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Clair L. Wyatt

Roy W. Esplin

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Multiplexed Dispersive Spectrometers Using Reduced Background Infrared Detectors

Clair L. Wyatt and Roy W. Esplin

The application of multiplex spectrometry to cryogenically cooled LWIR extrinsic photodetectors is limited by system noise. This noise limitation results in a detector NEP that is directly proportional to bandwidth. Therefore, multiplex schemes that require increased bandwidth are not productive of real advantage. However, doubly encoded systems that are based on $2n - 1$ or $n + N - 1$ measurements have the potential to provide a real throughput gain proportional to the number of elements used on the throughput matrix.

I. Introduction

In recent years, various multiplex schemes have been applied to spectrometry. Fourier transform spectroscopy has been used with great success in both the visible and ir regions of the spectrum.[1,2] More recently the possibility of applying optical coding based on Hadamard or modified Hadamard matrices has been noted.[3]

The idea of multiplex coding is to gain advantage over a sequential scanning spectrometer by allowing the photodetector to sense a multiplicity of wavelengths simultaneously. This idea goes back to the original work of Golay[4] and Fellgett.[5]

A sequentially scanning spectrometer having a single entrance and exit slit is used to measure n wavelengths by moving the exit slit through n positions in a total time T. The time for each measurement is $\tau = T/n$. In a typical multiplex scheme it is possible, for example, to use an exit slit encoding mask that results in having $n/2$ of the slits open (on the average) for each of n mask positions, for a total time T. This is roughly equivalent to increasing the measurement time for any one wavelength to $T/2$. The signal gain of such a multiplex scheme, relative to the sequential technique, is obtained by taking the ratio of the measurement time; in this example the gain is $n/2$. However, the effects of noise also must be considered to find the over-all advantage.

The gain of a spectrometer can be divided into two classes. For a singly encoded system (either input or exit slit) the gain is referred to as *multicolor,*[6] *multiplex*, or *Fellgett's*[5] *advantage*. For a doubly encoded system the additional gain is referred to as *étendue (throughput)* or *luminosity gain*.[7] Usually the mask with the larger number of slits determines the multiplex gain, and the mask with the fewer number of slits determines the throughput gain.

Two forms of limiting noise mechanisms are considered in the literature, they are *detector-noise-limited* systems and *photon-noise-limited* or *noise-in-signal limited* systems.[6,7]

The purpose of this paper is to explore the applicability of multiplexing schemes to cryogenically cooled long wavelength ir (LWIR) systems operating under effectively zero thermal background conditions. It is shown that in this special case a third form of noise must be considered that limits the multiplex advantage in a different way. This form of noise is described as *system noise*.

II. Detector Conditions

The operation of liquid-helium-cooled extrinsic detectors at reduced background results in *enhanced*[8] operation that has been applied in rocketsonde spectrometry.[9] The relative capability of detecting low level incident radiation is improved, and the detector impedance becomes very high. These effects result from the reduction of the background to essentially zero levels by cooling the system to near liquid helium temperatures.

The limiting noise is *system generated* because background noise in the detector is essentially nonexistent. For such a detector the detectivity D^* does not apply since the limiting noise is independent of detector area. The most significant detector parameter is the noise equivalent power (NEP)

The authors are with Utah State University, Department of Electrical Engineering, Logan, Utah 84321.

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$$\mathrm{NEP} = P_i/\mathrm{SNR} = e_n/R, \tag{1}$$

where P_i is the incident power (watts), SNR is the signal-to-noise ratio, e_n is the limiting noise (rms volts), and R is the detector voltage responsivity (rms volts per watt). For these high impedance detectors the responsivity is given by the product of the current responsivity R_i (ampere per watt) times the transfer impedance R_b (ohms).

The ultimate enhancement is obtained at very low frequencies, where the limiting noise voltage is thermal noise of the bias resistor (or transfer impedance resistor) given by the Johnson-Nyquist equation,

$$e_{\mathrm{jn}} = (4kTR_b\Delta f)^{1/2}, \tag{2}$$

where k is Boltzmann's constant (1.38×10^{-23} J K^{-1}), T is the absolute temperature (K), R_b is the bias resistance (ohms), and Δf is the noise bandwidth (Hz).

At these low frequencies, the NEP is given by substituting Eq. (2) into Eq. (1),

$$\mathrm{NEP} = \{(4kT\Delta f)^{1/2}/[R_i(R_b)^{1/2}]\}. \tag{3}$$

Equation (3) illustrates that minimum NEP is obtained by reducing the temperature T of the bias resistor and by making R_b as large as possible. Values of R_b as high as 10^{11} Ω have been used resulting in very low values of NEP, but such systems are frequency-response-limited by the input time constant R_bC_{in}.

For a thermal-noise-limited system, the noise and signal roll off at 6 dB per octave above the break frequency; consequently, the SNR is constant and Eq. (3) applies until at some relatively high frequency the thermal noise becomes less than the amplifier noise. At this frequency and for higher frequencies, the SNR and NEP deteriorate at the rate of 6 dB per octave. This frequency response roll off can be compensated for by using equalization or feedback techniques without changing the SNR or NEP limitations as outlined above.

The system noise e_{sn} for the transimpedance feedback amplifier (TIA) is given approximately by[10]

$$e_{\mathrm{sn}} = e_{\mathrm{ss}}2\pi fR_bC_{\mathrm{in}}, \tag{4}$$

where $2\pi fR_bC_{\mathrm{in}} > 1$, e_{ss} is the amplifier short circuit noise voltage, R_b and C_{in} are the bias resistor and input capacitance, respectively, and f is the frequency. The limiting frequency for which NEP [Eq. (3)] is valid is given by equating e_{sn} to e_{jn} [Eqs. (2) and (4)], solving for f,

$$f = \{(4kT\Delta f)^{1/2}/\ 2\pi e_{\mathrm{ss}}(R_b)^{1/2}C_{\mathrm{in}}]\}. \tag{5}$$

Examination of Eqs. (3) and (5) shows that both f and NEP are proportional to $(R_b)^{-1/2}$; thus, NEP is directly proportional to the limiting frequency, the noise equivalent power increasing in direct proportion to system bandwidth for system-noise-limited systems, i.e., NEP = constant × f.

III. Photon Noise

In addition to the effects discussed above with respect to enhanced operation of ir detectors under low backgrounds, it is necessary to examine the possibility of photon noise-in-signal effects.

Photon noise of magnitude equal to the system noise occurs for signal levels equal to the NEP photon rate squared. In terms of power, this power level, P_m, for which photon noise equals system noise is many orders greater than NEP and is never a significant problem in system-noise-limited LWIR systems.

IV. Multiplex Advantage

The multiplex advantage of system-noise-limited cryogenic LWIR systems can be approximated and visualized using a heuristic argument. In each scheme it is assumed that the S^t matrix is used, which on the average has $n/2$ open slits for each of n positions of the mask.

For a singly encoded system with an S^t matrix on the exit position, the signal at any wavelength will, on the average, be measured $n/2$ times that of a sequentially scanning spectrometer. The inverse transform is roughly equivalent to a co-adding scheme where the signal increases $n/2$ times and the noise increases $n^{1/2}$ times. For singly encoded systems, the total time T is the same for n measurements as it is for a sequentially scanning system, so the electrical bandwidth is the same. Thus, the over-all multiplex advantage is given by the ratio of signal increase to noise increase, or $n^{1/2}/2$.

Several doubly encoded systems are considered for a system using N input slit elements and n exit slit elements. In each case the total time is held constant at T for comparison purposes. It is also necessary to increase the bandwidth of the system to make more than n measurements, so NEP is increased proportionately. Table I gives the signal gain, the noise increase and resultant SNR, the NEP change, and the over-all multiplex advantage.

The data contained in Table I lead to the following three conclusions. First, the doubly encoded scheme of nN measurements requires an increase in bandwidth of N, which degrades NEP by that factor. The result is an over-all gain of $n^{1/2}/(4N^{1/2})$, which degenerates to the singly encoded scheme with the maximum gain for $N = 1$. Second, the doubly encoded scheme of $2n - 1$ requires a bandwidth increase of the factor 2, which degrades NEP by the factor 2, resulting in an over-all gain of $n^{1/2}N/[2(32)^{1/2}]$, and requires that $N = (32)^{1/2}$ to yield a gain equal to that of the singly encoded scheme. Third, the doubly encoded scheme of $n + N - 1$ measurements requires negligible increase in bandwidth, where $N < n$, and results in an approximate gain of $n^{1/2}N/4$. These last two schemes appear to yield a true *throughput* gain using input encoding.

Phillips and Harwit reported[11] for an $n = 19$ spectral element system a mean-square noise 4.2 times higher for the $2n - 1$ mode than for the nN mode. It

Table I. Signal Gain, Noise Increase, SNR, NEP Change, and Over-all Advantage for Various Multiplex Schemes in LWIR System-Noise-Limited Spectrometers

Scheme	Singly encoded	Doubly encoded		
Number of measurements	n	nN	$2n-1$	$n+N-1$[a]
Signal factor	$n/2$	$nN/4$	$nN/4$	$nN/4$
rms noise factor	$n^{1/2}$	$(nN)^{1/2}$	$\sim(2n)^{1/2}$	$\sim(n)^{1/2}$
SNR gain	$n^{1/2}/2$	$n^{1/2}N^{1/2}/4$	$\sim n^{1/2}N/(32)^{1/2}$	$\sim n^{1/2}N/4$
NEP quotient	1	N	2	1
Over-all advantage	$n^{1/2}/2$	$n^{1/2}[4N^{1/2}]$	$\sim n^{1/2}N/[2(32)^{1/2}]$	$\sim n^{1/2}N/4$

[a] $n > N > 1$.

is likely that the noise statistics would vary with the particular encoding scheme used and would have to be worked out for each case.

The $n + N - 1$ scheme should work well where $n > N > 1$ for the central $n - N$ unknowns.[12] The encoding schemes and noise distribution have not been worked out; however, the scheme looks very promising for system-noise-limited ir spectrometer systems.

V. Conclusions

The limiting noise form in cryogenically cooled LWIR spectrometers is system noise. This limiting noise results in a system NEP that is directly proportional to the bandwidth required. Photon noise is never a problem in system-noise-limited systems.

Thus, singly encoded multiplex spectrometers have an over-all multiplex advantage of about $n^{1/2}/2$. Doubly encoded systems have no advantage when an nN measurement scheme is used. However, the schemes of $2n - 1$ or $n + N - 1$ measurements have real potential throughput gains proportional to N because these schemes require very little bandwidth increase over a sequentially scanning system. The encoding schemes and noise distribution must be worked out to verify the feasibility of these promising applications of multiplex spectrometry to the long-wave ir region.

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