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## Abstract

Two rationality arguments are used to justify the link between conditional and unconditional preferences in decision theory: *dynamic consistency* and *consequentialism*. Dynamic consistency requires that ex ante contingent choices are respected by updated preferences. Consequentialism states that only those outcomes which are still possible can matter for updated preferences. We test the descriptive validity of these rationality arguments with a dynamic version of Ellsberg's three color experiment and find that subjects act more often in line with consequentialism than with dynamic consistency.

Keywords: Non expected utility preferences, ambiguity, updating, dynamic consistency, consequentialism, experiment

JEL-Codes: C91, D81

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# 1 Introduction

Since the seminal work of Daniel Ellsberg (1961) it is acknowledged that missing information about probabilities, in his terminology *ambiguity*, affects subjects' betting behavior. A majority is reluctant to bet on events with unknown probabilities. This reluctance, termed *ambiguity aversion*, violates not only the subjective expected utility theory of Savage (1954), but also the more general theory of probabilistic sophistication in the sense of Machina and Schmeidler (1992). However, despite the overwhelming empirical evidence on ambiguity aversion (surveyed by Camerer and Weber (1992)), there is very little literature investigating experimentally how ambiguity averse subjects behave in a dynamic choice situation. To fill that gap we run a dynamic version of the classical 3-color Ellsberg experiment. The only other dynamic extension of the Ellsberg urn experiment that we know of is by Cohen, Gilboa, Jaffray, and Schmeidler (2000).

In the last two decades several theories of non expected utility have been suggested to model ambiguity aversion, for instance the Choquet expected utility of Schmeidler (1989) and the maxmin expected utility of Gilboa and Schmeidler (1989).<sup>1</sup> To make these models tractable for economic and game theoretic applications a growing amount of literature extends the notion of ambiguity aversion to dynamic choice problems. In dynamic choice situations subjects receive information at consecutive points in time and formulate a contingent plan of action for the remaining time periods by updating their preferences. A central question that arises in this context is how updated preferences, which govern future choices, are related to choices made ex ante. Two properties underpin theories of updated preferences: *dynamic consistency*, and *consequentialism*. Dynamic consistency requires that ex ante contingent choices are respected by updated preferences. Consequentialism states that only those outcomes that are still possible can matter for updated preferences. We show

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<sup>1</sup>Recently various generalizations of Choquet expected utility and maxmin expected utility have been suggested: for instance invariant biseparable preferences by Ghirardato, Maccheroni, and Marinacci (2004), variational preferences by Maccheroni, Marinacci, and Rustichini (2006), and smooth ambiguity preferences by Klibanoff, Marinacci, and Mukerji (2005).

that the dynamic Ellsberg urn offers a straight forward tool to investigate whether subjects facing ambiguity behave consistent with either dynamic consistency or consequentialism.

It is well known (see Ghirardato (2002)) that dynamic consistency and consequentialism imply that preferences are of expected utility form and beliefs are updated according to the Bayes rule. This result implies that by going beyond expected utility models one of these rationality arguments must be relaxed. All subjects displaying preferences for bets with known probabilities must violate either dynamic consistency or consequentialism, or both.

The existing theoretical literature on dynamic extensions of ambiguity models has not yet reached consensus which of these rationality concepts is more plausible. Sarin and Wakker (1998), Epstein and Schneider (2003) and Eichberger, Grant, and Kelsey (2005) show that it is possible to maintain both rationality arguments, however at the cost of imposing restrictions on the domains of acts and conditioning events over which preferences are defined. Other theories focus on one property. For instance Hanany and Klibanoff (2007) and Eichberger and Kelsey (1996) assume dynamic consistency and drop consequentialism, whereas Gilboa and Schmeidler (1993), Pires (2002) and Eichberger, Grant, and Kelsey (2007) drop dynamic consistency and retain consequentialism. As our main result we observe that a significant majority of ambiguity averse subjects violate dynamic consistency rather than consequentialism. This evidence favors consequentialism as the more plausible rationality argument in the presence of ambiguity.

The dynamic 3-color experiment can also be seen as a tool to robustify the observations of the static Ellsberg experiment. A not negligible fraction of subjects, classified as ambiguity neutral in the static Ellsberg experiment, violate either consequentialism or dynamic consistency after arrival of new information. These subjects would be identified as probabilistic subjects in the static versions of Ellsberg's experiment, but are in fact inconsistent with expected utility.

Furthermore, we suggest a new method of how to deal with indifferent subjects. We ask subjects about their confidence in their choices by marking a number on a

scale from 0 (nil) to 5 (very strong). This allows us to separate indifferent subjects, without distorting incentives. We also find that subjects who violate dynamic consistency or consequentialism are less confident in their choices after receiving new information.

The rest of the paper is organized as follows. The next section presents the static Ellsberg three color experiment. In section 3 the notion of consequentialism and dynamic consistency is defined and the dynamic version of three color experiment is presented. Section 4 describes the experimental design. In section 5 the empirical results are presented and discussed. Finally we conclude in section 6.

## 2 Ellsberg's three color experiment

The most prominent theory of decision making under uncertainty is the subjective expected utility theory developed by Savage (1954). According to this theory one can deduce a unique subjective probability distribution over events with unknown probabilities from choice behavior. Object of choices are acts, denoted by  $f$ , which are mappings from the state space,  $\Omega$ , to the set of possible outcomes,  $X$ . An event  $E$  is a subset of  $\Omega$ . For instance an act  $f_Eg$  assigns the outcome  $f(\omega)$  to each state of nature  $\omega \in E$  and the outcome  $g(\omega)$  to each  $\omega \in \Omega \setminus E$ . Subjects are characterized by preferences  $\succsim$  over a set of all possible bets  $\mathcal{F}$ . Savage showed that if preferences  $\succsim$  over bets satisfy certain axioms then subjects will have a cardinal utility function over outcomes and a subjective probability distribution over events. Moreover, subjects will rank bets by maximizing expected values of their utility with respect to their subjective probability distribution.

However, Ellsberg (1961) challenged the Savage's view. He pointed out that missing information about probabilities, in his terminology *ambiguity*, will affect individuals betting behavior, which can not be explained by subjective expected utility. To confirm this conjecture he suggested an experiment similar to the following one. Consider an urn containing 30 balls, 10 of which are known to be yellow (Y) and 20 of which are somehow divided between blue (B) and green (G), with

no further information on the distribution. One ball will be drawn at random from the urn. Subjects face two choice situations, *I* and *II*, in which they are asked to choose between bets paying off 4 or 0, depending on the color of the drawn ball. For instance, in the first choice situation, *I*, a subject is asked to decide whether she prefers to bet on the yellow color or on the blue color. Table 1 summarizes the two relevant choice problems in the Ellsberg experiment.

		<i>Yellow</i>	<i>Blue</i>	<i>Green</i>
<i>Choice I</i>	$f_1$	4	0	0
	$f_2$	0	4	0
<i>Choice II</i>	$f_3$	4	0	4
	$f_4$	0	4	4

Table 1: Static Ellsberg experiment

The observable choices reveal subjects attitude towards ambiguity. Altogether there are four possible patterns of preferences (see table 2).

		<i>Ambiguity Attitude</i>		
		<i>Averse</i>	<i>Neutral</i>	<i>Loving</i>
<i>Choice I</i>	$f_1$	$f_1$	$f_2$	$f_2$
<i>Choice II</i>	$f_4$	$f_2$	$f_4$	$f_3$

Table 2: Ambiguity attitudes in Ellsberg's 3-color experiment

Each column depicts the chosen bet in each of the two relevant choice problems. The choices depicted in the first and fourth column reflect subjects' sensitive attitude towards ambiguity that is incompatible with subjective expected utility theory. In particular, for subjects displaying either of these two patterns of choices there is no probability distribution that can adequately represent their beliefs. For instance, consider the first column in which subjects prefer  $f_1$  to  $f_2$  and  $f_4$  to  $f_3$ . If we assume that these subjects would have a subjective probability distribution, then preferring

$f_1$  to  $f_2$  implies that they have a higher subjective probability for a yellow ball being drawn than for a blue ball being drawn. But the fact that they prefer  $f_4$  to  $f_3$  implies that they have a higher subjective probability for blue being drawn than for yellow being drawn. These two deductions are contradictory. Subjects displaying such preferences are called *ambiguity averse*, since they are reluctant to bet on events with unknown probabilities. Conversely, in the last column, subjects exhibit *ambiguity loving* behavior, since they prefer  $f_2$  to  $f_1$  and  $f_3$  to  $f_4$  and therewith they favor to bet on events with unknown probabilities.

In order to accommodate different ambiguity attitudes, various generalizations of subjective expected utility theories have been proposed. The most prominent are the Choquet expected utility model of Schmeidler (1989), which allows subjects' beliefs to be represented by not necessarily additive measures, called capacities, and the maxmin expected utility with multiple priors model of Gilboa and Schmeidler (1989), which allows subjects' beliefs to be represented by set of probabilities. However, since our investigations are conducted in a model free setup we are not restricted to a particular class of non expected utility models.

### 3 Conditional preferences

Moving to dynamic choice problems, a central question that arises is how preferences are updated to incorporate new information. Since updated preferences govern future choices it is important to know how they are related to choices made ex ante. We restrict our attention to non null events. An event  $N \subset \Omega$  is Savage null if for any bet  $f, g \in \mathcal{F}$  it holds that  $f_N g \sim g$ , otherwise it is non null. After being informed that an event  $E$  has occurred subjects construct conditional preferences over  $\mathcal{F}$ , represented by  $\succsim_E$ . Before arrival of any information subjects preferences over bets are represented by  $\succsim$  as usual.

Two rationality arguments are used to justify the link between ex ante preferences and preferences updated according to interim information. The first property, called *dynamic consistency* directly links conditional and unconditional preferences. It

requires that choices made ex ante are consistently implemented in the future and vice versa.<sup>2</sup>

**(DC) Dynamic Consistency:** For any non null event  $E$  and all bets  $f, g \in \mathcal{F}$ , such that  $f(\omega) = g(\omega)$  for each  $\omega \in \Omega \setminus E$ ,  $f \succsim_E g$  implies  $f \succsim g$ .

The essence of dynamic inconsistency in the sense Machina (1989) involves reversals. He writes (pp. 1636-7) "... behavior ... will be dynamically inconsistent, in the sense that ... actual choice upon arriving at the decision node would differ from ... planned choice for that node."

The second property, called *consequentialism* and introduced by Hammond (1988), concerns solely the conditional preference relation. It requires that preferences conditional on a non null event  $E$  are not affected by the outcomes outside the conditional event,  $\Omega \setminus E$ . Intuitively, once the subject is informed that the event  $E$  occurred, only the uncertainty about all subevents of  $E$  matters for conditional preferences.

**(C) Consequentialism:** For any non null event  $E$  and all bets  $f, g \in \mathcal{F}$ ,  $f(\omega) = g(\omega)$  for each  $\omega \in E$  implies  $f \sim_E g$ .

Now consider a simple dynamic version of Ellsberg's three color experiment. As a mind experiment it was described by Hanany and Klibanoff (2007) and Ghirardato, Maccheroni, and Marinacci (2008). In the dynamic version there is an interim stage, where subjects are informed whether or not the drawn ball is green. Moreover, subjects are allowed to condition their choices on the revealed information. Depending on their choices in the interim stage one can conclude whether subjects behave consistently with either dynamic consistency or consequentialism. Table 3 depicts implications on dynamic consistency and consequentialism resulting from choices made ex ante and choices made on the interim stage. The columns refer to choices made in the static Ellsberg experiment. Correspondingly, rows refer to choices made after being informed that the drawn ball is not green.

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<sup>2</sup>Our experimental design fits also to the weakest version of dynamic consistency suggested by Hanany and Klibanoff (2007).



		<i>Ambiguity Attitude</i>			
		<i>Averse</i>	<i>Neutral</i>		<i>Loving</i>
		$(f_1; f_4)$	$(f_1; f_3)$	$(f_2; f_4)$	$(f_1; f_4)$
<i>Interim Choice</i>	$(f_1; f_4)$	$DC, \neg C$	$\neg DC, \neg C$	$\neg DC, \neg C$	$\neg DC, \neg C$
	$(f_1; f_3)$	$\neg DC, C$	$DC, C$	$\neg DC, C$	$\neg DC, C$
	$(f_2; f_4)$	$\neg DC, C$	$\neg DC, C$	$DC, C$	$\neg DC, C$
	$(f_2; f_3)$	$\neg DC, \neg C$	$\neg DC, \neg C$	$\neg DC, \neg C$	$DC, \neg C$

Table 3: Dynamic consistency and consequentialism in the dynamic 3-color experiment

Consider for instance an ambiguity averse subject (first column with  $f_1; f_4$ ), who after the arrival of information prefers  $f_1$  to  $f_2$  and  $f_3$  to  $f_4$  (second row with  $f_1; f_3$ ). Because of the preference reversal at the interim stage in the second choice problem, she violates dynamic consistency (henceforth  $\neg DC$ ). However, her preferences satisfy consequentialism since  $f_1 = f_3$  and  $f_2 = f_4$  on the event  $\{Y, B\}$  and  $f_1$  is preferred to  $f_2$  and  $f_3$  is preferred to  $f_4$ . Consider again an ambiguity averse subject, who at the interim stage prefers  $f_1$  to  $f_2$  and  $f_4$  to  $f_3$  (first row with  $f_1; f_4$ ). Choices made ex ante coincide with choices made at interim stage, thus satisfying the property of dynamic consistency. However, again, since  $f_1 = f_3$  and  $f_2 = f_4$  on  $\{Y, B\}$ , and  $f_1$  is preferred to  $f_2$  and  $f_4$  is preferred to  $f_3$ , she behaves in a inconsequential way (henceforth  $\neg C$ ). Note that ambiguity averse or ambiguity loving subjects can not maintain both properties on the same time.

## 4 Experimental design

The experiment was conducted in December 2008 in Mannheim in the experimental lab of SFB504. A total of 90 subjects participated in 4 sessions, with each subject participating only once. 46 participants were male, 44 female; all but 1 subject were students from various majors. Subjects were recruited via ORSEE (Greiner (2004)) and paid in private and cash directly after the experiment. On average they earned

14.00 Euro in about 60 minutes.

The urn was represented by a bucket with white table tennis balls (with yellow, blue or green stickers on them). Before making their choices, subjects were shown the closed bucket and one ball of each color. The bucket remained in the room for the whole experiment and after the drawings were finished, subjects had the opportunity to look at the balls inside the bucket. After receiving and reading the instructions detailing the complete experiment, all subjects were handed the decision sheet, on which they marked their bets. Each correct answer paid 4 Euro.

A particular problem in ambiguity related experiments is how to deal with indifference. One possible solution is to force subjects to make a choice, the drawback being that some data points will reflect indifferent subjects, such that inferences from the Ellsberg decisions could be wrong (e.g. what looks like a preference reversal is not inconsistent with subjective expected utility theory if the subject was indifferent). On the other hand, including an explicit *indifferent* option raises problems in incentivised experiments: How will the subjects marking indifferent be paid? Choosing any rule, such as “the experimenter flips a coin” turns the problem into a decision with three alternatives, the coin flip being one of them. Subjects who prefer the coin flip need not be identical with those who are indifferent in the original two alternative decision. To solve this problem, we did not offer an indifferent option. However, additionally to each decision, subjects were asked to mark “How strong is your liking for the alternative you choose?” on a scale ranging from 0 (nil) to 5 (very strong). We interpret subjects who marked zero as having no confidence that their choices are better than the alternatives, that is, as being indifferent. These subjects were paid according to their decision, but discarded from the analysis.

When everyone had finished their decisions, subjects took part in a timed 10 minute statistics and cognitive ability test, with 9 questions in total (3 questions from Shane Frederick’s cognitive ability test (Frederick (2005)), the Wason selection task (Wason and Shapiro (1971)) and 5 simple statistic questions). Each correct answer was paid with 1 Euro. Finally, subjects were asked to answer an unpaid

questionnaire which included demographics.<sup>3</sup>

The draws took place at the end of the experiment. A randomly selected subject blindly drew a ball for each question. The balls were returned to the bucket after being shown to all subjects, so that all drawings were with replacement. Regarding question three and four, the following was stated in the instructions and implemented if needed: “If the first drawn ball happens to be green, we will continue drawing balls till a non-green ball is drawn.” After the drawings were done, each subject was paid according to his/her decisions and answers and the experiment ended.

## 5 Results

Out of our 90 subjects, 6 marked a confidence of nil for at least one of their choices. We interpret these subjects as indifferent and drop them from the following analysis since we are interested in strict preferences, leaving us with 84 data points.

First, we look at the choices in the first two questions, which replicate the static Ellsberg experiment. The last row in table 4 shows the proportion of ambiguity averse, neutral and loving subjects. We confirm previous observations (see Camerer and Weber (1992)) that a majority of people are ambiguity averse in this decision task: 54.8% prefer to bet on colors with known probabilities; 7.1% are ambiguity loving, while 38.1% exhibit ambiguity neutral behavior.

According to the responses in the third and fourth question, we can classify 21 as both dynamically consistent and consequentialist, 44 as not dynamically consistent, but consequentialist, 6 as dynamically consistent but not consequentialist and 13 as neither dynamically consistent, nor consequentialist.<sup>4</sup> Taken together, 32.1% are dynamically consistent, while 77.4% are consequentialist. This difference is highly significant using a McNemar test. This result does not change when we look only

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<sup>3</sup>See the appendix for translated instructions. The original instructions in German are available from the authors upon request.

<sup>4</sup>Note that in our experiment, it is not possible for subjects to be ambiguity averse/loving, dynamically consistent and consequentialist at the same time. Similar, there are no choice combinations that allow subjects to be ambiguity neutral, dynamically consistent, but not consequentialist.

at subjects who are ambiguity averse or ambiguity loving according to the first two questions.

	<i>Ambiguity Attitude</i>			<i>Total</i>
	<i>Averse</i>	<i>Neutral</i>	<i>Loving</i>	
<i>DC, C</i>	-	21	-	21
$\neg DC, C$	35	<b>3</b>	6	44
<i>DC, <math>\neg C</math></i>	6	-	0	6
$\neg DC, \neg C$	5	<b>8</b>	0	13
<i>Total</i>	46	32	6	84

Table 4: Distribution of dynamically consistent/consequentialist and ambiguity averse/neutral/loving subjects

The two bold numbers in the table 4 highlight subjects who would be classified as ambiguity neutral in the static Ellsberg urn, yet who turn out to be not bayesian in the dynamic urn. Thus, we find additional violations of subjective expected utility theory in the dynamic experiment.

The results in table 4 suggest that when subjects are not both dynamically consistent and consequentialist, they rather drop dynamic consistency than consequentialism. However, due to the design of the urn, there are more combinations of choices which are consequentialist than dynamically consistent. To check this result for robustness, we list in table 5 the hypothetical distributions we would expect if all our subjects would choose purely random and compare them to the observed results. Looking at all subjects, there are more consequentialist and dynamically consistent choices than under a random distribution. However this result is significant only for consequentialism. The difference is even more pronounced when we restrict the analysis to non-ambiguity neutral subjects. Now significantly *less* subjects than under random choice are dynamically consistent, while, still, there are, clearly and very significantly, more consequentialist ones.

Regarding the way subjects update preferences, Dubois and Prade (1994) distin-

		<i>Random</i>	<i>Observed</i>	<i>Binomial test</i> <i>two – sided</i>
<i>All subjects</i>	<i>DC</i>	25%	32%	.132
	<i>C</i>	50%	77%	.000
<i>Non – neutral</i>	<i>DC</i>	25%	12%	.024
	<i>C</i>	50%	79%	.000

Table 5: Fraction of dynamically consistent and consequentialist subjects

guish two different approaches, learning and focusing, which coincide in the additive case thank to the Bayes rule, but need not coincide outside of subjective expected utility. They consider two different updating rules: Maximum-Likelihood updating and Full Bayesian updating.<sup>5</sup> Intuitively, in the case of learning, the decision maker learns something about the composition of the urn. In this case, Dubois and Prade (1994) argue for the use of the Maximum-Likelihood rule. On the other hand, focusing is a situation in which no information is provided regarding the composition of the urn, as it is the case in our experiment. Dubois and Prade (1994) argue that in this situation of focusing the Full Bayesian rule should be used. In their paper,

	<i>Averse</i>	<i>Loving</i>
<i>Full Bayesian</i>	82.9%	66.7%
<i>Maximum – Likelihood</i>	17.1%	33.3%

Table 6: Full Bayesian vs Maximum-Likelihood

Cohen, Gilboa, Jaffray, and Schmeidler (2000) test whether subjects follow the Full Bayesian or the Maximum-Likelihood updating rule using a design very similar to ours. The questions they use are identical to our questions one, two and four. Then,

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<sup>5</sup>Roughly speaking the Full Bayesian updating rule is a rule where the decision maker updates all the probabilistic scenarios she has in mind and derives the conditional preference relation from these updated probabilities. According to the Maximum-Likelihood updating rule the decision maker updates only the probabilities that maximise the event which has occurred.

ambiguity averse agents using the Maximum-Likelihood rule would choose blue in question four and while those updating according to Full Bayesian updating would choose yellow. However, Cohen et al. assume that subjects are consequentialist.<sup>6</sup> We can repeat their test using only our consequentialist subjects. Similar to their results, we find significantly more support for the Full Bayesian updating rule (p-value  $< 0.001$ , signed rank test) among ambiguity averse subjects. The result for ambiguity loving subjects is not significant, very likely due to the small number of ambiguity loving subjects in our experiment.

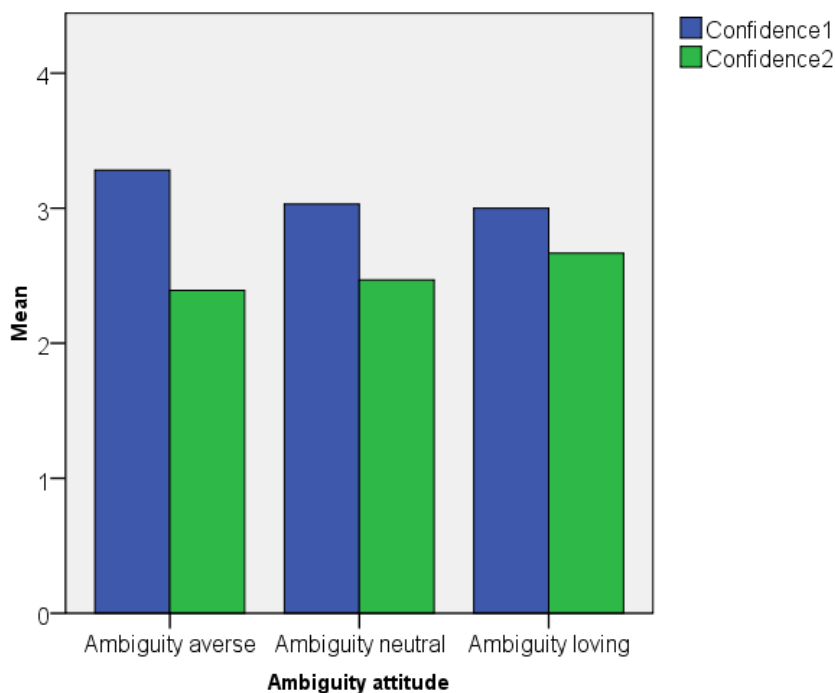


Figure 1: Confidence and ambiguity attitudes

Moreover, we asked all subjects about their confidence in their choices for each question. Apart from using these responses to discard indifferent subjects from the analysis, it is also interesting to look at the different levels of confidence for each question. Again, we start by looking at the first two questions, the static Ellsberg case.

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<sup>6</sup>Another difference between their paper and ours is that our subjects are paid, while Cohen et al. use hypothetical questions.

As figure 1 shows, all subjects are less confident in their second answer compared to the first one. This difference is significant at the 1% level for ambiguity averse and ambiguity neutral subjects, but not significant for ambiguity loving subjects in a Wilcoxon test. However, the “amount” of confidence that subjects lose depends on their choices: ambiguity averse subjects lose more confidence than ambiguity neutral ones.<sup>7</sup>

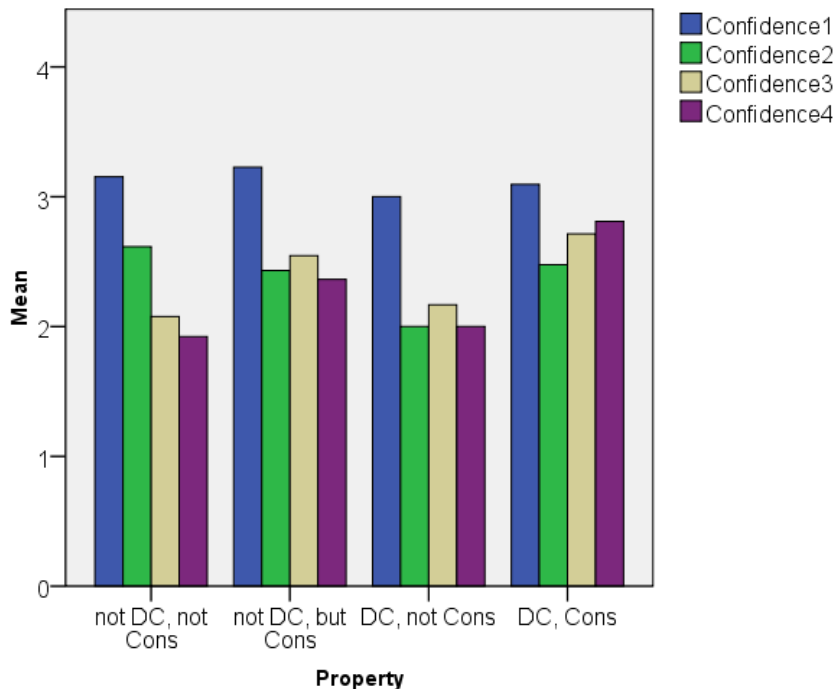


Figure 2: Confidence in the dynamic 3-color experiment

Next, we turn to confidence levels for all four answers. Figure 2 depicts the confidence levels for subjects depending on their adherence to the dynamic consistency and consequentialism. To evaluate the impact of going from a static to a dynamic Ellsberg urn, we look at the difference in average confidence in the first two compared to the last two questions:  $confidence\ loss = (confidence1 + confidence2) - (confidence3 + confidence4)$ . The first impression that subjects who adhere to the

<sup>7</sup>The two-sided p-value of a Mann-Whitney-U-Test on  $confidence1 - confidence2$  comparing ambiguity averse with ambiguity neutral subjects is 0.032. No comparison with ambiguity loving subjects is significant. In both cases, the insignificant results for ambiguity loving subjects might be due to their small number in our experiment.

rationality arguments lose less confidence in the dynamic case is confirmed. As table 7 shows, they have a significantly lower confidence loss than those subjects who violate one or both properties. This result is also confirmed when we use a multi-

	$DC, C$	$\neg DC, C$	$DC, \neg C$	$\neg DC, \neg C$
$DC, C$	-	-	-	-
$\neg DC, C$	0.024	-	-	-
$DC, \neg C$	0.011	0.371	-	-
$\neg DC, \neg C$	0.000	0.455	0.01	-

Table 7: Significance levels from two-sided MW test on updating confidence loss

nominal logistic regression to control for demographics and subjects' score in our cognitive ability questions (see appendix). Our results for subjects' confidence make sense if one assumes that subjects are more confident in their choice if they know of a way to rationally argue in favor of that choice. The probabilistic bayesian theory is the most mathematically simple and arguably the only one which our subjects might *consciously* use in the experiment. We find the highest levels of confidence for choices two to four exactly for those subjects who behave probabilistic bayesian.

## 6 Conclusion

People who display the Ellsberg paradox can not be dynamically consistent and consequentialist at the same time. We conduct a dynamic extension of Ellsberg's 3-color experiment and find that, in our setup, significantly more subjects behave in accordance with consequentialism rather than dynamic consistency. As such, our results can be seen as support for theories which retain consequentialism.

We observe that being ambiguity neutral when facing the static Ellsberg urn does not necessarily imply that subjects always behave bayesian. Several subjects who are classified as ambiguity neutral in the static choice situation can not be described by subjective expected utility theory in the dynamic extension.



Furthermore, we propose a new method of measuring indifference in ambiguity experiments, which resolves the conflict between the aim to exclude indifference from the analysis and the need to pay all subjects. This measure can also be used to show differences in confidence for different types of subjects: While all subjects are more confident in their first choice, ambiguity neutral subjects lose less confidence in later choices than ambiguity averse ones and bayesian subjects lose less confidence compared to those who violate dynamic consistency and consequentialism.

We hope that the dynamic extension of the Ellsberg urn will provide new insights for the discussion about behavior under ambiguity and will be a first step towards further experimental evaluation.

# Appendix

## A Regression

	Variable	Coef.	Std. Err.	z	P >  z	95% Conf. Interval	
<b>not DC, C</b>	<i>Religious</i>	-0.271	0.718	-0.38	0.705	-1.679	1.136
	<i>Male</i>	-0.802	1.058	-0.76	0.448	-2.877	1.272
	<i>Size</i>	-0.009	0.057	-0.17	0.869	-0.122	0.103
	<i>Gambling</i>	0.127	0.467	0.27	0.786	-0.789	1.043
	<i>Cog. Ability</i>	0.203	0.270	0.75	0.451	-0.325	0.732
	<i>Conf. loss</i>	-0.630	0.302	-2.09	0.037	-1.222	-0.038
	<i>Cons.</i>	3.772	10.292	0.37	0.714	-16.402	23.945
<b>DC, not C</b>	<i>Religious</i>	-1.055	1.203	-0.88	0.381	-3.414	1.304
	<i>Male</i>	0.924	1.690	0.55	0.585	-2.389	4.237
	<i>Size</i>	0.077	0.095	0.81	0.419	-0.110	0.264
	<i>Gambling</i>	1.329	0.628	2.12	0.034	0.098	2.559
	<i>Cog. Ability</i>	0.099	0.481	0.21	0.836	-0.843	1.042
	<i>Conf. loss</i>	-0.668	0.523	-1.28	0.201	-1.693	0.357
	<i>Cons.</i>	-15.310	17.173	-0.89	0.373	-48.970	18.349
<b>DC, C</b>	<i>Religious</i>	-0.548	0.843	-0.65	0.516	-2.199	1.104
	<i>Male</i>	-0.583	1.211	-0.48	0.630	-2.957	1.791
	<i>Size</i>	0.033	0.068	0.50	0.618	-0.099	0.166
	<i>Gambling</i>	0.473	0.515	0.92	0.359	-0.537	1.483
	<i>Cog. Ability</i>	-0.229	0.325	-0.70	0.481	-0.865	0.407
	<i>Conf. loss</i>	-1.092	0.373	-2.93	0.003	-1.823	-0.361
	<i>Cons.</i>	-3.833	12.124	-0.32	0.752	-27.595	19.929
<i>Number of Obs.</i> = 84				<i>LR</i> $\chi^2$ (18) = 31.01			
<i>Log likelihood</i> = -82.151742				<i>Prob</i> > $\chi^2$ = 0.0287			
				<i>Pseudo R</i> <sup>2</sup> = 0.1588			
<i>Property = not DC, not C is the base outcome</i>							

Table 8: Multinomial Logistic Regression

## B Instructions

### **Instructions**

Welcome to our Experiment! Please read these instructions carefully. The instruction is identical for all participants. During the entire experiment, we want to ask you to be quiet and not to talk with the other participants. Please turn your mobile phone off and keep it turned off till the end of the experiment. If you have any questions, please raise your hand and one of the experimenters will come to you.

### **Goal of the experiment**

This experiment includes decisions under uncertainty. In the decision phase, there are no “right” or “wrong” decisions. Only your personal preferences count. Depending on your preferences, it could well be that the decision will be very easy for you. The alternatives are real and not only hypothetical. Every participant will be privately paid in cash. The decisions of the other participants have no influence on your payment.

### **Structure of the experiment**

At the start of the experiment, we will answer questions regarding the instructions. Afterwards we start the decision phase. Decisions in this phase are real. They do have an impact on your payment. Please take your time in answering, the experiment only continues once all participants are done. At the end, the payments for the decision phase will be determined and all participants are paid.

Overall, the experiment will take approximately 60 minutes.

### **Bucket**

The bucket contains 30 table tennis balls. Every table tennis ball has a colored sticker, which determines the color of the ball. There are 10 yellow table tennis balls. The other 20 table tennis balls are either blue or green. The exact number of the blue and green table tennis balls is unknown. However, taken together, there are exactly 20 blue and green balls.

## Decision phase

At the end of the experiment, 4 independent draws (with replacement) will be taken from the bucket – one draw for each of the 4 questions, which you answer on the decision sheet. Your payment depends on your answers and on the result of the draws.

On the decision sheet, you have to choice 4 times between 2 alternatives. The alternatives are as follows:

- Alternative W: You receive a payment of 4€, if a *yellow or green* ball is drawn.
- Alternative X: You receive a payment of 4€, if a *blue or green* ball is drawn.
- Alternative Y: You receive a payment of 4€, if a *yellow* ball is drawn.
- Alternative Z: You receive a payment of 4€, if a *blue* ball is drawn.

## Questionnaire 1

The decision phase is followed by questionnaire 1. Here right and wrong answers exist! In total, you have 10 minutes to answer all questions. For each correct answer, you will be paid 1€ at the end of the experiment.

## Questionnaire 2

Questionnaire 2 collects some personal data. This information will only be used for the evaluation of this experiment. The answers in questionnaire 2 do have no influence on your payment.

## Draws

In the end, there will be 4 draws, one for each question from the decision phase. After each draw, the table tennis ball will be put back into the bucket. The draws will be taken by a randomly chosen participant.

It it happens that the first drawn ball is green for question 3 or question 4, there will be additional draws till the drawn ball is not green.

## Payment

For each draw, you receive a payment if and only if the color of the drawn table tennis ball matches the color of the answer you marked. Additionally, you receive 1€ for each correctly answered question in questionnaire 1.

## C Decision Sheet

### Decision Sheet

ID: \_\_\_\_\_

- Alternative W: You receive a payment of 4€, if a *yellow or green* ball is drawn.
- Alternative X: You receive a payment of 4€, if a *blue or green* ball is drawn.
- Alternative Y: You receive a payment of 4€, if a *yellow* ball is drawn.
- Alternative Z: You receive a payment of 4€, if a *blue* ball is drawn.

#### Question 1

What do you like more?:

W       X

How strong is your liking for the alternative you choose?

Nil                  Very strong

#### Question 2

What do you like more?:

Y       Z

How strong is your liking for the alternative you choose?

Nil                  Very strong

#### Question 3

What do you like more, *if you learn that the drawn ball is not green*:

W       X

How strong is your liking for the alternative you choose?

Null                  Very strong

#### Question 4

What do you like more, *if you learn that the drawn ball is not green*:

Y       Z

How strong is your liking for the alternative you choose?

Null                  Very strong

## D Questionnaires

### Questionnaire 1

ID: \_\_\_\_\_

**Page 1: 5 minutes maximum**

Please assume for all questions that dice are six-sided and fair.

Answer

<b>Question 1:</b> What is the probability that the number in a throw of a die is smaller or equal 2?	
<b>Question 2:</b> What is the probability that in two throws, the number is both times equal to 4?	
<b>Question 3:</b> Look at a single throw. Assume that the result is an even number. What is the probability that the number is equal to 2?	
<b>Question 4:</b> Assume that the number 3 was thrown 5 times in a row. What is the probability that the next throw will result in a 3?	
<b>Question 5:</b> Assume 4 dice are thrown and the numbers added. What is the total number on average?	

# Questionnaire 1

Page 2: 5 minutes maximum

<b>Question 6:</b> A bat and a ball cost \$1.10 in total. The bat costs \$1.00 more than the ball. How much does the ball cost?				
<b>Question 7:</b> If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets?				
<b>Question 8:</b> In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?				
<b>Question 9:</b> Assume you see 4 double sided cards in front of you. Each card has a number on one side and a letter on the other side. Which card or cards do you have to turn around to test whether the following assertion is true: "If there is a vowel (A,E,I,O,U) on one side, there is an even number on the other side."				
<b>E</b>	<b>K</b>	<b>4</b>	<b>7</b>	
Card 11	Card 12	Card 13	Card 14	

## Questionnaire 2

ID: \_\_\_\_\_

The questions on this questionnaire are not payoff relevant.

**Question 1:** Please give an estimate, how many balls are in the urn:

\_\_\_\_\_ blue balls          \_\_\_\_\_ yellow balls          \_\_\_\_\_ green balls

**Question 2:** What is your gender?     male                       female

**Question 3:** What is your size?                      \_\_\_\_\_ cm

**Question 5:** What is your major? \_\_\_\_\_  not a student

**Question 6:** Did you participate in a statistics course before?     yes                       no

**Question 7:** Would you call yourself politically left wing or right wing?

Left                                                                                          Right

**Question 8:** Are you religious?     yes                       no

**Question 9:** Which of the following game do you play occasionally?

- Lottery
- Roulette
- Poker
- Sports bets
- Lottery scratch tickets
- others: \_\_\_\_\_



## References

- CAMERER, C., AND M. WEBER (1992): “Recent Developments in Modelling Preferences: Uncertainty and Umbiguity,” *Journal of Risk and Uncertainty*, 5, 325–370.
- COHEN, M., I. GILBOA, J. JAFFRAY, AND D. SCHMEIDLER (2000): “An Experimental Study of Updating Ambiguous Beliefs,” *Risk, Decision and Policy*, 5, 123–133.
- DUBOIS, D., AND H. PRADE (1994): “Focusing versus Updating in Belief Function Theory,” in *Advances in the Dempster-Shafer Theory of Evidence*, ed. by M. Fedrizzi, J. Kacprzyk, and R. Yager, pp. 71–96. Wiley and Sons.
- EICHBERGER, J., S. GRANT, AND D. KELSEY (2005): “CEU Preferences and Dynamic Consistency,” *Mathematical Social Science*, 49, 143–151.
- (2007): “Updating Choquet Beliefs,” *Journal of Mathematical Economics*, 43, 888–899.
- EICHBERGER, J., AND D. KELSEY (1996): “Uncertainty Aversion and Dynamic Consistency,” *International Economic Review*, 37, 625–640.
- ELLSBERG, D. (1961): “Risk, Ambiguity, and the Savage axioms,” *Quarterly Journal of Economics*, 75, 643–669.
- EPSTEIN, L., AND M. SCHNEIDER (2003): “Recursive Multiple-Priors,” *Journal of Economic Theory*, 113, 1–31.
- FREDERICK, S. (2005): “Cognitive Reflection and Decision Making,” *Journal of Economic Perspectives*, 19(4), 24–42.
- GHIRARDATO, P. (2002): “Revisiting Savage in a Conditional World,” *Economic Theory*, 20, 83–92.
- GHIRARDATO, P., F. MACCHERONI, AND M. MARINACCI (2004): “Differentiating Ambiguity and Ambiguity Attitude,” *Journal of Economic Theory*, 118, 133–173.

- (2008): “Revealed Ambiguity and Its Consequence: Updating,” in *Advances in Decision Making under Risk and Uncertainty*, pp. 3–18. Springer.
- GILBOA, I., AND D. SCHMEIDLER (1989): “Maximin Expected Utility with Non-Unique Prior,” *Journal of Mathematical Economics*, 18, 141–153.
- GILBOA, I., AND D. SCHMEIDLER (1993): “Updating Ambiguous Beliefs,” *Journal of Economic Theory*, (59), 33–49.
- GREINER, B. (2004): “An Online Recruitment System for Economic Experiments,” in *Forschung und wissenschaftliches Rechnen 2003*, pp. 79–93. Ges. für Wiss. Datenverarbeitung, Göttingen.
- HAMMOND, H. (1988): “Consequentialist Foundations for Expected Utility,” *Theory and Decision*, 25, 25–78.
- HANANY, E., AND P. KLIBANOFF (2007): “Updating Preferences with Multiple Priors,” *Theoretical Economics*, 2, 261–298.
- KLIBANOFF, P., M. MARINACCI, AND S. MUKERJI (2005): “A Smooth Model of Decision Making under Ambiguity,” *Econometrica*, 73, 1849–1892.
- MACCHERONI, F., M. MARINACCI, AND A. RUSTICHINI (2006): “Ambiguity Aversion, Robustness, and the Variational Representation of Preferences,” *Econometrica*, 74, 1447–1498.
- MACHINA, M. (1989): “Dynamic Consistency and Non-expected Utility Models of Choice under Uncertainty,” *Journal of Economic Literature*, (27), 1622–1668.
- MACHINA, M., AND D. SCHMEIDLER (1992): “A More Robust Definition of Subjective Probability,” *Econometrica*, 60, 745–780.
- PIRES, C. P. (2002): “A Rule for Updating Ambiguous Beliefs,” *Theory and Decision*, 53, 137–152.
- SARIN, R., AND P. WAKKER (1998): “Dynamic Choice and Nonexpected Utility,” *Journal of Risk and Uncertainty*, 17, 87–119.

SAVAGE, L. J. (1954): *The Foundation of Statistics*. Willey, New York.

SCHMEIDLER, D. (1989): "Subjective Probability and Expected Utility without Additivity," *Econometrica*, 57, 571–587.

WASON, P. C., AND D. SHAPIRO (1971): "Natural and Contrived Experience in a Reasoning Problem," *Quarterly Journal of Experimental Psychology*, 23, 63–71.