $\mathcal{L} \in \mathcal{H}$: $Ch_{\mathcal{L}\backslash\{L\},v}(S \setminus \{i\}) = Ch_{\mathcal{L},v}(S \setminus \{i\})) \; \forall i \in L, L \in \mathcal{L}$.

Assume that network $\tilde{\mathcal{L}}$ maximizes the Potential function. We therefore know that $P(\tilde{\mathcal{L}}) - P(\tilde{\mathcal{L}}\backslash\{L\}) \geq 0$ and $P(\tilde{\mathcal{L}}) - P(\tilde{\mathcal{L}} \cup \{L\}) \geq 0 \; \forall \; L$ and all players $i \in N$. Hence no set of players is going to deviate and condition (i) and (ii) of Definition 3.2. are

[45] For the concept of a Potential game see Monderer and Shapley (1996). The allocation rule does not depend on the player i.

satisfied.
To show the second part of the theorem, we use a result which was proven in Jackson and Watts (2002, Theorem 1).

Claim 3.1. *If there exists a function* $w : \mathcal{H} \rightarrow \mathbb{R}$ *such that* $[\mathcal{L}$ *defeats* $\mathcal{L}'] \Leftrightarrow [w(\mathcal{L}) > w(\mathcal{L}')$ *and* $\mathcal{L}$ *and* $\mathcal{L}'$ *are adjacent*$]$, *then there are no improvement cycles.*

Such a function is given by equation (11). It implies that along an improving path P must be increasing. This increasing path must lead to a local maximizer which is multilaterally stable by definition. Of course, there may be more than just one local maximizer such that more than one multilaterally stable network may exist. Therefore the Myerson value has no cycles and all improving paths that emanate from any network $\mathcal{L}$ have to lead to a multilaterally stable network. □

The Myerson value belongs to a class of allocation rules that satisfy condition (11). According to Chakrabarti and Gilles (2007, Definition 3.1 *b*)) we can define:

Definition 3.16. *Let* $Y : \mathcal{H} \times \mathcal{V} \rightarrow \mathbb{R}^n$ *be an allocation rule. An allocation rule* Y *admits an exact network potential if there exists a function* $P : \mathcal{H} \rightarrow \mathbb{R}$ *such that for every network* $\mathcal{L} \in \mathcal{H}$, *every player* $i \in N$, *and every link* $L \in \mathcal{L}_i(\mathcal{L})$:

$$Y_i(\mathcal{L}, v) - Y_i(\mathcal{L}\backslash\{L\}, v) = P(\mathcal{L}) - P(\mathcal{L}\backslash\{L\}).$$

We already mentioned that with equation (11) the Myerson value admits an exact network potential. Can we guarantee for each member of this class of allocation rules that there always exists at least one multilaterally stable network and no cycle? We can show an even stronger result.

Definition 3.17. *Let* $Y : \mathcal{H} \times \mathcal{V} \rightarrow \mathbb{R}^n$ *be an allocation rule. An allocation rule* Y *admits an ordinal network potential if there exists a function* $P : \mathcal{H} \rightarrow \mathbb{R}$ *such that for every network* $\mathcal{L} \in \mathcal{H}$, *every player* $i \in N$, *and every link* $L \in \mathcal{L}_i(\mathcal{L})$:

$$P(\mathcal{L}) > P(\mathcal{L}\backslash\{L\}) \Leftrightarrow Y_i(\mathcal{L}, v) > Y_i(\mathcal{L}\backslash\{L\}, v);$$
$$P(\mathcal{L}) < P(\mathcal{L}\backslash\{L\}) \Leftrightarrow Y_i(\mathcal{L}, v) < Y_i(\mathcal{L}\backslash\{L\}, v);$$
$$P(\mathcal{L}) = P(\mathcal{L}\backslash\{L\}) \Leftrightarrow Y_i(\mathcal{L}, v) = Y_i(\mathcal{L}\backslash\{L\}, v).$$

This definition makes clear that the class of allocation rules that admits an exact network potential is a subset of the class of allocation rules that admit an ordinal network potential. We can now show the next theorem.

Theorem 3.4. *If the allocation rule Y admits an ordinal network potential, then the following properties hold:*

a) There exists at least one multilaterally stable network.

b) There are no cycles.

Proof. First we show that when the allocation rule exhibits an ordinal potential there exist no cycles.
With Claim 3.1. we have to find such a function w. With the definition of an ordinal network potential this function is given by P. Therefore if Y exhibits an ordinal network potential, then there are no cycles.
Theorem 3.4.a) follows from Theorem 3.4.b) and Lemma 3.4. which states that then there has to exist at least one multilaterally stable network. □

As shown above, the Myerson value exhibits an exact network potential. Furthermore, Chakrabarti and Gilles (2007) show that in the symmetric connection model of Jackson and Wolinsky (1996) the function $P(\mathcal{L}) = \sum_{i \in N} Y_i(\mathcal{L}, v)$ is an ordinal network potential for values $c < \delta - \delta^2$ as in this cost range the formation of an additional bilateral link between two players increases both players' payoffs when linking costs are very low. Furthermore as the formation of a bilateral link between two players may even decrease distances of third players to other players, it makes none of the other players worse off. In the following we shall analyze the allocation rule when we allow additional multilateral link formation.

Example 3.6. *Consider the symmetric connection model of section* 2.3 *with* $n = 5$*, values* $0 < c < \delta - \delta^2$ *and network* $\tilde{\mathcal{L}} = \{\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,4\},\{2,4\}\}$*. An additional global link among all players increases player* 5*'s payoff since* $Y_5(\tilde{\mathcal{L}}, v) < Y_5(\tilde{\mathcal{L}} \cup \mathcal{L}^G, v) = 4 \cdot \delta - 4 \cdot c$*. But with* $P(\mathcal{L}) = \sum_{i \in N} Y_i(\mathcal{L}, v)$ *for all* $\mathcal{L} \in \mathcal{H}$ *we can have* $P(\tilde{\mathcal{L}}) = 12 \cdot \delta - 12 \cdot c > P(\tilde{\mathcal{L}} \cup \mathcal{L}^G) = 20 \cdot \delta - 32 \cdot c$*. When this is the case, we also have that* $\delta < 4 \cdot c$ *such that the payoff for all the other players* i*,* $i \neq 5$*, decreases from an additional global link. From this we can conclude that* $P(\mathcal{L}) = \sum_{i \in N} Y_i(\mathcal{L}, v)$ *is not an ordinal potential function for* Y*.*

3.2.6 Discussion of the Stability Notion

What one has to mention first is that the stability notion as defined above is a very weak concept in the sense that it only considers deviations of single links. We can interpret multilateral stability as a necessary but not sufficient condition. Still, it narrows the set of possible stable structures that one can obtain and demonstrates that a conflict between the set of efficient and the set of stable networks occurs even under this relatively weak stability concept. As we could see in the connection model this notion often leads to many stable networks. One might think of a stability notion in which a player can sever more than just one link at a time. A corresponding equilibrium concept is suggested by Chakrabarti and Gilles (2007) for bilateral linking networks. They formulate the idea that in a stable network no player will sever any subset of his existing bilateral link, and denote this condition as strong link deletion proofness. We can elaborate an equivalent definition for hypergraphs in which we allow single players to sever more than just one link at a time and allow link formation by groups of players.

Definition 3.18. *A network $\mathcal{L} \in \mathcal{H}$ on N with $\mathcal{L} = \{L_1, ..., L_m\}$ is called strongly multilaterally stable, if*

$$(i)\;\; Y_i(\mathcal{L}, v) \geq Y_i(\mathcal{L} \backslash \mathcal{L}', v)\; \forall i \in N,\; \textit{for every } \mathcal{L}' \subseteq \mathcal{L}_i(\mathcal{L}) \textit{ and}$$

$$(ii)\;\; Y_i(\mathcal{L} \cup \{L\}, v) > Y_i(\mathcal{L}, v) \Rightarrow \exists j \in L,$$

$$\textit{such that } Y_j(\mathcal{L} \cup \{L\}, v) < Y_j(\mathcal{L}, v) \quad \forall L \notin \mathcal{L}$$

In the above definition condition (i) requires that no players will improve by severing any subset of his links. One can see that Definition 3.18 is a direct strengthening of the multilateral stability concept of Definition 3.2 which allows to reconsider the results in Proposition 3.1. As the multilateral stability concept only allows single deviation, fewer deviations are allowed than in the strong multilateral stability notion such that a strongly multilaterally stable network implies a multilaterally stable network.

Usually, it would be expected that the set of possible stable networks shrinks under a stronger stability concept. The following result shows that, when players' payoffs are given by the symmetric connection model with linear costs, stability even implies strong stability.

Proposition 3.4. *In the symmetric connection model of section* 2.3 *multilateral stability implies strong multilateral stability.*

In general this result cannot be extended to all payoff functions. In particular, when we allow non-linear cost functions as in equation (7) the reversal may no longer hold.
Note that the notion of stability allows only one deviation with respect to the formation of new links at a time. Another strengthening of the stability notion would allow simultaneous formation and severance of more than just one link such as the severance of one link by a player with a simultaneous formation of some new links by groups of players.

3.2.7 Discussion of the Efficiency Notion

The strong efficiency concept we introduced in section 2.2 of this chaper is a very strong concepts in the way that the set of efficient network structures is relatively small as compared to the set of stable network structures.
Another notion would be if we think of Pareto-efficient networks in which one cannot alter the network structure such that at least one player is better off without making another player worse off. Obviously this notion is a very weak concept and we will compare the results for the connection model with stability.

Definition 3.19. *A network $\mathcal{L}$ is Pareto-efficient relative to v and Y if there does not exist another network $\mathcal{L}' \in \mathcal{H}$ such that $Y_i(\mathcal{L}', v) \geq Y_i(\mathcal{L}, v)$ for all i with strict inequality for at least one i.*

It is obvious that Pareto efficiency is a relatively weak notion compared to strong efficiency since the strong efficiency notion allows redistribution of the players' payoffs whereas under Pareto efficiency we consider a specific allocation rule. It is clear that a strongly efficient network is therefore also Pareto efficient, since even when payoffs can be redistributed no player can improve by altering the structure without reducing the payoff of any other player. And this is exactly the definition of Pareto efficiency given a certain allocation rule. In this notion we fix the allocation rule whereas in the strong efficiency notion payoffs can arbitrarily be allocated and transferred among the players. The next notion of efficiency lies in-between the other two notions.

Definition 3.20. *A network $\mathcal{L}$ is called constrained efficient relative to v if there does not exist any network $\mathcal{L}'$ and a component efficient and anonymous allocation rule Y such that $Y_i(\mathcal{L}', v) \geq Y_i(\mathcal{L}, v)$ for all i with strict inequality for at least one i.*

This notion is stronger than Pareto efficiency because here we demand that for all allocation rules that are anonymous and component efficient we cannot improve a single player without reducing the payoff of any other player. In Pareto efficiency we only demand this condition for a specific allocation rule and whereas constrained efficiency considers a class of allocation rules.

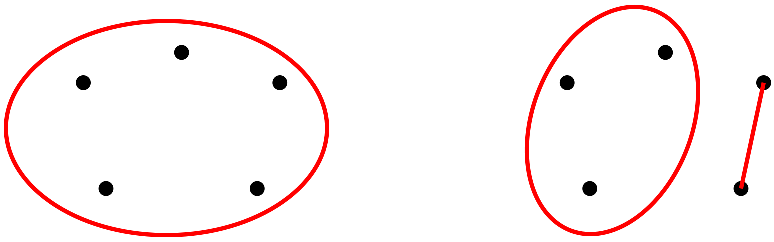
We can easily verify that there always exists a Pareto efficient network and a constrained efficient network since there always exists a strongly efficient network and every strongly efficient network is Pareto efficient as well as constrained efficient.

To understand the relationship between the three stability notions we will give examples in the following. First we demonstrate the difference between the set of constrained efficient networks and strong efficient networks.

Example 3.7. *Consider a network with $n = 5$ and any component additive and anonymous value function v where the network that consists of a global link between all players has a value of 10, a bilateral link between any two players has a value of 2 and a hyperlink between three players has a value of 9. All the other networks have a value of 0. We can easily verify that a network that consists of two connected components where one is a bilateral link and one is a hyperlink between three players is the unique efficient structure with a total value of 11. The global link between all players cannot be strongly efficient whereas we can show that it is constrained efficient.*

Consider any anonymous and component efficient allocation rule that allocates in a global network the payoff 2 to each player. In the network with two connected components each player in the bilateral link would receive a payoff of 1 from each anonymous and component efficient allocation rule such that they would be worse off. Therefore the global network is constrained efficient.

To see that the network that consists of two connected components is constrained



Constrained Efficient but not Strongly Efficient.

Constrained Efficient and Strongly Efficient.

Figure 13: Constrained Efficiency versus Strong Efficiency.

efficient consider that from any anonymous and component efficient allocation rule each player in the hyperlink between three players will receive a payoff of 3 *whereas in the global network they obtain a payoff of* 2*. Therefore they will be worse off under the global link and therefore the network that consists of two components is also constrained efficient. This is summarized in Figure* 13*.*

The next example clarifies the relationship between constrained efficient networks and Pareto efficient networks.

Example 3.8. *Consider $n = 3$ and an anonymous allocation rule where a global link between three players has value* 9 *and a network that consists of two bilateral links has value* 8*. All the other networks have value* 0*. The allocation rule is component efficient and anonymous. It allocates the payoff of* 3 *to each player in the global link and in each network with two bilateral links it allocates the payoff of* 4 *to the player with two links and the payoff of* 2 *to both end players.*
Since the total value is maximal in the global network, it is Pareto efficient and constrained efficient. We can also verify that a network that consists of two bilateral links is Pareto efficient since any other network will result in a lower payoff for at least one of the players. But we can also verify that it is not constrained efficient since we can find another anonymous and component efficient allocation rule, such that all players will be better off under the global network. This can be seen when we allocate the payoff of $\frac{8}{3}$ to each player in the network and therefore all players will

Constrained Efficient and Pareto Efficient.

Pareto Efficient but not Constrained Efficient.

Figure 14: Constrained Efficiency versus Pareto Efficiency.

improve with the global network. This is shown in Figure 14.

In the symmetric connection model of section 2.3 we observed in Proposition 2.1 and 2.2 that the set of efficient networks and stable networks do not always coincide for certain parameter values. When linking costs are very high, $c > \delta + \frac{(n-2)}{2} \cdot \delta^2$, we obtain that the only stable and efficient network is the empty network. For very low costs each efficient network can be stable. One problematic case occurs when linking cost are at an intermediate level. Here we can observe that there are stable structures that are not efficient. For $\delta < c < \delta + \frac{n-2}{2} \cdot \delta^2$ a star network is efficient but not stable. In this cost range an empty network is stable. In the following we shall investigate whether the star network can be efficient when we relax the efficiency condition and determine the Pareto efficient networks. Furthermore we observed that for low costs of links there are stable networks that are not strongly efficient. The next result characterizes the set of Pareto efficient networks in the symmetric connection model.

Proposition 3.5. *In the symmetric connection model we find that*

(i) *for very high linking costs the empty network is Pareto efficient.*

(ii) *Whenever* $\delta < c < \delta + \frac{n-2}{2} \cdot \delta^2$ *the empty network is stable but not necessarily Pareto efficient.*

(iii) *If* $\delta - c > \delta^2 > 0$ *the set of strongly efficient and Pareto efficient networks coincides.*

This result shows that for middle costs to link some stable networks are not even Pareto efficient. Consider the network $\mathcal{L} = \{\{1,2,3\},\{2,3,4\},\{1,4\}\}$ of section 2.3 with $n = 4$. With $K = 2$ and $\delta - 2c > \delta^2 > 0$, $\mathcal{L}$ is stable but not efficient. To see that $\mathcal{L}$ is not even Pareto efficient player 2 and player 3 can improve without reducing player 1's and player 4's payoffs with $\mathcal{L}' = \{\{1,2,3\},\{2,4\},\{1,4\},\{3,4\}\}$.

Whenever $\delta < c < \delta + \frac{n-2}{2} \cdot \delta^2$ we find that stable network are not necessarily Pareto efficient. In the appendix we provide an example that shows that for certain parameter values a line network Pareto dominates the empty network.

3.2.8 Conclusion

We have developed a theoretic framework that allows us to study which multilateral link structures will emerge in equilibrium. We have introduced the notion of hypergraphs to describe linking structures that allow multilateral links among agents. In order to predict the hypergraph structures that are going to emerge at equilibrium a new solution concept has been proposed: multilateral stability.
The idea of multilateral stability is that adding a new multilateral link needs the consent of all players involved in the multilateral link whereas deletion can be done unilaterally. We have shown that the possibility for players to form multilateral links leads to different results in the connection model of Jackson and Wolinsky (1996). One major result was that it yields a larger number of efficient network structures in the case of linear costs whereas for a decreasing cost function a complete network can no longer be efficient. Moreover, we showed that in the trading example which precludes pairwise stable network the introduction of multilateral link formation leads to the existence of multilaterally stable networks.

This chapter provides a first attempt to extend the literature of network games towards hypergraphs that allow more than just two players to form a link. It is shown that many of the results may not apply in the multilateral linking case. This suggests that multilateral network formation may be a new analytical tool to recon-

sider many of the results in network games. There are a number of direction in which the framework can be extended. We did not consider dynamic stability concepts as stochastic stability of network structures. Further this framework provides many opportunities to extend models of directed link formation towards hypergraphs.
Moreover, this framework provides an adequate analytical tool to investigate the strategic formation of multilateral trade agreements in many economic fields as in the context of firm collaboration networks where firms form multilateral collaborations or social organization networks where an organization consists of more than just two members.

Another economic field in which the formation of multilateral links plays an important role is international trade. In the following chapter we shall introduce an international trade framework in which players can form multilateral and bilateral trade agreements. First, we investigate the strategic stability of trading networks in which the nodes represent countries and links between countries represent trading agreements between the countries. We allow countries to endogenously form trade agreements when the payoff for each player is defined as a country's welfare function. Further, the value of a network is defined as the sum of all players' payoff and therefore describes the total world welfare. Second, we investigate whether the formation of bilateral trade agreements enhances incentives for multilateral trade liberalization. Therefore the following chapter contributes to the regionalism debate reviewed in the introduction.

3.3 Appendix

Proof of Proposition 3.1. (i) It follows from the fact that in this cost range any pair of players that has no direct link is going to form a link as a direct link is more beneficial than an indirect link with $\delta - c > \delta^2$. Furthermore, no additional links are formed as they don't increase the payoff of any of the players. Furthermore, with Lemma 3.1. we only consider networks $\mathcal{L}$ in which no link $L_1 \in \mathcal{L}$ is a subset of any other link $L_2 \in \mathcal{L}$ with $L_1 \subseteq L_2$ since these links also don't produce an additional payoff for any of the players.

(ii) It is easy to see that for $c > \delta > 0$ the empty network is multilaterally stable. Since we consider single deviations no group of players has an incentive to form a link and condition (ii) of Definition 3.2 is satisfied. But this is not necessarily the unique stable structure. Consider for $n = 5$ the network $\mathcal{L} = \{\{1,2\},\{2,3\},\{3,4\},\{4,5\},\{5,1\}\}$. From this network each player obtains a payoff of $2 \cdot \delta - 2 \cdot c + 2 \cdot \delta^2$. Since direct links are not beneficial no player will form an additional link. But comparison with a line network $\mathcal{L}' = \{\{1,2\},\{2,3\},\{3,4\},\{4,5\}\}$ shows that no player has an incentive to sever one of his links whenever $\delta - c + \delta^2 > \delta^3 + \delta^4$. In this cost range $\mathcal{L}$ is multilaterally stable.

(iii) From the cost range we know that it is most beneficial to be indirectly linked to any other player as the payoff from an indirect link is larger than the payoff from a direct link as no linking costs are incurred. Furthermore, with $\delta - c > 0$ the center player has no incentive to sever his direct links. But it is also possible that a line is stable. For $n = 4$ the payoff for the players on the loose end is given by $Y_i = \delta + \delta^2 - c$. This can be larger than the payoff these two players obtain by forming a circle if $\delta^3 > \delta - c$.

[iv) To check condition (ii) of multilateral stability it can be verified that with Lemma 3.1. no additional link is formed. Furthermore, no player has an incentive to sever the global link as $Y_i(\mathcal{L}^G) = (n-1) \cdot \delta - (n-1) \cdot c > 0 = Y_i(\mathcal{L}^e)$ whenever $\delta - c > 0$.

□

Proof of Proposition 3.2. (i) Given that $0 < \delta^2 < \delta - c$, a direct link is always more beneficial than an indirect link. Calculations show that the global network and the complete network produce the same overall welfare with

$$\sum_i Y_i(\mathcal{L}^N) = \sum_i Y_i(\mathcal{L}^G) = n \cdot ((n-1) \cdot \delta - (n-1) \cdot c).$$

Hence, beneficial direct links make every player want to form as many links as possible and thus every network in which every player has a direct link to each other player is efficient.

From Lemma 3.2 it is clear that each player has at most one direct link to every other player as an additional direct connection generates the same total payoff but a player has to bear the additional costs.

(ii) For the star network we obtain:

$$\sum_i Y_i(\mathcal{L}^S) = (n-1)(\delta - c) + (n-1)((n-2)\cdot\delta^2 + \delta - c)$$
$$= (n-1)(2\cdot\delta - 2c + (n-2)\cdot\delta^2).$$

$\delta^2 > \delta - c$ means that a player obtains a higher value from an indirect link than from a direct link such that the costs are low enough so that an empty network is not efficient. Then a star network will be efficient. Thus, it is efficient to have as many indirect links as possible. It is a well known result from graph theory that the star is a minimal connected network with $n-1$ links.

(iii) Comparison of the star and the empty network yields:

$$(n-1)(2\cdot\delta - 2c + (n-2)\cdot\delta^2) > 0$$
$$\Longleftrightarrow \quad \delta + \frac{n-2}{2}\cdot\delta^2 > c,$$

thus for high values of c the empty network is efficient.

□

Proof of Proposition 3.3. Computing the overall welfare of the star and comparing with the empty network yields:

$$\sum_i Y_i(\mathcal{L}) = 2(n-1)\cdot\delta - 2(n-1)\cdot c + (n-1)(n-2)\cdot\delta^2 < 0$$
$$\Leftrightarrow \quad \delta + \frac{(n-2)}{2}\cdot\delta^2 < c.$$

Comparing the star network with the global network yields:

$$2(n-1)\cdot\delta - 2(n-1)\cdot c + (n-1)(n-2)\cdot\delta^2 > n(n-1)\cdot\delta - n\sqrt{n-1}\cdot c$$
$$\Leftrightarrow \quad \delta^2 - \delta > \frac{2(n-1) - n\sqrt{n-1}}{(n-2)\cdot(n-1)}\cdot c,$$

where $f(n) := \frac{2(n-1)-n\sqrt{n-1}}{(n-2)\cdot(n-1)}$ is a function with values $f(n) \in (-1, 0) \;\; \forall\, n > 3$ □

Proof of Theorem 3.1. Since the set of graphs is a subset of hypergraphs we can extend the example of Jackson and Wolinsky (1996) for $n \geq 3$ players. The proof proceeds in the way that we can find a value function for each component efficient and anonymous allocation rule such that no multilaterally stable network is efficient. The value function is calculated in the following way:
Let $n = 3$ and $v(\mathcal{L}^N) = v(\mathcal{L}^G) = v(\mathcal{L}^G \cup \mathcal{L}^N) = 12 = v(\{\{1,2\}\}) = v(\{\{1,3\}\}) = v(\{\{2,3\}\})$, $v(\{\{1,2\},\{2,3\}\}) = v(\{\{1,2\},\{1,3\}\}) = v(\{\{1,3\},\{,2,3\}\}) = 13$. The value of the empty network is equal to zero. We further have $v(\mathcal{L}^G \cup \{\{i,j\}\}) = 12\ \forall\{i,j\} \in \mathcal{L}^N$ and $v(\mathcal{L}^G \cup \{\{i,j\},\{j,k\}\}) = 13\ \forall i,j$ and k. Thus the efficient network are of the kind $\{\{i,j\},\{j,k\}\}$ and $\mathcal{L}^G \cup \{\{i,j\},\{j,k\}\}$ with $i,j,k \in N$ and $i \neq j \neq k$. Since the allocation rule is component efficient and anonymous the payoff allocated to each player will be $Y_i(\mathcal{L}^N, v) = Y_i(\mathcal{L}^G, v) = Y_i(\mathcal{L}^G \cup \mathcal{L}^N, v) = 4$ and $Y_i(\{\{i,j\}\}, v) = Y_j(\{\{i,j\}\}, v) = 6\ \forall i,j \in N,\ i \neq j$. We further set $Y_i(\mathcal{L}^G \cup \{\{i,j\}\}, v) = Y_j(\mathcal{L}^G \cup \{\{i,j\}\}, v) = 5.5$ and $Y_k(\mathcal{L}^G \cup \{\{i,j\}\}, v) = 1$.
Multilateral stability of $\{\{i,j\},\{j,k\}\}$ requires that $Y_j(\{\{i,j\},\{j,k\}\}, v) \geq 6$ because otherwise player j would sever one of his bilateral links. For player i and k multilateral stability requires that $Y_i(\{\{i,j\},\{j,k\}\}, v) = Y_k(\{\{i,j\},\{j,k\}\}, v) \geq 4$ because otherwise they would form an additional link since their payoff in a complete network is 4. Thus we obtain that multilateral stability requires $Y_i(\{\{i,j\},\{j,k\}\}, v) + Y_j(\{\{i,j\},\{j,k\}\}, v) + Y_k(\{\{i,j\},\{j,k\}\}, v) \geq 16$ which is a contradiction since for the component efficient and anonymous allocation rule Y we have $v(\{\{i,j\},\{j,k\}\}) = 13$.
Multilateral stability of $\mathcal{L}^G \cup \{\{i,j\},\{j,k\}\}$ requires that $Y_j(\mathcal{L}^G \cup \{\{i,j\},\{j,k\}\}, v) \geq 5.5$ and $Y_i(\mathcal{L}^G \cup \{\{i,j\},\{j,k\}\}, v)$ and $Y_k(\mathcal{L}^G \cup \{\{i,j\},\{j,k\}\}, v) \geq 4$ because otherwise i and k would form a link. This will lead to $Y_j(\mathcal{L}^G \cup \{\{i,j\},\{j,k\}\}, v) + Y_i(\mathcal{L}^G \cup \{\{i,j\},\{j,k\}\}, v) + Y_k(\mathcal{L}^G \cup \{\{i,j\},\{j,k\}\}, v) \geq 13.5$ which is a contradiction to $v(\mathcal{L}^G \cup \{ij, jk\}) = 13$.
We can easily extend the results to $n \geq 3$ by setting the value of each network that contains a link including a player other than i, j and k equal to zero. □

Proof of Lemma 3.3. Assume $\mathcal{L}$ is multilaterally stable and efficient and L is a critical link of $\mathcal{L}$ which means that, $\forall i \in L$, i does not have an incentive to sever link L. Henceforth $Y_i = \frac{v(\mathcal{C})}{|N(\mathcal{C})|} \geq \frac{v(\mathcal{C}_1)}{|N(\mathcal{C}_1)|} \forall i \in N(\mathcal{C}_1)$,..., $Y_j = \frac{v(\mathcal{C})}{|N(\mathcal{C})|} \geq \frac{v(\mathcal{C}_k)}{|N(\mathcal{C}_k)|} \forall j \in N(\mathcal{C}_k)$.

This implies that $\frac{v(\mathcal{C})}{|N(\mathcal{C})|} \geq max[\frac{v(\mathcal{C}_1)}{|N(h_1)|}, \frac{v(\mathcal{C}_2)}{|N(\mathcal{C}_2)|}, ..., \frac{v(\mathcal{C}_k)}{|N(\mathcal{C}_k)|}]$.

Next consider $\mathcal{L}$ to be efficient for any component additive v such that $v(\mathcal{L}) = \sum_{\mathcal{C} \in C(\mathcal{L})} v(\mathcal{C}) \geq \sum_{\mathcal{C} \in C(\mathcal{L}')} v(\mathcal{C}) \quad \forall \mathcal{L}' \in \mathcal{H}$. Furthermore, the critical link property is satisfied: $v(\mathcal{C}) \geq v(\mathcal{C}_1)+v(\mathcal{C}_2)+...+v(\mathcal{C}_k)$ implies that $\frac{v(\mathcal{C})}{|N(\mathcal{C})|} \geq max[\frac{v(\mathcal{C}_1)}{|N(\mathcal{C}_1)|}, \frac{v(\mathcal{C}_2)}{|N(\mathcal{C}_2)|}, ..., \frac{v(\mathcal{C}_k)}{|N(\mathcal{C}_k)|}]$. Assume $\mathcal{L}$ is not stable and a critical link is severed. The only components that change will be the ones obtained from the severance of a critical link such that we obtain for these components by efficiency and component additivity that $v(\mathcal{C}) \geq v(\mathcal{C}_1) + v(\mathcal{C}_2) + ... + v(\mathcal{C}_k)$ which implies that $\frac{v(\mathcal{C})}{|N(\mathcal{C})|} \geq max[\frac{v(\mathcal{C}_1)}{|N(\mathcal{C}_1)|}, \frac{v(\mathcal{C}_2)}{|N(\mathcal{C}_2)|}, ..., \frac{v(\mathcal{C}_k)}{|N(\mathcal{C}_k)|}]$.

Consider next the addition of a critical link that connects components $\mathcal{C}_1$, $\mathcal{C}_2$, ...,$\mathcal{C}_k$ into one component $\mathcal{C}$. With component additivity of v and efficiency we have that $v(\mathcal{C}) \leq v(\mathcal{C}_1) + v(\mathcal{C}_2) + ... + v(\mathcal{C}_k)$ where $\mathcal{C}$ is the component that results from adding a critical link. Suppose that $\mathcal{L}$ is not stable and $\frac{v(\mathcal{C})}{|N(\mathcal{C})|} > \frac{v(\mathcal{C}_1)}{|N(\mathcal{C}_1)|}$ and $\frac{v(\mathcal{C})}{|N(\mathcal{C})|} \geq \frac{v(\mathcal{C}_i)}{|N(\mathcal{C}_i)|} \forall i \in \{2, ..., k\}$. Multiplying the first inequality with $\frac{|N(\mathcal{C}_1)|}{|N(\mathcal{C})|}$ and the ith inequality with $\frac{|N(\mathcal{C}_i)|}{|N(\mathcal{C})|} \forall i \in \{2, ..., k\}$ we find that $v(\mathcal{C}) > v(\mathcal{C}_1) + ... + v(\mathcal{C}_k)$, which is a contradiction and implies that $\mathcal{L}$ is stable.
For any non-critical link the addition or deletion does not change the structure of the components and since $\mathcal{L}$ is efficient with component addidivity of v the network $\mathcal{L}$ is stable.

□

Proof of Theorem 3.2. Anonymity of Y implies that whenever network $\mathcal{L}$ is fully connected such that $\mathcal{L} := \{L \subseteq 2^N || L| \geq 2\}$ contains all possible links between players in N it must be the case that $Y_i(\mathcal{L}) = \frac{v(\mathcal{L})}{n}$. Assume that network $\mathcal{L}$ is not fully connected and that there exists a player i with $i \in L$ and $L \notin \mathcal{L}$ such that $Y_i(\mathcal{L}, v) > \frac{v(\mathcal{L})}{n}$. We define the value functions v and w such that w and v coincide everywhere except on $\mathcal{L} \cup \{L\}$. We set $w(\mathcal{L} \cup \{L\}) > v(\mathcal{L})$ and show that this results in a contradiction with Y being pairwise monotonic. By induction, for a network with k links we have that $Y_i(\mathcal{L} \cup \{L\}, v) = \frac{w(\mathcal{L} \cup \{L\})}{n}$. We shall now show that whenever Y satisfies the equal split rule for k links it also satisfies the equal split rule for $k-1$ links. With independence of potential links we have that $Y_i(\mathcal{L}, w) = Y_i(\mathcal{L}, v) > \frac{v(\mathcal{L})}{n}$. When we choose $w(\mathcal{L} \cup \{L\}) - v(\mathcal{L})$ very small we obtain that i is going to sever his link L. Henceforth $\mathcal{L} \cup \{L\}$ is defeated by $\mathcal{L}$ under w with $Y_i(\mathcal{L}, w) > \frac{v(\mathcal{L})}{n}$. Since we assumed $w(\mathcal{L} \cup \{L\}) > w(\mathcal{L})$ this contradicts pairwise monotonicity.

When we assume that i is not fully connected with $Y_i(\mathcal{L}, v) < \frac{v(\mathcal{L})}{n}$, we can select a link L with $L \notin \mathcal{L}, i \in L$. If $Y_j(\mathcal{L}, v) > \frac{v(\mathcal{L})}{n}$ for any $j \in L$ we obtain a contradiction as above. Hence $Y_j(\mathcal{L}, v) \leq \frac{v(\mathcal{L})}{n}$ $\forall j \backslash \{i\} \in L$. Now w again coincides with v everywhere except on $\mathcal{L} \cup L$ such that $v(\mathcal{L}) = w(\mathcal{L} \cup \{L\})$. By induction we have that $Y_i(\mathcal{L} \cup \{L\}, w) = \frac{w(\mathcal{L} \cup \{L\})}{n}$ since the equal split rule is fulfilled for all networks with at least k links. This implies that $Y_i(\mathcal{L} \cup \{L\}, w) = \frac{w(\mathcal{L} \cup \{L\})}{n} = \frac{v(\mathcal{L} \cup \{L\})}{n} > Y_i(\mathcal{L}, v) = Y_i(\mathcal{L}, w)$ where the second equality derives from the assumption of the independence of potential links. This implies that $\mathcal{L} \cup \{L\}$ is defeated by $\mathcal{L}$ and the pairwise monotonicity implies that $w(\mathcal{L}) > w(\mathcal{L} \cup \{L\})$ which contradicts the assumption of the independence of potential links.

To conclude the proof we have to show that the equal split rule is also fulfilled for fully connected networks which is satisfied since we assumed Y and v to be anonymous and Y to satisfy efficiency. □

Proof of Proposition 3.4. We assume that network $\mathcal{L}$ is multilaterally stable such that condition (i) of Definition 3.2 implies:

$$
\begin{aligned}
& Y_i(\mathcal{L}) - Y_i(\mathcal{L} \backslash \{\tilde{L}\}) \\
=& \sum_{j \neq i} \delta^{t_{ij}(\mathcal{L})} - \sum_{L \in \mathcal{L}_i(\mathcal{L})} (\mid L \mid -1) \cdot c - \sum_{j \neq i} \delta^{t_{ij}(\mathcal{L} \backslash \{\tilde{L}\})} + \sum_{L \in \mathcal{L}_i(\mathcal{L} \backslash \{\tilde{L}\})} (\mid L \mid -1) \cdot c \geq 0 \\
\Leftrightarrow & \sum_{j \neq i} (\delta^{t_{ij}(\mathcal{L})} - \delta^{t_{ij}(\mathcal{L} \backslash \{\tilde{L}\})}) - (\mid \tilde{L} \mid -1) \cdot c \geq 0, \qquad \forall \tilde{L} \in \mathcal{L} \text{ and } i \in \tilde{L}.
\end{aligned}
$$

This implies:

$$
\begin{aligned}
& \sum_{j \neq i} (\delta^{t_{ij}(\mathcal{L})} - \delta^{t_{ij}(\mathcal{L} \backslash \{\tilde{L}\})}) - (\mid \tilde{L} \mid -1) \cdot c \geq 0 \quad \forall \tilde{L} \in \mathcal{L}' \subseteq \mathcal{L} \\
\Rightarrow & \sum_{\tilde{L} \in \mathcal{L}'} (\sum_{j \neq i} (\delta^{t_{ij}(\mathcal{L})} - \delta^{t_{ij}(\mathcal{L} \backslash \{\tilde{L}\})})) - \sum_{\tilde{L} \in \mathcal{L}'} ((\mid \tilde{L} \mid -1) \cdot c) \geq 0 \\
\Leftrightarrow & \sum_{j \neq i} (\delta^{t_{ij}(\mathcal{L})} - \delta^{t_{ij}(\mathcal{L} \backslash \mathcal{L}')}) - \sum_{\tilde{L} \in \mathcal{L}'} (\mid \tilde{L} \mid -1) \cdot c \geq 0 \\
\Leftrightarrow & Y_i(\mathcal{L}) - Y_i(\mathcal{L} \backslash \mathcal{L}') \geq 0, \qquad \forall i \in N, \text{ for every } \mathcal{L}' \subseteq \mathcal{L}_i(\mathcal{L}).
\end{aligned}
$$

To conclude the proof it can be seen that condition (ii) of Definition 3.2. equals condition (ii) of Definition 3.18. □

Proof of Proposition 3.5. (i) The proof follows from Proposition 3.2.(iii). For $\delta^2 > \delta - c$ and $c > \delta + \frac{(n-2)}{2} \cdot \delta^2$ the empty network is efficient and therefore also Pareto efficient.

(ii) Proposition 3.1 showed that the empty network is the unique stable network. To see that it is not necessarily Pareto efficient consider the following example borrowed from Jackson (2003, p. 28): Let $n = 4$ and $\delta < c < \delta + \frac{\delta^2}{2}$. In this case the empty network is Pareto dominated by a line network $\mathcal{L} = \{\{1, 2\}, \{2, 3\}, \{3, 4\}\}$. The payoff for players 1 and 4 is $\delta + \delta^2 + \delta^3 - c$ which is greater than 0 and for player 3 and 2 the payoff is $2 \cdot \delta + \delta^2 - 2c$ which is grater than 0 with $c < \delta + \frac{\delta^2}{2}$.

(iii) The proof follows from Proposition 3.2(i) as the set of efficient networks is also Pareto efficient. Furthermore, in each strongly efficient network each player is directly connected to each other player exactly once. The formation of an additional link makes no player better off but merely increases linking costs for some players. Therefore we cannot find a Pareto dominating network.

□

4 The Strategic Formation of Trade Agreements

In chapter 1 of the thesis we provided an overview of the theoretic models that analyze the impact of PTAs on multilateral liberalization and vice versa[46]. Krishna (1998) shows that the formation of a PTA lowers countries' incentives for multilateral tariff reduction. Freund (2000) addresses the question of how multilateralism can impact bilateralism. She observes that multilateral tariff reduction can affect the formation of PTAs, and therefore shows that multilateral trade liberalization enhances the incentives to form PTAs. The conclusion is that many of the current PTAs may be a result of the success of the GATT in lowering tariffs. Andriamananjara (2002) addresses the question as to whether the current wave of regionalism and bilateralism will lead to a division of the world into a number of competing inward-looking blocs or to a more open multilateral trading system. His paper investigates the possibility of achieving global free trade through the expansion of PTAs. He finds that global free trade can only be achieved when membership of trading blocs is open for each country.

The importance of this issue motivates an examination of the incentives of countries to form such agreements and of the strategic stability of different structures when linking decisions depend on countries' payoffs. This present thesis uses a network formation approach to model the formation of trade agreements. Goyal and Joshi (2006a) were the first to analyze the formation of trade agreements as a network formation game. By assuming that countries are symmetrical with respect to market size, they show that a network formation process in which players are allowed to form bilateral free trade agreements leads either to a complete network or to an almost complete network. Furusawa and Konishi (2007) use a similar approach but with a differentiated product market. They show that the complete network is stable. Although these papers investigate whether the bilateral formation of PTAs alone achieves global free trade, they put aside whether the formation of PTAs lowers incentives for multilateral tariff cooperation within the scope of the GATT as multilateral trade efforts and PTAs interact with each other. Their approach dif-

[46] See for example Krishna (1998) and Freund (2000). For excellent overviews of preferential trade vs. multilateralism see Panagariya (2000) as well as Bhagwati and Panagariya (1996).

fers significantly from our model, since we allow players to form multilateral trade agreements in addition to bilateral links. We analyze the coevolution of multilateral and bilateral trade agreement formation and investigate the question as to whether PTAs (here we assume that countries can form bilateral FTAs in addition to a multilateral trade agreement) hinder the process of multilateral liberalization. In our model countries are assumed to be heterogenous with respect to market size. In this chapter we focus on how differences in market size across countries impact their incentives to strategically form trade agreements.[47]

To examine the stability of different trading structures we use the multilateral stability concept, defined in chapter 3 of the thesis, which is an extension of the pairwise stability concept for bilateral link formation introduced by Jackson and Wolinsky (1996). The idea of multilateral stability is that players can form, as well as sever, bilateral links and multilateral links which include more than two players. The formation of any of these links requires the consent of all players included, but severance can be carried out unilaterally. To the best of our knowledge this is the first approach to apply models of network formation in economics when players have the opportunity to form multilateral links.

Applying the strategic formation of multilateral trade agreements in international trade models seems reasonable, as we can observe a world trading system in which multilateral and bilateral trade agreement coexist. Countries are members of the GATT and have additional preferential trade agreements. Furthermore, the number of GATT members increases and they all agree on reducing tariffs. However, due to Article XXIV of the GATT members can still form additional PTAs in which tariffs against trading partners have to be eliminated. The extension of network formation games towards hypergraphs allows us to model the formation of both multilateral

[47] During the 1980s, the formation of a Canada-U.S. free trade agreement was pushed by the relatively smaller trading partner Canada with the aim to promote productivity, full employment and to encourage foreign direct investment. As the U.S. had been Canada's largest trading partner, the Canadians wanted to improve and secure access in the larger U.S. market and to strengthen the competetiveness of Canadian firms. (http://www.canadianeconomy.gc.ca/english/economy/1989economic.html)

and bilateral trade agreements, where we model multilateral trade agreements between more than two countries by means of hyperlinks whereas bilateral FTAs are modelled by means of bilateral links.

We observed in chapter 1 of the thesis that during the last three decades tariff reduction as negotiated within the GATT does not satisfy the requested trade conditions and that countries have started to form additional PTAs. Starting with an exogenously given multilateralized world, existing literature on PTAs investigates what incentives countries have to form additional bilateral links and tries to answer the question of whether additional PTAs increase individual and global welfare. The literature on trading blocs investigates whether the effect of regional integration is positive or negative. Furthermore, the time path approach formulated by Bhagwati (1993) investigates whether regionalism leads to multilateral free trade for all through continued expansion of the regional blocs. These questions provide the motivation to introduce the endogenous formation of preferential trade agreements[48] in international trade models. However, Baldwin (2006) and, earlier, Deardorff and Stern (1994, p. 27) state that regional and bilateral tariff reduction went hand in hand with multilateral liberalization and preferential trade agreements coexisted with multilateralism from the start. In our model countries can simultaneously form bilateral FTAs, as one special case of a PTA,[49] and multilateral links. We want to know what structures will emerge when countries choose bilateral and multilateral links simultaneously and whether the increasing number of FTAs lead to a more open multilateral trading system when we consider strategic link formation of countries. In this context our model helps to understand how the coexistence of PTAs and GATT emerged.

In the following, we introduce a three-country setting and an imperfect competitively produced good that is traded among the three countries. In each country there is a single firm competing as a Cournot oligopolist in each market. Markets in different countries are assumed to be perfectly segmented as in Krishna (1998), so

[48]See e.g. Grossman and Helpman (1995) and Krishna (1998).

[49]In this model we concentrate on FTAs as one special case of a PTA in which partner countries negotiated to mutually eliminate tariffs against each other.

that each firm regards each country as a separate market. Welfare gains from trade stem from the additional competition in the domestic country. If two countries have signed a bilateral free trade agreement, each of them offers the other a zero tariff access to its respective domestic market. If two countries are connected via a multilateral link, they offer each other access to their respective domestic market at a medium tariff. If a trade agreement among a pair of countries does not exist, each of them imposes a high tariff on the imports of the other market.

Goyal and Joshi (2006a) also investigate strategic stability of trading structures in a segmented market Cournot competition model where countries can also gain from increased competition that is generated by free trade agreements. In their model they consider bilateral link formation of homogenous countries, in which tariffs are set to zero if two countries have signed a trade agreement. Furthermore they show that a network with an isolated country and the rest having a free trade agreement is a stable state. In the case of three countries, this implies that the situation in which one country is isolated and the other two have a free trade agreement is stable. In contrast to Goyal and Joshi (2006a), we find that due to country heterogeneity, a trade agreement between two countries with relatively small market size is stable but a bilateral trade agreement between two countries of different market size cannot be stable. Moreover, we can show that a multilateral link without additional bilateral trade agreements cannot be stable whereas global free trade is stable. When we consider endogenous tariff formation we assume that countries choose their welfare maximizing tariffs that they levy on foreign countries. We find that global free trade is the unique stable network. The complete network without an additional multilateral agreement can no longer be achieved, which implies that the GATT agreement stabilizes the trading structure and is even necessary for stability. We also observe that a conflict between overall welfare efficiency and stability can occur in a heterogeneous country model.

The results are driven by three different welfare effects that a trade agreement has on an importing country. The first effect is that an importing country gains from increased competition and increased consumer surplus in the domestic market. The second effect is that a country's welfare also increases since the domestic firm gets greater access to the foreign market. The third effect is that due to increased competition, the domestic firm gets a lower profit in its own market. We will see that the additional welfare gains from free trade are very high for countries with small market size, since a bilateral trade agreement allows small countries' firms to access the market of the large country and thus increases the profit of the domestic firm.

The remainder of the chapter is organized as follows: In section 1, we introduce the model of international trade as a three-country setting and define a notion for stability and efficiency of hypergraph structures, which is modelled as an extension of the pairwise stability concept introduced by Jackson and Wolinsky (1996). The stable and efficient trading structures are analyzed in section 2 in which tariffs are exogenously given. Section 3 extends the framework of section 2 in three directions. In section 3.1 we generalize the social welfare function, in 3.2 tariffs are endogenous and countries choose their optimal tariffs with respect to imports from the foreign markets and in section 3.3 we extend the framework to an arbitrary number of countries. Section 4 of chapter 4 provides the conclusion.

4.1 The Model

4.1.1 Trading Systems

We consider a three-country setting where countries are involved in bilateral and multilateral trade agreements. The collection of trading agreements determines the trading system between the three countries.

We apply the notion of hypergraphs from chapter 3 to allow the coexistence of multilateral and bilateral trade agreements, in which the set of players is now a set of countries and hypergraphs represent trading systems.

Let $N = \{A, B, C\}$ be the set of countries. $\mathcal{L}$ is a set of subsets of N, $\mathcal{L} \subseteq 2^N$, and is called a trading system on N.

As in the model of section 1 in chapter 2, $L \in \mathcal{L}$ with $L \subseteq N$ represents a trading agreement between all countries in L such that a trading system describes the tra-

ding agreements that exist between countries in N.
Since each country is linked with itself we restrict our attention to trading systems $\mathcal{L}$ with $\mathcal{L} \subseteq \{L \in 2^N || L| \geq 2\}$.
Whenever the trading system contains only one trade agreement that encompasses all players such that $\mathcal{L} = \{N\}$, it is called global and is denoted with $\mathcal{L}^G$. In our three-country setting the global trading system is presented in Figure 6a).
The complete trading system $\mathcal{L}^N$ is the family of subsets of N with $\mathcal{L}^N = \{L \in 2^N || L| = 2\}$. The complete trading system is shown in Figure 6b) and represents a trading system in which each pair of players has a bilateral trade agreement.
The *star* trading system, which we denote by $\mathcal{L}_i^S$, has only bilateral links from the central country i to each of the other countries with $\mathcal{L}_i^S = \{L \in 2^N || L| = 2 \text{ and } i \in L\}$. In the star trading system, the country which is directly linked to the other countries is called the *hub* country, while the other countries are called the *spoke* countries.
The empty trading system $\mathcal{L}^e$ corresponds to a trading system in which no trade agreement exists.
Furthermore let $N_i(\mathcal{L})$ denote the set of countries that are directly linked with country i in the trading system $\mathcal{L}$ with $i \in N_i(\mathcal{L})$ and $\eta_i(\mathcal{L}) = |N_i(\mathcal{L})|$ denotes the number of players that have a trade agreement with country i.

4.1.2 The Model

In each country $i \in N = \{A, B, C\}$ there is a firm producing a homogenous good with marginal cost of production c. Each firm has the opportunity to sell in the foreign markets[50]. Its supply in the foreign market depends on the tariffs faced by the firm on its exports. The tariffs a firm faces depend on the nature of trading agreements between the home and the foreign market and the trading system as a whole. Given the trading system, firms choose the quantities that they supply in the domestic as well as in the foreign markets.
With q_i^j we denote the output produced by firm j in country i, and $q_i = \sum_j q_i^j$ denotes the total sales of all firms in country i. The price of the good in country i's

[50] Our model is a variant of the model used by Brander and Krugman (1983) and Krishna (1998).

market is given by a linear function:

$$P_i = \alpha_i - q_i,$$

where $\alpha_i > 0$ for all $i \in N$.

We assume that firms compete as Cournot oligopolists in each country such that each firm maximizes its profit in each country separately as in Krishna (1998). Furthermore, we introduce the following exogenous tariff structure:

$$t_j^i(\mathcal{L}) = \begin{cases} 0 & \text{if } i \text{ and } j \text{ have a bilateral trade agreement,} \\ T & \text{if there is no trade agreement between } i \text{ and } j \text{ in } \mathcal{L}, \\ t & \text{otherwise,} \end{cases}$$

where $t_j^i(\mathcal{L})$ denotes the tariff faced by firm i in country j in $\mathcal{L}$ for each quantity supplied, where $T > t > 0$. Here, t is the tariff that countries impose when they are linked multilaterally without a bilateral link.

When two countries have signed a bilateral trade agreement they offer the trading partner free access to the domestic market. The assumption that two countries that are involved in a bilateral trade agreement face a tariff of zero is supported by the GATT Article XXIV that permits the formation of PTAs if tariffs are eliminated between the trading partners. When two countries are linked via a multilateral link and have no additional bilateral trade agreement, they offer their trading partners a medium tariff level of t. Since the global trade agreement represents the GATT, all countries have to offer the same tariff to all members of the global trade agreement due to the GATT principles of reciprocity and non-discrimination (MFN principle). In Figure 16a) a multilateral trade agreement ($L = \{A, B, C\}$) exists between all three countries and B and C have formed an additional bilateral trade agreement ($L = \{B, C\}$). Country B and C impose a tariff of t on market A whereas B and C face zero tariffs among themselves. In Figure 16b) countries B and C set zero tariffs against each other but imposed a high tariff on market A. Figure 16c) represents a trading system under free trade.

Firms are assumed to maximize profits, taking other firms' outputs as given, with all firms choosing their quantities simultaneously. First we assume that $T > \alpha_i \ \ \forall i$ to make sure that firms will only sell in another country if at least a multilateral link exists between two countries. A firm j chooses the quantity that it supplies in country i with respect to maximizing profit such that the equilibrium quantity that firm j supplies in country i in the trading system ($\mathcal{L}$) is given by:

$$q_i^j(\mathcal{L}) = \frac{(\alpha_i - c)}{(\eta_i(\mathcal{L}) + 1)} + \frac{\sum_{k \in N} t_i^k(\mathcal{L})}{(\eta_i(\mathcal{L}) + 1)} - t_i^j(\mathcal{L}),$$

where $\eta_i(\mathcal{L})$ denotes the number of firms active in country i and $k = A, B, C$. We restrict the parameter t to $0 < t < \frac{\alpha_i - c}{3} \ \ \forall i$ to concentrate on the case in which there is a positive quantity traded between two countries that share a multilateral agreement.

A firm j's profit in country i with $j \in N_i(\mathcal{L})$ can be calculated as:

$$\pi_i^j(\mathcal{L}) = {q_i^j}^2(\mathcal{L}).$$

We define country i's welfare function as the sum of consumer surplus, producer surplus and tariff revenue. Governments choose the tariffs as well as the linking decision with respect to maximizing social welfare. The objective function[51] is:

$$Y_i(\mathcal{L}) = \frac{1}{2}{q_i}^2(\mathcal{L}) + [(P_i(\mathcal{L}) - c)q_i^i(\mathcal{L}) + \sum_{j \neq i}(P_j(\mathcal{L}) - c - t_j^i(\mathcal{L}))q_j^i(\mathcal{L})] + \sum_{j \neq i} t_i^j(\mathcal{L})q_i^j(\mathcal{L}).$$

The first term represents consumer surplus in country i. The second and third term are firm i's profit in its own market and in the foreign markets, respectively. The last term is country i's tariff revenue. This formulation of social welfare places equal weight on consumer surplus and the firm's profit.

The total profit of a firm j is given by the sum of all the profits the firm j makes in all countries:

$$\Pi^j = \sum_{i=1}^{n} \pi_i^j = \sum_{i=1}^{n} (\frac{(\alpha_i - c)}{(\eta_i(\mathcal{L}) + 1)} + \frac{\sum_k t_i^k(\mathcal{L})}{(\eta_i(\mathcal{L}) + 1)} - t_i^j(\mathcal{L}))^2.$$

[51]This function describes a country's payoff depending on the trading system. More generally, in chapter 3 of the thesis the function that allocates for each hypergraph a payoff to each player was called the *allocation rule*.

Since countries only trade and supply in another country when a trade agreement exists, the social welfare function[52] is reduced to:

$$Y_i(\mathcal{L}) = \frac{1}{2}{q_i}^2(\mathcal{L}) + \sum_{j \in N_i(\mathcal{L})} (\alpha_j - c - q_j(\mathcal{L}) - t_j^i(\mathcal{L})) \cdot q_j^i(\mathcal{L}) + \sum_{j \in N_i(\mathcal{L})} t_i^j(\mathcal{L}) \cdot q_i^j(\mathcal{L}). \quad (12)$$

4.1.3 Stable and Efficient Networks

We assume that countries are able to strategically form and sever trading agreements. Each pair of countries can sign a bilateral free trade agreement and all countries can decide whether to form a multilateral trade agreement. In order to analyze the strategic stability of different trading structures and to determine the shape of stable trading systems we define an equilibrium concept that selects the trading system that are resistent with respect to countries' deviations. We adopt the multilateral stability notion of chapter 3 and introduce the following notations:

- For $L \notin \mathcal{L}$, $\mathcal{L} \cup \{L\}$ is the trading system we obtain from $\mathcal{L}$ when we form the trading agreement L.
- For $L \in \mathcal{L}$, $\mathcal{L} \backslash \{L\}$ is the trading system we obtain from $\mathcal{L}$ when we sever the trading agreement L, if $L \in \mathcal{L}$.

As in chapter 3 the formation of a trade agreement requires the consent of all countries involved, but severance can be carried out unilaterally.

Definition 4.1. *A trading system $\mathcal{L}$ on N is called multilaterally stable, if*

$$\begin{aligned} &(i) \;\; Y_i(\mathcal{L}) \geq Y_i(\mathcal{L}\backslash\{L\}) \; \forall L \in \mathcal{L}, \quad \forall i \in L \textit{ and} \\ &(ii) \; Y_i(\mathcal{L} \cup \{L\}) > Y_i(\mathcal{L}) \Rightarrow \exists j \in L, \\ &\qquad \textit{such that } Y_j(\mathcal{L} \cup \{L\}) < Y_j(\mathcal{L}) \quad \forall L \notin \mathcal{L}. \end{aligned}$$

The above definition describes a situation in which no country has an incentive to sever any of its existing trade agreements and no subset of countries has an incentive to form an additional agreement. This definition allows the formation of trade agreements with more than just two countries and therefore also allows the

[52]Note that the social welfare function defines each player's outcome in the trading system $\mathcal{L}$ and corresponds to the allocation rule defined in chapter 3.

formation of a multilateral trade agreement between all three countries. For a given trading system $\mathcal{L}$ countries base their decision of whether to form or sever trade agreements on their level of social welfare.

In order to analyze the efficiency of different trading systems, we need to consider global welfare which is given by the sum of all countries' payoffs and represents the total value generated from $\mathcal{L}$[53].

Definition 4.2. *A trading system $\mathcal{L}^*$ is said to be (strongly) efficient, if $v(\mathcal{L}) = \sum_{i \in N} Y_i(\mathcal{L}) \leq \sum_{i \in N} Y_i(\mathcal{L}^*) = v(\mathcal{L}^*)\ \forall \mathcal{L}$.*

4.2 Stability of Trading Structures and Market Size Asymmetries

We assume asymmetry with respect to countries' market size, which is expressed by different values of the parameter α_i, to answer the question as to how the variation of market size across countries affects their incentives for establishing trade agreements and whether this will lead to different stable trading structures.

4.2.1 The Symmetrical Model

First we investigate possible stable structures with $\alpha_A = \alpha_B = \alpha_C = \alpha$ and observe that our results are in line with Goyal and Joshi (2006a), who show that in the context of bilateral link formation the complete trading system is the unique stable structure. Introducing multilateral link formation the equilibrium trading systems are:

Proposition 4.1. *Global free trade is a stable trading system.*

Global free trade is represented by a complete trading system and a trading system that consists of a complete trading system and an additional global trade agreement. The proof of the result is shown in the appendix. Intuitively, no country

[53]The total world welfare aggregates the outcome of all countries and corresponds to the value function defined in chapter 3.

wants to sever any of its bilateral trade agreements under free trade. This will reduce consumer surplus as the prices increase due to lower competition in the market, domestic firm's profit in the foreign market decreases as it supplies smaller amounts due to an increase in tariffs, whereas tariff revenue increases from the severance of a bilateral link. In addition, domestic firm's profit in its own market increases due to lower competition but in total the additional profit in the domestic market and the additional tariff revenue cannot compensate for the loss in consumer surplus and the loss in profit in the foreign market. This implies that no country has an incentive to sever any of its bilateral links.

Furthermore it can be observed that a global link with a bilateral trade agreement between one pair of countries ($\mathcal{L} = \{N, \{i, j\}\}$) can be stable for certain parameter constellations. This is always fulfilled whenever t is relatively large compared to the market size. Intuitively, for large values of t country k's tariff revenue in $\mathcal{L} = \{N, \{i, j\}\}$ is very high, such that additional gains in profit due to the higher access in market i in $\mathcal{L} \cup \{\{i, k\}\}$ cannot compensate for the loss in tariff revenue.
The star trading system cannot be stable since the two countries which have only one link have an incentive to link to each other. The empty trading system cannot be stable since a pair of countries always gains from forming a bilateral trade agreement.

4.2.2 Two Small Countries and One Large Country

In the following we investigate whether the result will change when we consider asymmetries among countries.
We start with the assumption that $\alpha_A > \alpha_B = \alpha_C = \alpha$ which implies that country A's market size is relatively large compared to country B's and C's market size.

The analysis of possible stable structures leads to the next result that supports the observation that countries tend to form free trade agreements in addition to multilateral trade agreements.

Lemma 4.1. *The global trading system cannot be stable.*

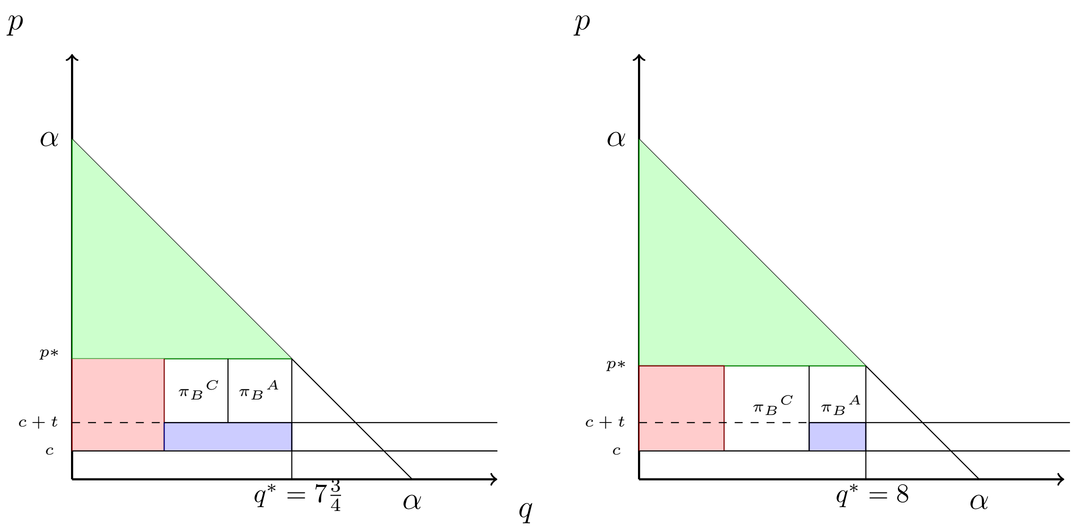


Figure 15: Demand function in country B; example with $\alpha = 12$, $c = t = 1$. Right side: global link with a free trade agreement between B and C and $\pi_B{}^C = 9$. Left side: global link with $\pi_B{}^C = (\frac{9}{4})^2$.

The global trading system without any bilateral trade agreements cannot be stable, since country B and C will both gain from an additional bilateral link between one another for the following reason: Consumer surplus in both countries increases due to increased competition and lower prices. We can also observe that a bilateral trade agreement leads to an increase in firm B's profit in country C's market and vice versa. However, there is a small negative effect on countries' welfare, since the domestic firm's profit in its own market decreases. Given that the two positive effects are higher than the negative effect, the overall welfare effect of an additional bilateral trade agreement is positive in both countries and thus B and C will deviate. The intuition is shown in Figure 15 in which the green area represents consumer surplus, the red area is domestic firm's profit and the blue area is tariff revenue. It also shows that the profit of firm A in market B ($\pi_B{}^A$) decreases due to the bilateral trade agreement whereas the profit of firm B in market A remains unchanged.

The next result provides a full description of possible stable trading structures.

Proposition 4.2. *We have three possible stable structures:*

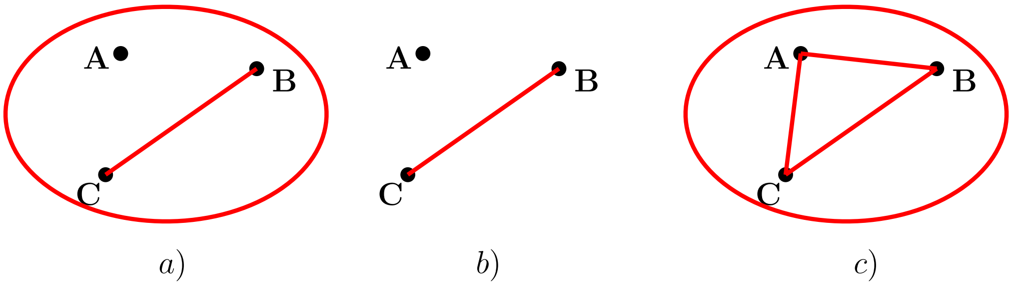


Figure 16: **Stable Networks.** a) The global trading system with a bilateral trade agreement between B and C, b) bilateral trade agreement between B and C and c) The complete trading system with a multilateral trade agreement (global free trade).

(i) For any parameter values global free trade is a stable state, [Figure 16c)*].*
(ii)For values $(\alpha_A - c)^2 > 1,5(\alpha - c)^2$ *and* $12(\alpha_A - c)t + 4(\alpha - c)^2 + 16t^2 < 24(\alpha - c)t + (\alpha_A - c)^2$ *a bilateral trade agreement between* B *and* C *is stable, [Figure* 16b)*].*
(iii)For values $12(\alpha_A - c)t + 4(\alpha - c)^2 + 16t^2 > 24(\alpha - c)t + (\alpha_A - c)^2$ *the global trading system with a bilateral trade agreement between* B *and* C *can be stable, [Figure* 16a)*].*

Figure 16 illustrates the structures that can be stable.

To give an intuition of the result, note that there are three direct effects at work when two countries sign a bilateral free trade agreement with zero import tariffs: First, the domestic firm faces greater competition from a foreign firm in the domestic market. Second, the domestic firm gets greater access to the foreign market. Third, domestic consumers benefit from greater competition in terms of lower prices. Therefore, the empty and the global trading system cannot be stable since B and C will form a bilateral trade agreement. The welfare effect in both countries from a bilateral link is positive.

From condition (ii) and (iii) we can observe that a threshold exists for which country A will deviate from the trading system described in Figure 16 a). To understand country A's linking decision we start with a global trading system in which country A's firm earns $(\frac{\alpha - c}{4} - \frac{t}{2})^2$ from its operations in B or C, respectively. Since the countries B and C increase domestic welfare by forming a bilateral trade agreement,

the profit of country A's firm reduces to $(\frac{\alpha-c}{4} - \frac{3 \cdot t}{4})^2$ and thus the foreign markets are less attractive to firm A. As a consequence, country A has an incentive to sever the multilateral link with B and C if its welfare effect is positive. This is the case for $12(\alpha_A - c)t + 4(\alpha - c)^2 + 16t^2 < 24(\alpha - c)t + (\alpha_A - c)^2$. In this case a bilateral trade agreement between B and C is stable if A has no incentive to form an additional bilateral link with B or C which is the case for $4(\alpha_A - c)^2 > 6(\alpha - c)^2$.
Proposition 4.2 gives a full characterization of multilaterally stable trading systems when the markets of country B and C are relatively small compared to the market of country A. However, the following question arises: Will we get the same results if we define country B and C as countries with large markets and A as a country with a relatively small market?

4.2.3 One Small Country and Two Large Countries

In the following, we analyze the question as to which trading systems are multilaterally stable if C and B are the countries with the largest markets (thus $\alpha_A < \alpha_B = \alpha_C = \alpha$). In particular, we are interested in the question as to whether country B and C still have incentives to maintain their bilateral free trade agreement or whether other structures will emerge in equilibrium. One possibility one might think of is that country A's firm additionally gains from a bilateral link as the additional market demand from country B (respectively C) is higher than the loss in market demand in its own market and therefore a bilateral free trade agreement between B and C cannot be stable anymore. In contrast to Proposition 4.2 we make the following observation:

Lemma 4.2. *The global trading system with a free trade agreement between country B and C cannot be stable. A free trade agreement between country B and C cannot be stable.*

To get an idea of the result, consider country B and C being involved in a bilateral trade agreement. Intuitively country A and country B may have an incentive to deviate by forming an additional bilateral link. As country B is the hub country, this seems plausible, but why does country A benefit? The argument is similar to Lemma 4.1: Due to increased competition in the domestic market, consumer surplus

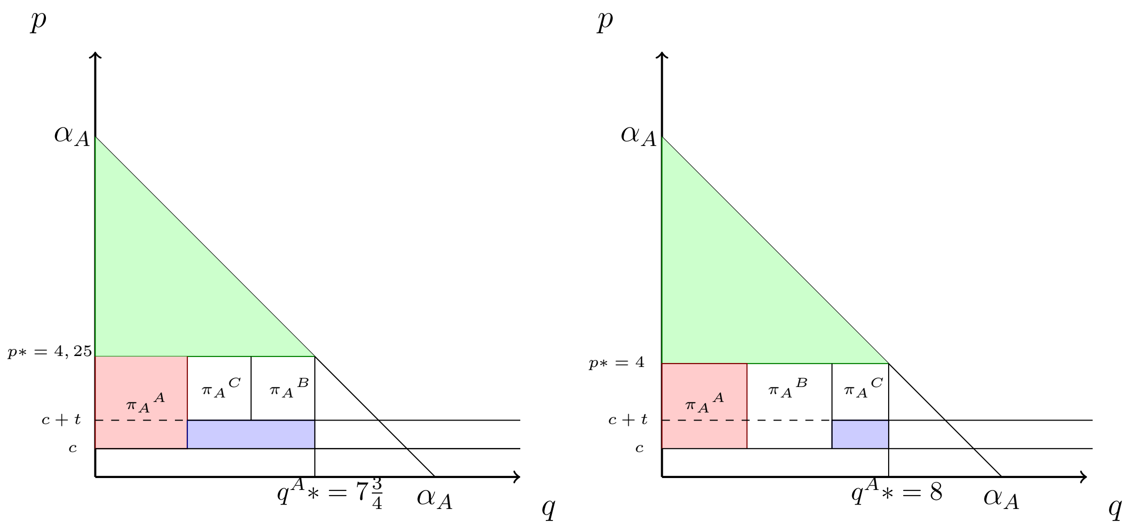


Figure 17: Demand function in country A; example with $\alpha_A = 12$, $c = t = 1$. Left side: global link with a free trade agreement between B and C. ${\pi_A}^C = {\pi_A}^B = (\frac{9}{4})^2$ and ${\pi_A}^A = (\frac{13}{4})^2$. Right side: The global trading system with a star trading system with hub country B. Here we can calculate that ${\pi_A}^A = {\pi_A}^B = 9$ and ${\pi_A}^C = 4$.

in A increases. Since we consider the case of a large market B, firm A obtains access to the large market of country B and therefore the additional firm's profit in B is higher than the reduced profit in A's domestic market. This intuition can as well be verified in Figure 17 which shows that country A obtains a consumer surplus of 32 on the right side, whereas without a bilateral trade agreement with country B country A's surplus is $\frac{1}{2}(\frac{31}{4})^2$. The additional profit in market B is given by $(\frac{11}{4})^2 - 4 = 3.5625$. The loss in tariff revenue is given by $2 - \frac{18}{4}$ and the loss in its own market profit: $9 - (\frac{13}{4})^2$. Since the total welfare effect is positive the small country has an incentive to form a bilateral trade agreement with a large market country.
The next result characterizes the set of stable trading systems:

Proposition 4.3. *Global free trade is the unique stable trading system.*

The result differs from Proposition 4.2, where we obtain a large variety of possible stable structures. Lemma 4.2 already shows that a bilateral trade agreement between B and C cannot be stable anymore. The reason why a bilateral link between country A and country B (respectively C) is not stable is that B and C have an incentive to form a bilateral trade agreement, since $Y_C(\mathcal{L}_B^S) > Y_C(\{\{A,B\}\})$ and $Y_B(\mathcal{L}_B^S) > Y_B(\{\{A,B\}\})$. The sum of the increase of consumer surplus in country C and an increase of firm C's profit generated in market B exceeds the decrease of firm C's profit in its domestic market. Hence the overall welfare effect on country C is positive. A similar argument explains why a global trading system with a bilateral free trade agreement between A and B is not stable either.

4.2.4 The Asymmetrical Case

In the following we will demonstrate that under total asymmetry with $\alpha_A > \alpha_B > \alpha_C$ it is more complicated to describe the nature of stable trading systems. The next examples illustrate the difficulties.

Consider first the empty trading system and a pair of countries i and j consider forming a bilateral link. Therefore it has to be fulfilled that $Y_i(\mathcal{L}^e) - Y_i(\{\{i,j\}\}) = \frac{3}{8}(\alpha_i - c)^2 - \frac{1}{3}(\alpha_i - c)^2 - \frac{1}{9}(\alpha_j - c)^2 < 0$ and $Y_j(\mathcal{L}^e) - Y_j(\{\{i,j\}\}) = \frac{3}{8}(\alpha_j - c)^2 - \frac{1}{3}(\alpha_j - c)^2 - \frac{1}{9}(\alpha_i - c)^2 < 0$. Since $\alpha_i \neq \alpha_j \; \forall i \neq j$ we can therefore show that for the smaller market, let's say market j, j will always have an incentive to form a bilateral link whereas i only deviates if $\frac{(\alpha_i - c)^2}{24} > \frac{(\alpha_j - c)^2}{9}$. The underlying intuition is that for market i the additional profit made in market j would be too small compared to the profit loss in the home market.

We can exclude a set of trading systems that cannot be stable. One structure that cannot be stable is a free trade agreement between B and A. To see this, consider that the small country C always has an incentive to form a bilateral link with a larger market country whereas the larger market always has an incentive to be the hub country in a star trading system. The same applies to a bilateral trade agreement between C and A, since B will have an incentive to form a trade agreement with A.

We can further exclude each star trading system $\mathcal{L}_i^S$, because for both spoke countries we have: $Y_j(\mathcal{L}_i^S) - Y_j(\mathcal{L}^N) = \frac{(\alpha_j - c)^2}{3} + \frac{(\alpha_i - c)^2}{16} - \frac{11(\alpha_j - c)^2}{32} - \frac{(\alpha_i - c)^2}{16} - \frac{(\alpha_k - c)^2}{16} < 0$.

One stable trading system is the complete trading system since, as shown above, no star trading system can be stable, and since the complete trading system with a global link generates the same payoffs to each country. We can further elaborate that the complete trading system with a global link is not necessarily stable, since country A can increase its welfare by severing its free trade agreement with C. The nature of stable trading systems is given by:

Proposition 4.4. *The complete trading system is stable.*

We will present the complete proof in the appendix.
To see why the complete trading system with a global link is not necessarily stable, consider that country i might have an incentive to sever its free trade agreement to any country j with $\alpha_i > \alpha_j$ if $Y_i(\mathcal{L}^G \cup \mathcal{L}^N) - Y_i(\mathcal{L}^G \cup \mathcal{L}^S_k) < 0$ which is equivalent to $2 \cdot t(\alpha_i - c) > 4 \cdot t(\alpha_j - c) + t^2$. Therefore we cannot guarantee its stability. Intuitively we can argue again that a bilateral trade agreement with a smaller market country might increase country i's firm profit in the foreign market to a lesser degree than the reduction of firm i's profit in the home market. And this will induce country i to sever the link with market j.

4.2.5 Efficiency

We next examine the nature of efficient trading systems. We thus have to evaluate the total welfare of different trading structures. Therefore with $t < \frac{\alpha_i - c}{3}$ $\forall \alpha_i$:

Proposition 4.5. *For any parameter values of α_i and c global free trade is an efficient trading structure.*

Proposition 4.5 can easily be verified if we compare different welfare levels as calculated in the appendix. We can elaborate that the total welfare in the complete trading system is always larger than in any arbitrary trading system and two different trading structures maximize the overall welfare level.
In contrast to stability there are just two efficient trading systems that produce the same total output. Hence, we observe a conflict between efficiency and stability in the symmetrical case and when there are two small countries and one large country as we obtain a larger set of possible stable trading systems, whereas there is no conflict when there are two large countries and one small country. In the case of total asymmetry the efficient trading systems can be stable.

4.3 Generalizations

The above results encourage the examination of a more generalized setting. We generalize the model into three directions. First, we allow for a more general social welfare measure with arbitrary weights on consumer surplus, profits and tariff revenue. Second, we extend the model such that the tariffs are chosen endogenously by the countries. Third, we give some implications on what will happen when we allow an arbitrary number of countries to form free trade agreements.

4.3.1 Generalized Social Welfare Function

First, we allow arbitrary weights on consumer surplus, firms' profit and tariff revenue and define a more general social welfare function from (12) with:

$$Y_i(\mathcal{L}) = \beta(\frac{1}{2}{q_i}^2(\mathcal{L})) + \gamma(\sum_{j \in N_i(\mathcal{L})} (\alpha - c - q_j(\mathcal{L}) - t_j^i(\mathcal{L})) \cdot q_j^i(\mathcal{L})) + \delta(\sum_{j \in N_i(\mathcal{L})} t_i^j(\mathcal{L}) \cdot q_i^j(\mathcal{L})). \tag{13}$$

In the framework of section 2 the welfare function places equal weight on profit, consumer surplus and tariff revenue with $\beta = \gamma = \delta = 1$. Here we assume that countries are symmetrical with respect to market size. In a political economy context we might be interested in what structures will emerge when the objective function depends only on firms' profit which implies $\gamma = 1$ and $\beta = \delta = 0$. Henceforth, social welfare is given by:

$$Y_i(\mathcal{L}) = \sum_{j \in N_i(\mathcal{L})} (\alpha - c - q_j(\mathcal{L}) - t_j^i(\mathcal{L})) \cdot q_j^i(\mathcal{L}).$$

One observation that can be made is that firm j's profit in market i decreases with the number of firms that are active in market i. Let us assume i and j have a bilateral trade agreement and market i forms a bilateral link with k. This reduces country j's welfare since welfare is given exclusively by firm profit. This observation provides the intuition for the next result.

Proposition 4.6. *When countries only care about producer profit the only stable trading systems are free trade and the empty trading system.*

First we investigate welfare in the empty trading system. Firm i's profit is given by $\frac{(\alpha-c)^2}{4}$ since it only supplies to market i. When countries only care about producer profit it can be shown that starting with an empty trading system no pair of countries has an incentive to form a link. The additional loss in profit due to increased competition is higher than the additional profit in the foreign market. This suggests that the empty trading system is stable.

What can also be observed is that under the complete trading system no country i, $i \in N$ will sever any of its links with country j, $j \neq i \in N$, since the reduction of profits in market j is higher than the additional profit obtained due to lower competition in the domestic market. Complete proof is provided in the appendix.

Next we investigate what structures can emerge when welfare is given by consumer surplus such that $\beta = 1$, $\gamma = \delta = 0$. Country i's welfare is now:

$$Y_i(\mathcal{L}) = \frac{1}{2}[(\sum_{j \in N_i(\mathcal{L})} (\frac{(\alpha - c)}{(\eta_i(\mathcal{L}) + 1)} + (\frac{\sum_k t_i^k(\mathcal{L}) - (\eta_i(\mathcal{L}) + 1) \cdot t_i^j(\mathcal{L})}{(\eta_i(\mathcal{L}) + 1)})))^2].$$

When we consider a trading system without the global link we can show that for an arbitrary network $\mathcal{L}$ consumer surplus from an additional bilateral trade agreement between country i and country j with $\{(i,j)\} \notin \mathcal{L}$ is given by

$$\begin{aligned}&Y_i(\mathcal{L} \cup \{\{i,j\}\}) - Y_i(\mathcal{L})\\ =&\frac{1}{2}[\frac{(\eta_i(\mathcal{L}) + 1)^2(\alpha - c)^2}{(\eta_i(\mathcal{L}) + 2)^2} - \frac{(\eta_i(\mathcal{L}))^2(\alpha - c)^2}{(\eta_i(\mathcal{L}) + 1)^2}] > 0,\end{aligned}$$

since $t_i^j(\mathcal{L}) = T$ for any pair of countries without trade agreement and $t_i^j(\mathcal{L}) = 0$ with a bilateral link between i and j. This implies that an additional bilateral link is always profitable and countries form as many links as possible.

When countries have a multilateral trade agreement social welfare is given by:

$$Y_i(\mathcal{L}) = \frac{1}{2}(\frac{n(\alpha - c)}{(n+1)} - \frac{(n - \tilde{\eta}_i(\mathcal{L})) \cdot t}{(n+1)})^2,$$

where $\tilde{\eta}_i(\mathcal{L})$ denotes the number of countries that are bilaterally linked with country i under MFN where $i \in \tilde{\eta}_i(\mathcal{L})$ and $\tilde{\eta}_i(\mathcal{L}) =\mid \tilde{N}_i(\mathcal{L}) \mid$.

The first derivative implies: $\frac{\partial Y_i(\mathcal{L})}{\partial \tilde{\eta}_i(\mathcal{L})} = (\frac{n(\alpha-c)}{(n+1)} - \frac{(n-\tilde{\eta}_i(\mathcal{L}))\cdot t}{(n+1)}) \cdot (\frac{t}{n+1}) > 0$ with $t < \frac{(\alpha-c)}{3}$.

It is attractive for countries to form as many bilateral trade agreements as posssible under MFN.

Proposition 4.7. *When social welfare is given by consumer surplus, global free trade is stable.*

In the following we will assume that countries' welfare is given by tariff revenue such that

$$Y_i(\mathcal{L}) = \sum_{j \in N_i(\mathcal{L})} t_i^j(\mathcal{L}) \cdot q_i^j(\mathcal{L}).$$

Since the tariff between two countries in a bilateral link is zero and between two countries in a multilateral link is t, with $t > 0$, we have $Y_i(\mathcal{L}) = (\eta_i(\mathcal{L}) - \tilde{\eta}_i(\mathcal{L})) \cdot t \cdot (\frac{\alpha - c}{\eta_i(\mathcal{L})+1} + \frac{(\eta_i(\mathcal{L}) - \tilde{\eta}_i(\mathcal{L})) \cdot t - (\eta_i(\mathcal{L})+1) \cdot t}{\eta_i(\mathcal{L})+1})$, where $\tilde{N}_i(\mathcal{L})$ denotes the set of countries that have a bilateral trade agreement with country i under the global link, $i \in \tilde{N}_i(\mathcal{L})$ in the trading system $\mathcal{L}$. We observe that $\frac{\partial Y_i(\mathcal{L})}{\partial \tilde{\eta}_i(\mathcal{L})} < 0$ such that welfare decreases with the number of bilateral links. It is therefore intuitive that the global trading system maximizes welfare and the proof can be omitted. The severance of the global link results in the empty network and tariff revenue for each country is zero. Under these considerations we can conclude:

Proposition 4.8. *When countries only care about tariff revenue, the only stable trading system is the global network.*

Next we combine the analysis and allow arbitrary values for β, γ and δ. Therefore, under the general welfare function as given in (13):

Proposition 4.9. *Under general welfare as given by (13) the empty network, the global network and global free trade can be stable.*

This result follows directly from Propositions 4.6. – 4.8..
Intuitively, for certain values of δ, countries tend to maintain as few bilateral free trade agreements as possible and maintain the global link to receive as many tariff revenues as possible. The lower δ and the higher γ countries prefer no links at all, since the additional competition will decrease firms' profit and therefore countries will sever all their links. Intuitively, for very high values of β additional competition in the markets is profitable for consumer surplus and therefore countries will form the complete network. We can further show that a star network and the global link with a star network cannot be stable. However, the next two examples show that a

bilateral free trade agreement and a global link with a free trade agreement between one pair of countries can be stable.

Example 4.1. *We set* $\beta = \frac{1}{2}$, $\gamma = 1$ *and* $\delta = \frac{1}{4}$. *With* $(\alpha - c) = 4$ *and* $t = 1$ *we get* $Y_i(\{\{i,j\}\}) = Y_j(\{\{i,j\}\}) = \frac{16}{3} > 5 = Y_i(\mathcal{L}^e) = Y_j(\mathcal{L}^e)$. *Furthermore, we have for country* k *with* $k \neq i \neq j$ *that* $Y_k(\{\{i,j\}\}) = 5 > Y_k(\{\{i,j\},\{i,k\}\}) = \frac{656}{144}$ *and* $Y_k(\{\{i,j\}\}) = 5 > Y_k(\{\{i,j\}\} \cup \mathcal{L}^G) = \frac{67}{16}$ *such that the conditions for multilateral stability of* $\mathcal{L} = \{\{i,j\}\}$ *are fulfilled.*

Example 4.2. *We set* $\beta = \frac{1}{2}$, $\gamma = \frac{1}{2}$ *and* $\delta = \frac{3}{4}$. *With* $(\alpha - c) = 4$ *and* $t = 1$ *we have* $Y_i(\{\{i,j\}\} \cup \mathcal{L}^G) = Y_j(\{\{i,j\}\} \cup \mathcal{L}^G) = \frac{241}{64} > 3\frac{11}{16} = Y_i(\mathcal{L}^G) = Y_j(\mathcal{L}^G)$ *and* $Y_i(\{\{i,j\}\} \cup \mathcal{L}^G) = Y_j(\{\{i,j\}\} \cup \mathcal{L}^G) = \frac{241}{64} > \frac{64}{18} = Y_i(\{\{i,j\}\}) = Y_j(\{\{i,j\}\})$ *such that neither country* i *nor country* j *has an incentive to sever the global link. Furthermore, we can show for country* k *with* $k \neq i \neq j$ *that* $Y_k(\{\{i,j\}\} \cup \mathcal{L}^G) = \frac{56}{16} > Y_k(\{\{i,j\},\{i,k\}\} \cup \mathcal{L}^G) = 3\frac{25}{64}$ *and* $Y_k(\{\{i,j\}\} \cup \mathcal{L}^G) = \frac{56}{16} > Y_k(\{\{i,j\}\}) = 3$ *such that country* k *has no incentive to deviate. This proves that* $\mathcal{L} = (\mathcal{L}^G \cup \{\{i,j\}\})$ *is stable.*

4.3.2 Stability of Trading Structures and Endogenous Tariffs

In the following we shall address the question that we already raised in the introduction of the thesis, whether the formation of PTAs hinders or facilitates multilateral tariff cooperation. In particular, we analyze the effects of a PTA on multilateral tariffs.

In section 2 we have considered the case in which tariffs in a bilateral trade agreement and in a multilateral trade agreement are exogenously given. In reality countries negotiate tariffs and due to the most favoured nation clause (MFN) a country within the GATT levies the same tariffs on each GATT member. PTAs are tolerated under Article XXIV that permits the formation of a PTA between two GATT members when they eliminate tariffs against each other. Furthermore, as Bagwell and Staiger (1999a) have shown, optimal tariff choice depends crucially on the trading system. When countries trade under GATT they choose different optimal tariffs as in a trading system without GATT due to the non-discrimination principle (MFN) under GATT. A multilateral GATT agreement is represented by a multilateral trade

Figure 18: *a)* $\mathcal{L} = \{\{A, B\}, \{A, C\}, \{B, C\}\}$ without MFN.
b) $\mathcal{L} = \{\{A, B, C\}, \{A, B\}, \{B, C\}, \{A, C\}\}$ Global free trade under MFN.

agreement between all three countries ($L = \{A, B, C\}$). A bilateral trade agreement under GATT represents a PTA and tariffs are eliminated whereas in a bilateral trade agreement without GATT countries choose non-cooperative tariffs against each other.

We introduce the following setting: Countries choose their optimal tariffs $t(\mathcal{L})$ with respect to maximizing domestic welfare in a trading system $\mathcal{L}$. Given that all countries are linked multilaterally, they impose the same tariffs on each of the other countries. When countries form additional PTAs (here we consider FTAs) they eliminate tariffs against each other. When countries are linked bilaterally without a multilateral trade agreement (i.e. no GATT rules exist) countries choose their non-cooperative and welfare maximizing Nash tariff level. The different links are represented in Figure 18. In Figure 18*a*) each pair of countries is linked bilaterally without MFN such that in this trading setting each country imposes non-cooperative tariffs on the other markets. In Figure 18*b*) under MFN (represented by a global link) each pair of countries has signed a FTA and countries have eliminated tariffs. A FTA is represented by a dashed line between two countries. The optimal tariff depends on the trading structure and, based on the tariffs and trading structure, each country obtains a welfare level $Y_i(t(\mathcal{L}), \mathcal{L})$. A comparison of different welfare levels determines the set of stable trading systems. In this setting global free trade is represented in Figure 18*b*) by a global trading system in which each pair of countries has formed a FTA.

We next investigate the following questions: what are the optimal tariffs when countries choose their GATT tariffs with respect to maximizing the welfare level, and what are the stable trading structures? Moreover, do countries raise tariffs against third countries as they form additional PTAs? Throughout the analysis we assume that countries are symmetrical with respect to market size such that $\alpha_i = \alpha \ \forall i \in N$. Due to the MFN clause we impose for the tariff of the global link:

$$t_j^i(\mathcal{L}) = t_j^k(\mathcal{L}) = t_j(\mathcal{L}), \ \forall i, k, \neq j,$$

where $t_j(\mathcal{L})$ is the tariff that country j imposes on foreign firms in the trading system $\mathcal{L}$. When countries have a FTA within the GATT agreement they have zero tariffs due to Article XXIV. Social welfare is given by:

$$\begin{aligned} Y_i(t(\mathcal{L}), \mathcal{L}) =& \frac{1}{2}(\frac{n \cdot (\alpha - c) - (n - \tilde{\eta}_i(\mathcal{L})) \cdot t_i(\mathcal{L})}{(n+1)})^2 \\ &+ \sum_{j \notin \tilde{N}_i(\mathcal{L})} (\frac{(\alpha - c)}{(n+1)} - \frac{(\tilde{\eta}_j(\mathcal{L}) + 1) \cdot t_j(\mathcal{L})}{(n+1)})^2 \\ &+ \sum_{k \in \tilde{N}_i(\mathcal{L})} (\frac{(\alpha - c) + (n - \tilde{\eta}_k(\mathcal{L})) \cdot t_k(\mathcal{L}))}{(n+1)})^2 \\ &+ (n - \tilde{\eta}_i(\mathcal{L})) \cdot t_i(\mathcal{L}) (\frac{(\alpha - c) - (\tilde{\eta}_i(\mathcal{L}) + 1) \cdot t_i(\mathcal{L})}{(n+1)}), \end{aligned} \tag{14}$$

whereas $\tilde{N}_i(\mathcal{L})$ denotes the set of countries that have a FTA with country i in the trading system $\mathcal{L}$, $\mid \tilde{N}_i(\mathcal{L}) \mid = \tilde{\eta}_i(\mathcal{L})$, with $i \in \tilde{N}_i(\mathcal{L})$. With $n = 3$ country i's optimal tariff is given by:

$$t_i(\mathcal{L}) = \frac{3(\alpha - c)}{(11 \cdot \tilde{\eta}_i(\mathcal{L})) - 1}. \tag{15}$$

We observe that the tariffs on third parties within the GATT decrease with the number of FTAs that country i has, which contradicts the result of Krishna (1998), who suggests that PTAs lower countries' incentives for multilateral liberalization. We will provide an explanation for this result later in the section.

When there is no GATT agreement, countries non-cooperatively choose an external tariff to levy on those countries with whom they are linked bilaterally. Social welfare is given by:

$$Y_i(t(\mathcal{L}), \mathcal{L}) = \sum_{j \in N_i(\mathcal{L})} t_i^j(\mathcal{L}) \cdot (\frac{(\alpha - c)}{(\eta_i(\mathcal{L}) + 1)} + \frac{\sum_{k \in N_i(\mathcal{L})} t_i^k(\mathcal{L})}{(\eta_i(\mathcal{L}) + 1)} - t_i^j(\mathcal{L})) \tag{16}$$
$$+ \frac{1}{2}(\frac{\eta_i(\mathcal{L}) \cdot (\alpha - c)}{(\eta_i(\mathcal{L}) + 1)} - \frac{\sum_{k \in N_i(\mathcal{L})} t_i^k(\mathcal{L})}{(\eta_i(\mathcal{L}) + 1)})^2$$
$$+ \sum_{j \in N_i(\mathcal{L})} (\frac{(\alpha - c)}{(\eta_j(\mathcal{L}) + 1)} + \frac{\sum_{k \in N_j(\mathcal{L})} t_j^k(\mathcal{L})}{(\eta_j(\mathcal{L}) + 1)} - t_j^i(\mathcal{L}))^2$$

Countries choose optimal non-cooperative tariffs with respect to maximizing domestic welfare on the other countries with whom they are linked bilaterally. We can show that due to country symmetry $t_i^j(\mathcal{L}) = t_i^k(\mathcal{L}) = t_i(\mathcal{L})\ \forall j, k \in N_i(\mathcal{L}) \backslash \{i\}$:

$$t_i^*(\mathcal{L}) = \frac{3(\alpha - c)}{7 + \eta_i(\mathcal{L})}. \tag{17}$$

This implies that when the number of bilateral trading partners increases for country i, the tariffs levied on the foreign markets decrease. This effect is called the tariff complementarity effect and was first mentioned by Bagwell and Staiger (1999b), who showed that, as the tariffs imposed on the foreign market due to an additional bilateral trade agreement decrease, tariffs on third parties also decrease. Assuming that country i and j are linked bilaterally, when i forms a bilateral trade agreement with k, i automatically lowers tariffs on j.
With the optimal tariffs we are able to calculate welfare level for different trading structures and can completely characterize the nature of stable trading systems. With endogenous tariffs stability is given by:

Definition 4.3. *A trading system $\mathcal{L}$ on N is called multilaterally stable, if*

(i) $Y_i(t(\mathcal{L}), \mathcal{L}) \geq Y_i(t(\mathcal{L} \backslash \{L\}), \mathcal{L} \backslash \{L\})\ \forall L \in \mathcal{L},\ \ \forall i \in L$ *and*

(ii) $Y_i(t(\mathcal{L} \cup \{L\}), \mathcal{L} \cup \{L\}) > Y_i(t(\mathcal{L}), \mathcal{L}) \Rightarrow \exists j \in L,$
such that $Y_j(t(\mathcal{L} \cup \{L\}), \mathcal{L} \cup \{L\}) < Y_j(t(\mathcal{L}), \mathcal{L})\ \ \forall L \notin \mathcal{L}.$

Here, social welfare of country i depends on the optimal tariff choices of all countries.

Proposition 4.10. *Global free trade is stable. Each structure with a global link and a FTA between one pair of countries is stable.*

Hence the complete trading system cannot be stable anymore. The complete proof of the result is shown in the appendix. We first calculate welfare level for each structure and then compare them with respect to our stability notion to characterize the stable trading systems. Here, we offer a short intuition for the result. First, we investigate why the global link with a FTA between two countries can be stable, consider tariffs of each country within the global structure: $t_i(\mathcal{L}^G) = \frac{3(\alpha-c)}{10} > \frac{(\alpha-c)}{7} = t_i(\mathcal{L}^G \cup \{\{B,C\}\})$,$i = B, C$, which is larger than a global link with an additional FTA between, say, B and C. Tariffs on A decrease as B and C form a bilateral link. The negative relationship between the number of FTAs and the tariffs on third parties is due to different effects that an increase in tariffs has on the welfare level. We observe that with a higher number of FTAs an increase in tariffs generates a higher loss in consumer surplus. That means B and C prefer a lower t_B so that the loss in consumer surplus is smaller. To see this we calculate the first derivative of country B's welfare with respect to tariffs:

$$\begin{aligned}\frac{\partial Y_i(t(\mathcal{L}),\mathcal{L})}{\partial t_i(\mathcal{L})} =& -\frac{n-\tilde{\eta}_i(\mathcal{L})}{n+1}[\frac{n(\alpha-c)-(n-\tilde{\eta}_i(\mathcal{L}))t_i(\mathcal{L})}{(n+1)}] \\ &+ \frac{n-\tilde{\eta}_i(\mathcal{L})}{n+1}[\frac{2(\alpha-c)+2(n-\tilde{\eta}_i(\mathcal{L}))t_i(\mathcal{L})}{(n+1)}] \\ &+ \frac{n-\tilde{\eta}_i(\mathcal{L})}{n+1}[(\alpha-c)-2(\tilde{\eta}_i(\mathcal{L})+1)t_i(\mathcal{L})].\end{aligned}$$

Another effect of an increase in tariffs is on firm B's profit in its own market. The positive effect of a rise in tariffs is higher when the number of FTAs is lower, so that in the global trading system it is more attractive for countries to impose high tariffs. This can be seen in the second line of the above equation. Thus, when B and C form an additional FTA, it is less attractive to raise tariffs on A.

When we calculate the impact of a FTA between B and C on the welfare level of A, we can observe that due to lower external tariffs, country A's welfare increases.

$$\begin{aligned}&Y_A(t(\mathcal{L}^G \cup \{\{B,C\}\}), \mathcal{L}^G \cup \{\{B,C\}\}) - Y_A(t(\mathcal{L}^G), \mathcal{L}^G) \\ =&2\cdot(\frac{(\alpha-c)-\frac{3}{7}(\alpha-c)}{4})^2 - 2\cdot(\frac{(\alpha-c)-\frac{3}{5}(\alpha-c)}{4})^2 > 0.\end{aligned}$$

We can observe that due to decreasing tariffs of B and C on A, firm A's profit in market B and C increases. Thus the overall effect is positive.

We can also observe that country B and C improve by forming the bilateral link, since the free entry to the foreign market leads to an increase in both countries' payoffs.
The reduction of tariffs on country A and the resulting increase in welfare induces country A to maintain its global link with B and C, but an additional bilateral FTA with either of the two countries would reduce its welfare.
The complete trading system cannot be stable, since all countries will deviate and form a global link, because in the global trading system with a FTA between each pair of countries the GATT agreement reduces tariffs to zero such that all countries are better off and increase welfare.
Without the GATT agreement countries choose non-cooperative external tariffs on the other countries and this leads to a mutual reduction of welfare, whereas the GATT stabilizes the underlying structure and Article XXIV leads to an increase in each country's welfare. Furthermore, due to the MFN clause countries have to offer each member the same tariffs. The reduction of tariffs imposed on third parties, after a FTA is in place, suggests that PTAs increase countries' incentives for multilateral liberalization.

Definition 4.4. *A trading system $\mathcal{L}^*$ is said to be (strongly) efficient, if $v(\mathcal{L}) = \sum_{i \in N} Y_i(t(\mathcal{L}), \mathcal{L}) \leq \sum_{i \in N} Y_i(t(\mathcal{L}^*), \mathcal{L}^*) = v(\mathcal{L}^*) \; \forall \mathcal{L}$.*

We can further characterize efficient trading systems.

Proposition 4.11. *Global free trade is the unique efficient trading system.*

For any trading system $\mathcal{L}$ we can calculate the overall welfare level under optimal tariffs $t_i^j * (\mathcal{L})$. We can observe that the complete trading system is no longer efficient because tariffs in the complete trading system $t_i^*(\mathcal{L}^N) = \frac{3(\alpha - c)}{10}$ against foreign firms lead to a reduction of firm's profits and consumer surplus. The payoff for each country in a complete trading system is given by $\frac{21}{50}(\alpha - c)^2$ whereas in the efficient trading system total payoff for each country is given by $\frac{15}{32}(\alpha - c)^2$. With non-cooperative tariffs countries mutually reduce their welfare level.

4.3.3 Arbitrary Number of Countries

Let us provide an insight into possible implications for stable trading structures when we increase the number of countries and assume that countries are symmetrical with respect to market size. In this framework we allow the total set of countries to form a multilateral link $L = \{1, ..., n\}$ and each pair of countries to form a bilateral link. We further assume endogenous tariffs as calculated in section 3.2 of this chapter for an arbitrary number of countries.

We shall learn something about the implications under endogenous tariffs with respect to tariffs imposed on third parties. Can the complete trading system still be stable or do endogenous tariffs induce that a multilateral link is essential for stability (cf. Proposition 4.10)?

With equation (15) we could observe that tariffs on third parties decrease within GATT when the number of FTAs increases. For an arbitrary number of countries country i's optimal tariff is given by:

$$t_i(\mathcal{L}) = \frac{3(\alpha - c)}{(2n+5) \cdot \tilde{n}_i(\mathcal{L}) - (n-2)}. \tag{18}$$

The optimal tariffs in country i for an arbitrary number of countries without MFN remain as in equation (17) as they depend merely on the number of PTAs and not on the total number of countries. This equation suggests that with an increasing number of PTAs of country i the tariffs on third parties decrease. These observations are helpful to understand the implications for the general case with $n \geq 3$.

The trading system with a single multilateral link is again the global trading system with $\mathcal{L}^G = \{\{1, 2, ..., n\}\}$. First we show that free trade is still a stable state and that the complete trading system cannot be stable.

Proposition 4.12. *For an arbitrary number of countries, global free trade is a stable trading system.*

The proof proceeds in the way that it first demonstrates that starting from the stable trading system a country decreases welfare under the GATT regime when it severs any of its FTAs. Secondly, it shows that all countries are worse off without GATT in the complete trading system and this completes the proof. Therefore it is obvious that starting from a complete trading system all countries have an incentive

to form a multilateral link and the complete trading system is not stable.
We can further conclude that for an arbitrary number of countries neither the empty nor the global trading system can be stable. We show that the empty trading system cannot be stable since a pair of countries will deviate and form a FTA with:

$$Y_i(t(\{\{i,j\}\}),\{\{i,j\}\}) - Y_i(t(\mathcal{L}^e),\mathcal{L}^e) = \frac{1}{2}(\frac{2(\alpha-c)-\frac{3}{9}(\alpha-c)}{3})^2$$
$$+(\frac{(\alpha-c)+\frac{3}{9}(\alpha-c)}{3})^2+(\frac{(\alpha-c)+\frac{3}{9}(\alpha-c)-(\alpha-c)}{3})^2+(\frac{(\alpha-c)+\frac{3}{9}(\alpha-c)}{3})^2$$
$$+(\frac{3(\alpha-c)}{9})(\frac{(\alpha-c)+\frac{3}{9}(\alpha-c)-(\alpha-c)}{3})-\frac{3}{8}(\alpha-c)^2>0.$$

Two countries get a higher payoff when they form a trade agreement.
Under GATT without any FTAs it can be shown that countries have an incentive to form an additional FTA:

$$Y_i(t(\mathcal{L}^G\cup\{\{i,j\}\}),\mathcal{L}^G\cup\{\{i,j\}\}) - Y_i(t(\mathcal{L}^G),\mathcal{L}^G)$$
$$=\frac{1}{2}\cdot(\frac{n\cdot(\alpha-c)-(n-2)\cdot(\frac{3(\alpha-c)}{2(2n+5)-(n-2)})}{(n+1)})^2+(n-2)\cdot(\frac{(\alpha-c)-2\cdot(\frac{3\cdot(\alpha-c)}{(2n+5)-(n-2)})}{(n+1)})^2$$
$$+2(\frac{(\alpha-c)+(\frac{3(\alpha-c)}{2(2n+5)-(n-2)})(n-2)}{(n+1)})^2$$
$$+(n-2)(\frac{3(\alpha-c)}{2(2n+5)-(n-2)})(\frac{(\alpha-c)-3(\frac{3(\alpha-c)}{2(2n+5)-(n-2)})}{(n+1)})$$
$$-\frac{1}{2}(\frac{n(\alpha-c)-(n-1)(\frac{3(\alpha-c)}{(2n+5)-(n-2)})}{(n+1)})^2-(n-1)(\frac{(\alpha-c)-2(\frac{3(\alpha-c)}{(2n+5)-(n-2)})}{(n+1)})^2$$
$$-(\frac{(\alpha-c)+(n-1)(\frac{3(\alpha-c)}{(2n+5)-(n-2)})}{(n+1)})^2$$
$$-(n-1)(\frac{3(\alpha-c)}{(2n+5)-(n-2)})(\frac{(\alpha-c)-2(\frac{3(\alpha-c)}{(2n+5)-(n-2)})}{(n+1)})$$
$$=3(\alpha-c)^2\frac{5n+32}{(n+7)^2(n+4)^2}>0.$$

When countries are linked multilaterally they tend to form additional FTAs and a global trading system without FTAs cannot be stable.
It is difficult to characterize the full set of stable trading systems but we can observe two more features to narrow the set of possible stable states:

- Under GATT the trading system in which each pair of countries has a bilateral FTA is the unique stable trading system in the class of symmetrical trading systems.

- GATT is necessary for stability.

The second observation also means that each stable trading system includes the MFN principle and no country has an incentive to cancel the GATT. We denote a trading system $\mathcal{L}$ to be symmetrical if each country has the same number of bilateral links such that $\tilde{\eta}_i(\mathcal{L}) = \tilde{\eta}_j(\mathcal{L})$ $\forall i, j \in N$ under MFN.
The proof of the first observation is similar to the proof of Proposition 8 in Goyal and Joshi (2006a) and can be omitted. The proof of the second observation is shown in the appendix and proceeds in the way that it shows that without MFN an additional bilateral trade agreement among two countries always increases welfare for both countries. In the following we shall analyze the dimension of the welfare effect without MFN when two countries i and j decide to form a bilateral trade agreement. We therefore calculate the expression $Y_i(t(\mathcal{L} \cup \{i,j\}), \mathcal{L} \cup \{i,j\}) - Y_i(t(\mathcal{L}), \mathcal{L})$. We use a simulation with $n = 100$ and $(\alpha - c) = 100$. The results are plotted in Figure 19. The simulation suggests that a bilateral trade agreement is always beneficial for all values of $\eta_i(\mathcal{L})$. In Figure 19a) the welfare effect is plotted when the number of active firms in the foreign market before the bilateral trade agreement with i is given by $\eta_j(\mathcal{L}) = 1$ whereas in Figure 19b) we set the number of firms in the foreign market before the bilateral trade agreement to $\eta_j(\mathcal{L}) = 99$. The simulation demonstrates that a welfare effect from an additional trade agreement is highest when the number of active firms in the domestic and in the foreign market is very low. With this in mind we can conclude:

Proposition 4.13. *In the class of symmetrical trading systems global free trade is the unique stable trading structure.*

This of course confirms the observation that neither the empty nor the global trading system can be stable.

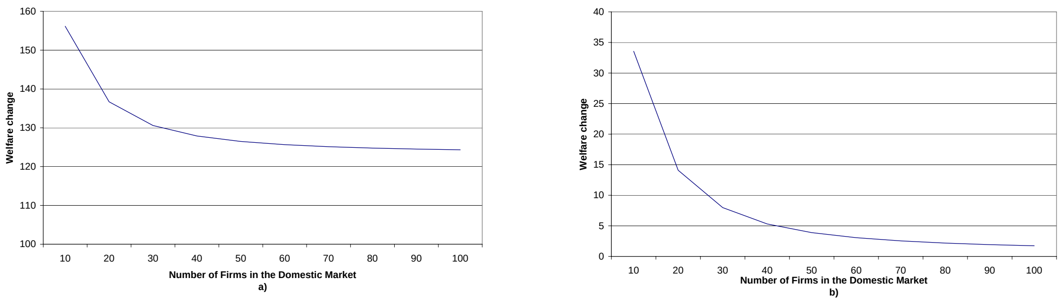


Figure 19: Welfare change from an additional bilateral trade agreement.

4.4 Conclusion

What structures will emerge when countries have the opportunity to form multilateral and bilateral trade agreements? Will the increasing number of PTAs lead to a more open multilateral trading system when we consider strategic link formation of countries? We have used a network formation approach to answer these questions and introduced an equilibrium concept called multilateral stability to investigate stable trading structures where countries can form multilateral as well as bilateral links. The idea of multilateral stability is that countries can form bilateral and multilateral trade agreements which include more than two countries. The formation of a new links requires the consent of all countries included but severance can be done unilaterally.

We have used a three-country model of imperfect competition, with a single firm in each country producing a homogenous good. Each firm competes as a Cournot oligopolist in each market and markets in the different countries are assumed to be perfectly segmented. Welfare gains from trade are obtained by increased competition in the countries. We have shown that if countries are asymmetrical with respect to market size, a complete trading system with a multilateral link is multilaterally stable. We investigate different cases with respect to market size to analyze what effect heterogeneity can have on countries' linking strategies. It was shown that trading systems, that are not efficient, can be the result of a strategic linking formation process. However, global free trade is always an efficient state.

When we introduce endogenous tariffs, we find that the complete trading system cannot be stable, since countries choose their tariffs non-cooperatively whereas in a multilateral trade agreement, as under the GATT, countries have to eliminate tariffs as they form a bilateral trade agreement. This increases countries' welfare level and a multilateral trade agreement stabilizes the world trading system. This result suggests that PTAs can coexist with multilateral liberalization and, moreover, the GATT is necessary for stability. This model shows that the GATT and its MFN clause still play an important role in stabilizing the world trading system

Although this is a simple three-country setting, we think it provides valuable insights into countries' decisions to form trade agreements. There are three different welfare effects that a trade agreement has on an importing country. By investigating the welfare effects of different trading agreements, we can fully describe the set of stable states. When we analyze the effect of a FTA on multilateral tariffs we find that, as a country increaseas its number of PTAs, the tariffs it imposes on third countries decrease.

In this chapter we characterized strategic and efficient trading systems when countries are asymmetrical with respect to market size.
During the current regionalism debate yet another issue may occur. As it is often argued that we hardly observe free trade agreements among low and high-income countries we will in the next chapter investigate countries' incentives to form FTAs and to liberalize multilaterally when we consider heterogeneity with respect to income level. Furthermore, it is argued that the formation of regional trade agreements among developed countries may undermine the multilateral liberalization with low-income countries and less-developed countries are inferior in the world trading system. The next chapter analyzes the formation and stability of trading systems when countries are asymmetrical with respect to income.
Furthermore, it should be noted that multilateral tariff negotiations often take place after the trading system is in place. In chapter 1 we extensively reviewed the role of

the multilateral trading rounds in the world trading system and their current failure during the Doha round negotiations. Furthermore, it was stated that China as a relatively strong bilaterally linked country blocks multilateral tariff reduction whereas countries like Brazil, that are included in very few regional trade agreements, are trying to push WTO members towards an agreement that further reduces multilateral tariffs. This gives reasons to believe that countries may differ in their incentives to force multilateral liberalization as their trading structure and thus the number of regional trade agreements differs. The following chapter presents an international trade model based on the model of Bagwell and Staiger (1999b) that we presented in chapter 2. We model a bargaining stage over multilateral tariffs after the trading system is in place and investigates countries incentives for multilateral cooperation subject to the network of trading agreements.

4.5 Appendix

First we report the welfare levels of countries $i, j, k \in \{A, B, C\}$ with $i \neq j \neq k$ and the overall welfare level of different trading structures.

The empty trading system

$Y_i(\mathcal{L}^e) = \frac{3(\alpha_i - c)^2}{8}$.

The star trading system with hub country i

$Y_i(\mathcal{L}_i^S) = \frac{11(\alpha_i - c)^2}{32} + \frac{(\alpha_j - c)^2}{9} + \frac{(\alpha_k - c)^2}{9}$ $\quad Y_j(\mathcal{L}_i^S) = \frac{(\alpha_j - c)^2}{3} + \frac{(\alpha_i - c)^2}{16}$.
$Y_k(\mathcal{L}_i^S) = \frac{(\alpha_k - c)^2}{3} + \frac{(\alpha_i - c)^2}{16}$.

The global trading system

$Y_i(\mathcal{L}^G) = \frac{11(\alpha_i - c)^2}{32} + \frac{3(\alpha_i - c) \cdot t}{8} + \frac{(\alpha_j - c)^2}{16} + \frac{(\alpha_k - c)^2}{16} - \frac{(\alpha_j - c) \cdot t}{4} - \frac{(\alpha_k - c) \cdot t}{4} - \frac{t^2}{8}$.

The complete trading system

$Y_i(\mathcal{L}^N) = \frac{11(\alpha_i - c)^2}{32} + \frac{(\alpha_j - c)^2}{16} + \frac{(\alpha_k - c)^2}{16}$.

Bilateral trade agreement between i and j

$Y_i(\{\{i, j\}\}) = \frac{(\alpha_i - c)^2}{3} + \frac{(\alpha_j - c)^2}{9}$ $\quad Y_j(\{(i, j)\}) = \frac{(\alpha_j - c)^2}{3} + \frac{(\alpha_i - c)^2}{9}$.
$Y_k(\{\{i, j\}\}) = \frac{3(\alpha_k - c)^2}{8}$.

Global trading system with a bilateral trade agreement between i and j

$Y_k(\mathcal{L}^G \cup \{\{i,j\}\}) = \frac{(\alpha_i-c)^2}{16} + \frac{(\alpha_j-c)^2}{16} + \frac{11(\alpha_k-c)^2}{32} + \frac{3(\alpha_k-c)\cdot t}{8} - \frac{3(\alpha_i-c)\cdot t}{8} - \frac{3(\alpha_j-c)\cdot t}{8} + \frac{t^2}{2}$.
$Y_i(\mathcal{L}^G \cup \{\{i,j\}\}) = \frac{11(\alpha_i-c)^2}{32} + \frac{(\alpha_j-c)^2}{16} + \frac{(\alpha_k-c)^2}{16} + \frac{3(\alpha_i-c)t}{16} + \frac{4(\alpha_j-c)t}{32} - \frac{8(\alpha_k-c)t}{32} - \frac{11t^2}{32}$.
$Y_j(\mathcal{L}^G \cup \{\{i,j\}\}) = \frac{11(\alpha_j-c)^2}{32} + \frac{(\alpha_i-c)^2}{16} + \frac{(\alpha_k-c)^2}{16} + \frac{3(\alpha_j-c)t}{16} + \frac{4(\alpha_i-c)t}{32} - \frac{8(\alpha_k-c)t}{32} - \frac{11t^2}{32}$.

Global trading system with a star trading system with hub country i

$Y_i(\mathcal{L}^G \cup \mathcal{L}_i^S) = \frac{(\alpha_j-c)^2}{16} + \frac{(\alpha_k-c)^2}{16} + \frac{11(\alpha_i-c)^2}{32} + \frac{2(\alpha_j-c)\cdot t}{16} + \frac{2(\alpha_k-c)\cdot t}{16} + \frac{4\cdot t^2}{32}$.
$Y_j(\mathcal{L}^G \cup \mathcal{L}_i^S) = \frac{11(\alpha_j-c)^2}{32} + \frac{(\alpha_i-c)^2}{16} + \frac{(\alpha_k-c)^2}{16} + \frac{6(\alpha_j-c)\cdot t}{32} - \frac{3(\alpha_k-c)\cdot t}{8} - \frac{3\cdot t^2}{32}$.
$Y_k(\mathcal{L}^G \cup \mathcal{L}_i^S) = \frac{11(\alpha_k-c)^2}{32} + \frac{(\alpha_i-c)^2}{16} + \frac{(\alpha_j-c)^2}{16} + \frac{3(\alpha_k-c)\cdot t}{16} - \frac{3(\alpha_j-c)\cdot t}{8} - \frac{3\cdot t^2}{32}$.

Proof of Proposition 4.1. For the proof of Proposition 3.1 consider that in this case $\alpha_i = \alpha\ \forall i$. We obtain with countries' welfare levels as calculated above:
The empty trading system cannot be stable, since each pair of countries has an incentive to deviate and with $Y_i(\mathcal{L}^e) = \frac{3(\alpha-c)^2}{8} < Y_i(\{\{i,j\}\}) = \frac{4(\alpha-c)^2}{9} < \frac{163(\alpha-c)^2}{288} = Y_i(\{\{i,j\},\{i,k\}\})$ and with $Y_k(\{\{i,j\},\{i,k\}\}) > Y_k(\{\{i,j\}\})\ \forall i,j,k\ i \neq j \neq k$ the star network with hub country i is formed.
The complete trading system is formed since $Y_k(\{\{i,j\},\{i,k\}\}) < Y_k(\mathcal{L}^N) = \frac{15(\alpha-c)^2}{32}$ and $Y_j(\{\{i,j\},\{i,k\}\}) < Y_j(\mathcal{L}^N) = \frac{15(\alpha-c)^2}{32}$. In the global trading system with a bilateral trade agreement between each pair of countries each country gets the same payoff as in the complete trading system. Furthermore, it is stable since no country has an incentive to sever any of its bilateral links. Thus the complete trading system and the global trading system with a bilateral trade agreement between each pair of countries are stable. To show that the global link with a bilateral trade agreement between B and C can be stable consider that country A does not have an incentive to form an additional bilateral link with either B or C if $19t > 6(\alpha - c)$. Furthermore, for country A to keep the global link requires $3(\alpha-c)^2 + 16t^2 > 12t(\alpha-c)$. And for country B (respectively C) to maintain the global link requires $7(\alpha-c)^2 + 18(\alpha-c)t > 99t^2$. For value $(\alpha - c) = 1$ and $t = 0,33$ it can be verified that all conditions for stability are satisfied. The global link is not stable since at least one pair of countries

improves by forming a bilateral link. □

Proof of Lemma 4.1. In the following we will compare country B's and country C's welfare level in a global trading system with the welfare they get when they form an additional bilateral link. To prove that the global trading system is not stable we have to show that $Y_i(\mathcal{L}^G) < Y_i(\mathcal{L}^G \cup \{\{C,B\}\})$ for all $i \in \{B,C\}$. This induces:

$$\frac{4(\alpha-c)\cdot t}{32} + \frac{11\cdot t^2}{32} - \frac{10(\alpha-c)\cdot t}{32} - \frac{4\cdot t^2}{32} < 0 \iff 7\cdot t < 6(\alpha-c).$$

Since $\frac{(\alpha-c)}{3} > t$, this equation is always fulfilled. Hence B and C will deviate and form an additional bilateral trade agreement such that condition (ii) of multilateral stability is not satisfied. □

Proof of Proposition 4.2. As shown in Lemma 4.1 a global trading system cannot be stable. The empty trading system is not stable since, as in the global structure, countries B and C will deviate and form a bilateral trade agreement with:

$$Y_B(\mathcal{L}^e) = Y_C(\mathcal{L}^e) = \frac{3(\alpha-c)^2}{8} < \frac{4(\alpha-c)^2}{9} = Y_B(\{\{B,C\}\}) = Y_C(\{\{B,C\}\}).$$

Country A will not form an additional bilateral trade agreement with B if

$$Y_A(\{\{B,C\}\}) > Y_A(\{\{B,C\},\{A,B\}\}) \iff (\alpha_A - c)^2 > 1,5(\alpha-c)^2,$$

whereas A will not have an incentive to form a global link with countries C and B if:

$$\begin{aligned} & Y_A(\{\{B,C\}\}) > Y_A(\mathcal{L}^G \cup \{\{B,C\}\}) \\ \iff & 12(\alpha_A - c)\cdot t + 4(\alpha-c)^2 + 16\cdot t^2 < 24(\alpha-c)\cdot t + (\alpha_A - c)^2. \end{aligned}$$

These conditions are sufficient for stability of $\mathcal{L} = \{\{B,C\}\}$ to hold.
If the reverse is true, then country A would like to form the global link. This is a necessary condition for stability of the global trading system with a bilateral trade agreement between B and C.
If A also has an incentive to form a bilateral link with B, that is $4(\alpha_A - c)^2 \leq 6(\alpha-c)^2$, A and C will also form a bilateral trade agreement and the complete trading system is reached. This is stable, since no country will have an incentive to sever any of its

bilateral links and the severance of $L = \{A, B, C\}$ will not make any of the countries better off. Since $Y_i(\mathcal{L}^N) = Y_i(\mathcal{L}^N \cup \mathcal{L}^G) \ \ \forall\, i \in N$ the complete trading system with a global link is also stable. □

Proof of Lemma 4.2. For a bilateral trade agreement between B and C to be stable we need the condition that country A and B will not have an incentive to form an additional bilateral link. In the following we will demonstrate that both countries will gain from an additional bilateral trade agreement between them. With $\alpha_A < \alpha$ we can show:

$$Y_A(\{\{A,B\}\}) = \frac{3(\alpha_A - c)^2}{8} < \frac{(\alpha_A - c)^2}{3} + \frac{(\alpha - c)^2}{16} = Y_A(\{\{A,B\},\{B,C\}\}).$$

For country B:

$$Y_B(\{\{B,C\}\}) = \frac{4(\alpha - c)^2}{9} < \frac{11(\alpha - c)^2}{32} + \frac{(\alpha_A - c)^2}{9} + \frac{(\alpha - c)^2}{9} = Y_B(\{\{A,B\},\{B,C\}\}),$$

such that A and B have an incentive to form an additional bilateral link and condition (ii) of multilateral stability is not satisfied.
For the global trading system with a bilateral trade agreement between B and C to be stable, the following four conditions have to be fulfilled:

(i) $6(\alpha - c) \cdot t > 15 \cdot t^2 + 12(\alpha_A - c) \cdot t$,

(ii) $18(\alpha_A - c)^2 + 90(\alpha - c) \cdot t > 11(\alpha - c)^2 + 72(\alpha_A - c) \cdot t + 99 \cdot t^2$,

(iii) $6(\alpha_A - c) \cdot t + 19 \cdot t^2 > 12(\alpha - c) \cdot t$,

(iv) $12(\alpha_A - c) \cdot t + 4(\alpha - c)^2 + 16 \cdot t^2 > 24(\alpha - c) \cdot t + (\alpha_A - c)^2$.

From condition (i) and (iii):

$$6(\alpha_A - c) \cdot t + 19 \cdot t^2 > 12(\alpha - c) \cdot t > 30 \cdot t^2 + 24(\alpha_A - c) \cdot t,$$

which results in a contradiction. Therefore, the global network with a bilateral trade agreement between B and C cannot be a stable structure. □

Proof of Proposition 4.3. We start with an empty trading system which cannot be stable since B and C have an incentive to deviate and form a bilateral trade agreement. As shown in Lemma 4.2 this cannot be stable either and thus an additional bilateral link between B and A is formed. A bilateral trade agreement between B and C and B and A cannot be stable. We show that A and C have an incentive to form a link:

$$Y_C(\mathcal{L}^N) = \frac{13(\alpha - c)^2}{32} + \frac{(\alpha_A - c)^2}{16} > \frac{19(\alpha - c)^2}{48}$$

and

$$Y_A(\mathcal{L}^N) = \frac{11(\alpha_A - c)^2}{32} + \frac{(\alpha - c)^2}{8} > \frac{(\alpha_A - c)^2}{3} + \frac{(\alpha - c)^2}{16},$$

and thus the complete trading system is formed, where $Y_i(\mathcal{L}^N) = Y_i(\mathcal{L}^N \cup \mathcal{L}^G)\ \ \forall\, i \in N$. None of the countries has an incentive to sever one of their links and we can thus conclude that this is a stable structure. The global trading system cannot be stable as shown in Lemma 4.1. With

$$Y_B(\{\{A, B\}\}) = \frac{(\alpha - c)^2}{3} + \frac{(\alpha_A - c)^2}{9} < Y_B(\mathcal{L}^S_B) = \frac{11(\alpha - c)^2}{32} + \frac{(\alpha_A - c)^2}{9} + \frac{(\alpha - c)^2}{9}$$

and

$$Y_C(\{B, A\}\}) = \frac{3(\alpha - c)^2}{8} < Y_C(\mathcal{L}^S_B) = \frac{19(\alpha - c)^2}{48}$$

a bilateral trade agreement between B and A (respectively C and A) cannot be stable. This completes the proof. □

Proof of Proposition 4.4. Condition (ii) of multilateral stability is trivially satisfied, since adding the global link makes no country better off. Condition (i) is satisfied, since the severance of any of the existing links will result in a star trading system, where the payoff for any of the two spoke countries i is given by $\frac{1}{3}(\alpha_i - c)^2 + \frac{1}{16}(\alpha_j - c)^2$ with hub country j, which is smaller than $Y_i(\mathcal{L}^N) = \frac{11}{32}(\alpha_A - c)^2 + \frac{1}{16}(\alpha_j - c)^2 + \frac{1}{16}(\alpha_k - c)^2\ \forall i \neq j \neq k$ such that the complete trading system is stable. □

Proof of Proposition 4.5. Total welfare of the complete trading system is given by:

$$\sum_{i \in N} Y_i(\mathcal{L}^N) = \sum_{i \in N} \frac{1}{2}(\frac{(\alpha_i - c)n)}{n+1})^2 + \sum_{i \in N} \sum_{j \in N_i(\mathcal{L}^N)} (\frac{(\alpha_j - c)}{(n+1)})^2.$$

By comparison, in an arbitrary trading system the total welfare is given by:

$$\begin{aligned}\sum_{i\in N} Y_i(\mathcal{L}^N) = \sum_{i\in N}[\frac{1}{2}((\frac{(\alpha_i - c)\eta_i(\mathcal{L})}{\eta_i(\mathcal{L})+1})^2 + (\sum_{j\in N_i(\mathcal{L})} \frac{\sum_{k\in N} t_i^k(\mathcal{L}) - (\eta_i(\mathcal{L})+1)t_i^j(\mathcal{L})}{\eta_i(\mathcal{L})+1})^2 \\ +2\cdot \sum_{j\in N_i(\mathcal{L})} \frac{(\alpha_i - c)(\sum_{k\in N} t_i^k(\mathcal{L}) - (\eta_i(\mathcal{L})+1)t_i^j(\mathcal{L}))}{\eta_i(\mathcal{L})+1}) \\ + \sum_{j\in N_i(\mathcal{L})} [(\frac{(\alpha_j - c)}{(\eta_j(\mathcal{L})+1)} + \frac{\sum_{k\in N} t_j^k(\mathcal{L}) - (\eta_j(\mathcal{L})+1)t_j^i(\mathcal{L})}{\eta_j(\mathcal{L})+1}) \\ (\frac{(\alpha_j - c) + \sum_{k\in N} t_j^k(\mathcal{L}) - (\eta_j(\mathcal{L})+1)t_j^i(\mathcal{L})}{\eta_j(\mathcal{L})+1})] \\ + \sum_{j\in N_i(\mathcal{L})} t_i^j(\mathcal{L})(\frac{(\alpha_i - c) + \sum_{k\in N} t_i^k(\mathcal{L}) - (\eta_i(\mathcal{L})+1)t_i^j(\mathcal{L})}{\eta_i(\mathcal{L})+1})],\end{aligned}$$

where the first two lines represent consumer surplus, the third and fourth line firms' profit and the last line tariff revenues. With $\sum_{i\in N}\sum_{j\in N_i(\mathcal{L})}(\sum_{k\in N} t_j^k(\mathcal{L}) - (\eta_j(\mathcal{L}) + 1)t_j^i(\mathcal{L})) \leq 0$ for an arbitrary trading system $\mathcal{L}$ we can directly see that the complete trading system maximizes total welfare when players are linked multilaterally, as $\eta_i(\mathcal{L}) = n\ \forall i$.

Without the multilateral link global welfare is given by:

$$\begin{aligned}\sum_{i\in N} Y_i(\mathcal{L}) &= \sum_{i\in N}[\frac{1}{2}(\frac{(\alpha_i - c)\eta_i(\mathcal{L})}{\eta_i(\mathcal{L})+1})^2 + \sum_{j\in N_i(\mathcal{L})}(\frac{(\alpha_j - c)}{(\eta_j(\mathcal{L})+1)})^2] \\ &= \sum_{i\in N} \frac{(\alpha_i - c)^2[\eta_i(\mathcal{L})^2 + 2\eta_i(\mathcal{L})]}{2(\eta_i(\mathcal{L})+1)^2} \leq \sum_{i\in N} Y_i(\mathcal{L}^N).\end{aligned}$$

□

Proof of Proposition 4.6. Welfare in the empty trading system is given by: $Y_i(\mathcal{L}^e) = \frac{(\alpha-c)^2}{4}$. Addition of a bilateral trade agreement between any pair of countries levies: $Y_i(\mathcal{L}^e) - Y_i(\{\{i,j\}\}) = \frac{(\alpha-c)^2}{4} - \frac{2\cdot(\alpha-c)^2}{9} > 0$ such that no bilateral link is formed. For the global link we have: $Y_i(\mathcal{L}^e) - Y_i(\mathcal{L}^G) = \frac{(\alpha-c)^2}{4} - \frac{3\cdot(\alpha-c)^2}{16} - \frac{3}{4}\cdot t^2 + \frac{t\cdot(\alpha-c)}{4} > 0$ with $t < \frac{(\alpha-c)}{3}$. This implies that the empty trading system is stable.

Now consider a star trading system with hub country A. Players B and C's payoff is given by $\frac{(\alpha-c)^2}{16} + \frac{(\alpha-c)^2}{9}$. In the complete trading system each country obtains a payoff of $\frac{3\cdot(\alpha-c)}{16}$. In the global trading system with a star trading system with hub

country A, country B and C's payoffs are given by: $\frac{3\cdot(\alpha-c)^2}{16} + \frac{5}{8}\cdot t^2 - \frac{t\cdot(\alpha-c)}{4} < \frac{3\cdot(\alpha-c)^2}{16}$ such that the complete trading system with a global link is formed.
A global link with a bilateral trade agreement between a pair of countries is not stable since the country without any bilateral link has an incentive to form a bilateral trade agreement with $\frac{3\cdot(\alpha-c)^2}{16} + \frac{22}{16}\cdot t^2 - \frac{t\cdot(\alpha-c)}{2} < \frac{3\cdot(\alpha-c)^2}{16} + \frac{5}{8}\cdot t^2 - \frac{t\cdot(\alpha-c)}{4}$ since $t < \frac{(\alpha-c)}{3}$. □

Proof of Proposition 4.10. In order to calculate each country's welfare level for a given trading system we can insert the optimal tariff level under GATT (equation(15)) and the optimal tariff level without GATT (equation (17)) into formula (14) and (16), respectively. With $t_i(\mathcal{L}^N) = \frac{3(\alpha-c)}{10}$ we get that $Y_i(t(\mathcal{L}^N), \mathcal{L}^N) = \frac{21}{50}(\alpha-c)^2$ for all i whereas tariffs are zero in the global link with a FTA between each pair of countries and $Y_i(t(\mathcal{L}^G \cup \mathcal{L}^N), \mathcal{L}^G \cup \mathcal{L}^N) = \frac{15}{32}(\alpha-c)^2 \ \ \forall i$ such that all countries are better off under the GATT regime and the complete trading system cannot be stable. Furthermore, $Y_i(t(\mathcal{L}^G \cup \{\{B,C\}\}), \mathcal{L}^G \cup \{\{B,C\}\}) = \frac{2199}{4900}(\alpha-c)^2 \ \forall i \in \{B,C\}$ and $Y_A(t(\mathcal{L}^G \cup \{\{B,C\}\}), \mathcal{L}^G \cup \{\{B,C\}\}) = \frac{108}{245}(\alpha-c)^2$ and a FTA with country B will reduce country A's payoff. Moreover, a single bilateral trade agreement between any two countries is not stable since all countries are better off under the GATT regime with $Y_i(t(\{\{i,j\}\}), \{\{i,j\}\}) = \frac{4}{9}(\alpha-c)^2 < \frac{2199}{4900}(\alpha-c)^2 \ \forall i,j$ and $Y_k(t(\{\{i,j\}\}), \{\{i,j\}\}) = \frac{3}{8}(\alpha-c)^2 < \frac{108}{245}(\alpha-c)^2$. With $Y_k(t(\mathcal{L}^G \cup \{\{i,j\}\}), \mathcal{L}^G \cup \{\{i,j\}\}) = \frac{108}{245}(\alpha-c)^2 > \frac{345}{784}(\alpha-c)^2 = Y_k(t(\mathcal{L}^G \cup \{\{i,j\},\{j,k\}\}), \mathcal{L}^G \cup \{\{i,j\},\{j,k\}\})$ a global link with a FTA between country i and j is stable.
The empty trading system is not stable since any arbitrary pair of countries has an incentive to deviate with $Y_i(t(\{\{i,j\}\}), \{\{i,j\}\}) = \frac{4}{9}(\alpha-c)^2 > \frac{3}{8}(\alpha-c)^2 = Y_i(t(\mathcal{L}^e), \mathcal{L}^e)$.

□

Proof of Proposition 4.12. We show that under the GATT regime each country's payoff decreases from the severance of any of its FTAs. With $t_i^*(\mathcal{L}^G \cup \mathcal{L}^N \backslash \{\{i,j\}\}) =$

$t_j^*(\mathcal{L}^G \cup \mathcal{L}^N \backslash \{\{i,j\}\}) = \frac{3\cdot(\alpha-c)}{11\cdot(n-1))-1}$ we have:

$$
\begin{aligned}
&Y_i(t(\mathcal{L}^G \cup \mathcal{L}^N), \mathcal{L}^G \cup \mathcal{L}^N) - Y_i(t(\mathcal{L}^G \cup \mathcal{L}^N \backslash \{\{i,j\}\}), \mathcal{L}^G \cup \mathcal{L}^N \backslash \{\{i,j\}\})\\
=&\frac{1}{2}(\frac{n(\alpha-c)}{n+1})^2 + n\cdot(\frac{(\alpha-c)}{n+1})^2 - \frac{1}{2}(\frac{n(\alpha-c)-(\frac{3\cdot(\alpha-c)}{11\cdot(n-1))-1})}{n+1})^2\\
-&(\frac{(\alpha-c)-((n-1)+1)\cdot(\frac{3\cdot(\alpha-c)}{11\cdot(n-1))-1})}{n+1})^2 - (n-2)\cdot(\frac{(\alpha-c)}{n+1})^2\\
-&(\frac{(\alpha-c)+(\frac{3\cdot(\alpha-c)}{11\cdot(n-1))-1})}{n+1})^2 - (\frac{3\cdot(\alpha-c)}{11\cdot(n-1))-1})\cdot(\frac{(\alpha-c)-n\cdot(\frac{3\cdot(\alpha-c)}{11\cdot(n-1))-1})}{n+1}) > 0
\end{aligned}
$$

with $n \geq 3$. Without GATT we can conclude that countries in the complete trading system have an incentive to form a multilateral link. Therefore, the complete trading system is not stable. With $t_i^i = 0$ and $t_i^j(\mathcal{L}^N) = \tilde{t}\ \forall j \neq i$:

$$
\begin{aligned}
Y_i(t(\mathcal{L}^N), \mathcal{L}^N) = &\frac{1}{2}(\frac{n(\alpha-c)}{n+1} - \frac{(n-1)\cdot\tilde{t}}{n+1})^2 + (n-1)\cdot(\frac{(\alpha-c)}{n+1} - \frac{2\cdot\tilde{t}}{n+1})^2\\
&+(n-1)\cdot\tilde{t}(\frac{(\alpha-c)}{n+1} - \frac{2\cdot\tilde{t}}{n+1}) + (\frac{(\alpha-c)+(n-1)\cdot\tilde{t}}{n+1})^2.
\end{aligned}
$$

We can therefore show that

$$
\begin{aligned}
&Y_i(t(\mathcal{L}^G \cup \mathcal{L}^N), \mathcal{L}^G \cup \mathcal{L}^N) - Y_i(t(\mathcal{L}^N), \mathcal{L}^N)\\
=&\frac{1}{2}(\frac{(n-1)^2\cdot\tilde{t}^2}{(n+1)^2}) + (\frac{(n-1)(\alpha-c)\cdot\tilde{t}}{(n+1)^2}) > 0.
\end{aligned}
$$

□

Proof of Proposition 4.13. The first part of the proof is to show that under the GATT regime in a symmetrical trading system two countries who share no FTA improve by forming a link. This result can be shown similarly to the second part of the proof of Proposition 8 in Goyal and Joshi (2006a).

We next demonstrate that without GATT two countries also gain by forming a link: Consider any symmetrical trading system $\mathcal{L}$ with $\eta_i(\mathcal{L}) = \eta_j(\mathcal{L})\ \forall i,j \in N$ where the link $L = \{i, j\} \notin \mathcal{L}$. The change in consumer surplus is given by:

$$\begin{aligned}
&\frac{1}{2}\Big(\frac{(\eta_i(\mathcal{L})+1)(\alpha-c)-(\eta_i(\mathcal{L}))\cdot(\frac{3(\alpha-c)}{7+(\eta_i(\mathcal{L})+1)})}{(\eta_i(\mathcal{L})+2)}\Big)^2\\
&-\frac{1}{2}\Big(\frac{(\eta_i(\mathcal{L}))(\alpha-c)-(\eta_i(\mathcal{L})-1)\cdot(\frac{3(\alpha-c)}{7+\eta_i(\mathcal{L})})}{(\eta_i(\mathcal{L})+1)}\Big)^2\\
&=4(\alpha-c)^2\frac{\eta_i(\mathcal{L})^2+11\cdot\eta_i(\mathcal{L})+26}{(7+\eta_i(\mathcal{L}))^2(8+\eta_i(\mathcal{L}))^2}. \qquad (19)
\end{aligned}$$

For firm i's profit in market i we have:

$$\begin{aligned}
&\Big(\frac{(\alpha-c)+(\eta_i(\mathcal{L}))\cdot(\frac{3(\alpha-c)}{7+(\eta_i(\mathcal{L})+1)})}{(\eta_i(\mathcal{L})+2)}\Big)^2-\Big(\frac{(\alpha-c)+(\eta_i(\mathcal{L})-1)\cdot(\frac{3(\alpha-c)}{7+\eta_i(\mathcal{L})})}{(\eta_i(\mathcal{L})+1)}\Big)^2\\
&=-16(\alpha-c)^2\frac{15+2\eta_i(\mathcal{L})}{(7+\eta_i(\mathcal{L}))^2(8+\eta_i(\mathcal{L}))^2}. \qquad (20)
\end{aligned}$$

The sum of firm i's profit in market j plus the change in tariff revenue is:

$$\begin{aligned}
&\Big(\frac{(\alpha-c)-2\cdot(\frac{3(\alpha-c)}{7+(\eta_j(\mathcal{L})+1)})}{(\eta_j(\mathcal{L})+2)}\Big)^2+\Big(\frac{(\alpha-c)-2\cdot(\frac{3(\alpha-c)}{8+\eta_i(\mathcal{L})})}{(\eta_i(\mathcal{L})+2)}\Big)\cdot\frac{3(\alpha-c)}{7+(\eta_i(\mathcal{L})+1)}\cdot\eta_i(\mathcal{L})\\
&-\Big(\frac{(\alpha-c)-2\cdot(\frac{3(\alpha-c)}{7+\eta_i(\mathcal{L})})}{(\eta_i(\mathcal{L})+1)}\Big)\cdot\frac{3(\alpha-c)}{7+\eta_i(\mathcal{L})}\cdot(\eta_i(\mathcal{L})-1)\\
&=\frac{(\alpha-c)^2}{(\eta_j(\mathcal{L})+8)^2}-3(\alpha-c)^2\frac{\eta_i(\mathcal{L})^2-\eta_i(\mathcal{L})-64}{(\eta_i(\mathcal{L})+7)^2(\eta_i(\mathcal{L})+8)^2}. \qquad (21)
\end{aligned}$$

From equation (19), (20) and (21) we can conclude:

$$\begin{aligned}
&Y_i(t(\mathcal{L}\cup\{\{i,j\}\}),\mathcal{L}\cup\{\{i,j\}\})-Y_i(t(\mathcal{L}),\mathcal{L})\\
&=(\alpha-c)^2\frac{\eta_i(\mathcal{L})^2+15\eta_i(\mathcal{L})+\eta_j(\mathcal{L})^2+16\eta_i(\mathcal{L})+120}{(8+\eta_j(\mathcal{L}))^2(7+\eta_i(\mathcal{L}))(8+\eta_i(\mathcal{L}))}>0.
\end{aligned}$$

Hence, countries i and j will deviate and without a multilateral link and under non-cooperative tariffs no stable trading system exists. □

5 Bargaining Networks in International Trade

In the previous chapter we analyzed an international trade model were countries are asymmetrical with respect to market size. We have analyzed the strategic formation of bilateral and multilateral trade agreements as an application to the network formation framework presented in chapter 3 of the thesis. We showed that under MFN when tariffs are endogenously chosen, the tariff a country imposes on other GATT members decreases with the number of FTAs. We have found that even when countries are asymmetrical the global free trade network can be achieved as a stable state. Furthermore, we have found that there may be stable trading systems in which no free trade between all countries exists. The result was driven by the asymmetry that we imposed on the market size of the countries. The present chapter investigates countries' incentives to form trade agreements when countries are asymmetrical with respect to income.

We observed that multilateral tariff negotiations fail and multilateral tariff reduction is substituted by RTAs. Tariffs negotiated at the WTO are not welfare improving for some countries and they rather form bilateral and regional trade agreements. Whether, and to what extent, multilateral tariff cooperation benefits WTO member countries may depend on their regional trading network. In this chapter we provide a trade model in which we analyze countries' multilateral bargaining tariffs when countries choose cooperatively multilateral tariff reduction with respect to the Nash bargaining solution.

As in the previous chapter we consider strategic link formation of countries and investigate stable trading structures. We allow countries to form bilateral as well as multilateral links. In this aspect our model differs from two further papers that use a network formation approach to investigating the strategic stability of trading regimes, Goyal and Joshi (2006a) and Furusawa and Konishi (2007). Furthermore these papers assume external tariff rates as exogenously fixed whereas in reality countries optimally adjust their tariffs when they sign a new trade agreement. In our model we let countries endogenously adjust their optimal tariffs with respect to different network structures to investigate how these tariff adjustments affect countries' incentives to form FTAs.

Our approach also differs in the aspect that in the second part of the chapter we introduce a bargaining stage over multilateral tariffs after the network is in place and determine countries' optimal bargaining solutions with respect to the network of the world trading system. We use the Nash bargaining solution to model simultaneous tariff negotiations and consider the case when all countries have the same number of FTAs and the situation where countries are asymmetrical with respect to linking structure. We address the question as to what extent network structure and therefore the number of FTAs that a country has formed influences a country's incentives for multilateral liberalization.

Barganing in networks was analyzed by Corominas-Bosh (2004). She investigates a model where prices for a good are determined by an alternation offers bargaining process. A link is necessary for a buyer and seller to bargain over the unit of good hold by the seller. She models the bargaining process as a variation of the Rubinstein bargaining model where the expected payoff of a buyer and a seller can be calculated as a function of the network structure where each player's bargaining power depends on his position in the network. In her model the bargaining network is exogenous. Calvo-Armengol (2003) was the first to introduce bargaining during the endogenous formation of network structures. In a two stage model he introduces bilateral bargaining where in the first stage a player chooses which his linked neigbours to bargain with whereas in the second stage a pair of players bargain over a unit of a good. In the second stage the unique subgame perfect outcome is the standard Rubinstein bargaining outcome. Each player receives an expected payoff as a function of the network structure. He then applies the pairwise stability notion to characterize the set of stable networks.

In this chapter we study an international trade setting as in Bagwell and Staiger (1999b) with three countries and three different goods such that each country is endowed with two of the goods but has a positive demand for each of the goods. The reason for trade is given by the positive demand function and each country is served by two competing exporters. This model was presented briefly in chapter 2 where we demonstrated how the outcome of a country is determined by the network of trading agreements that encompasses it. In the following we extend the model of Bagwell and Staiger (1999b) in the way that we allow an asymmetrical allocation of endowments such that countries may differ with respect to their income level and we analyze the strategic formation of trade agreements. Countries can sign bilateral trade agreements and a multilateral trade agreement and determine optimal tariffs on the imports of the goods. Later we extend the framework to a multi-country setting and calculate cooperative bargaining solutions under a given network structure. We then investigate countries' incentives to form trade agreements when they are asymmetrical. We allow for asymmetry with respect to income, where we differentiate a high-income country from a low-income country by means of different amounts of the endowed goods; a high-income country is assumed to have a larger amount of endowed goods as compared to the low-income country.

The main results of the chapter are the following: When countries are symmetrical with respect to income, global free trade, which corresponds to the network structure in which each country has a free trade agreement with each country and all countries have formed a multilateral link, is a stable state. This is because it can be shown that two countries always gain from signing a free trade agreement. Whereas when countries are asymmetrical global free trade can only be a stable state, if the difference in income is relatively low between all countries. Furthermore, we can show that while starting from an empty network a bilateral trade agreement is always more profitable for the lower-income country. When we increase the number of countries we can show that global free trade is still a stable state. When countries have the same number of FTAs, global free trade is an optimal bargaining result under MFN. In the asymmetrical case we have the situation that countries with few FTAs can benefit more from multilateral tariff reduction and depend more on multilateral cooperation when multilateral tariffs are sufficiently low.

The chapter proceeds as follows: First, we present the basic model in section 1. In section 2, we define stability and efficiency of trading structure and calculate equilibrium tariffs for each given network structure. Then we characterize the stable and efficient networks first in the case of symmetrical countries and later in the case of asymmetrical countries, when countries' endowment level differs. In section 3, we introduce the bargaining stage after the network is in place and determine Nash bargaining solutions for different network structures. In section 4, we extend the framework of section 2 and allow an arbitrary number of countries.

5.1 The Model

5.1.1 Overview

In the following we will consider a three-country model with three goods. Each country is endowed with two of the three goods and each country is served by two competing exporters.

We will denote the set of countries by $N = \{A, B, C\}$ and the set of goods will be denoted by $M = \{a, b, c\}$. We will assume that country $j \in N$ is endowed with zero units of good $J \in M$, $\frac{x_j}{2}$ units of good I and $\frac{x_j}{2}$ units of good K. Country $i \in N$ is endowed with zero units of good I and $\frac{x_i}{2}$ units of the other two goods, whereas country $k \in N$ is endowed with zero units of good K and $\frac{x_k}{2}$ units of good I and good J. In general we say that all countries demand each of the three goods, such that all countries $j \in N$ have to import good J from country i and k and each country's market is served by two competing exporters, whereby j exports good I to country i and good K to country k.[54]

Figure 20 illustrates the pattern of trade. Note that in this setup it is not possible for one country to import one good from a country and export it to another country. As Bagwell and Staiger (1999b) argue this assumption can be justified given that usually fixed costs accrue when serving a new market which implies that a firm has to supply a very high amount in the new market to compensate for the fixed costs. They argue that normally small changes in tariffs alter the volume of imports in the foreign markets but not the pattern of trade. Furthermore, this framework simplifies

[54]This model is based on the partial equilibrium model of Bagwell and Staiger (1999b). Our model is richer in two important aspects. First, we assume asymmetry with respect to endowment and hence income level. Furthermore, we consider strategic link formation of countries where optimal tariffs under different trading structures are determined endogenously.

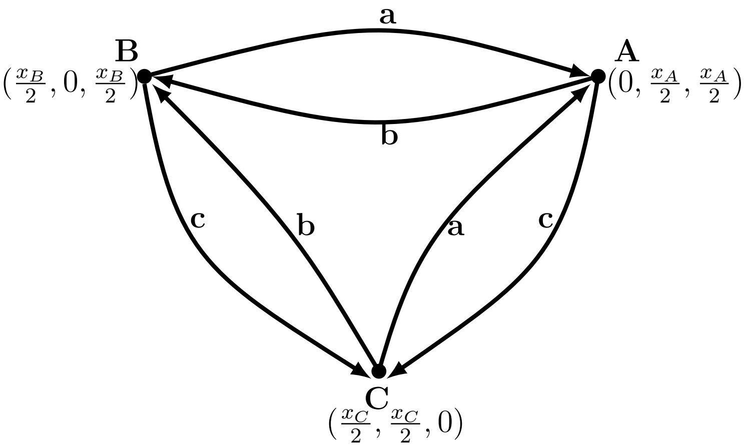


Figure 20: Pattern of trade between country A, B and C.

the analysis in a way such that we can concentrate on important effects of PTAs on multilateral tariffs.

Each country i's demand for good J is given by[55]:

$$D(P_i^J) = \alpha - \beta \cdot P_i^J. \tag{22}$$

Analogous to chapter 2 (equation (1)-(4)) we have the following conditions:
The no-arbitrage conditions for good J relate the price of good J in the importing country to the price in the exporting countries.

$$P_j^J = P_i^J + t_i^J = P_k^J + t_k^J,$$

where t_i^J and t_k^J are the tariffs that country j imposed on the imports of good J from country i and country k, respectively.
The import function of good J in country j is given by:

$$IM_j^J(P_j^J) = D(P_j^J),$$

[55]It is well known that this demand function can be derived from a quasilinear utility function that represents a representative consumer's preferences, which is quadratic and additively separable in each of the goods. In this case the price of good J does not depend on the price of the other goods. The purpose of the quasilinear utility function is extensively discussed in section 2.2 of this chapter.

whereas country j's exports of good I are given by:

$$X_j^I(P_j^I) = \frac{x_j}{2} - [\alpha - \beta \cdot P_j^I].$$

The market clearing condition for good J can be written as:

$$IM_j^J(P_j^J) = X_i^J(P_i^J) + X_k^J(P_k^J),$$

such that the total exports of good J from country i and country k have to equal the total imports of good J in market j.

5.1.2 Trading Systems

Equilibrium prices and tariffs imposed on the other markets as well as the goods traded between the countries depend on the underlying trading system. As in chapter 4 of the thesis we assume that countries only trade when there is a trade agreement between them. When no trade agreement between the countries exists, no trade takes place and each country consumes its endowment. As soon as a trade agreement between two countries exists each country exports its good to the foreign market. Due to the setup of the model, it is not possible to import from one source and export it to another. A trade agreement between country i and j enables country i to export good J into market j. The quantity supplied in the foreign market depends on further trade agreements of country i and j and hence on the trading system as a whole. Equilibrium prices and tariffs imposed on the exports are determined by the optimization problem of the countries and depend on the underlying trading system as well.

In the following we will describe the set of possible trading systems that may exist between countries A, B and C.

The following definition has been adopted from the previous chapter:

Let $N = \{A, B, C\}$ be the set of countries. $\mathcal{L}$, where $\mathcal{L} \subseteq 2^N$, is called a trading system on N and $L \in \mathcal{L}$, with $L \subseteq N$, represents a trade agreement between the countries in L.

As in chapter 2 (Figure 3a)), Figure 21a) represents a trading system under MFN in which a multilateral trade agreement $\mathcal{L} = \{N\}$ between all three countries exists that represents the GATT. Analogous to chapter 2 and 3 the trading structure in

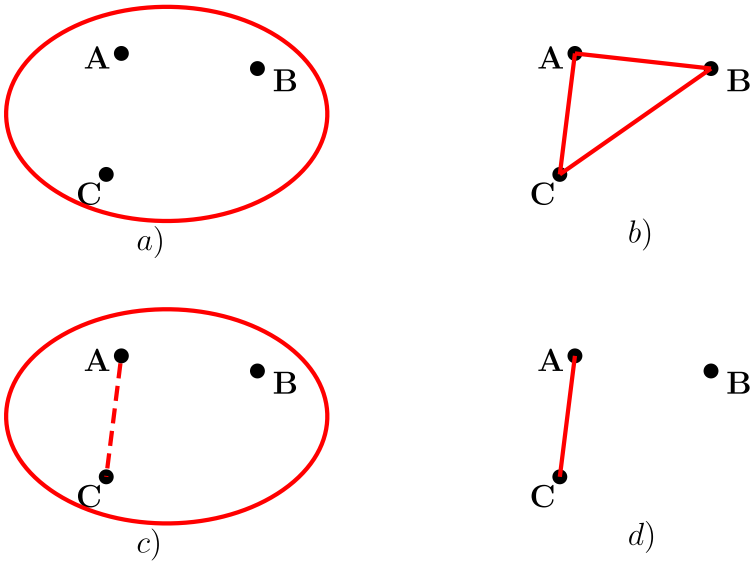


Figure 21: a) Multilateral GATT between countries A, B and C. $\mathcal{L} = \{\{A, B, C\}\}$. b) Bilateral trade agreements with $\mathcal{L} = \{\{A, B\}, \{B, C\}, \{A, C\}\}$. c) GATT with a FTA between A and C, $\mathcal{L} = \{\{A, B, C\}, \{A, C\}\}$. d) A bilateral trade agreement between A and C with $\mathcal{L} = \{\{A, C\}\}$.

Figure 21a) is called global and is denoted with $\mathcal{L}^G$. We will later see how a multilateral link, and therefore the fact that countries trade under MFN, effects countries' optimal tariff decision. The complete trading system $\mathcal{L}^N$ is the family of subsets of N with $\mathcal{L}^N = \{L \in 2^N || L| = 2\}$. This structure is represented in Figure 21b) where each pair of countries is linked bilaterally without MFN.

A trading system without any trade agreements represents the case of autarky. In this trading system each country consumes its own endowment and no trade takes place. This is represented by an empty trading system and is denoted by $\mathcal{L}^e$.

The model proceeds as follows:
First, for each trading system $\mathcal{L}$ we calculate each country i's tariffs on the imports from foreign markets on good I, $t^I(\mathcal{L}) = (t^I_j(\mathcal{L}), t^I_k(\mathcal{L}))$. Next, given each trading system and the optimal tariffs we can calculate each country's payoff that is given by its welfare in the trading system $\mathcal{L}$, $Y_i(t(\mathcal{L}), \mathcal{L})$, which is calculated as the sum

of producer surplus, consumer surplus and tariff revenue over all goods.

$$Y_i(t(\mathcal{L}), \mathcal{L}) = CS_i(t(\mathcal{L}), \mathcal{L}) + TR_i(t(\mathcal{L}), \mathcal{L}) + \Pi_i(t(\mathcal{L}), \mathcal{L}). \quad (23)$$

The tariff structure is determined as follows: When the set of countries has a global link that contains each of the countries as in Figure 21a), each country chooses its welfare maximizing tariffs with respect to the MFN clause, such that each country levies the same tariffs on each of the countries with which they are linked multilaterally, due to the non-discrimination requirement of the MFN clause. Countries are also able to form additional bilateral links. In a bilateral link under MFN represents a FTA and both trading partners eliminate tariffs due to Article XXIV of the GATT. In Figure 21c) countries A and C have formed a FTA[56] and eliminated tariffs against each other. Without the MFN clause each country imposes its non-cooperative, welfare maximizing Nash tariffs on its trading partner. In this case a country's optimal tariff on one trading partner might differ from the optimal tariff on another trading partner. In this situation tariff discrimination may take place and depends crucially on the endowment and linking structure of the other countries. Where there is no trade agreement between two countries, neither bilateral nor multilateral, we assume that no trade takes place. In Figure 21d) countries A and C have formed a bilateral trade agreement but as no trade agreement with country B exists, countries A and C do not import from market B. In this case, country B merely consumes its endowment. Two main questions that this chapter addresses is whether the formation of PTAs increases or decreases tariffs on third parties and whether the process of strategic trade agreement formation results in global free trade.

In the following we will introduce the notion of efficiency and multilateral stability from the previous chapters to characterize trading structures.

[56]As in the previous chapters, in this model we consider FTAs as a special case of a PTA.

5.2 Stable and Efficient Trading Structures

Stability

The notion of stability that we adopt in order to investigate which trading system can be a stable state is the multilateral stability notion of chapter 3. The formation of a trade agreement requires the consent of all countries involved, but severance can be carried out unilaterally.

Definition 5.1. *A trading system $\mathcal{L}$ on N is called multilaterally stable, if*

$$(i)\ \ Y_i(t(\mathcal{L}), \mathcal{L}) \geq Y_i(t(\mathcal{L}\backslash\{L\}), \mathcal{L}\backslash\{L\})\ \forall L \in \mathcal{L},\ \ \forall i \in L \text{ and}$$

$$(ii)\ Y_i(t(\mathcal{L} \cup \{L\}), \mathcal{L} \cup \{L\}) > Y_i(t(\mathcal{L}), \mathcal{L}) \Rightarrow \exists j \in L,$$

$$\text{such that } Y_j(t(\mathcal{L} \cup \{L\}), \mathcal{L} \cup \{L\}) < Y_j(t(\mathcal{L}), \mathcal{L})\ \ \forall L \notin \mathcal{L}.$$

The above definition describes a situation in which no country has an incentive to sever any of its existing trade agreements and no subset of countries has an incentive to form an additional agreement. This definition allows the formation of trade agreements with more than just two countries and therefore also allows the formation of a multilateral trade agreement between all three countries.

Efficiency

In order to analyze the efficiency of different trading systems, we again consider global welfare, which is given by the sum of all countries' payoffs.

Definition 5.2. *A trading system $\mathcal{L}^*$ is said to be (strongly) efficient, if* $\sum_{i \in N} Y_i(t(\mathcal{L}), \mathcal{L}) \leq \sum_{i \in N} Y_i(t(\mathcal{L}^*), \mathcal{L}^*),\ \forall \mathcal{L}.$

In the following we will consider stable trading systems when countries adjust their tariffs for each new link that is formed and therefore when countries choose optimal tariffs.

5.2.1 Symmetrical Countries

We will assume that countries are symmetrical with respect to income such that we set $x_A = x_B = x_C = x$. In this section we obtain three major findings. First, we show that under MFN tariffs on third parties decrease when a country increases its number of FTAs. Second, we can show that global free trade can be achieved as a stable trading structure but it is not the unique stable trading structure. Third, we can show that global free trade maximizes total world welfare and is therefore efficient.

We can calculate each country's equilibrium welfare level under each trading structure and check which of the trading systems is stable under the multilateral stability notion.

From the no arbitrage condition and market clearing condition we can yield without MFN the prices in country i for good J in the trading system $\mathcal{L}$ as a function of the optimal tariffs and the trading system $\mathcal{L}$:

$$P_i^J(t(\mathcal{L}),\mathcal{L}) = \frac{\alpha}{\beta} - \frac{2\cdot x}{6\beta} + \frac{t_k^J(\mathcal{L})}{3} - \frac{2t_i^J(\mathcal{L})}{3}, \quad \text{whenever} \;\; i \;\; \text{and} \;\; k \in N_j(\mathcal{L}),$$

$$P_i^J(t(\mathcal{L}),\mathcal{L}) = \frac{\alpha}{\beta} - \frac{x}{4\beta} - \frac{t_i^J(\mathcal{L})}{2}, \quad \text{whenever} \;\; i \in N_j(\mathcal{L}) \;\; \text{and} \;\; k \notin N_j(\mathcal{L}),$$

and

$$P_i^J(t(\mathcal{L}),\mathcal{L}) = \frac{\alpha}{\beta} - \frac{x_i}{2\beta},$$

whenever there is no trade agreement between country i and j, where $N_i(\mathcal{L})$ denotes the set of countries that are linked with country i in $\mathcal{L}$ without MFN and $i \in N_i(\mathcal{L})$. The last equation follows from the fact that whenever two countries share no trade agreement, they do not export and consume all of their endowment.

First, we can calculate a country's welfare level in the trading system $\mathcal{L}$. From consumer surplus we have with $i \neq j$, $j \neq k$ and $i \neq k$ for $i,j,k \in N$:

$$CS_i(t(\mathcal{L}),\mathcal{L}) = \sum_{J\in M}[\frac{1}{2\beta}(\alpha - \beta P_i^J(t(\mathcal{L}),\mathcal{L}))^2].$$

Tariff revenue in country i in $\mathcal{L}$ is given by

$$TR_i(t(\mathcal{L}),\mathcal{L}) = \sum_{j\in N_i(\mathcal{L})\setminus\{i\}} X_j^I(\mathcal{L})\cdot t_j^I(\mathcal{L}),$$

and profit:

$$\begin{aligned}
&\Pi_i(t(\mathcal{L}),\mathcal{L})\\
=&X_i^J(\mathcal{L})\cdot P_i^J(t(\mathcal{L}),\mathcal{L})+X_i^K(\mathcal{L})\cdot P_i^K(t(\mathcal{L}),\mathcal{L})\\
+&(\frac{x}{2}-X_i^J(\mathcal{L}))\cdot P_i^J(t(\mathcal{L}),\mathcal{L})+(\frac{x}{2}-X_i^K(\mathcal{L}))\cdot P_i^K(t(\mathcal{L}),\mathcal{L})\\
=&\frac{x}{2}\cdot P_i^J(t(\mathcal{L}),\mathcal{L})+\frac{x}{2}P_i^K(t(\mathcal{L}),\mathcal{L}),
\end{aligned}$$

which is calculated as the profit country i generates in its own market from good J and K plus the profit generated from selling in market k and j.

In the following we calculate equilibrium tariffs for the case without MFN and we can now show that the tariff imposed on a country's trading partner j depends on the tariffs imposed on the other trading partners' imports of good I and vice versa.

$$t_j^I(\mathcal{L})=\frac{(4\eta_i(\mathcal{L})+2)\beta\sum_{k\in N_i(\mathcal{L})\setminus\{i\}\cup\{j\}}t_k^I(\mathcal{L})+x}{(4\eta_i^2(\mathcal{L})-4\eta_i(\mathcal{L})-2)\beta},\tag{24}$$

where $\eta_i(\mathcal{L})=|N_i(\mathcal{L})|$[57]. Equation (24) reflects an interesting complementarity as already pointed out by Bagwell and Staiger (1999b) as the tariffs imposed on country j increase with a rise in tariffs on another trading partner. It is more attractive for a country to raise tariffs on one of its trading partners when tariffs on its other trading partner are already high. This complementarity effect is due to three welfare effects that induce a country to increase its tariffs on another country when the tariffs on third parties are high. First, when the tariff on one trading partner t_j^I are high, imports from the other trading partner k increase such that with an increase of the tariffs on the other trading partner t_k^I tariff revenues from the imports increase. And this induces country i to raise its tariffs on country k. Second, the rise in t_k^I, in turn, motivates an increase of the tariff on market j to increase tariff revenues from j. A third effect concerns the consumer surplus. When t_j^I is high, the consumption of good I in market i is low. When market i increases tariffs on market k, the loss in consumer surpus due to increased prices is lower, the higher the tariffs on market j, t_j^I. Therefore, high import tariffs on market j induce a high import tariff on market k.[58]

[57]Note that whenever $j\in N_i(\mathcal{L})$ such that country i imposes a tariff t_j^I on j, we have that $\eta_i(\mathcal{L})\geq 2$ with $i\in N_i(\mathcal{L})$.

[58]For a detailed discussion see Bagwell and Staiger (1999b, p. 7).

Without MFN, countries non-cooperatively choose welfare maximizing tariffs on the imports from countries with which they have a bilateral trade agreement. Since countries are symmetrical we can directly solve for the equilibrium tariffs in equation (24) where $t^I_j = t^I \ \forall j \neq i$:

$$t^I(\mathcal{L}) = \frac{x}{(2 \cdot \eta_i(\mathcal{L}) + 2)\beta} \quad \forall i \in N. \tag{25}$$

A country imposes higher tariffs when it has fewer trade agreements and as a result of an additional bilateral trade agreement country i reduces external tariffs on third countries. This is due to the tariff complementarity effect since a reduction on one of country i's trading partners increases country i's incentives to reduce tariffs on its other trading partner. Note that in equilibrium the optimal tariff does not depend on the optimal tariffs country j and country k impose on i.

Under MFN we set tariffs $t^J_i = t^J$ for all countries $i \in N$ that have no additional FTA with country j. The prices in equilibrium are given by:

$$P^J_i(t(\mathcal{L}), \mathcal{L}) = \frac{\alpha}{\beta} - \frac{2 \cdot x}{6\beta} + \frac{(3 - \tilde{\eta}_j(\mathcal{L}))t^J(\mathcal{L})}{3} - t^J(\mathcal{L}),$$

whenever i and j have no FTA and

$$P^J_i(t(\mathcal{L}), \mathcal{L}) = \frac{\alpha}{\beta} - \frac{2 \cdot x}{6\beta} + \frac{(3 - \tilde{\eta}_j(\mathcal{L}))t^J(\mathcal{L})}{3}$$

else, where $\tilde{N}_j(\mathcal{L})$ denotes the set of countries that have preferential access to market j and $\tilde{\eta}_j(\mathcal{L}) = |\tilde{N}_j(\mathcal{L})|$ denotes the number of FTAs of country j with $j \in \tilde{N}_j(\mathcal{L})$. First, we can calculate a country's welfare level in $\mathcal{L}$. From consumer surplus we have with $i \neq j$, $j \neq k$ and $i \neq k$, $i, j, k \in N$:

$$CS_i(t(\mathcal{L}), \mathcal{L}) = \sum_{J \in M} [\frac{1}{2\beta}(\alpha - \beta P^J_i(t(\mathcal{L}), \mathcal{L}))^2].$$

Tariff revenue in country i in the trading system $\mathcal{L}$ is given by

$$TR_i(t(\mathcal{L}), \mathcal{L}) = \sum_{j \in N \setminus \tilde{N}_i(\mathcal{L})} X^I_j \cdot t^I(\mathcal{L}),$$

and producer surplus:

$$\Pi_i(t(\mathcal{L}), \mathcal{L}) = \sum_{J\in M} \Pi_i^J(t(\mathcal{L}), \mathcal{L})$$
$$= \frac{x}{2}[\sum_{j\notin \tilde{N}_i(\mathcal{L})} (\frac{\alpha}{\beta} - \frac{2\cdot x}{6\beta} + \frac{(3-\tilde{\eta}_j(\mathcal{L})))t^J(\mathcal{L})}{3} - t^J(\mathcal{L}))$$
$$+ \sum_{j\in \tilde{N}_i(\mathcal{L})\backslash\{i\}} (\frac{\alpha}{\beta} - \frac{2\cdot x}{6\beta} + \frac{(3-\tilde{\eta}_j(\mathcal{L}))t^J(\mathcal{L})}{3})],$$

where the last term is derived from the fact that when two countries i and j have no trade agreement, then country i will consume all of its endowment of good J by itself.

First we consider the impact of a FTA between countries i and j, in which they both agree to reduce tariffs to zero, on the external tariffs on country k. We can show that under MFN country i's tariffs on the other markets in $\mathcal{L}$ is given by :

$$t^I(\mathcal{L}) = \frac{x}{2\beta(7\tilde{\eta}_i(\mathcal{L}) - 3)}. \tag{26}$$

This result shows that the tariff imposed on the foreign markets decreases when countries i and j increase their number of FTAs[59]. Therefore, this result is contrary to the results in Krishna (1998), who showed by means of a political economy model that PTAs decrease incentives for multilateral liberalization and tariffs on third parties increase. Furthermore, we can see that a country's optimal tariff does not depend on the decision of any of the other countries. Therefore, country k's choice of external tariffs is not affected by a FTA between i and j.

The next two examples adopted from chapter 2 demonstrate how the equilibrium prices and tariffs are calculated under a given trading regime.

Example 5.1. *Autarky, in which no trade takes place, is represented by an empty trading system $\mathcal{L}^e$. All countries consume their endowments and the price of good J in market i is given by:*

$$\frac{x}{2} = \alpha - \beta \cdot P_i^J(t(\mathcal{L}^e), \mathcal{L}^e).$$

[59]Here, $\tilde{\eta}_i(\mathcal{L}) \leq 3$ for all $i \in N$. In section 4 we shall see that this result still holds when we increase the number of countries.

Since $P_i^J(t(\mathcal{L}^e), \mathcal{L}^e) = \frac{\alpha - \frac{x}{2}}{\beta}$ *and tariff revenues are zero social welfare is given by:*

$$Y_i(t(\mathcal{L}^e), \mathcal{L}^e) = \frac{1}{2\beta}[(\frac{x}{2})^2 + (\frac{x}{2})^2] + \frac{x}{2}(\frac{\alpha - \frac{x}{2}}{\beta}) + \frac{x}{2}(\frac{\alpha - \frac{x}{2}}{\beta}) = \frac{4\alpha x - x^2}{4\beta}.$$

Example 5.2. *In the global trading system all countries are members of the GATT and impose the same tariff on each trading partner. The tariff country i imposes on j and k is given by:*

$$t^I(\mathcal{L}^G) = \frac{x}{8\beta},$$

and prices in market i for good J are given by:

$$P_i^J(t(\mathcal{L}^G), \mathcal{L}^G) = \frac{\alpha}{\beta} - \frac{x}{3\beta} - \frac{t^J(\mathcal{L}^G)}{3} = \frac{\alpha}{\beta} - \frac{x}{3\beta} - \frac{x}{24\beta}.$$

Social welfare is given by:

$$Y_i(t(\mathcal{L}^G), \mathcal{L}^G) = \frac{1}{2\beta}[(\frac{6 \cdot x}{24\beta})^2 + (\frac{9 \cdot x}{24\beta})^2 + (\frac{9 \cdot x}{24\beta})^2] + \frac{x}{2}(\frac{2\alpha}{\beta} - \frac{18 \cdot x}{24\beta}) + 2 \cdot \frac{x}{8\beta}(\frac{x}{2} - \frac{x}{24} - \frac{x}{3}),$$

where the first term is consumer surplus, the second term is firm i's profit and the last term reflects tariff revenues.

In the following we will consider stable trading systems when countries adjust their tariffs for each new link that is formed and therefore when countries choose optimal tariffs.

The first result on stability that we shall describe, concerns the question of whether global free trade can be achieved as a stable state.

Proposition 5.1. *When countries are symmetrical, global free trade (the complete trading system with a global link) is a stable trading system.*

Intuitively, severance of a link will decrease competition in the domestic market but increase tariff revenue. No country will sever any of its trade agreements as the loss in consumer surplus due to increased domestic prices and the loss in profit in the foreign market would be higher than the gains from tariff revenue and the profit in the domestic market.

Proposition 5.2. *Global free trade and the global link with a FTA between one pair of countries are the unique stable states.*

For an intuition of this result we first show that without MFN each pair of countries always gains from an additional trade agreement and therefore has an incentive to form as many links as possible:

$$\begin{aligned}
&Y_i(t(\mathcal{L}),\mathcal{L}) - Y_i(t(\mathcal{L}\backslash\{\{i,j\}\}),\mathcal{L}\backslash\{\{i,j\}\}) \\
=&\frac{1}{2\beta}(\frac{(\eta_i(\mathcal{L})-1)x}{2\eta_i(\mathcal{L})} - \frac{(\eta_i(\mathcal{L})-1)x}{\eta_i(\mathcal{L})(2\eta_i(\mathcal{L})+2)})^2 - \frac{1}{2\beta}(\frac{(\eta_i(\mathcal{L})-2)x}{2(\eta_i(\mathcal{L})-1)} - \frac{(\eta_i(\mathcal{L})-2)x}{(\eta_i(\mathcal{L})-1)2\eta_i(\mathcal{L})})^2 \\
+&\frac{x}{2}(\frac{\alpha}{\beta} - \frac{(\eta_j(\mathcal{L})-1)x}{2\eta_j(\mathcal{L})\beta} + \frac{(\eta_j(\mathcal{L})-2)x}{\eta_j(\mathcal{L})\beta(2\eta_j(\mathcal{L})+2)} - \frac{(\eta_j(\mathcal{L})-1)x}{\eta_j(\mathcal{L})\beta(2\eta_j(\mathcal{L})+2)}) \\
-&\frac{x}{2}(\frac{\alpha}{\beta} - \frac{x}{2\beta}) + \frac{(\eta_i(\mathcal{L})-1)x}{\beta(2\eta_i(\mathcal{L})+2)}(\frac{x}{2\eta_i(\mathcal{L})} - \frac{x}{\eta_i(\mathcal{L})(2\eta_i(\mathcal{L})+2)}) \\
-&\frac{(\eta_i(\mathcal{L})-2)x}{2\beta\eta_i(\mathcal{L})}(\frac{x}{2(\eta_i(\mathcal{L})-1)} - \frac{x}{(\eta_i(\mathcal{L})-1)2\eta_i(\mathcal{L})}) \\
+&\frac{1}{2\beta}(\frac{(\eta_j(\mathcal{L})-1)x}{2\eta_j(\mathcal{L})} - \frac{(\eta_j(\mathcal{L})-2)x}{\eta_j(\mathcal{L})(2\eta_j(\mathcal{L})+2)} + \frac{(\eta_j(\mathcal{L})-1)x}{(2\eta_j(\mathcal{L})+2)\eta_j(\mathcal{L})})^2 - \frac{1}{2\beta}(\frac{x}{2})^2 \\
=&\frac{x^2}{4\beta(\eta_i(\mathcal{L})+1)^2\eta_i(\mathcal{L})^2}(\eta_i(\mathcal{L})+\eta_i(\mathcal{L})^2) + \frac{x^2}{8\beta(\eta_j(\mathcal{L})+1)^2} > 0.
\end{aligned}$$

Therefore, two countries always have an incentive to sign a bilateral trade agreement. Since Proposition 5.1 showed that the complete trading system cannot be stable, we can conclude that without MFN no stable trading system can exist. This implies that each stable trading system includes MFN and no country has an incentive to cancel MFN.

Furthermore, we can observe that global free trade is not a unique stable trading system. Another stable trading structure consists of a FTA between one pair of countries under MFN, e.g. country B and C. To see why this is a stable trading system, consider that first starting from the multilateral trade agreement countries B and C will benefit from forming a free trade agreement. But given a FTA between B and C, country A prefers not to form a FTA with either B or C. The reason is that an additional FTA between country A and B would decrease tariffs on market C and therefore reduce country A's tariff revenue. Furthermore, a FTA would eliminate tariffs on the imports of good a from market B. This would lead to lower prices of good a in market A and to higher consumer surplus whereas the increase

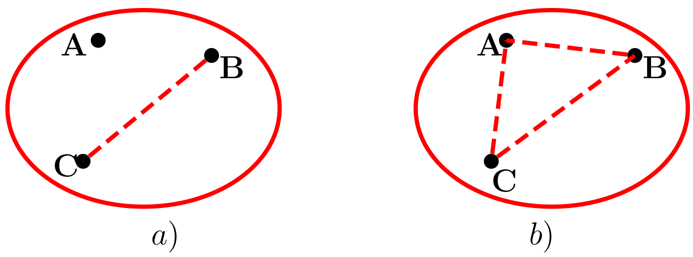


Figure 22: **Stable Trading Systems.** $a)$ $\mathcal{L} = \{\{A, B, C\}, \{B, C\}\}$, $b)$ $\mathcal{L} = \{\{A, B, C\}, \{A, B\}, \{B, C\}, \{A, C\}\}$

of exports of good b in market B would imply a higher price of good b in market A and a decrease in consumer surplus in market A. Another effect will be a decrease of profit of good b in market A. The total effect of a FTA between A and B on market A will be negative as the additional gain in consumer surplus in good a cannot compensate for the loss in consumer surplus in good b and tariff revenue. Furthermore the severance of the multilateral trade agreement is not profitable either since tariff revenue decreases. The existing FTA between B and C causes country A's welfare losses to be higher than its welfare gains that country A obtains from the opening of its market with either B or C.

Efficiency

Next we investigate the nature of efficient trading systems.

Proposition 5.3. *Global free trade is the unique efficient trading system.*

We can calculate for each trading system the overall welfare level and compare the values of the networks. Complete proof is provided in the appendix.

5.2.2 Asymmetrical Countries

The linear demand function of equation (22) for each good can be derived from a quasilinear utility function U which is quadratic and additively separable in each good $I \in M$ and linear in a numeraire good x_0.[60] Without loss of generality we

[60] This structure is relatively standard in the regionalism literature and has been applied by Krishna (1998) and Freund (2000) among others.

assume $x_A < x_B < x_C$ and each country's endowment of the numeraire good equals e. Since demand functions are symmetrical across goods and countries, prices of all non-numeraire goods are equal across countries under free trade. From the no arbitrage condition and market clearing condition we can calculate without MFN the prices in country i for good J in the trading system $\mathcal{L}$, $i \neq j$,

$$P_i^J(t(\mathcal{L}), \mathcal{L}) = \frac{\alpha}{\beta} - \frac{x_i + x_k}{6 \cdot \beta} + \frac{t_k^J(\mathcal{L})}{3} - \frac{2t_i^J(\mathcal{L})}{3} \qquad \text{whenever } i \text{ and } k \in N_j(\mathcal{L}),$$

$$P_i^J(t(\mathcal{L}), \mathcal{L}) = \frac{\alpha}{\beta} - \frac{x_i}{4 \cdot \beta} - \frac{t_i^J(\mathcal{L})}{2} \qquad \text{whenever } i \in N_j(\mathcal{L}) \text{ and } k \notin N_j(\mathcal{L})$$

and

$$P_i^J(t(\mathcal{L}), \mathcal{L}) = \frac{\alpha}{\beta} - \frac{x_i}{2\beta}$$

else, where the last equation expresses that whenever no trade agreement between i and j exists, i will consume all of its endowment.

Under free trade country A faces the largest volume of imports of good a ($\frac{x_B+x_C}{6}$) and country C faces the lowest volume of imports of good c under free trade ($\frac{x_B+x_A}{6}$). As Country A exports $\frac{2x_A-x_C}{6}$ units of good b to country B and $\frac{2x_A-x_B}{6}$ units of good c to country C, the volume of imports does not equal the volume of exports. To balance the trade country A additionally exports the numeraire good x_0 to countries C and B. To guarantee that the value of the total import volume of country A ($P_A^a \cdot IM_A^a$) is larger than the value of imports of market B ($P_B^b \cdot IM_B^b$) and market C ($P_C^c \cdot IM_C^c$), $6\alpha > x_A + x_B + 2x_C$ has to be satisfied. This condition is assumed to be satisfied in the following model.

With the free trade price without any tariffs in each country for good J, P^J, we can calculate with $x_A < x_B < x_C$ each country's income as $E_A = \frac{x_A}{2}P^b + \frac{x_A}{2}P^c + e < \frac{x_B}{2}P^a + \frac{x_B}{2}P^c + e < \frac{x_C}{2}P^b + \frac{x_C}{2}P^a + e = E_C$ such that we can interpret country A as the low-income country and country C as the high-income country.[61]
We can calculate for each country $i \in N$ its tariff without MFN on imports of good I from country $j \in N$ whenever i and j have a trade agreement:

$$t_j^I(\mathcal{L}) = \frac{x_j}{6\beta}, \tag{27}$$

[61] This interpretation is adopted from Saggi (2009).

whenever country $k \notin N_i(\mathcal{L})$, and

$$t_j^I(\mathcal{L}) = \frac{3 \cdot x_j - x_k}{16\beta} \quad (28)$$

whenever $k \in N_i(\mathcal{L})$, such that the tariffs depend on both countries' income level but not on the other countries' tariffs. We can observe that country i sets a higher tariff on the imports of good I when the importing country j has a higher income level relative to country k.

Starting with an empty trading system we analyze whether two countries i and j have an incentive to form a bilateral trade agreement. Here we can make the first observation:

Lemma 5.1. *Starting from an empty trading system, a trade agreement between two countries is always more profitable for the lower income country.*

This can be verified, when one considers that the change in consumer surplus in the low-income market is higher, as the quantity supplied by the high-income country in the low-income market is higher. The high-income country offers a larger amount in the foreign market due to its large endowment such that the market clearing prices in the low-income country are low compared to the market clearing prices in the high-income country and the gains in consumer surplus are higher. The change in tariff revenue in the low-income country is larger as the high-income market offers a higher amount in the foreign market. On the other hand, the change in firm profit in the low-income market is lower, as it suppplies a smaller amount in the foreign market. Overall, the two positive effects are larger than the negative effect such that it can be concluded, that a low-income country benefits more from the formation of a bilateral trade agreement than a higher-income country.

Next we investigate the incentives for two countries i and j to sign a FTA under MFN tariffs. We consider the MFN tariffs in a trading structure where countries i and j have a FTA versus the MFN tariffs for country i and j under the trading system without a FTA. This analysis is similar to the one in chapter 4. Under MFN each country imposes the same tariffs on each MFN member with respect to maximizing social welfare, i.e.:

$$\max_{t^I(\mathcal{L})} Y_i(t(\mathcal{L}), \mathcal{L}) = CS_i(t(\mathcal{L}), \mathcal{L}) + TR_i(t(\mathcal{L}), \mathcal{L}) + \Pi_i(t(\mathcal{L}), \mathcal{L}).$$

This yields country i's optimal tariff on the imports from markets j and k under MFN

$$t^I(\mathcal{L}^G) = \frac{x_k + x_j}{16 \cdot \beta}, \tag{29}$$

such that each country levies the same tariffs on each of its trading partners.
The tariff country i imposes on country k when i and j have a FTA is given by:

$$t^I(\mathcal{L}^G \cup \{\{i,j\}\}) = \frac{5x_k - 4x_j}{22 \cdot \beta}. \tag{30}$$

First we investigate the effect of the opening of the markets on the welfare components. The change in tariff revenue for country i is given by:

$$\begin{aligned} \triangle TR_i =& TR_i(t(\mathcal{L}^G \cup \{\{i,j\}\}), \mathcal{L}^G \cup \{\{i,j\}\}) - TR_i(t(\mathcal{L}^G), \mathcal{L}^G) \\ =& (\frac{5x_k - 4x_j}{22 \cdot \beta})(\frac{4x_k - x_j}{22}) - (\frac{(x_j + x_k)^2}{16 \cdot 8\beta}). \end{aligned} \tag{31}$$

For country j this is given by:

$$\begin{aligned} \triangle TR_j =& TR_j(t(\mathcal{L}^G \cup \{\{i,j\}\}), \mathcal{L}^G \cup \{\{i,j\}\}) - TR_j(t(\mathcal{L}^G), \mathcal{L}^G) \\ =& (\frac{5x_k - 4x_i}{22 \cdot \beta})(\frac{4x_k - x_i}{22}) - (\frac{(x_i + x_k)^2}{16 \cdot 8\beta}). \end{aligned} \tag{32}$$

A comparison of equation (31) and (32) shows that whether the change in tariff revenue from the opening of the markets is higher for the lower income country or for the higher income country depends on the relation of all three countries' market size. We cannot in general say that either the lower or the higher-income market yields a higher tariff revenue loss from the opening of the market.
The change in profit of country i is given by:

$$\triangle \Pi_i = \Pi_i(t(\mathcal{L}^G \cup \{\{i,j\}\}), \mathcal{L}^G \cup \{\{i,j\}\}) - \Pi_i(t(\mathcal{L}^G), \mathcal{L}^G) = \frac{x_i}{6}(\frac{5x_k - 4x_i}{22\beta} + \frac{x_i + x_k}{16\beta}). \tag{33}$$

The effect of a FTA on producer surplus is positive, whenever $x_i < x_k$ and negative else and thus depends on the relation of the income level between market i itself and the income level of the market that does not belong to any FTA.
Country i's change in consumer surplus is given by:

$$\triangle CS_i = \frac{1}{2\beta}[(\frac{5x_j + 2x_k}{22})^2 - (\frac{(x_j + x_k)}{8})^2 + (\frac{5x_i + 2x_k}{22})^2 - (\frac{9(x_i + x_k)}{48})^2]. \tag{34}$$

From equation (31), (33) and (34) we can now calculate the change in welfare level of country i from a FTA with country j.

$$\triangle Y_i = -\frac{1}{61\,952}\frac{-46x_ix_k + 721x_i^2 - 632x_j^2 + 1296x_kx_j - 1015x_k^2 - 512x_ix_j}{\beta}.$$

The above expression shows that whether the effect is positive or negative depends on the relationship between all three countries' income level. For the case of symmetrical countries we could show in section 2.1 of this chapter that this effect is always positive. Whenever the income level between all three countries differs a lot this expression might become negative for one of the countries and a FTA under MFN is not formed.

Observation 5.1. *Under MFN starting from a global link two countries always benefit from the formation of a FTA whenever their income levels are relatively similar.*

To see whether the lower or the higher-income country has a higher gain in an additional FTA we have to compare the change in country j's welfare level with the change in country i's welfare level.

$$\triangle Y_i - \triangle Y_j = -\frac{1}{5632}\frac{-122x_ix_k + 123x_i^2 - 123x_j^2 + 122x_kx_j}{\beta} > 0.$$

Therefore, we can conclude that country i benefits more from a FTA than country j and derives a larger benefit from the opening of the foreign market, whenever $x_i > \frac{122}{123}x_k - x_j$.

Proposition 5.4. *Global free trade is a stable state whenever the income levels of all three countries are not too different.*

The first condition of multilateral stability is obviously fulfilled since no additional link can be added.
The second condition for stability can be derived when we calculate a country's welfare change when the global link is severed and when a FTA is cancelled. The complete proof is shown in the appendix. When countries are symmetrical we argued

that there is a loss in welfare from the severance of a trade agreement. When countries are asymmetrical, however, suppose that the income level of country i (x_i) and j (x_j) are large as compared to market k's income level. In this case, the tariff that market i imposes on the imports of good I from market j increases with equation (30). As market j will export less, this rises the prices of good I in market j and decreases consumer surplus in good I. At the same time, when x_i is large as compared to x_k market j increases tariffs on market i such that the consumer surplus in good J in market i increases. In addition, the larger x_j the higher the the tariff revenue of country i from the severance of the FTA with country j. Conversely, when j imposes a higher tariff on market i due to the higher income level (see equation (30)) i will export less to market j and the profit of country i in market j decreases. However, it is possible that the loss in consumer surplus in good I together with the loss in profit is smaller than the gains in consumer surplus together with a higher tariff revenue such that the severance of the FTA with country j is profitable for country i.

Remark 5.1. *When countries' income levels differ a lot it is not quite clear which structure is stable. This depends largely on the relationship between all three countries' income levels. For example a bilateral link between A and C can be stable, whenever country B does not have an incentive to add an additional bilateral link with country A. In the symmetrical case we observed that this will never be the case since countries always form more bilateral trade agreements until the complete trading system is formed. In the asymmetrical case we have that* $Y_B(t(\{\{A,C\}\}), \{\{A,C\}\}) - Y_B(t(\{\{A,C\},\{A,B\}\}), \{\{A,C\},\{A,B\}\}) = -\frac{1}{1536}\frac{64x_A^2+3x_C^2+27x_B^2-18x_Bx_C}{\beta}$ *which can be positive whenever* x_C *is sufficiently large.*

5.3 Bargaining under Most Favoured Nation (MFN) Clause

In the following section we assume that countries are symmetrical with respect to income ($x_A = x_B = x_C = x$). The tariffs in equation (26) reflect the prisoner's dilemma in trade policy, since efficiency requires free trade in contrast to non-cooperative Nash tariffs. In reality, countries are involved in a bargaining process within the WTO to agree on a commonly reduced multilateral tariff $t^*(\mathcal{L})$. We investigate a bargaining stage after the trading system is in place and investigate whether under the Nash bargaining solution all countries improve or whether the solution is below the threat point.

We consider Nash bargaining because it is a generally accepted bargaining model which enables us to find a reasonable solution that is symmetrical (if countries have the same threat point, they receive the same bargaining outcome), non-dictatorial (no country can enforce its will without the consent of the other countries) and Pareto-optimal. The Nash bargaining solution will be adopted by all countries if and only if it gives each country an outcome that is at least as large as the outcome each country obtains if negotiations fail. This approach is consistent with the WTO negotiation rounds as international trade negotiations take place without strict negotiation rules that constitute which country goes first, who makes the last offer, and so on. Another justification for this bargaining approach is that we assume that there are no cooperation possibilities between two GATT members on their tariffs against third member-countries, which is consistent with the way GATT negotiations proceed.

In the following, we will assume that the threat point of country i under $\mathcal{L}$ is given by its non-cooperative welfare level as calculated in section 2. We shall investigate the effect of the trading structure on the bargaining solution and whether countries that are asymmetrically linked in the sense that some countries have more bilateral trade agreements than others benefit differently from the bargaining solution.
We start with the case in which two symmetrical countries share a bilateral link with non-cooperative tariffs and investigate the cooperative Nash bargaining solution. We have to maximize the Nash bargaining product with $\mathcal{L} = \{\{i, j\}\}$:

$$\max_{t^N(\mathcal{L})} V(\mathcal{L}) = (Y_i(t^N(\mathcal{L}), \mathcal{L}) - Y_i(t(\mathcal{L}), \mathcal{L}))(Y_j(t^N(\mathcal{L}), \mathcal{L}) - Y_j(t(\mathcal{L}), \mathcal{L})), \qquad (35)$$

such that $Y_i(t^N(\mathcal{L}), \mathcal{L}) > Y_i(t(\mathcal{L}), \mathcal{L})\ \forall i \in N$. $Y_i(t(\mathcal{L}), \mathcal{L})$ denotes the threat point for country i in $\mathcal{L}$ that corresponds to the non-cooperative welfare level under MFN if negotiations fail. $Y_i(t^N(\mathcal{L}))$ denotes the welfare level under Nash bargaining tariffs. In this formula it is already assumed that countries have equal bargaining power. Due to the symmetry of the countries and the linking structure, the solution to problem (35) is the tariff that maximizes $Y_i(t^N(\mathcal{L}), \mathcal{L}) + Y_j(t^N(\mathcal{L}), \mathcal{L})$.

As shown in Proposition 5.3. with $t^N \geq 0$ we obtain a unique bargaining solution $t^N = 0$ such that the welfare function is maximized under free trade with a total welfare level of $\frac{1}{20736}\frac{x^2}{\beta^2}$ and each of the linked countries obtains a welfare gain from the bargaining tariffs of $Y_i(0,\mathcal{L}) - Y_i(t(\mathcal{L}),\mathcal{L}) = \frac{x^2}{144\cdot\beta^2}$.

Next we investigate which tariff will be negotiated when countries share a multilateral global link. Now the Nash welfare function is given by:

$$\max_{t^N(\mathcal{L})} V(\mathcal{L}) = \prod_{i\in N}(Y_i(t^N(\mathcal{L}),\mathcal{L}) - Y_i(t(\mathcal{L}),\mathcal{L})).$$

Due to symmetry of the countries we have to find the equilibrium tariff $t^N(\mathcal{L})$ that maximizes $Y_i(t^N(\mathcal{L}),\mathcal{L}) + Y_j(t^N(\mathcal{L}),\mathcal{L}) + Y_k(t^N(\mathcal{L}),\mathcal{L})$. Analogous, with Proposition 5.3. the unique solution is $t^N(\mathcal{L}) = 0$ such that $Y_i(0,\mathcal{L}^G) - Y_i(t(\mathcal{L}^G),\mathcal{L}^G) = \frac{x^2}{192\beta} > 0$. When countries are linked multilaterally and have no additional FTAs zero tariffs are an optimal solution for each country.
The next result summarizes the findings.

Proposition 5.5. *When countries are symmetrical we obtain the following bargaining solution that maximizes the total welfare function:*

(i) When the linking structure is such that two countries are linked bilaterally the optimal tariff is given by $t^* = 0$.

(ii) When all countries share a global multilateral link the optimal bargaining solution is given by $t^* = 0$ such that the total welfare is maximized.

We can see that when countries are symmetrical and all bargaining partners have the same number of bilateral trade agreements, free trade is always an optimal solution. Multilateral bargaining selects the free trade solution as a Pareto-efficient bargaining outcome.

We can observe that whenever countries are not symmetrically linked such that some countries have a large number of FTAs within the GATT and some other countries belong to very few FTAs this result will change. Above we have assumed that all negotiation parties have the same number of trade agreements. In the

following we will assume asymmetries with respect to linking structure and investigate whether a country that has fewer FTAs has a higher incentive for multilateral liberalization since it will benefit more. With respect to income all countries are symmetrical and we consider the situation where under MFN regime one country is not linked bilaterally and the other two countries share a FTA. Since countries are symmetrical with respect to market size we can concentrate on the case in which country A and B belong to the same FTA.
We have the following maximization problem for $\mathcal{L} := \mathcal{L}^G \cup \{\{A, B\}\}$.

$$\max_{t^N(\mathcal{L})} V(\mathcal{L}) = \prod_{i \in N} (Y_i(t^N(\mathcal{L}), \mathcal{L}) - Y_i(t(\mathcal{L}), \mathcal{L})), \tag{36}$$

such that $Y_i(t^N(\mathcal{L}), \mathcal{L}) - Y_i(t(\mathcal{L}), \mathcal{L}) \geq 0 \; \forall i \in N$, because otherwise the bargaining solution results in a welfare level which lies below the threat point for at least one country and negotiations fail.
The payoff of country A and country B can be expressed as $Y_A(t^N(\mathcal{L}), \mathcal{L}) = Y_B(t^N(\mathcal{L}), \mathcal{L})$. It can be shown that:

Proposition 5.6. *For values of $t^N(\mathcal{L})$ that are not prohibitive the following holds:*

(i) Welfare change in market C is a linear, continous and decreasing function in $t^N(\mathcal{L})$.

(ii) Welfare change in market B and market A is a continous and concave function in $t^N(\mathcal{L})$ and reaches a maximum at $t^N(\mathcal{L}) = \frac{x}{18\beta}$.

(iii) The welfare change for country C will be greater than the welfare change for country A and B under multilateral tariff reduction if $t^N(\mathcal{L})$ is sufficiently small.

The welfare effect due to multilateral tariffs increases for the country without FTAs the smaller the multilateral tariffs. Furthermore, country C's welfare change is highest when countries multilaterally negotiate free trade tariffs.
From (ii) we can follow that for positive multilateral tariffs with $t^N(\mathcal{L}) < \frac{x}{18\beta}$ tariffs country A's and country B's welfare change is positive but decreases with an decrease in tariffs. This result gives an insight why countries with few FTAs depend more on multilateral liberalization than strongly networked countries and regionalism can

serve as a substitute for multilateral cooperation.

In section 2 we have observed that a global link with one FTA is a stable state, but so far we have not investigated whether this can still be achieved as a stable state under bargaining or whether in this setting global free trade is a unique stable trading system. Calvo-Armengol (2003) introduced bargaining during the formation of network structures and characterized the stable and efficient bargaining structures. To fully characterize the effects of PTAs and multilateral tariff negotiations on the world trading system one should additionally check for stability of bargaining networks.

5.4 Many Country Extension

We shall investigate implications for stable trading systems when we increase the number of countries. In the following we allow an arbitrary number of countries to form bilateral links with all the other countries and the whole country set can form a multilateral link. We shall investigate whether MFN is essential for stability or whether we can achieve stability without MFN. The set of countries is given by N where $\mid N \mid = n$ and n different goods are traded among these countries. Countries are symmetrical such that each country $i \in N$ is endowed with $\frac{x}{2}$ units of each good except for good $I \in M$ itself. Furthermore, we shall investigate the effect on multilateral tariffs and multilateral liberalization when countries form more and more FTAs. In equation (26) we could observe that tariffs on third parties decrease when we increase a country's number of FTAs. Will these results still hold when we increase the number of countries?

Without MFN welfare in country i under $\mathcal{L}$ is given by:

$$
\begin{aligned}
& Y_i(t(\mathcal{L}),\mathcal{L}) \\
&= \sum_{j\in N_i(\mathcal{L})} t_j^I(\mathcal{L})\Big(\frac{x}{2} - \frac{(\eta_i(\mathcal{L})-1)x}{2\eta_i(\mathcal{L})} + \frac{\beta\sum_{k\in N_i(\mathcal{L})\backslash\{i\}\cup\{j\}} t_k^I(\mathcal{L})}{\eta_i(\mathcal{L})} - \frac{(\eta_i(\mathcal{L})-1)\beta t_j^I(\mathcal{L})}{\eta_i(\mathcal{L})}\Big) \\
&+\frac{x}{2}\Big[\sum_{j\in N_i(\mathcal{L})\backslash\{i\}}\Big(\frac{\alpha}{\beta} - \frac{(\eta_j(\mathcal{L})-1)x}{2\eta_j(\mathcal{L})\beta} + \frac{\sum_{k\in N_j(\mathcal{L})\backslash\{j\}} t_k^J(\mathcal{L})}{\eta_j(\mathcal{L})} - t_i^J(\mathcal{L})\Big) + \sum_{j\notin N_i(\mathcal{L})}\Big(\frac{\alpha}{\beta}-\frac{x}{2\beta}\Big)\Big] \\
&+\frac{1}{2\beta}\sum_{j\notin N_i(\mathcal{L})}\Big(\frac{x}{2}\Big)^2 + \frac{1}{2\beta}\sum_{j\in N_i(\mathcal{L})\backslash\{i\}}\Big(\frac{(\eta_j(\mathcal{L})-1)x}{2\eta_j(\mathcal{L})} - \frac{\beta\sum_{k\in N_j(\mathcal{L})\backslash\{j\}} t_i^J(\mathcal{L})}{\eta_j(\mathcal{L})} + \beta t_i^J(\mathcal{L})\Big)^2 \\
&+\frac{1}{2\beta}\Big(\frac{(\eta_i(\mathcal{L})-1)x}{2\eta_i(\mathcal{L})} - \frac{\beta\sum_{k\in N_i(\mathcal{L})\backslash\{i\}} t_k^I(\mathcal{L})}{\eta_i(\mathcal{L})}\Big)\Big)^2.
\end{aligned}
$$

Without the global link countries form bilateral links without MFN and the tariff complementarity effect still holds with equation (24). Country i's optimal tariff on the imports from country j is still given by:

$$
t^I(\mathcal{L}) = \frac{x}{\beta(2\eta_i(\mathcal{L})+2)}, \qquad \forall i \in N. \tag{37}
$$

Therefore, the tariffs on other countries decrease with the number of bilateral trade agreements and without MFN a country does not impose higher tariffs when it increases its number of trade agreements.

Furthermore, under MFN all countries are linked multilaterally and each country imposes the same tariffs on imports of good I on each other country, except on those that belong to a FTA with country i. We denote the set of countries that have a FTA with country i in the trading system $\mathcal{L}$ by $\tilde{N}_i(\mathcal{L})$ with $i \in \tilde{N}_i(\mathcal{L})$ and its cardinality by $\mid \tilde{N}_i(\mathcal{L}) \mid = \tilde{\eta}_i(\mathcal{L})$.

Under MFN we can calculate country i's welfare in $\mathcal{L}$ with

$$
\begin{aligned}
& Y_i(t(\mathcal{L}),\mathcal{L}) \\
&= \frac{1}{2\beta}\sum_{j\notin \tilde{N}_i(\mathcal{L})}\Big(\frac{(n-1)x}{2n} + \frac{\beta t^J(\mathcal{L})\tilde{\eta}_j(\mathcal{L})}{n}\Big)^2 + \frac{1}{2\beta}\sum_{j\in \tilde{N}_i(\mathcal{L})}\Big(\frac{(n-1)x}{2n} - \frac{\beta t^J(\mathcal{L})(n-\tilde{\eta}_j(\mathcal{L}))}{n}\Big)^2 \\
&+ t^I(\mathcal{L})(n-\tilde{\eta}_i(\mathcal{L}))\Big(\frac{x}{2n} - \frac{\beta\tilde{\eta}_i(\mathcal{L})t^I(\mathcal{L})}{n}\Big) \\
&+\frac{x}{2}\Big[\sum_{j\notin \tilde{N}_i(\mathcal{L})}\Big(\frac{\alpha}{\beta} - \frac{(n-1)x}{2n\beta} - \frac{\tilde{\eta}_j(\mathcal{L})t^J(\mathcal{L})}{n}\Big) + \sum_{j\in \tilde{N}_i(\mathcal{L})\backslash\{i\}}\Big(\frac{\alpha}{\beta} - \frac{(n-1)x}{2n\beta} + \frac{(n-\tilde{\eta}_j(\mathcal{L}))t^J(\mathcal{L})}{n}\Big)\Big].
\end{aligned}
$$

When all countries are symmetrical we can calculate a country's optimal tariff by:

$$t^I(\mathcal{L}) = \frac{x}{2\beta(2n\tilde{\eta}_i(\mathcal{L}) - n + \tilde{\eta}_i(\mathcal{L}))}, \tag{38}$$

for all j and $k \in N \backslash \tilde{N}_i(\mathcal{L})$. This shows that with an increasing number of FTAs the MFN-tariffs on the other countries decrease and therefore FTAs enhance the incentives for multilateral liberalization.
First, we shall investigate the change in welfare from an additional FTA with respect to the number of FTAs of the foreign market. It can be observed that:

Observation 5.2. *Under MFN country i's welfare change from an additional FTA with country j is lower, the more FTAs country j already has, and higher the more FTAs country i belongs to. Therefore a country's incentive to sign a FTA with country j is highest if the foreign market does not belong to any FTA and country i has a FTA with every other country.*

A country's incentive to sign a FTA depends on the linking structure of the foreign market. The more linked a foreign market is, the less market i has an incentive to form a FTA with the foreign market. This result strengthens the result of Proposition 5.2 which states that a global link with a FTA between one pair of countries is stable since the country without any FTAs has no incentives to sign an additional trade agreement as the other two markets are already linked bilaterally.
The support for Observation 5.2 comes from Figure 23 and 24. To get an idea of the actual value of the welfare change in country i we use a simulation. In the following we set $n = 100$, $x = 200$ and $\beta = 1$. Figure 23 depicts the relationship between the number of FTAs of country j on the welfare change of country i from an additional FTA with country j for two different values of $\tilde{N}_i(\mathcal{L})$. $\tilde{N}_i(\mathcal{L})$ represents the number of FTAs of country i after the formation of the FTA with country j in $\mathcal{L}$.
It is shown that the welfare change decreases with the number of FTAs of country j.
In Figure 24 we can observe the influence of country i's linking structure on its own welfare change from a FTA. The more linked country i already is, the higher the welfare gains from an additional FTA.
Now we seek results on stability and investigate whether global free trade can still

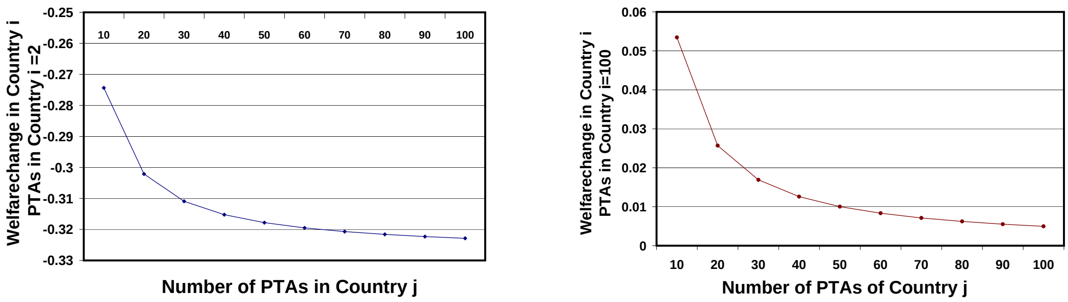


Figure 23: The effect of an increasing number of FTAs of the foreign country on country i's welfare.

be achieved as a stable trading structure even if we allow an arbitrary number of countries.

Proposition 5.7. *Global free trade is a stable state. Each stable trading system includes MFN.*

The second result shows that we cannot have stability without the MFN clause, since without MFN all countries form as many bilateral links as possible. The MFN clause has positive welfare effects on member countries and GATT stabilizes the trading system.[62]

The proof is shown in the appendix and it proceeds in the way that we first show that no country has an incentive to sever any of its bilateral links and later that no country has an incentive to sever the global link. For the second part we show that under each trading system without MFN each country improves by forming an additional bilateral link and therefore the linking structure converges to the complete trading system. Since with part one we have already proved that the complete trading system is not stable and this completes the proof.

[62]There is another strand of literature that concentrates on the question of how MFN effects the prospects of multilateral tariff cooperation and whether MFN facilitates multilateral cooperation (see e.g. Bagwell and Staiger (1999b) and Saggi (2009)).

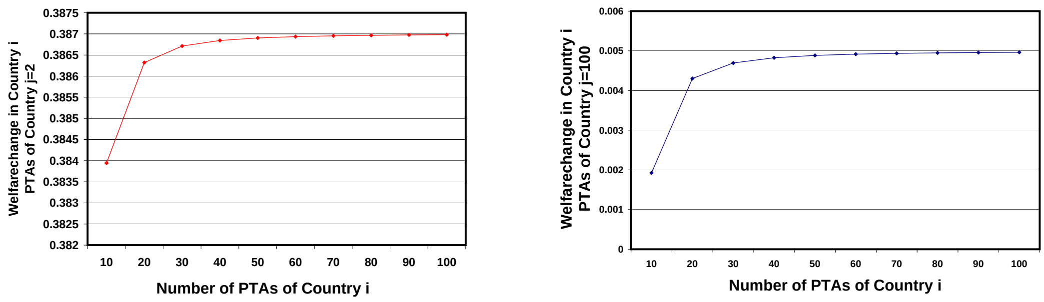


Figure 24: The effect of an increasing number of FTAs of the domestic country on its welfare

5.5 Conclusion

One characteristic of global trade is the occurrence of PTAs. There is hardly any country that is not involved in at least one PTA and the tendency is increasing that the world of trade agreements is becoming more and more complex.

This chapter helps to understand why such trading structures form and assumes that trading structures arise endogenously. We allow countries to form bilateral free trade agreements and multilateral links and investigate whether bilateralism facilitates or hinders global free trade. One main result is that each stable trading system includes MFN and no country has an incentive to cancel MFN.

We show that global free trade is a stable state and that multilateral tariffs on third parties decrease with the number of FTAs that a country forms. We allow for heterogeneous countries and show that free trade can still be stable whenever countries' income level is relatively similar.

When countries can negotiate tariffs to multilaterally reduce tariffs on each other after the trading system is in place we find that whenever each country has the same number of trade agreements, the unique efficient bargaining solution is zero tariffs and free trade is welfare improving for all countries. Whenever countries are linked asymmetrically such that some countries have more FTAs than others, free trade is not a bargaining solution and less linked countries can profit more from multilateral tariff reduction when tariffs are sufficiently low.

In section 3 of chapter 5 we assumed that the trading structure is exogenously given and calculated Nash bargaining tariffs under different trading regimes. Calvo-Armengol (2003) introduced bargaining during the formation of trading structures and characterized the stable and efficient bargaining structures. For further research we suggest investigating the nature of stable bargaining networks in an international trade framework.

5.6 Appendix

Proof of Proposition 5.1. We have to show multilateral stability for the free trade system. Condition (i) of multilateral stability is satisfied since no additional link can be formed. We have to show that no country has an incentive to sever any of its trade agreements. First, it can be shown that no country wants to sever the global link since this will result in the complete trading system and the loss in welfare gains will be $\triangle Y_i = Y_i(t(\mathcal{L}^G \cup \mathcal{L}^N), \mathcal{L}^G \cup \mathcal{L}^N) - Y_i(t(\mathcal{L}^N), \mathcal{L}^N) = \frac{505x^2}{8712\beta} > 0$ for each country $i \in N$. We can additionally observe that no country will sever any of its bilateral links since this will again result in a lower welfare as $Y_i(t(\mathcal{L}^G \cup \mathcal{L}^N), \mathcal{L}^G \cup \mathcal{L}^N) > Y_i(t(\mathcal{L}^G \cup \mathcal{L}^S_j), \mathcal{L}^G \cup \mathcal{L}^S_j)$ for all $j \neq i$. □

Proof of Proposition 5.3. Under global free trade total world welfare is given by:

$$\sum_{i \in N} Y_i(t(\mathcal{L}^G \cup \mathcal{L}^N), \mathcal{L}^G \cup \mathcal{L}^N) = \sum_{i \in N} [\frac{1}{2\beta}((\frac{x}{3})^2 + (\frac{x}{3})^2 + (\frac{x}{3})^2)] + \sum_{i \in N} \frac{x}{2}(\frac{2\alpha}{\beta} - \frac{2x}{3\beta})$$
$$= \frac{3\alpha x}{\beta} - \frac{x^2}{2\beta}.$$

In comparison, under MFN it can be verified that world welfare in $\mathcal{L}$ can be calculated from equation (26):

$$\begin{aligned}
&\sum_{i\in N} Y_i(t(\mathcal{L}),\mathcal{L})\\
=&\sum_{i\in N}\frac{1}{2\beta}[\tilde{\eta}_i(\mathcal{L})\cdot(\frac{x}{3}-\frac{(3-\tilde{\eta}_i(\mathcal{L}))x}{2(7\tilde{\eta}_i(\mathcal{L})-3)3})^2+(3-\tilde{\eta}_i(\mathcal{L}))(\frac{x}{3}+\frac{x\cdot\tilde{\eta}_i(\mathcal{L})}{2(7\tilde{\eta}_i(\mathcal{L})-3)\cdot 3})^2]\\
+&\sum_{i\in N}(3-\tilde{\eta}_i(\mathcal{L}))(\frac{x}{2\beta(7\tilde{\eta}_i(\mathcal{L})-3)})(\frac{x}{6}+\frac{x(3-\tilde{\eta}_i(\mathcal{L}))}{3\cdot 2(7\tilde{\eta}_i(\mathcal{L})-3)}-\frac{x}{2(7\tilde{\eta}_i(\mathcal{L})-3)})\\
+&\sum_{i\in N}[(3-\tilde{\eta}_i(\mathcal{L}))(\frac{x}{2}(\frac{\alpha}{\beta}-\frac{x}{3\beta}+\frac{(3-\tilde{\eta}_i(\mathcal{L}))x}{3\cdot 2(7\tilde{\eta}_i(\mathcal{L})-3)}-\frac{x}{2(7\tilde{\eta}_i(\mathcal{L})-3)})\\
+&(\tilde{\eta}_i(\mathcal{L})-1)(\frac{x}{2}(\frac{\alpha}{\beta}-\frac{x}{3\beta}+\frac{(3-\tilde{\eta}_i(\mathcal{L}))x}{3\cdot 2\beta(7\tilde{\eta}_i(\mathcal{L})-3)})]\\
=&\frac{3\alpha x}{\beta}-\frac{x^2}{8}\sum_{i\in N}(\frac{65\tilde{\eta}_i(\mathcal{L})^2-55\tilde{\eta}_i(\mathcal{L})+12}{\beta(7\tilde{\eta}_i(\mathcal{L})-3)^2}).
\end{aligned}$$

This is maximal, whenever $\tilde{\eta}_i(\mathcal{L})=3$ for all $i\in N$.
Without MFN global welfare can by calculated as:

$$\begin{aligned}
&\sum_{i\in N} Y_i(t(\mathcal{L}),\mathcal{L})\\
=&\sum_{i\in N}\frac{1}{2\beta}[(\frac{(\eta_i(\mathcal{L})-1)x}{2\eta_i(\mathcal{L})}-\frac{(\eta_i(\mathcal{L})-1)x}{(2\eta_i(\mathcal{L})+2)\eta_i(\mathcal{L})})^2\\
+&(\eta_i(\mathcal{L})-1)(\frac{(\eta_i(\mathcal{L})-1)x}{2\eta_i(\mathcal{L})}-\frac{x}{(2\eta_i(\mathcal{L})+2)\eta_i}+\frac{(\eta_i-1)x}{(2\eta_i+2)\eta_i})^2]\\
+&\sum_{i\in N}\frac{x}{2}[(3-\eta_i(\mathcal{L}))(\frac{\alpha}{\beta}-\frac{x}{2\beta})\\
+&(\eta_i(\mathcal{L})-1)(\frac{\alpha}{\beta}-\frac{(\eta_i(\mathcal{L})-1)x}{2\beta\eta_i(\mathcal{L})}+\frac{(\eta_i(\mathcal{L})-2)x}{\beta\eta_i(\mathcal{L})(2\eta_i(\mathcal{L})+2)}-\frac{(\eta_i(\mathcal{L})-1)x}{\beta(2\eta_i+2)\eta_i(\mathcal{L})})]\\
+&\frac{x(\eta_i(\mathcal{L})-1)}{(2\eta_i(\mathcal{L})+2)\beta}(\frac{x}{2\eta_i(\mathcal{L})}+\frac{x}{(2\eta_i(\mathcal{L})+2)\eta_i(\mathcal{L})}-\frac{(\eta_i(\mathcal{L})-1)x}{(2\eta_i(\mathcal{L})+2)\eta_i(\mathcal{L})})+(\frac{x}{2})^2(\frac{(3-\eta_i(\mathcal{L}))}{2\beta})\\
=&\frac{3\alpha x}{\beta}-\frac{x^2}{8}\sum_{i\in N}(\frac{\eta_i(\mathcal{L})^4-12\eta_i(\mathcal{L})^3-3\eta_i(\mathcal{L})^2+9\eta_i(\mathcal{L})-9}{\beta\eta_i(\mathcal{L})^2(\eta_i(\mathcal{L})+1)^2}).
\end{aligned}$$

It can easily be verified that $\sum_{i\in N} Y_i(t(\mathcal{L}),\mathcal{L})$ is maximal whenever $\eta_i(\mathcal{L})=3\ \forall i$ such that the complete trading system generates the highest welfare with $\sum_{i\in N} Y_i(t(\mathcal{L}^N),\mathcal{L}^N)=\frac{3\alpha x}{\beta}-\frac{33x^2}{64\beta}<\sum_{i\in N} Y_i(t(\mathcal{L}^G\cup\mathcal{L}^N),\mathcal{L}^G\cup\mathcal{L}^N)$. This completes the proof. □

Proof of Lemma 5.1. Let's consider a bilateral trade agreement between i and j. Country i's welfare increase will be $\triangle Y_i = \frac{3x_j^2}{72\beta} + \frac{x_i^2}{72\beta} > 0$ whereas for country j we get $\triangle Y_j = \frac{3x_i^2}{72\beta} + \frac{x_j^2}{72\beta}$ which is smaller than $\triangle Y_i$ whenever $x_i < x_j$. Therefore, the lower income country benefits more from a trade agreement.

□

Proof of Proposition 5.4.

$$
\begin{aligned}
&Y_i(t(\mathcal{L}^G \cup \mathcal{L}^N), \mathcal{L}^G \cup \mathcal{L}^N) - Y_i(t(\mathcal{L}^G \cup \mathcal{L}^N \backslash \{\{i,j\}\}), \mathcal{L}^G \cup \mathcal{L}^N \backslash \{\{i,j\}\})\\
=&\frac{x_i}{2}(\frac{\alpha}{\beta} - \frac{x_i + x_k}{6\beta}) + \frac{x_i}{2}(\frac{\alpha}{\beta} - \frac{x_i + x_j}{6\beta})\\
+&\frac{1}{2\beta}(\frac{x_j + x_k}{6})^2 + \frac{1}{2\beta}(\frac{x_i + x_k}{6})^2 + \frac{1}{2\beta}(\frac{x_j + x_i}{6})^2\\
-&\frac{1}{2\beta}(\frac{x_j + x_k}{6} - \frac{(5x_j - 4x_k)}{66})^2 - \frac{1}{2\beta}(\frac{x_i + x_k}{6} + \frac{2(5x_i - 4x_k)}{66})^2 - \frac{1}{2\beta}(\frac{x_j + x_i}{6})^2\\
-&\frac{x_i}{2}(\frac{\alpha}{\beta} - \frac{x_i + x_j}{6\beta}) - \frac{x_i}{2}(\frac{\alpha}{\beta} - \frac{x_i + x_k}{6\beta} + \frac{5x_i - 4x_k}{66\beta} - \frac{3(5x_j - 4x_k)}{66\beta})\\
-&\frac{5x_k - 4x_z}{22\beta}(\frac{x_j}{3} - \frac{x_k}{6} - \frac{3(5x_j - 4x_k)}{66} + \frac{(5x_j - 4x_k)}{66})\\
=&-\frac{1}{8712}\frac{-340x_i^2 + 412x_ix_k - 440x_jx_k + 275x_j^2 + 64x_k^2}{\beta}.
\end{aligned}
$$

$$
\begin{aligned}
&Y_i(t(\mathcal{L}^G \cup \mathcal{L}^N), \mathcal{L}^G \cup \mathcal{L}^N) - Y_i(t(\mathcal{L}^N), \mathcal{L}^N)\\
=&\frac{1}{2\beta}(\frac{x_j + x_k}{6})^2 + \frac{1}{2\beta}(\frac{x_i + x_k}{6})^2 + \frac{1}{2\beta}(\frac{x_j + x_i}{6})^2\\
-&\frac{1}{2\beta}(\frac{5x_i + x_k}{16})^2 - \frac{1}{2\beta}(\frac{x_j + x_k}{8})^2 - \frac{1}{2\beta}(\frac{5x_i + x_j}{16})^2\\
-&(\frac{3x_j - x_k}{16\beta})(\frac{3x_j - x_k}{16}) - (\frac{3x_k - x_j}{16\beta})(\frac{3x_k - x_j}{16}) + \frac{x_i}{2}(\frac{14x_i - 5x_k - 5x_j}{48\beta})\\
=&\frac{1}{4608}\frac{350x_i^2 - 202x_ix_k - 97x_k^2 - 97x_j^2 + 272x_jx_k - 202x_ix_j}{\beta}.
\end{aligned}
$$

□

Proof of Proposition 5.6. Assume $t^N(\mathcal{L})$ is the cooperative solution to the maximization problem of (36).

We first show (i) and (ii).

$$\begin{aligned}
&Y_A(t^N(\mathcal{L}),\mathcal{L}) - Y_A(t(\mathcal{L}),\mathcal{L}) = Y_B(t^N(\mathcal{L}),\mathcal{L}) - Y_B(t(\mathcal{L}),\mathcal{L})\\
=&t^N(\mathcal{L})(\frac{x}{6} - \frac{2t^N(\mathcal{L})\cdot\beta}{3}) + \frac{2}{2\cdot\beta}(\frac{x}{3} - \frac{\beta\cdot t^N(\mathcal{L})}{3})^2\\
+&\frac{1}{2\cdot\beta}(\frac{x}{3} + \frac{\beta\cdot t^N(\mathcal{L})}{3})^2 - \frac{x^2}{3\cdot\beta} + \frac{2615\cdot x^2}{15488\cdot\beta}\\
=&-\frac{1}{2}\beta t^N(\mathcal{L})^2 + \frac{1}{18}x\cdot t^N(\mathcal{L}) + \frac{101}{46\,464}\frac{x^2}{\beta}.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
&Y_C(t^N(\mathcal{L}),\mathcal{L}) - Y_C(t(\mathcal{L}),\mathcal{L})\\
=&2t^N(\frac{x}{6} - \frac{t^N\cdot\beta}{3}) + \frac{1}{2\cdot\beta}(\frac{x}{3} - \frac{2\cdot\beta t^N}{3})^2 + \frac{2}{2\cdot\beta}(\frac{x}{3} + \frac{2\cdot\beta t^N}{3})^2\\
&-\frac{x^2}{3\cdot\beta} - \frac{2\cdot x\cdot t^N}{3} + \frac{327x^2}{1936\cdot\beta}\\
=&-\frac{1}{17\,424}x\frac{1936t^N\beta - 39x}{\beta}.
\end{aligned}$$

The first derivative of country C's welfare change with respect to t^N is $\frac{-1936\cdot x}{17424} < 0$. The first derivate of country A's welfare change with respect to t^N is $\frac{x}{18} - \beta\cdot t^N$. The second derivate of country A's welfare change is $-\beta < 0$ such that the function $Y_A(t^N(\mathcal{L}),\mathcal{L}) - Y_A(t(\mathcal{L}),\mathcal{L})$ reaches a maximum at $t^N = \frac{x}{18\cdot\beta}$ and is strictly concave.

To verify (iii) we first calculate each country's welfare change at $t^N(\mathcal{L}) = 0$. We can easily verify that $Y_C(0,\mathcal{L}) - Y_C(t(\mathcal{L}),\mathcal{L}) > Y_A(0,\mathcal{L}) - Y_A(t(\mathcal{L}),\mathcal{L})$. Since $Y_C(t^N(\mathcal{L}),\mathcal{L}) - Y_C(t(\mathcal{L}),\mathcal{L})$ is a linear, decreasing and continous function in $t^N(\mathcal{L})$ as shown in part (i) of the Proposition. Furthermore, $Y_A(t^N(\mathcal{L}),\mathcal{L}) - Y_A(t(\mathcal{L}),\mathcal{L})$ is a continuous and concave function in $t^N(\mathcal{L})$ as shown in part (ii) of the Proposition, for values $t^N(\mathcal{L})$ smaller than t^* for that $Y_C(t^*,\mathcal{L}) - Y_C(t(\mathcal{L}),\mathcal{L}) = Y_A(t^*,\mathcal{L}) - Y_A(t(\mathcal{L}),\mathcal{L})$ country C's welfare effect is larger than country A's and country B's welfare effect. □

Proof of Proposition 5.7. Condition (ii) of Definition 5.1 is trivially satisfied since no additional link can be added.

Under free trade we know that tariffs on good I are $t^I* = 0$ for all $i \in N$ such that total welfare reduces to:

$$Y_i(t(\mathcal{L}^N \cup \mathcal{L}^G), \mathcal{L}^N \cup \mathcal{L}^G) = \frac{n}{2\beta}(\frac{(n-1)x}{2n})^2 + \frac{x}{2}(\frac{\alpha}{\beta} - \frac{(n-1)x}{2n\beta})(n-1).$$

When one player i severs one of his bilateral free trade agreements $\{i, j\}$ we will have $t^I = t^J = \tilde{t}$ and i's welfare will be:

$$\begin{aligned}
&Y_i(t(\mathcal{L}^N \cup \mathcal{L}^G \backslash \{\{i,j\}\}), \mathcal{L}^N \cup \mathcal{L}^G \backslash \{\{i,j\}\}) \\
=&\frac{(n-2)}{2\beta}(\frac{(n-1)x}{2n})^2 + \frac{1}{2\beta}(\frac{(n-1)x}{2n} + \frac{\beta\tilde{t}(n-1)}{n})^2 \\
+&\frac{1}{2\beta}(\frac{(n-1)x}{2n} - \frac{\beta\tilde{t}}{n})^2 + \frac{x}{2}(\frac{\alpha}{\beta} - \frac{(n-1)x}{2n\beta})(n-2) \\
+&\frac{x}{2}(\frac{\alpha}{\beta} - \frac{(n-1)x}{2n\beta} - \frac{(n-1)\tilde{t}}{n}) + \tilde{t}(\frac{x}{2n} - \frac{\beta(n-1)\tilde{t}}{n}).
\end{aligned}$$

This yields:

$$\begin{aligned}
&Y_i(t(\mathcal{L}^N \cup \mathcal{L}^G), \mathcal{L}^N \cup \mathcal{L}^G) - Y_i(t(\mathcal{L}^N \cup \mathcal{L}^G \backslash \{\{i,j\}\}), \mathcal{L}^N \cup \mathcal{L}^G \backslash \{\{i,j\}\}) \\
=&\frac{1}{8}x^2 \frac{2 - 11n^2 + 6n + 4n^3}{\beta\left(2n^2 - 2n - 1\right)^2 n^2} > 0,
\end{aligned}$$

$\forall n \geq 3$. Therefore, no player has an incentive to sever any of his bilateral links. In the complete trading system $t^I = \tilde{t}\ \forall i \in N$. To show that no player has an incentive to sever the global link we calculate:

$$\begin{aligned}
&Y_i(t(\mathcal{L}^N), \mathcal{L}^N) \\
=&\frac{1}{2\beta}(\frac{(n-1)x}{2n} - \frac{\beta\tilde{t}(n-1)}{n})^2 + \frac{1}{2\beta}(n-1)(\frac{(n-1)x}{2n} - \frac{\beta(n-1)\tilde{t}}{n} + \beta\tilde{t})^2 \\
+&(n-1)\frac{x}{2}(\frac{\alpha}{\beta} - \frac{(n-1)x}{2n\beta} + \frac{(n-1)\tilde{t}}{n} - \tilde{t}) \\
+&(n-1)\tilde{t}(\frac{x}{2} - \frac{(n-1)x}{2n} + \frac{\beta(n-2)\tilde{t}}{n} - \frac{(n-1)\beta\tilde{t}}{n}).
\end{aligned}$$

Since

$$Y_i(t(\mathcal{L}^N \cup \mathcal{L}^G), \mathcal{L}^N \cup \mathcal{L}^G) - Y_i(t(\mathcal{L}^N), \mathcal{L}^N) = \frac{1}{2}\frac{(n-1)}{n}\tilde{t}^2\beta > 0,$$

no player has an incentive to sever the global link. Thus condition (i) is satisfied and this completes the proof of the first part. Since we have already shown that the complete trading system results in global free trade, it is enough to show for the second part of Proposition 5.7 that in each arbitrary trading system without MFN each player will increase his welfare level when he forms an additional bilateral link. Since with the first part the complete trading system cannot be stable either, this completes the proof.

Consider any arbitrary trading system $\mathcal{L}$ without MFN with $\eta_j(\mathcal{L})$ and $\eta_i(\mathcal{L})$ and any link $\{i,j\} \in \mathcal{L}$:

$$\begin{aligned}
&\Pi_i(t(\mathcal{L}),\mathcal{L}) - \Pi_i(t(\mathcal{L}\backslash\{\{i,j\}\}),\mathcal{L}\backslash\{\{i,j\}\}) \\
=&\frac{x}{2}(\frac{\alpha}{\beta} - \frac{(\eta_j(\mathcal{L})-1)x}{2\eta_j(\mathcal{L})\beta} + \frac{(\eta_j(\mathcal{L})-2)x}{\eta_j(\mathcal{L})\beta(2\eta_j(\mathcal{L})+2)} - \frac{(\eta_j(\mathcal{L})-1)x}{\eta_j(\mathcal{L})\beta(2\eta_j(\mathcal{L})+2)}) - \frac{x}{2}(\frac{\alpha}{\beta} - \frac{x}{2\beta}) \\
=&\frac{x}{2}(\frac{x}{\beta(2\eta_j(\mathcal{L})+2)}).
\end{aligned}$$

Change in consumer surplus for good I:

$$\begin{aligned}
&CS_i^I(t(\mathcal{L}),\mathcal{L}) - CS_i^I(t(\mathcal{L}\backslash\{\{i,j\}\}),\mathcal{L}\backslash\{\{i,j\}\}) \\
=&\frac{1}{2\beta}(\frac{(\eta_i(\mathcal{L})-1)x}{2\eta_i(\mathcal{L})} - \frac{(\eta_i(\mathcal{L})-1)x}{\eta_i(\mathcal{L})(2\eta_i(\mathcal{L})+2)})^2 - \frac{1}{2\beta}(\frac{(\eta_i(\mathcal{L})-2)x}{2(\eta_i(\mathcal{L})-1)} - \frac{(\eta_i(\mathcal{L})-2)x}{(\eta_i(\mathcal{L})-1)2\eta_i(\mathcal{L})})^2 \\
=&\frac{x^2}{2\beta}(\frac{4\eta_i^2(\mathcal{L})4\eta_i(\mathcal{L})-4)}{4(\eta_i(\mathcal{L})+1)^2\eta_i^2(\mathcal{L})})\eta_i(\mathcal{L}).
\end{aligned}$$

For the loss in tariff revenue:

$$\begin{aligned}
&TR_i(t(\mathcal{L}),\mathcal{L}) - TR_i(t(\mathcal{L}\backslash\{\{i,j\}\}),\mathcal{L}\backslash\{\{i,j\}\}) \\
=&\frac{(\eta_i(\mathcal{L})-1)x}{\beta(2\eta_i(\mathcal{L})+2)}(\frac{x}{2\eta_i(\mathcal{L})} - \frac{x}{\eta_i(\mathcal{L})(2\eta_i(\mathcal{L})+2)}) \\
-&\frac{(\eta_i(\mathcal{L})-2)x}{2\beta\eta_i(\mathcal{L})}(\frac{x}{2(\eta_i(\mathcal{L})-1)} - \frac{x}{(\eta_i(\mathcal{L})-1)2\eta_i(\mathcal{L})}) \\
=&\frac{x^2}{2\beta}(\frac{(-\eta_i^2(\mathcal{L})+3\eta_i(\mathcal{L})+2)}{2(\eta_i(\mathcal{L})+1)^2\eta_i^2(\mathcal{L})}).
\end{aligned}$$

For the change in consumer surplus from good J:

$$
\begin{aligned}
&CS_i^J(t(\mathcal{L}),\mathcal{L}) - CS_i^J(t(\mathcal{L}\backslash\{\{i,j\}\}),\mathcal{L}\backslash\{\{i,j\}\}) \\
=&\frac{1}{2\beta}(\frac{(\eta_j(\mathcal{L})-1)x}{2\eta_j(\mathcal{L})} - \frac{(\eta_j(\mathcal{L})-2)x}{\eta_j(\mathcal{L})(2\eta_j(\mathcal{L})+2)} + \frac{(\eta_j(\mathcal{L})-1)x}{(2\eta_j(\mathcal{L})+2)\eta_j(\mathcal{L})})^2 - \frac{1}{2\beta}(\frac{x}{2})^2 \\
=&\frac{x^2}{2\beta}(\frac{-2\eta_j(\mathcal{L})-1)}{4(\eta_j(\mathcal{L})+1)^2}).
\end{aligned}
$$

For the total change in welfare we can calculate:

$$
\begin{aligned}
&Y_i(t(\mathcal{L}),\mathcal{L}) - Y_i(t(\mathcal{L}\backslash\{\{i,j\}\}),\mathcal{L}\backslash\{\{i,j\}\}) \\
=&\frac{x^2}{4\beta(\eta_i(\mathcal{L})+1)^2\eta_i^2(\mathcal{L})}(\eta_i(\mathcal{L})+\eta_i^2(\mathcal{L})) + \frac{x^2}{8\beta(\eta_j(\mathcal{L})+1)^2} > 0.
\end{aligned}
$$

Starting from any arbitrary trading system each player improves by forming a bilateral link. This completes the proof. □

6 Allocation Rules for Hypergraph Games

In the previous chapters we analyzed the shape of stable and efficient networks and concentrated on the question of whether efficient networks are stable. One can also look in detail at the axiomatic foundations of some allocation rules. Leaving aside the question on how network structures form and the internal interaction among players in the network, in this chapter we concentrate on the question of given a network and a value that is generated from the players in the network, how should the value be allocated?

The axiomatic literature largely grew out of the cooperative game theory literature and mostly followed cooperative games with communication structures (conference structures).[63] The cooperative game theory literature with communication structure started with the seminal paper of Myerson (1977) in which players in a cooperative game meet together in a set of conferences where payers can communicate and negotiate plans such that the conference structure describes the communication possibilities between players and a conference structure is described by means of a bilateral graph. The papers of Myerson (1977) and Myerson (1980) have been extended towards hypergraphs by van den Nouweland et al. (1992). Taking the conference structure as fixed, they analyze how the value generated by a coalition in an underlying conference structure should be allocated among the players.

In the following we will concentrate on the most prominent allocation rule, the Myerson value allocation rule, in which we assume that all players gain equally from each link and we also demand from an allocation rule that within a conference all players have equal voting power (position value).
Myerson's (1977) approach considers a set of players that are connected, independent on how exactly they are connected. In economic situation as in international trading networks, different trading structures lead to a different economic outcome. The following chapter addresses the question of how the value is supposed to be allocated. It shows that when payoff is supposed to be allocated fair and compo-

[63] Much of the literature on cooperative games with communication structures is surveyed in van den Nouweland (2005).

Figure 25.

nent efficient, then the unique possibility is when it is allocated with respect to the Myerson value. Fair in this context means that when two countries form a trade agreement, we want them to gain equally from the agreement. Efficiency demands that the total value that is generated from a component of players is supposed to be allocated among them.

A shortcoming of Myerson's approach is that the value function of the cooperative game assigns the same value to a connected set of players regardless of how they are connected. This implies that in Figure 25 coalition $\{A, B, C\}$ obtains the same value in both networks. Jackson and Wolinsky (1996) introduce a network game in which players can form bilateral non-directed links and the value function does not depend on a set of players but on the network structure itself. In the corresponding hypergraph game we allow that the value function depends on the network structure and therefore on how exactly the players are connected.
In this chapter we extend the approach of Jackson and Wolinsky (1996) towards hypergraphs where we allow linking structures with links that are not only bilateral but also multilateral. As the previous chapters have shown, multilateralism and bilateralism may lead to different optimal decisions of players and different economic outcome. When we allow for multilateral link formation we can show that the Shapley value of a corresponding characteristic function game, where the value function is now defined on hypergraphs, is the unique allocation rule that satisfies efficiency and fairness.[64]

[64] Durieu et al. (2005) introduce games played on hypergraphs where network formation is considered as an application.

Hypergraph conference structures may appear in many economic situations such as international trade. Countries have trade agreements that are partly multilateral and partly bilateral. Countries are members of the World Trade Organization (WTO) and have additional regional trade agreements. Consider for example Canada, the U.S. and Mexico, who are all members of the WTO and additionally members of the North American Free Trade Agreement (NAFTA). When we ask how the value generated in a conference structure should be allocated among the countries, economic concepts such as fairness and efficiency may play an important role. An important issue often discussed in international economics is whether trade agreements negotiated among heterogenous countries benefit all parties equally. (As Bhagwati claims, the formation of a regional trade agreement between a developed and an undeveloped country benefits the developed countries more.) In this context fairness can apply as a reasonable way to allocate the value in the sense that a trade agreement between a group of countries should favour all trading partners equally.

While the Myerson value concentrates on the role of a single player, the position value as introduced by Meessen (1988) focuses on the role of a single link in a conference structure. Whereas the Myerson value is defined on a so-called *point game*, the communicative strength of a conference is measured by means of a *link game*. Assuming that in a conference structure each player has a veto power in each of his participating links, we shall divide the value of each link equally among all its members. The total payoff that a player obtains from a given conference structure is then called the position value. The position value was characterized axiomatically by van den Nouweland et al. (1992) for the class of cycle-free networks. We characterize the position value in the context of cycle-free hypergraph games. The influence property characterizes allocation rules in which in each conference each player has the same power whereas efficiency demands that the total value generated by a connected component of the conference structure should be completely allocated among its members.

Chapter 6 is organized as follows. In section 1 we present the basic notation and definition that describe conferences and value functions. In section 2 we present a characterization of the Myerson value in the context of hypergraph games. In section 3 we characterize the position value by additivity, efficiency, the superfluous link property and the influence property in the class of cycle-free hypergraphs. In section 4 we give a corresponding core definition in the context of hypergraph games to analyze stability when the allocation of value and hypergraph formation happen simultaneously.

6.1 Preliminaries

The following definitions are based on chapter 3 of the thesis in which we consider a finite number of identical players $N = \{1, ..., n\}$ and assume $n \geq 3$.
Hypergraphs are defined as set systems that consist of subsets of the player set N.

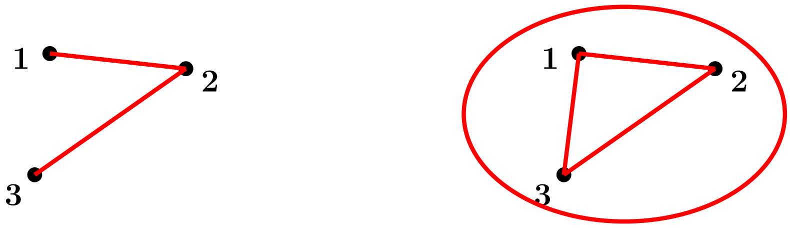
> Let $N = \{1, ..., n\}$ be a finite set of nodes. A set system $\mathcal{L}$, where $\mathcal{L} = \{L_1, ..., L_m\}$ is a set of links, $\mathcal{L} \subseteq 2^N$, is called a hypergraph on N.

We again restrict our attention to hypergraphs $\mathcal{L}$ with $\mathcal{L} \subseteq \{L \in 2^N || L| \geq 2\}$ since we assume that each player is linked with himself. A conference structure $\mathcal{L}$ can be seen as a set of links that describe all communication possibilities among players in N.[65] Players can only communicate with each other when they are both in the same conference. The set of all possible conference structures that satisfy the definition is denoted with $\mathcal{H}$.

As in chapter 3, if $L \in \mathcal{L}$, we say that all players $i \in L$ have a direct link. $N(\mathcal{L})$ denotes the set of players $i \in N$ that have at least one direct link in $\mathcal{L}$, $N(\mathcal{L}) = \{i \in N \mid \exists L \in \mathcal{L} : i \in L\}$.

Myerson (1977) and van den Nouweland et al. (1992) define the value function on a set of players such that the same set of connected players generates the same value, regardless of how the players are connected. As already noted by Jackson (2005), cooperative game theory characteristic functions do not capture economic

[65] Here a conference structure is described by a hypergraph that describes all communication possibilities between players.



a) *b)*

Figure 26

situations where conference structures are important. This can be seen with the following example. Consider the set of players $N = \{1, 2, 3\}$. In Figure 26 we find two different hypergraph structures that connect the same set of players. In cooperative game theory the value generated by the player set N is the same among both structures. In b) the hypergraph consists of all possible subsets of the player set with $\mathcal{L} = \{L \in 2^N || L| \geq 2\}$ whereas in a) the players are connected via a less connected hypergraph. As in an international trade context where the total volume of trade induced by both hypergraph structures differs, it should generally be the case that the exact linking structure matters. As in chapter 3 we will introduce a value function that directly depends on the conference structure. This allows the value generated by $N = \{1, 2, 3\}$ to differ between a) and b).
The value of a hypergraph is therefore represented by a real valued function $v : \mathcal{H} \to \mathbb{R}$, which specifies for each hypergraph $\mathcal{L} \in \mathcal{H}$ the total value $v(\mathcal{L})$ generated by $\mathcal{L}$ and will be the aggregate of individual payoffs or productions of a hypergraph. We assume $v(\emptyset) = 0$ and the set of all possible value functions is denoted as $\mathcal{V}$. In chapter 4 and 5 the value function was given by world welfare.

A hypergraph game is then a pair, (N, v), of a set of players and a value function defined on the set $\mathcal{H}$.

An allocation rule Y is a function $Y : \mathcal{H} \times \mathcal{V} \to \mathbb{R}^n$ that describes how the value of a conference structure is distributed among the players. The payoff of player $i \in N$ in $\mathcal{L}$ with a value function v under the allocation rule Y is denoted by $Y_i(\mathcal{L}, v)$. In chapter 4 and 5 the allocation rule was given by a country's welfare function that was defined on the set of trading systems.

A path in $\mathcal{L}$ connecting players i and j is a set of distinct nodes $\{i = i_1, i_2, ..., i_n = j\} \subseteq N(\mathcal{L})$ such that $\{i_1, i_2\} \subseteq L_1.....\{i_{n-1}, i_n\} \subseteq L_{n-1}$ and $\{L_1, ..., L_{n-1}\} \subseteq \mathcal{L}$.
A nonempty hypergraph $\mathcal{L}' \subseteq \mathcal{L}$ is a component of $\mathcal{L}$ if $\forall i \in N(\mathcal{L}')$ and $j \in N(\mathcal{L}')$ a path exists in $\mathcal{L}'$ connecting i and j, and for any $i \in N(\mathcal{L}')$ and $j \notin N(\mathcal{L}')$ a path does not exist in $\mathcal{L}$ between i and j. We will denote the set of components in $\mathcal{L}$ with $C(\mathcal{L})$. $\mathcal{L}$ is said to be fully connected if each pair of players i and j, $\forall i, j \in N$, is connected in $\mathcal{L}$. Then the hypergraph $\mathcal{L}$ consists of one component with $C(\mathcal{L}) = \mathcal{L}$. It is clear that a conference structure $\mathcal{L}$ within N determines a partition of links in $\mathcal{L}$ into communication components The following two sections extend the Myerson value and the position value towards hypergraphs.

6.2 The Myerson Value

Myerson (1977) was the first to show that in a cooperative game setting a variation of the Shapley value is the unique allocation rule that satisfies fairness and efficiency. This value was later referred to as the Myerson value. In the following we will redefine the Myerson value in the context of hypergraph games. The following two properties of value functions are borrowed from Definition 3.7. and Definition 3.8..

A value function $v : \mathcal{H} \to \mathbb{R}$ is component additive if $v(\mathcal{L}) = \sum_{\mathcal{C} \in C(\mathcal{L})} v(\mathcal{C})$.

This condition on value functions rules out externalities across components. With $S/\mathcal{L} = \{L \in \mathcal{L} \mid L \subseteq S\}$ we denote the set of links in $\mathcal{L}$ that contain only players in S, $S \subseteq N$, whereas $C(S/\mathcal{L})$ denotes the set of components in $S/\mathcal{L}$.

An allocation rule $Y : \mathcal{H} \times \mathcal{V} \to \mathbb{R}^n$ is called component efficient if for any component additive v, $\mathcal{L}$ and all components $\mathcal{C} \in C(N/\mathcal{L})$

$$\sum_{i \in N(\mathcal{C})} Y_i(\mathcal{L}, v) = v(\mathcal{C}).$$

Thus component efficiency requires the total value of a component to be allocated among the members of the component. This is satisfied in situations in which the value naturally results from a production process or utility functions of players and players have no incentive to allocate their value to players outside of a component.

An allocation rule $Y : \mathcal{H} \times \mathcal{V} \to \mathbb{R}^n$ satisfies equal bargaining power if for all v, $\mathcal{L}$, $L \in \mathcal{L}$

$$Y_i(\mathcal{L}, v) - Y_i(\mathcal{L}\backslash\{L\}, v) = Y_j(\mathcal{L}, v) - Y_j(\mathcal{L}\backslash\{L\}, v) \quad \forall i, j \in L,$$

where with $\mathcal{L}\backslash\{L\}$ we denote the hypergraph that we obtain from the deletion of link L. Equal bargaining power requires that all players in a link L gain or lose equally from its deletion.[66]

The Shapley value $SV(v)$ of v is defined by:

$$SV_i(v) = \sum_{S \subseteq N} \frac{(n - |S|)!(|S| - 1)!}{n!}(v(S) - v(S\backslash\{i\})),$$

for all $i \in N$.
We define a characteristic function game $Ch_{\mathcal{L},v}$ with

$$Ch_{\mathcal{L},v}(S) = \sum_{\mathcal{C} \in C(S/\mathcal{L})} v(\mathcal{C}),$$

where $S/\mathcal{L}$ is the set of links in $\mathcal{L}$ that contain only the players in S and the function $Ch_{\mathcal{L},v}$ assigns to each coalition the value the coalition can achieve in the conference structure $\mathcal{L}$ and is defined as the sum of the value of connected components in $\mathcal{L}$ that contain only players in S. We will refer to the characteristic function game as the *point game*.
The following result extends the approach of Jackson and Wolinsky (1996) towards hypergraphs in which the Myerson value is defined as the Shapley value of the characteristic function game $Ch_{\mathcal{L},v}$.

Theorem 6.1. *If v is component additive, then the Shapley value of the characteristic function game $Ch_{\mathcal{L},v}$, $MV_i(\mathcal{L}, v) = SV_i(Ch_{\mathcal{L},v}) = \sum_{S \subseteq N} \frac{(n-|S|-1)!|S|!}{n!}(Ch_{\mathcal{L},v}(S) - Ch_{\mathcal{L},v}(S\backslash\{i\}))$ is the unique allocation rule that satisfies equal bargaining power and component efficiency.*

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[66] In particular, we should notice that the equal bargaining power definition corresponds to the fairness definition on graphs in Myerson (1977).

The proof of the theorem is provided in the appendix. It proceeds by first showing that there exists only one allocation rule that satisfies equal bargaining power and efficiency and second, it shows that the Myerson value MV satisfies both properties.
Note that with fixed v, equal bargaining power is equivalent to what Myerson (1977) calls fairness, whereas the definition of component efficiency is equivalent to efficiency in his context.

6.3 The Position Value

The position value as proposed by Meessen (1988) concentrates rather on the role of a link in a conference structure than on the role of a player. The idea is that first the Shapley value of a characteristic function game based on conferences measures the strength of each link in the conference structure and then each player obtains a payoff from each of the conferences that he is a member in. Since we shall give each member in a conference veto power in each of his participating links we allocate the Shapley value of each conference equally among its members.
Based on the definition of Meessen (1988) we find a corresponding definition of the position value on hypergraph games. The position value for hypergraph games is defined by means of the so-called *link game* $Ch_{N,v}$ for all $\mathcal{L} \in \mathcal{H}$ and all $\mathcal{L}' \subseteq \mathcal{L}$ with

$$Ch_{N,v}(\mathcal{L}') := Ch_v(\mathcal{L}', N) = \sum_{\mathcal{C} \in C(N/\mathcal{L}')} v(\mathcal{C}),$$

where $Ch_{N,v}(\mathcal{L}')$ denotes the value allocated to the conference structure $\mathcal{L}'$.
We can now formalize the position value for hypergraph games with $\mathcal{L} \in \mathcal{H}$, such that $\forall i \in N$:

$$PV_i(\mathcal{L}, v) := \sum_{L \in \mathcal{L}_i} \frac{1}{\mid L \mid} \cdot SV_L(\mathcal{L}, Ch_{N,v}), \tag{39}$$

where $\mathcal{L}_i = \{L \in \mathcal{L} \mid i \in L\}$. The idea of the position value is that first the Shapley value is allocated to each link $L \in \mathcal{L}$ and afterwards the Shapley value of the link is allocated equally among all players in the conference. The position value of player i is the value that a player obtains from each conference in which he participates.

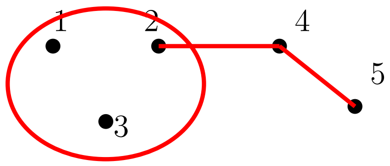

Figure 27

The following example demonstrates the difference between the Myerson value and the position value.

Example 6.1. *We define the (link) unanimity game on $\mathcal{L}' \subseteq \mathcal{L}$ by:*

$$u_{\mathcal{L}'}(\mathcal{A}) = \begin{cases} 1, & \text{if } \mathcal{L}' \subseteq \mathcal{A}, \\ 0, & \text{else.} \end{cases}$$

Consider the conference structure described in Figure 27, with $N = \{1,2,3,4,5\}$, $\mathcal{L} = \{\{1,2,3\},\{4,5\},\{2,4\}\}$ and $v = u_{\{\{1,2,3\},\{4,5\}\}}$. Then we can calculate the link game:

$$Ch_{N,v}(\mathcal{L}') = \begin{cases} 1, & \text{if } \mathcal{L}' = \mathcal{L}, \\ 0, & \text{else.} \end{cases}$$

For the Shapley value of a link $L \in \mathcal{L}$ we obtain $SV_L = \frac{1}{3}$ $\forall L \in \mathcal{L}$ and therefore:

$$PV_1 = PV_3 = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}, \quad PV_2 = \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} = \frac{5}{18},$$
$$PV_4 = \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{3}, \quad PV_5 = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}.$$

For the Myerson value with $v = u_{\{1,2,3,4,5\}}$ we calculate the point game

$$Ch_{\mathcal{L},v}(S) = \begin{cases} 1, & \text{if } S = N, \\ 0, & \text{else.} \end{cases}$$

We obtain: $MV_i(\mathcal{L}, v) = SV_i(Ch_{\mathcal{L},v}) = \frac{1}{5}$ $\forall i$.
We can see that $MV(\mathcal{L}, v) \neq PV(\mathcal{L}, v)$.

In order to characterize the position value on hypergraph networks axiomatically we introduce some further properties of allocation rules.

Definition 6.1. *An allocation rule* $Y : \mathcal{H} \times \mathcal{V} \to \mathbb{R}^n$ *is called additive if for all* $\mathcal{L}$ *and value functions* $v^1 : \mathcal{H} \to \mathbb{R}$ *and* $v^2 : \mathcal{H} \to \mathbb{R}$

$$Y(\mathcal{L}, v^1 + v^2) = Y(\mathcal{L}, v^1) + Y(\mathcal{L}, v^2).$$

Definition 6.2. *Consider any* $\mathcal{L} \in \mathcal{H}$ *and* $v \in \mathcal{V}$*. A link* $L \in \mathcal{L}$ *is called superfluous for* $\mathcal{L}$ *and* v*, if* $\forall \mathcal{L}' \subseteq \mathcal{L}$

$$Ch_v(\mathcal{L}' \setminus \{L\}, N) = Ch_v(\mathcal{L}', N).$$

The last definition states that the deletion of a superfluous link L from a set of links $\mathcal{L}'$ does not change the value generated from $\mathcal{L}'$.
We therefore define:

Definition 6.3. *An allocation rule* $Y : \mathcal{H} \times \mathcal{V} \to \mathbb{R}^n$ *has the superfluous link property if for all* $\mathcal{L} \in \mathcal{H}$*,* $v \in \mathcal{V}$ *and all* $L \in \mathcal{L}$ *that are superfluous for* $\mathcal{L}$ *and* v *we have:*

$$Y(\mathcal{L} \backslash \{L\}, v) = Y(\mathcal{L}, v).$$

Definition 6.4. *The influence of a player* i *in* $\mathcal{L}$ *is defined by:*

$$I_i(\mathcal{L}, v) = \sum_{L \in \mathcal{L}_i} \frac{1}{\mid L \mid},$$

where I_i *measures the importance of player* i *in the conference structure* $\mathcal{L}$*.*

Here, we want to assume that each player in a conference has the same power and the influence of a player in each conference structure depends on the number of links in which he participates.

Definition 6.5. *A hypergraph* $\mathcal{L}$ *is called link anonymous if a function exists* $f : \{0, 1, ..., \mid \mathcal{L} \mid\} \to \mathbb{R}$ *such that* $\forall \mathcal{L}' \subseteq \mathcal{L}$

$$Ch_v(\mathcal{L}', N) = f(\mid \mathcal{L}' \mid), \tag{40}$$

with $v \in \mathcal{V}$*.*

We can further define

Definition 6.6. *An allocation rule* $Y : \mathcal{H} \times \mathcal{V} \to \mathbb{R}^n$ *has the influence property if for each link anonymous hypergraph* $\mathcal{L} \in \mathcal{H}$ *an* $\alpha \in \mathbb{R}$ *exists such that for all* $i \in N$, $v \in \mathcal{V}$

$$Y_i(\mathcal{L}, v) = \alpha \cdot I_i(\mathcal{L}, v).$$

This axiom claims that all players in each link obtain the same payoff, and the total payoff that a player obtains is proportional to his influence in the conference structure.

To fully characterize the position value for conference structures we further consider the class of cycle-free hypergraphs that we denote with $\mathcal{H}*$. A cycle $\mathcal{L}$ is a chain $(i_1, L_1, i_2, L_2, ..., L_{k-1}, i_k)$ of length k, where $k \geq 2$, $L_1, ...L_{k-1} \in \mathcal{L}$ and $i_1 = i_k$. We further say that $\tilde{\mathcal{L}}$ is contained within a connected component of $\mathcal{L}$. We define with $H(\tilde{\mathcal{L}}) := \cap\{\mathcal{L}' \mid \tilde{\mathcal{L}} \subseteq \mathcal{L}' \subseteq \mathcal{L}, \mathcal{L}'$ is a connected subgraph of $\mathcal{L}\}$ the unique set system such that $\tilde{\mathcal{L}} \subseteq \mathcal{L}' \subseteq \mathcal{L}$ and $\mathcal{L}'$ is a connected subgraph.

According to Theorem 4 in Owen (1986), we can conclude that in a cycle-free hypergraph the intersection of connected subgraphs is connected and are able to show the following result:

Lemma 6.1. *Let* $\mathcal{L}$ *be element of* $\mathcal{H}^*$. *For any subgraph* $\tilde{\mathcal{L}} \subseteq \mathcal{L}$ *a unique minimal connected* $\mathcal{L}'$ *exists, such that* $\tilde{\mathcal{L}} \subseteq \mathcal{L}'$.

Proof. With $H(\tilde{\mathcal{L}})$ as defined above we obviously have that $\tilde{\mathcal{L}} \subseteq H(\tilde{\mathcal{L}})$ and the set $H(\tilde{\mathcal{L}})$ is a connected subgraph. Furthermore, we can argue that the set $H(\tilde{\mathcal{L}})$ is the subset of any connected subgraph $\mathcal{L}'$, and therefore it has to be minimal. □

Clearly $\tilde{\mathcal{L}} = H(\tilde{\mathcal{L}})$ whenever $\tilde{\mathcal{L}}$ is connected.

We can prove the following result:

Theorem 6.2. *The position value* $PV_i(\mathcal{L}, v)$ *satisfies additivity, component efficiency, the superfluous link property and the influence property. Furthermore, the position value is the unique allocation rule on* $\mathcal{H}^*$ *that satisfies additivity, component efficiency, the superfluous link property and the influence property.*

Remark 6.1. *It is important to note that the Position value does not satisfy equal bargaining power. From Example* 6.1. *we can calculate:*

$$\begin{aligned} PV_1(\mathcal{L}, v)) - PV_1(\mathcal{L}\backslash\{\{1,2,3\}\}, v) &= \frac{1}{9} - 0 = \frac{1}{9} \\ \neq PV_2(\mathcal{L}, v) - PV_2(\mathcal{L}\backslash\{\{1,2,3\}\}, v) &= \frac{5}{18} - 0 = \frac{5}{18}. \end{aligned}$$

6.4 The Core of Hypergraph Games

The question of whether the allocation of a hypergraph game as proposed by the Myerson and the position value is resistant with respect to coalitional deviations has not been addressed in this chapter so far. Under the Myerson value the conference structure is implicitly assumed as fixed when value is being allocated. In the context of conference structures in economic applications such as international trade an allocation rule is required which is resistant with respect to individual deviations that alter the conference structure. By introducing the following concept we shall investigate whether the Myerson and the position value satisfy this condition.

Given a value function v and a hypergraph $\mathcal{L}$, its monotonic cover is defined by

$$\hat{v}(\mathcal{L}) = \max_{\mathcal{L}' \subseteq \mathcal{L}} v(\mathcal{L}').$$

A monotonic cover for any v and $\mathcal{L}$ describes the maximal value that can be generated from any hypergraph $\mathcal{L}'$ that exclusively consists of links in $\mathcal{L}$.
In situations where players decide on the allocation of the value of a conference structure at the same time that they are forming the structure there is a natural core definition that captures some constraints on the allocation of value that would be required to avoid certain forms of instability. Jackson (2005) previously defined the core on bilateral network games which we extend for hypergraphs in the following. The following definition specifies hypergraph-allocation pairs in which no coalition of players benefits from altering the conference structure and reallocating the value such that all players of the coalition improve.

Definition 6.7. *A hypergraph-allocation pair* $\mathcal{L} \subset \mathcal{L}(N)$ *and* $Y \in \mathbb{R}^n$ *is in the core* *of the hypergraph game* (N, v) *if* $\sum_{i\in N} Y_i(\mathcal{L}, v) \leq v(\mathcal{L})$ *and* $\sum_{i\in S} Y_i(\mathcal{L}, v) \geq \hat{v}(\mathcal{L}(S))$ *for all* $S \subseteq N$*,*

where $\mathcal{L}(S) = \{L \subseteq 2^N \mid \text{for all } i \in L : i \in S\}$ so that $\mathcal{L}(S) \in \mathcal{H}$ is the hypergraph that contains all possible links among players of S. $\sum_{i\in N} Y_i(\mathcal{L}, v) \leq v(\mathcal{L})$ describes some feasibility condition whereas $\sum_{i\in S} Y_i(\mathcal{L}, v) \geq \hat{v}(\mathcal{L}(S))$ states that no coalition can improve by altering the network structure and reallocating the value among them. The definition says that a feasible allocation is not in the core whenever a subset S of the player set can deviate by forming a new network structure and reallocating the value such that all players of S can do better.

In contrast to the core of a hypergraph game the core of a coalitional game consists of all efficient allocation vectors such that members of each coalition collectively get at least the value they obtain by coordination there action.[67] A core hypergraph-allocation can also describe players' behaviour and allocation of value, when the formation of links and the payoff allocation happen simultaneously. In contrast, the notion of a coalitional game does not allow for deviations of the network structure but allocates the same value to each coalition of players independent of the linking structure between players of the coalition. Therefore, the core of a hypergraph game generalizes the core notion of a coalitional game.

Definition 6.8. *An allocation rule is core consistent, if for any* v *such that the core is nonempty, at least one* $\mathcal{L}$ *exists such that* $(\mathcal{L}, Y(\mathcal{L}, v))$ *is in the core.*

Next we investigate whether the Myerson value and the position value lie in the core of a hypergraph game to discuss its applicability as a solution concept in models of hypergraph games.

Example 6.2. *For* $N = \{1, 2, 3\}$ *consider a value function* v *defined by:* $v(\{\{1, 2, 3\}, \{1, 3\}\}) = 1$*,* $v(\{\{1, 2, 3\}, \{1, 2\}\}) = 1$*,* $v(\{\{1, 2, 3\}, \{1, 2\}, \{1, 3\}\}) = x$ *and* $v(\mathcal{L}) = 0$ *for all other networks. In this example the core is never empty but the core allocation depends on the value of* x*. Whenever* $x < 1$ *the core networks*

[67] The core of a coalitional game (N, v), where $v : N \to \mathbb{R}$, is $core(N, v) = \{Y \in \mathbb{R}^n \mid \sum_{i\in T} Y_i \geq v(T) \text{for all} T \subseteq N \text{and} \sum_{i\in N} Y_i = v(N)\}$.

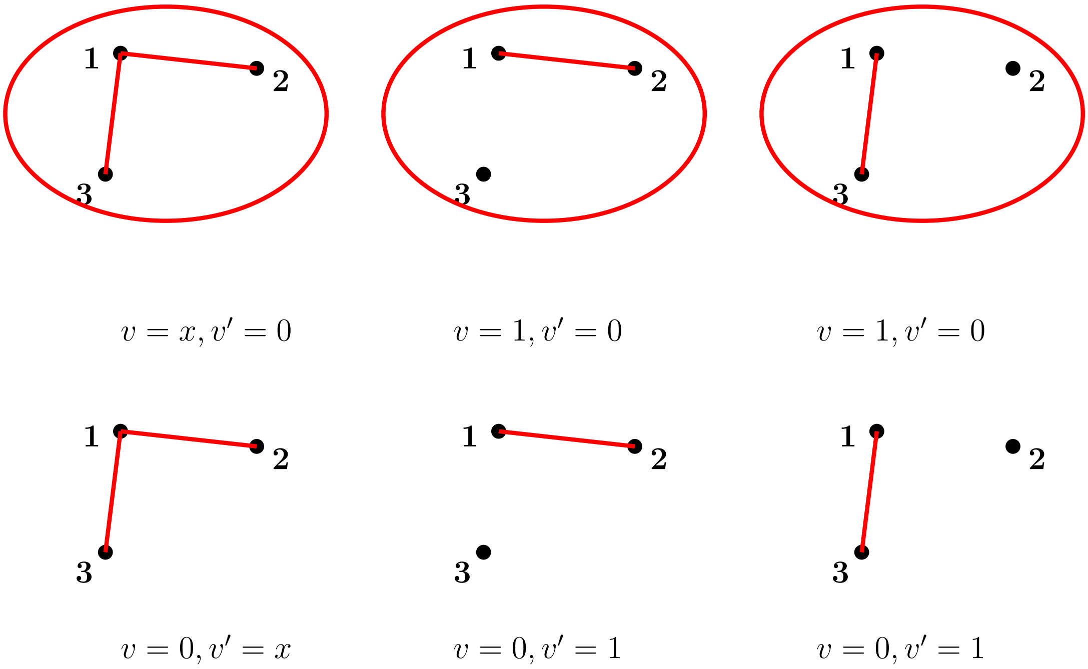

Figure 28

are $\mathcal{L} = \{\{1,2,3\},\{1,2\}\}$ *and* $\mathcal{L} = \{\{1,2,3\},\{1,3\}\}$ *with the set of allocations* $Y = (Y_1, Y_2, Y_3)$ *for that* $\sum_i Y_i = 1$. *Whenever* $x > 1$ *the only core hypergraph is given by*
$\mathcal{L} = \{\{1,2,3\},\{1,2\},\{1,3\}\}$ *where* $\sum_i Y_i = x$.
The Myerson and the position value of the corresponding value function are given by $MV_1(v) = \frac{x}{3}$, $MV_2(v) = MV_3(v) = \frac{x}{6}$ *and* $PV_1 = \frac{4}{9}x - \frac{1}{18}$, $PV_2 = PV_3 = \frac{5}{18}x + \frac{1}{36}$, *respectively. Whenever* $x < 1$ *with core-hypergraph* $\{\{1,2,3\},\{1,2\}\}$ *we have* $MV_i(v) = \frac{1}{3}$ *and* $PV_1 = PV_2 = \frac{5}{12}$ *and* $PV_3 = \frac{1}{6}$. *For any value of* x *the Myerson value and the position value are always in the core of the hypergraph game.*
Consider now the value function v' *given by* $v'(\{\{1,3\}\}) = v'(\{\{1,2\}\}) = 1$,
$v'(\{\{1,2\},\{1,3\}\}) = x$ *and* $v'(\mathcal{L}) = 0$ *for all other conference structures. The set of core allocations shrinks for values* $x \leq 1$ *and is now given by* $(1,0,0)$ *and networks* $\{\{1,3\}\}$ *and* $\{\{1,2\}\}$. *Whenever* $x > 1$ $\mathcal{L} = \{\{1,2\},\{1,3\}\}$ *is in the core together with any allocation* Y *such that* $Y_1 + Y_2 \geq 1$ *and* $Y_1 + Y_3 \geq 1$. *The reason is that player* 1 *is in a more influential situation under* v' *since deviation for player* 1 *to the other core-hypergraph requires a coalition with only one of the other players, whereas under*

v it requires the whole player set to cooperate and change the conference structure. To investigate whether the Myerson and the position value lie in the core of the game for values $x > 1$ we can calculate $MV_1(v) = \frac{x}{3} + \frac{1}{3}$, $MV_2(v) = MV_3(v) = \frac{x}{3} - \frac{1}{6}$, $PV_1(v) = \frac{x}{2}$ and $PV_2(v) = PV_3(v) = \frac{x}{4}$. We can observe that for certain values of x the Myerson value and the position value do not lie in the core of the hypergraph game.

This example shows that the Myerson value and the position value may fail to be core allocations in hypergraph games[68].

As in cooperative game theory, the core of a hypergraph game may be empty[69]. Consider the following game:

Example 6.3. *Let $N = \{1, 2, 3\}$ and the value function is given by $v(\{\{1, 2\}, \{1, 3\}, \{2, 3\}\}) = 300$, $v(\{\{1, 2\}\}) = v(\{\{1, 3\}\}) = v(\{\{2, 3\}\}) = 300$. In this game, the value available to the complete network can be achieved by any two players who can deviate and form a bilateral link. A reasonable outcome should be an allocation of $(100, 100, 100)$ as all three players are symmetric in this game. However, player 1 and 2 can improve by forming the network $\{\{1, 2\}\}$ and allocate the value $(150, 150, 0)$. In this case player 3 would be willing to convince, let's say player 1, to form the network $\{\{1, 3\}\}$ and propose an allocation $(200, 0, 100)$. In this game one will always find a pair of players that can deviate, change the network structure and both players will be better off.*[70]

[68]The failure of the Myerson and the position value to be core allocations also appears in the cooperative game theory literature. However, van den Nouweland and Borm (1991) characterized necessary and sufficient conditions on the conference structure such that the Myerson value and the Position value are in the core, when the corresponding cooperative game is convex.

[69]Bondareva (1963) and Shapley (1967) provide a characterization of cooperative games with an nonempty core. They show that a cooperative game has a nonempty core if and only if the game is balanced.

[70]This example is adopted from Myerson (1991), who defined a cooperative game in which the grand coalition obtains a value of 300 and any pair of players can deviate and form a new coalition and be strictly better off by allocating the value among themselves.

6.5 Conclusion

This chapter extends axiomatic characterizations of allocation rules towards network games on hypergraphs. We have extended the Myerson and the position value towards hypergraph games and proposed characterizations of both allocation rules in the context of hypergraph games.
Furthermore, we defined the core of hypergraph games as a hypergraph-allocation pair and showed that the Myerson and the position value are not necessarily in the core of the hypergraph game. Moreover, we showed that the core of a hypergraph game can be empty. Further research will have to provide a characterization of hypergraph games in which the core is nonempty.

6.6 Appendix

Excursus - The Shapley Value
The Shapley value $SV(v)$ of v is defined by[71]:

$$SV_i(v) = \sum_{S \subseteq N} \frac{(n-|S|)!(|S|-1)!}{n!}(v(S) - v(S \backslash \{i\})),$$

for all $i \in N$.
A carrier $\tilde{S}$ of N is a subset of N such that $v(\tilde{S} \cap S) = v(S) \forall S \subseteq N$. The following axioms are used in the proof of Theorem 6.1.
The Shapley value has, among others, the following properties:

- **Additivity Axiom:** For any two value functions v and w, $SV_i(v+w) = SV_i(v) + SV_i(w)\ \forall i \in N$.
- **Carrier Axiom:** For any carrier $S \subseteq N$ and value function v we have that $\sum_{i \in S} SV_i(v) = v(N)$.

Proof of Theorem 6.1. First we show uniqueness.
Suppose two allocation rules exist Y^1 and Y^2 and consider $\mathcal{L} \neq \emptyset$ as the hypergraph with a minimal number of links such that $Y^1(\mathcal{L}, v) \neq Y^2(\mathcal{L}, v)$. Then for any $L \in \mathcal{L}$

[71] See Shapley (1953).

we have that $Y^1(\mathcal{L}\backslash\{L\},v) = Y^2(\mathcal{L}\backslash\{L\},v)$. With equal bargaining power we obtain $\forall i,j \in L$:

$$\begin{aligned}&Y_i^1(\mathcal{L},v) - Y_j^1(\mathcal{L},v) = Y_i^1(\mathcal{L}\backslash\{L\},v) - Y_j^1(\mathcal{L}\backslash\{L\},v)\\=&Y_i^2(\mathcal{L}\backslash\{L\},v) - Y_j^2(\mathcal{L}\backslash\{L\},v) = Y_i^2(\mathcal{L},v) - Y_j^2(\mathcal{L},v)\end{aligned}$$

By rearranging we get that $Y_i^1(\mathcal{L},v) - Y_i^2(\mathcal{L},v) = Y_j^1(\mathcal{L},v) - Y_j^2(\mathcal{L},v)$. We also obtain with any other $\tilde{L} \in \mathcal{L}$ and $i,k \in \tilde{L}$

$$\begin{aligned}&Y_i^1(\mathcal{L},v) - Y_k^1(\mathcal{L},v) = Y_i^1(\mathcal{L}\backslash\{\tilde{L}\},v) - Y_k^1(\mathcal{L}\backslash\{\tilde{L}\},v)\\&= Y_i^2(\mathcal{L}\backslash\{\tilde{L}\},v) - Y_k^2(\mathcal{L}\backslash\{\tilde{L}\},v) = Y_i^2(\mathcal{L},v) - Y_k^2(\mathcal{L},v).\end{aligned}$$

We can thus write $Y_i^1(\mathcal{L},v) - Y_i^2(\mathcal{L},v) = Y_k^1(\mathcal{L},v) - Y_k^2(\mathcal{L},v) = Y_j^1(\mathcal{L},v) - Y_j^2(\mathcal{L},v)$ and thus $Y_i^1(\mathcal{L},v) - Y_i^2(\mathcal{L},v) = d_{\mathcal{C}}(\mathcal{L}) \forall i \in N(\mathcal{C})$ where $\mathcal{C} \in C(\mathcal{L})$ is a connected component in $\mathcal{L}$ and $d_{\mathcal{C}}(\mathcal{L})$ only depends on the component and on the conference structure but not on player i itself. With component efficiency we have:

$$\sum_{i\in N(\mathcal{C})} Y_i^1(\mathcal{L},v) = \sum_{i\in N(\mathcal{C})} Y_i^2(\mathcal{L},v) = v(\mathcal{C}) \;\; \forall \mathcal{C} \in C(\mathcal{L}).$$

Therefore we obtain that $0 = \sum_{i\in N(\mathcal{C})}(Y_i^1(\mathcal{L},v) - Y_i^2(\mathcal{L},v)) = |N(\mathcal{C})| \cdot d_{\mathcal{C}}(\mathcal{L})$, which implies $d_{\mathcal{C}}(\mathcal{L}) = 0$.

Hence, $Y_i^1(\mathcal{L},v) = Y_i^2(\mathcal{L},v)$ $\forall i \in N(\mathcal{C})$, $\forall \mathcal{C} \in C(\mathcal{L})$, $\mathcal{L} \in \mathcal{H}$, which is a contradiction with $Y^1 \neq Y^2$.

To show equal bargaining power consider any $\mathcal{L} \in \mathcal{H}$. With component additivity of v we obtain for any component $\mathcal{C}$ of $\mathcal{L}$ that $v(\mathcal{L}) = v(\mathcal{C}) + v(\mathcal{L}\backslash\mathcal{C})$.

We define a new function with $w := Ch_{\mathcal{L},v} - Ch_{\mathcal{L}\backslash\{L\},v}$. With $S/\mathcal{L} = S/(\mathcal{L}\backslash\{L\})$ if $L \not\subseteq S$ we obtain for all $S \subseteq N$:

$$w(S) = Ch_{\mathcal{L},v}(S) - Ch_{\mathcal{L}\backslash\{L\},v}(S) = \sum_{\mathcal{C}\in C(S/\mathcal{L})} v(\mathcal{C}) - \sum_{\mathcal{C}\in C(S/(\mathcal{L}\backslash\{L\}))} v(\mathcal{C}) = 0.$$

Therefore, the coalitions with zero value in w are the ones when $S/(\mathcal{L}\backslash\{L\}) = S/\mathcal{L}$, which is the case for all coalitions where at least one player of L is not member of S. The only coalitions with nonzero value are the ones that contain all members of L. This implies that even for i, $j \notin S$: $w(S \cup \{i\}) = w(S \cup \{j\}) = 0$. This implies the symmetry of the Shapley value and we obtain $SV_j(w) = SV_i(w)$. Since SV

satisfies additivity we obtain $SV_i(Ch_{\mathcal{L},v}) - SV_i(Ch_{\mathcal{L}\backslash\{L\},v}) = SV_i(w) = SV_j(w) = SV_j(Ch_{\mathcal{L},v}) - SV_j(Ch_{\mathcal{L}\backslash\{L\},v})$.

For component efficiency we fix any $\mathcal{L} \in \mathcal{H}$ and let $\tilde{S} \in P(N/\mathcal{L})$, where we denote with $P(N/\mathcal{L})$ the partition of players in N that are connected by means of a connected component of $\mathcal{L}$. First, we split the characteristic function game $Ch_{\mathcal{L},v}$ into two games $Ch^{\tilde{S}}(S)$ and $Ch^{N\backslash\tilde{S}}(S)$ with

$$Ch^{\tilde{S}}(S) := \sum_{\mathcal{C}\in C(S\cap\tilde{S}/\mathcal{L})} v(\mathcal{C}),$$

and

$$Ch^{N\backslash\tilde{S}}(S) := \sum_{\mathcal{C}\in C(S\backslash\tilde{S}/\mathcal{L})} v(\mathcal{C}),$$

for all $S \subseteq N$. Since $\tilde{S} \in P(N/\mathcal{L})$ we obtain that $Ch^{\tilde{S}}(S) + Ch^{N\backslash\tilde{S}}(S) = Ch_{\mathcal{L},v}(S)$. For the Shapley value of $Ch^{N\backslash\tilde{S}}(S)$ we have:

$$SV_i(Ch^{N\backslash\tilde{S}}(S)) = \sum_{S\subseteq N} \frac{(n-|S|-1)!|S|!}{n!}\Big(\sum_{\mathcal{C}\in C(S\backslash\tilde{S}/\mathcal{L})} v(\mathcal{C}) - \sum_{\mathcal{C}\in C(S\backslash(\{i\}\cup\tilde{S})/\mathcal{L})} v(\mathcal{C})\Big) = 0,$$

$\forall i \in \tilde{S}$. We now obtain with the definition of the Shapley value:

$$\sum_{i\in\tilde{S}} MV_i(\mathcal{L},v) = \sum_{i\in\tilde{S}} SV_i(Ch_{\mathcal{L},v}(S)) = \sum_{i\in\tilde{S}} SV_i(Ch^{\tilde{S}}(S)+Ch^{N\backslash\tilde{S}}(S)) = \sum_{i\in\tilde{S}} SV_i(Ch^{\tilde{S}}(S)),$$

where the last equality follows from the additivity of the Shapley value.

Since $\tilde{S}$ is a carrier of $Ch^{\tilde{S}}$ with $Ch^{\tilde{S}}(\tilde{S}\cap S) = Ch^{\tilde{S}}(S)$ we obtain with the carrier axiom and $\tilde{S} \in P(N/\mathcal{L})$: $\sum_{i\in\tilde{S}} SV_i(Ch^{\tilde{S}}) = Ch^{\tilde{S}}(\tilde{S})$. Therefore $\sum_{i\in\tilde{S}} MV_i(\mathcal{L},v) = \sum_{i\in\tilde{S}} SV_i(Ch^{\tilde{S}}) = Ch^{\tilde{S}}(\tilde{S}) = \sum_{\mathcal{C}\in C(\tilde{S}/\mathcal{L})} v(\mathcal{C}) = v(\mathcal{C})$. □

Proof of Theorem 6.2. First, we can show additivity. We obtain that due to the additivity axiom of the Shapley value: $PV_i(\mathcal{L},v^1+v^2) = \sum_{L\in\mathcal{L}_i} \frac{1}{|L|}\cdot SV_L(\mathcal{L},v^1+v^2) = \sum_{L\in\mathcal{L}_i} \frac{1}{|L|}\cdot SV_L(\mathcal{L},v^1) + \sum_{L\in\mathcal{L}_i} \frac{1}{|L|}\cdot SV_L(\mathcal{L},v^2) = PV_i(\mathcal{L},v^1) + PV_i(\mathcal{L},v^2)$.

Next we prove the superfluous link property. Assume $L \in \mathcal{L}$ is a superfluous link such that $Ch_{N,v}(\mathcal{L}') = Ch_{N,v}(\mathcal{L}'\backslash\{L\})$ for all $\mathcal{L}' \subseteq \mathcal{L}$. We therefore have that $SV_L(\mathcal{L},Ch_{N,v}) = 0$. Since $SV_{\hat{L}}(\mathcal{L},Ch_{N,v}) = SV_{\hat{L}}(\mathcal{L}\backslash\{L\},Ch_{N,v})$ for all $\hat{L} \in \mathcal{L}\backslash\{L\}$

and all $i \in N$ we obtain:

$$PV_i(\mathcal{L}, v) = \sum_{\hat{L} \in \mathcal{L}_i} \frac{1}{\mid \hat{L} \mid} \cdot SV_{\hat{L}}(\mathcal{L}, Ch_{N,v}) =$$
$$\sum_{\hat{L} \in \mathcal{L}_i \setminus \{L\}} \frac{1}{\mid \hat{L} \mid} \cdot SV_{\hat{L}}(\mathcal{L} \setminus \{L\}, Ch_{N,v}) = PV_i(\mathcal{L} \setminus \{L\}, v).$$

To show component efficiency we fix any $\mathcal{L} \in \mathcal{H}$ and any component $\mathcal{C} \in C(N/\mathcal{L})$. We then obtain

$$\sum_{i \in N(\mathcal{C})} PV_i(\mathcal{L}, v) = \sum_{i \in N(\mathcal{C})} \sum_{L \in \mathcal{L}_i} \frac{1}{\mid L \mid} \cdot SV_L(\mathcal{L}, Ch_{N,v}) = \sum_{L \in \mathcal{C}} SV_L(\mathcal{L}, Ch_{N,v})$$
$$= \sum_{L \in \mathcal{C}} SV_L(\mathcal{C}, Ch_{N,v}) = Ch_{N,v}(\mathcal{C}) = v(\mathcal{C}).$$

Where the first equality follows from the definition, the second equality follows from the fact that C is a component. The third equality follows from $Ch_{N,v}(\mathcal{L}' \cup \{L\}) - Ch_{N,v}(\mathcal{L}') = Ch_{N,v}((\mathcal{C} \cap \mathcal{L}') \cup \{L\}) - Ch_{N,v}((\mathcal{C} \cap \mathcal{L}'))$ $\forall \mathcal{L}' \subseteq \mathcal{L}$ and all $L \in \mathcal{C}$.
To show the influence property we first consider any link anonymous $\mathcal{L} \in \mathcal{H}$ and we define a function f as in equation (40). Since all links are symmetrical we have

$$SV_L(\mathcal{L}, Ch_{N,v}) = Ch_{N,v}(\mathcal{L}) \cdot \frac{1}{\mid \mathcal{L} \mid},$$

$\forall L \in \mathcal{L}$. Such that we obtain:

$$PV_i(\mathcal{L}, v) = \sum_{L \in \mathcal{L}_i} \frac{1}{\mid L \mid} \cdot SV_L(\mathcal{L}, Ch_{N,v}) = I_i(\mathcal{L}, v) \cdot \alpha,$$

with $\alpha = Ch_{N,v}(\mathcal{L}) \cdot \frac{1}{|\mathcal{L}|}$.
To show uniqueness we assume that if an allocation rule Y exists that satisfies component efficiency, additivity, the superfluous link property and the influence property it has to be PV. In the following we will define the value function $w := \beta \cdot u_{\tilde{\mathcal{L}}}$ with the (link) unanimity game $u_{\tilde{\mathcal{L}}}$.
We will differentiate between two cases:
Let $\mathcal{C} \in C(N/\mathcal{L})$ be a connected component and $\tilde{\mathcal{L}}$ such that $\tilde{\mathcal{L}}$ is not contained in a connected component of $\mathcal{L}$. We obtain

$$PV_i(\beta \cdot u_{\tilde{\mathcal{L}}}, \mathcal{L}) = \sum_{L \in \mathcal{L}_i} \frac{1}{\mid L \mid} SV_L(\mathcal{L}, Ch_{N,\beta \cdot u_{\tilde{\mathcal{L}}}}(\mathcal{L})) = 0,$$

$\forall i \in N$, where $Ch_{N,\beta\cdot u_{\tilde{\mathcal{L}}}}(\mathcal{L}) := Ch_{\beta\cdot u_{\tilde{\mathcal{L}}}}(\mathcal{L}, N) = \sum_{\mathcal{C}\in C(N/\mathcal{L})} w(\mathcal{C}) = \sum_{\mathcal{C}\in C(N/\mathcal{L})} \beta \cdot u_{\tilde{\mathcal{L}}}(\mathcal{C})$ and $\tilde{\mathcal{L}} \not\subseteq \mathcal{C}$. Since the allocation rule Y also satisfies the superfluous link property, we have that $Y_i(\mathcal{L}) = Y_i(\emptyset)$. With the influence property we have that $Y_i(\beta \cdot u_{\tilde{\mathcal{L}}}, \emptyset) = \alpha \cdot I_i(\emptyset) = 0$.

For the second part we assume that, given the hypergraph $(N/\mathcal{L})$, a connected component $\mathcal{C} \in C(N/\mathcal{L})$ exists such that $\tilde{\mathcal{L}} \subseteq \mathcal{C}$. Since $H(\tilde{\mathcal{L}})$ is a connected subgraph of $\mathcal{L}$ we obtain with $w := \beta \cdot u_{\tilde{\mathcal{L}}}$ and for all $\mathcal{L}' \subseteq \mathcal{L}$ that:

$$Ch_w(N, \mathcal{L}') = \sum_{\mathcal{C}^0 \in C(N/\mathcal{L}')} \beta \cdot u_{\tilde{\mathcal{L}}}(\mathcal{C}^0) = \begin{cases} \beta, \text{if } H(\tilde{\mathcal{L}}) \subseteq \mathcal{L}', \\ 0, \text{else}, \end{cases}$$

where

$$u_{\tilde{\mathcal{L}}}(\mathcal{C}^0) = \begin{cases} 1, \text{if } \tilde{\mathcal{L}} \subseteq \mathcal{C}^0, \\ 0, \text{else}. \end{cases}$$

This implies for the Shapley value:

$$SV_L(\mathcal{L}, Ch_w) = \begin{cases} \beta \cdot \frac{1}{|H(\tilde{\mathcal{L}})|}, \text{if } L \in H(\tilde{\mathcal{L}}), \\ 0, \text{else}. \end{cases}$$

We obtain for the position value $PV_i(N, \beta \cdot u_{\tilde{\mathcal{L}}}, \mathcal{L}) = \sum_{L\in\mathcal{L}_i} \frac{1}{|L|} SV_L(\mathcal{L}, Ch_w(N, \mathcal{L})) = \sum_{L\in\mathcal{L}_i\cap H(\tilde{\mathcal{L}})} \frac{1}{|L|}\beta\frac{1}{|H(\tilde{\mathcal{L}})|} = \beta\frac{1}{|H(\tilde{\mathcal{L}})|}\sum_{L\in\mathcal{L}_i\cap H(\tilde{\mathcal{L}})} \frac{1}{|L|}$.

Now consider any Y and the underlying conference structure $\mathcal{L}$. Since each link $L \notin H(\tilde{\mathcal{L}})$ is superfluous and Y satisfies the superfluous link property we can follow that $Y(N, \beta\cdot u_{\tilde{\mathcal{L}}}, \mathcal{L}) = Y(N, \beta\cdot u_{\tilde{\mathcal{L}}}, H(\tilde{\mathcal{L}}))$. With $Ch_w(N, \mathcal{L}'\backslash\{L_1\}) = Ch_w(N, \mathcal{L}'\backslash\{L_2\}) = 0$ if $L_1, L_2 \in \mathcal{L}'$ and $\mathcal{L}' \subseteq H(\tilde{\mathcal{L}})$ as well as $Ch_w(N, H(\tilde{\mathcal{L}})) = f(\mid H(\tilde{\mathcal{L}}) \mid) = \beta$ and $Ch_w(N, H(\tilde{\mathcal{L}})\backslash\{L\}) = f(\mid \emptyset \mid) = 0 \ \forall L \in H(\tilde{\mathcal{L}})$ we can conclude that $Y(N, \beta \cdot u_{\tilde{\mathcal{L}}}, H(\tilde{\mathcal{L}}))$ is link anonymous.

Since in a link anonymous conference structure all conferences are equally important, the strength of a player can be measured by his influence. This implies:

$$Y_i(N, \beta \cdot u_{\tilde{\mathcal{L}}}, H(\tilde{\mathcal{L}})) = \alpha \cdot I_i(H(\tilde{\mathcal{L}}), \beta \cdot u_{\tilde{\mathcal{L}}}), \tag{41}$$

where for all $i \in N\backslash N(H(\tilde{\mathcal{L}}))$ we have $Y_i(N, \beta \cdot u_{\tilde{\mathcal{L}}}, H(\tilde{\mathcal{L}})) = 0$. With component efficiency we obtain with $\mathcal{C} \in C(N/\mathcal{L})$:

$$\sum_{i\in N(\mathcal{C})} Y_i(N, \beta \cdot u_{\tilde{\mathcal{L}}}, \tilde{\mathcal{L}}) = \sum_{i\in N(\tilde{\mathcal{L}})} Y_i(N, \beta \cdot u_{\tilde{\mathcal{L}}}, \tilde{\mathcal{L}}) = \beta u_{\tilde{\mathcal{L}}}(\mathcal{C}) = \beta,$$

since we assumed that $\tilde{\mathcal{L}} \subseteq \mathcal{C}$.
With $PV_i(N, \beta \cdot u_{\tilde{\mathcal{L}}}, \mathcal{L}) = \beta \frac{1}{|H(\tilde{\mathcal{L}})|} \sum_{L \in \mathcal{L}_i \cap H(\tilde{\mathcal{L}})} \frac{1}{|L|} = 0$ if $i \notin N(H(\tilde{\mathcal{L}}))$ and $PV_i(N, \beta \cdot u_{\tilde{\mathcal{L}}}, \mathcal{L}) = \beta \frac{1}{|H(\tilde{\mathcal{L}})|} \sum_{L \in \mathcal{L}_i \cap H(\tilde{\mathcal{L}})} \frac{1}{|L|} = \beta \cdot \frac{I_i(H(\tilde{\mathcal{L}}), \beta \cdot u_{\tilde{\mathcal{L}}})}{\sum_{j \in N(H(\tilde{\mathcal{L}}))} I_j(H(\tilde{\mathcal{L}}), \beta \cdot u_{\tilde{\mathcal{L}}})}$ if $i \in N(H(\tilde{\mathcal{L}}))$ it follows with (41) and $\alpha = \frac{\beta}{\sum_{j \in N(H(\tilde{\mathcal{L}}))} I_j(H(\tilde{\mathcal{L}}), \beta \cdot u_{\tilde{\mathcal{L}}})}$ that $PV_i(N, \beta \cdot u_{\tilde{\mathcal{L}}}, \mathcal{L}) = Y_i(N, \beta \cdot u_{\tilde{\mathcal{L}}}, \mathcal{L})$ $\forall i \in N$. $\square$

7 Conclusion

This chapter summarizes the main results of the thesis and provides an insight into possible further research topics.

7.1 Main Results

The thesis has analyzed the strategic formation of bilateral and multilateral links and the interaction of multilateral and bilateral links in the context of international trade agreements. First, a theoretic framework has been provided which allows us to model the strategic formation of bilateral and multilateral trade agreements. We used the theory of hypergraphs to describe networks in which links between any number of players exist. Furthermore, we provided an equilibrium concept which enabled us to investigate the strategic stability of networks. Last, allocation issues have been addressed in the context of hypergraph games and axiomatic foundations of different allocation rules have been provided.

In chapter 4 and chapter 5 of the thesis we have applied the theoretic concepts and investigated the strategic formation of trade agreements between countries. The models we presented helped to address the following issues. First, we showed that in spite of the increasing trend towards regional trade agreements it is still necessary to maintain the GATT. Moreover, we can even observe that the number of GATT members increases. One reason might be that outside countries face a higher tariff from countries inside the WTO as the tariffs are not protected by the MFN clause. Furthermore, chapters 4 and 5 showed that the GATT and its MFN clause play an important role in stabilizing the world trading system. Our models suggest that MFN guarantees stability as no country has an incentive to cancel the GATT.

When we allow countries to form multilateral and bilateral trade agreements simultaneously we find that PTAs and the multilateral trading system can coexist. We observe stability of trading structures that have a coexistence of a multilateral trade agreement and PTAs and in which countries do not trade under free trade. Due to the complementarity effect discussed in chapter 5 in which countries lower tariffs on third parties after the formation of a FTA some countries are better off without free trade.

The literature survey in chapter 1 emphasized that some of the leading researchers in international trade have shown that a PTA lowers incentives for multilateral liberalization. Krishna (1998) among others concluded that the new wave of regionalism can be harmful for multilateral tariff cooperation. Our models suggest that an increasing number of FTAs lowers multilateral tariffs. Moreover, when a country grants another country free access to its own domestic market, it has an incentive to lower tariffs on countries outside the FTA. In chapter 4 and chapter 5 we analyzed the effect of a FTA on multilateral tariffs and found that, as a country increases its number of PTAs, the tariffs it imposes on third countries decrease.

7.2 Outlook

This thesis is only a first attempt to introduce hypergraphs into the literature of network games and to analyze economic models when players are able to form bilateral and multilateral links. It helped to gain further insights into the nature of stable trading networks and showed that it can be a very helpful tool to model strategic behaviour in many economic settings.

This thesis provided a first approach to model the formation of networks when countries have the opportunity to form trade agreements that contain a larger group of countries. Up to now the new theoretic framework has not been applied in other economic fields. Still there are a lot of other possibilities in which hypergraph games and corresponding equilibrium concepts can be applied. For once, it can be used to model networks in financial markets. In financial markets a network between multinational banks can be modelled in which nodes in the network represent multinational banks and the connectivity between banks and traders may contribute to the understanding of the development of the financial crisis.
Hypergraph games may help us to understand the formation of strategic firm collaboration networks. Reasons such as increasing returns to scale technologies may influence a firm's decision to form multilateral collaboration agreements with a larger

group of firms. Goyal and Joshi (2003) have already analyzed the strategic formation of bilateral firm alliances. Their framework can be extended towards multilateral alliances.

The models presented in the previous chapters select a large number of stable network structures, which were partly not desirable from an efficiency point of view. Stronger stability concepts can be defined to analyze the shape of stable network structures and to narrow the set of possible stable networks. One attempt has been made in chapter 3 with the notion of strong multilateral stability, where players are allowed to sever any subset of their existing links. One may also define a stronger stability concept for hypergraphs like *strong stability*, which was first analyzed by Jackson and van den Nouweland (2005) for bilateral graphs. The idea of *strong stability* is that changes in a bilateral network structure can be made by a coalition of players, without the need of consent of any player outside of the coalition. The definition of *strong stability* allows a coalition of players to change any number of bilateral links between them at a time. Jackson and van den Nouweland (2005) noticed that strong stability implies pairwise stability but not vice versa. One may define a corresponding notion for hypergraphs where strong stability also implies multilateral stability.

During the whole thesis, we have assumed that players react myopic in the sense that they are not farsighted and sever and form links one by one. Furthermore, we assumed that players have perfect information about the shape of the network structure and the payoffs all players receive. However, for networks with a very large number of players it seems more reasonable to assume that players can make mistakes. In our formulation of the stability concept changes in network structure are intended and it is assumed that players' decisions include no errors. Jackson and Watts (2002) extended the concept of an improving path that allows for unintended mutation in the network structure which may be due to miscalculations of the players or exogenous forces acting on the network structure. They introduced the notion of stochastic stability to analyze the convergence of networks. The definition of hypergraph games and multilateral stability allows an extension towards random mistakes of individuals and one can model the evolution of the network as a stochastic process. Stochastic stability of hypergraphs analogously to Jackson and Watts (2002) and the evolution of the network structure in discrete time can be analyzed.

To summarize, hypergraph games open a new possibility for modelling strategic behaviour of players in many economic fields. It allows us to introduce and examine the influence of features not present in standard theoretical models. This thesis has taken one step to understand strategic stability of network structure in prominent models of network formation and in international trade, when players are allowed to form multilateral and bilateral links. It has answered some questions, but much more research is necessary, before the role and the meaning of multilateral link formation in contrast to bilateral link formation in economics are clarified.

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