Estimating Forage Production Following Pinyon-Juniper Control: A Probabilistic Approach

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ESTIMATING FORAGE PRODUCTION FOLLOWING
PINYON-JUNIPER CONTROL:
A PROBABILISTIC APPROACH

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INTRODUCTION

An important management practice within the 60-80 million acres of Pinyon-Juniper woodlands is to convert these woodlands to open rangelands. The success or failure of seeding adapted grasses in place of trees is contingent upon the revegetation techniques employed and upon fortuitous weather patterns. In order to formulate policy for Pinyon-Juniper control decisions, persons responsible for such policies need to know the risks of introducing range grasses into given areas. This thesis is essentially a hypothesis concerning the magnitude of such risks.

Pinyon-Juniper control has been practiced widely in the five state areas of Arizona, Colorado, Nevada, New Mexico, and Utah. Justification for control and conversion of the woodlands to open grazing areas rests on the assumption that the trees have little apparent utility and, therefore, they should be replaced by a grass resource which has relatively greater value. In its extreme form the assumption involves the notion that the trees are actually detrimental to the productivity of the land. They become a hindrance to the growth of various forms of plant life considered desirable.

Considerable investment, primarily at the expense of society, has been incurred in the transformation process. Until recent years control projects have been limited to the most accessible sites and areas "invaded" by Pinyon-Juniper trees. At present the conversion pace has slackened due to the limited number of remaining accessible sites in some areas, but more importantly to the fact that the projects have had a history of mixed success. While certain rules of thumb have been put forth to explain the nature of the factors influencing seedling emergence and increased forage production, these are so volatile that an apprehension of failure exists among land managers.

The intensity of investment can only be balanced against risk levels if there is basic understanding of the roles played by the variables influencing seedling establishment and forage increase. Both policy and nonpolicy (not subject to human manipulation) variables must be identified and their influences upon success measured.

The analysis that follows begins by setting forth the objectives to be achieved in evaluating the tree conversion process and its associated risk. Next a theory of range grass seedling establishment is presented. The appropriate variables are identified and an "establishment" model is applied to the empirical data. A third section specifically treats weather as a major influence upon seedling emergence and forage production. A model expressed in probablistic terms, employing the Markov property, is applied to available data to evaluate weather index movements. Finally, having dealt with emergence, a theory is developed to explain expected forage production in the period following emergence. The parameters of the associated model are obtained from empirical data.

OBJECTIVES

The paramount issue is the identification of those factors which can or cannot be manipulated to influence expected forage production. However, the important prerequisite for any increase in forage production is to obtain seedling emergence and vigorous seedling establishment. The investment decision requires land managers to make a decision to incur control and seeding costs based on the odds they will give that successful establishment can be achieved in the future (as determined by the fortuitous variable[s]). These odds can be altered, sometimes considerably, by manipulation of revegetation techniques to help tip the balance of nature in favor of seedling emergence. Therefore, the objectives are set forth as:

1. To isolate those factors that will, when observed in a combination of quantified magnitude, determine seedling establishment.

2. To estimate the probability of weather movement.

3. To isolate those factors affecting subsequent production.

Expected forage increase is built upon the answers to two questions: (a) What is the probability that given revegetation techniques will result in initial seedling establishment, and (b) will weather patterns following establishment be such that the grass stand will endure? To formulate the establishment process define:

$Pr(A)$ = the probability of production.

$Pr(B)$ = the probability of seedling establishment where $0.50 < Pr(B) < 1.0$.

$Pr(B')$ = the probability of failure in seedling establishment where $0 < Pr(B') \leq .50$ and event A and B' are necessarily mutually exclusive.

Assuming event A (probability of production) is dependent upon event B (probability of establishment), the conditional production process can be expressed in probabilistic terms as:

$$Pr(A|B) = \frac{Pr(A|B)}{Pr(B)} , \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and it follows by the general rule of multiplication that:

$$Pr(A|B) = Pr(B) \; . \; Pr(A|B) . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The probability of establishment failure is expressed as:

$$Pr(A|B') = \frac{Pr(A|B')}{Pr(B')} = \text{null set.} \quad . \quad . \quad . \quad . \quad (3)$$

SEEDLING ESTABLISHMENT

Theoretical considerations

The viewpoint adopted in this study is that to comprehend the economic behavior of rangeland managers and to formulate policy, it is necessary to understand the biological environment within which their decisions are encompassed or circumscribed. Both economists and agronomists utilize as fundamental building blocks, production functions relating plant and animal growth to increments of water, fertilizers, feed mixes, or other variable inputs. This identity of interest is exemplified in the works of Heady et al. (1961), Hildreth (1957), Edwards (1963), and other agricultural economists who have cooperated with biological scientists.

It is generally conceded that where biological processes are involved in agronomic models, economic specification is rendered extremely difficult because the assumption of independence among variables and fixed measurement are relaxed. Such is the case in the present study of range grass production.

The formation of the seed leaf is of particular importance in determining future plant development. This process depends on germination success. Germination in turn is dependent upon interaction of the quality and amount of food stored in the seed with optimum temperature and moisture conditions. Germination occurs when the soil temperature ranges between 68 to 78 degrees Fahrenheit. This, of course, assumes that there is an "adequate" supply of moisture.

Water serves three functions in facilitating plant growth. It provides a source of raw materials which the plant utilizes in the photosynthesis process. Water also acts as a vehicle for conveying chemical elements and compounds to and from the center of photosynthesis activity. Finally, water serves as a cooling agent within the plant to protect the tissues from desiccation.

The mechanism by which plants obtain water must also be considered. The relationship between plants and surrounding soil is identical to that between two solutions separated by a permeable membrane. The sap in the plant cells is one solution, and the soil water containing soluble salts is the other. If the two solutions are of different densities, the one least concentrated will flow through the membrane and dilute its more concentrated neighbor until equilibrium is reached. This flow, under normal conditions, is from soil to plant, although the reverse is not precluded. This same phenomenon, known as osmosis, also occurs between cells within individual plants. As a result, the water distribution in the cells reaches an equilibrium with water in the soil. This assumes no tendency for the water to be lost somewhere in the system.

Competing with photosynthesis is the important phenomenon of transpiration. This is the process by which water is passed from the plant into the surrounding atmosphere. The principle of flows induced by differential solution densities is also of relevance. The external solution in this case is the atmosphere, a solution of gases in water vapor. The density of this solution is almost always greater than the adjoining cellular solutions of plants; hence, the direction of water flow is from plant to air in turn inducing flow from soil to plant. The magnitude of the existing pressure differential governs the rate at which transpiration occurs, and any factor affecting the density of either the cellular or atmospheric solution affects the rate of water flow. The density of the surrounding atmospheric solution is a function of temperature and the potential vapor pressures. The latter is predetermined leaving temperature as the causal variable in the flow of water.

Important also is the evaporation of water from or just below the soil-air interface of the soil. Water evaporation from the soil, like transpiration, is dependent upon temperature. The combination of transpiration and evaporation called "evapotranspiration" represents the reverse of precipitation. As the water supply increases, evapotranspiration rises to a maximum depending on the environmental climate. This maximum, if reached, is called "potential evapotranspiration." Thornthwaite (1948), in studies on weather crop relationships, attempted measurements on "potential evapotranspiration" and found it impossible to measure. Penman (1948) has since developed a more precise measure in his studies. Thornthwaite found the evapotranspiration rate to be dependent on the interdependent factors of climate, soil moisture supply, plant cover, and land management. His experiments indicated the existence of a growth inhibiting factor directly proportional to temperature.

A growth equation has been proposed by Thornthwaite by generalizing Van't Hoff's law of physics written as:[1]

$$V = a \frac{bCe^{ct}}{(e^{ct}+b)^2}, \qquad \qquad (4)$$

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[1]Van't Hoff's law of physics expresses the velocity of a chemical reaction as an exponential function of temperature. Van't Hoff's law can be conceptualized as: $A = n/V\ RT$, where: A = osmotic pressure. R = universal gas constant. T = absolute temperature. n = concentration of a solute in moles per liter.

where:

t = temperature in centigrade.

a, b, and c = constants.

e = base of Naperian logarithms.

V = percentage of the optimum growth rate.

Also, a moisture ratio expressing humidity or aridity during a given period in a given location was developed by Thornthwaite as:

$$m = \frac{P - e}{e} = \frac{P}{e} - 1, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

expressing the difference between precipitation and "potential evapotranspiration" divided by "potential evapotranspiration." In (4) optimum temperature for growth is at the point where numerator and denominator are equal. The second relationship (5) approaches zero as water supply equals water need. Series of the growth index (4) are not available thus making it nonoperational.

Up to this point nothing has been mentioned about interplant relationships. Beginning with a simple plant per unit area and increasing the density of the plant population from that point, it is obvious that over a considerable range of plant density, aggregate emergence (also eventual production) is a linear function of the number of plants. As the number of plants on a fixed soil increases, the growth of each plant will become more and more dependent upon the growth of other plants. This dependence is primarily competitive in the case of range grasses versus Pinyon-Juniper trees. Particularly critical is the competitive disadvantage the grasses have in obtaining moisture in dense stands of trees. Since a plant is unable to resist atmospherically induced pressures toward water loss, such losses will be incurred regardless of the occurrence of growth. The larger plant systems, therefore, can survive any periods of high "evapotranspiration" because of large storage capacity.

Because of this disadvantage, the introduction of range grasses (wheatgrasses, lovegrasses, and sandrop, etc.) requires an interplant system providing a favorable competitive position for the grasses in the system. This suggests, as is being carried out, the artificial manipulation of the interplant system to tip the balance of nature toward a favorable environment for these range grasses.

Development of the variables

The theoretical consideration has suggested the critical variables in the emergence process of the range grasses. These variables, in order to be of use in a model of seedling establishment, must be measurable.

Water, the important nutrient, vehicle, and cooling agent, is measurable in the form of precipitation. Temperature, the underlying causal variable of the flow of water through the plant, is likewise measurable. The difficulty is their interrelationship when being used as single factors affecting seedling emergence.

Sunlight, a difficult factor to measure, is considered constant for two reasons. First, the emergence process takes place during the time the photosynthetic process is active. Second, transpiration is largely confined to the daylight hours because transpiration takes place on the undersurface of the leaves through specialized cells called stomata. These specialized cells are photo-sensitive, closing in the absence of sunlight.

Climatological theory suggests inverse relationship between precipitation and temperature. Oury (1965a) found a negative correlation of -.667 between these two variables. de Martonne (1926) suggested a ratio of precipitation adding a constant to the denominator to avoid negative values:

$$I = \frac{P}{T + 10} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

Ångström (1936) later suggested a modification of the aridity index which was proportional to precipitation amount and inversely proportional to an expotential function of temperature:

$$I = \frac{P}{1.07^t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

The agreement with physical theory is much closer and is specifically in agreement with Van't Hoff's law. The denominator doubles with each rise of 10 degrees centigrade in temperature. For empirical purposes it has the advantage of being continuous for negative values. Oury (1965a) found the Ångström aridity index to be particularly favorable for aggregation purposes.

The manipulation of nature's balance to make the environment favorable for seedling emergence can be measured in three policy variables. The seeding projects first call for the removal of the existing stand of Pinyon-Juniper trees. The success of this removal is measured by the percent of live trees to that of the total trees on a tenth-acre plot.[1]

Second, the measure for plant numbers can be obtained by using the variable of seed rate on a per-acre basis for the seeding projects.

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[1] The measurements of the competition removal have been taken by a research team of the Bureau of Land Management attached to the Utah State University.

The suggestion from the theory discussed previously is that moisture and its availability to plants via the soil is a critical factor influencing both germination and production. Therefore, the third policy variable that measures how successful management is in using this fact in maximizing establishment is the depth at which the seed is sown. Also related to this is the timing in planting different grass species. It may be that no cover is required if the planting is done at a time when adequate amounts of moisture and optimum temperatures exist. The effect of timing is assumed to be taken care of in the aridity index (6) and (7), if the index is associated with the emergence time of a particular species.

Seedling establishment, the dependent variable, can be measured in different ways such as plants per square foot, percent of the total seed planted which emerged, or a more qualitative measure of success and failure in establishment. The latter measure presents the difficulties of a dichotomous dependent variable as well as subjective opinion about success or failure of a grass stand. The available data indicates that seeding establishment in the Pinyon-Juniper type is of this dichotomous nature. The records of the seeding projects only show the seeding to be classified as successful or as failures.

The generalized model

The work of others illustrates the difficulties inherent in the formulation and interpretation of qualitative data. At the same time, their efforts show promise for the present study because a number of suitable analytic techniques have been developed for analogous situations (Goldberger, p. 249).

Ordinarily, the dichotomous regression problem is treated as a regular linear regression problem, taking $E(Z)$ to be a linear function of the regressors, $Z = X'X\beta + \epsilon$, with $E(\epsilon) = 0$. Then classical least squares estimators are obtained. Two distinct difficulties arise. One, the assumption of homoskedasticity is untenable. For a particular set of x's (a row of X'X), $\epsilon = Z_i - (X'X)_i \beta$. Since Z_i is either 0 or 1, ϵ_i must be either $-(X'X)^{-1}{}_i\beta$ or $1-(X'X)^{-1}\beta$. The distribution, if ϵ_i is to have zero expectations is $f(\epsilon_t) = 1-(X'X)^{-1}{}_t\beta$ for $\epsilon_t = -(X'X)^{-1}{}_t\beta$ and $f(\epsilon_t) = (X'X)^{-1}{}_t\beta$ for $\epsilon_t = 1-(X'X)^{1}{}_t\beta$ will variance, $E(\epsilon_t{}^2) = [(X'X)^{-1}{}_t\beta][1-(X'X)^{-1}{}_t . \beta] = [E(Z_i)], [1-E(Z_i)]$, since $E(Z_i) = (X'X)^{-1}\beta$ (Goldberger, p. 249). Thus the disturbance varies systematically with $E(Z_i)$ and hence with $(X'X)_i$. Second, the linear probability function allows inconsistency with the conditions required by the definition of $E(Z_i)$, that is, $0 \leq E(Z_i) \leq 1$, since a linear function is unbounded.

To avoid such difficulties, an alternative method to restrict the unbounded linear function and to explain the movement of the disturbance term must be found. A means is suggested by probit analysis employed by biometricians and modified by economists (Goldberger, 1964, p. 250). Let C be an index which is a linear function of the regressors $(X_1 \ldots Xn)_i$ and expressed as:

$$C_i = X'X\beta. \qquad (8)$$

Let C* be $\sim$ N(0,1) and determine Z by:

$$Z_i = \begin{cases} 1 \text{ if } C_i \geq C_i^* \\ 0 \text{ if } C_i < C_i^* \end{cases} \qquad (9)$$

Each Z_i is a function of the X_i's, via the C_i, and of the C_i^*. The C_i^*'s play the role of disturbances, and they may be interpreted as critical values of C. The critical values, C_i^*, must stay within the 0,1 interval since this interval is determined by observed seeding establishment results.

Since C_i* is $\sim$ N(0,1), let $F(Z_{io})$ equal the value of the standard normal cumulative distribution at Z_{io}, then:

$$Pr\left\{Z = 1 \middle| C\right\} = \left\{Pr\ C^* \leq C \middle| C\right\} = F(C). \quad . \quad . \quad . \quad . \quad . \qquad (10a)$$

$$Pr\left\{Z = 0 \middle| C\right\} = \left\{Pr\ C^* > C \middle| C\right\} = 1-F(C) \quad . \quad . \quad . \quad . \quad . \qquad (10b)$$

The probabilities via C_i are functions of the β_i's and suggest use of maximum likelihood estimation of the β_i's. If the sample is ordered such that the first R observations are those of establishment success and the remaining S-R observations are of establishment failure, then the likelihood function of the sample is:

$$L = F(C_1) \cdots F(C_R)\ .\ \left[1-F(CR+1)\right] \cdots \left[1-F(C_s)\right], \quad . \quad . \qquad (11a)$$

with logarithmic likelihood as:

$$L = \sum^{R} \ln F(C_i) + \sum^{S}_{i\ =\ R+1} \ln \left[1-F(C_j)\right] \quad . \quad . \quad . \quad . \quad . \qquad (11b)$$

Each term now is a function of the β_i's and the standard normal cumulative distribution:

$$F(C_i) = \frac{1}{2\pi} \int_{-\infty}^{X'X\beta} e^{-\frac{U^2}{2}}\, du. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11c)$$

Setting the derivatives with respect to the β's equal to zero gives the nonlinear normal equations necessary to determine the maximum likelihood estimates.

In the probit model, conditional expectation is given as:

$$E(Z_i | C_i) = Fr\left\{Z_i = 1 \middle| C_i\right\} = F(C_i), \quad . \quad . \quad . \quad . \qquad (12)$$

when the estimated expectation, $F(C_i)$, is:

$$\hat{Z}_i = F(X'X\beta). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

$F(C_i)$ has the same properties as that of the cumulative normal distribution, and thus falls in the unit interval.

Having solved for the β's, the model expressing $\hat{C}_i$ as a function of the regressors and parameters is written as:

$$\hat{C}_i = \sum_{i=1}^{4} \beta_i X_i, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where:

$\hat{C}_i$ = the index following the probit model.

β_i = the estimated parameters.

X_1 = percent removal as tree competition.

X_2 = depth of seed cover.

X_3 = seeding rate per acre.

X_4 = Ångström aridity index for the critical period of establishment.

Given the $\hat{C}_i = F(X'X\beta)$, the probabilities of establishment (10a) and (10b) can be found by the cumulative normal distribution. The probabilities found measure seedling establishment in relative terms, since no specific measure (plants per square foot 1 percent of total seed which germinated, etc.) is obtainable in the data from the Pinyon-Juniper type.

Empirical applications

In this section the generalized model (14) is applied to three range improvement areas. Two of these areas are under the jurisdiction of the Bureau of Land Management and the other area includes projects, some of which are under the jurisdiction of the Bureau of Indian Affairs at the Ft. Apache Indian Reservation and others under the jurisdiction of the Forest Service in the Sitgreaves National Forest. The areas were split in this manner because the weather patterns are differentiated and the management practices and grass species are different between the areas.

The first area analyzed includes the fourth district in Colorado (Durango) combined with the sixth and ninth districts in Utah (Monticello).

In this area most of the improvement projects were aerial seeded and seeded at a rate of 6-7 pounds, mainly of Crested Wheatgrass. For this reason the depth of cover and seeding rate variables were excluded since they are constant for all practical purposes. Application of the model to the Monticello-Durango data yields:

$$\hat{C} = -1.47 + 2.57X_1 + .009X_4 - .01X_5, \quad . \quad . \quad . \quad (15)$$

where:

$\hat{C}$ = the index following the probit model.

X_1 = percent removal of tree competition.

X_4 = Āngström aridity index for the critical period of establishment.

$X_5 = X_1 X_2$.

The Āngström index measurement was taken for the period of time critical to seedling establishment adjusted for the time of planting. The adjustment was based on the observation of land managers regarding this critical moisture period and the time of planting. For example, it was mentioned, in talking to most of the range managers of the three areas, that if the planting time was in middle to late October, the seeding had full advantage of March-June moisture the following spring. On the other hand, if planting was completed in May, the seeding only had the low moisture conditions of June and July to draw from in emergence in the Colorado, Utah, and Nevada areas. In the areas of Arizona and New Mexico the critical moisture period is August and September for the gramagrasses and lovegrasses. Crested Wheatgrasses are not well adapted in these southern areas because they germinate in the spring and low moisture conditions exist at that time. In order to choose the critical period, correlations were run with seedling establishment on the Ångström aridity index corresponding to March-June, June-July, August-September, and September-October using Colorado, Utah, and Nevada data. The highest partial correlation of .737 corresponded to the March-June period. Then for the New Mexico and Arizona data, using the same periods with seedling establishment, the highest partial correlation coefficient of .694 corresponded with the August-September period. The empirical computation of the aridity index, using monthly mean temperatures and total precipitation data from Weather Bureau publications, (1954, 1955) is:

$$I = \frac{P \times 12}{1.07^t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

where temperature is converted to degrees centigrade. A program developed for the IBM 1620 digital computer was used for the computations. Details of this program are found in Appendix B.

Multicollinearity occurred between the competition removal variable and the Ångström aridity index. This was expected in the Monticello-Durango area since low tree removal percentages were associated with low indexes producing failures. Thus, for the area, the interaction term was included in the model. Because of the multicollinearity, no interpretation of the affect of a single variable on the index following the probit model can be made. Thus the significance of the regression coefficients is a test on the coefficients jointly where the null hypothesis is $\beta_0 = \beta_1 = \beta_4 = \beta_5 = 0$, following the F-distribution. The calculated F of 98.26, using 2 and 21 degrees of freedom, gave evidence at the $\alpha = .01$ level that the regression coefficients were significantly different from zero. A coefficient of determination of .903 was obtained for the model. More informative is the computations of the conditional probabilities according to (10a) and (10b) as tabulated in Table 1.

Table 1. Probit model index and associated probabilities of seedling establishment for selected percentages of tree removal and Ångström aridity index for the Monticello-Durango area

Percentage tree removal	Critical period Ångström aridity index	Estimated probit index	Conditional probability
38	52	-.69	.2451
40	50	-.20	.4207
50	21	-.08	.4681
56	30	.09	.5359
50	71	.10	.5398
63	28	.26	.6026
65	72	.42	.6628
70	163	.74	.7704
77	131	.75	.7734
89	88	.82	.7939
80	282	.87	.8078
80	235	2.91	.9982
99	190	2.95	.9984
99	197	3.02	.9995

Mean tree removal percentages of 74, 83, and 90 were obtained for the single chaining, dozing, and double chaining techniques respectively. From Table 1 it is seen that the higher tree removal percentages are generally associated with higher aridity indexes indicating better management practices with regards to planting time and providing a favorable competitive position for the new seeding.

The second area analyzed is that of the fifth district in Nevada and the fourth district in Utah and portions of the sixth and ninth districts in Utah. Hereafter this area will be referred to as the Cedar City-Caliente area. The establishment model was applied with all variables except seed rate per acre which was considered a constant for the area at 5-6 pounds. The estimated establishment equation was computed as:

$$\hat{C} = -.186 + .030X_1 + .703X_3 + .009X_4 - .006X_5, \quad . \quad . \quad . \quad (17)$$

where: $\hat{C}$ and X_1 are as in (14) and

X_3 = depth of the seed cover.

X_4 = Ångström aridity index for the critical period of establishment, adjusted for time of planting.

$X_5 = X_2X_3$.

As was the case in (15), multicollinearity enters the model. In this case the multicollinearity is between depth of seed cover and the aridity index, which is expected in this area since the practice of drilling the seed was associated with proper time of the planting to take full advantage of spring moisture conditions. The joint test $\beta_0 = \beta_1 = \beta_3 = \beta_4 = \beta_5 = 0$ yielded a calculated F-value of 188.46 using 2 and 21 degrees of freedom, which gives evidence that the coefficients, jointly, are significantly different from zero at the $\alpha = .01$. However, a look at the coefficient of the tree removal variable alone shows no significance. The data for percentage tree removal are grouped around high values of 99 percent, then 70 percent, and still lower at 59-60 percent. This is due to the management policy regarding the technique to be used in tree removal. The coefficient of determination for the model is .735. Table 2 shows the conditional probabilities for selected values of tree removal percent, depth of cover, and aridity index computed from (16).

From Table 2 it is shown that the values indicate cover to be an important factor influencing seedling establishment and is particularly critical when moisture conditions are relatively low, i.e., an aridity index below 100. Even at low tree removal percentages, the probabilities still maintain values greater than 0.50 if a reasonable aridity index occurs and the seed is covered.

Table 2. Probit model index and associated probabilities of seeding establishment for selected percentages of tree removal, depth of seed cover, and Āngström aridity index for the Cedar City-Las Vegas area

Percentage tree removal	Depth of seed cover (inches)	Critical period Āngström aridity index	Estimated probit index	Conditional probability
28	0.00	2	.16	.4364
40	.20	23	.03	.5120
99	.20	9	.05	.5190
95	0.00	26	.08	.5319
59	0.00	41	.20	.5793
19	.20	52	.23	.5910
75	0.00	55	.33	.6293
99	.75	23	.47	.6808
99	1.00	61	.73	.7673
60	0.00	102	.75	.7734
99	.75	89	.77	.7794
80	1.50	197	.89	.8133
90	1.50	80	.90	.8159
70	.50	176	1.24	.8925

The third area analyzed is the combined Sitgreaves National Forest-Ft. Apache Indian Reservation area. This area has some projects which were seeded to Crested Wheatgrass, but most of the projects were seeded to the lovegrasses and gramagrasses more adapted to the area. Many of the control projects were merely Pinyon-Juniper tree removal projects to allow the native gramagrasses to recover to a favorable competitive position without seeding. No significant cover of the seed was accomplished, and for the most part, the seed rate per acre is a constant at 6-7 pounds. The estimated model for the area is:

$$\hat{C} = -.982 + 1.387X_1 + .003X_2, \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

where: $\hat{C}$ is the same as in (15) and (17) and

X_1 = percent removed of tree competition.

X_4 = Ångström aridity index for the critical period of establishment adjusted for planting time.

The coefficient of determination for the model is .809, and the joint test on the regression coefficients yielded a calculated F-value of 35.942 using 2 and 17 degrees of freedom. A test was also made on the individual regression coefficients, and the calculated F-values are 4.67, 19.88, and 23.95 for the null hypothesis $\beta_0 = 0$, $\beta_1 = 0$, and $\beta_2 = 0$, respectively, using 1 and 17 degrees of freedom. In all cases evidence supported the alternative hypothesis that the regression coefficients are significantly different from zero at the $\alpha = .01$ level. The conditional probabilities for selected values of tree removal percentage and aridity index are shown in Table 3.

Of the areas studied, the Monticello-Durango area yielded the highest probability estimates but had the widest range of probabilities. For the most part the seed was not covered, but success in removing the Pinyon-Juniper trees was indicated by the data. Favorable spring moisture conditions exist as indicated by the aridity index in Table 1.

The Sitgreaves-Ft. Apache area has favorable late summer-fall moisture conditions. The seeding failures in this area were mainly due to planting cool season grasses requiring spring moisture conditions which are less favorable. This area was relatively successful in removing the Pinyon-Juniper trees.

The Cedar City-Caliente area has less favorable moisture conditions during the emergence time which explains why seed cover is important in the improvement method and is not independent of the aridity index. This area yielded the lowest probability estimates due primarily to the fact that only a few of the projects had the seed drilled to provide cover.

Table 3. Probit model index and associated probabilities for selected percentage of tree removal and Ångström aridity index for the Sitgreaves National Forest-Ft. Apache area

Percentage tree removal	Critical period Ångström aridity index	Estimated probit index	Conditional probability
45	68	-.15	.4404
60	28	-.06	.4761
55	67	-.02	.4920
55	82	.04	.5160
55	109	.12	.5478
95	70	.14	.5557
99	46	.53	.7019
95	109	.67	.7486
85	161	.70	.7580
95	133	.75	.7704
99	139	.82	.7939
90	190	.85	.8023
95	188	.92	.8212
99	334	1.43	.9236

WEATHER IN THE DECISION PROCESS

Theoretical process

Many theories have been advanced regarding weather patterns, and much has been done to calculate normal weather conditions for given areas. "Normals" are valuable when taken into consideration in agricultural production decisions. In the case of the Pinyon-Juniper control decision, knowledge of weather movements from one state of condition to another is also needed. The costs of control and seeding are incurred in one season (fall), and the grass germinates and emerges in another season (spring).

It is generally conceded that spring moisture conditions are determined by the winter precipitation patterns. This relationship, however, does not help in the control decision since the removal of Pinyon-Juniper trees and planting must take place at an earlier time than winter in Colorado, Utah, and Nevada. Also in the Arizona and New Mexico Pinyon-Juniper areas the late summer-early fall moisture patterns are quite different than the spring-early summer patterns because of the temperature patterns. In these areas the so-called "Monsoon" moves in from Southwest to Northwest, the degree of which depends on the temperatures. As the moist air moves into the area, it is forced to rise by oreographical life and condenses as the warm air is cooled at higher elevations where the temperatures are cooler. The temperatures in the plateau areas of New Mexico and Arizona vary from year to year during the late summer-early fall season quite independently of the preceding season.

It is hypothesized, therefore, that the most appropriate decision tool is to obtain an estimate of the weather movement from a particular weather state, i, in year t-1 to the same weather state, i, or different state, j, in year t. Land managers can obtain, in advance of control decision, the aridity index measure of the precipitation-temperature relationship and determine the weather conditions in year t-1 for a particular season such as the months March-June or August-September. From this movement scheme, a control decision can be made based on the probability that state i moves to state i or j in a finite number of moves.

The probability model

The hypothesis suggested from the theoretical consideration is analogous to the basic assumption underlying a Markov chain. Any sequence of trials that can be subjected to probabilistic analysis is called a stochastic process. Thus for a stochastic process, it is assumed that movements of objects from one state to another are governed by a probabilistic mechanism. The finite Markov process is a finite stochastic process such that for any statement, q, whose value depends only on the outcomes before the n^{th}, the Markov property is expressed as:

$$\Pr\left\{F_n = S_j \middle| (F_{n-1}),\ q\right\} = \Pr\left\{F_n = S_j \middle| F_{n-1} = S_i\right\}, \quad . \quad . \qquad (19)$$

where:

F_n = a sequence of outcome functions, n = 0, 1, 2....

S_i = state i.

S_j = state j.

Assuming $F_{n-1} = S_i$ and q are consistent and that the outcome of the last experiment is known, then the n^{th} step transition probabilities, denoted as $P_{ij}(n)$ are:

$$P_{ij}(n) = Pr(S_j | S_i). \quad . \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

The transition matrix for a Markov chain is the matrix P whose elements consist of P_{ij}. Then given probability, $P_{(o)j}$, that the process starts in S_i, the initial probability vector π_o, representing the probabilities of the starting states, may be formulated. This vector together with P determines the chain process.

For the present purposes the observed movement of the Ångström aridity index from state i of year t-1, to state j of year t, can best be described as an ergodic chain consisting in its entirety of a single ergodic set. An ergodic chain is a Markov chain such that it is possible to go from any state to any other state with period 1 and all sufficiently high powers of P positive. The transition probabilities are estimated using the method derived by Anderson and Goodman (1957) as:

$$\hat{P}_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (21)$$

where:

P_{ij} = the probability of moving from state i to j.

n_{ij} = the number of moves from state i to j.

The limiting matrix, A, is found by computing $\lim_{n \to \infty} P^n = A$ and is such that each row is the same probability vector, W. The limiting vector, W, indicates the long-run probabilities associated with S_i. The mean recurrence time for any starting state will be the column vector 1/W'.

The fundamental matrix, Z, is derived by:

$$Z = (I - P + A)^{-1}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (22)$$

Since I - Z = A - PZ, the mean first passage matrix, M, can be found as:

$$M = (I - Z + ADZ_{dg})\ D, \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where:

D = the reciprocal of the diagonal of A.

Z_{dg} = the diagonal of Z.

For a given control technique a certain magnitude of the aridity index is necessary to estimate a favorable probability of seedling establishment from the probit model (14). Thus the initial states producing the Markov chain are defined by choosing a desired probability of establishment, then each state is defined as the interval of aridity index magnitude which gives that probability or greater given each different control technique.

Application of the model: Monticello-Durango

The establishment model (15) for the Monticello-Durango area is employed to decide the intervals of aridity index magnitude, giving a probability of 0.75 for seedling establishment. Four tree removal techniques are generally used in the Pinyon-Juniper woodlands. These are single chaining, cabling, bulldozing, and double chaining. The mean percent tree removal is 74 percent for single chaining. The cabling technique is considered the same as single chaining, with respect to mean tree removal percent, in the areas of Colorado, Utah, and Nevada. The mean tree removal percent is 83 percent for bulldozing and 90 percent for double chaining. Also considered is the worst experience for the area with only 38 percent of the trees removed. Substituting these values into (15) and solving for the Ångström aridity index, which produces a .75 probability or greater, defines three states for the aridity index, I, for the given techniques as:

1. $S_1 = I \geq 203$, for the lowest tree removal experience.
2. $S_2 = 110 \leq I < 203$, for the single chaining technique.
3. $S_3 = 13 \leq I < 110$, for the bulldozing technique.

When the double chaining technique is used, the value of the aridity index required is negative. The lowest March-June grouped aridity index for the area was at zero; therefore, only three states are defined.

Precipitation and temperature data to compute the Ängström aridity index are taken for five weather stations in the area: Cortez and Northdale, Colorado; Blanding, La Sal, and Monticello, Utah. A Markov chain then was developed for each using the three initial states previously described. Then each of the transition matrices is computed by the method of maximum likelihood. A computer program, developed for the IBM digital computer, is used to compute the limiting matrix from the transition matrix. Details of this program are found in Appendix B.

The transition matrix, P, for Monticello is:

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .6000000 & .2666667 & .1333333 \\ .4545455 & .3636364 & .1818181 \\ .2500000 & .5000000 & .2500000 \end{bmatrix} \end{array} \quad . \quad . \quad . \qquad (24)$$

The transition matrix indicates that relatively favorable moisture conditions exist in the spring for seeding projects near Monticello, Utah. In fact an aridity index of 203 or greater is not a rare occurrence. The probability of moving from an aridity index of 203 or greater in the spring of year t-1, to the same state in the spring of year t, is .60. The probability of moving from state 3, the state requiring the lowest aridity index, to state 1 is .25. There is a greater probability,

.45, of moving from state 2 to state 1. If weather conditions are such that the aridity index is in state 3, half of the time the index will be in state 2 the next year. From state 2 the aridity index will be in state 1 the next year for nearly half of the time. This is not to infer that the index moves from state 3 to state 2 and then to state 1 in that order. The first order Markov chain only infers the probability of transition from i to j without consideration of how the system arrived in i.

The transition matrix for Northdale is computed as:

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .7058820 & .1764710 & .1176470 \\ .2222222 & .3333333 & .4444445 \\ .5714290 & .1428570 & .285714 \end{bmatrix} \end{array} , \quad . \quad . \quad (25)$$

indicating favorable March-June moisture conditions for projects near this station. A probability of approximately .71 of moving from an aridity index of 203 or greater to the same state is the highest for the weather stations of the Monticello-Durango area. There exists an approximate .57 probability of moving from an index between -13 and 109 to an index of 203 or greater.

At Blanding the moisture conditions for March-June are considerably less favorable. The transition matrix for Blanding is:

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .2000000 & .0000000 & .8000000 \\ .2000000 & .2000000 & .6000000 \\ .1250000 & .2083333 & .6666667 \end{bmatrix} \end{array} . \quad . \quad . \quad (26)$$

The highest transition probability is approximately .80 of being in state 3 after having been in state 1 initially. Low probabilities of movement from both state 2 and state 3 to state 1 indicate a pattern of low aridity indexes.

The Cortez transition matrix indicates a transition probability of approximately .55 of recurrence in state 3. The transition matrix is computed as:

$$P = \begin{matrix} & S_1 & S_2 & S_3 \\ S_1 & .1818181 & .5454540 & .2727273 \\ S_2 & .4766666 & .4166666 & .1666667 \\ S_3 & .3636364 & .0909090 & .5454546 \end{matrix} \quad . \quad . \quad . \quad (27)$$

Moisture conditions at La Sal are comparable to those of Cortez as indicated by the transition matrix:

$$P = \begin{matrix} & S_1 & S_2 & S_3 \\ S_1 & .3846150 & .3846150 & .2307700 \\ S_2 & .625000 & .125000 & .250000 \\ S_3 & .4166667 & .0833333 & .5000000 \end{matrix} \quad , \quad . \quad . \quad (28)$$

with the exception of the even distribution of probabilities of moving from state 1 to state 2 and the favorable probability of moving to state 1 given after the initial state 2 has occurred. As the projects move from the Monticello and Northdale vicinity north to the more desert-like La Sal vicinity and south and southeast to Blanding and Cortez, tree removal becomes more important because the odds for favorable spring moisture conditions are less.

The asymptotic behavior of the Markov chains indicates compliance with the assumptions that the system of aridity index movement is a first order Markov chain. The limiting matrix, A, for the Monticello weather station, computed by $\lim_{n \to \infty} P^n$, is:

$$A = \begin{array}{c} \\ S_3 \\ S_2 \\ S_1 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .4913298 & .3391136 & .1695565 \\ .4913298 & .3391136 & .1695565 \\ .4913298 & .3391136 & .1695565 \end{bmatrix} \end{array}, \quad . \quad . \quad (29)$$

with limiting vector, W = [.4913298, .3391136, .1695565]. Raising the transition matrix (24) to successive powers up to P^{10} and comparing with the limiting matrix, A, indicates relatively fast convergence as:

$$P^{10} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .4913297 & .3391134 & .1695564 \\ .4913296 & .3391135 & .1695564 \\ .4913294 & .3391137 & .1695565 \end{bmatrix} \end{array}. \quad . \quad . \quad (30)$$

The limiting vector, W, reveals the long-run probability of each state occurring. These probabilities are approximately .49, .34, and .17 for state 1, state 2, and state 3 respectively. The probability of an aridity index of 203 or greater occurring is nearly one-half.

The limiting matrix for Northdale also has the asymptotic properties, complying with the first order assumptions. The limiting matrix is:

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .5792920 & .2005244 & .2201835 \\ .5792920 & .2005244 & .2201835 \\ .5792920 & .2005244 & .2201835 \end{bmatrix} \end{array}, \quad . \quad . \quad (31)$$

with limiting vector, W = [.5792920, .2005244, .2201835]. Comparison of the transition matrix, P, raised to P^{10} as:

$$P^{10} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .5792919 & .2005242 & .2201834 \\ .5792919 & .2005243 & .2201833 \\ .5792918 & .2005242 & .2201835 \end{bmatrix} \end{array}, \quad . \quad . \quad . \quad (32)$$

with $\lim_{n \to \infty} P^n$ (28) indicates rapid convergence. The long-run probabilities of each state are approximately .58, .20, and .22 for states 1, 2, and 3 respectively. Of the five stations studied in the Monticello-Durango area, Northdale has the highest long-run probability for state 1.

The Blanding weather situation as described by the Markov scheme is opposite that of either Monticello or Northdale. The limiting matrix is:

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .149343 & .175698 & .674685 \\ .149343 & .175698 & .674685 \\ .149343 & .175698 & .674685 \end{bmatrix} \end{array}, \quad . \quad . \quad . \quad . \quad (33)$$

with, $W = [.149343, .175698, .674685]$, indicating a .67 probability of obtaining an aridity index between -13 and 110. The asymptotic behavior of the transition matrix (26) indicates convergence at a higher power, P^{15}, which is comparable to the limiting matrix (33) as:

$$P^{15} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .1493848 & .1757465 & .6748684 \\ .1493849 & .1757466 & .6748683 \\ .1493847 & .1757468 & .6748684 \end{bmatrix} \end{array}. \quad . \quad . \quad . \quad (34)$$

The limiting matrix for La Sal is:

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .4498269 & .2283733 & .3217997 \\ .4498269 & .2283733 & .3217997 \\ .4498269 & .2283733 & .3217997 \end{bmatrix} \end{array}, \quad . \quad . \quad . \qquad (35)$$

and the limiting vector, W = [.4498269, .2293733, .3217997], indicating similar probabilities to those of Monticello. Cortez has even a distribution of long-run probabilities between the three states as revealed by the limiting matrix:

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .3235294 & .3529409 & .3235296 \\ .3235294 & .3529409 & .3235296 \\ .3525294 & .3529409 & .3529409 \end{bmatrix} \end{array}, \quad . \quad . \quad . \qquad (36)$$

and limiting vector, W = [.3235294, .3529409, .3235296].

The mean first passage time, an important tool to determine the timing for seeding projects to be initiated, is next computed from the transition matrix, fundamental matrix, and the limiting matrix. The fundamental matrix has important use in computation of the mean first passage time, as well as other uses which will be seen later. The fundamental matrix, Z, is derived by $Z = (I - P - A)^{-1}$ from (22) and is shown only for the Monticello weather station at this point as:

$$Z = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} 1.1367487 & .0920116 & -.0460060 \\ .0920116 & 1.0298757 & -.0155732 \\ -.3065058 & -.2043355 & 1.1008981 \end{bmatrix} \end{array}. \quad . \quad . \quad . \qquad (37)$$

Z has similar properties as that of P, the transition matrix, but does not necessarily have all non-negative entries.

The fundamental matrix (37) is employed in the derivation of the mean first passage matrix, M, for Monticello as:

$$M = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} 2.0352927 & 2.7656340 & 6.7641412 \\ 2.0352927 & 2.9489643 & 6.5846564 \\ 2.6352927 & 3.6345213 & 5.8977391 \end{bmatrix} \end{array}, \quad . \quad . \quad . \quad (38)$$

where the main diagonal is the same as $D = \frac{1}{W'}$, the mean recurrence time for each state. The recurrence of states 1 and 2, implying an aridity index above 110, is frequent giving corroborative evidence of the favorable spring moisture patterns of this vicinity. Also the mean first passage time for either states 1 or 2 is relatively frequent. Thus if a particular year were marked by failure in terms of the aridity index, the mean time to pass for the first time from state 3 to state 1 is approximately two and one-half years.

For Northdale the pattern is similar to that of Monticello, for the recurrence time of state 1. The mean first passage time matrix:

$$M = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} 1.7262451 & 5.7728724 & 5.3825580 \\ 4.0467813 & 4.9869242 & 3.4863684 \\ 1.9941340 & 6.0061238 & 4.5416663 \end{bmatrix} \end{array}, \quad . \quad . \quad . \quad (39)$$

indicates less frequent passage of time from state 2 to state 1 and more frequent entry from state 2 to state 3. However, regardless of the initial entrance of the aridity index in state 3, a frequent move from state 3 to 1 is indicated by (39).

Blanding, as previously indicated by (26) and (33), has moisture patterns opposite of Monticello and Northdale. Planning the time of Pinyon-Juniper control and seeding becomes crucial in this vicinity as indicated by the mean first passage matrix:

$$M = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} 6.6959950 & 4.8508630 & 1.3552840 \\ 7.4699041 & 5.6915840 & 1.6512640 \\ 7.2766110 & 5.6959386 & 1.4821730 \end{bmatrix} \end{array}, \quad . \quad . \quad . \quad (40)$$

where the mean recurrence time of state 1 is approximately seven years as opposed to one and one-half years for state 3. The mean first passage time matrices:

$$M = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} 2.2230773 & 3.4545531 & 4.2257975 \\ 3.5454590 & 4.3787955 & 4.1612840 \\ 2.2999991 & 4.8787957 & 3.1075230 \end{bmatrix} \end{array}, \quad . \quad . \quad . \quad (41)$$

and,

$$M = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} 3.0909092 & 2.6666707 & 4.5151187 \\ 3.2968423 & 2.8333355 & 4.2970070 \\ 4.1513525 & 4.9221931 & 3.0909072 \end{bmatrix} \end{array}, \quad . \quad . \quad . \quad (42)$$

for the La Sal and Cortez stations respectively. La Sal is similar to Monticello, and Cortez shows approximately equal mean first passage times for the movement i to j.

Three years may seem to be a relatively frequent passage time from state 3 to state 1. However, in terms of moisture demanded by the seedling emergence process, three years of unfavorable moisture conditions insures failure, as the young seedlings cannot survive that long before they become desiccated.

The fundamental matrix is also of importance in deriving the limiting variance for the number of times in a state in the first n steps. Also an important use is made of the Central Limits Theorem for Markov chains, the proof of which can be found in Feller (1957, pp. 271-272). In essence the Theorem states that for an ergodic chain, let $Y^{(n)}_j$ be the number of times in state s_j in the first n steps, and let $W = [W_j]$ and $V = [V_j]$ be respectively the steady state vector and limiting variance vector, then if $V_j \neq 0$, for any numbers $r < S$:

$$\Pr\left[r < \frac{Y^{(n)}_j - nW_j}{\sqrt{nV_j}} < S \right] \rightarrow \frac{1}{\sqrt{2\pi}} \int_r^S e^{\frac{-U^2}{2}}\, dx, \quad . \quad . \quad (43)$$

as $n \rightarrow \infty$ for any starting state. The Theorem says that for large n, $\frac{Y^{(n)}_j - nW_j}{\sqrt{nV_j}}$ would have approximately a normal distribution. The limiting variance is derived by:

$$V_j = \{W_j(2Z_{jj} - 1 - W_j)\}, \quad . \quad . \quad . \quad . \quad . \quad (44)$$

where the Z_{jj} are the main diagonal elements of the fundamental matrix and V_j, W_j are as defined previously.

The limiting variance vectors for the Monticello, Northdale, Blanding, La Sal, and Cortez weather stations respectively are:

$$V = [.384,\ .244,\ .175], \quad . \quad . \quad . \quad . \quad . \quad (45)$$

$$V = [.413,\ .107,\ .195], \quad . \quad . \quad . \quad . \quad . \quad (46)$$

$$V = [.146,\ .159,\ .259], \quad . \quad . \quad . \quad . \quad . \quad (47)$$

$$V = [.182,\ .207,\ .375], \quad . \quad . \quad . \quad . \quad . \quad (48)$$

and,

$$V = [.147,\ .308,\ .452]. \quad . \quad . \quad . \quad . \quad . \qquad (49)$$

Thus, for example, using the Monticello limiting variance vector, the Central Limit Theorem says that $\frac{Y^{(n)}{}_1 - nW_1}{\sqrt{.384}}$ would, for large n, have a normal distribution. The estimated number of years in 100 years having an aridity index of 203 or greater would be unlikely to deviate from .38 by more than 11 (probably about .077). Though the limiting variance is relatively large, it can be said that approximately one-third of the time the spring aridity index will have a value of 203 or greater.

Application of the model: Cedar City-Caliente

The establishment model (17) for this area introduces two policy variables to be taken care of in the decision model. The depth of seed cover, the additional policy variable, is set at two values: 88 inches for drilling the seed and zero for no cover. This defines two systems in terms of the aridity index required to generate a .75 or greater probability of establishment given the techniques of tree removal.

The states are again defined for the worst tree removal experience, single chaining, bulldozing, and double chaining. The mean success in tree removal for these techniques are respectively, 19 percent, 74 percent, 83 percent, and 90 percent. The first system is derived by holding depth of seed cover at zero. Substituting the values for tree removal and seed cover into (17) and solving for the value of the aridity index, I, required to generate .75 probability or greater, the states are defined as:

1. $S_1 = I \geq 96$, for the worst tree removal experience.
2. $S_2 = 94 \leq I < 96$, for single chaining.

3. $S_3 = 90 \leq I < 94$, for bulldozing and double chaining.

4. $S_4 = I < 90$.

States 3 and 4 vary from the previous definitions of the initial states. There is little differentiation of the aridity index magnitude required by either substituting 83 percent or 94 percent in (17). Hence the bulldozing and double chaining techniques are combined in state 3. State 4 allows a measure of the probability of occurrence of an index less than 90, since the data does have values in this range.

Precipitation and temperature data to compute the aridity index are used for four weather stations: Modena and Wah Wah Ranch, Utah; Caliente and Pioche, Nevada. Using the method of maximum likelihood, the transition matrices for the Modena, Wah Wah, Caliente, and Pioche weather stations are respectively:

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} .6476590 & .0000000 & .0588230 & .2941180 \\ 1.0000000 & .0000000 & .0000000 & .0000000 \\ 1.0000000 & .0000000 & .0000000 & .0000000 \\ .5000000 & .2000000 & .0000000 & .3000000 \end{bmatrix} \end{array}, \quad . \qquad (50)$$

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} .5000000 & .0000000 & .0000000 & .5000000 \\ 1.0000000 & .0000000 & .0000000 & .0000000 \\ 1.0000000 & .0000000 & .0000000 & .0000000 \\ .5000002 & .1666666 & .1666666 & .1666666 \end{bmatrix} \end{array}, \quad . \qquad (51)$$

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \\ \left[\begin{array}{cccc} .4285710 & .0000000 & .0000000 & .5714290 \\ .5000000 & .5000000 & .0000000 & .0000000 \\ 1.0000000 & .0000000 & .0000000 & .0000000 \\ .4285710 & .0000000 & .1428580 & .4285710 \end{array}\right] \end{array}, \quad . \qquad (52)$$

and,

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \\ \left[\begin{array}{cccc} .8181820 & .0000000 & .0909090 & .0909090 \\ 1.0000000 & .0000000 & .0000000 & .0000000 \\ .5000000 & .5000000 & .0000000 & .0000000 \\ .5000000 & .0000000 & .0000000 & .5000000 \end{array}\right] \end{array}, \quad . \qquad (53)$$

The Pioche (53) and Modena (50) transition matrices reveal high probabilities of moving from the initial state 1 to state 1 in the next move. Because of the small difference in the values of the aridity index between states 1, 2, and 3, the important inference from all the transition matrices is the probabilities of movement from state 4 to state 1. This movement is from a low aridity index, given a successful tree removal, to a high index, given that little success in tree removal is realized. In both cases it is assumed that nothing is done to cover the seed. The Wah Wah matrix (51) reveals a probability of .50 for moving from state 4 to state 1, with the lowest probability of recurrence of state 4. Pioche, the station having the highest probability of recurrence of state 1, has 50 percent chance of moving to state 1 from state 4 and the same probability of recurrence of state 3. Confidence cannot be placed in the probabilities of movement from state 2 and 3 in any of the matrices because of the lack of real differentiation between states in terms of the aridity index.

The limiting matrices for Modena, Wah Wah, Caliente, and Pioche are respectively:

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} .6397850 & .0537634 & .0376340 & .2688174 \\ .6397850 & .0537634 & .0376340 & .2688174 \\ .6397850 & .0537634 & .0376340 & .7688174 \\ .6397850 & .0537634 & .0376340 & .7688174 \end{bmatrix} \end{array}, \quad . \quad (54)$$

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} .5555561 & .0555553 & .0555553 & .3333333 \\ .5555561 & .0555553 & .0555553 & .3333333 \\ .5555561 & .0555553 & .0555553 & .3333333 \\ .5555561 & .0555553 & .0555553 & .3333333 \end{bmatrix} \end{array}, \quad . \quad (55)$$

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} .4666665 & .0000000 & .0666670 & .4666664 \\ .4666665 & .0000000 & .0666670 & .4666664 \\ .4666665 & .0000000 & .6666670 & .4666664 \\ .4666665 & .0000000 & .6666670 & .4666664 \end{bmatrix} \end{array}, \quad . \quad (56)$$

and,

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} .7586209 & .0344827 & .0689654 & .1379309 \\ .7586209 & .0344827 & .0689654 & .1379309 \\ .7586209 & .0344827 & .0689654 & .1379309 \\ .7586209 & .0344827 & .0689654 & .1379309 \end{bmatrix} \end{array}. \quad . \quad (57)$$

The row vector, W, which is any row of the limiting matrix, A, has as entries the long-run probabilities for each state. The limiting matrix (57) indicates more favorable spring moisture conditions for the Pioche vicinity than the other weather stations. Also the lowest probability of state 4 occurring is revealed by the Pioche matrix. Each transition matrix converged at P^{15}, which is comparable to $\lim_{n \to \infty} P^n$.

The mean first passage times for Modena, Wah Wah, and Pioche are respectively:

$$M = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 1.5670250 & 17.5549016 & 21.8524445 & 3.6349798 \\ .9500761 & 18.6001410 & 26.5562231 & 4.6409577 \\ .9965551 & 18.5738327 & 26.5717170 & 4.6446974 \\ 1.7266861 & 13.4387871 & 28.5865360 & 3.7199972 \end{bmatrix} \end{array}, \quad (58)$$

$$M = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 1.7999980 & 18.9474362 & 18.9474344 & 8.5263075 \\ 13.9262998 & 18.0000828 & 18.0000846 & 8.5263075 \\ 13.2603006 & 18.0000846 & 18.0000838 & 1.4210662 \\ 20.0841521 & 12.3158727 & 12.3158709 & 3.0000003 \end{bmatrix} \end{array}, \quad (59)$$

and,

$$M = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 1.3181814 & 26.9520832 & 11.9247463 & 19.7823234 \\ 3.9384153 & 29.0000493 & 14.1929373 & 16.1965961 \\ 2.9685076 & 16.0841088 & 14.5000246 & 15.3483047 \\ .9405156 & 23.3508542 & 9.8693895 & 7.2500070 \end{bmatrix} \end{array}. \quad (60)$$

Since no long-run probability is defined for state 2 in (52), the Caliente matrix of mean first passage times reduces to the mean recurrence, time for states 1 and 4 and the mean first passage times, from state 4 to state 1 and state 1 to state 4. These are respectively:

2.1428579, 2.1428583, 2.6522463, and 2.3207018.

The limiting variance vectors for Modena, Wah Wah, and Pioche are respectively:

$$V = [.219, .040, .036, 1.091], \quad . \quad . \quad . \quad . \quad . \quad (61)$$

$$V = [12.820, .032, .052, 1.331], \quad . \quad . \quad . \quad . \quad (62)$$

and,

$$V = [12.820, .030, .044, .578]. \quad . \quad . \quad . \quad . \quad (63)$$

The limiting variance for states 1 and 4 for Caliente are .684 and .326 respectively. No limiting variance is derived from the other two states since w_{22} is not defined in (56). Frequent passage time from state 4 to state 1 is indicated by (58) and (60). The mean recurrence time for state 1 in (59) is frequent, but from any other state a lapse of time as high as 13, 14, or 20 years is required before entry into state. Because of the small differentiation between states, the Caliente data does not coincide with the Markov scheme when the asymptotic properties are considered.

The introduction of the seed cover variable prescribes a different Markov chain, and new initial states are defined. As is expected when seed cover is substituted into the establishment model (17), a lower value for an aridity index is required to generate a .75 probability of establishment.

In fact the value of the aridity index required, if only 19 percent tree removal is realized and the seed is covered at .88 inches, is 65 or greater. In the previous scheme the value had to be 96 or greater for a 19 percent tree removal. A seed cover of .88 inches in depth is used, since this value is the mean for the seed drilling technique of the Cedar City-Caliente area. The new states are defined in terms of the aridity index, I, for the given tree removal techniques at the seed cover level of .88 inches as:

1. $S_1 = I \geq 65$, for the worst experience with tree removal.

2. $S_2 = 60 \leq I < 65$, for bulldozing and single chaining.

3. $S_3 = I < 60$, for double chaining.

Again, small differentiation between states is present, and the only real inference that can be made is the probability of movement from the third state to the first state.

The transition matrices for Modena, Wah Wah, Caliente, and Pioche, using the new initial states, are respectively:

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .7037028 & .0370370 & .2592592 \\ 1.0000000 & .0000000 & .0000000 \\ .8750000 & .1250000 & .0000000 \end{bmatrix} \end{array}, \quad . \quad . \quad . \qquad (64)$$

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .6250000 & .1250000 & .2500000 \\ .5000000 & .0000000 & .5000000 \\ 1.0000000 & .0000000 & .0000000 \end{bmatrix} \end{array}, \quad . \quad . \quad . \qquad (65)$$

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .6250000 & .1250000 & .2500000 \\ 1.0000000 & .0000000 & .0000000 \\ .4000000 & .0000000 & .6000000 \end{bmatrix} \end{array}, \quad . \quad . \quad . \qquad (66)$$

and,

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} .8125000 & .1250000 & .0625000 \\ .6666667 & .3333333 & .0000000 \\ .5000000 & .0000000 & .5000000 \end{bmatrix} \end{array} \quad . \quad . \quad . \qquad (67)$$

Generally these transition matrices indicate higher probabilities of moving from the lowest state in terms of the aridity index to the highest state on the next step than do (50), (51), (52), and (53). In (64) and (65) and probability of recurrence of the lowest state is reduced to zero but is increased in (66). Since spring moisture conditions are considerably more favorable in the vicinity of Pioche compared to the other station, the affect of cover on the probabilities generated is less. Only the transition matrices are presented since the asymptotic properties follow proportional to the changes in the transition matrices induced by the introduction of seed cover.

Application of the model: Sitgreaves-Ft. Apache

In the Sitgreaves National Forest-Ft. Apache Indian Reservation area the only policy variable of significance in the establishment model is tree removal. The aridity index used in the model is the grouped index for August-September. Substituting the various values for percent tree removal in (18) and solving for the aridity index, I, generating a .75 probability of establishment, the following states are defined:

1. $S_1 = I > 346$, for the worst tree removal experience of 45 percent.
2. $S_2 = 263 \leq I < 346$, for the cabling technique at a mean percent of 63.
3. $S_3 = 212 \leq I < 263$, for the single chaining technique at a mean percent of 74.

4. $S_4 = 170 \leq I < 212$, for the bulldozing technique at a mean percent of 83.

5. $S_5 = 137 \leq I < 170$, for the double chaining technique at a mean percent of 90.

6. $S_6 = I < 137$, which covers the remaining computed aridity indexes.

Precipitation and temperature data from Cibecue, Pinedale, and Show Low, Arizona are used to compute the aridity indexes. The transition matrices using the indexes for Cibecue, Pinedale, and Show Low are respectively:

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{array} \begin{array}{cccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{array} \left[\begin{array}{cccccc} .3750000 & .0000000 & .0000000 & .3650000 & .1250000 & .1350000 \\ .0000000 & .0000000 & .0000000 & .3333333 & .3333333 & .3333334 \\ .0000000 & .1666667 & .3333333 & .1666667 & .0000000 & .3333333 \\ .0000000 & .0000000 & .0000000 & .0000000 & .3333333 & .6666667 \\ .0000000 & .0000000 & .2500000 & .0000000 & .2500000 & .5000000 \\ .3571420 & .0000000 & .0714280 & .2142850 & .0000000 & .2857150 \end{array}\right], \quad (68)$$

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{array} \begin{array}{cccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{array} \left[\begin{array}{cccccc} .2000000 & .2000000 & .2000000 & .0000000 & .2000000 & .2000000 \\ .2500000 & .0000000 & .0000000 & .2500000 & .2500000 & .2500000 \\ .2000000 & .2000000 & .2000000 & .0000000 & .2000000 & .2000000 \\ .0000000 & .0000000 & .0000000 & .2500000 & .2500000 & .5000000 \\ .0000000 & .2500000 & .2500000 & .2500000 & .2500000 & .0000000 \\ .6000000 & .2000000 & .2000000 & .0000000 & .0000000 & .0000000 \end{array}\right], \quad (69)$$

and,

$$P = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \end{array} \begin{array}{c} \begin{array}{cccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \end{array} \\ \begin{bmatrix} .0000000 & .0000000 & .0000000 & .5000000 & .5000000 & .0000000 \\ .0000000 & .0000000 & .0000000 & .0000000 & .0000000 & 1.0000000 \\ .5000000 & .0000000 & .0000000 & .0000000 & .5000000 & .0000000 \\ .0000000 & .0000000 & .0000000 & .0000000 & .0000000 & 1.0000000 \\ .1111111 & .0000000 & .1111111 & .1111111 & .1111111 & .5555556 \\ .0000000 & .1111111 & .1111111 & .2222222 & .4444445 & .1111111 \end{bmatrix} \end{array} . \quad (70)$$

The Cibecue matrix (68) indicates approximately .38 probability of recurrence of state 1. The Pinedale matrix indicates a .60 probability of moving from an index of below 137 to one of over 346. Both (69) and (70) indicate no probability of recurrence of state 1. The high index of 346 is rarely obtainable except in Pinedale, indicating that a 45 percent tree removal is a sure gamble with establishment failure. Actually, the higher probabilities generally occur in the movements from states 4 and 5 to state 6, indicating the most frequent aridity index of some value less than 170. This suggests that the bulldozing and double chaining techniques should be the tree removal techniques used if the seed is not covered.

A further look at the asymptotic properties of this particular Markov scheme reveals the same outlook for projects in the vicinity of the three weather stations. The limiting vectors, w, taken from a row of the limiting matrix, A, reveal the long-run probabilities of each state for Cibecue, Pinedale, and Show Low as:

$$W = [.2833934,\ .0136785,\ .0820712,\ .1067980,\ .1007773,\ .4132813], \quad (71)$$

$$W = [.1753020,\ .1480327,\ .1480327,\ .1589404,\ .1885469,\ .1811453], \quad (72)$$

and,

$$W = [.0523077,\ .0461537,\ .0753845,\ .1476922,\ .2630771,\ .4153846]. \quad (73)$$

In (72) the probabilities are somewhat evenly distributed among the states. State 6 in (71) and (73) has the greatest probability of occurring. State 1 has low probability for all vectors.

The mean first passage times matrices indicate less frequent mean recurrence times and passage times for these Arizona weather stations than is indicated for the Colorado, Utah, and Arizona stations previously discussed. The mean first passage times matrices for Cibecue, Pinedale, and Show Low are respectively:

M = (74)

	S_1	S_2	S_3	S_4	S_5	S_6
S_1	3.5286636	64.3736593	4.8299964	.0916449	10.0778836	2.8580448
S_2	7.0620372	73.1074313	14.1325776	7.9450692	8.3045775	2.2370019
S_3	13.3975753	73.3314178	12.1845422	7.2055806	10.2130787	1.6653427
S_4	16.0769993	72.5543662	12.9994102	9.3534712	9.3423419	1.2309049
S_5	14.2911927	73.0733779	10.4254087	9.4854548	9.9228695	1.5205311
S_6	23.6666769	70.6409986	10.2313126	5.3337347	11.8365693	2.4196594

M = (75)

	S_1	S_2	S_3	S_4	S_5	S_6
S_1	5.7044414	5.5578963	6.9868360	6.4584000	5.7933887	4.5161369
S_2	4.5749973	6.7552642	8.1816578	6.0000698	5.5950412	4.2607618
S_3	9.0500073	5.3263237	6.7552642	7.7500201	5.7851239	4.7930081
S_4	5.2861125	6.7157999	8.1447437	6.2916665	5.8347105	3.1317135
S_5	5.0124139	5.3473697	3.3986781	5.9167360	5.3037201	5.3951694
S_6	2.8444507	5.4000062	6.8289479	7.6250267	6.7520658	5.5204302

and,

$$M = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{array} \begin{array}{c} \begin{array}{cccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{array} \\ \begin{bmatrix} 19.1176442 & 27.4540790 & 2.0499042 & 9.1600009 & 4.9453072 & 4.0204556 \\ 51.0451366 & 21.6667352 & 14.0002427 & 5.6338188 & 4.5559199 & .1227371 \\ 19.1179826 & 19.9333141 & 13.2653264 & 5.9151200 & 3.3333232 & 1.9285635 \\ 19.1183018 & 15.8636468 & .3032878 & 6.7708382 & 3.7394733 & 3.5505865 \\ 19.1182063 & 21.1527492 & .3803155 & 5.9686692 & 3.8011670 & 3.5505865 \\ 19.1182407 & 19.0213547 & .5447804 & 5.6605853 & 2.8892746 & 2.4074074 \end{bmatrix} \end{array} . \quad (76)$$

The Cibecue matrix (74) shows a relatively frequent mean recurrence time for state 1, but more frequent occurrence takes place at an aridity index value below 137. The Pinedale matrix (35) has a relatively even time interval for recurrence and passage from i to j, but the most frequent passage is still approximately three years. The limiting variances for Cibecue, Pinedale, and Show Low are respectively:

$$V = [3.248, .012, .058, .031, .116, .095], \quad . \quad . \quad (77)$$

$$V = [.111, .116, .147, .152, .003, .091], \quad . \quad . \quad (78)$$

and,

$$V = [.003, .037, .320, .089, .129, .078], \quad . \quad . \quad (79)$$

The limiting variances indicate that small confidence intervals about the mean number of years in n years that the system will be in each state. This is desirable for prediction and decision making.

FORAGE YIELD RESPONSE

Theoretical considerations

The grass plant, once it has been established as an independent organism, enters upon a stage of vegetative proliferation. This stage continues until the plant forms the reproductive organs and flowers. This is the longest growth period of the plant. What happens to the range grass plant in this vegetative state is of utmost importance to stockmen and rangeland managers.

As in seedling establishment, photosynthesis is the important growth activity. The rate of photosynthesis, as previously discussed, is influenced by temperature, while water provides the nutrients necessary to maintain growth if photosynthesis is active. Water also acts as the cooling agent at the same time transporting the nutrients from soil to cell. Excessively high temperatures mean a high rate of "evapotranspiration" which in turn means death to the grass plant. Low temperatures mean growth suspension. This assumes a given plant adaptability. It is reasonable to assume that even in the presence of adequate moisture growth as a function of temperature is initially subject to positive, but decreasing, marginal products. It is also reasonable to assume that as the plant obtains greater levels of growth, more of the input mix (nutrients) is needed to maintain the same rate of growth. More of the input mix is now required for maintenance of the present level of vegetative proliferation.

Important also is the utilization of vegetative portions of the grass plant by range animals. The region of the photosynthetic activity is located in the leaves. When the animals eat the leaves down to the crown, photosynthesis, and thereby the rate at which food is produced, is impaired. It is assumed, therefore, that as the stocking rate is increased for a fixed grazing area, grass production decreases.

Development of the variables

The theoretical consideration of the response of range grass to temperature, water, and forage utilization suggests the need for measurement of these variables. Water and temperature measurement has already been discussed. However, since only cross-sectional data for range grass production is available, the combination of precipitation and temperature into an index takes on a new form. For prediction purposes normal temperatures and precipitation are used. These normals are then combined in the Ångström aridity index (16) and grouped for the critical moisture period corresponding to the various areas of the study.

Forage utilization is measured by the percent of the average production per acre utilized at given average stocking rates. The stocking rate, usually in terms of acres per animal unit month, is converted to animal unit months per acre then multiplied by an animal unit month forage requirement (1000 pounds) to give the amount of forage utilized. This is then converted to a percentage.

Since all of the seeding projects had unequal establishment, there is need for a measurement of the affect of seedling establishment on subsequent growth. It is assumed that the probability generated by the establishment model provides a measure of attained level of growth from the emergence process. The associated establishment probability estimates are used as independent variables.

The dependent variable, grass production, is measured in terms of increased air-dry forage production per acre due to range improvement. All things equal, good establishment probabilities should be linked to relatively high production. But measured figures in vigorous grass stands could be low because a large percentage of individual sites might be covered with downed tree slash from the control process. Therefore, the forage estimates are adjusted for slash percentages.

The generalized model

To test the theory that grass yield is a function of forage utilization, seedling establishment, and weather affects, a linear model is employed using cross-sectional data. The model is expressed as:

$$\hat{Y} = \sum_{i=1}^{3} \beta_i X_i + \epsilon_i, \qquad (80)$$

where:

$\hat{Y}$ = gross yield in pounds per acre (dry-weight).

β_i = regression coefficients.

X_1 = average pattern of utilization from time of establishment to 1965 (expressed in percent of forage utilized).

X_2 = probability of seedling establishment as estimated from the probit model.

X_3 = Ångström aridity index using normal precipitation and temperatures.

ϵ = disturbance term.

The model, as expressed, estimates the response of forage yield to two variables capable of policy control and the uncontrolled variable, weather. Managers can, by manipulation of the policy variables, tip the balance of nature in favor of grass production within the limits of given expected weather conditions.

Empirical application of the model

The model is applied to data from the Monticello-Durango, Cedar City-Caliente, and Sitgreaves-Ft. Apache areas previously studied in the case of establishment and weather movement. The production data are for the 1965 growing season for all of the areas.

Application of the model to the Monticello-Durango area yields the estimated response equation:

$$\hat{Y} = 528.762 + 450.288X_2 + 6.289X_3, \quad . \quad . \quad . \qquad (81)$$

where:

X_2 = estimated probability of establishment March-June.

X_3 = Ångström aridity index using normal precipitation and temperatures.

Utilization, (X_1), for the Monticello-Durango data is not significant. The Monticello Bureau of Land Management office has cut the stocking rate on seedings of low productivity, and the Durango office stocks their seedings at the same rate. Therefore, no cross-sectional variation in utilization data is present. The estimated equation is such that production is zero when the estimated probability of establishment falls below .50 and the aridity index, computed from normals, is approximately 47. If the estimated probability is greater than .50, and if production is zero, the aridity index is less than 47.

The coefficient of determination for (81) is .557, and the partial correlation coefficients of increased production with the estimated probability of establishment and again with the aridity index are respectively .590 and .714. The test of the regression coefficients jointly, using the F-distribution, yields a calculated F-value of 8.794. This gives evidence that the regression coefficients jointly are significantly different from zero at the α = .01 level using 2 and 21 degrees of freedom. A test of the regression coefficients separably gives evidence that β_2 is significantly different from zero at the α = .10 level, and β_3 significantly different from zero at the α= .01 level, in both cases using 1 and 21 degrees of freedom.

The Cedar City-Caliente area has variation in the utilization data since three Bureau of Land Management offices have charge of the seeding projects and vary the stocking rates. The estimated response equation for this area is:

$$\hat{Y} = -1157.036 - 305.818X_1 + 2357.020X_2 + 2.715X_3, \qquad (82)$$

where:

X_1 = percent forage utilized.

X_2 = estimated probability of establishment.

X_3 = March-June Ångström aridity index using normal precipitation and temperatures.

The coefficient of determination for the model is .713 and the partial correlation coefficients of increased forage yield with forage utilization, estimated establishment probability, and the aridity index are -.396, .780, and .530 respectively. A joint test on the regression coefficients, using 3 and 20 degrees of freedom, yields a calculated F-value of 16.525. The model is significant at the α = .01 level. A test on the individual regression coefficients indicates β_1 to be significantly different from zero at the α = .10 level, β_2 significant at the α = .01 level, and β_3 again significant at the α = .10 level.

Again the estimated equation is such that production is zero if X_2 is held constant at .49 or a value below .49, and either X_1 or X_2 is held constant while the other is allowed to vary until the equation is driven to zero.

The response equation, when applied to the Sitgreaves-Ft. Apache area, yields the estimate:

$$\hat{Y} - 370.955 = 452.719X_1 + 51.247X_2 + .844X_3, \quad . \quad . \quad (83)$$

where X_1, X_2, and X_3 are the same as in (82). The coefficient of determination for the model is .869, and the partial correlation coefficients of increased forage yield with forage utilization, estimated establishment probability, and the aridity index are -.620, .698, and .812 respectively. A joint test on the regression coefficients yields a calculated F-value of 22.1809 which gives evidence that the model is significant at the $\alpha = .01$ level, using 3 and 10 degrees of freedom. Separate tests on the regression coefficients individually indicate β_1 to be significant at the $\alpha = .01$, β_2 significant at the $\alpha = .05$, and β_3 significant at the $\alpha = .05$ level. The equation is arranged the same as (81) and (82) with the exception of the positive intercept. The coefficient for the aridity index variable is smaller, since the aridity indexes are generally high for the area. Thus the intercept increases in value and becomes positive in (83).

EVALUATIONS AND CONCLUSIONS

The establishment model

Since no common measurement of seedling establishment among range managers exists, estimation of the relationship of factors influencing establishment can only be accomplished by the use of a dichotomous dependent variable. The weakness of this method is obvious. As the independent variables change in degree, the variance changes also. The probit model is superior to other approaches since limits are set and constant variance is forced. The applicability of the model is in the generated probabilities. The probabilities are a tool, useful for present improvement decision making with consideration of the chances for improvement success in the future. A more desirable model is an extension of the probit model to a measure of degree of success rather than being confined to the limits of a dichotomy. Insufficient data, however, dictates the use the dichtomous model.

The Markov chain

The Markov chain is an applicable tool for explanation of time-ordered phenomenon. In this study it explains the time-ordered movement of the aridity index relatively well. The asymptotic properties of the chains derived show that relatively fast convergence occurs as the transition matrices are raised to successive powers. The asymptotic properties also comply with the assumptions of the first order Markov chain. It must be kept in mind, however, that the chain derived is an indicative, not a conclusive, explanation of weather patterns. The initial states are defined in terms of the establishment model, both for a given range improvement technique and the aridity index used. The chain, when expressed in these terms, gives a good indication of what technique should be followed given the probabilities of aridity index movement. The limiting variances indicate relatively good estimation of the weather movement probabilities, in general, for the three areas studied. The Sitgreaves-Ft. Apache area did indicate a much smaller variance than the other two, however.

Forage response model

With the exception of the estimated equation for the Monticello-Durango area, all the estimated coefficients, corresponding to each variable hypothesized to influence grass production, indicate a significant relationship. Data dictated the reduction of the Monticello-Durango model to just the establishment probability and aridity index variable.

Forage utilization is a primary factor hypothesized to influence forage production in the Sitgreaves-Ft. Apache area. Grazing control is extremely hard on Indian Reservations and grazing is heavy. The estimated response equation for this area indicates that forage utilization is a primary factor. In the Monticello-Durango area relatively high probabilities occur. The estimated model indicates probability of establishment to be a prime factor explaining the high forage increases in this area. The estimated equations in all three cases indicate the expected sign for the regression coefficients. These coefficients show an inverse relationship of forage increase with forage utilization. Positive relationship is shown for forage increase with establishment probability and the aridity index.

Conclusions

The objectives of this thesis were to develop a theoretical framework for seedling establishment and production and to identify those variables that can or cannot be manipulated to influence establishment and production success. The approach taken has been to calculate the influence that specific policy variables have upon establishment and production of range grasses given the limits of weather patterns. The results of the analysis, using data for establishment, production, and the nonpolicy variable, weather, from the three Pinyon-Juniper control areas are encouraging. The conclusion that the theory developed herein is promising is, of course, conditional. There is need for demonstration that each of the three models explains its respective relationship better than any other model. This demonstration, however, is beyond the scope of this study.

The study did indicate that tree competition removal, depth of seed cover, and weather patterns have significant influence upon seedling establishment. Seed rate was a constant for all areas studied, and no measure of influence was obtainable. It is important to note the comparison of the three areas with respect to weather patterns. The establishment model for the Monticello-Durango area indicated that relatively high establishment probabilities could be obtained with little or no cover of the seed. The reason for such success, involving less seeding investment, is the favorable spring moisture conditions existing in this area. As indicated by the transition matrices for the five weather stations studied in the Monticello-Durango area, the transition probabilities for the recurrence of state 1, the state requiring the highest aridity index, are relatively high with the exceptions La Sal and Blanding, Utah. In the Cedar City-Caliente area, however, the spring moisture conditions are not as favorable and seed cover becomes more important. The Pioche, Nevada weather station is the only one that has comparable weather conditions to those of the Monticello-Durango area. In the Sitgreaves National Forest-Ft. Apache Indian Reservation area, the August-September moisture conditions are generally favorable for the three weather stations included in the analysis. However, a considerably higher aridity index is required for state 1 in the Markov chain. The mean first passage time for arriving in state 1 from the other states is less frequent than the same mean passage times in either the Monticello-Durango or Cedar City-Caliente area. Also the recurrence times of each state are considerably less frequent in this area compared to the other two areas. This means that if a particular year is marked with a low aridity index, as high as 70 years may pass before the highest aridity index occurs. This was indicated by the mean first passage matrix for the Cibecue, Arizona weather station. Timing and the problem of seeding nonadapted grass species has been the main reason for failure in the Sitgreaves-Ft. Apache area. The timing with respect to weather conditions has also contributed to the failures in the Cedar City-Caliente area.

The forage response models indicated that forage utilization, establishment probability, and the "normal" aridity index all have significant influence on forage production. The one exception was forage utilization in the Monticello-Durango area. No cross-sectional variation in the utilization variable occurred. The expected signs for each of the variables was estimated in each of the models for the three areas. Forage utilization is inversely related with forage production while establishment probability and the "normal" aridity index are directly related to forage production. The "normal" aridity is considerably higher for the August-September period in the Sitgreaves-Ft. Apache area than is the spring "normal" aridity index in the two other areas.

Suggestions for further research

The main criticism of the establishment model is that no estimate of the probability of different degrees of establishment success is made. For future research more accurate observation of the success of a seeding project could be made. The observation would have to be recorded as some standard evaluation measurement as plants per square foot, basal area, or some other suitable measure. Then the probit model can be used to estimate the probabilities of failure and success and an additional estimation of the probability of degree of success. Certainly this latter probability would have more meaning as a variable in the forage response model.

Another important probability measure is needed in the forage response model for reduction purposes. This is a measure of the probability of a certain aridity index occurring. In order to have this measure of probability, the probability distribution of the aridity index would have to be found. It is likely that a particular weather season would have to be designated to avoid the difficulties associated with the change of skewness of a particular distribution other than the normal distribution. Also further work with the Markov scheme and weather seasons could be pursued. This work could prove to be very helpful in the explanation of weather movements from season to season.

APPENDIXES

Appendix A: Review of literature

A review of all the literature pertinent to grass production and weather-yield relationships would be beyond the scope of this study. In keeping with the objectives of the study, a review is given of some preceding theoretical work concerning the affects of weather and certain policy variables on crop yields.

Attempts to measure weather affects on crop yield have taken one of two lines of research. One might be called the "traditional regression approach" and the other the theoretical approach. One of the earliest studies using the traditional approach is that of Mattice (1931). Twelve different functions relating various weather measurements to state and regional average corn yields were estimated. The variables of no significance were eliminated by a stepwise regression analysis. A priori hypothesis was made about the relationship of the weather variables with corn yield. The results of the analysis were dependent entirely upon the data used. There was no empirical test of any theory. Stallings (1958) also followed the traditional approach using check plot data from various crop experiments at agricultural experiment stations. Data were taken on specified crop varieties, grown under constant conditions of soil environment and cultural practices, where only weather was allowed to vary. Indexes were computed for the unexplained variation in crop yields from the experiments. The weather variables are not defined in this approach.

Edwards (1963) used a theoretical approach laying out a preliminary hypothesis of the variables influencing crop yields. A thorough description of the biological processes involved in plant growth is accomplished which suggests the variables to use and then a theory of the functional relationships is presented. The model then is applied to small grain data to make the empirical test of the theory. The conclusions of this study were that the theoretical model is superior to the "naive" models of the traditional approach.

Oury (1965) sets out a theory of weather affects on crop yields which reduces the weather affects to two variables, temperature and precipitation. Since water provides as well as transports nutrients to the areas of photosynthetic activity and since the flow of water is dependent on temperature, the priori hypothesis is that these are the basic two physical variables explaining growth. Because temperature and precipitation are inversely correlated, they are combined in an index following the work of de Martonne (1926) and Ångström (1936). The index then becomes inversely related to temperature and directly related to precipitation.

McConnen (1965) hypothesized that over time both grazing patterns and precipitation affect range grass yields. The study was based on experiments on range pastures using three levels of forage utilization. A logarithmic function of time was assumed for forage utilization, and annual precipitation was assumed to be the only weather variable influencing range grass production. The intercept of the logarithmic function was determined by the level of grazing.

Appendix B: Computation of the aridity index and Markov chain

The computation of all the aridity indexes was accomplished by means of a program developed for the IBM 1620 digital computer. Dr. Bernard Oury at North Carolina State University developed the program and allowed its use in this study. The aridity index computations were greatly reduced because of Dr. Oury's valuable help.

The input for the program is the accumulative monthly precipitation and monthly mean temperatures. These may be combined in either the de Martonne or the Ångström aridity index. The indexes are grouped for any arrangement of months desired. Each month's index is weighted by its variance before being grouped.

The Markov chain computations were greatly aided with the help of a program developed for the IBM 1620 digital computer. This program was developed by Dr. William R. Reilly at the University of Connecticut. The program computes the limiting vector and the limiting matrix. The transition matrix, P, can be raised to successively higher powers up to P^{99} for a 10 by 10 matrix. Also included for firm movement analysis is a prevision to compute a transitional state matrix and the n-stage vector. The input for the program is the initial state vector and the transition matrix which can be estimated by least squares, maximum likelihood, or quadratic programming.

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