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An Exploratory Study of Attributes, Affordances, Abilities, and Distance in Children's Use of Mathematics Virtual Manipulative iPad Apps

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AN EXPLORATORY STUDY OF ATTRIBUTES, AFFORDANCES, ABILITIES, AND DISTANCE IN CHILDREN'S USE OF MATHEMATICS VIRTUAL MANIPULATIVE IPAD APPS

by

Stephen I. Tucker

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Education
(Curriculum and Instruction)

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Logan, Utah

2015

ABSTRACT

An Exploratory Study of Attributes, Affordances, Abilities, and Distance in Children's Use of Mathematics Virtual Manipulative iPad Apps

by

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Utah State University, 2015

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This exploratory qualitative study investigated the presence of and relationships among constructs that contribute to children's interactions with educational technology, leading to the development of the Modification of Attributes, Affordances, Abilities, and Distance (MAAAD) for Learning framework. For this study, each of 10 fifth-grade children participated in one individual video-recorded semistructured interview session, during which they interacted with two mathematics virtual manipulative iPad apps and responded to follow-up questions. Video recordings and observation field notes were analyzed for evidence of attributes, affordance-ability relationships, distance, and relationships among these constructs.

Constant comparative data analysis using memoing and eclectic coding provided evidence of the presence of each focus construct. Further analysis and interpretation, including quantization of qualitative data for visualization using novel rhombus plots, also led to the identification of emergent themes related to each construct and revealed relationships among the constructs. Emergent themes included categorization, alignment, and modification of attributes, variations and interrelationships among affordance-ability relationships, and the identification of and interactions among mathematical and technological distance. Furthermore, each construct related to each other construct. The evidence and interpretations led to the development of the MAAAD for Learning framework.

The results of the study suggest that the MAAAD for Learning framework models relationships among attributes, affordance-ability relationships, and distance in the context of user-app interactions. The framework could serve as a tool for app developers designing apps, educators using apps to support children’s learning, and researchers characterizing user-app interactions and the outcomes of those interactions. The constructs, relationships, and framework identified in this study advance the literature on children’s interactions with educational technology tools, in particular literature concerning children’s interactions with mathematics virtual manipulative iPad apps.

(220 pages)

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Stephen I. Tucker

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DEDICATION

To Edgar, Jose, Kairy, and everyone else who has been
mad for learning with me.

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To my mentor and chair, Dr. Moyer-Packenham, another 100 pages would hardly seem enough to thank you, though you would easily make it a concise 50. You helped me become a scholar, but more importantly, you helped me become a better person. To my dissertation committee members, thank you for generously sharing your invaluable time and insights. To my doctoral siblings and cousins, thank you for helping me grow. My "big sisters," Arla Westenskow blazed the trail, Katie Anderson-Pence helped me get my head on straight, Jessica Shumway helped keep it there, and Jennifer Boyer-Thurgood helped ensure there was something meaningful in it. My "little sisters," Emma Bullock and Christina Watts, provided thought-provoking conversations and served as mentorship guinea pigs. My "cousins" are numerous, but I am especially privileged to have shared my Utah years with HyeKyoung Lee and Joel Alejandro Mejia.

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Stephen I. Tucker

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CHAPTER I

INTRODUCTION

Interactions between children and mathematics technology tools have important implications for learning. These technology tools, including touchscreen devices such as tablets (e.g., Apple iPad, Microsoft Surface), are important for exploring, visualizing, and representing mathematics concepts (National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). As touchscreen devices become increasingly popular in the facilitation of mathematics instruction, the importance of informed design and implementation of technology tools for learning mathematics increases accordingly. To understand appropriate design and implementation of technology tools, one must understand how mathematics learning takes place when using these tools. One particular type of technology, virtual manipulatives (Moyer, Bolyard, & Spikell, 2002), as set within mathematics iPad apps, served as the context for this study.

Background and Problem Statement

The purpose of this exploratory qualitative study was to conceptualize the relationships among attributes, affordances, abilities, and distance in a framework that describes the nature of children's interactions with technology to learn mathematics, here set in the context of children's interactions with mathematics virtual manipulative iPad apps. The results and interpretations from this study inform researchers, educators, and software designers about constructs that relate to children's mathematics learning while using technology.

Representation and embodied cognition serve as the theoretical foundations for much of the research on learning mathematics. The internalization and externalization of mathematical representations (Goldin & Kaput, 1996) via representational fluency (Zbiek, Heid, Blume, & Dick, 2007) plays a key role in the learning process. From an embodied cognition perspective, perception of and interaction with mathematics in the physical environment influence human cognition. These actions can be considered evidence of mathematical thinking, and changes in these interactions are evidence of mathematical learning (Nemirovsky, Kelton, & Rhodehamel, 2013). Studies show that fifth-grade students can independently interact with technology (Blumberg & Sokol, 2004), through which they can construct mathematical concepts (Arzarello, Robutti, & Bazzini, 2005). One can use various tools to facilitate the internalization and externalization of representations through physical interactions with the environment, including virtual manipulatives.

In the past quarter of a century, virtual manipulatives have become important tools for learning mathematics. Virtual manipulatives are "an interactive…visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer et al., 2002, p. 373). Research and implementation of these tools led Namukasa, Stanley, and Tuchtie (2009) to claim that "virtual manipulatives may be an invention that not only changes what it means to learn mathematics, but also may change what mathematics can be learned" (p. 283). The effectiveness of virtual manipulatives is well established. Moyer-Packenham and

Westenskow (2013) conducted a meta-analysis of studies comparing virtual manipulatives with other instructional treatments, in which 32 studies, some with multiple comparisons, generated 82 effect sizes that yielded a moderate (0.35) effect in favor of virtual manipulatives. The authors identified five categories of affordances of virtual manipulatives that contributed to student learning: simultaneous linking, efficient precision, focused constraint, motivation, and creative variation. Instruction using virtual manipulatives may also produce equalizing effects on achievement, as one study indicated that fewer demographic predictors of student performance existed in the virtual manipulative groups compared to the student groups that used textbooks and physical manipulatives (Moyer-Packenham, Baker, et al., 2014).

One can use virtual manipulatives on many platforms, including those with multi-touch capability, such as tablets. iPads and other tablets are becoming popular tools for teaching and learning mathematics, but little research has investigated how children's mathematics learning is influenced by use of touch-screen interfaces (Moyer-Packenham et al., 2015). Research suggests that many constructs may influence mathematics learning while using iPads, including interaction modalities (McKenna, 2012; Paek, 2012) and various types of feedback (e.g., Bartoschek, Schwering, Li, & Münzer, 2013; Blair, 2013; Paek, 2012). However, iPad use does not necessarily improve student achievement (e.g., Carr, 2012; L. Wilson, Nash, Wissinger, & Leidman, 2013). Research-based app evaluations have concluded that "many applications were little more than digital flash cards encouraging rote learning" (Larkin, 2014, p. 30), and that while few offered opportunities to manipulate multiple mathematical representations, none allowed students to construct mathematical content (Goodwin & Highfield, 2013; Highfield & Goodwin, 2013). However, some mathematics iPad apps contain virtual manipulatives; whereas, “other mathematics apps, such as flash cards and drill games…lack the interactive visual representations of dynamic objects” (Tucker, Moyer-Packenham, Shumway, & Jordan, 2014, p. 1).

Emergent research has examined how using mathematics virtual manipulative iPad apps can influence mathematics learning. Research suggests that instructional experiences using apps featuring virtual manipulatives had positive effects on achievement (e.g., Haydon et al., 2012; Riconscente, 2013; Zhang, Trussell, Gallegos, & Asam, 2015). In a large-scale mixed-methods study, Moyer-Packenham and colleagues (e.g., Boyer-Thurgood et al., 2014; Moyer-Packenham, Anderson, et al., 2014; Moyer-Packenham et al., 2015; Tucker, Moyer-Packenham, Boyer-Thurgood, et al., 2014) developed and implemented research tools to investigate learning performance, learning efficiency, and behavior patterns of 100 children aged 3 to 8 interacting with mathematics virtual manipulative iPad apps during 30- to 40-minute interviews. Results indicated that the preschool group increased efficiency but their performance was unchanged, the kindergarten group increased performance but their efficiency was unchanged, and the Grade 2 group improved performance and efficiency in skip counting without showing similar growth in place value (Moyer-Packenham et al., 2015). The researchers concluded that children in different age groups interacted differently with the apps, and that apps selected for the study influenced learning in various ways. Related research (e.g., Moyer-Packenham et al., in press; Tucker, Moyer-Packenham, Westenskow, &

Jordan, 2015) suggests that children access affordances of mathematics virtual manipulative iPad apps in a variety of ways, contributing to many outcomes related to performance, efficiency, and affordance-ability relationships. Given the variance in effectiveness for different students when using mathematics virtual manipulative iPad apps, questions arise about what constructs contribute to the learning process.

By examining children's mathematics learning while they use technology, theoretically grounded in representation and embodied cognition, and applied in the context of interactions with mathematics virtual manipulative iPad apps, this study supports the development theory on the interaction among constructs that contribute to that learning.

Significance of the Study

This study used a theoretical lens of representation and embodied cognition to focus on mathematics learning while using technology, set within the context of interactions with mathematics virtual manipulative iPad apps. This study conceptualized the relationships among attributes, affordances, abilities, and distance, which are constructs that contribute to children's mathematics learning while they interact with technology. This aids the interpretation children's learning in these situations, and can also influence design and analysis of mathematics education technology. Emergent research exists on the different experiences of students who use mathematics virtual manipulative iPad apps to learn mathematics, as well as constructs that contribute to the process of learning while interacting with technology.

Embodied cognition focuses on interactions with the mathematical representations in the physical world as methods for learning and changes in these interactions as evidence of learning. In-depth examinations of student interactions with mathematics virtual manipulative iPad apps suggest that student experiences may vary based on characteristics of the students and the apps (e.g., Tucker, Moyer-Packenham, Shumway, et al., 2014). Other studies of mathematics virtual manipulative iPad apps suggested they may positively influence performance and attitudes related to fractions (Riconscente, 2013), numeracy (Spencer, 2013), and multiplication (Paek, 2012). These results indicate that mathematics virtual manipulative iPads apps are promising tools for mathematics learning, but that their effects may vary. Thus, this study supports the development of theory through the investigation of constructs and the connections among those constructs that contribute to children's mathematics learning while they use technology such as mathematics virtual manipulative iPad apps.

An array of research exists concerning constructs relating to tools and users that may influence the learning process. These include attributes (e.g., Greeno, 1994), affordances (e.g., Gibson, 1986; Norman, 1988), and abilities (e.g., Gibson, 1986), which are all thought to contribute to the distance between the user and the technology tool (e.g., Sedig & Liang, 2006). Although some of these constructs have been discussed in relation to embodied cognition (e.g., Anderson, 2003; Hostetter & Alibali, 2008) and discussed in the context of learning using technology (e.g., Belland & Drake, 2013; McGrenere & Ho, 2000; Sedig & Sumner, 2006), no prior research could be found that coherently synthesized these constructs in relation to mathematics learning while using technology. This exploratory empirical study sought to conceptualize relationships among attributes, affordances, abilities, and distance in a framework that describes the nature of children's interactions with technology to learn mathematics. The results inform the interpretation of children's learning while they interact with technology, as well as the design and analysis of mathematics education technology. The study is important because it supports the development of theory by integrating multiple lines of mathematics education and technology research. This is necessary because mathematics education increasingly incorporates technology tools, but one must understand how children learn using technology in order to effectively design, choose, and implement these tools.

Research Questions

The purpose of this study was to conceptualize the relationships among attributes, affordances, abilities, and distance in a framework that describes the nature of children's interactions with technology to learn mathematics, here set within fifth-grade children's interactions with mathematics virtual manipulative iPad apps. The over-arching research question and subquestions were as follows.

What evidence of attributes, affordances, abilities, and distance is present in the context of fifth graders' interactions with mathematics virtual manipulative iPad apps?

1. Attributes:
    a. What evidence of app attributes is present in mathematics virtual manipulative iPad apps?
    b. What evidence of user attributes is present in user interactions with

mathematics virtual manipulative iPad apps?

2. Affordance-ability relationships: What evidence of affordance-ability relationships is present in user interactions with mathematics virtual manipulative iPad apps?

3. Distance: What evidence of distance is present in user interactions with mathematics virtual manipulative iPad apps?

4. Relationships: What evidence of relationships among attributes, affordances, abilities, and distance is present in user interactions with mathematics virtual manipulative iPad apps?

Summary of Research Study Design

In order to find evidence of attributes, affordances, abilities, and distance and relationships among these elements in the context of children's interactions with mathematics virtual manipulative iPad apps, this study employed an exploratory qualitative design (Marshall & Rossman, 2010). "Exploratory research seeks to provide new and previously overlooked explanations…by looking at reality from a new angle" (Reiter, 2013, p. 7). This design used qualitative methods to analyze children's interactions with mathematics virtual manipulative iPad apps, consistent with the embodied cognition focus on physical interaction. Each of 10 fifth-grade participants individually interacted with two mathematics virtual manipulative iPad apps during semistructured task-based interviews in an observation room at a university research center. Data collection included observations and video recordings of the semistructured task-based interviews. Constant comparative data analysis included qualitative and quantitized qualitative focusing on identifying evidence of attributes, affordances, abilities, distance, and relationships among these elements through development and interpretation of codes and categories, which supported theory development.

Assumptions and Scope of the Study

The researcher made several assumptions about the study based on theories of representation and embodied cognition. First, the researcher assumed that the participants would interact with the virtual manipulative iPad apps and that the video recordings would capture these interactions. Consistent with embodied cognition, the researcher assumed that participant interaction with this form of mathematical representations in the physical environment would provide data on interactions with touchscreen devices that could be coded for evidence of constructs related to learning mathematics.

The study was exploratory in nature because previous empirical research had not cohesively examined the constructs investigated herein in relation to mathematics learning while interacting with technology. The exploratory approach required the acknowledgement of several delimitations of this study. The sample was limited to fifth-grade participants, and potential differences by demographic characteristics were beyond the scope of this study. The inclusion of the focus constructs presented the possibility of fine-grained, complex relationships within and among the elements, such as extensive attribute lists, varying attribute changes in different contexts, minute changes in individual affordance-ability relationships, and situation-specific relationships among distance types, but these analyses were beyond the scope of this exploratory study.

The focus on evidence of the constructs meant that this phase of exploratory research did not seek to generate detailed characterizations of the quality or quantity of learning taking place. Other influences that contribute to learning mathematics, such as social context (e.g., Ladel & Kortenkamp, 2013), were also beyond the scope of this study. Furthermore, this study focused on a specific content area (mathematics) and tool (mathematics virtual manipulative iPad apps), and was not intended to delineate the nuances of how these specific content areas (e.g., algebra) and elements of the tools (e.g., game environment vs. virtual manipulative) influenced the individual constructs. Additionally, many of the constructs have multiple definitions, but this study focuses on the usages cited throughout this document. For example, motivation has been described from various perspectives, such as expectancy-value theory, which takes into account one's beliefs about potential outcomes of an activity and the degree to which one values the activity (Wigfield & Eccles, 2000). In this study, the term *motivation* is applied in a narrower sense to describe the affordance of motivation (i.e., offering features that influence affect, engagement, and interest), consistent with the description and application in the specific sense of affordances of virtual manipulatives by Moyer-Packenham and Westenskow (2013). More studies will be required to investigate these dimensions, as is standard for exploratory research (Stebbins, 2001).

Definition of Terms

The following definitions are pertinent to the study.

Virtual manipulative: "Interactive… visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer et al., 2002, p. 373).

Attribute: A quality or character considered to belong to or be inherent in a person or thing (online Oxford English Dictionary).

Affordance: Something that "relates attributes of something in the environment to an interactive activity by an agent who has some ability" based on its own attributes, which are characteristics of the environment or agent (Greeno, 1994, p. 383). Also, "cues of the potential uses of an artefact by an agent in a given environment" (Burlamaqui & Dong, 2014, p. 13).

Ability: Something that "relates attributes of an agent to an interactive activity with something in the environment that has some affordance" (Greeno, 1994, p. 383).

Distance: The "degree of difficulty in understanding how to act upon [something] and interpret its responses" (Sedig & Liang, 2006, p. 184).

CHAPTER II

LITERATURE REVIEW

This chapter reviews the research literature related to the proposed study. The first section presents a theoretical framework for the study, based mainly in representation and embodied cognition. The second section examines constructs that may contribute to users' experiences when interacting with mathematics education technology. The concluding section identifies areas for further research and the potential contributions of this study to the field of educational research, including app design and implementation.

Theoretical Framework

Literature concerning representations, as accessed through embodied cognition set in the context of interaction with multi-touch technology tools, served as the basis of the theoretical framework for this study.

Representation in Mathematics

Learning mathematics involves interactions between and the development of internal and external representations. Internal representations are individuals' mental configurations of mathematics that cannot be directly observed, while external representations are physically embodied configurations of mathematics that can be accessed by those with appropriate understandings of the representations (Goldin & Kaput, 1996). Interplay among representations can include internalizing external representations (e.g., interpreting graphs, symbols, and pictures) and externalizing internal representations (e.g., writing, speaking, manipulating concrete objects). Research has also shown that interactions with appropriate combinations of multiple external representations can enhance learning (Ainsworth, 2006). The processes of interacting with multiple representations and internalizing and externalizing representations involves representational fluency, which includes translation across representations, drawing meaning from different representations of a mathematical entity, and generalizing across representations (Zbiek et al., 2007). In other words, representational fluency involves the understanding of representations and the connections among multiple representations, which contributes to interactions between and development of internal and external representations.

Representational fluency is key to connecting and modifying representations, and thus to mathematical learning. Students with greater representational fluency show greater success in mathematical problem-solving and justification (Niemi, 1996), while representational fluency can be developed through interactions with technology such as virtual manipulatives that include multiple connected representations (Suh & Moyer, 2007). Representational fluency can both facilitate and result from mathematical learning (Heinze, Star, & Verschaffel, 2009; Nathan & Kim, 2007). This means that representational fluency is both an element of and an outcome of mathematical learning. This process can take place in many ways, including through physical interactions with representations, such as the physical interactions that students have with mathematics virtual manipulative iPad apps.

Embodied Cognition: Physically Interacting with Representations

Physical interaction with external representations involves embodied cognition, as cognitive processes relate to bodily interactions with the environment. In particular, it has been theorized that human cognition is rooted in sensorimotor processing (M. Wilson, 2002), which is the integration of perception of the environment using multiple senses with actions taken upon the environment. Human cognition is thus based in action and perception, and is grounded in the physical environment (Alibali & Nathan, 2012). Interactions with the physical environment influence human cognition, and the physical environment contains representations of mathematics. Interactions with mathematical representations in the physical environment influence the interplay between internal and external representations, and therefore influence learning. Accordingly, one can analyze physical interactions with representations of mathematics in the environment for evidence of mathematical learning.

Daghestani (2013), expanding on cognitive frameworks for learning with media (e.g., Mayer, 2002; Moreno, 2006), posited that visual, auditory, and tactile components are integral to the learning process, and asserted that the user plays an active role in selecting and manipulating tasks when interacting with multimedia technology. Nemirovsky et al. (2013) took this further, suggesting that “the intertwining of perceptual and motor aspects of tool use [is] *perceptuomotor integration*” (p. 373, emphasis in original), which allows a person to perceive and interact with representations in such a way that integrates action and thought. For these authors, mathematical thinking is equivalent to expressions of bodily activity, and mathematical learning consists of changes in learners' physical engagement in mathematical practices. Thus, perceptuomotor integration is the mechanism by which a person uses bodily activity to develop representational fluency and facilitate the interplay between internal and external representations. Changes in bodily engagement (external) in mathematical practices can provide evidence of changing (internal) representations of mathematics. Therefore, examining physical activity of children engaged in mathematics tasks can shed light on how children learn mathematics.

Using Multi-Touch Technology to Interact with Representations

Different technology tools offer varying levels of embodiment. Bodily engagement involves gestures, which include a variety of hand and body movements that stem from perceptual and motor underpinnings of embodied language and mental imagery (Hostetter & Alibali, 2008). The term "gestures" is used here to refer specifically to representational gestures, which are bodily actions that are used in the interplay between internal and external representations (Hostetter & Alibali, 2008; Segal, 2011). Gestures have been shown to help children retain and apply newly acquired knowledge within similar contexts (Cook, Mitchell, & Goldin-Meadows, 2008) when developmentally appropriate (Ginsburg, Jamalian, & Creighan, 2013; Shuler, 2009) and mapped to the specific content (Segal, 2011; Segal, Tversky, & Black, 2014). Multi-touch interfaces, such as those found on iPads and other tablet devices, offer the potential to support rich contexts in which to learn mathematics (Hegedus, 2013) and can be programmed to recognize a wide variety of input that many consider to be gestures (e.g., Hamon, Palanque, Silva, Deleris, & Barboni, 2013). However, while few apps take advantage of multi-touch capabilities (Byers & Hadley, 2013), apps that do use multi-touch capabilities may influence children's mathematical understandings and strategy development in unique ways (Baccaglini-Frank & Maracci, 2015). The potential for a range of gesture use means that multi-touch technology allows for greater embodiment than mouse-based interaction, as it can afford users more direct control over the manipulation of representations. Greater embodiment allows for a greater range of possible bodily engagement in mathematically meaningful gestures and practices, and thus more room for changes in this engagement that provide evidence of changing internal representations of mathematics, and therefore mathematical learning. However, one must appropriately design these tasks and the tools.

Faithful Technology Tools for Interacting with Representations

Researchers have theorized ways to design educational tools, including software such as apps, that facilitate mathematics learning (e.g., Ginsburg et al., 2013; Pelton & Francis Pelton, 2011). Many of these guidelines can be traced to Dick (2008), who recommended that designers of technology tools insure high levels of cognitive, pedagogical, and mathematical fidelity. Cognitive fidelity is the degree to which the mathematical representations of the tool align with the cognitive processes of the student. Pedagogical fidelity is the degree to which the tool aligns with design principles. Mathematical fidelity is the degree to which the tool appropriately represents mathematical content. Tools and tasks with high fidelity in all three areas are more likely to (a) accurately reflect the user's internal representations and methods of modifying these representations (cognitive fidelity), (b) allow users to perceive the tool as useful for learning mathematics (pedagogical fidelity), and (c) represent mathematical content in a way it is understood by the mathematical community (mathematical fidelity), supporting the development of representational fluency (Zbiek et al., 2007).

Tools and tasks, including those involving virtual manipulatives, vary in fidelity (Moyer-Packenham, Salkind, & Bolyard, 2008). Olive (2013) argued that the greatest challenge in designing digital tools for learning mathematics is to insure they are cognitively faithful to externalize students' mathematical thinking. Digital tools have the potential to offer "idealized" representations of some mathematical concepts that are more mathematically faithful than non-digital representations (Kirby, 2013), allowing users to interact with visualizations of concepts that were once only available in mental models (Carpenter, 2013). Pedagogical approaches of digital tools (e.g., instructive, manipulable, and constructive—Highfield & Goodwin, 2013; self-leveling, collaborative, and sandbox—Zanchi, Presser, & Vahey, 2013) frame discussions of pedagogical fidelity. Each type of fidelity influences the design of the tool and how the user perceives and interacts with the tool, thus influencing the internalization and externalization of representations via perceptuomotor integration.

Summary of the Theoretical Framework

Theories of representation and embodied cognition imply that learning mathematics involves the modification of internal representations, often through physical interaction with external representations. Perceptuomotor integration and representational fluency contribute to the transformation of internal representations, with gestures assisting the externalization and internalization of representations. Multi-touch technology allows for increased embodiment in human-computer interaction, and cognitive, pedagogical, and mathematical fidelity influence how users interact with technology. Thus, using an embodied cognition approach, one can investigate constructs involved in mathematics learning by examining how children physically interact with representations of mathematics using multi-touch technology.

Attributes, Affordances, Abilities, and Distance

This section explains the constructs examined in the study, set within the context of learning mathematics through physically embodied interactions with technology-based mathematical representations: attributes, affordances, abilities, and distance.

Attributes, Affordances, and Abilities

An attribute is a characteristic of a person or thing online Oxford English Dictionary). In the context of this study, the user (i.e., the participant) has attributes and the app has attributes. Based on Gibson’s work (e.g., 1986), Greeno (1994) defined an affordance as something that “relates attributes of something in the environment to an interactive activity by an agent who has some ability” based on its own attributes, which are characteristics of the environment or agent (p. 383). An ability, therefore, is something that “relates attributes of an agent to an interactive activity with something in the environment that has some affordance” (Greeno, 1994, p. 383). Greeno further asserted that affordances are graded properties, rather than being present or not present, and that an affordance exists only in relation to an ability, and vice versa. Chemero (2003) extended this, positing that affordances are coupled with abilities as part of a continuous system. However, Dotov, Nie, and de Wit (2012) noted that different fields (e.g., ecological psychology, cognitive psychology, and neuroscience) conceive of affordances differently. The variances are also evident within any given field, leading researchers to recognize that there are many controversial claims about affordances, aside from the idea that affordances are possible actions related to an agent (Burlamaqui & Dong, 2014). Some authors discuss the idea of constraints, but if an attribute has a feature that provides a constraint, that is part of what the app affords. In the context of this study, (a) apps have attributes that combine to provide affordances, (b) users have attributes that combine to create abilities, and (c) there is an affordance-ability relationship between user and app (see Figure 1).

Gaver (1991) brought Gibson's conception of affordances into the field of human-computer interaction, including how design suggests affordances. In contrast, McGrenere and Ho (2000) interpreted Norman's (e.g., 1988, 1999) application of affordances as one of perceived possibilities, wherein a user should be able to determine what to do without difficulty. This identified another difference between Gibson, who focused on perception

Figure 1. Affordance-ability relationship set within user-app interactions.

of the environment, and Norman, who focused on manipulability of the environment. Affordance typologies and applications vary, and include distinctions such as technological, social, and educational categories of what a given tool allows to be possible (Kirschner, Strijbos, Kreijns, & Beers, 2004). Sedig and Liang (2006) applied Norman's affordances to visual mathematical representations, stating that they should "clearly communicate their affordances to learners, making it easy for them to perceive and attend to the interactions that are possible" (p. 185). Perceiving and attending to these affordances depends on the abilities of the user. One can conceive of the ease or difficulty of taking advantage of an affordance as the distance between the user and the tool.

Distance

Sedig and Liang (2006) conceived distance as the "degree of difficulty in understanding how to act upon [something] and interpret its responses" (p. 184). This builds on the idea that there are two gulfs to be bridged between computer and user: the Gulf of Execution and the Gulf of Evaluation (Hutchins, Hollan, & Norman, 1985; Norman, 1986, 1991). The Gulf of Execution is the difficulty of interacting with the environment, which one can bridge by matching the mechanisms of the computer system with the thoughts and goals of the user. The Gulf of Evaluation is the difficulty of determining the state of the environment, which one can bridge by making the information displayed easily understandable. Distance determines the amount of cognitive load a user encounters, and ways to reduce this cognitive load include designing the tool to fit the learner's conceptions or by the learner bridging the difference by learning to use the tool (Sedig & Liang, 2006). Levels of cognitive, pedagogical, and mathematical fidelity (Dick, 2008) may also contribute to distance, as they influence both the tool design and how the user perceives the tool.

Types of distance. Sedig and Liang (2006) defined four types of distance: semantic, articulatory, conceptual, and presentation. Semantic distance described the level of matching between a user's intent and the types of interaction an object allows, such as whether one is able to move an object or if it must remain stationary. Articulatory distance referred to the difference in expression of input and output, such as direct or indirect manipulation. Conceptual distance referred to the gap between a user's understanding of how to act upon the mathematical model and how the technology allows for manipulation, such as permitting only reflection when a user only knows how to use rotation to manipulate shapes. Presentation distance referred to how learners are able to adjust a representation relative to the types of adjustments users can make to the representations, such as rotating a figure to allow for a different perspective that may prove easier for a user to interpret.

Maintaining distance. Sedig, Klawe, and Westrom (2001) argued that maintaining an appropriate amount of distance encourages reflective thinking and deeper reasoning. The authors explained that user efforts to bridge the gulfs of execution and evaluation affect reasoning and amount of mental effort, and thus the depth of learning. They concluded that purposeful, stepwise modification of distance by the tool is key to facilitating learning. The authors framed this in terms of the removal of scaffolding, in their example surrounding a visual mathematical representation. Scaffolding is the external control of task elements initially too difficult for the learner (Wood, Bruner, &

Ross, 1976). Sedig et al. (2001) explained that initial interactions with a technology-based visual representation to explore mathematics content are reflective, but become habitual as the learner progresses. The interface then removes scaffolding to disrupt the habituation, which leads to reflective interaction and cyclical repetition. In this model, the authors focused on the removal of scaffolding moving from a concrete experiential interaction toward abstract reflective interaction. However, the principle of stepwise adjustment need not be unidirectional.

Many aspects of interactive visual representations can be dynamically modified to maintain interactivity (Parsons & Sedig, 2014). The multidirectional adjustment of scaffolding is akin to the zone of proximal development (ZPD) developed by Vygotsky (1978) and applied to technology by Murray and Arroyo (2002) as progressive mastery of instructional objectives that takes place when material is neither too easy nor too difficult. Progressive mastery suggests that users also change to maintain appropriate amounts of distance. Thus, both users and technology change during interactions to maintain distance and facilitate the learning process.

Summary of Research on Attributes, Affordances, Abilities, and Distance

Literature suggests the existence of attributes, affordances, abilities, and distance in interactions between users and technology. Attributes of a user form abilities that provide varying access to affordances of technology, which are products of attributes of the technology. In this context, distance involves interpreting and responding to technology. Maintaining an appropriate amount of distance involves dynamic change of both user and technology. Each of the constructs influences how users interact with technology. These physically embodied interactions with technology-based representations of mathematics are mathematical thinking and can provide evidence of mathematical learning, thus setting the constructs within the theoretical framework of embodied cognition and representation.

Unique Contributions of the Current Study

Research exists on affordances, the effectiveness of virtual manipulatives, and children's learning of mathematics in connection with iPad app use. However, little research has combined all of these areas, and no research could be located that coherently investigated relationships among affordances, abilities, attributes, and distance. One can study this using the lens of embodied cognition for interaction with representations, applied to investigations of how children interact with technology. Thus, this exploratory study contributes to the field by supporting the development of theory based on an investigation of relationships among affordances, abilities, attributes, and distance, in the context of children's interactions with mathematics virtual manipulative iPad apps.

This study informs future research on user-app interactions through a closer examination of relationships among attributes, affordances, abilities, and distance in the context of children's interactions with technology-based representations of mathematics. The study also informs future research on the use of educational technology, such as mathematics virtual manipulative iPad apps, to learn mathematics. The study is significant because iPads and other tablets are becoming popular tools for learning mathematics. When app designers understand user-app interactions, this has the potential to inform future app creation. When app implementers, including teachers, understand user-app interactions, this has the potential to determine how best to employ apps for mathematics learning.

CHAPTER III

METHODS

This exploratory study used multiple qualitative methods to conceptualize the relationships among attributes, affordances, abilities, and distance in a framework that describes the nature of children's interactions with technology to learn mathematics, set within the context of fifth-grade children's interactions with mathematics virtual manipulative iPad apps. Researchers use exploratory qualitative research to develop conceptual frameworks and support theory development based on the description of phenomena evident in emergent patterns in the data (Marshall & Rossman, 2010). This design allows a focus on describing children's interactions with technology-based mathematical representations during semistructured task-based interviews, consistent with theories embodied cognition and representation.

During each semistructured task-based interview, a fifth-grade child interacted with virtual manipulative iPad apps and answered follow-up questions. The researcher collected data from video recordings of the sessions and observation field notes taken during the sessions. Qualitative data analysis included constant comparative techniques using eclectic coding that incorporated multiple iterative coding techniques to focus on attributes, affordances, abilities, distance, and relationships among these constructs, and quantitization of qualitative data to facilitate data visualization. Analysis led to identification of emergent patterns and supports theory development. The over-arching research question and subquestions were as follows.

What evidence of attributes, affordances, abilities, and distance is present in the context of fifth graders' interactions with mathematics virtual manipulative iPad apps?

1. Attributes:
    a. What evidence of app attributes is present in mathematics virtual manipulative iPad apps?
    b. What evidence of user attributes is present in user interactions with mathematics virtual manipulative iPad apps?
2. Affordance-ability relationships: What evidence of affordance-ability relationships is present in user interactions with mathematics virtual manipulative iPad apps?
3. Distance: What evidence of distance is present in user interactions with mathematics virtual manipulative iPad apps?
4. Relationships: What evidence of relationships among attributes, affordances, abilities, and distance is present in user interactions with mathematics virtual manipulative iPad apps?

Design

This study used an exploratory qualitative research design with qualitative data collection, data coding, and data analysis techniques after receiving appropriate Institutional Review Board (IRB) approval (see Appendix H). Exploratory research "emphasizes developing theory from data" (Stebbins, 2001, p. 5). Exploratory qualitative research is appropriate for theory development because it involves investigating and describing phenomena to generate hypotheses for future research, focusing on themes and patterns in data, and links between the patterns (Marshall & Rossman, 2010). Importantly, exploratory research is “an act of gradual, structured, and theory-led heuristic expansion from an original set of models, explanations, and questions” (Reiter, 2013, p. 11). This study expanded upon original models and explanations, supporting the development of theory by describing evidence of attributes, affordances, abilities, distance, and links among these constructs present in the context of fifth graders’ interactions with mathematics virtual manipulative iPad apps. The application of this design aligned with the theoretical framework of embodied cognition and representation, as the primary focus for data collection and analysis was on the children’s interactions with external, physical representations of mathematics.

The study used qualitative data collection, coding, and analysis techniques. Data collection included video recording of user-app interactions and responses to questions during semistructured task-based interviews (Goldin, 2000). This generated audiovisual records that could provide evidence of the target constructs and relationships among these constructs. The researcher employed a constant comparative technique to analyze the data, which involved integrated, iterative data collection and analysis (Glaser & Strauss, 1967; Merriam, 2009), as recommended for generating categories and building theories (Anfara, Brown, & Mangione, 2002). Data analysis included memoing and eclectic coding to incorporate the constructs and emergent themes (Saldaña, 2013), and quantitized qualitative analysis (Teddlie & Tashakkori, 2011) to create data visualizations for pattern identification. The data analysis methods were appropriate for addressing the research questions because they facilitated identification and description of relevant emergent themes and categories. Thus, the exploratory qualitative research design and the chosen design elements were appropriate to address the research questions in this study because the purpose of this study was to conceptualize the relationships among attributes, affordances, abilities, and distance in a framework that describes the nature of children's interactions with technology to learn mathematics.

Pilot Study

Before the proposed study, the researcher conducted a pilot study with ten students from fourth through sixth grade to test the methods, instruments, app choices, data collection and data analysis techniques. During the pilot study, the researcher created and refined a facilitation protocol to make the data collection flow smoothly. The researcher also developed and honed the observation protocol to focus on user-app interactions to address the research questions, as consistent with embodied cognition. The researcher tested data coding techniques to identify evidence of attributes, affordances, abilities, and distance, and relationships among these constructs. The Data Analysis section describes how preliminary analysis of pilot data influenced analysis in the proposed study.

Unexpected issues arose during the pilot study. A prompted think aloud was tested for the purposes of determining if participant narration would yield relevant data but produced few informative utterances without distracting the users from successfully completing their tasks, decreasing the focus on interactions that are emphasized in embodied cognition. To address this issue, the researcher changed to a semistructured task-based interview format to allow uninterrupted, independent user-app interactions while still providing supplementary verbal user responses. Computer-based recording provided a clearer audio and video record of the interactions and interviews than the wall-mounted camera, but the wall-mounted camera proved more reliable. To address this issue, the researcher used both recording systems for each semistructured task-based interview in this study. Additionally, participants requested a stand for the iPad to increase their comfort level, so the researcher used an iPad stand to prop up the iPad at a more accessible angle. The app selection process also occurred during the pilot study.

Selection of Materials

As part of the pilot study, the researcher determined inclusion criteria for choosing mathematics virtual manipulative iPad apps. Apps included in the pilot study: (a) were designed for the iPad and available through the Apple Appstore, (b) explored mathematics content using at least one virtual manipulative (as defined by Moyer et al., 2002), (c) included ages 10-12 (i.e., approximate age of a fifth grader) in the target age range stated by the developers, (d) featured mathematics content connected to at least one fifth-grade Common Core State Standards Content Standard (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), (e) were primarily manipulable, rather than primarily instructive (Goodwin & Highfield, 2013), (f) were organized in progressive levels of difficulty (i.e., they progressed through related mathematics content with distinct breaks between segments), and (g) required touching the screen as the primary mode of input. Seven apps were tested during the pilot study for the purpose of selecting two apps to be used in the study: Motion Math: Zoom, DragonBox Algebra 5+, DragonBox Algebra 12+, DragonBox Elements, Tiny Fractions, Symmetry School, Chicken Coop Fractions (Estimating Fractions).

The researcher eliminated piloted apps for consideration for the dissertation study for various reasons. The researcher excluded two apps with minimal piloting. It was not possible to reset Tiny Fractions for additional users, thus allowing some users to begin at more advanced levels, while DragonBox Algebra 5+ had a version available targeted more specifically at the focus age group (DragonBox Algebra 12+). The researcher excluded other apps if multiple pilot participants were unwilling to play for at least ten minutes. Participants described Symmetry School as “boring” and claimed that Chicken Coop Fractions (Estimating Fractions) was “too hard” because “we haven’t done [connecting fractions and decimals on a number line] yet.” The researcher eliminated DragonBox Elements because participants struggled to independently interact with the app. The participants repeatedly asked for help interacting with the app and interpreting or completing the tasks. Thus, the dissertation study included the two apps the pilot participants played independently for the longest average duration and responded to the most positively: DragonBox Algebra 12+ and Motion Math: Zoom.

Setting and Participants

The study took place in interview rooms in a public university in the Intermountain West region of the U.S. Participants were 10 fifth-grade children: 6 male and 4 female, 8 White and 2 Asian, 5 ten years old and 5 eleven years old. Researchers recommend that "sample sizes in qualitative research should not be so small as to make it difficult to achieve data saturation" but not so large as to hinder deep analysis (Onwuegbuzie & Collins, 2007, p. 290). Theory-based sampling techniques (Onwuegbuzie & Collins, 2007; Saumure & Given, 2008) were used to identify and recruit potential participants. Saumure and Given (2008) noted that theory-based sampling is "key to achieving saturation quickly. Here research participants are selected so that the resulting data help to build and validate the emerging theory" (p. 197). Guest, Bunce, and Johnson (2006) found that of 36 common codes formed from 60 interviews, 94% of the codes were identified within the first six interviews and 97% of the codes were identified within the first 12 interviews. Other methodologists recommend limiting sample sizes to between 5 and 10 participants to allow for deeper analysis (Miles, Huberman, & Saldaña, 2013). Thus, a sample size of 10 participants was sufficient for in-depth analysis in this exploratory study.

The study focused on participants in fifth grade for theoretical and mathematical reasons. In accordance with the theoretical framework of embodied cognition and representation set within the context of interactions with technology, research indicates that fifth-grade students benefit from directly manipulating animations while interacting with technology-based representations of content (Black, 2010), and these benefits include construction of mathematical concepts (Arzarello et al., 2005). Additionally, research indicated that fifth-grade students were aware of the relevance and importance of mathematics (Vanayan, White, Yuen, & Teper, 1997), and are capable of both independently completing technology-based tasks and of answering questions related to these tasks (Blumberg & Sokol, 2004). Recruitment involved distributing fliers to fifth-grade students through local elementary schools. During the scheduling of data collection, the researcher asked each parent if the potential participant had interacted with any version of either or both of the apps in this study to control for prior experience by only including participants who had not previously interacted with the apps.

Materials

The materials for the study include iPads and the two mathematics virtual manipulative iPad apps chosen during piloting: Motion Math: Zoom and DragonBox Algebra 12+.

Motion Math: Zoom

According to the developers, Motion Math: Zoom is an app recommended for children ages 5-12 (Motion Math, Inc., 2014). Motion Math: Zoom features content related to number comparisons, estimation, place value, and magnitude on the number line, including positive and negative numbers, integers to 10,000, and decimals to the thousandths place (see Figure 2). This interactive representation is a type of "idealized number line" (Kirby, 2013) that was not possible before digital tools (Carpenter, 2013), in this case featuring changeable scales and fluid movement to navigate the number line (Zhang et al., 2015). To interact with the app, users employ single-touch and multi-touch gestures to navigate the number line and pop bubbles to place target numbers in the correct empty spaces. Animals of varying sizes separate intervals proportionately between numbers. One or two interval ranges may be visible at a given time. For positive

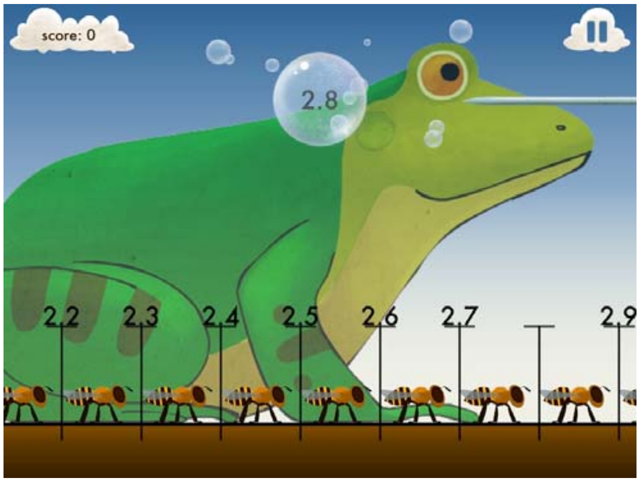

Figure 2. Screenshot of Motion Math: Zoom.

numbers, the animals face rightward; for negative numbers, the animals face leftward.

Users swipe or drag the number line left or right to view numbers along the line. To zoom in or out, a user must bring two fingers apart or together (“pinching”) horizontally. This decreases or increases the intervals between visible numbers accordingly (e.g., ones, tens, hundreds, etc.). Users can pinch with one finger on each hand or two fingers on the same hand. There are 24 levels, with all but the introduction consisting of 8-15 tasks. Users can complete levels non-sequentially, including by skipping some levels depending on how they perform on previous levels. Users can also elect to use the “needle,” which acts as a timer, popping the bubble to end the level if the user is too slow to place a given number. The default needle setting is off, but when users first quickly and accurately complete level 6, the app offers them the opportunity to try level 15 with the needle on.

During the pilot study, only one participant asked to stop playing Motion Math: Zoom before 15 minutes had elapsed. No participants appealed for researcher assistance. No participant completed a level beyond 16, with most reaching no higher than level 15, each of which involved decimals to the hundredths place. No participant completed all levels leading up to 16, and most participants chose to change levels when prompted by the app, with few choosing to do so their own volition.

DragonBox Algebra 12+

According to the developers, DragonBox Algebra 12+ is an app recommended for ages 9 and up (WeWantToKnow AS, 2014). DragonBox Algebra 12+ includes content related to operations, additive and multiplicative thinking, negative and positive values, solving expressions and equations, and fractions. The app consists of 10 20-level chapters in the context of growing a dragon in each chapter by completing levels. Each level requires the user to solve one equation or expression (see Figure 3). The app demonstrates new content via “new powers” before integrating the “powers” into subsequent levels. Users employ single-touch gestures to tap or drag tiles to complete each level. The app presents levels sequentially, but users may return to a previous level by accessing the menu. Users can choose to undo a move, restart a level, or watch a video of the solution to the level by selecting menu options within the level.

During the pilot study, no participant asked to stop playing DragonBox Algebra 12+ before fifteen minutes had elapsed. Participants rarely appealed for researcher assistance, then usually asking for help navigating the game (e.g., how to reset a level), rather than the mathematics content. All participants but one completed 30-40 levels. The participant who completed 49 levels had completed the previous version of the game (Dragon Box Algebra 5+), leading to the requirement that the full study exclude potential participants who have played a version of the app. Most participants reached a stage in

Figure 3. Screenshot of DragonBox Algebra 12+.

which pictures and letters were present as variables within pictorial representations of equations, but no operation symbols were present, as in more advanced levels.

Procedures

This study used a version of the task-based interview procedure, akin to other iPad and virtual manipulative studies (e.g., Moyer-Packenham et al., 2015). This process generated relevant data in the form of user-app interactions that consisted of users' embodied interactions with the physical representations of mathematics as part of the apps, as well as user verbal responses to follow-up questions, providing multiple avenues for data analysis. Task-based interviews involve a subject and an interviewer who interact in relation to tasks and questions which the interviewer introduces in a planned way (Goldin, 2000). Major differences between structured and unstructured task-based interviews are the explicit provision for contingencies (i.e., how to guide participants depending on their actions) and the deliberate design of the sequence and structure of the tasks in the former as opposed to the free problem solving and lack of assistance provided in the latter. Questions in task-based interviews can vary in the structure and amount of questioning, ranging from non-directive follow-up questions to metacognitive questions, both during and after the task completion. However educational research should not be limited by a particular research methodology and should instead use a diverse range of methods to investigate research questions (Kelly & Lesh, 2000). Thus, this study combined elements of structured and unstructured task-based interviews to form a semistructured task-based interview.

In the semistructured task-based interview, the researcher provided an environment in which to problem-solve (an app) that presented tasks in an organized manner (i.e., levels), but the researcher did not offer assistance during task completion. This semistructured task-based interview aligned with Goldin's (2000) recommendations for quality interviews: (a) the tasks were accessible to the participants because the tasks were targeted at their developmental level, (b) the tasks embodied rich representational structures because they included a variety of app-generated tasks using virtual manipulatives, (c) the tasks encouraged free problem solving without researcher guidance, and (d) the interview maximized interaction with the learning environment by emphasizing app interaction time. The interview thus focused on embodied interactions with representations of mathematics (i.e., physical manipulations of external mathematical representations), which are equivalent to mathematical cognition (Nemirovsky et al., 2013).

The semistructured task-based interview process (see Figure 4) began with the

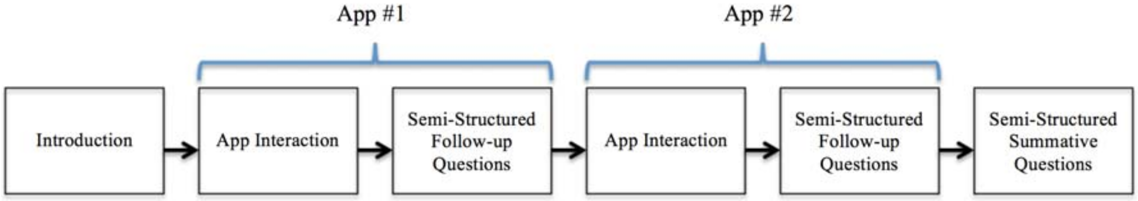

Figure 4. Outline of semistructured task-based interview procedures.

introduction, outlining the procedures to the participant and answering any questions the participant may have. The participant then interacted with the first app for up to 30 minutes. The participant could elect to stop interacting with the app before the 30 minutes had elapsed. The researcher allowed participants who were attempting a level when the 30 minutes had elapsed to finish the attempt. Following the app interaction time, the researcher asked follow-up questions about interactions with the first app. The process for the second app followed the same steps, with the follow-up questions focused on interactions with the second app. The follow-up questions were also semistructured to allow the researcher to bring up purposeful themes while remaining flexible enough to respond to participants' preferred directions (Rossman & Rallis, 2003). The purpose of the study (i.e., seeking evidence of the constructs and their relationships) and the participant's app interactions (e.g., "you played Level 10 many times; how did you figure out what to do?") informed the questions the researcher asked. The researcher asked questions only after the participant completed the app interaction portion of the interview to avoid distracting the participant during problem solving. Although some researchers suggest that prompting during tasks does not significantly distract the participant from completing the task (e.g., Cotton & Gresty, 2006, 2007), many app-generated tasks in this study had time requirements for successful completion. After completing both interaction-question sections, the session concluded with brief, semistructured summative questions.

Each session generated a maximum of 90 minutes of video-recorded data. In the dissertation study, the researcher excluded all data from the participant who played one of the apps for less than 10 minutes and recruited a replacement participant. The researcher chose 10 minutes as the minimum time to provide sufficient data for analysis that would lead to saturation. Half of the participants started with Motion Math: Zoom and the other half of the participants started with Dragon Box Algebra 12+ to control for possible effects of app order or participant fatigue. However, there were no noticeable effects of app order.

Facilitation Protocol

The researcher used a facilitation protocol to guide the video recording data collection process (see Appendix A). The facilitation protocol outlined the steps that the researcher followed to conduct the semistructured task-based interview. The facilitation protocol began with preparing the materials for the session. Upon participant arrival, the researcher distributed and explained the consent and assent forms, which the parent and participant completed. Next, the researcher showed the parent into the observation room and returned with the participant to the interview room. The semistructured task-based interview then proceeded as described in the Procedures section above. After the semistructured task-based interview, the researcher debriefed the parent and participant.

Data Sources and Instruments

The data sources for this study were video recordings and observation field notes, as in other research concerning mathematics virtual manipulative iPad app use (e.g., Moyer-Packenham, Anderson, et al., 2014; Moyer-Packenham et al., 2015). The video recordings captured user-app interactions and user responses to questions. The observation field notes verified data collected from video recordings. Thus, multiple data sources contributed to the description of evidence of the constructs and their relationships (Creswell & Plano-Clark, 2011).

Video Recordings

Video recordings were made of the entire semistructured task-based interview. Video recordings can provide a visual record of change over time (Lesh & Lehrer, 2000) and can be used to facilitate the development of theory (Hall, 2000). Researchers using an embodied cognition perspective often employ video recordings to produce a continuous visual record of physical interactions, such as during user-app interactions (e.g., Baccaglini-Frank & Maracci, 2015; Barendregt, Lindström, Rietz-Leppänen, Holgersson, & Ottosson, 2012; Segal, 2011). Two devices recorded each session. The primary recording device, the built-in FaceTime HD Camera and microphone on a Macbook Pro, was placed immediately adjacent to the iPad, raised from the table, and opposite the user to record a close-up view of the user-app interactions (see Figures 5 and 6). The secondary recording device was a wall-mounted video camera with room microphone. The camera focused on the area of the table where the researcher placed the iPad (see

Figure 5. Arrangement of computer and iPad.

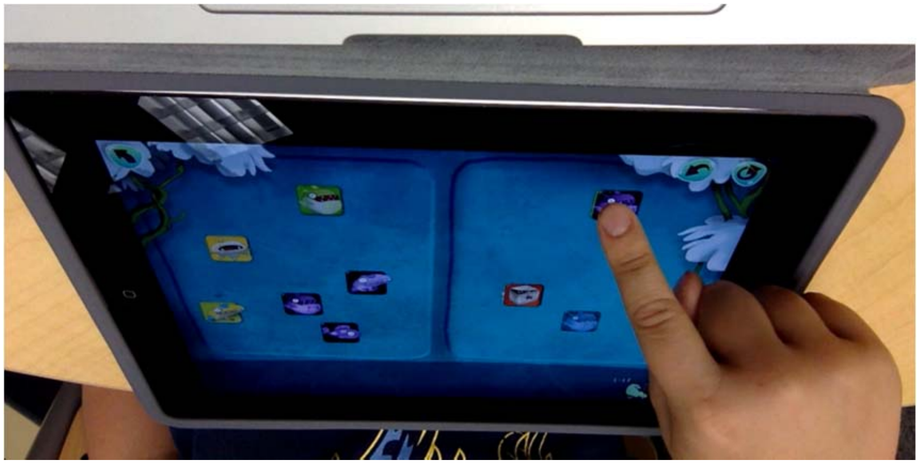

Figure 6. Screenshot of computer camera view (after reorientation).

Figure 7), recording video of the interactions between the user and the iPad app as the microphone recorded the audio. This is akin to other app-interaction studies (e.g., Holgersson, Barendregt, Rietz-Lepännen, Ottosson, & Lindström, 2013; Moyer-Packenham, Anderson, et al., 2014). The narrow focus of each recording device provided a clear picture of the two-way interaction between user and iPad app. The researcher used

Figure 7. Wall-mounted camera view of app interaction.

QuickTime software to reorient and organize the computer-based recordings. Milestone X Protect software automatically integrated wall-mounted video and room microphone audio recordings. The researcher labeled each recording file with the participant ID and camera type (e.g., PD01C for participant 1, computer recording; PD01R for participant 1, room recording) and backed up all recordings on external hard drives.

Observation Field Notes

The observation field notes were a secondary data source used to verify video data and explain occurrences that the video camera may have missed. Although video-recorded interviews should capture as much data as possible (Goldin, 2000), one must acknowledge that because the focus of the camera is limited (Lesh & Lehrer, 2000), one should include other data sources. Other factors might influence user-app interactions (e.g., interview environment) and some occurrences might be outside the frame of the video recordings. The observation field notes thus provided a source of context for the occurrences during the semistructured task-based interviews that might not be included in the video recordings.

During each semistructured task-based interview session, the researcher used a researcher-designed observation protocol (see Appendix B) to record written observation field notes. The observation field notes focused on user interactions with apps and user responses to questions as in other studies based in embodied cognition (e.g., Moyer-Packenham et al., 2015), as well as external influences beyond the frame of the video recording (Lesh & Lehrer, 2000). These observations also guided follow-up questions during the semistructured task-based interview session.

Validity and Reliability

Stebbins (2001) posited that exploratory research is initial research into a new field, and that "early weaknesses in sampling, validity, and generalizability tend to get corrected" through multiple related studies (p. 5). Methodologists recommend a variety of strategies to insure quality in qualitative research, including generating an audit trail, collecting rich data, and insuring researcher reflection (Gall, Gall, & Borg, 2007). This study addressed these areas by creating detailed records of decisions and analyses, using video recordings that allowed for revisiting data and revising interpretations, and analytic memoing to track reflections. Other methodologists suggest building qualitative validity through triangulation of data from several sources, such as the recordings and field notes in this study (Creswell & Plano-Clark, 2011).

However, many qualitative research methodologists disagree with the use of these terms and criteria, and definitions of validity and reliability in qualitative research vary greatly by context (Altheide & Johnson, 2011). Although one can consider both methodological and interpretive rigor of research, many researchers wonder if "there is no such thing as invalidity of data or method if someone can find it to be an accurate reflection of their interpretation of reality" (Lincoln, Lynham, & Guba, 2011, p. 115). Furthermore, reliability, such as intercoder agreement, "has limited meaning in qualitative research" and requires a predetermined coding scheme (Creswell & Plano-Clark, 2011, p. 212), which is inappropriate for exploratory theory development. Therefore, this study focused on trustworthiness, for which a researcher must "make practices visible, and therefore, auditable" (Rolfe, 2006, p. 305). This study used thorough description of the methods, consistent application of methods during data collection and data analysis, and thorough description of results and conclusions that are consistent with the theoretical framework to achieve trustworthiness and provide justifications for the resulting interpretations.

Data Analysis

The theoretical framework, pilot results, and research literature informed the processes used for the qualitative data analysis. Constant comparative data analyses included memoing and eclectic coding for evidence of attributes, affordances, abilities, distance, and relationships among these constructs. These processes were supported by analysis of quantitized qualitative data.

Influences of the Theoretical Framework, Pilot Results, and Research Literature on Data Analysis Process

In exploratory research, it is important for the researcher to acknowledge beliefs based on prior knowledge, such as theory and experience, as these influence the research process (Reiter, 2013). Initial synthesis of the researcher's prior knowledge suggested the need for further investigation of attributes, affordances, abilities, distance, and relationships among these constructs. Theories of embodied cognition and representation posit that interactions with physical representations of mathematics are mathematical thinking and changes in these interactions are mathematical learning (Nemirovsky et al., 2013). Piloting produced data that included these physical interactions with mathematical representations, and thus mathematical thinking and potentially mathematical learning. Presence of the focus constructs and relationships among the constructs as part of these embodied interactions with mathematical representations could provide evidence of their roles in mathematical thinking and learning.

Research literature and preliminary analysis of pilot data suggested the possibility of categories of attributes, affordance-ability relationships, and distance, as well as possible relationships among these constructs. Research literature established that attributes of an agent (or person, user) contribute to abilities used to access affordances offered by combinations of attributes of an environment (or technology tool, app) (Greeno, 1994), and that each affordance is coupled in a relationship with an ability (Chemero, 2003). Researchers also established categories of affordances of virtual manipulatives (Moyer-Packenham & Westenskow, 2013). Affordance access also varies by user, with patterns of affordance access emerging (e.g., Moyer-Packenham & Suh, 2012; Tucker, Moyer-Packenham, Shumway, & Jordan, 2014; Tucker et al., 2015).

Evidence from analysis of pilot data suggested that in addition to affordance categories, attribute categories for users and apps might also exist, including mathematical and technological attributes. For example, technological input requirements (e.g., tapping, dragging, pinching, single-touch vs. multi-touch) and mathematical representations (e.g., number line) varied by app. Interpretation of research literature implies the presence of both technological (e.g., Lao, Heng, Zhang, Ling, & Wang, 2009) and mathematical (e.g., Rick, 2012) attributes. User motor skills related to technological inputs (e.g., Ginsburg et al., 2013) and manifestations of conceptions of mathematical representations (e.g., Moyer-Packenham, Bolyard, & Tucker, 2014) also varied.

Distance stems from the difference in a person's interpretation of the environment and what the environment requires for interaction (Sedig & Liang, 2006). Modification of distance involves matching learner conceptions with tool design or cues (Sedig et al., 2001; Sedig & Liang, 2006). These conceptions and designs are based on attributes, which are characteristics of an environment or person. Therefore, literature implies that distance may relate to attributes. Literature also suggests the existence of multiple distance types (Sedig & Liang, 2006). Preliminary analysis of pilot study data provided evidence of possible novel distance types, including mathematical distance and technological distance, based on user and app attributes. When the app changed mathematical or technological attributes, tasks often became more difficult for users. With additional practice, users often became more adept at completing the tasks. This aligns with the concept of distance modification to support learning (Sedig et al., 2001). However, pilot results suggested that distance modification might vary across contexts. DragonBox Algebra 12+ involves simple single-touch gestures such as tapping and dragging, which is consistent with most tablet apps (Byers & Hadley, 2013), while Motion Math: Zoom includes relatively complex multi-touch gestures (e.g., pinching) that require the coordination of multiple fingers (Kammer, Henkens, Henzen, & Groh, 2013). Preliminary analysis of pilot study data suggested that distance and distance modification varied differently across the two apps, with users struggling more to use complex input gestures relative to simple input gestures.

Exploratory research can lead to evolution of ideas, models, and theories (Reiter, 2013), and iterative constant comparative analysis facilitates this process (Anfara et al., 2002; Merriam, 2009). Together, evidence from the literature and pilot study suggested the possibility of mathematical distance and technological distance, linked to mathematical attributes and technological attributes of users and apps. Furthermore, as attributes also relate to affordances and abilities (e.g., Greeno, 1994), relationships might also exist among affordances, abilities, and distance. However, further exploration of the constructs and relationships in the context of physically embodied interactions with representations of mathematics required collection and analysis of additional data. Thus, while acknowledging influences of the aforementioned conceptions from theory, research literature, and pilot data, the researcher expected that new categories, themes, and relationships would emerge throughout the data analysis process.

Data Analysis Process

This study used qualitative data coding, analysis and interpretation methods. This process focused on the constructs (i.e., attributes, affordances, abilities, and distance) set within the context of physically embodied interactions with mathematical representations. Miles et al. (2013) asserted that "coding *is* analysis…. Coding is a deep reflection about and, thus, deep analysis and interpretation of the data's meanings" (p. 72). Therefore, this study integrated iterative coding, analysis, and interpretation, including eclectic coding techniques integrating analytic memoing as part of constant comparative analysis. Constant comparative analysis involves integrated, iterative data collection and analysis (Glaser & Strauss, 1967), during which "data are compared and categories and their properties emerge or are integrated together" (Anfara et al., 2002, p. 32). Eclectic coding involves beginning with "an array of coding methods for a 'first draft' of coding" followed by "recoding decisions based on the learnings of the experience" (Saldaña, 2013, p. 188). Eclectic coding is appropriate for constant comparative analysis because it involves iterative applications of codes and coding techniques. In this study, coding methods included descriptive coding, provisional coding, magnitude coding, process coding, and theoretical coding, as appropriate for each step of data analysis. Quantitized qualitative analysis (Teddlie & Tashakkori, 2011) involved assigning values to magnitude codes to facilitate data visualization and analysis. Data analysis also included code mapping and networking to organize codes, assemble the framework, and support theory development.

Integral to coding was the writing of analytic memos, which record information and interpretations regarding the data and the analysis process (Miles et al., 2013; Saldaña, 2013). Continuous visual data are key to studies involving embodied cognition, which inherently require a focus on actions. Saldaña noted the complexity of coding visual data, but recommended that a researcher "generate language-based data that *accompany* the visual data" in the form of codeable analytic memos (p. 52). Therefore, the researcher first generated memos and then coded the memos. The content of the memos both informed the coding scheme and was informed by the coding scheme, as content of later memos reflect the shifting foci of the analysis process. For example, many of the initial memos for the first participant focused on attributes, with later stages of analyses of the same data yielding memos and codes focusing on relationships among constructs. However, some of the initial memos of the tenth participant included relationships among constructs, as these were emerging during the time that data for this participant were first analyzed. These methods are appropriate for exploratory qualitative research and theory development, as they facilitate flexible, iterative analysis of multiple types of data to address research questions concerning a variety of constructs (Saldaña, 2013). The researcher used QSR International's NVivo for Mac software (QSR International, 2014) to organize the data coding process.

Analyzing video data and observation field notes for each participant. Each participant's data consisted of the video data and observation field notes from the interview session in which the participant interacted with two different apps and answered follow-up questions pertaining to the interactions. The researcher began each analysis with the video data. First, the researcher wrote a brief analytic memo for each attempt that the participant made to complete a level within the first app, labeled with the attempt and level number for reference purposes (e.g., A1 L1 for attempt 1, level 1). Individual attempts to complete a level served as a manageable grain size for meaningful memos and aggregation across attempts, levels, and participants. This round of memoing provided written documentation of the researcher's examination of the videos to accompany the visual data, generating a written component for coding (Saldaña, 2013). The researcher then transcribed the follow-up questions and responses. Next, the researcher wrote an analytic memo about the participant's interactions with and questions related to the first app. The researcher then repeated these steps to analyze the data on the participant's interactions with the second app and the related questions. Next, the researcher transcribed the summative questions and responses. The researcher then wrote an analytic memo concerning the summative follow-up questions and responses. Next, the researcher wrote an analytic memo concerning all of the video data for the participant. Following this, the researcher wrote an analytic memo for the observation field notes. The researcher then coded the resulting memos, transcripts, and observation field notes, generating additional analytic memos as additional ideas emerged.

Stages of coding and analysis. The constant comparative data analysis process involved eight iterative, interrelated stages of eclectic coding and analysis during and after data collection (see Table 1). Stages were not exclusively linear and overlapped during ongoing data collection, analysis, and interpretation as codes led to categories and emergent themes that informed the development of codes, categories, and themes. The theoretical framework of embodied cognition and representation influenced data coding

Table 1

Data Analysis Stages Used to Address Each Research Question

Data analysis stage	1a: App attributes	1b: User attributes	2: Affordance-ability relationships	3: Distance	4: Relationships
1: Attribute determination	X	X			
2: Attribute organization	X	X			
3: Attribute clustering	X	X			
4: Distance coding				X	
5: Affordance-ability relationship coding			X		
6: Variation coding	X	X	X	X	
7: Construct relationship coding					X
8: Framework development					X

and analysis, as the process involved identifying evidence of the focus constructs set within children's physical interactions with mathematical representations and changes in these interactions. Some stages primarily focused on identifying the constructs within the interactions (e.g., Stage 5: affordance-ability relationship coding), while other stages built on this by examining changes in the constructs as part of changing interactions (e.g., Stage 6: variation coding).

The first stage of the coding process used descriptive coding to analyze the apps and the pilot data to determine an initial list of relevant app attributes and user attributes involved in interactions with each app. Descriptive coding involves coding for topics of the data, facilitating the production of a categorized inventory of the data’s contents (Saldaña, 2013). The second stage consisted of organizing the relevant attributes into categories using code mapping. Code mapping involves iterations of organizing codes into comprehensible categories for further coding (Saldaña, 2013). This stage also involved the creation of bins, which were groups of consecutive levels within each app that contained similar content and tasks, based on the attributes present. The third stage involved clustering attributes to form codes that could apply across multiple levels of interactions with the app. This stage began with an analysis of the apps and the pilot participants’ app interaction data. The codes were revised as they were applied to the memos from the video data and observation field notes for the first three participants’ interactions with the apps.

The fourth stage of the coding process involved development of distance codes from app analysis, pilot data analysis, and analysis of the data from the user-app interactions and follow-up questions and responses for the first three participants. Distance codes emerged based on analysis of the attribute clusters from stage three, and were thus developed before affordance-ability relationship codes. Distance coding also involved magnitude codes, which are used to indicate the degree or intensity of a coded construct (Saldaña, 2013). Distance magnitude codes were applied to data from all participants. The researcher used a four-step process to quantitize distance magnitude codes (see Appendix C), which facilitated data visualization and further analysis (Sandelowski, 2001; Teddlie & Tashakkori, 2011). First, the researcher assigned values to each distance magnitude code, ranging from 1 to 4, where 1 represented low attribute cluster alignment and 4 represented high attribute cluster alignment. Next, these values were used to determine overall attempt values for mathematical and technological attribute categories. The resulting overall attempt values were scaled to a four-point distance scale, with 1 representing the greatest amount of distance and 4 representing the least amount of distance. Finally, the distance scale values were paired to form the (mathematics, technology) distance score.

The fifth stage of the coding process involved development of affordance-ability relationship codes from app analysis, pilot data analysis, and analysis of the data from user-app interactions and follow-up questions and responses for the first three participants. Moyer-Packenham and Westenskow's (2013) categories of affordances of virtual manipulatives served as the basis for provisional affordance-ability relationship codes. Provisional coding involves using predetermined categories as a starting point for coding while allowing for emergent constructs (Saldaña, 2013). The resulting affordance-ability relationship codes were applied to data from the first six participants, with specific examples applied to data from the other four participants as focus examples emerged.

The sixth stage of the coding process involved developing variation codes for each construct based on analysis of the pilot data and emergent themes identified during analysis of the data from the first six participants. Revision of variation codes continued during application to data from the remaining four participants. This stage involved process coding using gerunds to indicate action and change over time (Charmaz, 2011; Saldaña, 2013), descriptive coding to characterize variants of constructs, and visual analysis of quantitized data. R software (R Core Team, 2014) was used to generate novel rhombus plots to visualize quantitized distance data (see Figure 8 for annotated example).

These rhombus plots arrange the data by (mathematical, technological) distance score. The degree of mathematical distance decreases from left to right (i.e., the mathematical distance value increases from 1 to 4), while the degree of technological distance decreases from bottom to top (i.e., the technological distance value increases from 1 to 4). Placement at the top of the rhombus plot indicates a low degree of both types of distance, while placement at the bottom of the rhombus plot indicates a high degree of both types of distance. Placement at the right of the rhombus plot indicates a

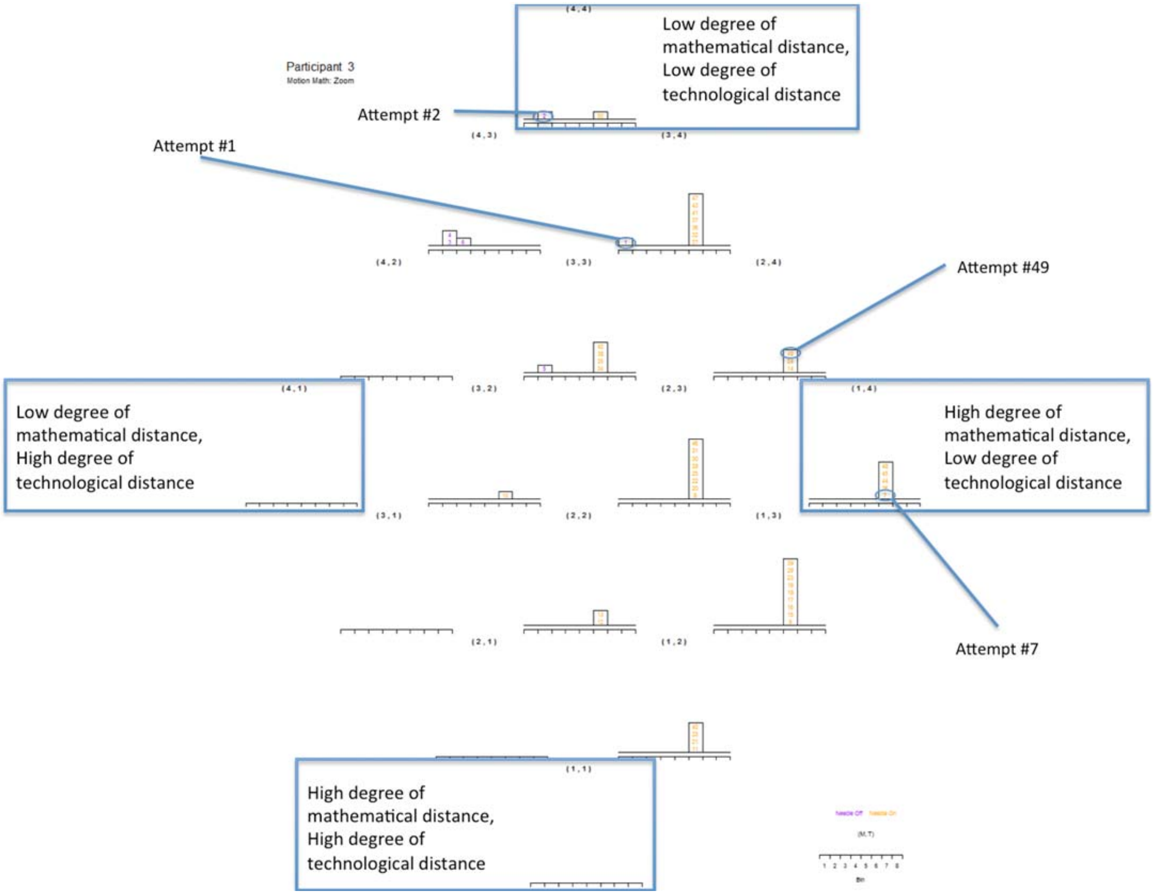

Figure 8. Annotated rhombus plot for Participant 3's interactions with Motion Math: Zoom. (M,T) indicates (mathematical, technological) distance value. Bin indicates level group.

high degree of mathematical distance and a low degree of technological distance, while placement at the left of the plot indicates a low degree of mathematical distance and a high degree of technological distance. Within each possible (mathematical, technological) distance score, placement on the axis indicates the bin (i.e., group of consecutive levels with similar content) from the app and numbers indicate each attempt to complete a level (e.g., 1, 2, … 49). Additionally, rhombus plots for Motion Math: Zoom interactions use different number colors to indicate attempts in which the needle function (i.e., timer) was active or inactive. In Figure 8, Attempt 1 was on a level in Bin 1 with the needle inactive, with a (mathematical, technological) distance value of (3, 4). Attempt 2 was on a level in Bin 2 with the needle inactive, with a (mathematical, technological) distance value of (4, 4). Later in the sequence, Attempt 7 was on a level in Bin 6 with the needle active, with a (mathematical, technological) distance value of (1,4). The final point in the sequence was Attempt 49, which was on a level in Bin 6 with the needle active, with a (mathematical, technological) distance value of (2,4).

The seventh stage of the coding process involved developing codes concerning relationships among constructs based on analysis of data from all participants. This stage integrated process coding, descriptive coding, and visual analysis of quantitized distance data to identify potential connections among constructs. The eighth stage of the analysis process involved analyzing the coding structure and the data using theoretical coding and networking, which supports the development of theory. Theoretical coding involves synthesizing and integrating the prior coding and analysis to develop theory (Saldaña, 2013). Networking facilitates this process, as it is used to indicate how the categories or constructs “interact and interplay in complex pathways to suggest interrelationship” (Saldaña, 2013, p. 252). In this context, theoretical coding and networking supported theory development in the form of a conceptual framework that integrates the constructs and their relationships.

CHAPTER IV

RESULTS

The purpose of this study was to conceptualize the relationships among attributes, affordances, abilities, and distance in a framework that describes the nature of children's interactions with technology to learn mathematics, here set within fifth-grade children's interactions with mathematics virtual manipulative iPad apps. The overarching research question guiding this study was: What evidence of attributes, affordances, abilities, and distance is present in the context of fifth graders' interactions with mathematics virtual manipulative iPad apps? Subquestions focused on: (1a) evidence of app attributes, (1b) evidence of user attributes, (2) evidence of affordance-ability relationships, (3) evidence of distance, and (4) evidence of relationships among attributes, affordances, abilities, and distance. The results presented in this chapter are organized by research questions and based on analysis of videos of user-app interactions and follow-up questions from 10 participants. These user-app interactions involved physically embodied interactions with representations of mathematics. The following sections integrate results and interpretation because the interrelated data coding, analysis, and interpretation generated codes and categories that led to the emergence of themes that informed development of further codes, categories, and themes.

The first section presents results related to attributes, including: (a) presence and categorization, (b) alignment, and (c) modification. The second section presents results related to affordance-ability relationships, including: (a) presence and categorization, (b) variations, and (c) interrelationships. The third section presents results related to distance, including: (a) presence and categorization, (b) change, and (c) interactions among types. The fourth section presents results related to relationships among constructs, including relationships between: (a) attributes and affordance-ability relationships, (b) attributes and distance, and (c) distance and affordance-ability relationships. These relationships led to the development of the modification of attributes, affordances, abilities, and distance for learning conceptual framework.

Research Question 1: Presence and Modification of Attributes

The first research question focused on the evidence of attributes in fifth graders' interactions with mathematics virtual manipulative iPad apps, with subquestions pertaining to app attributes and user attributes. As discussed in the Chapter III (Methods), the researcher addressed this research question by analyzing the two apps, video data of user-app interactions and follow-up questions, and observation field notes during the data analysis stages that included determining attributes, organizing attributes, clustering attributes, and analyzing emergent variations of attributes. The first analysis examined what attributes were present, as indicated by their relevance to user-app interactions. Data coding characterized and categorized attributes relevant for user-app interactions. Interpretation of categories and subcategories related to app attributes and user attributes led to the identification of emergent themes. Emergent themes included attribute categorization, attribute alignment, and attribute modification.

App Attributes

Analysis provided evidence of the presence of app attributes. Interpretation of this evidence revealed categories of app attributes: mathematical, technological, and structural. Table 2 shows examples of codes applied during app attribute analysis, organized by resulting categories and subcategories.

As seen in Table 2, development and interpretation of app attribute codes (e.g., *symbolic notation*) used to label app attribute descriptions (e.g., Arabic notation) informed categories (e.g., mathematical), which were refined to include subcategories (e.g., mathematical: representation). Mathematical attributes were characteristics pertaining to representations of mathematical content, including subcategories of content and representation. Mathematical content related to the conceptual underpinnings of the subject-matter information (e.g., decimals), as externalized via mathematical representations (e.g., number line). Technological attributes were characteristics pertaining to physically embodied interactions with the app, including subcategories of input range and input complexity. Input range related to the scope of gestures accepted by the app for a given function (e.g., tap to indicate selection), while input complexity related to intricacy of required gestures (e.g., coordination of multiple fingers for pinch input). Structural attributes were characteristics pertaining to nonmathematical presentation features of apps, including subcategories of feedback, context, and scaffolding. Feedback was responses of the app to the user input (e.g., symbol denoting a response as correct or incorrect). Context included aspects such as the pedagogical setting for the content (e.g., game involving advancing levels upon meeting certain criteria vs. free play) and the purpose of completing tasks in the app (e.g., earning points). Scaffolds were supports for completing tasks (e.g., a button that presents a worked example).

Table 2

App Attribute Categories with Examples from Motion Math: Zoom and DragonBox Algebra 12+[1]

Motion Math: Zoom	DragonBox Algebra 12+
Mathematical: Content	
Decimals: Introduction of tenths (L12)	*Additive identity:* X + 0 + Y, clear swirl/vortex that represents 0 to make X + Y (L1:01)
Negative numbers: Introduction of negative integers (L9)	*Additive inverse* • *Internal*[2]*:* 1 + -1 = 0 (L1:03) • *External*[3]*:* Where 1 + X = Y; possible to add 1 from outside equation space, change 1 to -1, add to equation space to make 1 + -1 + X = Y + -1 (L1:16)
Range: Choose where to zoom to ones to find 45 when number line shows 0-100, intervals of 10 (i.e., in which range is 45 located?) (L7)	*Additive equality:* Where X + -1 = Y; add 1 from outside equation space to make X + -1 + 1 = Y + 1) (L1:09)
Estimation: Choose where to zoom to tenths to find 2.8 when number line shows 1 to 4, intervals of 1 (i.e., approximately where is 2.8 located on the number line?) (L14)	*Multiplicative inverse:* • *Internal:* Divide X/X to make 1 (L2:01) • *External:* Where XY = 3, divide each term by Y from outside equation space to make XY/Y = 3/Y (L2:11)
Comparison: Find 12 when number line shows 5-8, intervals of 1 (L3)	*Multiplicative identity:* Where 1X is present (as one-dot connected to X), tap coefficient 1 to make X (L2:05)
Intervals: Choose which interval to travel by to reach 1,035 when number line shows 46 to 57, intervals of 1 (L18)	*Equality:* Two-sided equation space with instructions to “Get me [box] alone on ONE side” (L1:05)
Magnitude: Find 0.006 when number line shows 0.1-0.4, intervals of 0.1 (L19)	*Reverse order of operations:* Requirement of adding before dividing to completely simplify the equation (L2:13)
Mathematical: Representation	
Number line: Find 5 when number line shows 0-3, intervals of 1 (L1)	*Pictorial variables:* Picture tiles as variables (L1:01)
Symbolic notation: Arabic numerals (L1)	*Operation symbols:* Operation symbols (e.g., +, =) (L2:19)
Technological: Input range	
Multi-touch: Use multiple fingers to perform pinch gesture to zoom, changing intervals from tens to ones to find 15 within the range 10-20 (L1)	*Single-touch:* Tap only one swirl/vortex at a time to apply additive identity property (L1:02)
Input recognition: Horizontal pinch gesture recognized as zooming, but vertical or nearly vertical pinching is not recognized (L15)	*Input recognition:* Tap coefficient 1 to apply multiplicative identity property (L2:05)

(*table continues*)

Motion Math: Zoom	DragonBox Algebra 12+
Technological: Input Complexity	
Swipe: Swipe finger from right to left to increase along the number line or swipe finger from left to right to decrease along the number line (L5)	*Tap:* Tap swirl/vortex to apply additive identity property (L1:02)
Pinch (zoom): Using two fingers to pinch, a) bring together to increase intervals (zoom out), or b) move apart to decrease intervals (zoom in) (L14)	*Drag:* Drag and drop a tile onto the corresponding inverse tile to perform additive inverse property (L1:03)
Structural: Feedback	
Points: Earn ten points for correctly completing a task within a level (L9)	*Allow/Disallow move:* Disallow drag and drop a tile onto the incorrect tile when attempting to combine (e.g., X + Y disallowed) (L1:04)
Animations: Flea breakdances to acknowledge completion of level (L15)	*Stars:* Earn stars for successful level completion, simplification, and using an efficient number of moves to complete the level (L2:03)
Structural: Context	
Advance levels: Upon completion of level with required speed and accuracy, app unlocks additional levels (L6)	*Advance level:* Successful completion of this level unlocks the next level (L2:02)
Needle: From menu or app challenge prompt, needle is explicit timer that pops bubble to end level attempt if task completion is too slow	*Dragon growth:* Dragon "grows" through stages (new dragon for each chapter) (L1:01)
Structural: Scaffolding	
Hints: App prompts "want a hint?" if task completion is slow (L7)	*Solution video:* After restarting the level, a light bulb appears. Tapping the light bulb offers a solution video that demonstrates the solution level. (L1:15)
Fingers: App displays fingers to indicate motion and direction (e.g., drag left, zoom out) if task completion is slow (L12)	*Highlights:* White highlight appears in a variable tile to indicate that it can be combined with a tile the user is currently touching to perform the additive inverse property. (L2:04)

[1] Attributes for all levels of Motion Math: Zoom, where L indicates Level, and levels in first three chapters (1:01-3:20) of DragonBox Algebra 12+, where L indicates Chapter and Level (e.g., L1:03 is Chapter 1, Level 3). Chapters are groups of levels in DragonBox Algebra 12+.

[2] Internal refers to steps involving only tiles already present in the equation without moving the variable from one side of the equation to the other side of the equation

[3] External refers to steps involving bringing in variables from outside the equation

Most app attributes remained consistent within a level for each app, but often differed across levels. For example, in DragonBox Algebra 12+, attributes of Level 1:01 included: (a) the additive identity property, (b) recognition of tapping and dragging, and (c) sounds, animations, and earning a star. Attributes of Level 3:01 were different and included: (a) the additive inverse property, (b) recognition of dragging, and (c) sounds, animations, and earning multiple stars. It was also possible to control some structural attributes independently of the app level (e.g., needle timer in Motion Math: Zoom; solution video in DragonBox Algebra 12+). It was only possible to control the mathematical attributes and technological attributes of these apps by choosing a level involving those attributes. The analysis of the app attributes shows that one can organize app attributes using consistent categories across different apps. Attribute categories (e.g., mathematical, technological, structural) and subcategories (e.g., content, representation, flexibility) remained constant as specific attributes varied (e.g., mathematical content: decimals in Motion Math: Zoom vs. additive identity in DragonBox Algebra 12+).

User Attributes

Analysis provided evidence of the presence of user attributes. Interpretation of this evidence revealed three categories of user attributes that manifested in relation to requirements for interacting with the apps: mathematical, technological, and personal. Table 3 shows examples of codes applied during user attribute analysis, organized by resulting categories and subcategories.

As seen in Table 3, development and interpretation of user attribute codes (e.g., *symbolic notation*) used to label memo excerpts (e.g., “point 93”) informed categories

Table 3

User Attribute Categories with Examples from Interactions with Motion Math: Zoom and DragonBox Algebra 12+[1]

Motion Math: Zoom	DragonBox Algebra 12+
Mathematical: Content	
Decimals: Tries 1.00 for 0.10. At 0.10, zooms in to hundredths (P10 A33 L16N)	*Additive identity:* "You had to match it up with the same ones. Identity property is the same." (P03 DBFT)
Negative numbers: Smooth navigation for negative numbers. (P04 A13 L9)	*Additive inverse:* Tries to combine many incorrect "positive + positive" and "negative + negative" variables (P01 A12 L1:12)
Range: Zooms at 1.40 for 1.39 (precise zoom interval placement), but other times zooms anywhere within the range (P04 A7 L16)	*Additive equality:* Missing additive equality--repeatedly tries to interrupt. (P10 A13 L1:12)
Estimation: "I would go past one and estimate about how far past the bee would I zoom in to get onto the little ants." (P10 ZFT)	*Multiplicative inverse:* Multiplicative inverse [focus] leads to forgetting need for additive inverse and correct order (reverse order of operations)? (P08 A43 L1:21)
Comparison: Struggles with decimal magnitude and comparison. (P10 A44 L16N)	*Multiplicative identity:* Taps one-dot [coefficient] to apply multiplicative identity. (P09 A32 L2:05
Intervals: To 4.3, travels by tenths as inefficient travel interval (P06 A26 L14N)	*Equality:* As if seeing two separate equations (P07 A19 L1:14)
Magnitude: To find 1.47 [from 1.82], too slow by hundredths and needle pops. Not confident with comparison and magnitude for tenths and hundredths greater than 1 (P07 A21 L15N)	*Reverse order of operations:* Again starts with division instead of addition. (P09 A45 L2:13)
Mathematical: Representation	
Number line: "It was really not that hard… to find the numbers along the number line." (P04 ZFT)	*Pictorial variables:* "It was really fun to match up [pictures] but it was a little confusing" (P03 DBFT)
Symbolic notation: "It was easy when it was a whole number but when it's like 'point 93' and it's in between it's hard." (P10 ZFT)	*Operation symbols:* Addition symbols appear (P04 A49 L2:19)
Mathematical: Flexibility	
Transfer: Travels by 10s (starting interval) when hundreds or tens/hundreds combo might be more efficient. (Not transferring zoom [as applied to intervals and ranges] to new situation yet) (P09 A5 L5)	*Transfer:* Repeating same mistake—does not apply division as shown in prior level. (P06 A37 L2:12)
Perception: "I had to focus on how many hundredths were in a tenth and how many tenths in one, and how many ones in ten and how many tens are in a hundred." (P07 ZFT)	*Perception:* "There was absolutely no math in there…. I don't know why you gave this game to me; it's not a math game. I thought this was a study on math games." (P08 DBFT)

(*Table continues*)

Motion Math: Zoom	DragonBox Algebra 12+
Technological: Motor Skills	
Coordination: At times, zoom out fingers overlap. (P07 A19 L15N)	*Coordination:* Direct combo for box side but scaffold for non-box (attempted for second direct combo on box side but missed) (P09 A22 L1:15)
Technological: Input familiarity	
Input awareness: Needed repeat scaffold (with voice) for zoom to find 15 after repeated L-R movement w/ taps. (P06 A1 L1)	*Input awareness:* During demo, tries to drag instead of tap first. (P06 A16 L1:16)
Personal	
Affect: “It was hard but fun at the same time.” (P05 ZFT)	*Affect:* “Pretty fun…. I like brain games and figuring things out.” (P07 DBFT)
Goals: “I was trying to get to challenging things to see what it would be like.” (P05 ZFT)	*Goals:* “I play it so I can get one or two stars.” (P01 DBFT)
Persistence: “Finally!” [upon completing L15 on the 44th attempt] (P03 A50 L15N)	*Persistence:* “The second time I got stuck I watched part of the [solution] and then exited out so I could figure out the rest.” (P07 DBFT)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level, N indicates presence of needle. ZFT is Motion Math: Zoom Follow-Up Questions Transcript. DBFT indicates DragonBox Algebra 12+ Follow-Up Questions Transcript. Quotation marks indicate direct quotes from participants. Brackets indicate clarifications.

(e.g., mathematical), which were refined to include subcategories (e.g., mathematical: representation). User mathematical attributes pertained to representation of mathematical content, with subcategories of content, representation, and flexibility. Content and representation were similar to the corresponding categories of app attributes (e.g., content: decimals; representation: number line). User flexibility referred to how the user transferred across representations and situations (e.g., applying understanding of the quantities contained within the range of 0.1-0.2 to the range of 1.1-1.2). User technological attributes also pertained to physically embodied aspects of interactions with the app, but with different subcategories than app technological attributes. Motor skills referred to the facility with which a user performed the relevant physical actions (e.g., coordination used to control the pinching zoom input gesture), whereas input familiarity referred to how conversant a user was in a given input (e.g., awareness of tapping as indicator of intended answer). User personal attributes were characteristics of the user's personality that related to how the user interacted with the app. Personal attributes included affect (e.g., enjoyment), persistence (e.g., repeated attempts), and goals (e.g., seeking a challenge).

User attributes are representative of those that participants displayed in response to requirements for interacting with the apps and have some similar categories to app attributes. For example, interacting with Level 15 (needle active) in Motion Math: Zoom involved: (a) awareness of and understanding of magnitude using tenths and hundredths on the number line (mathematical attributes), (b) perception of and performance of input gestures of swiping to move the number line and zooming to change intervals (technological attributes), and (c) goals and persistence (personal attributes). Many attributes were evident in multiple forms (e.g., coordination to control pinch and drag inputs). The analysis of user attributes shows that one can organize user attributes using consistent categories. Attribute categories (e.g., mathematical, technological, personal) and subcategories (e.g., content, representation, flexibility) remained constant as specific attributes varied (e.g., mathematical content: decimals in Motion Math: Zoom vs. additive identity in DragonBox Algebra 12+). The analysis also shows that one can use similar categories to organize app attributes and user attributes.

Attribute Alignment

Analysis revealed an emergent theme of attribute alignment, supported by evidence indicating that alignment of clusters of mathematical attributes or technological attributes varied. As discussed in the Methods chapter, the researcher grouped related attributes into clusters that could apply across multiple levels of interactions with an app (e.g., “swipe input” included iterations of coordination and input awareness). Table 4 shows examples of codes applied during analysis concerning aligned and unaligned attribute clusters from participants’ interactions with Motion Math: Zoom.

Every participant’s interactions showed evidence of varying alignment for each attribute cluster related to Motion Math: Zoom. Participants often showed greater alignment of mathematical attribute clusters when tasks featured whole numbers (e.g., 1, not 1.00) than when tasks featured hundredths (e.g., 0.83). Unaligned attribute clusters

Table 4

Aligned and Unaligned Attribute Clusters from Interactions with Motion Math: Zoom

Cluster	Aligned	Unaligned
Mathematical attributes		
Comparison: Navigation	Pops [bubble] while [number line] still moving when [target number] should be nearby (P01 A2 L2)	For 0.13, again seemed confused as to where it would be placed and used tenths until limited around 1.0 (P03 A17 L15N)
Comparison: Target placement	Aligning bubble to target (P04 A2 L2)	Tries to place at 0.4 for 0.04 (P06 A8 L15N)
Magnitude: Within-interval travel choice	Uses most appropriate travel interval (only one mixed and self-corrected partway through) (P02 A7 L6)	From 500 to 784, seems to hesitate among ones, tens, hundreds, and settles on ones around 700 (P06 A5 L5)
Magnitude: Between-interval travel choice	Immediately to ideal zoom [range of 0-0.1] for 0.05 (P09 A11 L15N)	For 1.81 from 1.00, zoomed in to hundredths at 1.00. (P05 A5 L15)
Technological attributes		
Swipe input	Smooth navigation via swiping (P01 A2 L2)	One brief misread swipe. (P06 A2 L2)
Zoom input	Controlled zooming when close to interval level (P04 A8 L18)	Struggled to zoom using one hand diagonal (P02 A23 L15N)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level, N indicates presence of needle. Brackets indicate clarifications.

were often evident when the task required finding numbers between 1.00 and 2.00. Eight participants showed evidence of unaligned technological attribute clusters, struggling to efficiently apply the zoom input gesture during multiple consecutive attempts. However, other participants aligned these attributes, such as Participant 4, who honed the pinch gesture to precisely control the place and degree of zooming. Table 5 shows examples of codes applied during analysis concerning aligned and unaligned attributes from interactions with DragonBox Algebra 12+.

Every participant's interactions also showed evidence of varying alignment for each attribute cluster related to DragonBox Algebra 12+. All participants showed evidence of unaligned mathematical attribute clusters, such as when attempting mathematically incorrect moves. Unaligned technological attribute clusters were also evident. For example each of the nine participants who reached Level 2:05, which

Table 5

Aligned and Unaligned Attribute Clusters from Interactions with DragonBox Algebra 12+

Cluster	Aligned	Unaligned
Mathematical attributes		
Efficiency: moves accepted by app	Every step correct (P03 A22 L1:18)	63 recognized moves! (P01 A38 L2:03)
Elegance: simplification or leftovers	Completes both [sides] before sweep begins (P06 A11 L1:11)	Immediately clears swirl on box (right) side, resulting in leftovers. (P05 A7 L1:07)
Accuracy: (dis)allowed mathematics	Immediately divides correctly (P06 A26 L2:03)	Twice attempted to add instead of divide (P05 A54 L2:20)
Technological attributes		
Performance of input gestures	Reproduces demo with no difficulty of input (P01 A1 L1:01)	Misses drag/drop again (P03 A6 L1:06)
Choice of correct input gesture	Watches power demo, then duplicates. No longer dragging box. (P10 A3 L1:03)	Drag/tap mix (P10 A39 L2:13)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates chapter and level. Brackets indicate clarifications.

introduced tapping a coefficient of one to apply the multiplicative identity property, at times attempted to drag instead of tap when trying to apply the property on subsequent levels. Attribute cluster alignment and misalignment provided evidence of connections between user attributes and app attributes throughout participants' interactions with each app. Attribute cluster alignment was not static, nor was it the same for every participant.

Proactive Versus Reactive Attribute Modification

Evidence of changing attributes and changing attribute alignment led to the emergent theme that users and apps frequently modified attributes. Specifically, this attribute modification was either reactive or proactive. Table 6 shows examples of codes applied during analysis of attribute modification.

As seen in Table 6, interpretation of evidence of changing attributes (e.g., chose to turn off needle) and attribute alignment change (e.g., repeat level until complete) during participants' physically embodied interactions with the apps informed the development of the theme of attribute modification, which was refined to differentiate between reactive and proactive attribute modification. In *reactive attribute modification*, apps modified app attributes and in response, users applied and modified user attributes. When user and app attributes aligned and the user successfully completed tasks, the app responded by modifying app attributes. The user then attempted to align the relevant attributes by applying and modifying user attributes, continuing the cycle. In *proactive attribute modification*, apps modified app attributes and users applied and modified user attributes, but users also modified app attributes.

Table 6

Examples of Proactive and Reactive Attribute Modification

Modification	Examples
Motion Math: Zoom	
Proactive	Chose level (4 participants)
	• Returns to menu instead of accepting jump to 15 challenge w/needle (did not wait to interpret prompt). (P05 A13 L7)
	Chose to turn needle on or off (1)
	• At menu, pauses, turns on needle… (P08 A4 L15N)
	• Turns off needle at menu, returns to 15 (P08 A6 L15N)
Reactive	Skip from L6 to L15, turn on needle (7)
	• Accepts app-prompted 15N [from L6] (P09 A8 L15N)
	Repeat level (10)
	• App suggested repeat (P02 A3 L2)
	Move to next level (9)
	• Accepts 16 (P10 A32 L16N)
DragonBox Algebra 12+	
Proactive	Choose level (3)
	• Purposefully chose different level (P02 A37 L2:10)
	Watch solution (6)
	• Solution (part--did not watch full solution) (P01 A38 L2:03)
Reactive	Move to next level (10)
	• P05: A25 L2:01… A26 L2:02… A27 L2:03…
	Repeat level until complete (10)
	• P04: A42 L2:16 Restart… A43 L2:16 Restart… A44 L2:16 Restart… A45 L2:16 Restart… A46 L2:16 [completes]

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates attempt number, L indicates level or chapter and level. Brackets indicate clarifications.

Reactive attribute modification was more common than proactive attribute modification. For example, during Level 15, the user had to apply understandings of comparison and magnitude involving decimals to the hundredths place. Every participant repeated Level 15 multiple times, but only Participants 3, 4, 6, and 10 modified user attributes enough for the app to permit advancement to Level 16. Interactions involving DragonBox Algebra 12+ also provided evidence of reactive attribute modification, as every participant usually followed the app prompt to the next level and its content rather than choosing a different level.

Proactive attribute modification was relatively rare. While interacting with Motion Math: Zoom, four participants (4, 5, 6, 8) proactively chose levels (i.e., modified app mathematical content attribute), while only Participant 8 proactively controlled the needle timer (i.e., modified structural context attribute). While interacting with DragonBox Algebra 12+, six participants proactively used the Solution feature to demonstrate how to complete a level (i.e., activated structural scaffolding attribute). Of the three participants (1, 2, 10) who proactively chose levels, two (2, 10) built back through prior levels after encountering difficulty. Participant 2 explained this, saying “my parents tell me if something gets too hard stop on that and then go back and you’ll get better ideas.” However, many participants appeared unaware of the potential to proactively modify app attributes. Participant 2 wanted to “choose what [math] it would have you do” in Motion Math: Zoom, which was possible via the menu screen. Participant 3 suggested the addition of “hints or clues to help” in DragonBox Algebra 12+, which Participant 1 recognized as accessible via the Solution light bulb “because all light bulbs do that. A light bulb above your head gives you an idea.”

In summary, there was evidence that both users and apps have attributes. Categories of app attributes were the same across apps, and categories of user attributes were the same across users, though attributes within the categories and specific manifestations of the attributes varied. Alignment of attributes or clusters of attributes during user-app interactions provided evidence of relationships among categories of app attributes and user attributes. Attribute alignment was not static, nor was it identical for all participants. Attribute modification was a frequent occurrence, and reactive attribute modification was more common than proactive attribute modification.

Research Question 2: Presence and Variance of Affordance-Ability Relationships

The second research question concerned evidence of affordance-ability relationships in fifth graders' interactions with mathematics virtual manipulative iPad apps. As discussed in the Methods chapter, the researcher addressed this research question by analyzing the two apps, video data of user-app interactions and follow-up questions, and observation field notes during data analysis stages that included identifying examples of affordance-ability relationships and analyzing emergent variations related to affordance-ability relationships. Initial data analysis examined whether affordance-ability relationships were present, as indicated by accession of the affordances during participants' physically embodied interactions with the app and their comments related to affordance access. Data coding and analysis provided evidence that affordance-ability relationships were present in the user-app interactions and aligned with the categories of affordances of virtual manipulatives as defined by Moyer-Packenham and Westenskow's (2013). Further coding and analysis focused on specific types of affordance-ability relationships, and interpretation revealed emergent themes of: (a) variations in affordance-ability relationships and (b) interrelationships among affordance-ability relationships.

Presence of Affordance-Ability Relationships

Analysis of the data indicated that affordance-ability relationships were present during participants’ interactions with mathematics virtual manipulative iPad apps. The main categories of these relationships aligned with the five categories of affordances of virtual manipulatives as defined by Moyer-Packenham and Westenskow (2013): simultaneous linking, efficient precision, focused constraint, motivation, and creative variation. Table 7 shows examples of codes applied during analysis concerning affordance-ability relationships.

Some affordance-ability relationships were always evident as participants completed tasks with a particular app. For example, attempting tasks in Motion Math: Zoom always required accessing the simultaneous linking of actions and mathematical representations on the number line. Efficient precision was widespread, including every participant accessing iterations of guided tile placement during interactions with DragonBox Algebra 12+. Focused constraint was also common, as both apps limited the mathematical interactions to specific content in any given level. Evidence of accessing the affordance of motivation often appeared in participants’ comments (e.g., Participant 1: “wanna punch something”). Creative variation was relatively rare, and although some participants attempted to find innovative solution strategies (e.g., combining planning and multiple visible intervals to navigate the number line in Motion Math: Zoom), participants rarely iterated these strategies.

Variation in Affordance-Ability Relationships

Analysis indicated that affordance-ability relationships were not identical across

Table 7

Examples of Affordance-Ability Relationships

Motion Math: Zoom	DragonBox Algebra 12+
Simultaneous linking	
Actions + changes in number line "I pick an area between like 1 and 2 or 3 and 4 then zoom in to the area close enough [to find the number]." (P08 ZFT)	*Actions + changes in equation* "The one I was just playing on, it would let you switch the others to the other side." (P09 DBFT)
Efficient precision	
Timed, needle Tries 0.4 for 0.04, then zooms in immediately (non-ideal). Popped by needle. (P07 A15 L15N)	*Guided placement: highlights* Tries to drag variable across but stopped by app (focused constraint) then sees scaffold of lit/highlighted target combo (uses correctly) (P04 A5 L1:05)
Unlock next level if conditions met App suggested repeat (P02 A3 L2)	*Guided placement: squares* Used direct combo on box side, then scaffolded [square] placement on non-box (even though passed through yellow highlight on the way) (P10 A10 L1:10)
Planning Now planning [by zooming] out before next bubble arises (P09 A18 L15N)	
Multiple visible intervals To find 450 from 357, zooms to multiple visible intervals (1/10); stays at this to 402, 448, 500 (P10 A5 L5)	
Focused constraint	
Navigation restrictions: swipe left-right Repeatedly tries to swipe past 1.0 (P01 A11 L15N)	*Mathematics restrictions: correct* Tries to combine two terms with same denominators but different numerators (allowed later, but not here) (P02 A31 L2:11)
Navigation restrictions: zoom in-out Briefly stuck on a task when attempting to zoom further when not allowed (P04 A14 L18)	*Mathematics restrictions: incorrect* Repeated X + 0 = 0 attempts (P07 A18 L1:14)
Motivation	
Positive "I was really motivated to get to the challenge room and that got me really excited about playing it." (P05 SFT)	*Positive* "I thought that it was pretty fun and hooking. You get hooked to it…. Trying to get the dragon as big as you can." (P05 DBFT)
Negative "The whole thing is frustrating. Makes you wanna punch something or throw something at someone." (P01 ZFT)	*Negative* "Could make you wanna punch your screen or throw it at the wall because I can't… figure out which is which." (P01 DBFT)
Creative variation	
Multiple visible intervals To find 450 from 357, zooms to multiple visible intervals (1/10) [for the first time]; stays at this to 402, 448, 500…. Using multiple visible intervals was not the most efficient way to travel here, but worked nonetheless. (P10 A5 L5)	*Guess-and-check* Eventually complete correction after guess-and-checks. Many extra moves. (P09 A65 L3:06)
Planning Some planning evident already as moving while number falls into place. (No need to plan yet without zooming or time here) (P07 A2 L2)	*Systematic trials* Extended, systematic attempts to try nearly everything (with many math mistakes, extra moves, etc.) (P02 A36 L2:13)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level. DBFT indicates DragonBox Algebra 12+ Follow-Up Questions Transcript. ZFT indicates Motion Math: Zoom Follow-Up Questions Transcript. SFT indicates Summative Follow-Up Questions Transcription. Quotation marks indicate direct quotes from participants. Brackets indicate clarifications.

all participants and situations, leading to the theme of variation in affordance-ability relationships. Affordance access varied within and between participants by approach or degree. Examining accession of versions of affordances of efficient precision and motivation provided evidence of these variations. Within each of these examples, further variations were evident, including consistency, change, and outcome.

Variations in accession of efficient precision. Analysis indicated that accession of efficient precision related to guided placement to combine tiles using the additive equality property and the additive inverse property in DragonBox Algebra 12+ varied by approach. Consistency of approach to accessing this affordance varied and had favorable or unfavorable outcomes even when participants adopted the same approach. Table 8 shows codes applied during analysis concerning variations of efficient precision affordance-ability relationships.

As seen in Table 8, accession of efficient precision related to guided placement when applying the additive equality property and the additive inverse property could vary by approach, consistency, and outcome. For example, Participant 9 accessed all three ways DragonBox Algebra 12+ allowed one to perform the additive equality property and the additive inverse property in a given situation (see Appendix D, Figures D1-D5). This included: (a) using direct combinations whenever possible (i.e., simultaneous use of the additive inverse property and the additive equality property on both sides of the equation), (b) always using separate steps (i.e., complete the additive equality property before beginning the additive inverse property), or (c) using direct combination on the first side and separate steps on the other side. While completing the additive equality

Table 8

Variations of Guided Placement Efficient Precision Affordance-Ability Relationships in DragonBox Algebra 12+

Variation	Example
Variant: Approach	P09 Combined additive inverse and additive equality when possible • Purposeful direct combos. (A20 L1:14) Combined on first side, separate steps on second side • Direct combo first (non-box) then scaffolded (box) (A10 L1:10) Separate steps on both sides • Does not use direct combo… was [possible] on each side, each time (A27 L1:20)
Consistency: Consistent	Regular approach to affordance access: P10 • Direct combos for box side… and uses scaffolded placements for non-box side. (A19 L1:17) • Direct combo on non-box, scaffolded on box (direct possible). (A21 L1:19) • Direct combo on box side… scaffolded on non-box. (A22 L1:19) • Direct combo for box side… scaffolded for non-box side. (A31 L2:08)
Consistency: Inconsistent	Irregular approach to affordance access: P09 • Consistent in non-use of direct combos [so far] (A12 L1:11) • Accidental direct combo (does not repeat) (A18 L1:14) • Purposeful direct combos (A20 L1:14) • Direct combo for box side but scaffold for non-box (attempted for second direct combo on box side but missed) (A22 L1:15) • No direct combo for box side. (A23 L1:16) • First as direct combo, second barely, third not… from gesture precision (A24 L1:17) • Does not use direct combo… was [possible] on each side, each time (A27 L1:20)
Outcome: Favorable	Direct combination followed by separate steps, then completes additive inverse: • Direct combo first (non-box) then scaffolded (box) and combines correctly. Clears non-box then box. (P07 A10 L1:10)
Outcome: Unfavorable	Direct combination followed by separate steps, then does not complete additive inverse: • Almost direct combo on non-box first--saw yellow highlight--but used direct combo on box side, then scaffolded placement on non-box (even though passed through yellow highlight on the way). Did not address non-box side. (P10 A10 L1:10)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates chapter and level. Brackets indicate clarifications.

property by adding a duplicate variable to the opposite side of the equation space, a square target space appeared and the app disallowed other moves until the participant finished applying the additive equality property.

Even when participants applied the same approach to accessing the affordance, outcomes varied. Participants 7 and 10 frequently used the separate steps approach. However, while Participant 7 often completed the additive inverse property on the second side before finishing the level, Participant 10 rarely did so. Furthermore, after identifying the potential for combinations, some participants repeatedly attempted combinations that were mathematically incorrect (e.g., variable plus vortex/swirl, X + 0 = 0, see Figure 9) or in ways that the app did not permit until later levels (e.g., add variables with common denominators, see Figure 10). Some participants may have overgeneralized the potential for combinations and failed to recognize the connection to the additive inverse property.

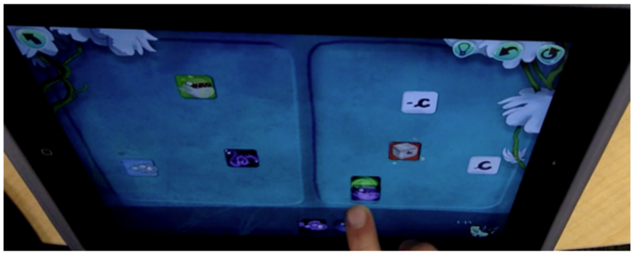

Figure 9. Attempting X + 0 = 0.

Figure 10. Attempting to add variables with common denominators.

Variations in accession of motivation. Analysis indicated that accession of the affordance of motivation varied by degree, could change by situation or by time, was consistent for some participants yet inconsistent for others, and could have had favorable or unfavorable outcomes even when the participants accessed similar degrees of the affordance of motivation, depending on the participant's relevant ability. Table 9 shows examples of codes applied during analysis concerning variations of motivation affordance-ability relationships.

As seen in Table 9, participants' interactions demonstrated a range of access to the affordance of motivation, from a high degree of access to positive motivation (e.g., Participant 7: "excited") through a low degree of access to motivation (e.g., Participant 4: "kinda boring"), to a high degree of access to negative motivation (e.g., Participant 1: "wanna punch something"). The degree of access to motivation did not remain static, varying by situation (e.g., across different levels) or over time (e.g., across consecutive repetitions of the same level). Accession of motivation could also be consistent (e.g., Participant 2: building challenge) or inconsistent (e.g., Participant 1: earning stars). Even when access to motivation was similar, outcomes were not. Whether showing high access to positive motivation (e.g., Participant 5 vs. Participant 7), low access to motivation (Participant 4 vs. Participant 3), or high access to negative motivation (Participant 2 vs. Participant 1), outcomes could be favorable or unfavorable in terms of continuing to interact with the app for the full amount of time permitted.

Table 9

Variations of Motivation Affordance-Ability Relationships

Variation	Example
Variant: Degree	High access to positive motivation • “I was really motivated to get to the challenge room and got me really excited about playing it.” (P05 SFT) Low access to motivation • “[It] was kinda boring” (P04 ZFT) High access to negative motivation • “The whole thing is frustrating. Makes you wanna punch something.” (P01 ZFT)
Change: Situation	Initially low access to motivation becomes high access to positive motivation once reaching 15N • “[Once it was timed] it was exciting. You were timed so you were freaked out so it was fun.” (P07 ZFT) Initially low access to motivation becomes high access to negative motivation after turning on the needle timer • “Being timed is putting a lot of pressure on you and you don’t really like that, no one really does.” (P08 ZFT)
Change: Time	Initially high access to positive motivation decreases to low motivation, then becomes high access to negative motivation through repetition of L15N: P03 • [First 15N attempt.] Completed no tasks, but excitable. (A7 L15N) • Verbal frustration evident here and before (A27 L15N) • “Frustrating…. When you’re about to press it but the needle pops it.” (P03 ZFT)
Consistency: Consistent	Regular approach to affordance access High access to positive motivation across situations • “It was fun…. ‘Cause the levels got harder.” (P02 ZFT)
Consistency: Inconsistent	Irregular approach to affordance access Motivation to earn stars; positive, neutral, negative: P01 • Repeated level (to earn missed star) (A20 L1:15) • Tapped dots and completely ignored division on non-box side [does not repeat level to earn stars] (A42 L2:06) • “[I decided to restart if] I want to beat the level…. I play it so I can get one or two stars. Because one star… makes you feel like you suck. So that’s why it makes you wanna play again.” (DBFT)
Outcome: Favorable	Full 30-minute interaction time with: High access to positive motivation • “I was really motivated to get to the challenge room and got me really excited about playing it.” (P05 SFT) Low access to motivation • “[It] was kinda boring” (P04 ZFT) High access to negative motivation • “That was hard! [It was very] frustrating.” “ (P02 ZFT)

(*table continues*)

Variation	Example
Outcome: Unfavorable	Ended interaction time early (interaction duration): High access to positive motivation • "You were timed so you were freaked out so it was fun." (P07 ZFT) (24 minutes) Access to both positive motivation and negative motivation • "It was really fun to match up but it was a little confusing because I don't think it told you to do that but it was a little tricky." (P03 DBFT) (24 minutes) High access to negative motivation • "The whole thing is frustrating. Makes you wanna punch something." (P01 ZFT) (28 minutes)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level. DBFT indicates DragonBox Algebra 12+ Follow-Up Questions Transcript. ZFT indicates Motion Math: Zoom Follow-Up Questions Transcript. SFT indicates Summative Follow-Up Questions Transcription. Quotation marks indicate direct quotes from participants. Brackets indicate clarifications.

Interactions among Affordance-Ability Relationships

Evidence indicated that multiple affordance-ability relationships could be simultaneously present in the same user-app interaction sequence, leading to the theme of interrelationships among affordance-ability relationships. User-app interactions could involve multiple affordance-ability relationships, such as simultaneous linking of actions and representations (e.g., values changing when swiping to move along the number line), and a degree of access to motivation (e.g., enjoyment). However, notable interactions among affordance-ability relationships required balancing accession of efficient precision, focused constraint, and creative variation. Table 10 shows examples of memo excerpts identified during analysis concerning interactions among these affordances.

As seen in Table 10, interactions with each app provided evidence of interactions among efficient precision, focused constraint, and creative variation. Motion Math: Zoom afforded creative variation to apply novel strategies to complete tasks, such as having

Table 10

Interactions Among Efficient Precision, Focused Constraint, and Creative Variation

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Motion Math: Zoom

Popped at 1.00 to 1.XX…. planning involved zooming in to hundredths at 0.01 to find 1.51--paused around 0.5 and immediately tried to zoom out but too late (P02 A28 L15N)

P09

- Using [navigation restriction] to stop swipe at 0. (A16 L15N)
- Fine adjustments to use multiple intervals, which took [too much] time (A17 L15N)
- Planning out before next bubble arises, for 1.64 from 1.00 (at tenths) (A18 L15N)
- Uses tenths for 1.00 (A21 L15N)
- Tenths to 1.00 (A22 L15N)

P06

- Planning zooms help but anticipatory taps are often inaccurate and actually slow overall [process] (A12 L15N)
- Far off for 0.13--tries to swipe beyond 1.0 at tenths. ([App] restricts further rightward navigation) (A15 L15N)
- Planning sometimes efficient but sometimes slows overall process (A17 L15N)
- Planning zoom out not always effective in this level [because] of new prompt and does not continue [planning] (A20 L12N)

DragonBox Algebra 12+

Begins with incorrect [addition instead of multiplication]…. rearranging and failed combining attempts…. many disallowed combinations (some of which were mathematically possible). Some gestures… placed/executed poorly. Five minutes of guess and check.... Ends with 34 moves (excluding from undo) before Restart. (P10 A39 L2:13)

P01

- Attempted to clear combo before adding to both sides [resulting in completed level with missing stars] (A19 L1:15)
- Repeated level (to earn missed star) (A20 L1:15)
- P04
- Tries to drag variable across but stopped by app (A5 L1:05)
- Tries to drag one-dot across instead of combining (A24 L2:01)
- New power--drag across (A54 L3:01)
- Uses drag across for direct combos (app counts as two moves) as example of efficient precision ([vs.] not allowed much earlier--focused constraint) (A55 L3:02)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level. Brackets indicate clarifications.

multiple intervals visible at the same time (e.g., 1s and 10s, see Figure 11), and planning ahead by navigating in anticipation of the next task (see Figure 12). These strategies could also contribute to efficient precision, leading to quick and accurate task completion. Conversely, these creative strategies could hinder progress if a participant's relevant abilities did not permit efficient, precise use. As a participant modified relevant abilities, this could modify the balance of affordance accession. For example, when first attempting Level 15 with the needle timer on, Participant 9 struggled to efficiently plan using multiple visible intervals to navigate from 1.00 to 1.XX (e.g., 1.00 to 1.53),

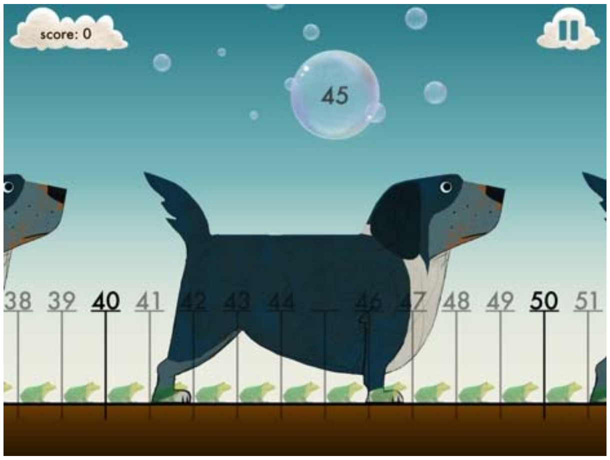


Figure 11. Multiple visible intervals.

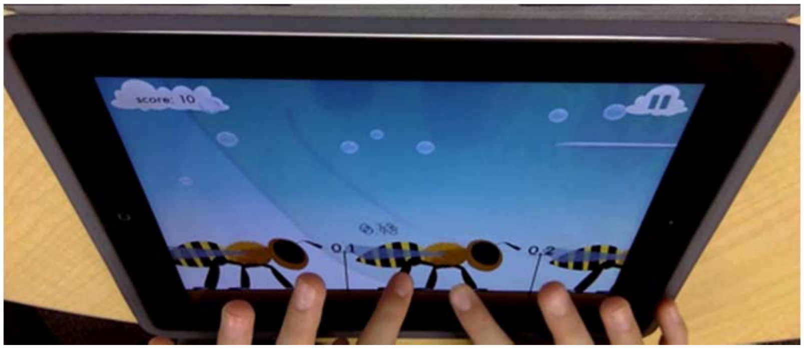


Figure 12. Planning by navigating before the next prompt appears.

resulting in the needle popping the bubble. Later, Participant 9 efficiently planned without using multiple visible intervals, placing 1.0 when the screen showed intervals of 0.1 beginning at 1.0, in preparation for finding 1.XX and contributing to successful task completion. Throughout these interactions, accession of focused constraint influenced accession of both efficient precision and creative variation. For example, navigation restrictions on Level 15 prohibited zooming in beyond hundredths or swiping into negative numbers, which limited potential exploration. To locate 0.05 when presented with intervals of 0.1, Participant 9 moved the number line leftward and allowed the app to stop the motion at 0, then zoomed in and allowed the app to stop motion at hundredths. Thus, the app constrained creativity while affording efficiency.

Interactions with DragonBox Algebra 12+ also provided evidence of interactions among efficient precision, focused constraint, and creative variation. The app afforded creative variation because participants could use guess-and-check or systematic trials to complete a level, such as the array of attempted moves made by Participant 10 during Level 2:13. However, the app emphasized efficient precision over creative variation, awarding stars for simplified solutions made in a minimal number of moves. The app emphasized focused constraint over efficient precision and creative variation in some situations, disallowing moves that were otherwise mathematically correct. Some of these moves were permitted during more advanced levels, such as when Participant 4 was not permitted to drag a variable to the other side of the equation until after the app officially demonstrated this “new power” during Level 3:01.

In summary, affordance-ability relationships were present throughout user-app interactions. However, affordance-ability relationships were not identical across all participants and situations, and affordance access often varied by approach or degree. Furthermore, user-app interactions involved multiple affordance-ability relationships and these affordance-ability relationships interacted with one another.

Research Question 3: Presence and Modification of Distance

The third research question concerned evidence of distance in fifth graders' interactions with mathematics virtual manipulative iPad apps. As discussed in the Methods chapter, the researcher addressed this research question by analyzing the video data of user-app interactions and follow-up questions, and the observation field notes during data analysis stages that included identifying examples of distance and analyzing emergent variations related to distance. Data were coded to identify the presence of distance, as indicated by interactions and comments concerning differences between what participants did to interact with the app and what was required to successfully interact with the app. Qualitative analysis revealed two emergent types of distance: mathematical and technological. Qualitative and quantitized qualitative analyses indicated that: a) distance can change over time, and b) mathematical distance and technological distance can influence each other.

Presence of Two Types of Distance

Analysis provided evidence of the presence of distance. Interpretation of this evidence revealed two types of distance: mathematical distance and technological distance. Mathematical distance was the degree of difficulty of the mathematical aspects of interactions between the user and the tool (e.g., a mathematics virtual manipulative iPad app), whereas technological distance was the degree of difficulty of the technological aspects of interactions between the user and the tool. The degree of distance present varied. Table 11 shows examples of memo excerpts identified during analysis concerning mathematical and technological distance.

Mathematical distance and technological distance were always present in these interactions, but the degree of each distance depended on the relationship between user and app in the specific context. For example, during interactions with Level 15 with the needle active in Motion Math: Zoom, Participant 2 showed a high degree of mathematical

Table 11

Examples of Mathematical and Technological Distance

Motion Math: Zoom	DragonBox Algebra 12+
Mathematical distance: High degree	
[Needle] popped first task (0.05); tried 0.5 placement even after app filled the empty space (P02 A8 L15N)	Ends up adding all [variables] from outside and trying to combine across or within for unlike [variables]. (P03 A13 L1:12)
Mathematical distance: Low degree	
[Chooses] ideal intervals and ranges (P10 A4 L4)	Replicates solution. Audible deep sigh. (P07 A31 L1:14)
Technological distance: High degree	
Mixed up zoom in/out gestures (P03 A8 L15N)	Misses drag/drop again (P03 A6 L1:06)
Technological distance: Low degree	
Controlled zooming when close to interval level (P04 A8 L18)	Watches new power once, correctly replicates tap. (P06 A28 L2:05)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level. Brackets indicate clarifications.

distance while struggling to find the appropriate place on the number line for 0.05, whereas Participant 3 showed a high degree of technological distance, switching the zoom in and zoom out input gestures. In other situations, participants showed evidence of a low degree of distance, such as when they did not struggle to find correct placements on the number line (i.e., low degree of mathematical distance), and some participants managed to precisely control zoom input gestures (i.e., low degree of technological distance). While attempting Level 15, all 10 participants at times confused tenths and hundredths (e.g., 0.04 vs. 0.4) and became lost on the number line when attempting to find numbers greater than one whole that included hundredths (e.g., 1.82).

Interactions with DragonBox Algebra 12+ also provided evidence of distance. In terms of mathematical distance, some participants repeatedly struggled to apply the correct properties to complete a level, other participants managed to complete the level with less difficulty. For example, to complete Level 1:14, six participants required one attempt, three participants required 3-5 attempts, and Participant 7 required 17 attempts. A common example of technological distance in DragonBox Algebra 12+ occurred beginning at Level 2:05, which involved tapping a coefficient of 1 to apply the multiplicative inverse property. All nine participants who progressed beyond Level 2:05 at times struggled to appropriately apply the tapping gesture. However, relatively high degrees of technological distance were more common during interactions with Motion Math: Zoom than during interactions with DragonBox Algebra 12+. This may have been because interactions with DragonBox Algebra 12+ involved only single-touch input (e.g., tapping and dragging), whereas interactions with Motion Math: Zoom also involved multi-touch input (e.g., pinching to zoom).

Changes in Distance

Qualitative analyses and visual analysis of the quantitized qualitative data provided evidence of changes in mathematical distance and technological distance. Table 12 shows examples of memo excerpts coded as changes in distance.

Distance did not remain static throughout these interactions. Few participants struggled to swipe when beginning to interact with Motion Math: Zoom, showing a low degree of technological distance. However, technological distance increased for six participants as they initially struggled to zoom when first required to regularly do so. All participants improved their facility with the zoom input gesture and decreased technological distance to some degree. Mathematical distance also changed, such as during interactions with DragonBox Algebra 12+. Many participants effectively applied some attributes but struggled to apply others, so mathematical distance varied, in part, depending on attributes required to interact with the level of the app. For example, no participants consistently simplified both sides of the equation. Level 1:14 required participants simplify only one side of the equation, and every participant completed Level 1:14 with a low degree of mathematical distance, although four participants required repeated attempts to do so. However, Level 1:15 involved simplifying both sides of the equation, yet few participants did more than the minimum required to finish the level, simplifying only the X side of the equation, leading to a higher degree of mathematical distance. Only Participant 8 simplified both sides of the equation, completing Level 1:15 with a low degree of mathematical distance.

Table 12

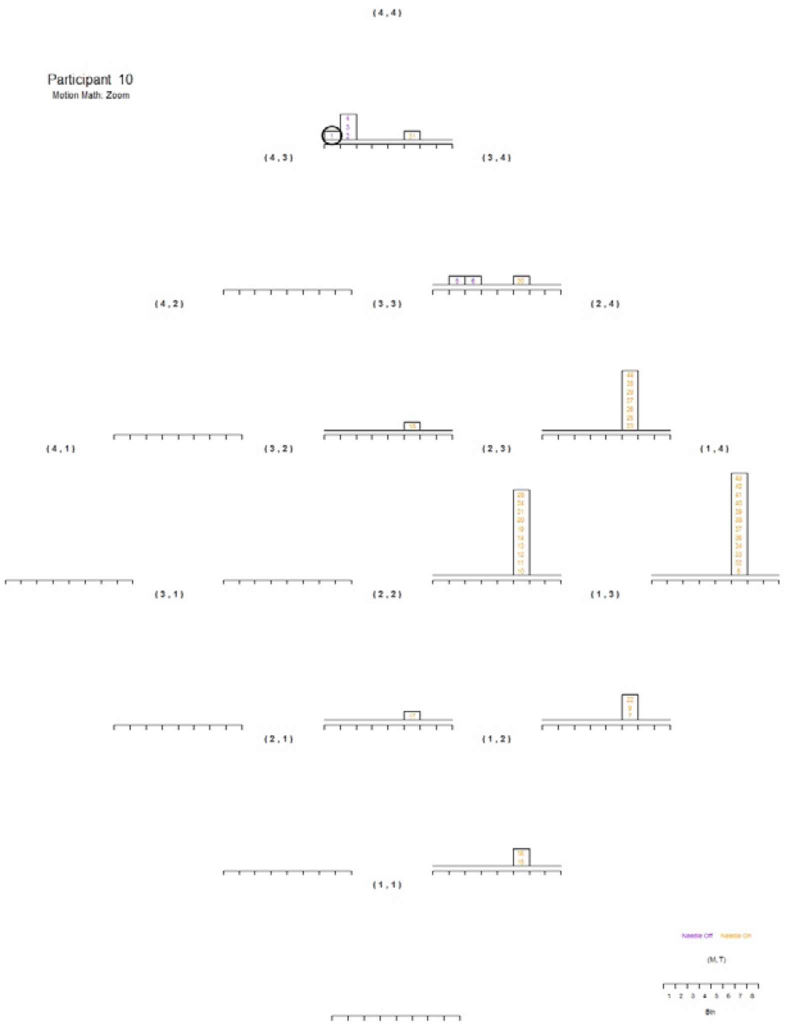
Examples of Changing Distance

Motion Math: Zoom	DragonBox Algebra 12+
Mathematical distance: Decreasing	
P05	P09
• Much L[eft]-R[ight] confusion…. Imprecise, non-ideal choices… often travels for extended time with inefficient interval, but usually in correct direction (A11 L5) • Chooses some inefficient intervals… to travel within… [but] accurate completion (A12 L6)	• Begins with incorrect unneeded addition variable…. Eventually places all variables and attempts some impossible combos. (A11 L1:11) • Quickly correct and complete. (A12 L1:11)
Mathematical distance: Increasing	
P06	P04
• Correct/ideal for 0.05…. For 1.00 to 1.53, chooses appropriate place to zoom in…. Balance of sufficient accuracy with lots of speed--and memory of type of upcoming task for planning. (A17 L15N) • 0.10 from tenths (0.7)--right first, then zoomed in at 0.5 to travel by hundredths. (A18 L16N)	• Combo inside, direct combo from outside [completes simplified solution] (A15 L1:14) • Direct combo from outside…. [does not simplify] opposite side (A16 L1:15)
Technological distance: Decreasing	
P03	P02
• Struggled to zoom out [because] of mixing up zoom in/out gestures. (A13 L15N) • Zoomed out for 0.3 with multiple intervals showing. Zoomed in for 0.04 at 0.1-0.2…. [No] in/out mixups. (A14 L15N)	• Swipe swirls [instead of tap], app did not read every time (A5 L1:05) • Now using correct tech input (A12 L1:12)
Technological distance: Increasing	
P08	P10
• Thumb swipe at corner of screen for 5, 21, 12. Scaffolded zoom for 15. (A1 L1) • Nearly vertical zoom… slows progress…. Scaffolded zoom in replication difficulties…. For 0.01 from 1.XX, travels by hundredths. Tries to zoom out but fails. (A2 L15)	• While holding [variable], sees white highlight on [inverse], drags closer, sees yellow highlight, combines (A38 L2:13) • Some gestures misread or placed/executed poorly (A39 L2:13)

Note: Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level. Brackets indicate clarifications.

Analysis of quantitized qualitative data using rhombus plots. To aid comparison of distance across situations and participants, the researcher quantitized qualitative distance data in the form of (mathematical, technological) distance values for each attempted level, organized by bins, and visualized using rhombus plots (see Methods section). The researcher determined bins by analyzing app attributes and grouping consecutive levels by similarity of attributes (see Appendix E). For example, in Motion Math: Zoom, Bin 2 included levels 2-5, which focused on integers to 1,000 with little changing of intervals from one task to the next, and mainly required swipe input. In the same app, Bin 3 included levels 6-8, which focused on integers to 10,000 with frequent changing intervals of intervals from one task to the next, and required both more zoom input. Within each bin, mathematical content usually slightly increased in difficulty from one level to the next level (e.g., range of 0-20 followed by range of 0-40).

Visual analysis of the rhombus plots and tables provided numerical and graphical evidence of change in distance. The researcher examined the rhombus plots (a) individually, (b) arranged in small multiples by user, and (c) arranged in small multiples by app. Small multiples involves presenting smaller versions of graphs together to allow for visual comparison, which is effective when everything is in a fixed location in the graph and only the data change (Heer, Bostock, & Ogievetsky, 2010; Tufte, 2001). Figures 13-19 (each discussed and shown separately) illustrate connections between the rhombus plots and tabular data used to identify changes in distance. Appendix F includes rhombus plots arranged in small multiples by app. Appendix G includes the data used to generate the rhombus plots.

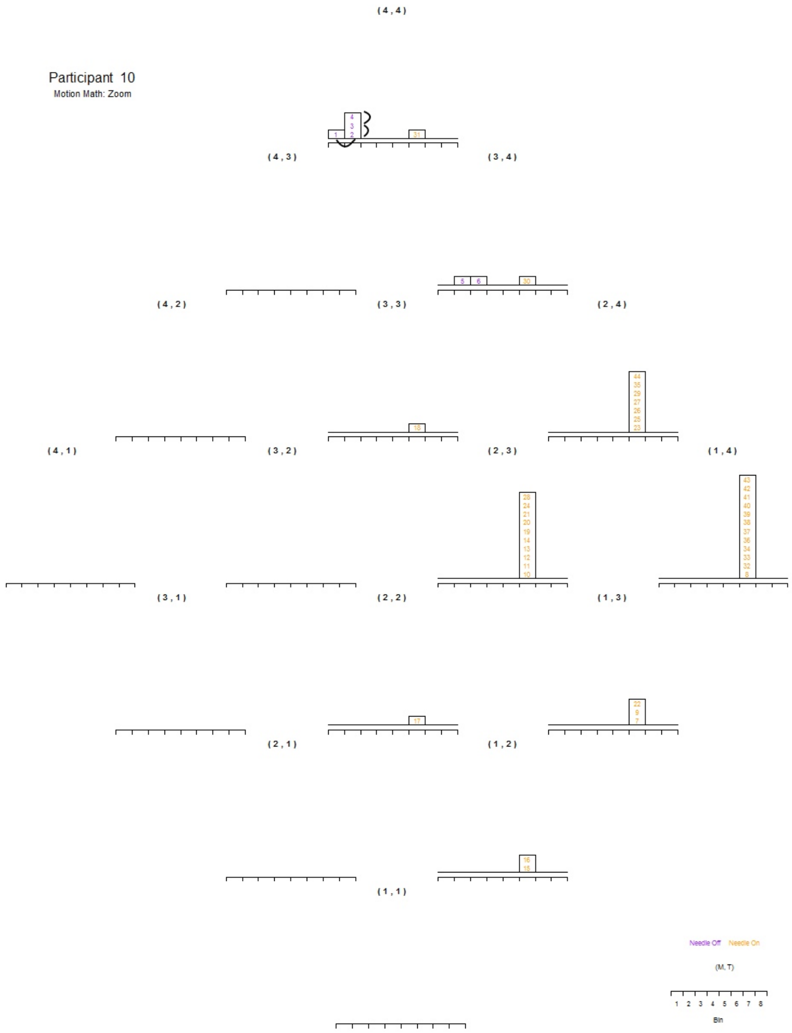


A	B	L	N	M	T
1	1	1	0	4	4
2	2	2	0	4	4
3	2	3	0	4	4
4	2	4	0	4	4
5	2	5	0	3	4
6	3	6	0	3	4
7	6	15	1	1	3
8	6	15	1	1	4
9	6	15	1	1	3
10	6	15	1	2	3
11	6	15	1	2	3
29	6	15	1	2	4
30	6	15	1	3	4
31	6	15	1	4	4
32	6	16	1	1	4
33	6	16	1	1	4
40	6	16	1	1	4
41	6	16	1	1	4
42	6	16	1	1	4
43	6	16	1	1	4
44	6	16	1	2	4

Key: **A**ttempt, **B**in, **L**evel, **N**eedle (0 inactive, 1 active); Distance Values: **M**athematical, **T**echnological

Figure 13. First annotated rhombus plot and data table of Participant 10's interactions with Motion Math: Zoom.

Figure 13 depicts the rhombus plot and data table excerpt for Participant 10's interactions with Motion Math: Zoom, annotated to emphasize Attempt 1, Bin 1, Level 1, needle timer off, (mathematical, technological) distance value (4,4). This is the starting point of the interactions, indicated by the attempt label of 1. Figure 14 illustrates the second excerpt in the sequence.

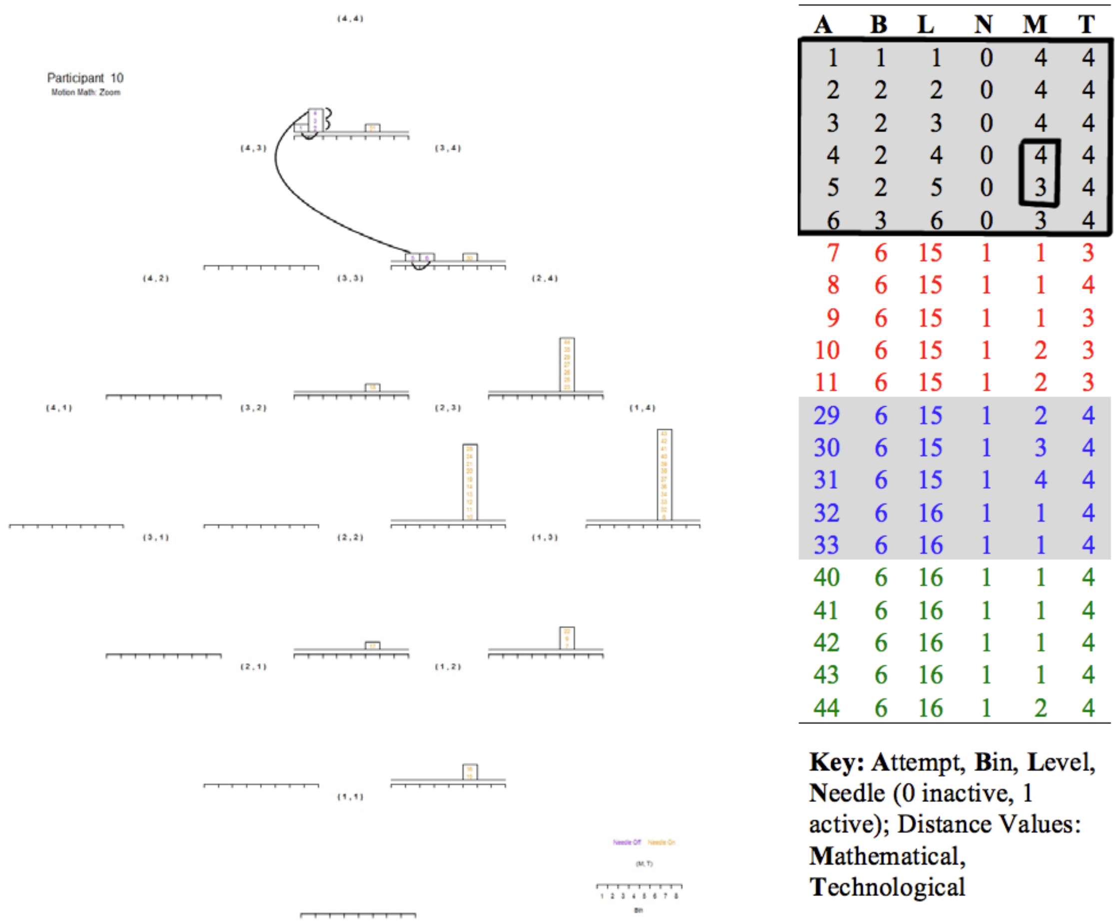


A	B	L	N	M	T
1	1	1	0	4	4
2	2	2	0	4	4
3	2	3	0	4	4
4	2	4	0	4	4
5	2	5	0	3	4
6	3	6	0	3	4
7	6	15	1	1	3
8	6	15	1	1	4
9	6	15	1	1	3
10	6	15	1	2	3
11	6	15	1	2	3
29	6	15	1	2	4
30	6	15	1	3	4
31	6	15	1	4	4
32	6	16	1	1	4
33	6	16	1	1	4
40	6	16	1	1	4
41	6	16	1	1	4
42	6	16	1	1	4
43	6	16	1	1	4
44	6	16	1	2	4

Key: Attempt, **B**in, **L**evel, **N**eedle (0 inactive, 1 active); Distance Values: **M**athematical, **T**echnological

Figure 14. Second annotated rhombus plot and data table of Participant 10's interactions with Motion Math: Zoom.

Figure 14 depicts the rhombus plot and data table excerpt for Participant 10's interactions with Motion Math: Zoom, annotated to emphasize Attempts 1-4. In particular, as Participant 10 moved from a level in Bin 1 to a level in Bin 2, the (mathematical, technological) distance values did not change. In the table, Attempts 1 and 2 changed bins and levels but the mathematical and technological distance values remained the same. In the rhombus plot, Attempt 1 was in Bin 1 whereas Attempt 2 was in Bin 2 on the same axis. Figure 15 illustrates the third excerpt in the sequence.

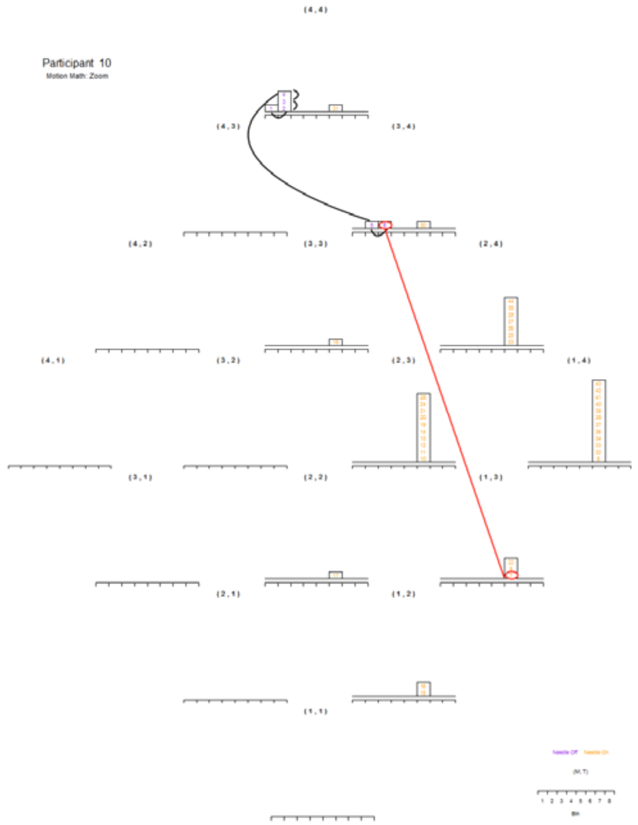


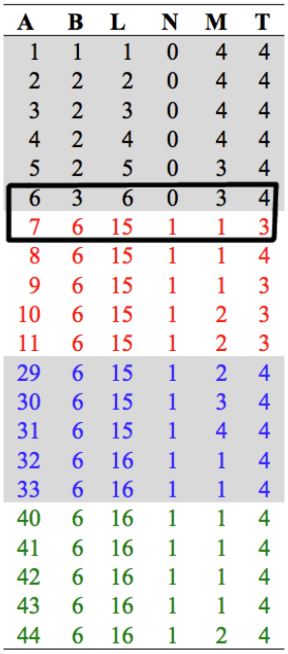
A	B	L	N	M	T
1	1	1	0	4	4
2	2	2	0	4	4
3	2	3	0	4	4
4	2	4	0	4	4
5	2	5	0	3	4
6	3	6	0	3	4
7	6	15	1	1	3
8	6	15	1	1	4
9	6	15	1	1	3
10	6	15	1	2	3
11	6	15	1	2	3
29	6	15	1	2	4
30	6	15	1	3	4
31	6	15	1	4	4
32	6	16	1	1	4
33	6	16	1	1	4
40	6	16	1	1	4
41	6	16	1	1	4
42	6	16	1	1	4
43	6	16	1	1	4
44	6	16	1	2	4

Key: **A**ttempt, **B**in, **L**evel, **N**eedle (0 inactive, 1 active); Distance Values: **M**athematical, **T**echnological

Figure 15. Third annotated rhombus plot and data table of Participant 10's interactions with Motion Math: Zoom.

Figure 15 depicts the rhombus plot and data table excerpt for Participant 10's interactions with Motion Math: Zoom, annotated to emphasize Attempts 1-6. In particular, as Participant 10 moved from Attempt 4 to Attempt 5 on levels within Bin 2, mathematical distance increased. In the table, Attempt 4 had mathematical distance value of 4, whereas Attempt 5 had a mathematical distance value of 3. In the rhombus plot, Attempt 4 was in Bin 2 on the (4,4) axis, whereas Attempt 5 was in Bin 2 on the (3,4) axis. The annotation shows a downward trajectory as mathematical distance increased. Figure 16 illustrates the fourth excerpt in the sequence.



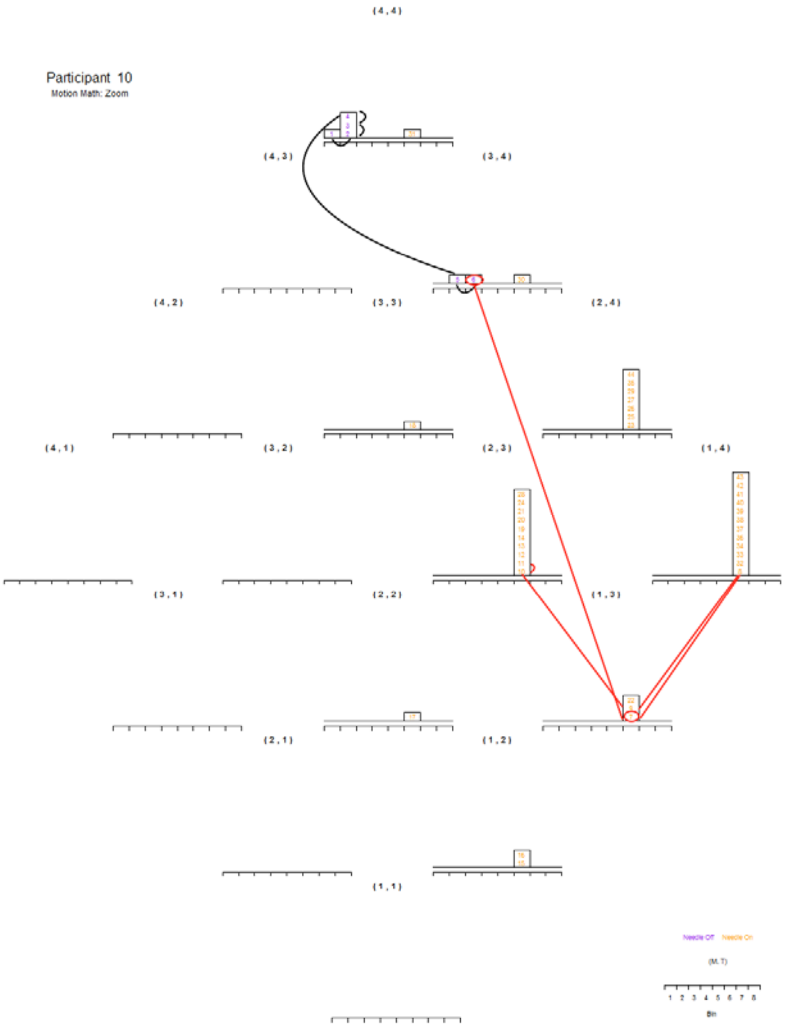


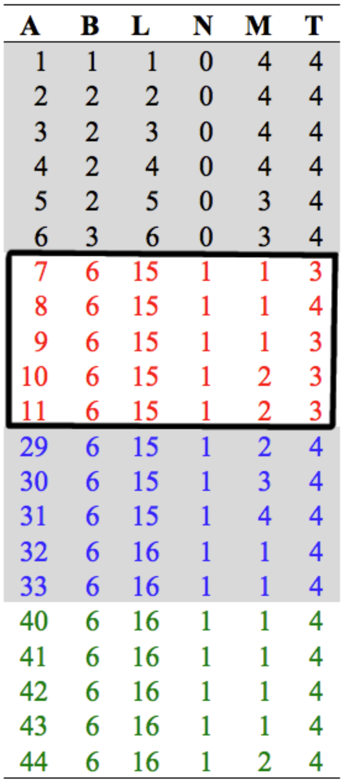
A	B	L	N	M	T
1	1	1	0	4	4
2	2	2	0	4	4
3	2	3	0	4	4
4	2	4	0	4	4
5	2	5	0	3	4
6	3	6	0	3	4
7	6	15	1	1	3
8	6	15	1	1	4
9	6	15	1	1	3
10	6	15	1	2	3
11	6	15	1	2	3
29	6	15	1	2	4
30	6	15	1	3	4
31	6	15	1	4	4
32	6	16	1	1	4
33	6	16	1	1	4
40	6	16	1	1	4
41	6	16	1	1	4
42	6	16	1	1	4
43	6	16	1	1	4
44	6	16	1	2	4

Key: **A**ttempt, **B**in, **L**evel, **N**eedle (0 inactive, 1 active); Distance Values: **M**athematical, **T**echnological

Figure 16. Fourth annotated rhombus plot and data table of Participant 10’s interactions with Motion Math: Zoom.

Figure 16 depicts the rhombus plot and data table excerpt for Participant 10’s interactions with Motion Math: Zoom, annotated to emphasize Attempts 6 and 7. In particular, there was an increase in mathematical distance and technological distance. In the table, Attempt 6 was in Bin 3, Level 6, needle function inactive, and (mathematical, technological) distance value of (3,4), while Attempt 7 was in Bin 6, Level 15, needle function active, and (mathematical, technological) distance value of (1,3). In the rhombus plot, Attempt 6 was in Bin 3 on the (4,3) axis with the Needle Off color, whereas Attempt 7 was in Bin 6 on the (1,3) axis with the Needle On color. The annotation shows a downward trajectory as mathematical and technological distance increase. Figure 17 illustrates the fifth excerpt in the sequence.



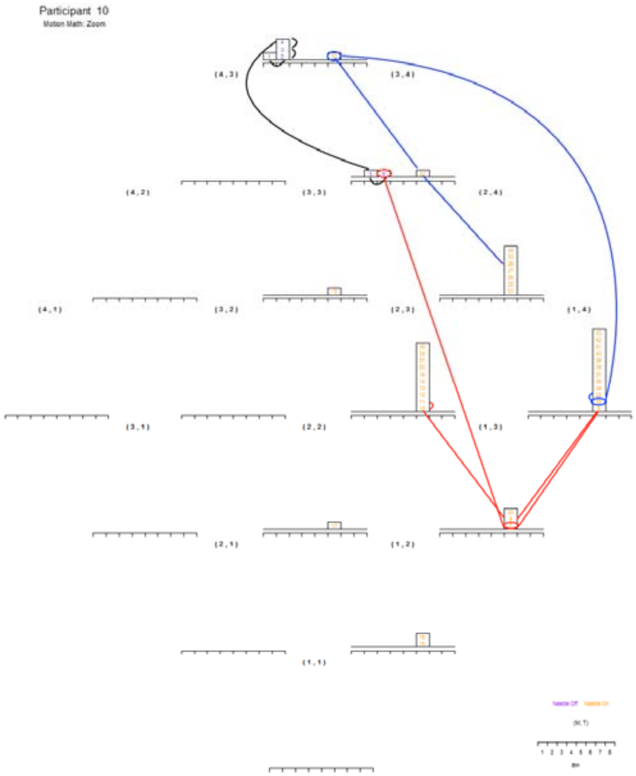


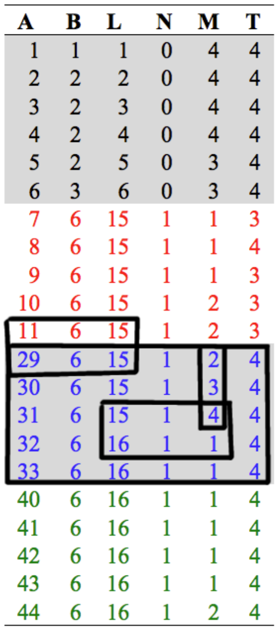
A	B	L	N	M	T
1	1	1	0	4	4
2	2	2	0	4	4
3	2	3	0	4	4
4	2	4	0	4	4
5	2	5	0	3	4
6	3	6	0	3	4
7	6	15	1	1	3
8	6	15	1	1	4
9	6	15	1	1	3
10	6	15	1	2	3
11	6	15	1	2	3
29	6	15	1	2	4
30	6	15	1	3	4
31	6	15	1	4	4
32	6	16	1	1	4
33	6	16	1	1	4
40	6	16	1	1	4
41	6	16	1	1	4
42	6	16	1	1	4
43	6	16	1	1	4
44	6	16	1	2	4

Key: Attempt, **B**in, **L**evel, **N**eedle (0 inactive, 1 active); Distance Values: **M**athematical, **T**echnological

Figure 17. Fifth annotated rhombus plot and data table of Participant 10's interactions with Motion Math: Zoom.

Figure 17 depicts the rhombus plot and data table excerpt for Participant 10's interactions with Motion Math: Zoom, annotated to emphasize Attempts 7-11. Each attempt was on Bin 6, Level 15, needle function active, and had high degrees of mathematical distance and lower degrees of technological distance. The annotated path on the rhombus plot shows a small range of changes to mathematical and technological distance at the lower right side of the plot. Figure 18 illustrates the sixth excerpt in the sequence.



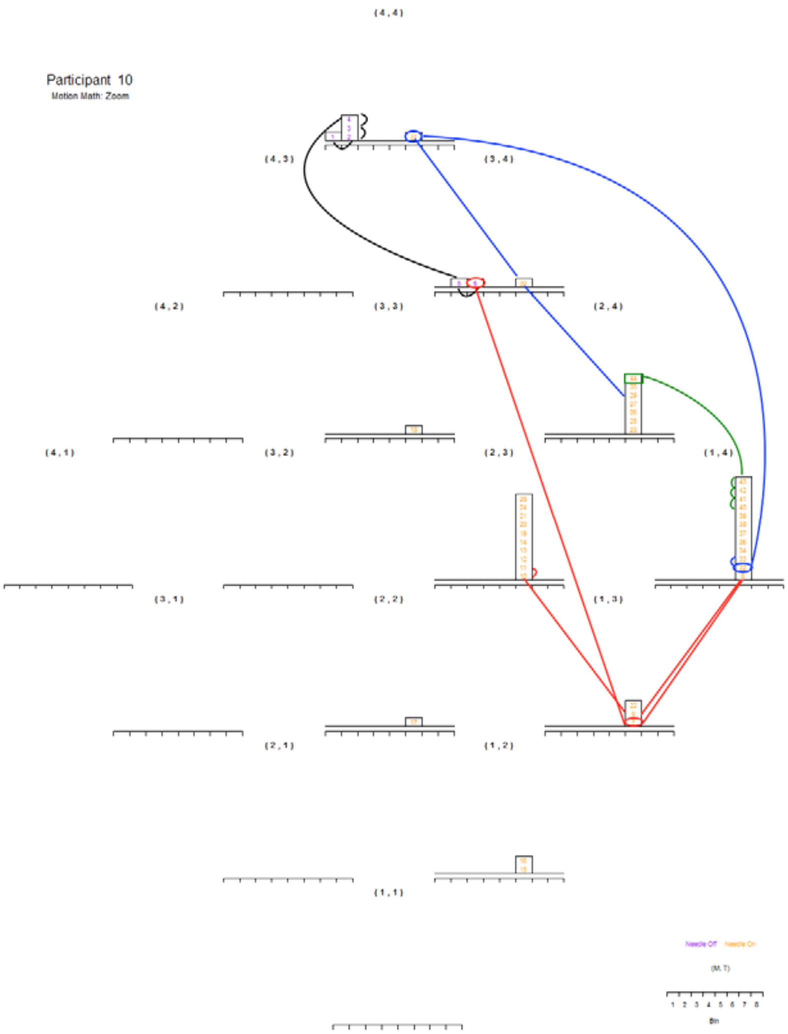


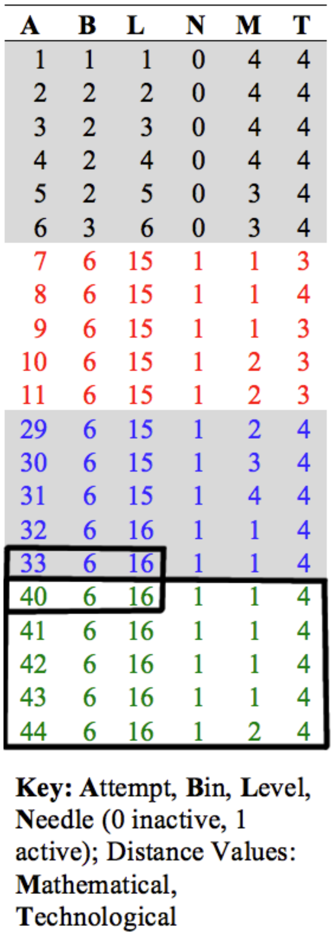
A	B	L	N	M	T
1	1	1	0	4	4
2	2	2	0	4	4
3	2	3	0	4	4
4	2	4	0	4	4
5	2	5	0	3	4
6	3	6	0	3	4
7	6	15	1	1	3
8	6	15	1	1	4
9	6	15	1	1	3
10	6	15	1	2	3
11	6	15	1	2	3
29	6	15	1	2	4
30	6	15	1	3	4
31	6	15	1	4	4
32	6	16	1	1	4
33	6	16	1	1	4
40	6	16	1	1	4
41	6	16	1	1	4
42	6	16	1	1	4
43	6	16	1	1	4
44	6	16	1	2	4

Key: Attempt, **B**in, **L**evel, **N**eedle (0 inactive, 1 active); Distance Values: **M**athematical, **T**echnological

Figure 18. Sixth annotated rhombus plot and data table of Participant 10’s interactions with Motion Math: Zoom.

Figure 18 depicts the rhombus plot excerpt for Participant 10’s interactions with Motion Math: Zoom, annotated to emphasize the completion of Bin 6, Level 15. The table annotations show that Participant 10 was still attempting Bin 6, Level 15 by Attempt 29. The upward trajectory of the rhombus plot path from Attempts 29-31 align with the table data showing that Participant 10 decreased mathematical distance and completed Level 15. The downward trajectory of the rhombus plot path between circled attempts aligns with the table data showing that mathematical distance then increased as Participant 10 followed Attempt 31, Bin 6, Level 15, needle active, (mathematical, technological) distance value (4,4) with Attempt 32, Bin 6, Level 16, (mathematical, technological) distance value (1,4). Figure 19 illustrates the final excerpt in the sequence.





A	B	L	N	M	T
1	1	1	0	4	4
2	2	2	0	4	4
3	2	3	0	4	4
4	2	4	0	4	4
5	2	5	0	3	4
6	3	6	0	3	4
7	6	15	1	1	3
8	6	15	1	1	4
9	6	15	1	1	3
10	6	15	1	2	3
11	6	15	1	2	3
29	6	15	1	2	4
30	6	15	1	3	4
31	6	15	1	4	4
32	6	16	1	1	4
33	6	16	1	1	4
40	6	16	1	1	4
41	6	16	1	1	4
42	6	16	1	1	4
43	6	16	1	1	4
44	6	16	1	2	4

Key: Attempt, **B**in, **L**evel, **N**eedle (0 inactive, 1 active); Distance Values: **M**athematical, **T**echnological

Figure 19. Seventh annotated rhombus plot and data table of Participant 10's interactions with Motion Math: Zoom.

Figure 19 depicts the rhombus plot and data table excerpt for Participant 10's interactions with Motion Math: Zoom, annotated to emphasize the final excerpt from the interaction session. The table shows that by Attempt 40, Participant 10 was still attempting Bin 6, Level 16, with a high degree of mathematical distance and a low degree of technological distance. Participant 10 continued to attempt Bin 6, Level 16 and slightly decreased mathematical distance by the end of the interaction session. The annotations on the rhombus plot show this decrease in mathematical distance as Attempt 43 was in Bin 6 on the (1,4) axis, whereas Attempt 44 was in Bin 6 on the (2,4) axis. Attempt 44 was the last attempt in the interaction session, as indicated by the greatest attempt label in the plot: 44.

Patterns emerged in analysis of data from individual participants and comparison between participants. Table 13 shows excerpts from the data tables used to generate one of the plots, and Figures 20 and 21 are the rhombus plots showing the (mathematical, technological) distance values for Participant 10's interactions with each app.

Table 13

Excerpts from Distance Data Used to Generate Rhombus Plot for Participant 10's Interactions with Motion Math: Zoom

A	B	L	N	M	T
7	6	15	1	1	3
8	6	15	1	1	4
9	6	15	1	1	3
10	6	15	1	2	3
11	6	15	1	2	3
29	6	15	1	2	4
30	6	15	1	3	4
31	6	15	1	4	4
32	6	16	1	1	4
33	6	16	1	1	4
40	6	16	1	1	4
41	6	16	1	1	4
42	6	16	1	1	4
43	6	16	1	1	4
44	6	16	1	2	4

Table key. A indicates attempt, B indicates bin, L indicates level or chapter and level. N indicates presence of needle (0 is inactive, 1 is active). M indicates mathematical distance value; T indicates technological distance value.

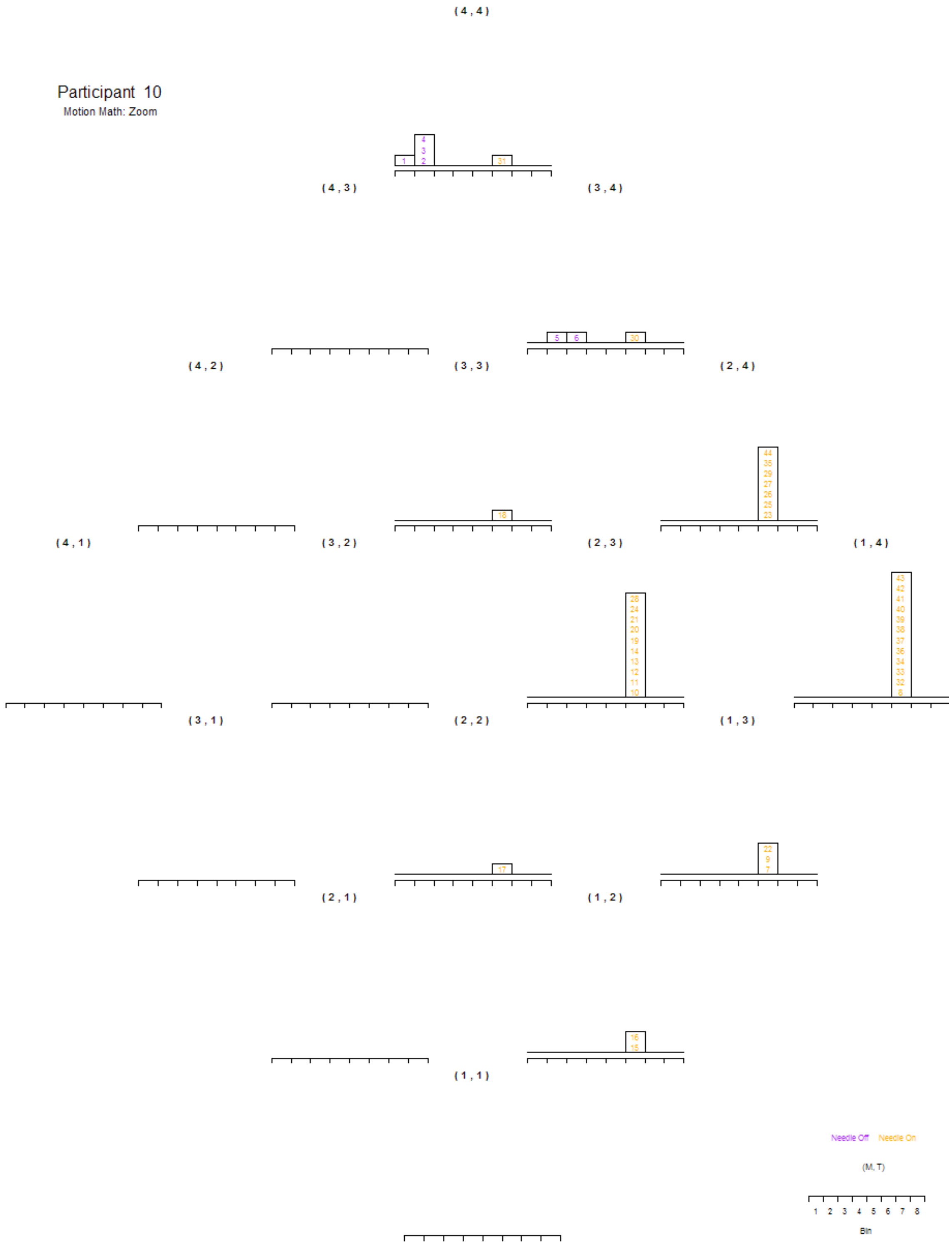


Figure 20. Rhombus plot of Participant 10's interactions with Motion Math: Zoom. (M,T) indicates (mathematical, technological) distance value. Bin indicates level group.

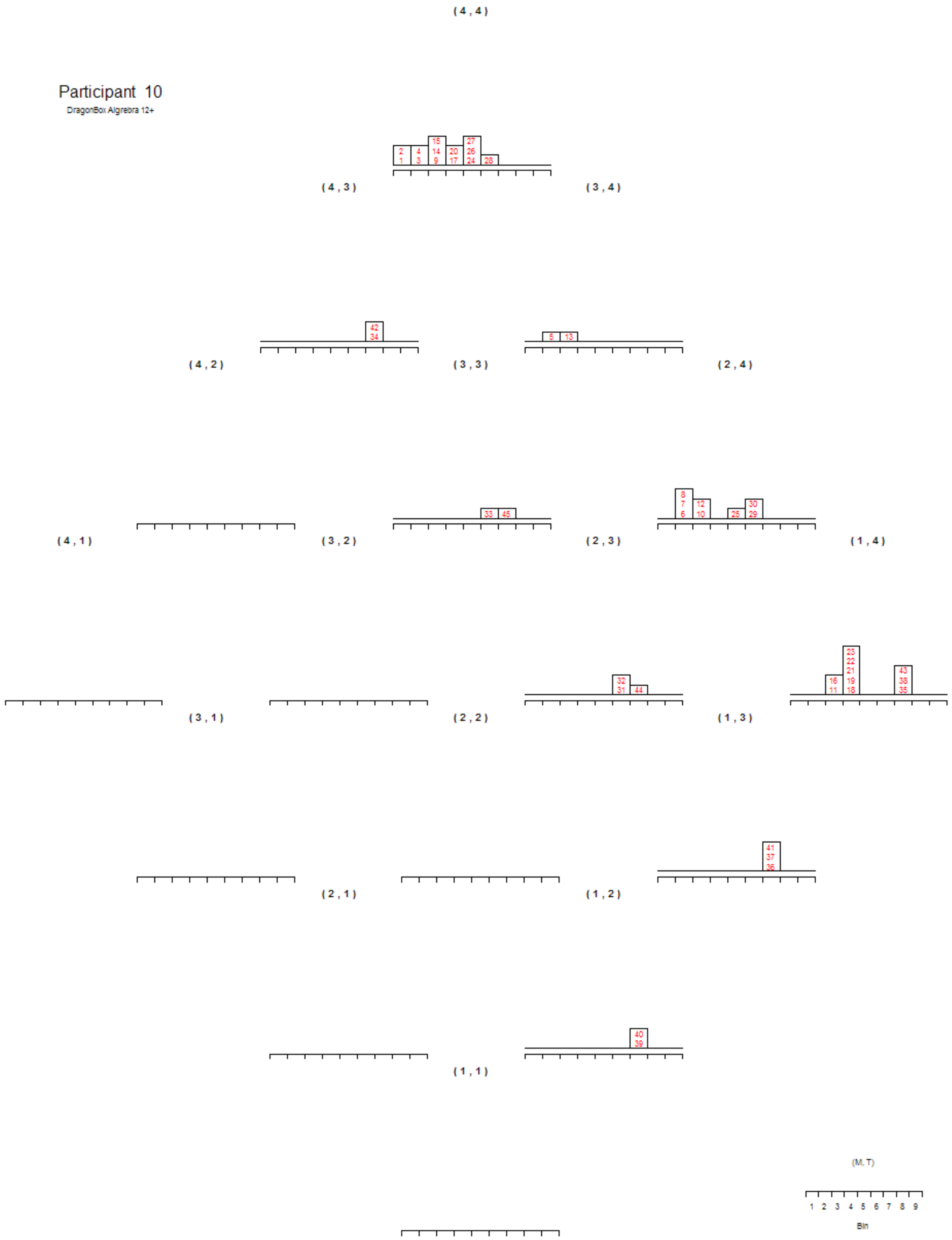


Figure 21. Rhombus plot of Participant 10's interactions with DragonBox Algebra 12+. (M,T) indicates (mathematical, technological) distance value. Bin indicates level group.

The distance values for Participant 10’s interactions with each app show that distance could change during user-app interaction. For example, during Participant 10’s interactions with Motion Math: Zoom, there was initially a low degree of both mathematical distance and technological distance (indicated by Attempts 1-6 appearing at or near the top of the plot in Figure 20). Upon reaching Bin 6, beginning with Level 15 with the needle activated, both types of distance increased (indicated by most attempts in Bin 6 appearing further from the top of the plot in Figure 20). Participant 10 eventually decreased both mathematical distance and technological distance until advancing to Level 16, whereupon there was a high degree of mathematical distance but technological distance remained low (see Table 13, Attempts 30-33). However, Participant 10’s interactions with DragonBox Algebra 12+ showed a relatively lower degree of technological distance (indicated by the greater portion of values on the upper right diagonal of the plot in Figure 21 than the plot in Figure 20) and more variation in the degree of mathematical distance (indicated by the greater concentration of values toward the bottom of the plot in Figure 20 than the plot in Figure 21). Similar to Participant 10, most participants encountered a relatively higher degree of technological distance while interacting with Motion Math: Zoom than while interacting with DragonBox Algebra 12+. There were also differences in distance between participants, such as those shown in the excerpts from the data tables (Table 14) used to generate the rhombus plots depicting the (mathematical, technological) distance values for Participant 6 and Participant 10’s interactions with Motion Math: Zoom (Figures 22 and 20, respectively).

Table 14

Excerpts from Distance Data Used to Generate Rhombus Plots for Participant 6 and Participant 10's Interactions with Motion Math: Zoom

Participant 6						Participant 10					
A	B	L	N	M	T	A	B	L	N	M	T
7	6	15	1	1	4	7	6	15	1	1	3
8	6	15	1	2	4	8	6	15	1	1	4
9	6	15	1	3	3	9	6	15	1	1	3
10	6	15	1	2	3	10	6	15	1	2	3
11	6	15	1	2	3	11	6	15	1	2	3
12	6	15	1	2	4	29	6	15	1	2	4
13	6	15	1	2	4	30	6	15	1	3	4
14	6	15	1	3	4	31	6	15	1	4	4
15	6	15	1	1	4	32	6	16	1	1	4
16	6	15	1	1	4	33	6	16	1	1	4
17	6	15	1	4	4	40	6	16	1	1	4
18	6	16	1	1	4	41	6	16	1	1	4
19	6	16	1	2	3	42	6	16	1	1	4
20	5	12	1	4	4	43	6	16	1	1	4
21	4	9	1	4	4	44	6	16	1	2	4

Table key. A indicates attempt, B indicates bin, L indicates level or chapter and level. N indicates presence of needle (0 is inactive, 1 is active). M indicates mathematical distance value; T indicates technological distance value.

The distance values for Participant 6 and Participant 10's interactions with Motion Math: Zoom provide evidence of differences in distance across their interactions with the app. Whereas Participant 10 required 25 attempts to sufficiently decrease mathematical distance and technological distance to complete Level 15 with the needle active, Participant 6 required only 11 attempts to do so (see Table 14). Participant 6 then attempted different levels in multiple bins with varying degrees of mathematical distance evident (indicated by the plot in Figure 22 showing attempts in every bin along the upper right diagonal). Participant 6 also maintained a relatively lower degree of technological

Figure 22. Rhombus plot of Participant 6's interactions with Motion Math: Zoom. (M,T) indicates (mathematical, technological) distance value. Bin indicates level group.

distance than Participant 10, which was especially evident during their interactions with levels in Bin 6 (indicated by the plot in Figure 22 showing a greater proportion of Bin 6 attempts on the upper right diagonal than the plot in Figure 20). Differences were also evident in the excerpts from the data tables (Table 15) used to generate the rhombus plots that depict the (mathematical, technological) distance values for Participant 6 and Participant 10's interactions with DragonBox Algebra 12+ (Figures 23 and 21).

Rhombus plots and tables for Participant 6 and Participant 10's interactions with DragonBox Algebra 12+ provide evidence of differences in distance across their

Table 15

Excerpts from Distance Data Used to Generate Rhombus Plots for Participant 6 and Participant 10's Interactions with DragonBox Algebra 12+

Participant 6					Participant 10				
A	B	L	M	T	A	B	L	M	T
35	7	2:12	1	4	35	7	2:12	1	4
36	7	2:12	1	3	36	7	2:12	1	3
37	7	2:12	1	3	37	7	2:13	1	3
38	7	2:12	1	4	38	7	2:13	1	4
39	7	2:12	4	4	39	7	2:13	1	2
40	7	2:13	1	3	40	7	2:13	1	2
41	7	2:13	1	4	41	7	2:13	1	3
42	7	2:13	4	4	42	7	2:11	4	3
48	7	2:17	1	4	43	7	2:12	1	4
49	7	2:17	1	4	44	7	2:12	2	3
50	7	2:17	1	4	45	7	2:13	3	3
51	7	2:17	4	4					
52	7	2:18	1	3					
53	7	2:18	1	4					
54	7	2:18	1	4					
55	7	2:18	3	4					

Table key. A indicates attempt, B indicates bin, L indicates level or chapter and level. N indicates presence of needle (0 is inactive, 1 is active). M indicates mathematical distance value; T indicates technological distance value.

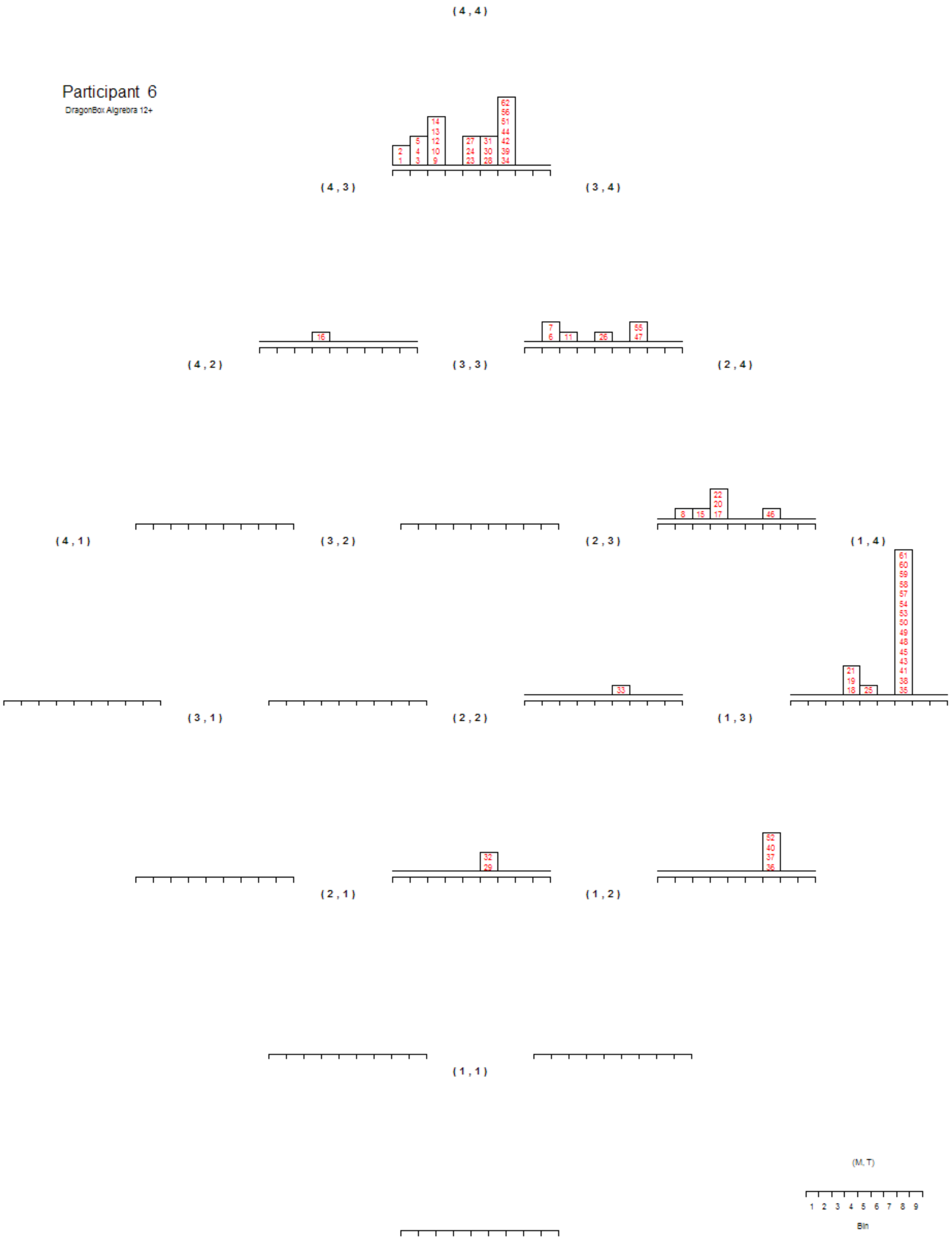


Figure 23. Rhombus plot of Participant 6's interactions with DragonBox Algebra 12+. (M,T) indicates (mathematical, technological) distance value. Bin indicates level group.

interactions with the app. Both participants exhibited the common occurrence of generally maintaining a low degree of technological distance that slightly increased shortly after beginning Bin 6 (Level 2:05) (indicated by the presence of a greater proportion of attempts outside of the upper right diagonal in Bins 6-9 than in Bins 1-5 in Figures 21 and 23). However, while Participant 6 decreased the degree of technological distance (indicated by the presence of few attempts outside of the upper right diagonal on the plot in Figure 23), Participant 10 consistently had higher degree technological distance after this point (indicated by the presence of a greater portion of attempts in Bins 6-9 outside of the upper right diagonal than on the right diagonal on the plot in Figure 21). Both participants had similar degrees of mathematical distance during interactions with levels in the same bins, though Participant 6 often had slightly less mathematical distance. Each participant reached levels where they encountered a high degree of mathematical distance, particularly in Bin 7 (specifically Levels 2:12 and 2:13), which required multiple attempts to decrease the degree of mathematical distance enough to successfully complete the levels (see Table 15). Participant 6 then continued in Bin 7 beyond Level 2:13, often cycling through degrees of mathematical distance, beginning a level with a high degree of mathematical distance and decreasing it after multiple attempts (see Table 15, Participant 6, Attempts 48-55). A similar pattern was evident by bin, wherein there was more likely to be a high degree of mathematical distance during interactions with advanced levels in a bin compared with earlier levels in the same bin. Both qualitative and quantitized qualitative analyses showed that distance did not remain static throughout user-app interactions.

Interactions between Mathematical Distance and Technological Distance

Data analysis provided evidence of interactions between mathematical distance and technological distance. Table 16 shows examples of excerpts from memos and transcripts coded as interactions between the two types of distance.

Table 16

Interactions between Mathematical Distance and Technological Distance

Motion Math: Zoom	DragonBox Algebra 12+
P04 "The intro… was really easy so I kept kinda playing… whole numbers was really easy so I changed to decimals. Hundredths was still fairly easy and thousandths…. was a little bit harder because there was more zooming in and sometimes it got a little confusing. Same with negatives cuz like I'm so used to positives where you go forward I was not used to going backward to get to a higher number." (P04 ZFT)	P02 • Tries to divide by appropriate [variable] but misses--ends up then trying to [add] (A35 L2:13) • Tries to use two [variables] at the same time (multi-touch) (A40 L2:13)
P03 • Gesture is two-hand mostly horizontal and quick--app struggles to read at times (may touch too lightly?). Chooses correct interval for zooming, but zoom becomes diagonal and [difficult] for app to recognize (A4 L4) • Reverses zoom in/out gestures and ends up traveling at inefficient interval (A6 L6) • "I knew the math, but the zooming in and zooming out part is hard." (ZFT)	P06 • Multiple extra one-dot [coefficients made] when drag/tap input mistakes. Ends up with extra moves. (A32 L2:09) • Creates extra one-dot [coefficients] while trying to combine variables before clearing [coefficient] (A33 L2:10) • Accidentally makes one-dot [coefficient] while trying to drag (A36 L2:12)
P07 "In between zero and one there is a certain amount of hundredths.… [For] one and sixty-sixty hundredths, I would go past one and estimate about how far past the bee would I zoom in to get onto the little ants. If I got farther then I might have got to thousandths." (P07 ZFT)	P01 • Missed placement… drag/tap dot [drags instead of tapping] (A44 L2:07) • Drag/tap dot (app recognizes, but actually dragging or flicking) (A49 L2:10) • Moving quickly, gestures blur (A54 L2:13)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level. ZFT indicates Motion Math: Zoom Follow-Up Questions Transcript. Quotation marks indicate direct quotes from participants. Brackets indicate clarifications.

As seen in Table 16, user-app interactions and comments provided evidence of connections between mathematical distance and technological distance. For example, Participant 4 referred to comparison and magnitude in connection to changing intervals using a specific gesture, as well as using the navigational terms “forward” and “backward” for “higher” positive and negative numbers. Participant 3 implied that the technological distance encountered when zooming made it difficult to complete the mathematical tasks and minimize the mathematical distance. This was similar to evidence from interactions with DragonBox Algebra 12+, particularly those involving Level 2:05 and beyond. Participant 6 was one of nine participants who at times chose an inappropriate gesture to perform the multiplicative identity property, which often resulted in the generation of coefficients to address (i.e., 1*X became 1*1*X), decreasing the efficiency of the solution process. In each case, the degree of mathematical distance and technological distance could influence one another, whether or not the participants were aware this occurred.

In summary, mathematical distance and technological distance were present throughout user-app interactions. The degree of each type of distance could be quantitized, organized into tables, and visualized using novel rhombus plots, which showed that the degree of mathematical distance and the degree of technological distance changed over time for each participant. Furthermore, mathematical distance and technological distance influenced each other.

Research Question 4: Relationships among Constructs

The fourth research question addressed evidence of relationships among the major constructs—attributes, affordance-ability relationships, and distance in fifth graders' interactions with mathematics virtual manipulative iPad apps. As discussed in the Methods chapter, the researcher addressed this question by analyzing the video data of user-app interactions and follow-up questions, the observation field notes, and the results of prior stages of the coding process during data analysis stages that included identifying relationships among constructs, developing a framework, and contributing to the development of theory based on the evidence resulting from the entire analysis. Analysis of the data involved coding to examine whether any relationships were present, as indicated by the presence of multiple constructs in the same portion of the data, and emergent relationships. Analysis indicated the presence of relationships between: (a) attributes and affordance-ability relationships, (b) attributes and distance, and (c) distance and affordance-ability relationships. Further analysis led to the development of a conceptual framework based on these relationships.

Attributes and Affordance-Ability Relationships

Analyses provided evidence of relationships between attributes and affordance-ability relationships. Attribute modification could lead to modification of affordance-ability relationships and vice versa. Table 17 shows examples of memo excerpts coded as relationships between attributes and affordance-ability relationships.

Table 17

Examples of Relationships between Attributes and Affordance-Ability Relationships

Motion Math: Zoom	DragonBox Algebra 12+
Change in attributes leading to change in affordance-ability relationship	
P03: Changing user attributes changes ability to access to efficient precision affordance	P04: Changing app mathematical attributes changes focused constraint affordance
● Chooses accurate places to zoom but struggles at times as [P03] tries diagonal and may touch too lightly (A5 L5) ● Sometimes reverses zoom in/out gestures and ends up traveling at inefficient interval (A6 L6) ● [No] trouble zooming (A23 L15N) ● Started zooming in from 1.00 as 1.78 came up (A37 L15N)	● Tries to drag variable across but stopped by app (A5 L1:05) ● Tries to drag one-dot across (A24 L2:01) ● New power--drag across (A54 L3:01) ● Uses drag across for direct combos (app counts as two moves) as example of efficient precision (as opposed to when this is not allowed much earlier--focused constraint) (A55 L3:02)
Change in affordance-ability relationship leading to change in attributes	
P02: Changing affordance-ability relationship leads to change in app attributes	P04: Changing affordance-ability relationship leads to change in user attributes
• Only zoomed when forced to do so--otherwise, always traveled by 1 (A5 L4) • No longer waiting to be forced to zoom by the end of this level. (A6 L5) • Uses most appropriate travel interval…. correct direction and immediate responses for L-R and zoom in-out; consistent with choice of appropriate zoom interval (A7 L6) ● [Needle] popped first task (0.05); tried 0.5 placement even after app filled the empty space. (A8 L15N)	• Tries to drag variable across but stopped by app (A5 L1:05) • Tries to drag one-dot across (A24 L2:01) • New power--drag across (A54 L3:01) ● Uses drag across for direct combos (app counts as two moves) as example of efficient precision (as opposed to when this is not allowed much earlier--focused constraint) (A55 L3:02)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level. Brackets indicate clarifications.

As seen in Table 17, relationships between attributes and affordance-ability relationships were present throughout the interactions. Attribute modification contributed to modification of affordance-ability relationships. Modifying user attributes led to modification of ability, and thus also modification to affordance-ability relationships. For example, improving the motor skills required to hone the zoom input while interacting with Motion Math: Zoom contributed to Participant 3 choosing to incorporate planning as part of efficient precision. Modification to affordance-ability relationships could also lead to modification of app attributes, such as when Participant 2 increased accession of efficient precision by traveling using efficient intervals while interacting with Motion Math: Zoom, which contributed to advancement through levels with different content.

Participant 4's interactions with DragonBox Algebra 12+ provided examples of how modifying app mathematical attributes modified the focused constraint affordance, and how these modifications to the affordance-ability relationship led to modification of user attributes. Participant 4 initially tried to combine the additive inverse property and the additive equality property to move a variable from one side of an equation to the other, but the app prohibited this in early levels. After completing a series of levels during which the additive inverse property and the additive equality property were performed only in separate steps, the level content changed to focus on combining these properties, permitting the "drag across" move. After this, Participant 4 began combining the properties when possible, providing evidence of a change in the participant's mathematical attributes. Some attributes contributed to multiple affordance-ability relationships. For example, coordination (user technological: motor skills) contributed to accession of planning (efficient precision) and navigation restrictions (focused constraint). Improving coordination could lead to a different approach to planning and avoidance of encountering navigation restrictions. Thus, attribute modification often led to modified affordance-ability relationships, and vice versa.

Attributes and Distance

Analyses provided evidence of relationships between attributes and distance. Attribute modification often led to modification of distance, while modification of distance could also contribute to modification of attributes. Table 18 shows examples of memo excerpts coded as relationships between attributes and distance.

As seen in Table 18, interactions with each app provided evidence of relationships between attributes and distance, which were often present when either attributes or distance changed. When advancing to a new level, mathematical distance often increased. Participants then attempted to decrease mathematical distance by modifying user mathematical attributes. Some participants proactively modified app attributes to decrease mathematical distance, providing an environment in which to improve user attributes, leading to decreased distance when encountering levels that were initially too challenging (e.g., P10 A40-45). Users also modified user technological attributes to align with requirements for interacting with levels within the apps. Modification of structural attributes (e.g., needle timer) and personal attributes (e.g., goals) also influenced mathematical distance and technological distance during these interactions. Cross-referencing rhombus plots and data representing (mathematical, technological) distance values (e.g., rhombus plots: Appendix F; data: Appendix G) with the presence of attributes (e.g., Tables 2 and 3; level bins in Appendix E) and attribute modification (e.g., Table 6) also supported connections between distance and attributes. Table 19 includes data excerpts used to make annotated (mathematical, technological) distance rhombus plots (Figures 24 and 25) for Participant 5.

Table 18

Examples of Relationships Between Attributes and Distance

Mathematical attributes and mathematical distance

App attribute change: Advance level, increase distance: P05

- Navigates with multiple intervals (ones and tens) (A4 L3)
- Chose level 15…. Tried to place 0.04 at 0.4. Zoomed in at non-ideal range. For 1.81 from 1.00, zoomed in to hundredths at 1.00…. Non-ideal [range] selection for zooming in and out. Often out too far [and] never in where ideal. (A5 L15)

App attribute change: Reduce level, decrease distance: P03

- Mistakes in division lead to extended, systematic attempts to try nearly everything (with many math mistakes, extra moves, etc.) Eventually exits level. (A36 L2:13)
- Tries to divide top to bottom, then immediately corrects. [Makes errors but completes level] (A37 L2:10)

User attribute change: Improve mathematical content knowledge, decrease distance: P05

- Choice of [range] to zoom still far from ideal…. Rushes past a handful of intended targets in both directions. (A15 L10)
- Chooses appropriate travel interval (from zooming out) but inefficient zoom placement (zoom in range)…. By the end of the level, choosing more efficient ranges for between-interval travel. Much faster than previous attempt. (A16 L10)

Reduced distance allows for user attribute modification: P10

- Restart via Undo. (A40 L2:13)
- Exit to menu. (A41 L2:13)
- [Completes] level where “forgotten” power was demonstrated. (A42 L2:11)
- Correct division... [completes level] (A44 L2:12)
- Begins with addition… [completes level] (A45 L2:13)

Technological attributes and technological distance

App attribute change: Allow zoom input: P01

- Smooth navigation via swiping…. No choice for within or between interval travel (all swiping) (A2 L2)
- Zooms out/in/out/in to find 24 and uses two fingers on one hand…. odd choice of gesture--may have been trying to acclimate (A3 L3)

User attribute change: Perform precise tap input: P01

- Drag/tap dot difficulty (A41 L2:05)
- Tapped dots and completely ignored division on non-box side (A42 L2:06)

Reduced distance allows for user attribute modification: P05

- Zooms out instead of zooming in (A8 L15)
- Zooms out to find 10, then appropriate [range to] zoom in for 27, out for 35 with appropriate [range to zoom] in. [Zooming more constrained on this level] (A9 L3)

(*table continues*)

Structural attributes and distance

Presence of needle (timer) influences mathematical distance and technological distance: P08

- Not controlling zoom gesture makes difficult zoom placement (A3 L15)
- For 0.05, zooms in at 0.4, hesitates, moves toward 0.5, then toward 0.05 but nowhere close when popped by needle. (A4 L15N)
- Much faster zoom and swipe. Mixes zoom in/out at times, which influences math placement. Uncontrolled zooming gesture, especially out. Near ideal range attempts but sometimes misses because of zoom gesture. (A5 L15N)

Personal attributes and distance

Goal change (quick completion) increases mathematical distance: P09

- Makes into opposite before dividing (as if adding), then tries to interrupt... [divides] incorrectly again. Restarts. [No trouble with input] (A38 L2:11)
- New power (again) correct this time. Uses drag for [multiplicative identity. Rushes on repeat.] (A39 L2:11)

Goal change (planning) increases technological distance: P06

- Some hitches when zooming in/out… (deciding how far to zoom in and trying to do so) (A10 L15N)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level, N indicates presence of needle.

Table 19

Excerpts from Distance Data Used to Generate Rhombus Plots for Participant 5's Interactions with Motion Math: Zoom and DragonBox Algebra 12+

Motion Math: Zoom						DragonBox Algebra 12+				
A	B	L	N	M	T	A	B	L	M	T
3	2	2	0	3	4	35	7	2:11	4	4
4	2	3	0	4	3	36	7	2:12	1	4
5	6	15	0	2	3	39	7	2:13	1	4
6	6	15	0	2	3	43	7	2:14	4	4
7	6	15	0	1	3	44	7	2:15	1	4
8	6	15	0	1	2	46	7	2:16	4	4
9	2	3	0	3	3	47	7	2:17	4	3
10	2	4	0	3	3	48	7	2:18	1	3
						51	7	2:19	1	4
						53	7	2:20	1	4

Table key. A indicates attempt, B indicates bin, L indicates level or chapter and level. N indicates presence of needle (0 is inactive, 1 is active). M indicates mathematical distance value; T indicates technological distance value.

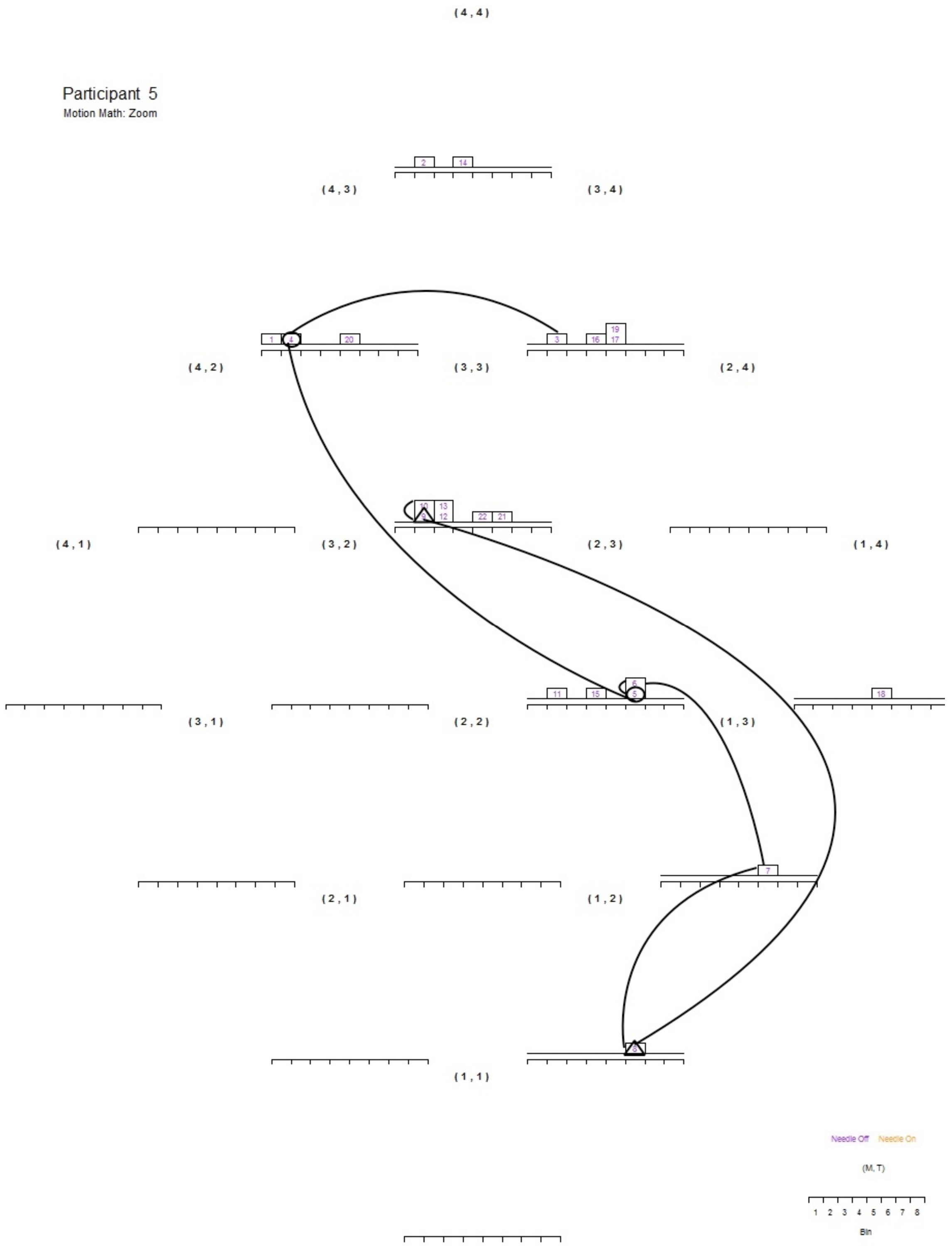


Figure 24. Annotated rhombus plot of Participant 5’s interactions with Motion Math: Zoom. (M,T) indicates (mathematical, technological) distance value. Bin indicates level group.

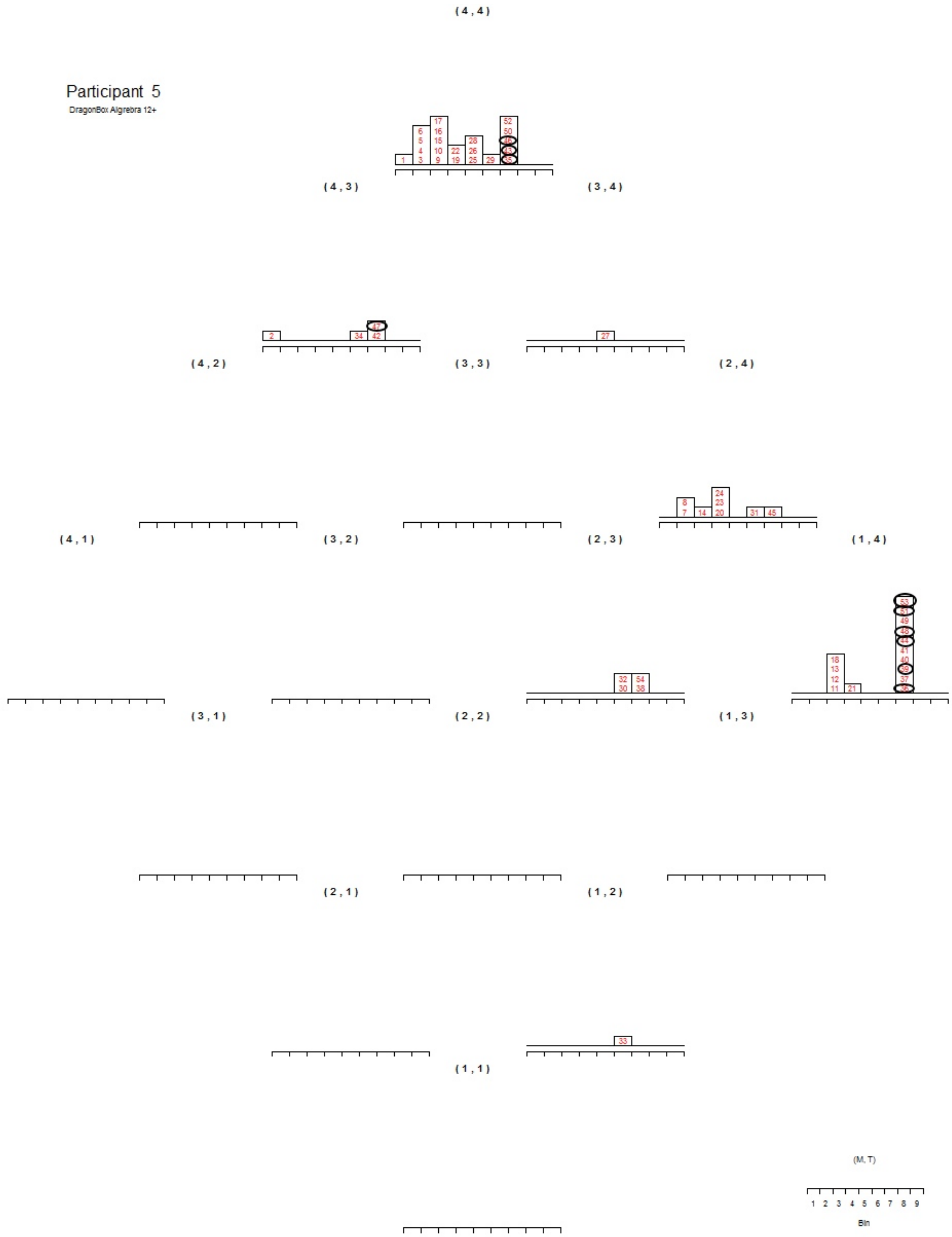


Figure 25. Annotated rhombus plot of Participant 5's interactions with DragonBox Algebra 12+. (M,T) indicates (mathematical, technological) distance value. Bin indicates level group.

As seen in Table 19, Figure, 24, and Figure 25 Participant 5's interactions with each app provided evidence of some common occurrences among the participants. For example, Participant 5's interactions with DragonBox Algebra from Bin 6 onward provided evidence of the common occurrence wherein technological distance increased after the introduction of tapping to perform the multiplicative identity property (as indicated in Figure 25, where only Bins 6 and 7 have multiple attempts that are not on the upper right diagonal). However, akin to most participants, Participant 5 encountered a relatively higher degree of technological distance while interacting with Motion Math: Zoom (as indicated by the greater proportion of attempts not on the upper right diagonal in Figure 24 than in Figure 25, and the T columns in Table 19), which required multi-touch input.

Figure 24 annotations and Table 19 Motion Math: Zoom data highlight another common occurrence seen while interacting with Motion Math: Zoom. Participant 5 encountered increased mathematical distance upon skipping from Bin 2 to Bin 6 (as indicated by the downward trajectory of the path from Attempt 4 to Attempt 5, denoted by oval endpoints, in Figure 24, and the corresponding Motion Math: Zoom data in Table 19), which corresponded with a change in app attributes from a focus on whole numbers to including decimals to the hundredths place. Upon proactively modifying app attributes to focus on whole numbers by returning to Level 3, mathematical distance decreased (as indicated by the upward trajectory of the path from Attempt 8 to Attempt 9, denoted by triangle endpoints in Figure 24, and the corresponding Motion Math: Zoom data in Table 19).

These relationships could also be cyclical, such as during interactions with DragonBox Algebra 12+, which introduced new properties through isolated demonstration followed by application in scenarios that generally became more complex until beginning the next cycle. Figure 25 annotations and Table 19 DragonBox Algebra 12+ data highlight this cycle. This cycle was particularly evident in Participant 5's interactions with levels from Bins 6 and 7, during which there was a low degree of mathematical distance on the introduction level, with initial attempts on subsequent levels in the bin often showing a higher degree of mathematical distance (as indicated by the circled attempts in Figure 25 and the DragonBox Algebra 12+ data in Table 19). In these instances, participants frequently required multiple attempts to decrease the mathematical distance enough to advance to the next level. Thus, attribute modification often led to distance modification, and vice versa.

Distance and Affordance-Ability Relationships

Analysis provided evidence of relationships between distance and affordance-ability relationships. Accession of affordances could influence distance, and distance could influence accession of affordances. Table 20 shows excerpts from transcripts and memos coded as examples of relationships between distance and affordance-ability relationships.

As seen in Table 20, relationships between distance and affordance-ability relationships were present throughout the interactions. Affordance-ability relationships could influence distance, such as when Participant 10 implied that the difficulty of levels in DragonBox Algebra 12+ depended on what combination of content was present. After

Table 20

Relationships between Distance and Affordance-Ability Relationships

Example	Description
"It starts off easy and then gets harder and it tells you what [math] to do at first and then you do that on your own on the next one." (P10 DBFT)	Focused constraint influencing mathematical distance
[It would be better] "if it wouldn't zoom in as far. You zoom in once and it goes way too far." (P03 ZFT)	Focused constraint influencing technological distance
Not direct combo (missed) first time--then not second time either, but did not attempt to direct combo. (May have thought not possible from imprecise tech input) (P08 A33 L2:03)	Technological distance influencing accession of efficient precision
"The whole thing is frustrating. Makes you wanna punch something or throw something at someone…. it [zooms] on the wrong spot and I have to start over [because the bubble is popped]. (P01 ZFT)	Technological distance influencing accession of motivation
"To make the app better they would have to make it so you can zoom better so if you wanna zoom out it perfectly zooms out where you wanna be—when you let go it just keeps going." (P01 ZFT)	Technological distance and mathematical distance influencing accession of simultaneous linking
Ideal for 0.05 (each time using [navigation restriction] to stop swipe at 0). (P09 A16 L15N)	Efficient precision influencing mathematical distance
[Quit before full time elapsed] "It was easy, but it just wouldn't let me pass the level I was on for some reason. Well it was easy, but it wouldn't give me enough time to do stuff because it was super-hard to get to areas you wanted to go to." (P08 ZFT)	Perception of low degree of mathematical distance and high degree of technological distance influencing accession of motivation

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level. DBFT indicates DragonBox Algebra 12+ Follow-Up Questions Transcript. ZFT indicates Motion Math: Zoom Follow-Up Questions Transcript. Quotation marks indicate direct quotes from participants. Brackets indicate clarifications.

a level focusing solely on introducing the new power (i.e., mathematical property), the app provided a more challenging level that required the user to apply the newest power with other powers to solve, usually resulting in an increase in mathematical distance.

Distance could also influence affordance-ability relationships. Participant 1's comments about the apps showed that a great degree of (technological) distance could influence access to (negative) motivation, while a great degree of mathematical and technological distance could influence access to simultaneous linking. Participant 8's comments about Motion Math: Zoom showed that perceived interactions between mathematical distance and technological distance could influence accession of motivation. Observations suggested that Participant 8 was not entirely fluent with the mathematical content and struggled with some of the technological input required for success in the chosen levels, and the degree of mathematical distance and technological distance contributed to Participant 8's decision to stop interacting with the app. Thus, distance modification often led to modified affordance-ability relationships, and vice versa.

Conceptual Framework

Overall analysis provided evidence of interconnected relationships among attributes, affordance-ability relationships, and distance throughout user-app interactions. These relationships were used to develop the proposed Modification of Attributes, Affordances, Abilities, and Distance (MAAAD) for Learning framework for user-tool interactions (see Figure 26).

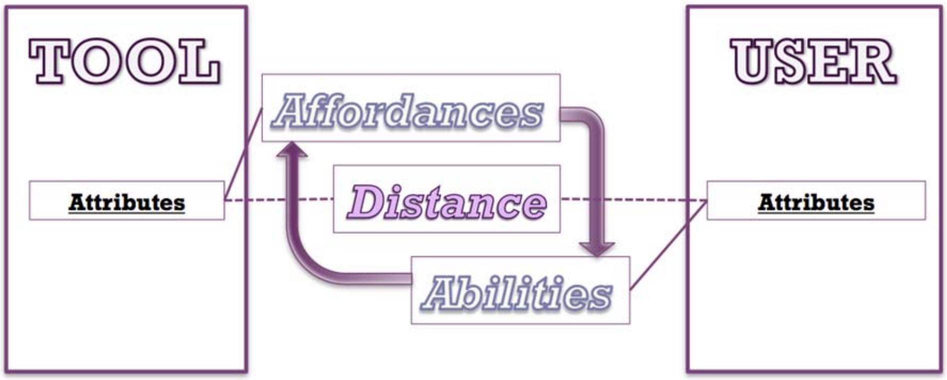


Figure 26. Modification of Attributes, Affordances, Abilities, and Distance for Learning framework.

The MAAAD for Learning framework begins with attributes. The difference between relevant tool and user attributes or attribute clusters forms distance. Attribute modification may align attributes (e.g., the user masters the content the tool presents) and decrease distance, or it may misalign attributes (e.g., the tool presents more challenging content when a user is successful), and increase distance. Clusters of user attributes form abilities, each of which relate to specific affordances stemming from clusters of tool attributes. Variations in user attributes lead to different approaches or degrees of affordance access. A given attribute may contribute to multiple affordance-ability relationships and to distance. Distance also influences affordance-ability relationships, as a greater degree of distance from misaligned attributes can lead to different affordance access than when attributes are aligned and a lesser degree of distance is present. An expanded version of the MAAAD for Learning framework as applied to learning mathematics through user-app interactions appears in Figure 27.

An expanded version of the MAAAD for Learning framework developed as a result of the data analyses of user interactions with mathematics virtual manipulative iPad apps in this study includes both the attribute categories and subcategories and the distance types identified from the results of the research questions. The difference between relevant clusters of app mathematical attributes and user mathematical attributes is the mathematical distance, while the difference between relevant clusters of app technological attributes and user technological attributes is the technological distance. Clusters of user mathematical attributes, user technological attributes, and personal attributes form abilities used to access app affordances, which stem from clusters of app

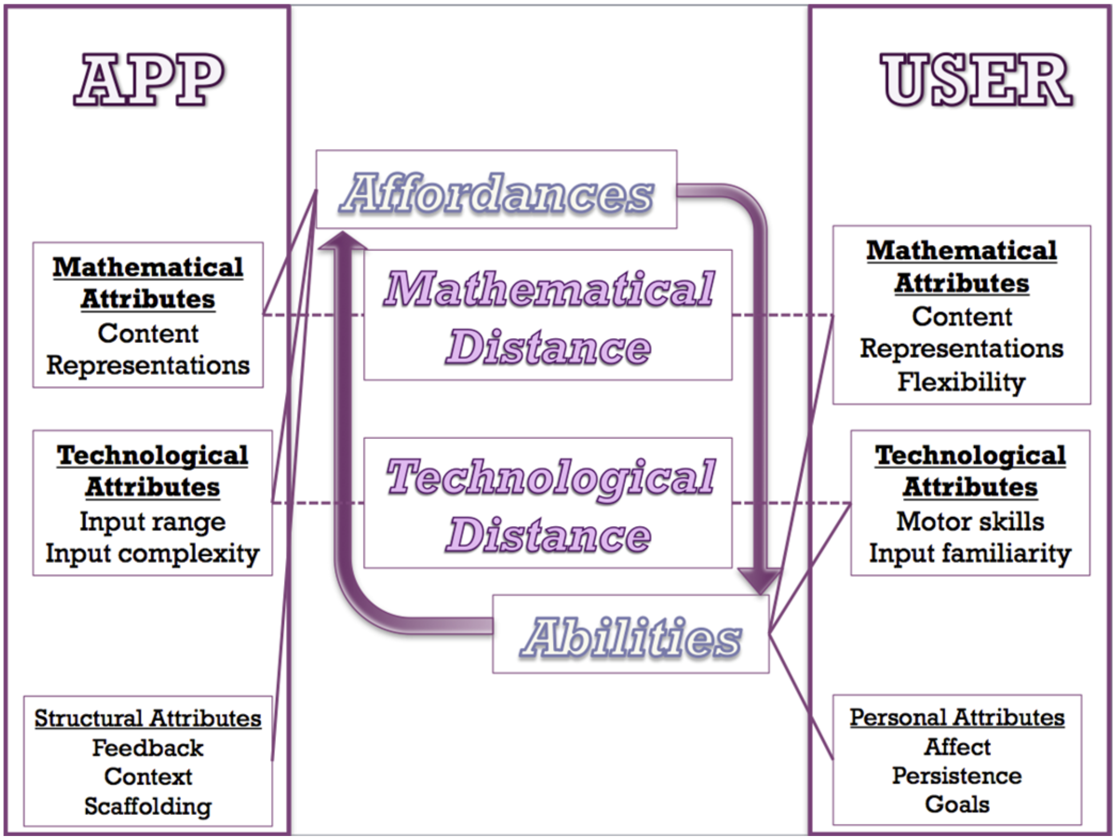


Figure 27. Expanded version of the Modification of Attributes, Affordances, Abilities, and Distance for Learning framework applied to learning mathematics through user-app interactions.

mathematical attributes, app technological attributes, and structural attributes. Affordance-ability relationships can influence other affordance-ability relationships, as each attribute can contribute to multiple affordance-ability relationships. Mathematical distance and technological distance can interact, and both types of distance can influence affordance-ability relationships, which can contribute to variations in affordance access.

Example of conceptual framework from interactions with DragonBox Algebra 12+. Participant 1's interactions with DragonBox Algebra 12+ provided evidence of the integrated MAAAD for Learning framework, as shown in Table 21 which pairs memo excerpts with (mathematical, technological) distance values.

Table 21

Excerpts from Participant 1's Interactions with DragonBox Algebra 12+ Paired with (Mathematical, Technological) Distance Values

Excerpt	(M,T)
Solution…. repeatedly tries to quickly tap--too quickly for app to recognize… (frustration?) (A53 L2:13)	(1,3)
Restart [at end]: fails to replicate solution…. ends up dividing far too many unneeded [variables], repeatedly tries to combine terms in denominator... [voices] frustration…. once frustrated [and] moving quickly, gestures blur together and repeatedly attempts to make disallowed moves (A54 L2:13)	(1,2)
Restart [at end]…. divides too soon--before clearing, lets it sit a moment, does not finish the incorrect division before using restart (A55 L2:13)	(1,4)
[Calmer.] Very hesitant to divide second side each time…. drag/tap dot (app recognizes, but actually dragging/flicking) [Completes level, ends app interaction] (A56 L2:13)	(4,4)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level. M indicates mathematical distance value; T indicates technological distance value. Distance values are 1-4, where 1 is a high degree of distance and 4 is a low degree of distance. Brackets indicate clarifications.

As seen in Table 21, excerpts from Participant 1's interactions with DragonBox Algebra 12+ highlight connections among the relationships that form the MAAAD for Learning framework. During Attempt 53, Participant 1 proactively used the solution scaffold to determine how to complete Level 2:13. However, Participant 1 did not succeed in replicating the solution during Attempt 54 and showed signs of frustration and a high degree of access to negative motivation. Furthermore, there was a higher degree of technological distance as Participant 1 blurred the tap and drag input gestures. During Attempt 55, Participant 1's goal changed to accurate completion. Participant 1 used deliberate, relatively precise gestures to decrease technological distance, and recognized the missed application of reverse order of operations. Upon restarting the level for Attempt 56, Participant 1 correctly applied the properties using the reverse order of operations, providing evidence of changing mathematical attributes leading to a decrease in mathematical distance. Participant 1 also continued to apply appropriate gestures, which contributed to decreasing technological distance and the ability to access the affordance of simultaneously linking actions with mathematical representations. Figures 28-31 set these interactions in the context of the MAAAD for Learning framework.

As Figures 28-31 demonstrate, relationships within the MAAAD for Learning framework were evident throughout Participant 1's interactions with DragonBox Algebra 12+. For example, Participant 1 initially showed a high degree of access to negative motivation. Through modification of user attributes (e.g., mathematical: reverse order; technological: tap and drag input; personal: goals of quick vs. accurate completion) and app attributes (structural: use of solution scaffolding), Participant 1 decreased both

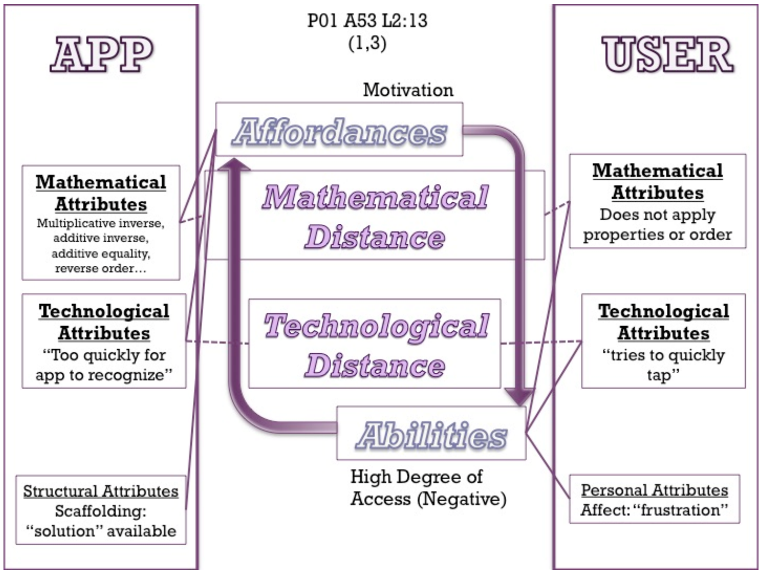


Figure 28. Applying MAAAD for Learning: Participant 1, Part 1. Participant 1 attempted to decrease mathematical distance due to unaligned mathematical attributes through proactive modification of app structural attributes (solution scaffolding). Participant 1 showed a high degree of access to negative motivation.

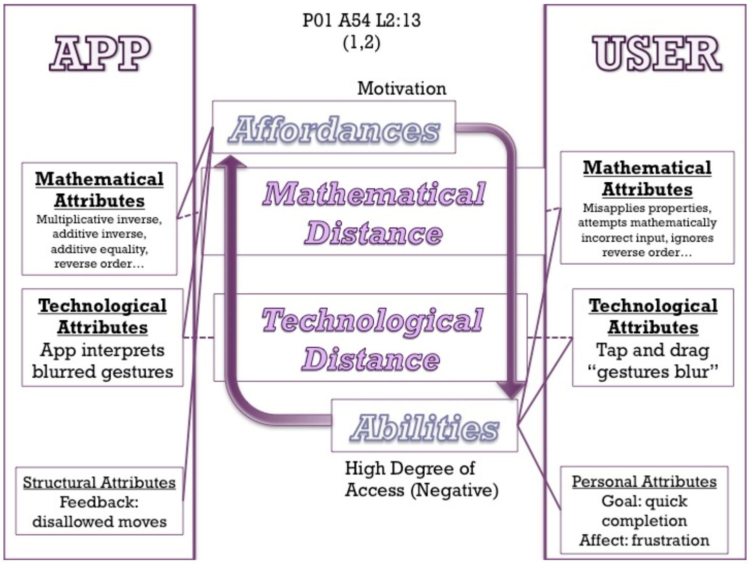


Figure 29. Applying MAAAD for Learning: Participant 1, Part 2. Participant 1 failed to correctly replicate solution while attempting to quickly complete the level. A high degree of mathematical distance remained and technological distance increased as Participant 1 struggled to make the app recognize some input gestures. Participant 1 continued to have a high degree of access to negative motivation and ended the attempt by resetting the level.

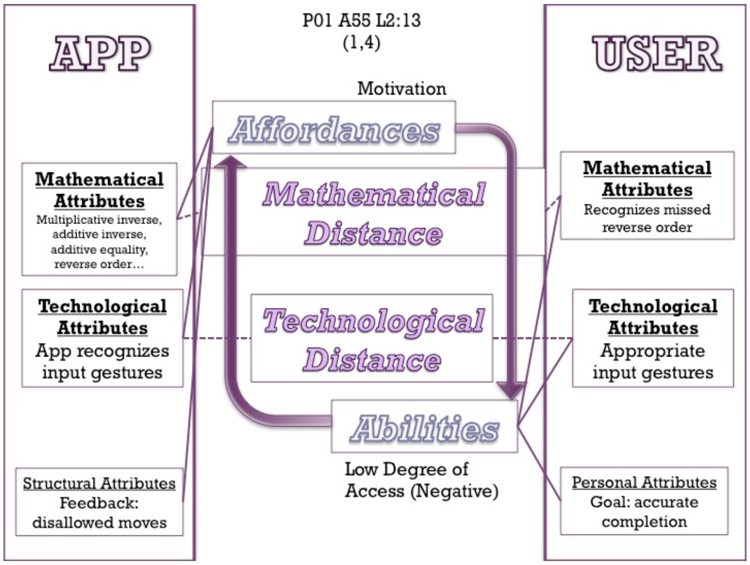


Figure 30. Applying MAAAD for Learning: Participant 1, Part 3. Participant 1 slowed and attempted to accurately complete the level, but failed to correctly replicate solution. However, Participant 1 noticed the missed use of the reverse order of operations for solving. A high degree of mathematical distance remained but technological distance decreased as Participant 1 produced recognizable input gestures. Participant 1 reduced the degree of access to negative motivation and ended the attempt by resetting the level.

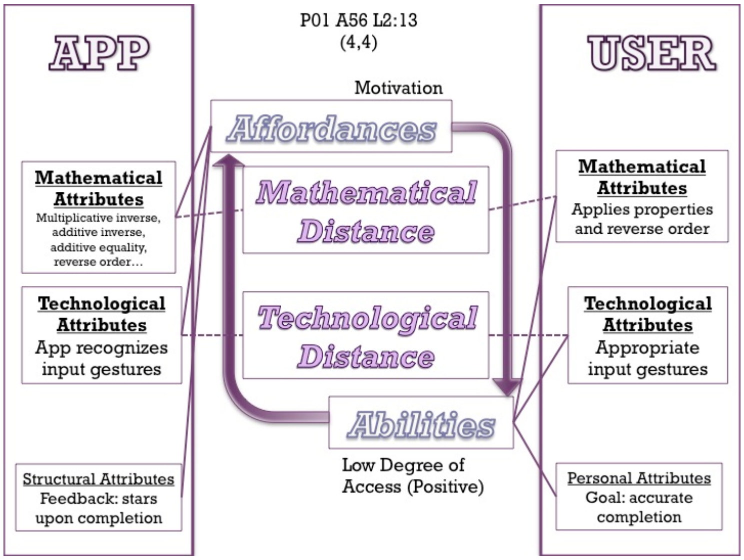


Figure 31. Applying MAAAD for Learning: Participant 1, Part 4. Participant 1 further slowed interactions and accurately completed the level, having changed mathematical attributes and reduced mathematical distance by correctly applying the properties in the correct (reverse) order. Participant 1 also reduced technological distance, completing the level without struggling to perform recognizable input gestures. Participant 1 showed a low degree of access to positive motivation.

mathematical distance and technological distance. This same process influenced access to motivation to shift from a high degree of negative motivation to a low degree of positive motivation.

Example of conceptual framework from interactions with Motion Math: Zoom. Participant 6's interactions with Motion Math: Zoom provided evidence of the integrated MAAAD for Learning framework, as shown in Table 22, which pairs memo excerpts with (mathematical, technological) distance values.

As seen in Table 22, excerpts from Participant 6's interactions with Motion Math: Zoom highlight connections among the relationships that form the MAAAD for Learning framework. During Attempt 15 (Level 15 with needle active), Participant 6 initially struggled to navigate using hundredths and tenths on the number line, leading to a high

Table 22

Excerpts from Participant 6's Interactions with Motion Math: Zoom Paired with (Mathematical, Technological) Distance Values

Excerpt	(M,T)
1/10 [complete] In correctly for 0.05 but far off for 0.13--tries to swipe beyond 1.0 at tenths. (Focused constraint restricts further rightward navigation) Rushing [while attempting to plan] (A15 L15N)	(1,4)
(All complete) Zooming inefficient as more planning sometimes efficient but sometimes slows overall process. For 1.00 to 1.53, chooses appropriate place to zoom in (not quite ideal) and pauses briefly at tenths to adjust. When placing 0.01, zooms out to 1 before [feedback] (Balance of sufficient accuracy with lots of speed--and memory of type of upcoming task for planning) (A17 L15N)	(4,4)
0/11 [complete] 0.10 from tenths (0.7)--right first, then zoomed in at 0.5 to travel by hundredths. (A18 L16N)	(1,4)
Returns to menu, chooses level 12. Completes. Planning zoom out not always effective in this level [because] of new prompt and does not continue [planning]. (A20 L12N)	(4,4)

Note. Alphanumeric sequences in parentheses indicate references to specific memos. P indicates participant, A indicates the attempt number, L indicates level or chapter and level. M indicates mathematical distance value; T indicates technological distance value. Distance values are 1-4, where 1 is a high degree of distance and 4 is a low degree of distance. Brackets indicate clarifications.

degree of mathematical distance. During Attempt 17 (Level 15 with needle active), Participant 6 decreased mathematical distance by effectively applying understandings of comparison and magnitude to hundredths on the number line. Modifying these attributes also helped Participant 6 modify the ability to access the planning affordance, shifting from creativity toward efficiency. The app then modified mathematical attributes, presenting new tasks using similar content in Level 16 (Attempt 18), resulting in increased mathematical distance as Participant 6 struggled to flexibly transfer the understandings used in the previous level. Participant 6's ability, based on these and other attributes, also led to a low degree of access to efficient precision of consistent range contents (e.g., 0.0-0.1 always contains 0.11, 0.12, etc.). Participant 6 then proactively modified app attributes, reducing the level (i.e., Level 12 from Level 16), which presented different mathematical content (i.e., tenths instead of hundredths). This decreased mathematical distance, as Participant 6 effectively applied understandings of comparison and magnitude to tenths on the number line to complete these tasks. Figures 32-35 show these interactions using the MAAAD for Learning framework.

As Figures 32-35 demonstrate, relationships within the MAAAD for Learning framework were evident throughout Participant 6's interactions with Motion Math: Zoom. For example, Participant 6 consistently attempted to plan when it was possible to do so. Participant 6 initially experimented with a creative approach to planning, but this contributed to failure to complete the level. Modifying the approach to planning to

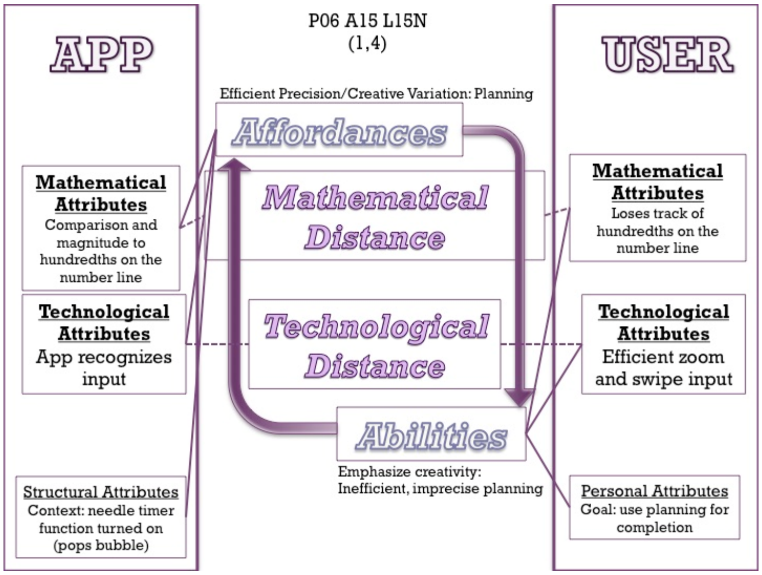


Figure 32. Applying MAAAD for Learning: Participant 6, Part 1. Participant 6 experimented with planning as part of completing the level but lost track of hundredths on the number line, showing a high degree of mathematical distance. Time ran out to complete a task in the level and the needle popped the bubble. There was a low degree of technological distance because the app recognized the efficient input gestures. Participant 6 restarted the level.

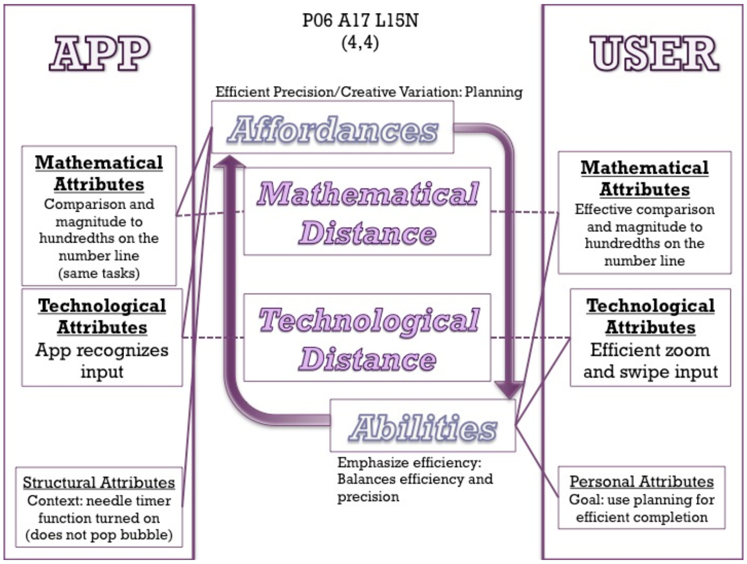


Figure 33. Applying MAAAD for Learning: Participant 6, Part 2. Participant 6 modified user mathematical attributes to effectively navigate hundredths on the number line and decreasing mathematical distance. Participant 6 changed approach to planning, planning with a balance of efficiency and precision instead of experimenting. Technological distance remained minimal.

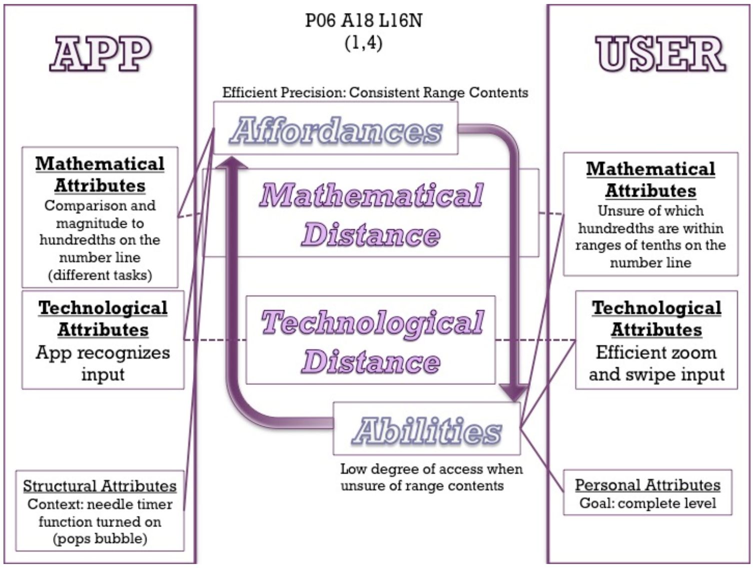


Figure 34. Applying MAAAD for Learning: Participant 6, Part 3. The app changed mathematical attributes, presenting different tasks that focused on similar content. Mathematical distance increased as Participant 6 was unable to effectively transfer understanding of ranges to the new tasks. Participant 6 struggled to access the affordance of efficient precision related to consistent range contents. Technological distance remained minimal.

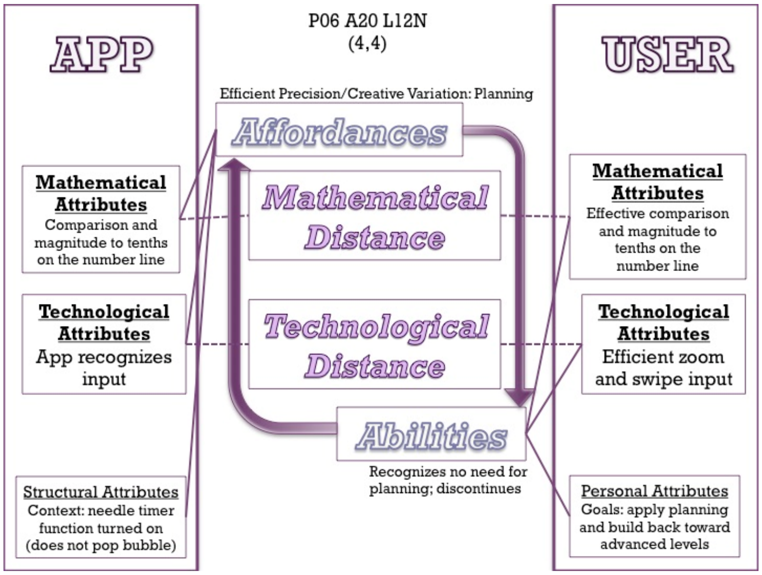


Figure 35. Applying MAAAD for Learning: Participant 6, Part 4. Participant 6 proactively modified app attributes by reducing the level and changing mathematical content to focus on tenths, decreasing mathematical distance. Technological distance remained minimal. During this attempt, Participant 6 discontinued planning after recognizing it was not necessary to complete the tasks.

emphasize efficiency, coupled with modification of mathematical attributes, contributed to decreasing mathematical distance and successful completion of Level 15. The degree of mathematical distance was so great during Participant 6’s first attempt at Level 16 that Participant 6 did not complete a single task and thus did not have the opportunity to plan. Upon decreasing mathematical distance by proactively changing app attributes via choosing to attempt Level 12, Participant 6 was once again able to access the planning affordance, but discontinued planning after realizing it was not necessary to do so. As shown in the figure sequences and coding excerpts, constructs and relationships that form the MAAAD for Learning framework were present throughout the participants’ embodied interactions with mathematics virtual manipulative iPad apps.

Summary of the Results

In summary, the results of this study showed that attributes, affordance-ability relationships, and distance were present when fifth-grade children interacted with mathematics virtual manipulative iPad apps, and that there were relationships among these constructs. The relationships were used to develop the MAAAD for Learning framework. The results of this study showed specific subcategories and variations of the constructs. Apps have mathematical attributes, technological attributes, and structural attributes, while users have mathematical attributes, technological attributes, and personal attributes. Attributes were not always aligned, and reactive attribute modification was common, but proactive modification also occurred. Affordance-ability relationships aligned with Moyer-Packenham and Westenskow's (2013) affordance categories, primarily varied by approach or degree, and could influence one another. Both mathematical distance and technological distance were present, and the degree of each distance varied throughout the interactions.

The results of the study also showed relationships among the focus constructs. A given attribute may contribute to multiple relationships. Clusters of user attributes form abilities in direct relation to specific affordances, which stem from clusters of app attributes. Differences between clusters of app attributes and user attributes formed distance. Attribute modification often led to modification of affordance-ability relationships or distance, and vice versa. Distance modification could also lead to modification of affordance-affordance-ability relationships and vice versa. The integration of these relationships is depicted by the MAAAD for Learning framework.

CHAPTER V

DISCUSSION

The increasing use of technology in mathematics education requires detailed examinations of constructs that influence users' mathematical interactions with technology, including mathematics virtual manipulative iPad apps. The purpose of this study was to conceptualize the relationships among attributes, affordances, abilities, and distance in a framework that describes the nature of children's interactions with technology to learn mathematics, here set within fifth-grade children's interactions with mathematics virtual manipulative iPad apps. Analyses focused on user-app interactions, which involved physical interactions with representations of mathematics to provide evidence of mathematical practices, including mathematical thinking and learning. Results indicated that the focus constructs were present in the interactions, and interpretation led to the development of the MAAAD for Learning framework. Emergent themes also included variations and change within the constructs and interrelationships among the constructs. Importantly, modification of examples of any of these constructs led to modification of relevant examples of the other constructs. The proposed MAAAD for Learning framework provides a structure for examining user-app interactions in mathematics, and the framework has broader implications for characterizing children's interactions with educational technology.

The discussion of the results has four sections. The first section discusses the results concerning individual constructs: attributes, affordances-ability relationships, and distance. The second section discusses the relationships among the constructs and the emergent MAAAD for Learning framework. The third section discusses the limitations of the study. The fourth section discusses the study's implications and potential applications for those who develop, implement, and research educational technology, particularly mathematics apps.

Attributes, Affordance-Ability Relationships, and Distance

The first group of results from the study concerned the presence of and patterns related to attributes, affordance-ability relationships, and distance in user-app interactions.

Attributes

The first research question addressed the presence of attributes in user-app interactions. User attributes and app attributes were consistently present during the user-app interactions and could be categorized using similar structures. Attribute modification to align or misalign attributes was common, but users were more likely to reactively modify attributes than to proactively modify attributes. Together, these results indicate that attributes and the modification of attributes were part of user-app interactions for the participants in this study. App attributes were mathematical (e.g., content: integers; representation: number line), technological (e.g., input range: multi-touch; input complexity: swipe), and structural (feedback: points; context: timer; scaffolding: demo). The presence of app attributes observed in this study is consistent with literature that describes mathematical content of apps (e.g., fraction models: Rick, 2012), technological capabilities of apps relating to input (e.g., input gesture range: Byers & Hadley, 2013), and structural aspects of apps (e.g., scaffolding: Belland & Drake, 2013; feedback: Blair, 2013). User attributes were mathematical (e.g., content: integers; representation: number line; flexibility: transfer), technological (e.g., motor skills: coordination; input familiarity: input recognition), and personal (affect, goals, persistence). The presence of user attributes observed in this study is consistent with literature that describes children's mathematical understandings (e.g., fraction models: Moyer-Packenham, Bolyard, et al., 2014), physical actions used to interact with apps (e.g., motor skills: Ginsburg et al., 2013), and personal characteristics of users (e.g., affect: Goldin, Epstein, Schorr, & Warner, 2011; persistence: Jong, Hong, & Yen, 2013). The specific attribute categorizations were a novel finding of this study.

Attributes or clusters of attributes could be aligned (e.g., performing appropriate gestures) or misaligned (e.g., attempting to add instead of divide). This led to attribute modification, which could be reactive (e.g., allowing the app to repeat a level) or proactive (e.g., purposefully choosing a level). Attribute modification has been reported in other research (e.g., Parsons & Sedig, 2014) and relates to progressive mastery, wherein learners continue to develop skills while interacting with technology (Murray & Arroyo, 2002). However, proactive and reactive attribute modification types are novel findings of this study. These results are important because they identify attributes as a construct that contributes to user-app interactions, aligning with findings and implications of several studies.

From an embodied cognition and representation perspective, results concerning attributes are important because they indicate that attributes play a role in mathematical thinking and learning. The recorded interactions provided evidence of mathematical thinking (i.e., bodily interactions with mathematical representations), and attributes contributed to these interactions (e.g., coordination and magnitude influencing navigation on the number line). Attributes included various app and user representations of mathematical content, and modification of attributes concerning mathematical representation could lead to changes in externalization of mathematical representations. In particular, attribute alignment and modification during these physically embodied interactions with mathematical representations imply that some children were learning, as their attributes and the behaviors (i.e., bodily engagement in mathematical practices) associated with manifesting these attributes were not static. For example, some reactive modification involved learning, as participants adapted the attributes they applied in response to the new content presented by the app. Proactive modification could also involve learning, both in adapting user attributes and in learning to choose app attributes (e.g., representations of mathematical content) that were more appropriate for the user's mathematical understandings. Thus, attributes are an important construct in the context of embodied cognition and representation.

Affordance-Ability Relationships

The second research question addressed the presence of affordance-ability relationships in user-app interactions. Affordance-ability relationships were consistently present during the user-app interactions and could be organized using Moyer-Packenham and Westenskow's (2013) affordance categories. Affordance access varied, most notably by approach and degree, and outcomes of accessing the same affordance could vary.

Consistency of approach to and degree of affordance accession also varied. Furthermore, there were interactions among affordance-ability relationships. Together, these results on affordance-ability relationships and the modification of affordance-ability relationships were part of user-app interactions. The presence of affordance-ability relationships documented in this study corroborates claims that affordances and abilities exist in relation to one another (Greeno, 1994) and are coupled in continuous systems (Chemero, 2003). Results included examples of all five of Moyer-Packenham and Westenskow's (2013) categories of affordances of virtual manipulatives (e.g., efficient precision: planning). Consistent with other literature (e.g., Moyer-Packenham & Westenskow, in press; Tucker et al., 2015) results indicated that creative variation was identified less frequently than other affordances with the participants in this study.

Results also showed that affordance access varied by approach (e.g., efficient precision: direct combinations or separate steps) and varied by degree (e.g., motivation: degrees of positive or negative motivation). However, accessing the same variation of an affordance could lead to different outcomes, such as accessing efficient precision of planning contributing to both success and failure. Several other studies (e.g., Kay, 2012; Moyer-Packenham et al., in press; Su, 2012) have reported variation in children's accession of affordances of technology. Emerging research also identified variation of affordance access by approach and degree, with similar affordance access leading to different outcomes (Tucker et al., 2015), which adds nuance to Greeno's (1994) assertion that affordances are graded. Affordance-ability relationships influenced each other, such as the balance of planning and multiple visible intervals as part of efficient precision, focused constraint, and creative variation during interactions with Motion Math: Zoom. Other researchers provided evidence of the interrelated nature of affordances, such as Burris (2010, 2013), who implied connections between simultaneous linking and creative variation. However, specific interrelationships are novel findings of this study. These results are important because they identify affordance-ability relationships as a construct that contributes to user-app interactions, aligning with findings and implications of other research.

From an embodied cognition and representation viewpoint, results concerning affordance-ability relationships are important because they indicate that affordance-ability relationships are involved in mathematical thinking and learning. Accessing an affordance as part of an affordance-ability relationship took place during embodied interactions with mathematical representations. Variations of affordance accession within and between participants during the user-app interactions provided evidence of differences in mathematical thinking and learning. Degrees of and approaches to affordance access, particularly in response to the same affordance (e.g., three approaches to accessing guided placement efficient precision affordance), indicated a range of mathematical thinking that might stem from different internal representations. Within the same participant, consistent changes in abilities as part of affordance-ability relationships (e.g., beginning to use navigation constraints for efficient precision and applying whenever appropriate) provided evidence of changing mathematical practices and thus mathematical learning. Thus, affordance-ability relationships are also an important construct in the context of embodied cognition and representation.

Distance

The third research question addressed the presence of distance in user-app interactions. Two types of distance were consistently present throughout the user-app interactions: mathematical distance and technological distance. The degree of each type of distance could change, and there were interactions between mathematical distance and technological distance. Together, these results indicate that distance and the modification of distance are part of user-app interactions. Mathematical distance included the degree of difficulty of applying the additive inverse property, while technological distance included the degree of difficulty of applying the pinching gesture to zoom across intervals. Sedig and Liang (2006) proposed four types of distance—semantic, articulatory, conceptual, and presentation—that mainly fit within technological distance in the MAAAD for Learning framework. The rhombus plots used to visualize the data in support of this analysis was a novel technique.

Distance could also change, such as when participants successfully completed a level after multiple attempts, decreasing mathematical distance. The changes in distance observed in this study align with Sedig et al.'s (2001) assertion that balancing the degree of distance occurs during interactions with educational technology tools. Interactions between mathematical distance and technological distance included high degrees of technological distance negatively influencing the degree of mathematical distance. This occurred during interactions with both apps, but was more common during interactions with Motion: Math Zoom, which required coordinating multi-touch input on an idealized number line. This relates to Byers and Hadley's (2013) observation that novel ways to interact with mathematics may be unfamiliar, even when valuable. This is akin to Sedig and Liang’s (2006) multiple distance types contributing to an overall concept of distance, and aligns with Rick’s (2012) implied relationships between mathematical distance and technological distance when users of a fraction app prioritized figuring out difficult input gestures over learning the mathematical content. However, mathematical and technological distance classifications and the interaction among these distance types are novel findings of this study. These results are important because they identify distance as a construct that contributes to user-app interactions and align with findings and implications of prior research.

Using a lens of embodied cognition and representation, results concerning distance are important because they indicate that distance is involved in mathematical thinking and learning. Evidence of distance was present throughout embodied interactions with mathematical representations. In the context of user-app interactions, distance is the difference between what is required for successful embodied practices and the actual enacted embodied practices. In terms of mathematical thinking and learning, mathematical distance provided evidence of mathematical thinking (e.g., knowing when to apply a given mathematical property). Interactions between the distance types implied that high degrees of technological distance might hinder mathematical learning (e.g., struggling to perform a gesture needed to apply a mathematical property and progress to different representations of the mathematical property). Thus, distance is also an important construct in the context of embodied cognition and representation. Results from the first three research questions indicated that each of the focus constructs contribute to user-app interactions and are relevant to the theoretical framework of embodied cognition and representation.

The Modification of Attributes, Affordances, Abilities, and Distance for Learning Framework

The fourth research question addressed the presence of relationships among attributes, affordance-ability relationships, and distance in user-app interactions. Results indicated that there were relationships among the focus constructs in user-app interactions, and these relationships form the MAAAD for Learning framework.

Relationships among attributes, affordances, abilities, and distance were present throughout user-app interactions. Results indicated that attributes relate to affordance-ability relationships. Based on Greeno’s (1994) definitions and set within the context of user-app interactions, clusters of app attributes form affordances, whereas clusters of user attributes form abilities. Results indicated that modification of attributes could lead to modification of affordance-ability relationships (e.g., honing zoom input gesture leads to relatively efficient planning), while modification of affordance-ability relationships could lead to modification of attributes (e.g., app changes constraints to allow focus on different mathematical content). These modifications could lead to changes in mathematical practices (e.g., planning mathematical actions; combining mathematical properties), which from an embodied cognition perspective of interacting with representations indicates mathematical learning. Similarly, other research found that students’ access to app affordances such as audio feedback (as part of motivation and simultaneous linking) decreased as they became adept at performing a certain task (Bartoschek et al., 2013; Paek, 2012). Furthermore, an attribute could contribute to multiple affordance-ability relationships. For example, coordination contributed to the ability to access planning as efficient precision and the ability to access navigation restrictions as focused constraint.

Results indicated that attributes relate to distance. Distance, being the degree of difficulty interacting with a tool (Sedig & Liang, 2006), in part relates to the cognitive fidelity of the tool (Dick, 2008). Distance and cognitive fidelity are both relationships between characteristics (i.e., attributes) of the tool (e.g., app) and the user. From a theoretical standpoint, the embodied interactions with mathematical representations provided visual evidence of distance between a user's mathematical thinking and the mathematical content represented by the app, based on user mathematical attributes and app mathematical attributes, respectively. For example, results indicated that modification of attributes influenced attribute alignment, which could lead to modification of distance. In response to the modification in distance, attribute modification could occur. These changes could manifest as changes in engagement in mathematical practices, providing evidence of mathematical learning. Sedig et al. (2001) reported similar implications in their example of the cycle modifying the presence of scaffolding to maintain an appropriate amount of distance, during which a tool decreases scaffolding as the user increases familiarity with the representation.

Results indicated that distance relates to affordance-ability relationships. Both distance and affordance-ability relationships are based on attributes and attribute alignment. The same attribute can contribute to distance and affordance-ability relationships, such as flexibility contributing to both mathematical distance and the ability to access planning as efficient precision. Distance can influence affordance-ability relationships (e.g., high degree of technological distance leading to high degree of access to negative motivation), and affordance-ability relationships can influence distance (e.g., constraining focus to whole numbers providing a level with a low degree of mathematical distance). Using a lens of embodied cognition and representation, these relationships influence how one engages in mathematical practices as evidence of mathematical thinking and learning (e.g., focused constraint intended to minimize mathematical distance influences which mathematical properties one can enact during a task). Multiple studies imply connections between distance and affordance-ability relationships, including affordance access varying by both students' mathematical proficiency (e.g., Gadanidis, Hughes, & Cordy, 2011; Moyer-Packenham & Suh, 2012) and technological proficiency (e.g., Rick, 2012; Tucker & Moyer-Packenham, 2014).

The results are important because they indicated that relationships among attributes, affordances, abilities, and distance contribute to the proposed MAAAD for Learning framework. Furthermore, these results and interpretations connect the framework to theoretical foundations, as the analyses focused on user-app interactions that involved physically embodied interactions with mathematical representations and provided evidence of mathematical thinking and learning. Syntheses of the literature also indicate the presence of each of these constructs, and some researchers have implied the existence of relationships among multiple constructs, such as Sedig and Liang (2006), who included affordances and distance among 12 interactivity factors of visual mathematical representations. However, before this study, no published research had identified all of these constructs in the context of user-app interactions, coherently articulated and integrated relationships among these constructs, nor grounded these relationships in embodied cognition and representation. Findings from this study indicate that the MAAAD for Learning framework provides a structure for examining user-app interactions.

Limitations

Characteristics of the exploratory design place delimitations on the study. At the time of this study, there was no previous research identifying and integrating these focus constructs to examine user-app interactions. Therefore, the purpose of this exploratory research was to describe these physically embodied user-app interactions to generate hypotheses and support theory development for future research (Marshall & Rossman, 2010). The study used specific definitions of terms (e.g., affordance of motivation: Moyer-Packenham & Westenskow, 2013), but using alternative definitions (c.f., motivation: Belland, Kim, & Hannafin, 2013) could lead to different interpretations of the results. The sample was limited to ten fifth-grade children from a local community, and it was beyond the scope of the study to generalize by specific demographic characteristics, including age, socioeconomic status, and ethnicity. Furthermore, this study occurred in a research lab using individual participants, but there are other contexts in which learning can take place (e.g., classroom with teacher and groups of students). Additional research is required to address these limitations, which are standard for exploratory research (Stebbins, 2001). Thus, the study focused on trustworthiness through thorough description, allowing evaluation by the reader (Rolfe, 2006), and laying the foundation for future investigations.

Implications and Potential Applications

The results of this study, including the MAAAD for Learning framework, align with theoretical foundations and have implications and applications for app developers, those who implement apps (e.g., educators), and researchers who study how users interact with apps and other educational technology.

Alignment with Theoretical Foundations

Development of the MAAAD for Learning framework was consistent with theories of embodied cognition and representation. The user-app interactions in which these constructs and relationships were identified were a form of perceptuomotor integration, wherein mathematical thinking was evident in participants' bodily activity as they physically interacted with mathematical representations. The changes that took place during these interactions provided evidence of transformations in participants' bodily engagement in mathematical practices, which is equivalent to mathematical learning. Evidence indicated that the constructs and relationships among the constructs related to both mathematical thinking and mathematical learning. Thus, the framework is both grounded in and has implications for theories of embodied cognition and representation, as it models specific constructs and relationships among constructs that contribute to physically embodied interactions with representations (i.e., mathematical practices) that constitute mathematical thinking and learning. Researchers could also consider MAAAD for Learning in relation to frameworks that approach human interactions with technology from other theoretical perspectives. These include frameworks based in activity theory (e.g., Artifact-centric activity theory: Ladel & Kortenkamp, 2013), complex cognitive activities (e.g., EDIFICE-AP: Sedig & Parsons, 2013), and multimedia learning (e.g., Interactive Multimedia Model for Cognitive Learning: Daghestani, 2013), and also inform design, implementation, and research related to educational technology.

Implications for Development

The MAAAD for Learning framework has implications and applications for developers of educational apps. For example, developers could consider the framework when designing and testing apps as a way to examine the attributes that contribute to the affordance-ability relationships involved in the user-app interactions, as well as the various ways these relationships may manifest. Designers could also consider the purposeful modification of the constructs involved in the framework, such as how and when an app could modify attributes that lead to modification of distance and affordance-ability relationships, as well as the potential outcomes of these modifications. Furthermore, designers could encourage proactive modification by clarifying for users which app attributes they can modify. Technology research and development groups could also consider these results and implications in relation to literature on human-computer interaction as applied to technology design, such as decision making, adaptive systems, and information visualization (e.g., Jacko, 2012).

Implications for Implementation

The MAAAD for Learning framework also has implications and applications for those who implement educational apps and those who train others to do so. For example, teachers and parents could consider the alignment of user attributes and app attributes when choosing an app for children. Because of the rarity of proactive attribute modification despite repeated attribute misalignment, teachers may wish to provide additional external scaffolding, such as by helping children recognize the potential for modifying app attributes and by supporting the performance of appropriate input gestures. Furthermore, attribute change was evident during these relatively short interactions, though participants were not always aware they were engaged in mathematics. Teachers may consider both monitoring user-app interactions for evidence of changing attributes and supporting recognition of the mathematics by facilitating intentional discussions of these mathematical interactions. Educators and professional development providers could consider these results and implications in relation to literature about teachers' use of educational technology, including Technological Pedagogical Content Knowledge (Mishra & Koehler, 2006) and teacher beliefs about technology integration (e.g., Ertmer, 2005).

Implications for Research

The MAAAD for Learning framework has many potential implications and applications for researchers of learning and technology. The theoretical alignment indicates that researchers can apply this framework to investigations grounded in embodied cognition and representation, or alternatively examine this framework using other theoretical lenses. Fine-grained applications of the framework include investigating specific mathematical learning trajectories (e.g., Sarama & Clements, 2009) or examining the role of particular examples of the constructs (e.g., efficient precision affordance).

Lateral applications of MAAAD for Learning include applying it to interactions with other apps, which could lead to additional examples of each of the constructs (e.g., distance: mathematical and technological) and further develop the emergent themes from this study (e.g., attribute modification: proactive and reactive). Other lateral applications include applying the framework to characterize interactions with other subject matter (e.g., science) and technology tools (e.g., video games), or using it for different settings (e.g., classroom) and participants (e.g., diverse learners). Broader applications of the framework include linking it to specific learning outcomes and implementing micro-longitudinal or longitudinal investigations. Furthermore, it is possible to combine these applications, such as by applying the framework to examine connections between novel attribute modification types and learning outcomes when college students use educational technology to learn physics content. The MAAAD for Learning framework has implications and applications for development, implementation, and research concerning mathematics virtual manipulative apps in particular and educational technology in general.

Conclusion

This study represents an integration of multiple constructs that contribute to children's experiences of interacting with educational technology. Extensive research exists on results of these interactions (e.g., Moyer-Packenham et al., 2015; Paek, 2012; Zhang et al., 2015), and some research has identified constructs that play a role in related interactions (e.g., Sedig & Liang, 2006; Tucker et al., 2015). However, research had not coherently examined relationships among these contributing constructs. The purpose of this study was to conceptualize the relationships among attributes, affordances, abilities, and distance in a framework that describes the nature of children's interactions with technology to learn mathematics. This study was built on the premise that (a) mathematics learning occurs when children physically interact with mathematical representations, including those that involve mathematics virtual manipulative iPad apps, and (b) attributes, affordance-ability relationships, and distance are involved in these interactions.

The results of this study indicated that the focus constructs were present in the user-app interactions and that the relationships among these constructs are explained with the MAAAD for Learning framework. During the user-app interactions, attributes, affordance-ability relationships, and distance were consistently present and often changing. Important emergent themes included proactive and reactive attribute modification, relationships among affordance-ability relationships, and the presence of mathematical distance and technological distance. Furthermore, each construct influenced the other constructs, with modifications to one construct leading to modifications of the connected constructs. These relationships form the MAAAD for Learning framework.

The results of this study suggest the MAAAD for Learning framework models relationships among attributes, affordance-ability relationships, and distance in the context of user-app interactions. The framework is a useful tool for developers, educators, and researchers. Developers designing technology tools can use the framework to consider relationships among constructs that contribute to the users' experiences when interacting with the tools. Educators implementing technology tools to support children's learning can use the framework to evaluate the appropriateness of the tool for the children, as well as a way to evaluate learning during children's interactions with educational technology in the classroom. Researchers can apply the framework when investigating constructs that play a role in children's learning while interacting with technology, as well as the potential outcomes of these interactions.

The constructs, relationships, and framework identified in this study advance the literature on children's interactions with educational technology tools, in particular literature concerning children's interactions with mathematics virtual manipulative iPad apps. Future investigations involving connections to learning outcomes, different contexts, diverse populations, additional content areas, and various technology tools will contribute to the development of the emergent themes and novel findings, as well as the application of the MAAAD for Learning framework.

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APPENDICES

Appendix A

Facilitation Protocol

Facilitation Protocol

Before Participant Arrival

- Set up materials (paperwork, cameras, iPad and apps)
- Begin room camera recording

Upon Participant Arrival

- Greetings/introductions
- Distribute, explain, and complete consent/assent form with video waiver

Introduction

- You'll play one app, I'll ask you some questions, you'll play a second app, and I'll ask you some more questions. No worries, there's no math quiz; I just want to learn more about what you were doing and thinking.
- You can play each app for up to thirty minutes, but let me know if you want to stop playing and we will move on.
- The cameras only record your interactions with the apps. They focus on your hands and the iPad, but they can't move, so please keep the iPad in place so the cameras can see.
- I'll be here the whole time, but I'm not here to help. I want to learn about what you do, not what I do. Just do your best. I'll take notes about so I can ask good questions.
- Do you have any questions? *Address questions.*
- I'm going to start the computer recording now. *Begin computer recording. Check angle!*

App Interaction & Follow-Up Questions #1 (*Open app, begin timer and observation protocol)*

- You may begin.
- *After 30 minutes or when the child requests to stop playing, ask some or all of the follow-up questions, depending on the situation. Adjust for occurrences and purposefully build on responses that include one or more of attributes, affordance-ability, and distance.*
    - What did you think of the app? Why?
        - *Easy/hard, etc.:* Was it always easy/hard or did it change? Why?
    - Why did you stop/would you have liked to play longer? Why?
    - How did you figure out what you were supposed to do (*if possible, in specific situations where stuck on math, tech, or both*)?
        - *Specific to Zoom:* How did you decide when to zoom instead of swipe?
    - How did you use what you already knew while playing this app?
    - Did you learn anything while playing?
        - How did you use that in the app?
        - How might you use that when you aren't playing the app?
    - What would make the app better?

App Interaction & Follow-Up Questions #2

- *Repeat similar for second app*

Summative Follow-Up Questions

- *Ask follow-up questions such as the following, adjusting for specific occurrences*
    - What mathematics did you notice in these apps?
    - Did you notice any features of the app that helped you to complete the app tasks? What were they and how did they help you?
    - Which app did you like better? Why?
    - Which app was easier/harder? Why?

Debriefing

- Debrief child and parent: answer any questions and thank them for participating.

Appendix B

Observation Protocol

Observation Protocol
(Print Version of Digital Excel Document)

Observation Field Notes Protocol				
Participant #			Date	
App 1:(app name)			App 2: (app name)	
Time (0+)	Occurrence		Time (0+)	Occurrence
App 1 Follow-Up Questions			App 2 Follow-Up Questions	
Q	A		Q	A
Integrated Follow-Up Questions				
	Q	A		

Appendix C

Distance Magnitude Codes and Quantitization Steps

Distance Magnitude Codes and Quantitization Steps

Step 1: Apply directly related attribute cluster codes with distance magnitude codes based for each attempt to complete level

Motion Math: Zoom distance magnitude coding structure with descriptors

- Mathematical Distance:
    - Comparison-Navigation (left-right choices)
        - NA: Accurate: few, brief mistakes, no extended; no scaffold; rare past off-screen
        - NG: Generally Accurate: more brief mistakes, rare extended (1/level) or POS; no scaffold
        - NO: Often Inaccurate: frequent brief mistakes, may multiple extended/POS; may include 1 scaffold
        - NI: Inaccurate: most tasks have inaccuracies, may multiple extended/POS; any 2+ scaffolds
    - Comparison-Target/Placement choices (correct answers: must be corrected to finish)
        - TA: Accurate: few mistakes or pauses
        - TG: Generally Accurate: more mistakes or pauses (<1/3 tasks)
        - TO: Often Inaccurate: frequent mistakes or pauses (appx. 1/2)
        - TI: Inaccurate: mistakes/pauses on most tasks
    - Magnitude-within interval (swipe interval choice—not direction)
        - WA: Accurate: ideal interval (nearly) every time; no scaffold
        - WG: Generally Accurate: occasional non-ideal interval (<1/3 tasks); no scaffold
        - WO: Often Inaccurate: frequently non-ideal interval (appx. 1/2); may 1 scaffold
        - WI: Inaccurate: rarely ideal interval; any 2+ scaffold
    - Magnitude-between/across interval (zoom interval choice—note if precise)
        - BA: Accurate: ideal intervals (nearly) every time; near when not ideal
        - BG: Generally Accurate: mixed ideal intervals; near ideal interval (nearly) every time
        - BO: Often Inaccurate: often not near ideal interval
        - BI: Inaccurate: rarely near ideal interval (i.e., choose whatever is closest); any uses hint
- Technological Distance
    - Swipe (efficiency when using)
        - SE: Efficient
        - SG: Generally efficient
        - SO: Often inefficient
        - SI: Inefficient
    - Zoom (efficiency when using)
        - ZE: Efficient

        - ZG: Generally efficient
        - ZO: Often inefficient
        - ZI: Inefficient

DragonBox Algebra 12+ distance magnitude coding structure with descriptors

- Mathematical Distance:
    - Moves (Efficiency: moves allowed by app)
        - MA: Accurate: Ideal
        - MG: Generally Accurate: 1 off or Ideal with Undo
        - MO: “Often” Inaccurate: 2-3 off or 1 off with Undo
        - MI: Inaccurate: 4+ off or 2+ off with Undo
    - Leftovers (Elegance: completely cleared)
        - LA: Accurate: Ideal
        - LG: Generally Accurate: Tries to finish while sweeping
        - LO: “Often” Inaccurate: No attempt to finish while sweeping, 1-2 left
        - LI: Inaccurate: No attempt to finish while sweeping, 3+ left
    - Accuracy (Disallowed math attempts)
        - AA: Accurate: Ideal
        - AG: Generally Accurate: 1-2 different errors, not repeated (separately)
        - AO: “Often” Inaccurate: 1-2 different errors, repeated (separately)
        - AI: Inaccurate: 3+ different errors (repeats or not)
- Technological Distance
    - Performance of input gestures (tap, drag, etc.)
        - PE: Efficient: smooth, no difficulties
        - PG: Generally efficient: few difficulties
        - PO: Often inefficient: frequent difficulties, one gesture
        - PI: Inefficient: frequent difficulties, multiple gestures
    - Choice of correct input gesture
        - CE: Efficient: Always appropriate choice
        - CG: Generally efficient: few inappropriate choices
        - CO: Often inefficient: frequently attempts one inappropriate gesture
        - CI: Inefficient: frequently attempts more than one inappropriate gesture

Step 2: Assign values to distance magnitude codes

Table C1

Distance Magnitude Code Values

Magnitude Code	Scale Value
A, E	4
G	3
O	2
I	1

Step 3: Determine overall attempt value

Overall attempt value determination for Motion Math: Zoom

- Mathematical Components: [(N + T + W + B) / 4] – (decimal of % of level incomplete)
  - Note: The value determination is adjusted to reflect the number of codes applied (e.g., if no “B” because there was no changing intervals, divide by 3 instead of 4)
  - Example A
    - NA, TA, WG, BG, completed 10 of 10 tasks
    - [(4 + 4 + 3 + 3) / 4] – 0.0 = 3.5
  - Example B
    - NO, TO, WI, BI, completed 3 of 10 tasks
    - [(2 + 2 + 1 + 1) / 4] – 0.7 = 1.8
- Technological Components: (S + Z)/2
  - Note: The value determination is adjusted to reflect the number of codes applied (e.g., if no “Z” because there was no zooming, divide by 1 instead of 2)
  - Example A
    - SE, ZE
    - (4 + 4) / 2 = 4
  - Example B
    - SE, ZI
    - (4 + 1) / 2 = 2.5

Overall attempt value determination for DragonBox Algebra 12+

- Mathematical Components: (M + L + A) / 3
  - Note: Ending an attempt by resetting the level using Restart, Solution, or Undo results in a mathematical distance score of 1 for the attempt
  - Example A
    - MA, LA, AG
    - (4 + 4 + 3) / 3 = 3.67

  - Example B
    - MG, LG, AI
    - (3 + 3 + 1) / 3 = 2.33
- Technological Components: (P + C)/2
  - Example A
    - PE, CE
    - (4 + 4) / 2 = 4
  - Example B
    - PG, CO
    - (3 + 2) / 2 = 2.5

Step 4: Scale attempt values to distance value

Table C2

Distance Value Scale

Distance scale value	Attempt value range
4	$3.67 \leq x$
3	$3 \leq x < 3.67$
2	$2 \leq x < 3$
1	$x < 2$

Examples

- Motion Math: Zoom
  - Example A
    - Mathematical: 3.5 scaled to 3
    - Technological: 4 scaled to 4
  - Example B
    - Mathematical: 1.8 scaled to 1
    - Technological: 4 scaled to 4
- DragonBox Algebra 12+
  - Example A
    - Mathematical: 3.67 scaled to 4
    - Technological: 4 scaled to 4
  - Example B
    - Mathematical: 2.33 scaled to 2
    - Technological: 2.5 scaled to 2

Step 5: Pair values to determine (Mathematics, Technology) distance value

- Motion Math: Zoom
  - Example A: (3, 4)
  - Example B: (1, 2)
- DragonBox Algebra 12+
  - Example A: (4, 4)
  - Example B: (2, 2)

Appendix D

Examples of Approach Variants for Efficient Precision

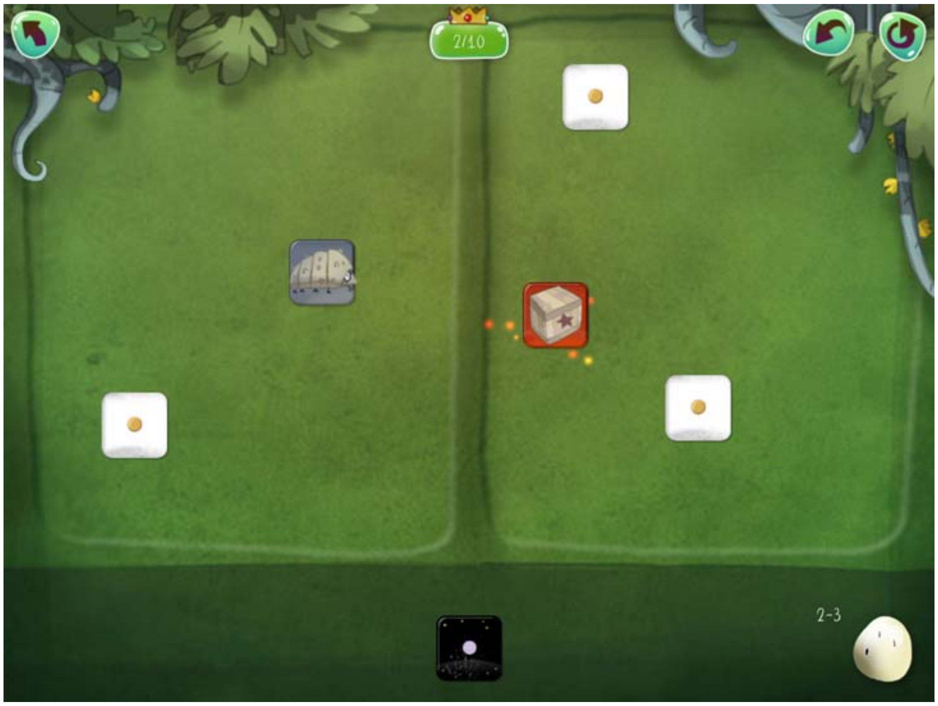


Figure D1. Situation where additive equality and additive inverse properties can be applied.

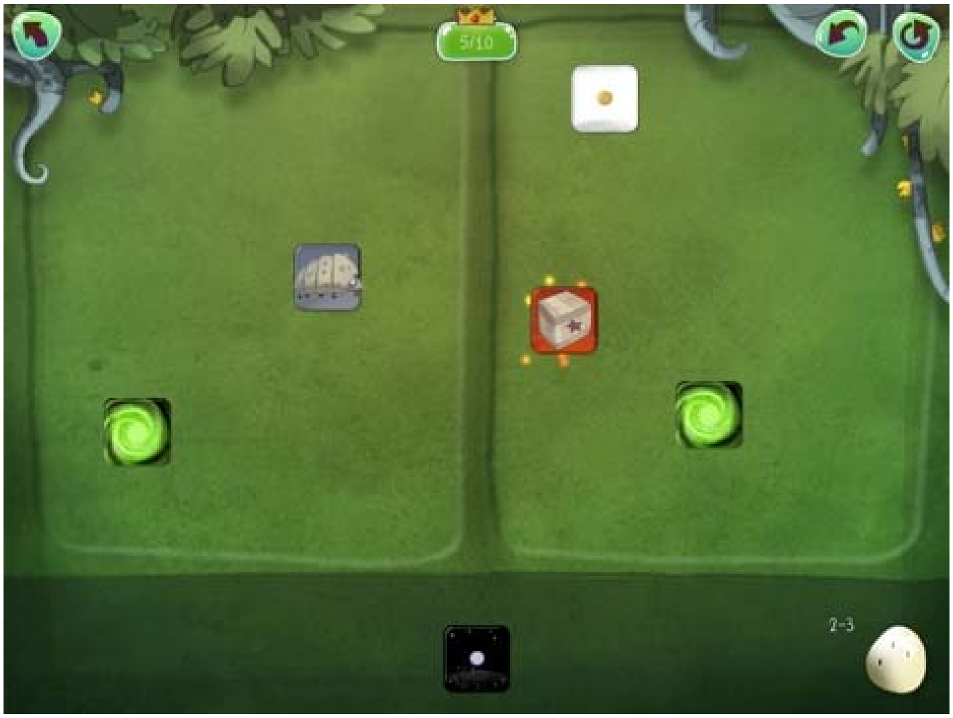


Figure D2. Direct combination on both sides.

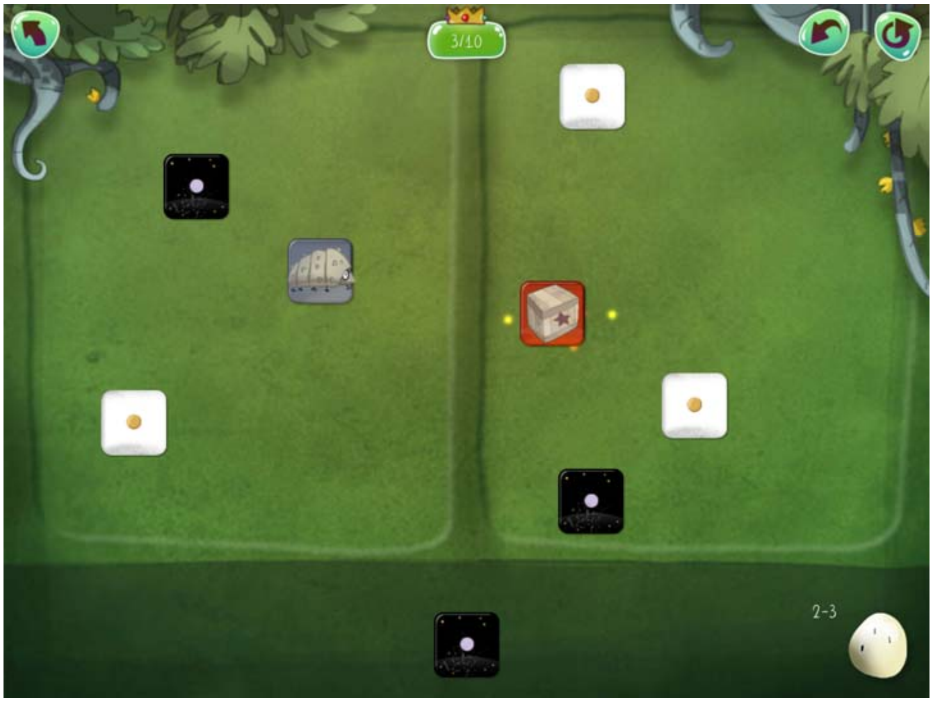


Figure D3. Separate steps on both sides.

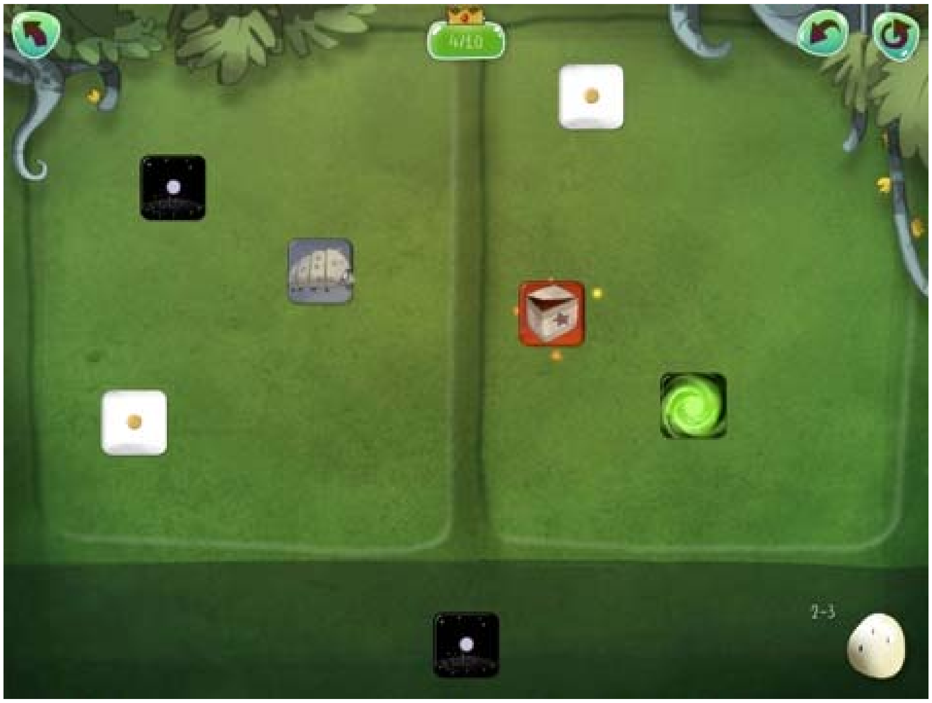


Figure D4. Direct combination on the first side and separate steps on the other side.

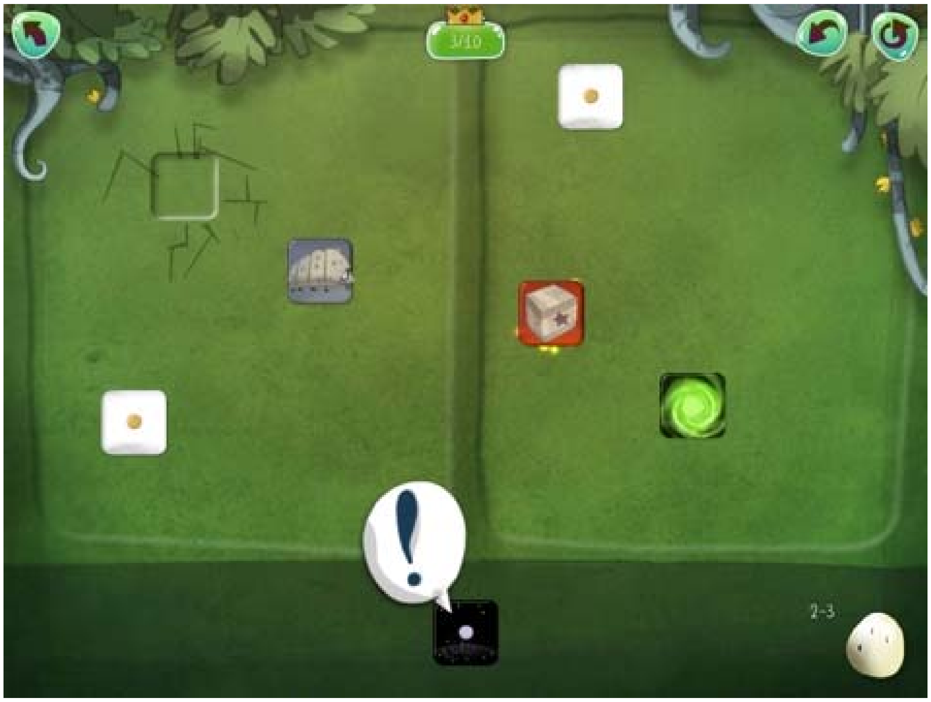


Figure D5. Partially complete additive equality property. Square target space, with additional animations in response to attempted interruption of the additive equality property after direct combination.

Appendix E

Level Bins

Table E1

Motion Math: Zoom Level Bins

Bin	Start level	Focus content
1	1	Intro
2	2	Integers to 1,000
3	6	Integers to 10,000 (Changing Intervals)
4	9	Positive and negative integers to 10,000
5	12	Decimals: Tenths
6	15	Decimals: Hundredths
7	19	Decimals: Thousandths
8	21	Challenge (all previous)

Table E2

DragonBox Algebra 12+Level Bins[1]

Bin	Start Level	New Content
1	1:01	Additive identity
2	1:03	Additive inverse (internal[2])
3	1:09	Additive equality (external[3])
4	1:16	Additive inverse (external)
5	2:01	Multiplicative inverse (internal)
6	2:05	Multiplicative identity (internal)
7	2:11	Multiplicative inverse (external)
8	3:01	Additive inverse (across[4])
9	3:07	Multiplicative inverse (across)

[1] First three chapters (levels 1:01-3:20) of DragonBox Algebra 12+

[2] Internal refers to steps involving only tiles already present in the equation without moving the variable from one side of the equation to the other side of the equation

[3] External refers to steps involving bringing in variables from outside the equation

[4] Across refers to steps involving moving a variable from one side of the equation to the other.

Appendix F

Rhombus Plots: Small Multiples

Rhombus Plots: Small Multiples

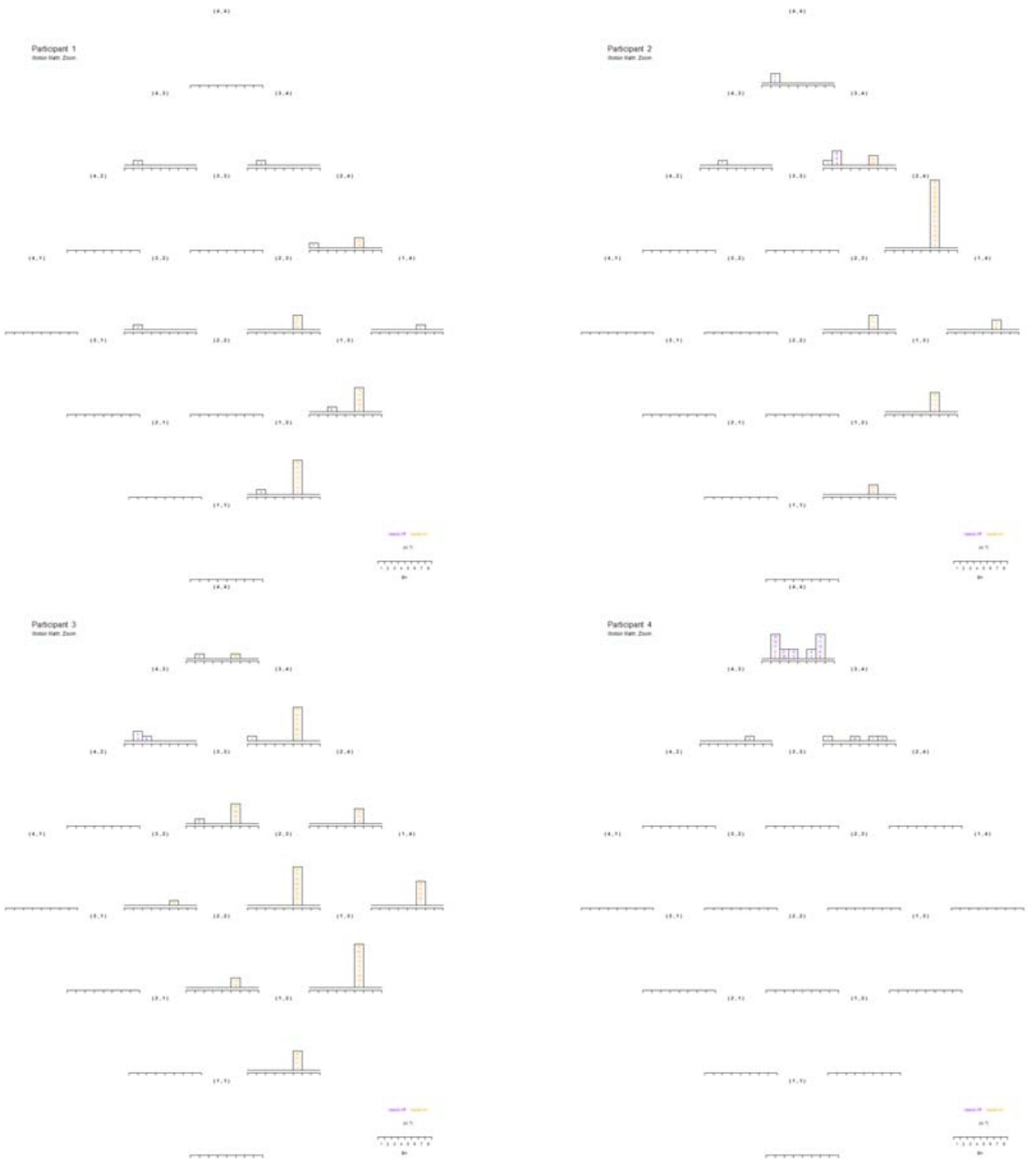
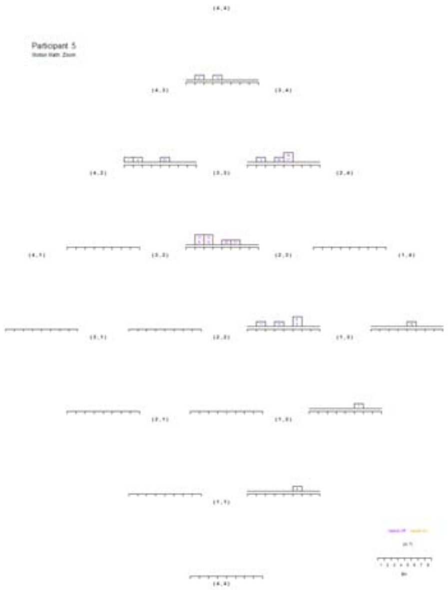
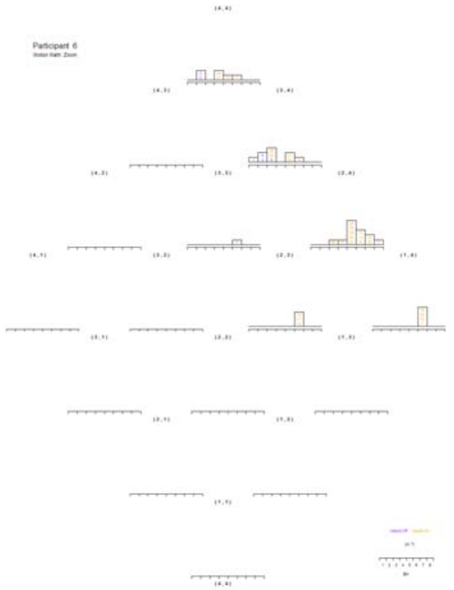
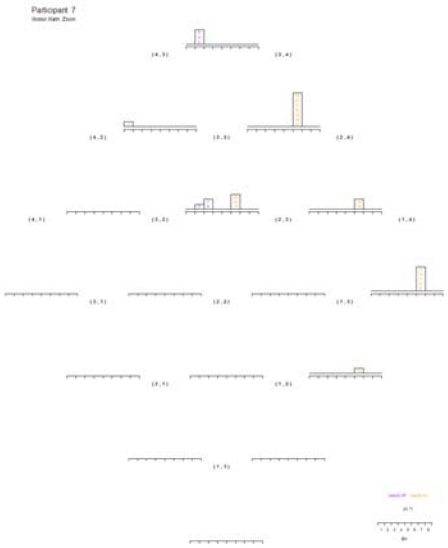


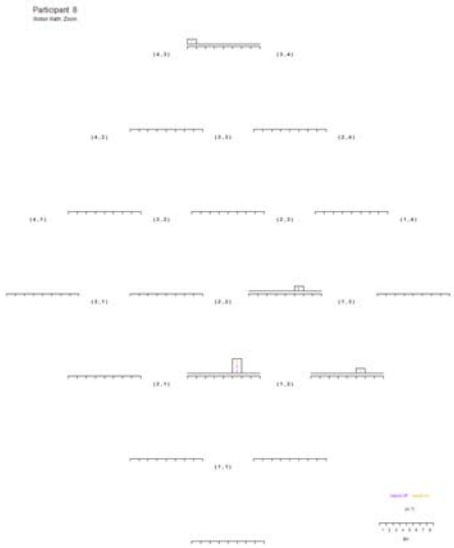
Figure F1. Rhombus plot small multiples for Motion Math: Zoom data. (M,T) indicates (mathematical, technological) distance value. Bin indicates level group.

(*Figure F1 continues)*

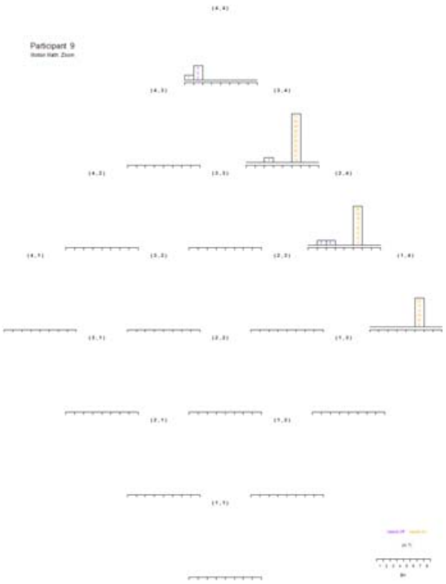


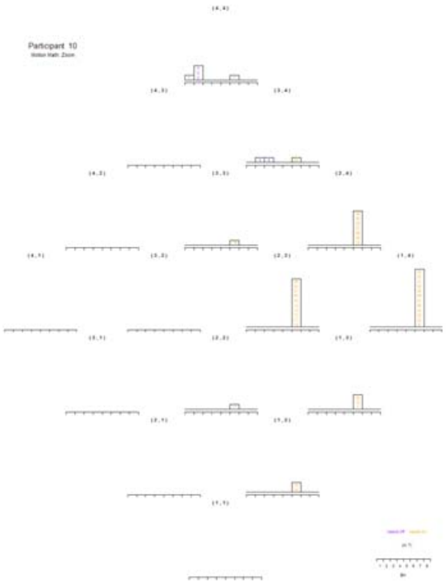






(Figure F1 continues)


(4,4)
Participant 9
Motion Math: Zoom
(4,3)
(3,4)
(4,2)
(3,3)
(2,4)
(4,1)
(3,2)
(2,3)
(1,4)
(3,1)
(2,2)
(1,3)
(2,1)
(1,2)
(1,1)
1 2 3 4 5 6 7 8


(4,4)
Participant 10
Motion Math: Zoom
(4,3)
(3,4)
(4,2)
(3,3)
(2,4)
(4,1)
(3,2)
(2,3)
(1,4)
(3,1)
(2,2)
(1,3)
(2,1)
(1,2)
(1,1)
1 2 3 4 5 6 7 8

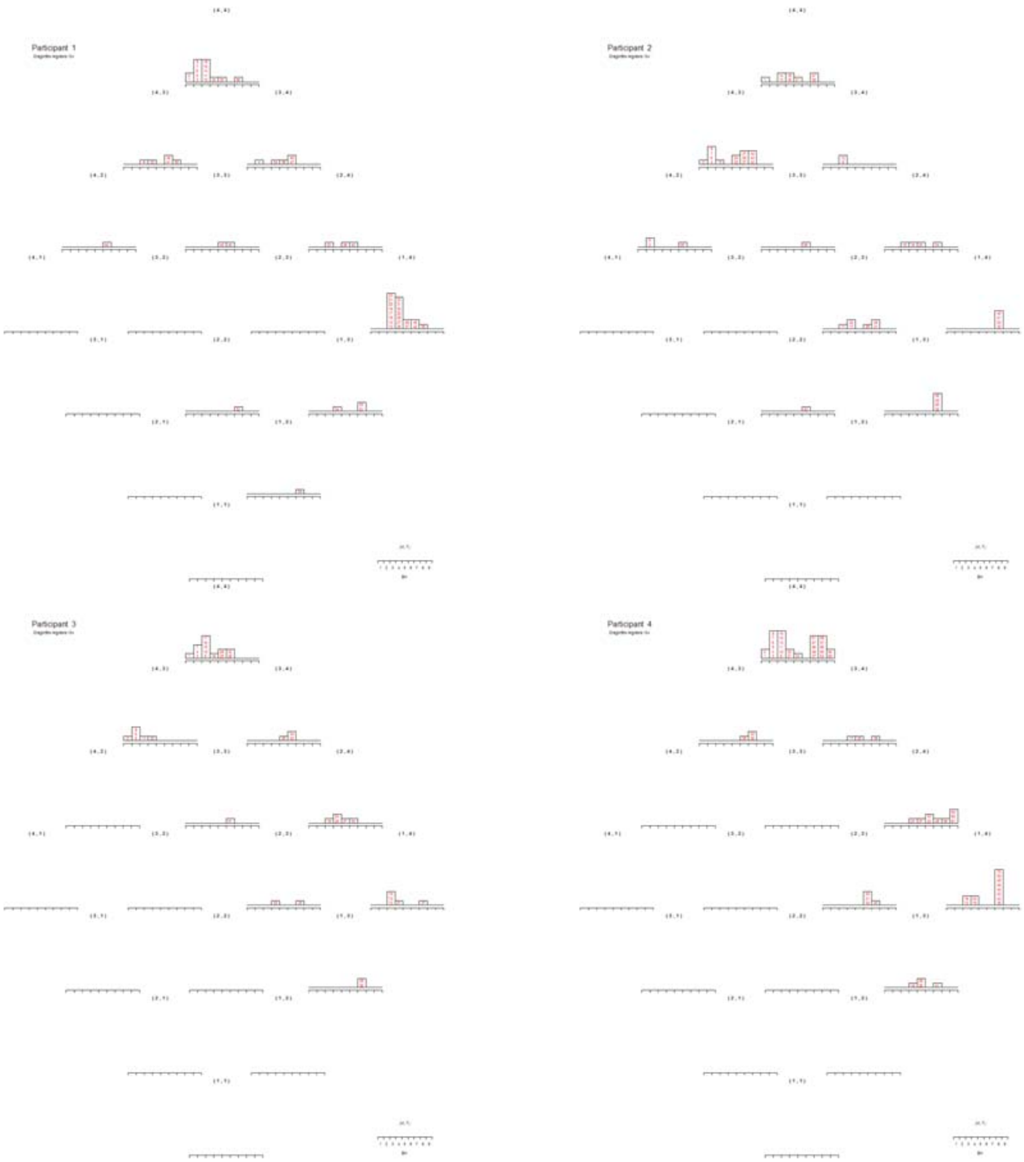
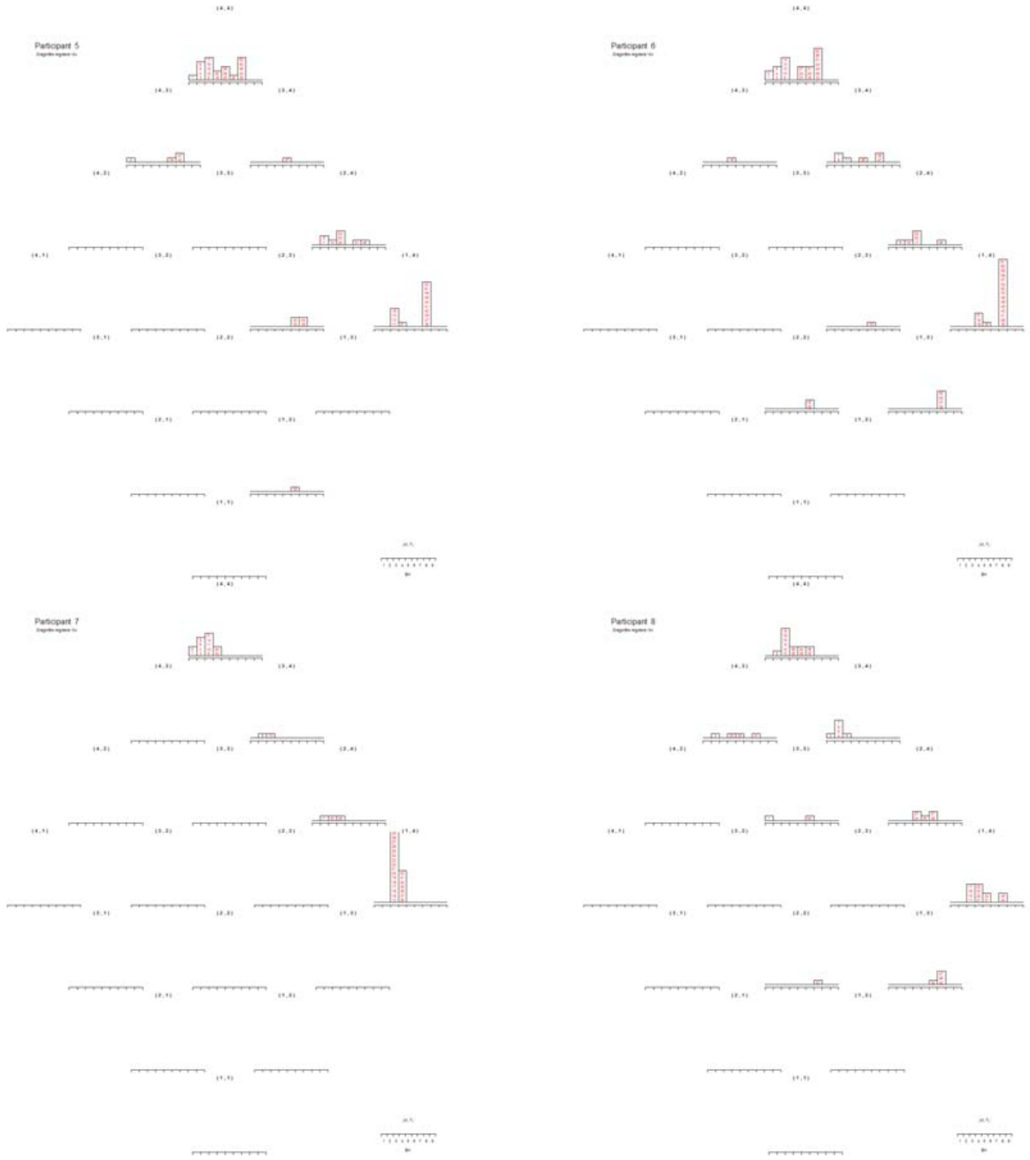
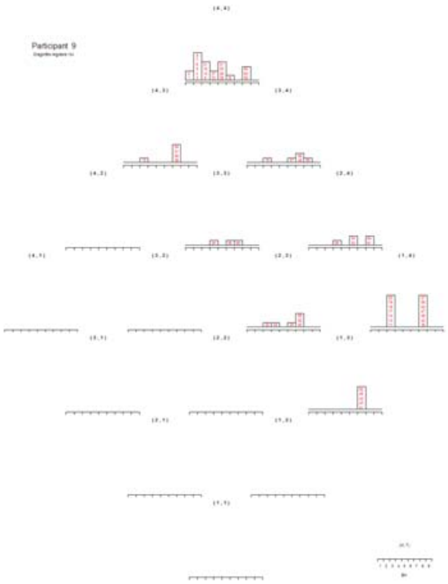


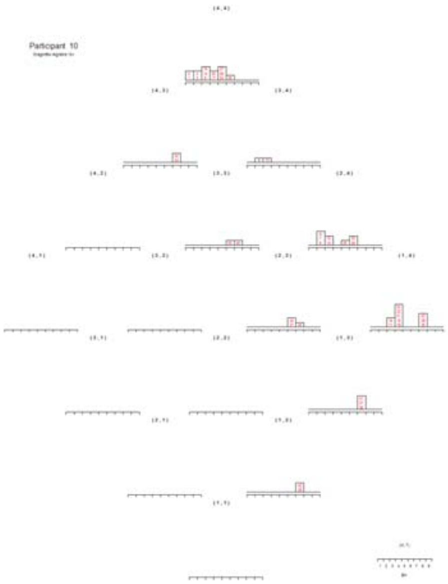
Figure F2. Rhombus plot small multiples for DragonBox Algebra 12+ data. (M,T) indicates (mathematical, technological) distance value. Bin indicates level group.

(Figure F2 continues)



(Figure F2 continues)

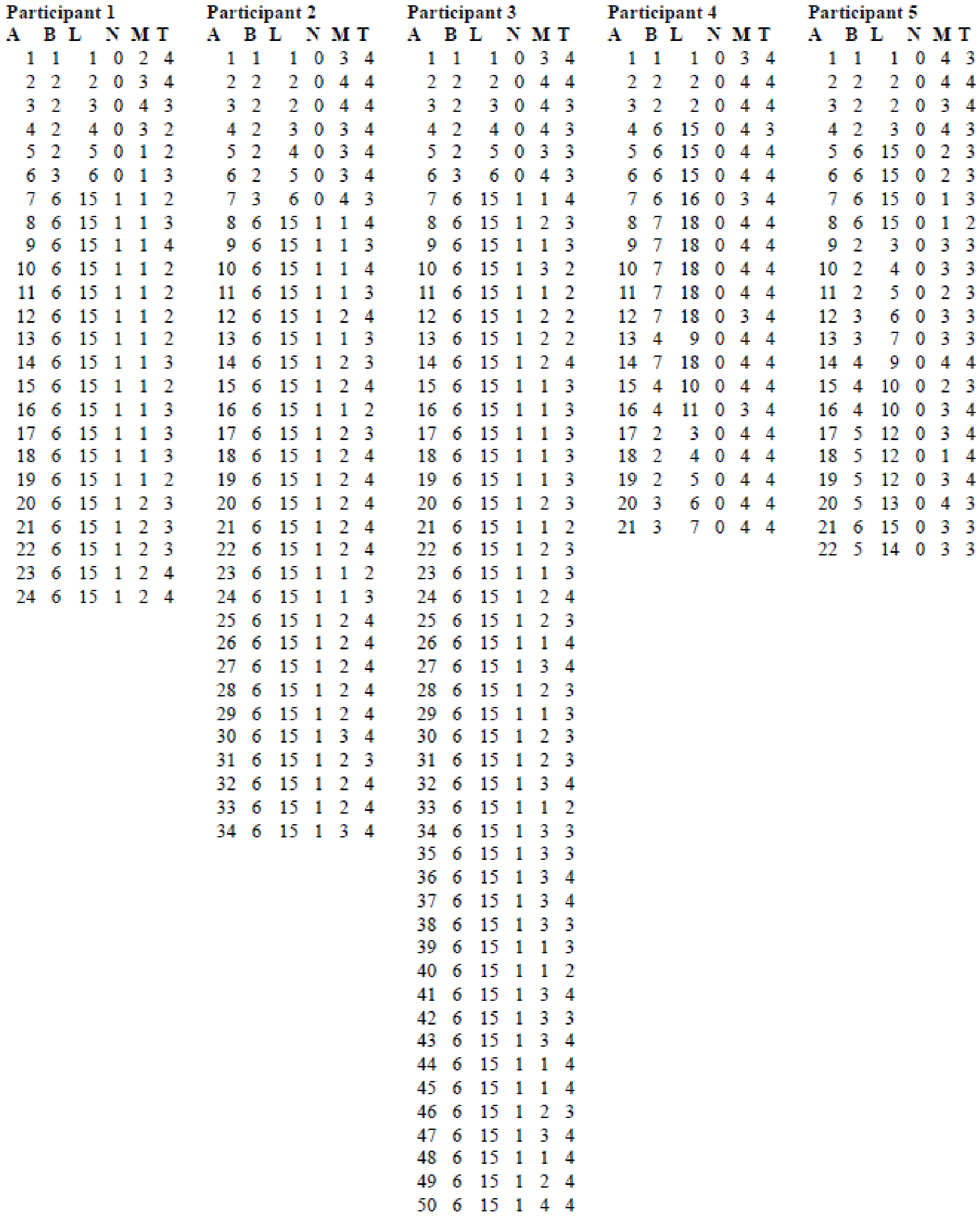

(4,4)
Participant 9
(4,3)
(3,4)
(4,2)
(3,3)
(2,4)
(4,1)
(3,2)
(2,3)
(1,4)
(3,1)
(2,2)
(1,3)
(2,1)
(1,2)
(1,1)
1 2 3 4 5 6 7 8 9


(4,4)
Participant 10
(4,3)
(3,4)
(4,2)
(3,3)
(2,4)
(4,1)
(3,2)
(2,3)
(1,4)
(3,1)
(2,2)
(1,3)
(2,1)
(1,2)
(1,1)
1 2 3 4 5 6 7 8 9

Appendix G

Quantitized Distance Data

Quantitized Distance Data Used to Generate Rhombus Plots for Interactions with Motion Math: Zoom



Participant 1

A	B	L	N	M	T
1	1	1	0	2	4
2	2	2	0	3	4
3	2	3	0	4	3
4	2	4	0	3	2
5	2	5	0	1	2
6	3	6	0	1	3
7	6	15	1	1	2
8	6	15	1	1	3
9	6	15	1	1	4
10	6	15	1	1	2
11	6	15	1	1	2
12	6	15	1	1	2
13	6	15	1	1	2
14	6	15	1	1	3
15	6	15	1	1	2
16	6	15	1	1	3
17	6	15	1	1	3
18	6	15	1	1	3
19	6	15	1	1	2
20	6	15	1	2	3
21	6	15	1	2	3
22	6	15	1	2	3
23	6	15	1	2	4
24	6	15	1	2	4

Participant 2

A	B	L	N	M	T
1	1	1	0	3	4
2	2	2	0	4	4
3	2	2	0	4	4
4	2	3	0	3	4
5	2	4	0	3	4
6	2	5	0	3	4
7	3	6	0	4	3
8	6	15	1	1	4
9	6	15	1	1	3
10	6	15	1	1	4
11	6	15	1	1	3
12	6	15	1	2	4
13	6	15	1	1	3
14	6	15	1	2	3
15	6	15	1	2	4
16	6	15	1	1	2
17	6	15	1	2	3
18	6	15	1	2	4
19	6	15	1	2	4
20	6	15	1	2	4
21	6	15	1	2	4
22	6	15	1	2	4
23	6	15	1	1	2
24	6	15	1	1	3
25	6	15	1	2	4
26	6	15	1	2	4
27	6	15	1	2	4
28	6	15	1	2	4
29	6	15	1	2	4
30	6	15	1	3	4
31	6	15	1	2	3
32	6	15	1	2	4
33	6	15	1	2	4
34	6	15	1	3	4

Participant 3

A	B	L	N	M	T
1	1	1	0	3	4
2	2	2	0	4	4
3	2	3	0	4	3
4	2	4	0	4	3
5	2	5	0	3	3
6	3	6	0	4	3
7	6	15	1	1	4
8	6	15	1	2	3
9	6	15	1	1	3
10	6	15	1	3	2
11	6	15	1	1	2
12	6	15	1	2	2
13	6	15	1	2	2
14	6	15	1	2	4
15	6	15	1	1	3
16	6	15	1	1	3
17	6	15	1	1	3
18	6	15	1	1	3
19	6	15	1	1	3
20	6	15	1	2	3
21	6	15	1	1	2
22	6	15	1	2	3
23	6	15	1	1	3
24	6	15	1	2	4
25	6	15	1	2	3
26	6	15	1	1	4
27	6	15	1	3	4
28	6	15	1	2	3
29	6	15	1	1	3
30	6	15	1	2	3
31	6	15	1	2	3
32	6	15	1	3	4
33	6	15	1	1	2
34	6	15	1	3	3
35	6	15	1	3	3
36	6	15	1	3	4
37	6	15	1	3	4
38	6	15	1	3	3
39	6	15	1	1	3
40	6	15	1	1	2
41	6	15	1	3	4
42	6	15	1	3	3
43	6	15	1	3	4
44	6	15	1	1	4
45	6	15	1	1	4
46	6	15	1	2	3
47	6	15	1	3	4
48	6	15	1	1	4
49	6	15	1	2	4
50	6	15	1	4	4

Participant 4

A	B	L	N	M	T
1	1	1	0	3	4
2	2	2	0	4	4
3	2	2	0	4	4
4	6	15	0	4	3
5	6	15	0	4	4
6	6	15	0	4	4
7	6	16	0	3	4
8	7	18	0	4	4
9	7	18	0	4	4
10	7	18	0	4	4
11	7	18	0	4	4
12	7	18	0	3	4
13	4	9	0	4	4
14	7	18	0	4	4
15	4	10	0	4	4
16	4	11	0	3	4
17	2	3	0	4	4
18	2	4	0	4	4
19	2	5	0	4	4
20	3	6	0	4	4
21	3	7	0	4	4

Participant 5

A	B	L	N	M	T
1	1	1	0	4	3
2	2	2	0	4	4
3	2	2	0	3	4
4	2	3	0	4	3
5	6	15	0	2	3
6	6	15	0	2	3
7	6	15	0	1	3
8	6	15	0	1	2
9	2	3	0	3	3
10	2	4	0	3	3
11	2	5	0	2	3
12	3	6	0	3	3
13	3	7	0	3	3
14	4	9	0	4	4
15	4	10	0	2	3
16	4	10	0	3	4
17	5	12	0	3	4
18	5	12	0	1	4
19	5	12	0	3	4
20	5	13	0	4	3
21	6	15	0	3	3
22	5	14	0	3	3

Key. A indicates attempt, B indicates bin, L indicates level or chapter and level. N indicates presence of needle (0 is inactive, 1 is active). M indicates mathematical distance value; T indicates technological distance value.

Participant 6

A	B	L	N	M	T
1	1	1	0	3	4
2	2	2	0	4	4
3	2	3	0	4	4
4	2	4	0	3	4
5	2	5	0	3	4
6	3	6	0	3	4
7	6	15	1	1	4
8	6	15	1	2	4
9	6	15	1	3	3
10	6	15	1	2	3
11	6	15	1	2	3
12	6	15	1	2	4
13	6	15	1	2	4
14	6	15	1	3	4
15	6	15	1	1	4
16	6	15	1	1	4
17	6	15	1	4	4
18	6	16	1	1	4
19	6	16	1	2	3
20	5	12	1	4	4
21	4	9	1	4	4
22	5	13	1	2	4
23	5	13	1	2	4
24	5	13	1	3	4
25	5	14	1	2	4
26	5	14	1	2	4
27	5	14	1	3	4
28	5	13	1	2	4
29	4	10	1	4	4
30	7	18	1	2	4
31	8	21	1	2	4
32	3	7	1	3	4
33	4	11	1	2	4
34	7	18	1	2	4
35	3	8	1	2	4
36	3	8	1	3	4

Participant 7

A	B	L	N	M	T
1	1	1	0	4	3
2	2	2	0	4	4
3	2	3	0	3	3
4	2	4	0	4	4
5	2	5	0	4	4
6	3	6	0	3	3
7	3	6	0	3	3
8	6	15	1	1	4
9	6	15	1	1	4
10	6	15	1	1	3
11	6	15	1	1	4
12	6	15	1	1	4
13	6	15	1	1	4
14	6	15	1	3	4
15	6	15	1	2	4
16	6	15	1	3	4
17	6	15	1	3	3
18	6	15	1	3	4
19	6	15	1	2	4
20	6	15	1	3	4
21	6	15	1	3	4
22	6	15	1	3	4
23	6	15	1	3	3
24	6	15	1	3	3
25	6	15	1	3	4

Participant 8

A	B	L	N	M	T
1	1	1	0	4	4
2	6	15	0	2	2
3	6	15	0	2	2
4	6	15	1	1	3
5	6	15	1	2	2
6	6	15	0	2	3

Participant 9

A	B	L	N	M	T
1	1	1	0	4	4
2	2	2	0	4	4
3	2	3	0	4	4
4	2	4	0	4	4
5	2	5	0	2	4
6	3	6	0	2	4
7	3	6	0	3	4
8	6	15	1	1	4
9	6	15	1	1	4
10	6	15	1	1	4
11	6	15	1	2	4
12	6	15	1	2	4
13	6	15	1	2	4
14	6	15	1	1	4
15	6	15	1	1	4
16	6	15	1	2	4
17	6	15	1	2	4
18	6	15	1	3	4
19	6	15	1	2	4
20	6	15	1	2	4
21	6	15	1	3	4
22	6	15	1	3	4
23	6	15	1	1	4
24	6	15	1	3	4
25	6	15	1	3	4
26	6	15	1	2	4
27	6	15	1	3	4
28	6	15	1	3	4
29	6	15	1	3	4
30	6	15	1	3	4
31	6	15	1	3	4

Participant 10

A	B	L	N	M	T
1	1	1	0	4	4
2	2	2	0	4	4
3	2	3	0	4	4
4	2	4	0	4	4
5	2	5	0	3	4
6	3	6	0	3	4
7	6	15	1	1	3
8	6	15	1	1	4
9	6	15	1	1	3
10	6	15	1	2	3
11	6	15	1	2	3
12	6	15	1	2	3
13	6	15	1	2	3
14	6	15	1	2	3
15	6	15	1	1	2
16	6	15	1	1	2
17	6	15	1	2	2
18	6	15	1	3	3
19	6	15	1	2	3
20	6	15	1	2	3
21	6	15	1	2	3
22	6	15	1	1	3
23	6	15	1	2	4
24	6	15	1	2	3
25	6	15	1	2	4
26	6	15	1	2	4
27	6	15	1	2	4
28	6	15	1	2	3
29	6	15	1	2	4
30	6	15	1	3	4
31	6	15	1	4	4
32	6	16	1	1	4
33	6	16	1	1	4
34	6	16	1	1	4
35	6	16	1	2	4
36	6	16	1	1	4
37	6	16	1	1	4
38	6	16	1	1	4
39	6	16	1	1	4
40	6	16	1	1	4
41	6	16	1	1	4
42	6	16	1	1	4
43	6	16	1	1	4
44	6	16	1	2	4

Key. A indicates attempt, B indicates bin, L indicates level or chapter and level. N indicates presence of needle (0 is inactive, 1 is active). M indicates mathematical distance value; T indicates technological distance value.

Quantitized Distance Data Used to Generate Rhombus Plots for Interactions with DragonBox Algebra 12+

Participant 1

A	B	L	M	T
1	1	101	4	4
2	1	102	4	4
3	2	103	4	4
4	2	104	3	4
5	2	105	4	4
6	2	106	4	4
7	2	107	4	4
8	2	108	4	4
9	3	109	4	3
10	3	110	4	4
11	3	111	4	4
12	3	112	1	4
13	3	112	1	4
14	3	112	4	4
15	3	113	4	4
16	3	114	1	4
17	3	114	1	4
18	3	114	4	4
19	3	115	1	4
20	3	115	1	4
21	3	115	1	4
22	3	115	1	4
23	3	115	2	4
24	4	116	3	4
25	4	117	4	4
26	4	118	1	4
27	4	118	1	4
28	4	118	1	4
29	4	118	1	4
30	4	118	1	4
31	4	118	1	4
32	4	118	4	3
33	4	119	1	4
34	4	120	1	3
35	5	201	4	4
36	5	202	3	4
37	5	203	1	4
38	5	203	1	4
39	5	203	2	4
40	5	204	3	3
41	6	205	4	3
42	6	206	2	4
43	6	207	1	4
44	6	207	4	2
45	6	208	3	3
46	6	209	1	4
47	6	209	3	4
48	6	209	3	4
49	6	210	4	3
50	7	211	4	3
51	7	212	1	3
52	7	212	2	2
53	7	213	1	3
54	7	213	1	2
55	7	213	1	4
56	7	213	4	4

Participant 2

A	B	L	M	T
1	1	101	4	4
2	1	102	4	3
3	2	103	4	2
4	2	104	4	3
5	2	105	4	2
6	2	106	4	3
7	2	107	4	3
8	2	108	4	3
9	3	109	3	4
10	3	110	4	3
11	3	111	2	3
12	3	112	3	4
13	3	113	4	4
14	3	114	4	4
15	3	115	2	4
16	4	116	4	4
17	4	117	2	3
18	4	118	4	4
19	4	119	2	4
20	4	120	2	3
21	5	201	4	4
22	5	202	4	3
23	5	203	2	4
24	5	204	4	3
25	6	205	4	2
26	6	206	2	3
27	6	207	4	3
28	6	208	4	3
29	6	209	3	3
30	6	210	2	2
31	7	211	2	3
32	7	212	1	4
33	7	212	1	4
34	7	212	2	4
35	7	213	1	3
36	7	213	1	3
37	6	210	4	3
38	7	211	4	4
39	7	212	2	3
40	7	213	1	3
41	7	213	1	4
42	7	213	4	3
43	7	214	4	3
44	7	215	4	3
45	7	216	1	4
46	7	216	1	3
47	7	216	4	4

Participant 3

A	B	L	M	T
1	1	101	4	4
2	1	102	4	3
3	2	103	4	4
4	2	104	4	4
5	2	105	4	3
6	2	106	4	3
7	2	107	4	4
8	2	108	4	3
9	3	109	4	4
10	3	110	4	4
11	3	111	4	3
12	3	112	1	4
13	3	112	1	4
14	3	112	1	4
15	3	112	4	4
16	3	113	4	4
17	3	114	4	4
18	3	115	2	4
19	4	116	4	4
20	4	117	2	4
21	4	118	1	4
22	4	118	4	3
23	4	119	2	3
24	4	120	2	4
25	5	201	4	4
26	5	202	3	4
27	5	203	2	4
28	5	204	4	4
29	6	205	4	4
30	6	206	2	4
31	6	207	3	3
32	6	208	4	4
33	6	209	3	4
34	6	210	3	4
35	7	211	2	3
36	7	212	1	3
37	7	212	1	4
38	7	212	1	3

Participant 4

A	B	L	M	T
1	1	101	4	4
2	1	102	4	4
3	2	103	4	4
4	2	104	4	4
5	2	105	4	4
6	2	106	4	4
7	2	107	4	4
8	2	108	4	4
9	3	109	4	4
10	3	110	4	4
11	3	111	4	4
12	3	112	1	4
13	3	112	4	4
14	3	113	4	4
15	3	114	4	4
16	3	115	1	4
17	4	116	3	4
18	4	117	1	3
19	4	117	1	4
20	4	118	1	4
21	4	118	4	4
22	4	119	4	4
23	4	120	2	4
24	5	201	4	4
25	5	202	3	4
26	5	203	1	3
27	5	203	2	4
28	5	204	1	3
29	6	205	4	3
30	6	206	2	3
31	6	207	2	3
32	6	208	2	4
33	6	209	2	4
34	6	210	2	3
35	7	211	1	4
36	7	211	4	3
37	7	212	4	4
38	7	213	4	4
39	7	214	3	4
40	7	215	2	3
41	7	216	1	3
42	7	216	1	4
43	7	216	1	4
44	7	216	1	4
45	7	216	1	4
46	7	216	4	4
47	7	217	4	4
48	7	218	2	4
49	7	219	1	4
50	7	219	1	4
51	7	219	4	4
52	7	220	1	4
53	7	220	4	3
54	8	301	4	4
55	8	302	4	4
56	8	303	4	4
57	8	304	4	4
58	8	305	4	4
59	8	306	2	4
60	9	307	4	4
61	9	308	2	4
62	9	309	2	4
63	9	310	2	4
64	9	311	4	4

Participant 5

A	B	L	M	T
1	1	101	4	4
2	1	102	4	3
3	2	103	4	4
4	2	104	4	4
5	2	105	4	4
6	2	106	4	4
7	2	107	2	4
8	2	108	2	4
9	3	109	4	4
10	3	110	4	4
11	3	111	1	4
12	3	111	1	4
13	3	111	1	4
14	3	111	2	4
15	3	112	4	4
16	3	113	4	4
17	3	114	4	4
18	3	115	1	4
19	4	116	4	4
20	4	117	2	4
21	4	118	1	4
22	4	118	4	4
23	4	119	2	4
24	4	120	2	4
25	5	201	4	4
26	5	202	4	4
27	5	203	3	4
28	5	204	4	4
29	6	205	4	4
30	6	206	2	3
31	6	207	2	4
32	6	208	2	3
33	6	209	1	2
34	6	210	4	3
35	7	211	4	4
36	7	212	1	4
37	7	212	1	4
38	7	212	2	3
39	7	213	1	4
40	7	213	1	4
41	7	213	1	4
42	7	213	4	3
43	7	214	4	4
44	7	215	1	4
45	7	215	2	4
46	7	216	4	4
47	7	217	4	3
48	7	218	1	4
49	7	218	1	4
50	7	218	4	4
51	7	219	1	4
52	7	219	4	4
53	7	220	1	4
54	7	220	2	3

Key: A indicates attempt, B indicates bin, L indicates level or chapter and level. M indicates mathematical distance value; T indicates technological distance value.

Participant 6

A	B	L	M	T
1	1	101	4	4
2	1	102	4	4
3	2	103	4	4
4	2	104	4	4
5	2	105	4	4
6	2	106	3	4
7	2	107	3	4
8	2	108	2	4
9	3	109	4	4
10	3	110	4	4
11	3	111	3	4
12	3	112	4	4
13	3	113	4	4
14	3	114	4	4
15	3	115	2	4
16	4	116	4	3
17	4	117	2	4
18	4	118	1	4
19	4	118	1	4
20	4	118	2	4
21	4	119	1	4
22	4	120	2	4
23	5	201	4	4
24	5	202	4	4
25	5	203	1	4
26	5	203	3	4
27	5	204	4	4
28	6	205	4	4
29	6	206	2	2
30	6	207	4	4
31	6	208	4	4
32	6	209	2	2
33	6	210	2	3
34	7	211	4	4
35	7	212	1	4
36	7	212	1	3
37	7	212	1	3
38	7	212	1	4
39	7	212	4	4
40	7	213	1	3
41	7	213	1	4
42	7	213	4	4
43	7	214	1	4
44	7	214	4	4
45	7	215	1	4
46	7	215	2	4
47	7	216	3	4
48	7	217	1	4
49	7	217	1	4
50	7	217	1	4
51	7	217	4	4
52	7	218	1	3
53	7	218	1	4
54	7	218	1	4
55	7	218	3	4
56	7	219	4	4
57	7	220	1	4
58	7	220	1	4
59	7	220	1	4
60	7	220	1	4
61	7	220	1	4
62	7	220	4	4

Participant 7

A	B	L	M	T
1	1	101	4	4
2	1	102	4	4
3	2	103	4	4
4	2	104	4	4
5	2	105	3	4
6	2	106	4	4
7	2	107	2	4
8	2	108	4	4
9	3	109	4	4
10	3	110	4	4
11	3	111	4	4
12	3	112	1	4
13	3	112	3	4
14	3	113	4	4
15	3	114	1	4
16	3	114	1	4
17	3	114	1	4
18	3	114	1	4
19	3	114	1	4
20	3	114	1	4
21	3	114	1	4
22	3	114	1	4
23	3	114	1	4
24	3	114	1	4
25	3	114	1	4
26	3	114	1	4
27	3	114	1	4
28	3	114	1	4
29	3	114	1	4
30	3	114	1	4
31	3	114	4	4
32	3	115	1	4
33	3	115	2	4
34	4	116	4	4
35	4	117	2	4
36	4	118	1	4
37	4	118	1	4
38	4	118	1	4
39	4	118	1	4
40	4	118	1	4
41	4	118	1	4
42	4	118	1	4
43	4	118	4	4

Participant 8

A	B	L	M	T
1	1	101	3	3
2	1	102	3	4
3	2	103	4	3
4	2	104	3	4
5	2	105	3	4
6	2	106	3	4
7	2	107	3	4
8	2	108	4	4
9	3	109	3	4
10	3	110	4	4
11	3	111	1	4
12	3	111	4	4
13	3	112	1	4
14	3	112	4	4
15	3	113	4	4
16	3	114	1	4
17	3	114	1	4
18	3	114	4	4
19	3	115	4	4
20	4	116	4	3
21	4	117	1	4
22	4	117	1	4
23	4	117	1	4
24	4	117	2	4
25	4	118	1	4
26	4	118	4	4
27	4	119	2	4
28	4	120	4	4
29	5	201	4	4
30	5	202	2	4
31	5	203	1	4
32	5	203	1	4
33	5	203	4	3
34	5	204	4	4
35	6	205	4	4
36	6	206	2	4
37	6	207	2	4
38	6	208	4	4
39	6	209	1	3
40	6	210	3	3
41	7	211	4	3
42	7	212	1	3
43	7	212	2	2
44	7	213	1	4
45	7	213	1	4
46	7	213	1	3
47	7	213	1	3

Participant 9

A	B	L	M	T
1	1	101	4	4
2	1	102	4	4
3	2	103	4	4
4	2	104	4	4
5	2	105	4	4
6	2	106	4	4
7	2	107	4	4
8	2	108	4	4
9	3	109	4	4
10	3	110	4	4
11	3	111	1	4
12	3	111	4	3
13	3	112	1	4
14	3	112	1	4
15	3	112	3	4
16	3	113	4	4
17	3	114	1	4
18	3	114	1	4
19	3	114	1	4
20	3	114	1	4
21	3	114	4	4
22	3	115	2	3
23	4	116	4	4
24	4	117	2	3
25	4	118	3	3
26	4	119	2	4
27	4	120	4	4
28	5	201	4	4
29	5	202	4	4
30	5	203	4	4
31	5	204	4	4
32	6	205	4	4
33	6	206	2	4
34	6	207	2	4
35	6	208	2	3
36	6	209	3	3
37	6	210	3	4
38	7	211	1	4
39	7	211	4	3
40	7	212	2	3
41	7	213	1	3
42	7	213	1	4
43	7	213	1	3
44	7	213	1	3
45	7	213	1	3
46	7	213	1	4
47	7	213	1	4
48	7	213	3	4
49	7	214	4	3
50	7	215	2	3
51	7	216	4	3
52	7	217	1	3
53	7	217	1	4
54	7	217	4	3
55	7	218	2	3
56	7	219	1	4
57	7	219	1	4
58	7	219	3	3
59	7	220	3	4
60	8	301	4	4
61	8	302	2	4
62	8	303	4	4
63	8	304	4	4
64	8	305	2	4
65	8	306	3	4

Participant 10

A	B	L	M	T
1	1	101	4	4
2	1	102	4	4
3	2	103	4	4
4	2	104	4	4
5	2	105	3	4
6	2	106	2	4
7	2	107	2	4
8	2	108	2	4
9	3	109	4	4
10	3	110	2	4
11	3	111	1	4
12	3	111	2	4
13	3	112	3	4
14	3	113	4	4
15	3	114	4	4
16	3	115	1	4
17	4	116	4	4
18	4	117	1	4
19	4	117	1	4
20	4	118	4	4
21	4	119	1	4
22	4	119	1	4
23	4	120	1	4
24	5	201	4	4
25	5	202	2	4
26	5	203	4	4
27	5	204	4	4
28	6	205	4	4
29	6	206	2	4
30	6	207	2	4
31	6	208	2	3
32	6	209	2	3
33	6	210	3	3
34	7	211	4	3
35	7	212	1	4
36	7	212	1	3
37	7	213	1	3
38	7	213	1	4
39	7	213	1	2
40	7	213	1	2
41	7	213	1	3
42	7	211	4	3
43	7	212	1	4
44	7	212	2	3
45	7	213	3	3

Key: A indicates attempt, B indicates bin, L indicates level or chapter and level. M indicates mathematical distance value; T indicates technological distance value.

Appendix H

Institutional Review Board (IRB) Certificate

UtahState
University
OFFICE OF RESEARCH AND GRADUATE STUDIES

Institutional Review Board

USU Assurance: FWA#00003308

Association for the Accreditation of Human Research Protection Programs, Inc.

Expedite #6

Letter of Approval

FROM:

Melanie Domenech Rodriguez, IRB Chair

True M. Rubal, IRB Administrator

True M. Rubal

To:	Patricia Moyer-Packenham, Stephen Tucker
Date:	January 29, 2015
Protocol #:	6362
Title:	An Exploratory Study Of Attributes, Affordances, Abilities, And Distance In Students' Use Of Virtual Manipulative Mathematics Ipad Apps
Risk:	Minimal risk

Your proposal has been reviewed by the Institutional Review Board and is approved under expedite procedure #6 (based on the Department of Health and Human Services (DHHS) regulations for the protection of human research subjects, 45 CFR Part 46, as amended to include provisions of the Federal Policy for the Protection of Human Subjects, November 9, 1998):

> Collection of data from voice, video, digital, or image recordings made for research purposes.

This approval applies only to the proposal currently on file for the period of one year. If your study extends beyond this approval period, you must contact this office to request an annual review of this research. Any change affecting human subjects must be approved by the Board prior to implementation. Injuries or any unanticipated problems involving risk to subjects or to others must be reported immediately to the Chair of the Institutional Review Board.

Prior to involving human subjects, properly executed informed consent must be obtained from each subject or from an authorized representative, and documentation of informed consent must be kept on file for at least three years after the project ends. Each subject must be furnished with a copy of the informed consent document for their personal records.

CURRICULUM VITAE

STEPHEN ISAAC TUCKER

School Address:
Utah State University
College of Education & Human Services
2605 Old Main Hill
Logan, UT 84322-2605

Contact Information:
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researchgate.net/profile/Stephen_Tucker6

EDUCATION

Ph.D. June 2015
Education, Utah State University. Logan, Utah.
Specialization: Curriculum and Instruction
Concentration: Mathematics Education and Leadership
Dissertation: *An Exploratory Study of Attributes, Affordances, Abilities, and Distance in Students' Use of Mathematics Virtual Manipulative iPad Apps*

M.A. June 2010
Mathematics Education K-6, Western Governors University, Utah.
Unpublished Thesis: *Hands-On Virtual Manipulatives and Their Affect on Third Graders' Performance and Attitudes in Geometry*

B.A. May 2006
Elementary Education, American University, Washington DC

EMPLOYMENT HISTORY
UTAH STATE UNIVERSITY

Instructor, Tutoring Intervention & Mathematics Enrichment (TIME) Clinic (2013-2014)
School of Teacher Education and Leadership, College of Education and Human Services, Utah State University, Logan, UT.
Researched, planned, and provided mathematics tutoring for students enrolled in the TIME Clinic.

University Teaching & Supervision, Elementary Education Program (2011-present).
School of Teacher Education and Leadership, College of Education and Human Services, Utah State University, Logan, UT.
Responsibilities included teaching graduate and undergraduate courses as the instructor of record in the elementary education program, collaborating with other instructors, developing course syllabi, revising courses based on course evaluation feedback, creating assessments using online course management system (Canvas), supervising pre-service elementary teachers during their practicum and student teaching placements, creating and conducting seminars based on student teachers' lessons, lesson plans, journal reflections, and questions, observing lessons and facilitating supervisor-cooperating teacher-student teacher conferences.

Graduate Research Assistant (2011-present)
School of Teacher Education and Leadership, College of Education and Human Services, Utah State

University, Logan, UT.
(2011-present) Dr. Patricia Moyer-Packenham, Mathematics Education
Provided research assistance in projects concerning virtual manipulatives and mathematics content, reviewed articles for possible inclusion in and maintained organization of research database, collaborated on papers for publication.

PUBLIC SCHOOL TEACHING EXPERIENCE – 5 YEARS

Elementary School Teacher, Grades 2 & 3, All subjects (2006-2011)
Charlotte Mecklenburg School District, Charlotte, North Carolina

AWARDS & PROFESSIONAL RECOGNITION

Emma Eccles Jones College of Education and Human Services Graduate Student Researcher of the Year Award, 2015, Utah State University, Logan, UT
Recognized as the most outstanding graduate student researcher from the largest college of the university, consisting of seven departments with more than 900 total graduate students. Nominated for the Robins Award as the University Graduate Student Researcher of the Year Award.

Department of Teacher Education and Leadership Graduate Research Assistant of the Year Award, 2015, Utah State University, Logan, UT
Recognized as the most outstanding graduate research assistant from the department.

Graduate Enhancement Award, 2014, Utah State University, Logan, UT
One of twenty recipients of a merit-based $4,000 award for “outstanding graduate students who have shown a track record of excellence” in research, teaching, and contributions to professional, campus, and local communities.

Young Researchers’ Day participant, 2014, International Group for the Psychology of Mathematics Education (PME) Conference, Vancouver, BC, Canada
One of approximately fifty scholars chosen to participate in the inaugural PME Young Researchers’ Day “to provide early career researchers with opportunities to develop their research skills in various fields, establish new contacts and build networks among themselves and to future PME conferences, and meet and work with international experts in the field.”

Oral Presentation Honorable Mention Award, 2013, Utah State University Research Week
Honorable mention citation (Engineering) for *A Loss to Explain, Autoethnographically;* research and methodology presentation.

Vice Presidential Research Fellowship, 2011-2012, Utah State University, Logan, UT
One of eleven recipients of the nomination-based one-year $15,000 fellowship awards for doctoral students pursuing research degrees including a thesis or dissertation

Albemarle Road Elementary School Teacher of the Year, Finalist (2010), Charlotte, NC

Albemarle Road Elementary School Teacher of the Year, Nominee (2009), Charlotte, NC

RESEARCH

RESEARCH INTERESTS

Learning mathematics using technology
Virtual manipulatives
Mathematics teacher development

PUBLICATIONS

Journal Articles (Refereed)

1. Moyer-Packenham, P. S., Bullock, E. P., Shumway, J. F., Tucker, S. I., Watts, C., Westenskow, A., Anderson-Pence, K. L., Boyer-Thurgood, J., Gulkilik, H., & Jordan, K. (in press, 2015). The role of affordances in children's learning performance and efficiency when using virtual manipulatives mathematics iPad apps. *Mathematics Education Research Journal.*

2. Tucker, S. I. (2015). Three men and a maybe: Identity and privilege in male preservice elementary school teachers. *The Journal of Men's Studies, 23*(1), 3-20.

3. Moyer-Packenham, P. S., Shumway, J. F., Bullock, E., Tucker, S. I., Anderson-Pence, K. L., Westenskow, A., Boyer-Thurgood, J., Maahs-Fladung, C., Symanzik, J., Mahamane, S., MacDonald, B., & Jordan, K. (2015). Young children's learning performance and efficiency when using virtual manipulative mathematics iPad apps. *Journal of Computers in Mathematics and Science Teaching*, *34*(1), 41–69.

4. Moyer-Packenham, P. S., Bolyard, J. J., & Tucker, S. I. (2014). Second graders' mathematical practices for solving fraction tasks. *Investigations in Mathematics Learning*, *7*(1), 54–81.

5. Tucker, S. I. (2014). REFractions: The representing equivalent fractions game. *Australian Primary Mathematics Classroom*, *19*(1), 29–34.

Conference Proceedings (Refereed)

1. Tucker, S. I., & Moyer-Packenham, P. S. (2014). Virtual manipulatives' affordances influence student learning. In S. Oesterle, C. Nicol, P. Liljedahl, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (Vol. 6, p. 251). Vancouver, Canada: PME.

2. Moyer-Packenham, P. S., Westenskow, A., Shumway, J. F., Bullock, E., Tucker, S. I., Anderson-Pence, K. L., Boyer-Thurgood, J., Maahs-Fladung, C., Symanzik, J., Mahamane, S., MacDonald, B., & Jordan, K., The Virtual Manipulatives Research Group at Utah State University. (2014, September). The effects of different virtual manipulatives for second graders' mathematics learning and efficiency in the touch-screen environment. *Proceedings of the 12th International Conference of the Mathematics Education into the 21st Century Project*, Herceg Novi, Montenegro.

3. Tucker, S. I., Moyer-Packenham, P. S., Boyer-Thurgood, J. M., Anderson, K. L., Shumway, J. F., Westenskow, A., Bullock, E. & The Virtual Manipulatives Research Group at Utah State University. (2014, January). Literature supporting investigations of the nexus of mathematics, strategy, and technology in second-graders' interactions with iPad-based virtual manipulatives. In *Proceedings of the 12th Annual Hawaii International Conference on Education (HICE)* (pp. 2338–2346). Honolulu, Hawaii, ISSN# 1541-5880.

4. Moyer-Packenham, P. S., Anderson, K. L., Shumway, J. F., Tucker, S. I., Westenskow, A., Boyer-Thurgood, J. M., Bullock, E., Mahamane, S., Baker, J. M., Gulkilik, H., Maahs-Fladung, C., Symanzik, J., Jordan, K. & The Virtual Manipulatives Research Group at Utah State University. (2014, January). Developing research tools for young children's interactions with mathematics apps on the iPad. In *Proceedings of the 12th Annual Hawaii International Conference on Education (HICE)* (pp. 1685-1694). Honolulu, Hawaii, ISSN# 1541-5880.

5. Tucker, S. I. (2014, January). Three men and a maybe: Identity and privilege in male preservice elementary school teachers. *Proceedings of the 12th Annual Hawaii International Conference on Education (HICE),* (p. 818), Honolulu, Hawaii, ISSN# 1541-5880.

6. Lee, H. K., & Tucker, S. I. (2014, January). Revisiting the professional identities of transnational foreign language teachers in the United States. *Proceedings of the 12th Annual Hawaii International Conference on Education (HICE),* (p. 2106), Honolulu, Hawaii, ISSN# 1541-5880.

7. Boyer-Thurgood, J. M., Tucker, S. I., Mejia, J. A., & Norman, P. (2014, January). The socio-cultural importance of writing and sharing autoethnographic research. *Proceedings of the 12th Annual Hawaii International Conference on Education (HICE),* (pp. 1115-1116), Honolulu, Hawaii, ISSN# 1541-5880.

8. Boyer-Thurgood, J. M., Moyer-Packenham, P. S., Tucker, S. I., Anderson, K. L., Shumway, J. F., Westenskow, A., Bullock, E., & The Virtual Manipulatives Research Group at Utah State University (2014, January). Kindergarteners' strategy development during combining tasks on the iPad. *Proceedings of the 12th Annual Hawaii International Conference on Education (HICE),* (pp. 1113-1114), Honolulu, Hawaii, ISSN# 1541-5880.

Other Publications

1. Lee, H.K., & Tucker, S. I. (2014, April). *Revisiting the professional identities of transnational foreign language teachers in the United States*. Paper presented at the annual American Educational Research Association Conference (AERA), Philadelphia, PA.

Unpublished Manuscripts

1. Tucker, S. I., Moyer-Packenham, P. S., Westenskow, A., & Jordan, K. E. (under review, 2015). *The Complexity of the Affordance-Ability Relationship when Second-Grade Children Interact with Virtual Manipulative Mathematics Apps*. Unpublished manuscript.

2. Tucker, S. I., Moyer-Packenham, P. S., Shumway, J.F., & Jordan, K. E. (under review, 2015). *Zooming in on students' thinking: How a virtual manipulative app revealed, concealed, and developed students' number understanding.* Unpublished manuscript.

3. Shumway, J. F., Westenskow, A., Moyer-Packenham, P. S., Anderson-Pence, K. L., Tucker, S. I., Boyer-Thurgood, J. M., & Baker, J. M. (under review, 2015). *Using open-response fraction problems to explore the relationship between instructional modalities and students' solution strategies*. Unpublished manuscript.

4. Tucker, S. I. (2013). *Implementing iPads in mathematics education: Affordances, attitudes, and achievement.* Unpublished manuscript.

Manuscripts under Development

1. Moyer-Packenham, P. S., Symanzik, J., & Tucker, S. I., & The Virtual Manipulatives Research Group

at Utah State University. (under development). *Examining Patterns in Second Graders' Mathematics Learning in the Touch-Screen Environment through Heatmap Analysis*. Unpublished manuscript.

2. Baker, J. M., Moyer-Packenham, P. S., Tucker, S. I., Shumway, J. F., Jordan, K., & Gillam, R. (under development). *Examining Patterns in Second Graders' Brain Activation while Using Virtual Manipulative Mathematics iPad Apps through Near-Infrared Spectroscopy*. Unpublished manuscript.

3. Lee, H.K. & Tucker, S. I. (under development, 2015). *Living and Teaching in Two Worlds: Professional Identity Development of Transnational Dual Language Immersion Teachers.* Unpublished manuscript

4. Tucker. S. I. (under development). *Preservice Elementary Teachers' Analysis of Mathematics Curriculum Materials*. Unpublished manuscript.

CURRENT RESEARCH ACTIVITIES

Captivated! Young Children's Learning Interactions with iPad Mathematics Apps (2013-present)
- Assisted in project conceptualization
- Assisted in grant development
- Led literature review
- Data collection: conducted and observed clinical interviews of pre-K, K, and second grade students
- Data analysis: Developed and applied coding schemes for video data of 33 second grade students
- Dissemination: leading or contributing to manuscripts and conference presentations

Preservice Elementary School Teachers' Analysis of Mathematics Curriculum Materials (2013-present)
- *Independent research project*
- Reviewed literature
- Conceptualized project
- Pilot study (Case study)
    - Developed instruments for data collection
    - Collected data (observations, interviews, journals)
    - Analyzed data using qualitative coding of emergent themes
    - Presented pilot findings with undergraduate colleagues
- Full study (Classroom & case study)
    - Revised data collection instruments
    - Collected data (observations, interviews, journals)
    - Analyzed data using qualitative coding of piloted and emergent themes
    - Developing manuscript for publication

Virtual Manipulatives Research Group, member (2011-present)
- Conceptualized and conducted research projects concerning virtual manipulatives
- Conceptualized and contributed to publications and presentations

Virtual Manipulatives Database, manager (2011-present)
- Maintained research database and review of articles for inclusion therein
- Developed and organized review and coding procedures for contributors

VM BRAIN Patterns: Virtual Manipulatives: Brain Research on Activation and Investigation of Neural Patterns. (2011-present)
- Developed fraction task sequence for pilot study
- Assisted in grant-writing process

- Interviewed and observed in clinical interviews
- Contributed to data analysis and manuscript development

GRANTS FUNDED

Graduate Student Professional Conference Awards. ($1,415: $700 Department of Teacher Education and Leadership, $300 Office of Research and Graduate Studies, and $415 Center for Women and Gender). (2014). Travel Funding awarded for presentations at the Hawaii International Conference on Education, Honolulu, HI.

Graduate Student Professional Conference Awards. ($600: $300 Office of Research and Graduate Studies and $300 Department of Teacher Education and Leadership). (2013). Travel Funding awarded for presentation at the NCTM Annual Conference in Denver, CO.

Graduate Research Assistant ($20,000). *Captivated! Young Children's Learning Interactions with iPad Mathematics Apps.* (2013-2014). Utah State University, Vice President for Research RC Funding. Project goal: build theory and knowledge about the nature of young children's ways of thinking and interacting with virtual manipulatives using touch-screen mathematics apps on the iPad. My role: assist in grant-writing process, design and conduct iPad-based interviews with participants, design study protocols and surveys, collect, code, and analyze data, review literature, and collaborate on writing papers for publication. (With Principal Investigator Patricia Moyer-Packenham, Co-PI Cathy Maahs-Fladung, and the Virtual Manipulatives Research Group)

PRESENTATIONS—SCHOLARSHIP

Invited Presentations

A Working Session on Virtual Manipulatives. 2014 International Group for the Psychology of Mathematics Education (PME) Conference, Vancouver, BC, Canada. Co-chaired by Dr. Patricia Moyer-Packenham & Dr. Jennifer Suh.

- Invited Presentation: *MAAAD for Learning: Modification of Attributes, Affordances, Abilities, and Distance to Learn Mathematics Using Technology*

International Presentations

Hawaii International Conference on Education

Tucker, S. I. (2014, January). *Three Men and a Maybe: Identity and Privilege in Male Preservice Elementary School Teachers*. Paper Session, Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

Tucker, S. I., Moyer-Packenham, P. S., Boyer-Thurgood, J. M., Anderson, K. L., Shumway, J. F., Westenskow, A., & Bullock, E. (2014, January). *The Nexus of Mathematics, Strategy, and Technology in Second-Graders' Interactions with an iPad-Based Virtual Manipulative*. Paper Session, Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

Tucker, S. I. & Lee, H.K. (2014, January). *Revisiting the Professional Identities of Transnational Foreign Language Teachers in the United States*. Paper Session, Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

Boyer-Thurgood, J. M., Tucker, S. I., & Mejia, J. A. (2014, January). *The Socio-Cultural Importance of Writing and Sharing Autoethnographic Research*. Workshop Session, Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

Boyer-Thurgood, J. M., Moyer-Packenham, P. S., Tucker, S. I., Anderson, K. L., Shumway, J. F., Westenskow, A., & Bullock, E. (2014, January). *Kindergarteners' Strategy Development during Combining Tasks on the iPad*. Paper Session, Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

Moyer-Packenham, P. S., Shumway, J. F., Westenskow, A., Tucker, S. I., Anderson, K. L., Boyer-Thurgood, J., & Bullock, E. (2014, January). *Young Children's Mathematics Interactions with Virtual Manipulatives on iPads*. Paper Session, Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

International Conference of the Mathematics Education into the 21st Century Project (ICME21)

Moyer-Packenham, P. S., Westenskow, A., Shumway, J. F., Bullock, E., Tucker, S. I., Anderson-Pence, K. L., Boyer-Thurgood, J., Maahs-Fladung, C., Symanzik, J., Mahamane, S., MacDonald, B., & Jordan, K., The Virtual Manipulatives Research Group at Utah State University. (2014, September). *The Effects of Different Virtual Manipulatives for Second Graders' Mathematics Learning and Efficiency in the Touch-Screen Environment.* Paper Presentation, 12th International Conference of the Mathematics Education into the 21st Century Project, Herceg Novi, Montenegro.

International Conference on Education in Mathematics, Science, & Technology (ICEMST)

Moyer-Packenham, P. S., Bullock, E., Watts, C., Tucker, S. I., Shumway, J. F., Anderson-Pence, K. L., Westenskow, A., Boyer-Thurgood, J., Gulkilik, H., Jordan, K. (2015, April). *The Relationship between Affordances of Virtual Manipulatives Mathematics Apps and Young Children's Learning Performance and Efficiency*. Paper presentation. International Conference on Education in Mathematics, Science, & Technology. Anatalya, Turkey.

International Group for the Psychology of Mathematics Education (PME)

Tucker, S. I. & Moyer-Packenham, P. S. (July, 2014) *Virtual Manipulatives' Affordances Influence Mathematical Understanding.* Short Oral Presentation, International Group for the Psychology of Mathematics Education (PME) Conference, Vancouver, BC, Canada.

National Presentations

American Education Research Association (AERA)

Tucker, S. I. (2015, April). *Three Men and a Maybe: Identities and Privileges of Male Preservice Elementary Teachers*. Paper presentation, Division K, Section 3, Paper session: Mirror Mirror on The Wall: Reflection on Identities and Practice in Teachers' Lives, American Education Research Association (AERA) Annual Conference, Chicago, Illinois.

Lee, H.K. & Tucker, S. I. (2014, April). *Revisiting the Professional Identities of Transnational Foreign Language Teachers in the United States*. Division I Poster Session, American Education Research Association (AERA) Annual Conference, Philadelphia, Pennsylvania.

Association of Mathematics Teacher Educators (AMTE)

Shumway, J. F., *Bostwick, A., Anderson, K., & Tucker, S. I. (2013, January). *Building Partnerships: A Collaborative Lesson-Study Experience in a Preservice Mathematics Methods Course*. Research Presentation, Association of Mathematics Teacher Educators Conference, Orlando, Florida.

**Inservice teacher*

National Council of Teachers of Mathematics: Annual Meeting and Exposition (NCTM)

Tucker, S. I. (2013, April) *REFractions: The Representing Equivalent Fractions Game*. Gallery Workshop Session, National Council of Teachers of Mathematics Conference, Denver, Colorado.

National Council of Teachers of Mathematics: Research Conference (NCTM-R)

Moyer-Packenham, P. S., Shumway, J. F., Tucker, S. I., Boyer-Thurgood, J. M., Hunt, J. & Bullock, E. (2014, April). *Young Children's Mathematics Interactions with Virtual Manipulatives on iPads*. Paper Presentation, National Council of Teachers of Mathematics Research Conference, New Orleans, Louisiana.

State & Regional Presentations

Utah Council of Teachers of Mathematics (UCTM)

Tucker, S. I. (2014, November). *Barbie and Friends Take the Measure of Mathematics for Social Justice*. Workshop Presentation, Utah Council of Teachers of Mathematics (UCTM), Annual State Conference, Davis County School District, Utah.

Tucker, S. I., *Weight, M., & *Olsen, K. (2013, November). *Exploring Mathematics Culture through Critical Analysis of Curriculum Materials*. Research Presentation, Utah Council of Teachers of Mathematics (UCTM), Annual State Conference, Salt Lake City, Utah.
**Undergraduate preservice teacher*

Tucker, S. I. (2012, October). *Investigating REFractions: The Representing Fractions Game*. Workshop Presentation, Utah Council of Teachers of Mathematics, Annual State Conference, American Fork, Utah.

Tucker, S. I. (2011, November). *Array of Hope: A Common Core-Focused Hands-On Exploration of Multiplication and Division*. Workshop Presentation, Utah Council of Teachers of Mathematics, Annual State Conference, Magna, Utah.

University Presentations

Utah State University (USU)

Moyer-Packenham, P. S., Shumway, J. F., Bullock, E., Tucker, S. I., Anderson-Pence, Katie L., Boyer-Thurgood, J., Maahs-Fladung, C., Symanzik, J., Mahamane, S., MacDonald, B., & Jordan, K. (2015, April). *Young Children's Learning Performance and Efficiency when Using Virtual Manipulative Mathematics iPad Apps.* Early Childhood Education Research Center Poster Session, Utah State University Research Week, Logan, Utah.

Tucker, S. I. (2014, April). *Three Men and a Maybe: Identity and Privilege in Male Preservice Elementary School Teachers*. Oral Presentation, Utah State University Research Week, Logan, Utah.

Lee, H.K. & Tucker, S. I. (2014, April). *Revisiting the Professional Identities of Transnational Foreign Language Teachers in the United States*. Poster Session, Utah State University Research Week, Logan, Utah.

Tucker, S. I., Boyer-Thurgood, J. M., Mejia, J. A., & Norman, P. (2013, April). *A Loss to Explain, Autoethnographically*. Research and Methodology Session, Utah State University Research Week, Logan, Utah. *Oral Presentation Honorable Mention Award.*

TEACHING

UNIVERSITY TEACHING

Utah State University, Logan, Utah (2011-present)
College of Education and Human Services

Lead Instructor

TEAL 6525/TEPD 5525 Mathematics for Teaching K-8: Data Analysis and Problem-Solving (Summer 2014)
Graduate Course. This course provides practicing K-8 teachers a deeper understanding of problem solving techniques, probability, and data representation and analysis. Integrated introductory social justice themes with data analysis and probability content. Delivered via broadcast distance education technology.

ELED 4060 – Teaching Mathematics & Practicum Level III (Spring 2012, Fall 2012, Spring 2013, Spring 2014; Fall 2014; Spring 2015)
Undergraduate Course. Relevant mathematics instruction in the elementary and middle-level curriculum; methods of instruction, evaluation, remediation, and enrichment. A field experience practicum is required.

Courses Co-Taught

TEAL 6521 – Mathematics for Teaching K-8: Numbers and Operations (Fall 2013)
Master's Course, Co-Taught with Dr. Amy Brown. This course, for K-8 teachers, covers the content of Number and Operations to develop comprehensive understanding of our number system, relating its structure to computation, arithmetic, algebra, and problem solving. Delivered via broadcast distance education technology.

ELED 5100 – Student Teaching – Primary (Grades 1-3) (Fall 2011)
ELED 5150 – Student Teaching – Elementary (Grades 4-6) (Fall 2011)
Undergraduate Course. Student teachers need to demonstrate competency and professionalism in teaching. Students begin their transition from university student to professional teacher.

Guest Instructor

SPED 5340 – Teaching Math to Students with Mild/Moderate Disabilities: Expressions, Equations, and Algebraic Thinking (January 14, 2014)
Undergraduate Course. Planned and delivered in person instruction on expressions, equations, and algebraic thinking in middle grades to students as guest instructor for Dr. Jessica Hunt for one three-hour class session.

TEAL 6524/TEPD 5524 – Mathematics for Teaching K-8: Geometry and Measurement: Transformations and Tessellations; Similarity and Congruence (March 19, 2013)
Master's Course. Planned and delivered instruction to students in person and via broadcast distance education technology as substitute instructor for Dr. Jim Barta for one three-hour class session.

CURRICULUM DEVELOPMENT

Utah State University, Logan, Utah (2011-present)
College of Education and Human Services

CCSSM Elementary Mathematics Teacher Academy, member (2012-present)

Developed course materials for master's level courses and online professional development for Utah State University's Elementary Mathematics Teacher Academy (EMTA). Courses designed to develop teachers' mathematical knowledge for teaching aligned with the Common Core State Standards for Mathematics. Materials developed included readings, video lectures, application assignments, and assessments for online course delivery. Assisted in development of program structure and badges system. Authored two mathematics content blog posts. Developed the following 27 curriculum modules:

4.NBT | Big Idea: Student-Generated and Alternative Algorithms (2013)
4.NBT.A.1-2 | Multi-Digit Place Value (2013)
4.NBT.A.1-3 | Multi-Digit Place Value with Rounding (2013)
4.NBT.B.4 | Fluent Addition and Subtraction (2013)
4.NBT.B.5 | Multi-Digit Whole Number Multiplication (2013)
4.NBT.B.6 | Multi-Digit Whole Number Division (2013)
4.NF | Big Idea: Proportional Reasoning (2013)
4.NF.A.1 | Equivalent Fractions (2013)
4.NF.A.2 | Comparing Fractions (2013)
4.NF.B.3a,d | Adding and Subtracting Fractions in Word Problems (2013)
4.NF.B.3b,d | Decomposing Fractions (2013)
4.NF.B.3c,d | Adding and Subtracting Mixed Numbers in Word Problem Contexts (2013)
4.NF.B.4a,c | Fractions as Multiples in Word Problem Contexts (2013)
4.NF.B.4b,c | Fractions as Multiples of Multiples in Word Problem Contexts (2013)
4.NF.C | Decimals, Notations, and Computation (2013)
4.OA.A.1-2 | Multiplicative Comparison (2013)
4.OA.A.3 | Multi-Step Word Problems (All Operations) (2013)
4.OA.B.4 | Factoring, Primes, and Composites (2013)
4.OA.C.5n | Patterns: Numbers (2013)
4.OA.C.5s | Patterns: Shapes (2013)
K.MD.A.1-2 | Describe and Compare Measurable Attributes (2014)
K.MD.B.3 | Classify and Count Data (2014)
K.NBT.A.1 | Composing and Decomposing Numbers (11-19) (2015)
K.OA.A.1 | Representing Addition and Subtraction (2015)
K.OA.A.2 | Solving Addition and Subtraction Word Problem (2015)
K.OA.A.3 | Composing and Decomposing Numbers (0-10) (2015)
K.CC.B.5 | Count to Answer "How Many?" (2015)

Reflective Mathematics Educators Group, member (2011-present)

Redesigned syllabus, assignments, and assessments, and planned instruction for ELED 4060 Elementary Mathematics Methods courses

SERVICE
PRESENTATIONS—SERVICE

University—Invited Presentations

University of Washington – Tacoma

Tucker, S. I., Boyer-Thurgood, J. M., Mejia, J. A., & Norman, P. (2014, April). *Experiencing Autoethnography.* Invited Presentation, TCORE 123C: The Autoethnographic Self. Instructor: Dr. Rich Furman. University of Washington – Tacoma.

Utah State University (USU)

Tucker, S. I., Mejia, J.A., & Norman, P. (2015, January). *From Many Stories, One: Reflecting on Autoethnographic Experiences.* Invited Presentation & Panel, EDUC 7780: Qualitative Methods II. Instructor: Dr. Sherry Marx. Logan, Utah.

Boyer-Thurgood, J. M., Tucker, S. I. & Mejia, J.A. (2014, January). *Experiences with Autoethnography.* Invited Presentation, EDUC 7780: Qualitative Methods II. Instructor: Dr. Sherry Marx. Logan, Utah.

Tucker, S. I. (2013, July). *Zotero: Getting Organized.* Invited Presentation, TEAL 7050: Instructional Leadership. Instructor: Dr. Susan Turner. Logan, Utah.

Tucker, S. I. (2012, March). *Analyzing and Implementing REFractions: The Representing Equivalent Fractions Game.* Invited Presentation, TEAL 6522: Mathematics for Teaching K-8: Rational Numbers and Proportional Reasoning. Instructor: Dr. Amy Brown, Logan, Utah.

University—Teaching Presentations

Utah State University (USU)

Tucker, S. I. (2015, March). *Barbie and Friends Take the Measure of Math for Social Justice.* Teaching Workshop Presentation for Preservice Elementary Teachers, Logan, Utah.

Tucker, S. I. (2015, February). *Analyzing and Implementing REFractions via Inquiry: The Representing Equivalent Fractions Game.* Teaching Workshop Presentation for Preservice Elementary Teachers, Logan, Utah.

Tucker, S. I. (2014, October). *Barbie and Friends Teach Math for Social Justice.* Teaching Workshop Presentation for Preservice Elementary Teachers, Logan, Utah.

Tucker, S. I. (2014, October). *Analyzing and Implementing REFractions via Inquiry: The Representing Equivalent Fractions Game.* Teaching Workshop Presentation for Preservice Elementary Teachers, Logan, Utah.

Tucker, S. I. (2014, February). *Analyzing and Implementing REFractions via Inquiry: The Representing Equivalent Fractions Game.* Teaching Workshop Presentation for Preservice Elementary Teachers, Logan, Utah.

Tucker, S. I. (2014, February). *Barbie and Friends Teach Math for Social Justice.* Teaching Workshop Presentation Workshop for Preservice Elementary Teachers, Logan, Utah.

Tucker, S. I. (2013, February). *Analyzing and Implementing REFractions: The Representing Equivalent Fractions Game.* Teaching Workshop Presentation for Preservice Elementary Teachers, Logan, Utah.

Tucker, S. I. (2012, October). *Investigating REFractions: The Representing Equivalent Fractions Game.* Teaching Workshop Presentation for Preservice Elementary Teachers, Logan, Utah.

Tucker, S. I. (2012, February). *Investigating REFractions: The Representing Equivalent Fractions Game.* Teaching Workshop Presentation for Preservice Elementary Teachers, Logan, Utah.

Tucker, S. I. (2011, October). *Fraction Interactions: Using Technology to Develop Fraction Concepts in the Elementary Classroom.* Three Presentations, ELED 4060: Elementary Mathematics Methods. Instructors: Dr. Dicky Ng, Katie Anderson, and Jessica Shumway, Logan, Utah.

PROFESSIONAL LEADERSHIP & SERVICE

INTERNATIONAL SERVICE

Session Chair	2014 Hawaii International Conference on Education o Early Childhood Education o Teacher Education

NATIONAL SERVICE

Reviewer (2015-present)	*Technology, Knowledge, and Learning*
Reviewer (2014-present)	*Journal of Teacher Education*
Reviewer (2012-present)	*Teaching Children Mathematics*, National Council of Teachers of Mathematics

INSTITUTIONAL LEADERSHIP & SERVICE

UTAH STATE UNIVERSITY
Institutional Service—University Level

Committee Membership

- Department Teaching Excellence Award Committee, graduate student representative (2011-2013)
  - Reviewed application proposals, observed instruction, and determined winners of a $20,000 university-wide award intended for the department that best demonstrated it values learning and teaching excellence. Revised review process, including measures, scales, and procedures.
- Graduate Student Interview Committee for Associate Vice President and Associate Dean of the School of Graduate Studies, member (2012)
  - Co-interviewed three candidates and provided feedback to inform the hiring process.
- Graduate Student Senate Research and Projects Grant Review Committee, member (2012)
  - Reviewed graduate student research and project grant proposals to determine recipients of awards up to $1,000.
- Graduate Robins Awards Selection Committee, member (2013)
  - Reviewed departmental nominations to determine university-wide winners of Graduate Researcher of the Year and Graduate Teaching Assistant of the Year awards.
- Graduate Enhancement Awards Selection Committee, member (2013)
  - Reviewed graduate student applications to determine 20 recipients of merit-based $4,000 grants.

Other Service

- USU Physics Day, Judge, Mathematics, Science, Engineering, Achievement (MESA) Prosthetic Arm Challenge Contest (2014)
    - Judged the throwing distance & accuracy component of the MESA Prosthetic Arm Challenge Contest, high school section.
- LGBTQA Allies, member (2013-present)
    - Trained and served as an LGBTQA Ally to help reduce homophobia and heterosexism on a personal and professional level.

Institutional Service—College Level

- Group Assessment Admissions Interviews, member (2013-present)
    - Collaborated to conduct group interviews and review candidates for entrance into the elementary education major.
- Student Teaching Portfolio Reviewer (2011-present)
    - Reviewed pre-service teacher student teaching portfolios, provided feedback and scores according to Utah Effective Teaching Standards.

CHARLOTTE-MECKLENBURG SCHOOL SYSTEM
Institutional Service—District Level

- Third Grade Math Alliance, East Learning Community, Member (2009-2010)

Institutional Service—School Level
CLEAR CREEK ELEMENTARY SCHOOL

- Teacher/Parent/Student Mathematics Skills Development, Founder/Instructor (2011)
- Third Grade Mathematics Tutoring, Co-founder/Instructor (2011)
- Clear Creek Elementary Technology Drive, Founder/Leader (2011)
- PTA Communications Committee, Member (2010-2011)
- School Scrabble Club, Creator/Instructor (2010-2011)

ALBEMARLE ROAD ELEMENTARY SCHOOL

- Maximizing Teacher Utilization of Smart Board Resources, Founder/Instructor (2009-2010)
- National Young Scholars Program Student Fundraiser, Founder/Leader (2009)
- Albemarle Road Elementary School Math Leadership Committee, Member (2008-2010)
- School Scrabble Club, Founder/Instructor (2008-2010)

PROFESSIONAL AFFILIATIONS & LEADERSHIP ROLES

- American Educational Research Association, Member (2014-present)
    - Division I: Education in the Professions (2014-present)
    - Division C: Learning and Instruction (2014-present)
    - Division K: Teaching and Teacher Education (2014-present)
    - Special Interest Group: Research in Mathematics Education (2014-present)
    - Special Interest Group: Technology, Instruction, Cognition, and Learning (2014-present)
- International Group for the Psychology of Mathematics Education, Member (2014-present)
- Association of Mathematics Teacher Educators, Member (2012-present)
- National Council of Teachers of Mathematics, Member (2011-present)
- Utah Council of Teachers of Mathematics, Member (2011-present)