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Electron Penetration Range for Diverse Materials

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Theory

The range of a material describes the maximum distance electrons can travel through said material, given an initial incident energy, before losing the entirety of its kinetic energy. The penetration depth is the resulting depth at which the electrons have come to rest.[1,2] Two factors affect this range of any given material -- firstly, energy loss through inelastic interactions and collisions, and secondly, the loss of electron energy through elastic interactions or backscattering of electron emissions.[3]

This idea is illustrated by a Lichtenburg discharged tree pictured in Figure 1. This "tree" is an example of a situation where an accelerated high voltage comes to rest and deposits electrons at a given range in an insulating material.[3] The side view of the Lichtenburg figure displays the melted plastic caused by the energy of the deposited incident electrons at a uniform penetration depth. Here the stored charge is dissipated through a discharge.[1]

Fig. 1. Front (Left) and side (Right) views of a Lichtenberg discharge tree. The white line (Right) indicates the narrow distribution of deposited charge from a ~1 MeV electron beam at R≈3 mm in a PMMA sample.

Original Model

A model previously developed by the Material Physics Group predicts the range 3, and 4) have been found[1] to describe the energy-dependent range, $R(E)_T$, as a function of incident electron energy for known materials. In a continuous composite analytic approximation to the range with a single fitting parameter spanning incident energies, E, from <10 eV to > 10 MeV, the following functions (Figure 2, 3, and 4) have been found[1] to describe the energy-dependent range, R(E).

$$R(E;N_V)=\begin{cases}\left[\frac{E}{E_m}\right]\frac{\lambda_{IMFP}(E_m)(1-exp[-1])}{\left(1-exp\left[-\frac{E}{E_m}\right]\right)^2} & ;E<E_m \text{ Low Energy}\\ \left[\frac{E}{E_m}\right]\frac{\lambda_{IMFP}(E)}{1-exp\left[-\frac{E}{E_m}\right]} & ;E_m\le E\le E_{HI} \text{ Medium Energy}\\ bE^n\left(1-\left[1+\left[\frac{E/N_V}{m_ec^2}\right]^2\right]^{-2}\right) & ;E>E_{HI} \text{ High Energy}\end{cases}$$

Fig. 2. Range formula for low energy, medium energy and high energy.

$$E_m = 2.8\left[E_{gap}^2 + E_p^2\right]^{\frac{1}{2}}$$

Fig. 3. Formula for geometric mean energy lost per collision.

$$E_p = \hbar\left(\frac{N_V N_A \rho m_e q_e^2}{m_e \varepsilon_0 M_A}\right)^{\frac{1}{2}}$$

Fig. 4. Formula for plasmon energy.

Fits to these initial equations and optimum values of N_v were found only using data for only a handful of well-known elements and compounds and were able to predict known N_v. Figure 5 demonstrates some of the fitting parameters that can be applied using known data selected from ESTAR database.[2]

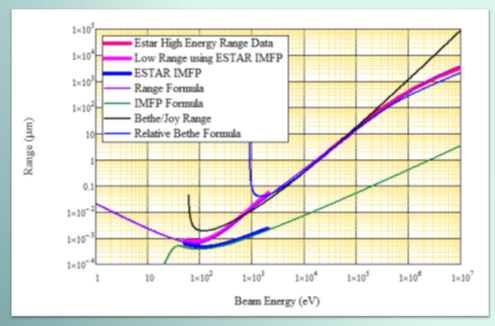


Figure 5. Comparison between several range approximations and the data from the ESTAR database for Au. The IMFP data for Au are also plotted along with the TPP-2M IMFP formula for $\lambda_{IMFP}(E)$.

References

1. Wilson, G., & Dennison, J.R. (2010). *Approximation of range in materials as a function of incident electron energy.*
2. Teancum Quist (with Greg Wilson and JR Dennison), "Compilation and Comparison of Electron Penetration Ranges as a Function of Effective Number of Valence Electrons," Utah State University, Logan, UT, April 2013.
3. Starley, A., Phillipps, L., Dennison, J.R., "Electron Range Penetration for Various Materials" U.R.C.O Proposal, Utah State University, Logan, UT, 2015.
4. J. Sempau, S.J. Wilderman, and A.F. Bielajew, 2000, "DPM, a fast, accurate Monte Carlo code optimizedfor photon and electron radiotherapy treatment planning dose calculations," Physics in Medicine and Biology 45, 8

Abstract

The penetration range of energetic electrons into diverse materials can be modeled approximately with a simple fit. This fit is a function of a single parameter, N_v, which describes the effective number of valence electrons. Using the Continuous-Slow-Down-Approximation (CSDA) for energy deposition in a material, a composite analytical formula has been developed which estimates the range or maximum penetration depth of incident electrons for energies from <10 eV to >10 MeV with an uncertainty of <20%. The fit also incorporates several common properties compiled for each material, including the mean atomic number, mean atomic weight, density, and band gap or Plasmon energy. The model has been fit to existing data for 247 materials collected from the ESTAR and IMFP databases compiled by NIST to determine N_v values. Comparison of N_v with the material's properties from this large material database may lead to the prediction of N_v for materials which have no supporting data.

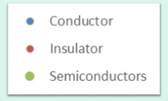
Applications

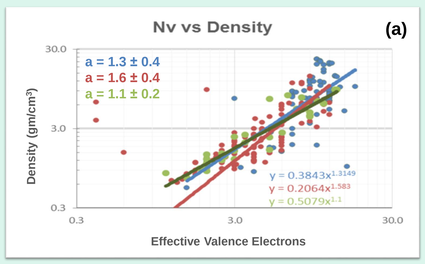
The range model developed predicts the penetration depth for various materials for different incident electrons. Its effects extends to spacecraft charging where the range is used to predict the distribution of incident electrons produced by the space plasma environment within materials as well as the energy deposited by the electrons as they travers through materials.[1] The range is also used in Electron Beam Therapy (Figure 6), the most common form of medical radiotherapy. Obtaining accurate, reliable, and efficient information on the range of electron penetration is, therefore, extremely important to the medical community.[4]

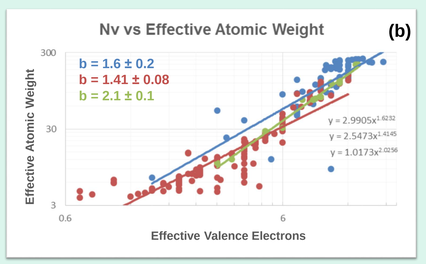
Fig. 6. Medical radiotherapy.

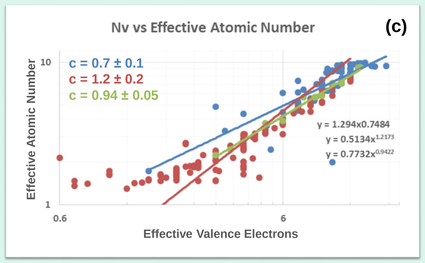
Effects of Parameterization

We initially looked at our single parameter N_v as a function of density, mean atomic weight, mean atomic number, plasmon energy or bandgap, conductivity, phase, and more. We fit the information gathered in our analysis, and in hopes of finding strong trends we added one or more of the parameterizations as illustrated in Figure 6.









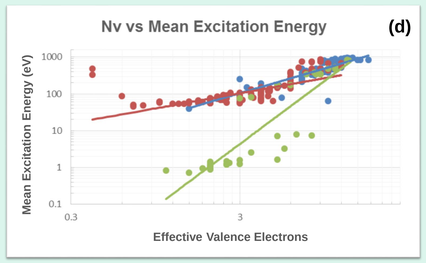


Fig. 6. N_v compared to different material properties and divided into further material subcategories: (a) Density (b) Effective Atomic Weight (c) Effective Atomic Weight (d) Mean Excitation Energy

Continual modification to the range model using our theoretical equation (Figure 7) could lead us to universal values for K, a, b, and c. Perhaps equations for conductors versus insulators and semiconductors would have somewhat different values of K, a, b, and c. Maybe materials that are solid at room temperature have a different range than materials that are liquid or gas at room temperature.

$$N_V = K\rho^a M_A^b Z^c$$

Fig. 7. Theoretical equation for effective number of valence electrons.

We gratefully acknowledge contributions from the Materials Physics Groups This work was supported through funding from SDL.

Scan code to access the USU Material Physics Group papers and presentations.

Expansion of Materials Database

To further validate the range formulations and to lead to possible new discoveries in range penetration, the Material Physics Group's material database needed to be expanded. A spreadsheet had previously been compiled with information on a number of materials. The spreadsheet's minor errors were corrected and both the total number of materials in the database and parameters for each material were extended. (e.g. considerations such as phase, color, and conductivity were added). The greater number of materials allowed for a more exact fit to be discovered with the CDSA. Adding more parameters offered the opportunity of discovering possible trends that might enable a discovery of an even more exact function to describe the range. Table 1 offers a small selection of the compiled materials, along with some of the materials' applicable physical properties.

Table 1. Representative materials and specific material properties.

Materials	Nv	Density (gm/cm³)	Mean Excitation Energy (eV)	Z* (Effective atomic number)	V_n (Effective Atomic Weight)
Chromium	3.0	7.18	257.0	4.898	51.996
Gadolinium	12.0	7.90	591.0	8.000	157.250
PI (Kapton)	3.6	1.42	79.6	2.132	9.768
Oxide	4.0	3.01	93.2	2.414	12.505
Nickle	7.0	8.90	311.0	5.290	58.690
Radium	15.0	5.00	826.0	9.380	226.025

Allowable Margin of Error

In order to perform range calculations, a value for the electron bandgap was needed for each material. While some material bandgaps were easier to find than others, it was necessary for a comparison to be made to see how much the fitting factor would change with a varying bandgap value. These calculations gave desirable results, showing that the fitting factor varied minimally with changing bandgaps. For an example, see Table 2, which uses alumina (Al_2O_3) as our chosen material. It was also realized that the error between the values in our calculations and the NIST provided values increased as the bandgap's value increased in distance from the true value.

Table 2. Alumina's variation in a fitting factor as the bandgap fluctuates.

Bandgap	Fitting Factor	Fit Error
6.0	4.02	6.07%
8.0	4.05	1.04%
8.4	4.05	0.19%
Original: 8.5	4.05	0.50%
8.6	4.06	0.82%
9.0	4.06	2.13%
11.0	4.11	9.93%

In order to put the fitting factor variances into perspective, Figure 8 shows what alumina's fit would look like if we had used a fitting factor of 0.10, 4.05 (the calculated value), and 8.00. Even with significant variance in the fitting factor, we can expect to find values that are reasonably accurate for most applications.

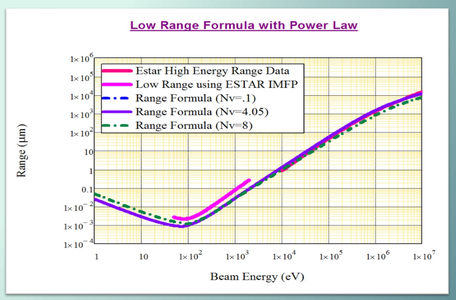


Fig. 8. Three different fitting factors are applied to alumina (0.10, 4.05, and 8.00), and the fits compared.

Future Work

Future work by the USU Materials Physics Group includes:

- Study fits based on the extended parameters of the material database.
- Develop a user friendly application to calculate the range verses incident energy for all materials in the database and for other arbitrary materials.
- Develop a general formula to predict values for N_v and the range for arbitrary materials, based on readily available materials properties.
- Compile findings onto a user friendly website.