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## Problem Set 7

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**PROBLEM SET 7****Problem 7.1**

Recall the mechanical system consisting of two coupled oscillators. The kinetic energy  $T$  for the system is defined as usual ( $T = \frac{1}{2}m(v_1^2 + v_2^2)$ ). The potential energy is denoted by  $V(x_1, x_2)$  and is defined so that the force,  $F_i$ , on the  $i^{\text{th}}$  particle ( $i = 1, 2$ ) is given by

$$F_i = -\frac{\partial V}{\partial x_i}.$$

Find the form of  $V$ , and prove that the total energy  $E = T + V$  is conserved, that is,  $\frac{dE}{dt} = 0$  for solutions of the equations of motion.

**Problem 7.2**

Solutions to the wave equation have a conserved momentum. The momentum density for a wave  $q(x, t)$  is defined by

$$\rho = \frac{\partial q}{\partial t} \frac{\partial q}{\partial x}.$$

Find the corresponding momentum current density  $j$  for the wave. (*Hint*: Use the continuity equations.)

**Problem 7.3**

Recall the Gaussian wave

$$q(x, t) = A \left[ e^{-(x-vt)^2} + e^{-(x+vt)^2} \right].$$

Compute the total energy contained in this wave by integrating the energy density  $\rho(x, t)$  over all  $x$  and show that the result does not depend upon the time  $t$ .

**Problem 7.4**

In the previous problem, it is shown that the total energy of the Gaussian wave is time independent. Explain this result by showing that the energy current density  $j$  vanishes as  $x \rightarrow \pm\infty$ .

**Problem 7.5**

Verify that

$$\rho(\vec{r}, t) = \frac{1}{2} \left[ \left( \frac{\partial q}{\partial t} \right)^2 + v^2 (\nabla q \cdot \nabla q) \right],$$

and

$$\vec{j}(\vec{r}, t) = -v^2 \frac{\partial q}{\partial t} \nabla q.$$

satisfy the continuity equation when  $q$  satisfies the (3-d) wave equation.

### Problem 7.6

If  $q(\vec{r}, t)$  depends only upon  $x$  and  $t$  (*i.e.*,  $q$  is independent of  $y$  and  $z$ ) show that the 3-dimensional forms for the energy density, energy current density, and continuity equation reduce to the 1-dimensional results.

### Problem 7.7

Use the divergence theorem (14.33) to derive (14.28).

### Problem 7.8

Verify (14.17). Show that the time rate of change of energy in the region is the net flux of energy into the region (14.18).

### Problem 7.9

Derive the approximate formula (14.40).

### Problem 7.10

Show that the quantity

$$\Pi(t) = \int_{-\infty}^{\infty} dx \frac{\partial q(x, t)}{\partial t}$$

is independent of  $t$  (*i.e.*, is a conserved quantity) for all solutions  $q$  of the one-dimensional wave equation whose first derivatives vanish at infinity,

$$\lim_{x \rightarrow \pm\infty} \frac{\partial q(x, t)}{\partial x} = 0.$$

### Problem 7.11

Compute the energy contained in a cylindrically symmetric wave within a cylinder about the  $z$  axis of radius  $R$  as a function of time. Compute the flux of the energy current density through the cylinder and verify conservation of energy.