5-1992

Comparison of Optimization Methodologies for Sustained-Yield Groundwater Pumping Planning in East Shore Area, Utah

Shu Takahashi
Utah State University

COMPARISON OF OPTIMIZATION METHODOLOGIES FOR SUSTAINED-YIELD GROUNDWATER PUMPING PLANNING IN EAST SHORE AREA, UTAH

by

Shu Takahashi

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Agricultural and Irrigation Engineering

UTAH STATE UNIVERSITY
Logan, Utah

1992

ACKNOWLEDGEMENTS

I wish to thank Professor Peralta for his guidance in my research, encouragement, and friendship. I also wish to thank the other members of my graduate committee, Drs. Willardson, Neale, Merkley, Glover, and Hughes, for their help and advice. I appreciate Nippon Koei Co., Ltd. for providing me the opportunity of this academic experience and for their financial support. I wish to extend my appreciation to all my friends at Utah State University and the staff members of the Agricultural and Irrigation Department for their support.

I am grateful to my wife, Misa, for her continuous support and understanding and to my daughter, Natsuko, and my parents for their love.

Shu Takahashi

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure **Page**

Figure **Page**

ABSTRACT

Comparison of Optimization Methodologies for Sustained-Yield Groundwater Pumping Planning In East Shore Area, Utah

by

Shu Takahashi, Doctor of Philosophy

Utah State University, 1992

Major Professor: Dr. Richard C. Peralta
Department: Biological and Irrigation Engineering

Combined simulation and optimization models, which are helpful for long-term groundwater planning of complex nonlinear aquifer systems, are developed using alternative modelling approaches. The models incorporate a representation of steady-state, quasi-three-dimensional head response to pumping within an optimization. An embedding model which describes exactly the nonlinear flow of an unconfined aquifer is presented. In contrast with the embedding models presented in the Utah State University Ground Water Model, it directly achieves the optimal solution without a "cycling." To address the nonlinearity of the flow system, response matrix models couple superposition with the cycling procedure. Their linear influence coefficients are generated using a modified McDonald and Harbaugh model.

First, these models are tested for a hypothetical, 625 cell, nonlinear aquifer system and compared in terms of computational accuracy and efficiency. All of the models achieve the same optimal solution. The fully nonlinear embedding model attains the same optimal solution regardless of how far the initial guess is from that solution. Thus, global optimality is probably obtained. A predictive program for comparing a priori the embedding and response matrix models in terms of computational size is also developed. This computes the required memory for running each model, an important factor in computational efficiency. It is based on the number of nonzero elements in the matrix of the optimization scheme.

The model most appropriate for a given aquifer and desired management scenarios is dependent upon required simulation accuracy, flow conditions (steady or unsteady), spatial scale, model computational resources requirement, and the computational capacity of available hardware and software. The linear embedding model coupled with a cycling procedure, as incorporated within a modified version of the USUGWM, is most appropriate for the subject reconnaissance-level study of the East Shore Area. Here, the demand for sufficient water of adequate quality is increasing. The underlying aquifer is three-layered, unconfined/confined and is discretized into 4,880 finite-difference cells. To overcome the difficulties of solving many nonsmooth functions describing evapotranspiration, discharge from flowing wells,

and drain discharge, a former cycling procedure is improved by optimizing the purely linearized models repeatedly. Using the modified version of the USUGWM, optimal sustained-yield pumping strategies are computed for alternative future scenarios in the East Shore Area.

(214 pages)

CHAPTER I

INTRODUCTION

In most water resource projects, surface water has been extensively developed as a main water source, and groundwater has been utilized as a supplemental water source. In the future, the use of groundwater will be increasingly important to meet the growing demand for water of sufficient quality and quantity. Groundwater has some advantages over surface water: greater dependability during droughts; generally higher quality; and less required investment for the facilities. However, once the adverse side effects of groundwater development occur, it takes a long time for the aquifer to recover because of the low velocity of groundwater flow. Therefore, some important considerations in water resource planning and management are how much and where groundwater can be supplied to the users of a given aquifer for a long time without causing adverse side effects.

Combined simulation and optimization models have been developed for groundwater management over the last two decades. The combined models predict the behavior of a given aquifer and determine the best management decision for the specified objectives and constraints. Most of the models have assumed a linear flow system. However, saturated thickness varies in an unconfined aquifer, and several hydrological flow processes represented by nonsmooth functions such as evapotranspiration are involved in the flow system. In such cases, appropriate methods are required to solve the nonlinearities. Additionally, it is not theoretically possible to prove whether an optimal solution of the nonlinear system is globally optimal.

Combined models are sometimes classified as utilizing either an embedding or a response matrix approach (Gorelick, 1983). In the embedding approach, finite-difference or finite-element approximations of hydraulic flow equations are contained directly in the management model and are required for all cells or nodes. The advantage of this approach is the straight forward representation of the flow equations in its management model. The disadvantage is mainly the computational difficulty resulting from the large optimization scheme.

By embedding nonlinear equations directly in the optimization modelling program, a nonlinear problem can be solved directly. However, it can be difficult to solve such nonlinear problems because of processing time requirements and extensive memory.

The USU Groundwater Management Model (USUGWM) overcame many of the previously reported disadvantages of the embedding approach (Gharbi et al., 1990). In the USUGWM, the cycling procedure, which repeatedly optimizes linearized formulas of the nonlinear flow terms, was presented to develop sustained-yield strategies for large and complex aquifer systems. The USUGWM included the fully and partially linearized embedding models. Both models successfully optimized groundwater pumping for the Salt Lake Valley of Utah. That underlying aquifer has two layers and is discretized into 1,086 cells. However, the fully linearized model sometimes fails to find a feasible solution for that case if initial guesses of head are far from the optimal heads. In the partially linearized model, nonlinear formulas of the nonsmooth functions are involved in the management model. This model avoids problems which occur in the fully linearized model. However, if the partially linearized model is applied to an unconfined aquifer, cycling is still needed to achieve the optimal solution of the original, nonlinear flow system because tranmissivity is computed from heads in the previous cycle. In addition, it is difficult to prove that the optimal solution from such linearized models is truly optimal for the original nonlinear flow system.

In the response matrix approach, superposition is used within its management model to compute heads only at specific heads. Head response to unit hydraulic stress (influence coefficients) is estimated using an external flow simulation model. This reduces the required memory in the management model. However, the optimization scheme size is rather large if the portion of cells having pumping decision variables or requiring head constraint is large (Peralta et al., 1991). The most important requirement for this approach is that the governing differential equation must be linear.

Several researchers have solved the problem of nonlinear flow in an unconfined aquifer while using the response matrix approach. However, none of the models considered external flows such as these addressed by nonsmooth functions in this work. Even if such nonsmooth functions are represented by simple linear segments, superposition cannot be used because the system linearity is violated when heads move from one linear segment to another.

In this study, several new modelling techniques are developed to overcome difficulties in incorporating nonlinear flow simulation within the embedding and response matrix approaches. They include: (1) improvements of the solution procedure in the linear version of the USUGWM, (2) a fully nonlinear embedding model, and (3) a response matrix model which can handle nonsmooth functions as well as transmissivity in an unconfined aquifer. The global optimality of the above models is also confirmed.

In addition, it is desirable to know a priori which type of model is most suitable for a specific situation since implementing any method requires much effort. Required computer memory and/or computer processing time are important factors in determining model desirability. A methodology for a priori comparison of the combined models based on the number of nonzero elements is presented.

The primal goal of this study is to construct a regional groundwater management model for the East Shore Area, Utah where the groundwater reservoir is a three-layer, confined/unconfined aquifer system. The East Shore Area aquifer is discretized into 4,880 cells (about five times the size of the Salt Lake Valley model). The model contains about 2,000 nonsmooth functions describing evapotranspiration, flow from flowing wells, and drain discharge. The fully linear embedding model, which is most appropriate for the East Shore aquifer system, is used to compute different perennial-yield pumping strategies for alternative future scenarios.

Objectives

The objectives in this study are the following:

1. To improve the solution procedure of nonsmooth functions, originally presented in the linear version of the USUGWM, to achieve a stable optimal solution.
2. To develop regional, sustained-yield, planning models suitable as alternatives to the USUGWM. The fully nonlinear embedding model and the response matrix model suitable for nonlinear flow systems are newly presented. All of the models, including the original USUGWM, are to be applied to a hypothetical area (625 cells) and compared in terms of computational efficiency and accuracy.
3. To confirm the global optimality for the alternative models. In the hypothetical aquifer

problem, it is necessary to confirm whether the models achieve the same optimal solution even if they are run with an initial guess far from the optimal solution.

4. To develop a predictive technique to determine the number of nonzero elements required by the embedding and response matrix approach models for complex, nonlinear systems.
5. To construct a modified version of the USUGWM, which uses a fully linear embedding approach, in optimizing sustained-yield planning for the East Shore Area and demonstrate its flexible abilities for alternative future scenarios.
6. To develop a preliminary "Decision Support System" for regional groundwater management so that the developed models and methodologies can be easily transferred to other study areas.

In order to describe the accomplishments of the above tasks, this paper consists of three parts. Chapter II compares alternative modelling approaches which can be used in planning for complex and nonlinear aquifer systems. These are computed for a hypothetical, three-layer, 625 cell, aquifer system. Chapter III presents the modified version USUGWM, which is coupled with cycling and fully linearized formulas for transmissivity and external flows described by a nonsmooth function. Its application to the East Shore Area and results of alternative future scenarios are also presented. These two chapters accomplish the primal goal. In addition, a preliminary structure of a Decision Support System (DSS) for the regional groundwater management is presented in Chapter IV.

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CHAPTER II

OPTIMIZING SUSTAINED-YIELD PUMPING PLANNING FOR NONLINEAR SYSTEMS: APPROACH COMPARISON

Abstract

Six alternative simulation/optimization models useful for computing optimal sustained-yield (steady-state) groundwater pumping strategies are compared in terms of formulation, solution procedure, accuracy, and computational efficiency. The different models require different computer processing time and memory. For the aquifer tested system, if more than 10% of the cells have pumping as a decision variable, a fully linearized embedding model will require less computer memory than any other model. All the models address linear and nonlinear steady-state flow in multilayer, unconfined/confined aquifers. They also address several types of nonsmooth external flows. Newly presented are response matrix models solving external flows described by nonsmooth functions through cycling, and a fully nonlinear embedding model that directly achieves an optimal solution without cycling. Models are tested using a hypothetical three-layer (unconfined/confined) aquifer system (3 layers x 15 rows x 15 columns = 675 cells). Empirically, globally optimal solutions seem to be obtained. All the models compute the same optimal pumping even if their optimizations are begun using vastly different initial guesses. This addresses a common concern that the solutions to nonlinear

problems are not necessarily globally optimal.

Introduction

Some groundwater management models can determine the best pumping strategy for a desired goal while simulating the aquifer response to that pumping. Such models generally use either the embedding or response matrix approach (Gorelick, 1983). Most models reported in the literature have been applied to linear systems or have assumed linearity. However, flow in many aquifers is nonlinear.

Numerical approximations of the saturated groundwater flow equation are either linear or nonlinear (for confined or unconfined, respectively). However, flows such as evapotranspiration, drain discharge, stream-aquifer interflow, and discharge from flowing (artesian) wells can be represented by nonsmooth functions which are not continuously differentiable. For such nonlinear flow systems, it is sometimes inappropriate to assume system linearity. Furthermore, it is sometimes not theoretically possible to prove that a solution is globally optimal.

The embedding approach directly incorporates numerical approximations of the groundwater flow equation in the model as constraints. It provides optimal solutions of head, pumping rate, and other variables at all cells simultaneously for the entire area. Some researchers have summarized or reported computational difficulties of optimization algorithms for the embedding models especially for transient problems (Gorelick, 1983; Tung and Kolterman, 1985; Yazdanian and Peralta, 1986).

Others have successfully used the embedding approach for large and/or complex aquifer systems (Cantiller et al., 1988; Gharbi et al., 1990; Peralta et al., 1991a). The MINOS software (Murtagh and Saunders, 1987) was used to perform the optimization in the latter models.

The USU groundwater management model, USUGWM (Gharbi, 1991), is the first embedding model optimally managing a large, multilayer, and nonlinear aquifer system under transient conditions. Gharbi applied it to the Salt Lake Valley aquifer, which is discretized into 1,086 cells. Constraints describing flow in the unconfined aquifer, contaminant transport, stream-aquifer interflow, and evapotranspiration are formulated both linearly and nonlinearly. The model was cyclically solved to reach the optimal solution of the original nonlinear flow system. USUGWM overcame previously reported disadvantages of the embedding approach. It used nonlinear formulations of nonsmooth flow functions. However, USUGWM also used a linear surrogate to address the nonlinear transmissivity of an unconfined layer.

The response matrix approach relies on the principle of superposition to simulate groundwater flow. Influence coefficients describing potentiometric head response to unit pumping are first generated for specified locations using an external groundwater flow simulation model. A response matrix consisting of these influence coefficients is then used with superposition to compute heads in the management model. Because only the influence coefficients for control locations are included, memory required by the response matrix optimization model can be minimized.

There have been many transient simulation or management models using the response matrix approach for various objectives. Among these models, Illangasekare and Morel-Seytoux (1982) presented a stream/aquifer simulation model using discrete kernels (influence coefficients). Illangasekare et al. (1984) also developed "reinitialization" and "scanning subsystem" techniques for creating and handling discrete kernels. These techniques can save computer storage to simulate in two dimensions the physical behavior of the large aquifer. These types of discrete kernels can be coupled with optimization problems.

Peralta and Kowalski (1986) used discrete kernels to determine optimal groundwater extraction strategies for the Grand Prairie of Arkansas. Peralta et al. (1988a) used resolvent influence coefficients for maximizing crop production in a hypothetical stream/aquifer system. These stream-stage and groundwater levels changed dynamically in response to pumping and inflow. Peralta et al. (1988b) used the response matrix approach to develop optimal groundwater extraction strategies including recharge basins for the study area. In that study, they used resolvent influence coefficients which expressed groundwater-level response to pumping and simultaneous interflow between a recharge basin and aquifer. Peralta et al. (1990) combined embedding, cell and well influence coefficients and superposition, with the stream-flow routine to represent dynamic stream stage and groundwater level interaction while optimizing conjunctive use.

Reichard (1987) used two types of influence coefficients, water level responses to a unit discharge and recharge, in the groundwater management model for the Salinas Valley of California. To address surface water-groundwater interaction, a river recharge function is embedded in the model.

Since superposition is most properly applicable to linear systems, assumptions or methods are required to apply it to nonlinear (unconfined) aquifers. Maddock (1974) developed a nonlinear, technological function for a one-dimensional unconfined aquifer system. The drawdown response to pumping is represented by an infinite power series. A nonlinear, technological function is computed using a finite sum of the power series. The number of terms needed to achieve a good approximation was determined by the ratio of drawdown to saturated thickness. When this nonlinear, technological function is used in an optimization model, the objective function becomes a nonlinear formula.

Heidari (1982) applied the normal response matrix approach to groundwater management in the Pawnee Valley of southcentral Kansas. A one-layer, unconfined aquifer system was approximated as a confined aquifer, and the drawdown correction for the unconfined aquifer was calculated using the approach of Jacob (1944).

Danskin and Gorelick (1985) developed a hydrologic-economic response model for the Livermore Basin in northern California. The underlying aquifer is a two-layer, unconfined/confined system. They used the response matrix approach coupled with the iterative method to address nonlinear transmissivity in the upper, unconfined layer. The influence coefficients are generated using the transient, quasi-three-dimensional, finite-difference model of Trescott (1976). The iterative approach linearizes the system and iterate a management model containing the linearized system. Others have termed this procedure as cycling.

Willis and Yeh (1987) presented a procedure to deal with flow in a small, one-dimensional, unconfined system using a response equation. This nonlinear response equation is a differential equation transformed from the Boussinesq equation (Willis, 1984). The nonlinear response equations are quasi-linearized using the generalized Taylor series. Because of this quasi-linearization, a series of optimizations is needed to achieve a solution of the original unconfined aquifer system.

Elwell and Lall (1988) used the response matrix approach for analyzing groundwater development in the Salt Lake Valley of Utah. They superimposed a two-dimensional finite-difference grid on the area of interest in the unconfined aquifer system. To address the nonlinearity of the unconfined, leaky, or stratified aquifer, the Girinski potential was used instead of head in the management model.

The approach most suitable for a given situation is dependent upon simulation accuracy, flow conditions (steady or unsteady), spatial scale (large-scale or small-scale), and the computational capacity of hardware and software (Gorelick 1983; Peralta et al., 1991b). Peralta et al. (1991b) provided a comparison regarding the required computer memory and the accuracy of the computed results using models designed to develop sustained-yield strategies in a hypothetical confined aquifer (11 x 9 = 99 cells). They concluded that the embedding model requires less computation time and computer memory than the response matrix model if the proportion of pumping cells and cells requiring head computation or constraints within the optimization is large.

The first objective of this study is to enhance the modelling approach originally presented in USUGWM and to make it completely applicable for fully nonlinear systems and for a steady-state condition. The original USUGWM contains both fully and partially linearized models. When the fully linear model is applied to a nonlinear system, heads from the previous cycle are used to compute transmissivity and to select the correct linear segments of equations for evapotranspiration (Et), river-aquifer interflow, and flow reduction. The model is re-optimized until the values of variables do not change with the cycles. In the nonlinear model, the above external flows are represented by their nonlinear formulation, but transmissivity is still treated linearly (Gharbi et al., 1990). When the nonlinear model is applied to an unconfined aquifer, cycling is necessary.

For illustration, now evapotranspiration (Et), described using piecewise linear equations (segments), is explained below. Et is a known maximum values if the water table elevation in an unconfined aquifer exceeds a certain elevation (proximity to the ground surface). Et is zero if the water table is beneath a certain elevation. Between these two elevations, Et changes linearly from the maximum value to zero. Et is a nonsmooth process because its equation is segmented and not continuously differentiable. To address this problem linearly requires deciding, before optimization, which linear segment of the Et equation to use.

Because of the pre-selection of the linear segments for nonsmooth external flow functions, the fully linearized model of the USUGWM would not necessarily converge to the optimal solution if the initial guess of the solution was far from that optimal solution. To address that problem, a USUGWM user should switch from the fully linearized model to the partially linearized model. In this study, the linearized model is improved so that it will always converge to the same solution regardless of its initial guess.

In addition, a fully nonlinear embedding model, in which transmissivity is represented as a nonlinear function of head, is newly developed. This model directly computes an optimal solution without cycling.

The second objective of this study is to construct response matrix models so that they have comparable ability to address nonlinear systems like the embedding approaches mentioned above. As described previously, several researchers have applied the response matrix approach to unconfined aquifers. However, none of these models contained external, hydrological flows described by nonsmooth functions such as drain discharge. If the flow equation contains these external flows and they are significant, then superposition cannot be used directly. In this paper, we show how to use linear superposition with cycling to address such nonlinear, nonsmooth flow systems.

The third objective attempts to increase the probability of achieving globally optimal solutions for these nonlinear systems. That involves two issues: (1) It is difficult to prove that the optimal solution to a linear surrogate of a nonlinear problem is also an optimal solution of the original nonlinear problem (Gorelick, 1983; Gharbi and Peralta, 1992). An approach to prove this is to successfully develop the fully nonlinear model and to compare solutions. (2) It is difficult to know whether the solution solved by a nonlinear model is local or global optimal. Here, for a selected system, we demonstrate that three types of embedding models (fully linear, partially linear, and fully nonlinear models) and the response matrix models all achieve the same optimal solution even if the models are run with different initial guesses chosen from a wide range. Empirically, perhaps global optimality is achieved.

The fourth objective is to compare alternative approaches computing sustained-yield pumping strategies for a complex nonlinear aquifer system. Alternatives include three embedding and three response matrix models. These models can replicate all of the steady-state simulation abilities of the USGS modular, three-dimensional, finite-difference, groundwater flow model, MODFLOW, (McDonald and Harbaugh, 1988) while computing optimal groundwater pumping strategies. The embedding models contain finite-difference approximations of a quasi-three-dimensional flow equation as constraints. The response matrix models compute heads using superposition and influence coefficients, generated by a modified McDonald and Harbaugh (MODFLOW). Also, a predictive technique for deciding which model is most appropriate for a specific situation based on required memory is demonstrated.

To achieve these goals, some definitions are first provided. Then the objective function is presented, followed by a discussion of the four steady-state optimization models being compared. All are tested for a hypothetical, three-layer system having unconfined and confined layers, a nonsmooth flow, and six potential pumping cells. Finally, memory requirements of each modelling approach are compared.

Iteration and Cycling

The following terms are used in subsequent sections and are defined below:

Iteration An iteration refers to the processing of solvers, such as the LP and DNLP solvers in the MINOS optimization software and the SIP (Strong Implicit Package) solver in MODFLOW. Many iterations might be required to find a solution.

Cycling Cycling is a recursive process of solving an optimization problem over and over. Between cycles, changes are made in assumed parameter values on utilized equations. For example, first, nonlinear formulas are linearized. Then the model containing the linearized formulas is optimized using initial guesses of variables. For the second cycle, parameters are recomputed, and the optimization model is rerun. The process of using the optimal solution from the previous run to initialize parameter values for the next optimization is repeated until the computed optimal variable values do not change with the cycles. Here, nonlinear terms include transmissivity in an unconfined aquifer and use external flows described by nonsmooth functions. For all presented models, except for the fully nonlinear embedding model, multiple cycles are usually required to achieve the true optimal solution when the models are applied to flow systems including a unconfined aquifer and/or nonsmooth functions.

Models Using Embedding Approach

In this section, three alternatives are presented. Alternatives E1 and E2 are prepared using the USU groundwater management model (USUGWM) initially developed for the Salt Lake Valley (Gharbi et al., 1990) but modified with the added ability to address flowing (artesian) wells (Takahashi and Peralta, 1991). Alternative E1 is fully linear. Alternative E2 is nonlinear for nonsmooth external flows but linear for transmissivity. Alternative E3 is a newly demonstrated fully nonlinear model which requires neither linearization nor the cycling procedure. All these models are written in General Algebraic Modeling System, GAMS (Brooke et al., 1988). Optimizations are performed using MINOS (Murtagh and Saunders, 1987).

Model Formulation

Objective Function

The objective function of each model is to maximize total steady groundwater extraction.

$$maximize \ \ z = \sum_{o=1}^{N} gp_o \tag{1}$$

where

gp_o groundwater pumping in cell o located in layer l, row i, and column j, (L^3/T);

N total number of cells with potential pumping wells.

In the model, discharge, i.e., groundwater pumping, is a positive value, and recharge is a negative value.

Constraints Describing the Physical Flow System

The steady-state, finite-difference form of the quasi-three-dimensional groundwater flow equation (McDonald and Harbaugh, 1988) is used as constraints (one for each cell and layer).

$$CR_{l,i,j+1/2}(H_{l,i,j+1}-H_{l,i,j})+CR_{l,i,j-1/2}(H_{l.i.j-1}-H_{l.i,j})$$
$$+CC_{l,i+1/2,j}(H_{l,i+1,j}-H_{l,i,j})+CC_{l,i-1/2,j}(H_{l,i-1,j}-H_{l,i,j})$$
$$+CV_{l+1/2,i,j}(H_{l+1,i,j}-H_{l,i,j})+CV_{l-1/2,i,j}(H_{l-1,i,j}-H_{l,i,j})$$
$$= \Sigma^{N}_{n=1} q^{*}_{l,i,j,n} \qquad (2)$$

where

$$CR_{l,i,j+1/2}=2dx_j(T^j_{l,i,j}T^j_{l,i,j+1})/(T^j_{l,i,j}dy_{i+1}+T^j_{l,i,j+1}dy_i) \qquad (3a)$$

$$CC_{l,i+1/2,j}=2dy_i(T^i_{l,i,j}T^i_{l,i+1,j})/(T^i_{l,i,j}dx_{j+1}+T^i_{l,i+1,j}dx_j) \qquad (3b)$$

$$CV_{l+1/2,i,j}=dx_jdy_i/\{(dz_l/2Kz_{l,i,j})+(dz_{l+1}/2Kz_{l+1,i,j})\} \qquad (3c)$$

$H_{l,i,j}$ potentiometric head, (L);

$T^i_{l,i,j}$ transmissivity in the row direction, (L^2/T);

$T^j_{l,i,j}$ transmissivity in the column direction, (L^2/T);

l,i,j layer, row, column indices of a finite-difference cell;

CR,CC	hydraulic conductances (harmonic averages of transmissivities) along x,y axes between the nodes, (L^2/T);
CV	vertical conductance between the nodes, (L^2/T);
dx,dy,dz	cell sizes in layer l, row i, and column j, (L);
$Kz_{l,i,j}$	vertical hydraulic conductivity, (L^2/T);
$q^*_{l,i,j,n}$	(n th) external flow term in a cell, (L^3/T).

Alternatives E1 and E2. For a confined layer, transmissivity is constant. Thus, hydraulic conductances CR, CC, and CV are constant, and the left-hand side (LHS) of Equation 2 is always linear. For an unconfined layer, transmissivity should most properly be a function of an unknown head and hydraulic conductivity (T = kh). In Alternatives E1 and E2, transmissivity ($T_{l,i,j} = k_{l,i,j}HFC^{n-1}_{l,i,j}$) is constant in a cycle by substituting a head HFC^{n-1} known from the former (n-1) cycle for an unknown head H in the present cycle. Thus, hydraulic conductances CR, CC, and CV are constant, and the LHS of Equation 2 becomes linear in each cycle. Cycling is continued until heads do not change with the cycles. Alternative E1 requires cycling to treat the nonsmooth flows and transmissivity of an unconfined aquifer. E2 uses cycling only to address transmissivity.

Alternative E3. For a confined aquifer, transmissivity is constant as in Alternatives E1 and E2. For an unconfined aquifer, $T_{l,i,j} = K_{l,i,j}\ H_{l,i,j}$ and one uses an unknown head H. As a result, hydraulic conductances CR and CC are nonlinear while CV is always linear. The LHS of Equation 2 is nonlinear.

The models also compute various external flow terms: (1) flow at sources or sinks such as pumping/recharge wells (gp), drains (q^d), or flowing wells (q^f), (2) other processes such as stream-aquifer interflow (q^s), flow across a general head boundary (q^g), evapotranspiration (q^e), flow reduction due to partial desaturation (q^{rd}), areal constant recharge (q^r), and flux across constant head boundary (q^c).

All external flows except for q^r are treated as variables. External flows dependent on head in the subject cell are formulated separately from the flow equation (Equation 2) as independent constraints. Based on their formula (linear or nonsmooth and dependent or independent of head), those external flow terms are classified into three types. This is important for subsequent explanations because the model development and solving procedure differ with each type.

Type 1. These external flows are assumed to be independent of groundwater head in the subject cell or to be dependent on a constant head.

$gp_{l,i,j}$ pumping rate in a cell, (L^3/T);

$q^c{}_{l,i,j}$ saturated flow across a constant head boundary cell, (L^3/T);

$q^r{}_{l,i,j}$ known constant recharge in a cell, which includes bedrock recharge, unsaturated canal seepage, irrigation seepage, precipitation in the recharge area, (L^3/T).

Type 2. This external flow is represented by a linear function of head in the subject cell.

- *Recharge/discharge through general head boundary for all alternatives:*

$q^{g}_{l,i,j}$ = saturated flow between the aquifer and a general head boundary in the cell, (L^3/T);

$$= \Gamma^{g}_{l,i,j}\,(H_{l,i,j} - h^{ls}_{l,i,j}) \qquad (4)$$

where

Γ^{g} hydraulic conductance between the aquifer and general head boundary cell, (L^2/T);

h^{ls} fixed water level such as that of the sea, (L).

Type 3. These external flows are assumed to be represented by a nonsmooth function of head in the subject cell. The function consists of two or three linear segments. For Alternative E1, the segment to be used is based on head from the previous cycle. In Alternatives E2 and E3, these flows are solved using (max or min (argument 1, argument 2)), a DNLP (nonlinear programming with discontinuous derivatives) option of MINOS.

- *Discharge from drains*

for Alternative E1:

$q^{d}_{l,i,j}$ = saturated flow leaving the aquifer in a cell with drains, (L^3/T);

$$= \Gamma^{d}_{l,i,j}(H_{l,i,j} - B^{d}_{l,i,j}) \quad \text{for } HFC^{n-1}_{l,i,j} > B^{d}_{l,i,j}$$

$$= 0 \qquad \text{for } HFC^{n-1}_{l,i,j} < B^{d}_{l,i,j} \qquad (5a)$$

for Alternatives E2 and E3:

$$q^{d}_{l,i,j} = \Gamma^{d}_{l,i,j}\,\max(H_{l,i,j} - d_{l,i,j},\ 0) \qquad (5b)$$

where

Γ^d hydraulic conductance between the aquifer and drains, (L^2/T);

B^d Bottom elevation of the drains, (L).

In 5(a), if a head HFC known from the former cycle is above the drain bottom (HFC > d), then $q^d = \Gamma^d(H-d)$, otherwise (HFC $\leq$ 0), $q^d = 0$. Since the linear segment is not selected using an unknown head H in the current cycle, this linear formula needs cycling to solve drain discharge.

In 5(b), the $\max(H_{1,i,j}-d_{1,i,j}, 0)$ selects the bigger of $(H_{1,i,j}-d_{1,i,j})$ and 0 while simultaneously performing the optimization. If an unknown head (H) in the current cycle is above the drain bottom ($H \geq d$), then $q^d = \Gamma^d(H-d)$, otherwise ($H < d$), $q^d = 0$. Thus, cycling is not necessary to solve this formula (Gharbi et al. 1990). Other Type 3 external flows are also solved in the same manner as this.

- *Evapotranspiration*

for Alternative E1 (Linear formula):

$q^e_{1,i,j}$ = distributed discharge from evapotranspiration in a cell, (L^3/T);

$$= E_o\, dx_j dy_i \qquad \text{for } HFC^{n-1}_{1,i,j} > h^s_{1,i,j}$$

$$= E_o\, dx_j dy_i\, \{H_{1,i,j}-(h^s_{1,i,j}-d_{1,i,j})\}/d_{1,i,j}$$

$$\text{for } h^s_{1,i,j}-d_{1,i,j} < HFC^{n-1}_{1,i,j} < h^s_{1,i,j}$$

$$= 0 \qquad \text{for } HFC^{n-1}_{1,i,j} < h_{s1,i,j}-d_{1,i,j} \qquad (6a)$$

for Alternatives E2 and E3 (DNLP formula):

$$q^e_{1,i,j} = E_o\, dx_j dy_i/d$$

$$\{\min(h^s_{1,i,j}, H_{1,i,j})-\min(h_{1,i,j}-d_{1,i,j}, H_{1,i,j})\} \qquad (6b)$$

where

E^o potential evapotranspiration, (L/T);

h^s potentiometric surface elevation below which evapotranspiration decreases, (L);

d extinction depth, (L);

- *Discharge from flowing wells*

for Alternative E1:

$q^f_{l,i,j}$ = discharge from flowing wells or springs in a cell, (L^3/T);

$= \Gamma^f_{l,i,j} \; (H_{l,i,j}-h^{gs}_{l,i,j})$ for $HFC^{n-1}_{l,i,j} > h^{gs}_{l,i,j}$

$= 0$ for $HFC^{n-1}_{l,i,j} < {}^{hgs}{}_{l,i,j}$ (7a)

for Alternatives E2 and E3:

$$q^f_{l,i,j} = \Gamma^f_{l,i,j} \max(H_{l,i,j}-h^{gs}_{l,i,j}, 0) \quad (7b)$$

where

Γ^f coefficient describing reduction in discharge rate of the flowing wells per 1 foot head decline, (L^2/T);

h^{gs} ground surface, (L).

- *Stream-aquifer interflow*

for Alternative E1:

$q^s_{l,i,j}$ = interflow between the aquifer and stream in a selected river cell, (L^2/T);

for saturated flow

$= \Gamma^s_{l,i,j}(H_{l,i,j}-\sigma_{l,i,j})$ for $HFC^{n-1}_{l,i,j} > B^s_{l,i,j}$

for unsaturated flow

$= \Gamma^s_{l,i,j}(B_{sl,i,j}-\sigma_{l,i,j}$ for $HFC^{n-1}_{l,i,j} < B^s_{l,i,j}$ (8a) for Alternatives E2 and E3:

$$q^{s}_{l,i,j} = \Gamma^{s}_{l,i,j} \max(H_{l,i,j}-\sigma_{l,i,j}, B^{s}_{l,i,j}-\sigma_{l,i,j}) \quad (8b)$$

where

Γ^{d} hydraulic conductance between the aquifer and river, (L^2/T);

σ elevation of the free water surface in the river, (L);

B^{s} bottom elevation of the river, (L).

If an elevation of the free water surface in the river can be assumed to be constant, then q^{s} is constant for unsaturated flow.

- *Vertical flow reduction*

for Alternative E1:

$q^{rd}_{l,i,j}$ = vertical flow reduction to correct overestimation in Equation 2 when the lower confined aquifer is desaturated (L^3/T);

$$= - CV_{l,i,j}(E^{top}_{l+1,i,j}-H_{l,i,j}) \quad \text{for } HFC^{n-1}_{l+1,i,j}<E^{top}_{l+1,i,j}$$

$$= 0 \qquad \text{for } HFC^{n-1}_{l+1,i,j}>E^{top}_{l+1,i,j} \quad (9a)$$

for Alternatives E2 and E3:

$$q^{rd}_{l,i,j} = - CV_{l,i,j} \max(H_{l,i,j}-E^{top}_{l+1,i,j}, 0) \quad (9b)$$

$$(q^{rd}_{l,i,j} = -q^{rd}_{l+1,i,j})$$

where

E^{top}_{l+1} elevation of the top of layer l+1, (L);

Bounds on Variables

For all three alternatives, bounds on pumping rate and head are described as:

$$h^{L}_{l,i,j} < h_{l,i,j} < h^{U}_{l,i,j} \tag{10}$$

$$gp^{L}_{l,i,j} < gp_{l,i,j} < gp^{U}_{l,i,j} \tag{11}$$

where

L,U denote lower and upper bounds, respectively.

Usually, bounds on head are used to avoid or minimize problems caused by unacceptable drawdowns, while bounds on pumping are set based on a well capacity and/or water demand. Other bounds can be added depending on the problem. For example, if flux across the constant head boundary must be restrained, the bounds are described as:

$$q^{cL}_{l,i,j} < q^{c}_{l,i,j} < q^{cU}_{l,i,j} \tag{12}$$

Solution Procedures

The steady-state finite-difference form of the quasi-three-dimensional groundwater flow equation (McDonald and Harbaugh, 1988) contains the following: (1) nonlinearity in an unconfined aquifer, where transmissivity is not constant but is a function of head, and (2) Type 3 external flows. These terms cannot be solved with the LP technique directly or without additional action. In Alternatives E1 and E2, the fully and partially linearized models, respectively, are formulated first. To achieve an optimal solution to a linear surrogate of a nonlinear problem, the models are solved repeatedly until variable values do not change with cycle (Gharbi et al., 1990). In Alternative E3, the above terms are formulated in a nonlinear manner and are solved using the

MINOS DNLP solver without the cycling procedure. Flow charts of solution procedures for the models are shown in Figure 1 and are described below.

Alternative E1

1. Read and prepare: read data files and set heads in the first cycle (HFC^0) equal to starting heads (STRT) which are initially guessed or given.
2. Formulate (start of cycle): using heads in the former (n-1 th) cycle (HFC^{n-1}), estimate the transmissivity (T) and conductances (CR, CC, and CV) and determine the linear segment of each Type 3 external flow. As a result, the transmissivity and conductances become constant. Additionally, the external flow is described as either (aH-b) or b (a and b are constant and H is variable). For example, drain discharge q^d is either (conductance (Γ^d) x unknown head (H) - Γ^d x B^d (drain bottom) or 0. Thus, the flow equation (Equation 2) and external flows become linear.
3. Solve: using the MINOS LP solver, solve the linear model, which includes the flow equation (Equation 2) and external flow linearized in step 2 as constraints. The LP solver uses an advanced simplex method. To commence, set initial values of head (H) equal to HFC^{n-1}.
4. Compare and converge (end of cycle): compare optimal solutions of variables such as head and pumping rate in the current (n th) cycle and those in the former (n-1 th)

cycle. If the difference between the optimal solutions of two consecutive cycles satisfies criteria which indicate the convergence of the variables, then go to step 6; otherwise, go to step 5.

5. Replace: the optimal solutions in the former (n-1 th) cycle are replaced with those in the current (n th) cycle. Go back to step 2 and continue through step 4.
6. Optimal solution: stop the cycle, and the true optimal solutions are found.

Alternative E2

The solving procedure of Alternative E2 is the same as Alternative E1 except for steps 2 and 3 which are described below.

2. Formulate (start of cycle): using heads in the former (n-1 th) cycle (HFC^{n-1}), estimate transmissivities (T) and conductances (CR, CC, and CV).
3. Solve: using the MINOS DNLP solver, solve the model, which includes the flow equation (Equation 2) linearized only with respect to transmissivities in step 2 and DNLP formulas of Type 3 external flows as constraints. The DNLP solver uses a reduced gradient method.

Alternative E3

1. Read and prepare: read data files including starting heads (STRT).
2. Formulate: using starting heads, estimate the transmissivity and hydraulic conductances (CRstrt and

CCstrt).

3. Solve: using the MINOS DNLP solver, solve the nonlinear model, which includes the nonlinear formula of the flow equation (Equation 2) and DNLP formulas of Type 3 external flow terms as constraints. In the system, initial values of H and the conductances (CR.L and CC.L) are set equal to STRT, CRstrt, and, CCstrt, respectively.
4. Optimal solution: the true optimal solutions are found.

In Alternative E3, the nonlinearities are formulated more ideally than in Alternatives E1 and E2. However, because of its nonlinearity, more memory and more strict programming requirements are necessary. These include better conception of an initial guess and bounds.

If Alternatives E1 and E2 are applied to a completely linear flow system, which includes neither an unconfined aquifer nor type 3 external flows, then the cycling procedure is skipped.

Global Optimality

The optimal solution of the fully linear model (E1 and a response matrix model) is globally optimal. However, it uses cycling, and the global optimality is guaranteed only in each cycle. On the other hand, the fully nonlinear model (E3) does not use cycling, but the DNLP solver looks for the local optimal solution. There are two problems concerning the optimality. First, it is difficult to know if the optimal solution to a linear surrogate of a nonlinear problem via cycling (in E1 or E2) is the solution of the original nonlinear problem (E3). Second, it is uncertain that the solution of nonlinear models (E2 or E3) is unique (globally optimal), meaning that a better solution exists. If the presented nonlinear problem is convex, it has only one optimal solution, the global optimum. In that case, all these models should achieve the same optimal solution.

Models Using Response Matrix Approach

Three response matrix models which can simulate groundwater flow in a complex nonlinear aquifer system using the principle of superposition are presented here. This alternative uses influence coefficients generated by a modified version of the MODFLOW model written by McDonald and Harbaugh (1988). The management models are written in GAMS and are solved with the MINOS LP solver.

The basic idea in solving the nonlinear flow system is the same as Alternative E1 except that superposition rather than embedding is used to compute heads. In this case, the flow equation (Equation 2) and constraints describing Type 3 external flows are treated linearly in each cycle and superposition is used. The cycling procedure is still used to ensure that final optimal equation segments and transmissivities are the same as those assumed commencing the cycle.

The size of the management model can be reduced drastically in some cases by using the response matrix approach instead of the embedding approach. To facilitate the use of the response matrix approach for nonlinear flow systems, MODFLOW is converted and modified into two independent external simulation models termed the Pre-Influence Coefficient Generator (Pre-ICG) and the Influence Coefficient Generator (ICG). The ICG is used to generate influence coefficients. The Pre-ICG computes heads for the ICG in the next (n+1) cycle.

Modified McDonald and Harbaugh (MODFLOW) Models

The objective is to gain the ability to use linear influence coefficients, superposition, and cycling to accurately represent head response to stimulus in a nonlinear system (unconfined aquifer and Type 3 external flow equations). The approach is presented after reviewing how the original MODFLOW works.

MODFLOW uses only linear equations. It selects which Type 3 equation segment (and transmissivity) to use based on values at the beginning of an iteration. Then it solves for those external fluxes based on their segments. Next, MODFLOW solves the entire flow equation with those external fluxes as knowns. There are many iterations and segment selections before the convergence to a solution.

Since we are using MODFLOW to generate influence coefficients, we must achieve compatibility between the management model and MODFLOW. To do this, assumptions used within a cycle of optimization modelling must be the same as those used in a single iteration in MODFLOW. Otherwise, the convergence of a solution would not always occur.

After using the same assumptions in developing influence coefficients and in subsequently computing the optimal strategy, some of the assumed equation segments of Type 3 external flows will be wrong (for the optimal pumping rates, although they are correct for the utilized unit pumping rates). However, segment assumptions will be corrected through cycling just as MODFLOW corrects these equations through iteration.

Pre-Influence Coefficient Generator (Pre-ICG)

The purpose of the Pre-ICG is to compute the heads needed by the ICG to calculate transmissivities and influence coefficients for the next cycle. Before describing how the Pre-ICG works, we present the common techniques used in normal simulation modelling.

Type A: Transmissivity is assumed constant through all time steps if the drawdown in an unconfined layer is relatively small compared with the saturated thickness. Less than 10% change in saturated thickness is usually acceptable for assuming system linearity (Reilly et al., 1987).

Type B: Transmissivity is assumed constant for each time step but is recomputed at the end of each time step. If this technique is applied to the steady-state, it is similar to Type A because a steady-state simulation uses either no time step (storage coefficient = 0) or only one very long time step.

Type C: Most groundwater flow simulation models, including MODFLOW, rely on iterative methods to solve the flow equation. These address the nonlinearity of an unconfined aquifer more realistically than the above techniques because transmissivity is assumed constant only in each iteration rather than in each time step. (There are many iterations within a time step.)

MODFLOW's steady-state solution procedure for nonlinear aquifer systems is discussed and shown in Figure 2(a). The steps are:

1. Read and prepare: read data files and set heads (HOLD) in the first time step (there is only one pseudo-time step for steady-state) equal to starting heads (STRT).
2. Prepare for iteration: set heads in the first iteration ($HNEW^0$) equal to HOLD.
3. Formulate (start of iteration): determine transmissivity (T), conductances (CR, CC, and CV), and external flow terms using heads in the former (m-1 th) iteration $HNEW^{m-1}$ for each nodes. As a result, the transmissivity and conductances are constant within an iteration. Additionally, the external flow term is described as either (a x $HNEW^m$ - b) or b (a and b are constant and HNEW is variable). Equation 2 is linear here.
4. Solve: compute a solution to the flow equation linearized in step 3 with one of the alternative solvers such as

Strong Implicit Procedure (SIP), a method for solving a large system of simultaneous linear equations by iteration.

5. Close (end of iteration): iteration proceeds until closure achieves (maximum $(HNEW^m - HNEW^{m-1}) \leq$ specified convergence criteria).
6. Final solution.

Type D: Another simulation procedure, which combines Type B and MODFLOW's simulation procedure to involve the LP technique in the management model, was used in USUGWM (Gharbi et al., 1990). In using USUGWM for transient optimization, transmissivity is estimated using hydraulic conductivity and optimal time varying head from the former cycle. As in Type B, transmissivity is assumed constant within a time step. However, transmissivity is recomputed for all time steps at the end of each cycle. This procedure is continued until transient heads do not change with the cycles. The simulation results of the USUGWM have been virtually identical to those of the MODFLOW.

In this study, MODFLOW is modified to be compatible with a Type D approach for steady-state. The solution procedure is presented in Figure 2(b) and is described as follows:

1. Read and prepare for time step: read data files and set heads (HOLD) in the first time step (but only one pseudo-time step for steady-state) to starting heads (STRT).
2. Prepare for cycle: set heads in the first cycling loop (HFC^0) to HOLD.

3. Formulate (start of cycle): determine transmissivity (T), conductances (CR, CC, and CV) and external flow terms using heads in the former (n-1 th) cycle HFC^{n-1}. As a result, transmissivity and conductances become constant. Additionally, the external flow term is either ($a \times HNEW^n - b$) or b. Equation 2 is linear here.
4. Prepare for iteration (start of iteration): set heads in the first iteration ($HNEW^0$) equal to HFC^{n-1}.
5. Solve: compute a solution to the linear equation in step 3 using a solver such as SIP.
6. Close (end of iteration): iteration proceeds until closure achieves (maximum ($HNEW^m - HNEW^{m-1}$)) $\leq$ specified convergence criteria.
7. Set heads in the current cycle (HFC^n) equal to heads solved through the iteration ($HNEW^s$).
8. Converge (end of cycle): cycling procedure proceeds until closure achieves (maximum ($HFC^n - HFC^{n-1}$)) $\leq$ specified convergence criteria.
9. Final solution: Stop the cycle.

Influence Coefficient Generator (ICG)

MODFLOW, in which the flow equation is linear at the beginning of each iteration, cannot be used directly as the Influence Coefficient Generator (ICG) for nonlinear flow systems. The ICG generates influence coefficients at the beginning of each cycle and is designed to perform as described below:

a. Read data files using mostly MODFLOW format. Added is a file identifying those cells for which head has to be computed within the optimization model.

b. Using SIP to generate influence coefficients for the entire system, solve the flow equation linearized at the beginning of each cycle.

c. Make a response matrix table containing influence coefficients.

The ICG calculates:

h^{um}_o Unmanaged head describing average head response over a cell *o* to known steady stresses (bed rock recharge, precipitation, etc.), (T/L^2);

$\delta_{o,m}$ Influence coefficient describing the average head response at cell *o* to a unit pumping in cell *m*, (T/L^2).

Computation of Head Using Influence Coefficients

The summation of influence coefficients times pumping is contained in the management model as a constraint to compute heads in specific cells.

$$h_o = h^{um}_o + \sum_{m=1}^{M} \delta_{o,m}\, q_m \qquad (13)$$

where

h_o average potentiometric head in cell *o*, (L);

q_m unit pumping in cell *m*, (L^3/T).

Model Formulation

The objective function and bounds on variables of the response matrix models are the same as in the embedding models. Their different forms are its constraints describing head. These are used only for specific cells, as opposed to being used for all cells as in the embedding approach.

Constraints Describing the Physical Flow System

To apply superposition and cycling to the nonlinear system and to calculate external flows, the following types of constant head cell (CHC) and variable head cell (VHC) are defined:

- CHC — constant head cell in which flow across constant head boundary (q^c) must be calculated.
- VHCc — variable head cell next to constant head cell (CHC).
- VHCf — variable head cell containing external flows which are functions of head.
- VHCs — variable head cell in cells surrounding a cell containing external flows. VHCs is defined only for Alternative R2 (specified later).
- VHCb — variable head cell in which head must be bounded to prevent unacceptable drawdown, salt-water intrusion, or other problems.
- VHCo — variable head cell in which there are no external flows, and head is not bounded but must be estimated for observation by the user.

VHCu variable head cell of an unconfined aquifer in which transmissivity must be estimated with optimal heads in the current (n th) cycle for the next (n+1 th) cycle optimization run.

Depending on the selected response matrix model, either Equation 2 (embedding flow equation) or Equation 13 (superposition) is applied to the above cells. The equation used is summarized in Table 1 and discussed below.

Alternative R1. The embedded flow equation (Equation 2) is applied to CHC, VHCf, and VHCu. Heads of cells surrounding a cell containing an external flow term must be calculated with Equation 13.

Alternative R2. The embedded flow equation (Equation 2) is applied only to constant head boundary cells, whether flux across the boundary is restrained or unrestrained. The physical boundary conditions are the following: (1) no flow (no flux), (2) constant flux, (3) restrained flux, and (4) unrestrained flux. In cases (1) and (2), cells on the boundary have variable heads. In cases (3) and (4), cells on the boundary have constant heads. Equation 13 is used as a constraint to compute heads in VHCc, VHCf, VHCb, VHCo, and VHCu cells.

Alternative R3. Equation 13 is used to compute head only for VHCb. Heads for other types of cells and external flows are computed externally by running the Pre-ICG with optimal pumping. However, if q^c is bounded or fixed, then Equation 2 for CHC and Equation 13 for VHCc are used. Also, if some external flow terms are bounded, then Equation 13 is used for VHCf.

Types 2 and 3 external flows are dependent upon head in the subject cell. In Alternatives R1 and R2, these flows are treated as variables and independently formulated as constraints, as in the embedding method models. In Alternative R3, those flows are used and independently formulated only if they require constraint.

Solution Procedures

Solution procedures of Alternatives R1, R2 and R3 are shown in Figure 3 (a) and (b), respectively, and described below.

Alternatives R1 and R2

In the management model:

1. Read and prepare: read data files and set heads (HFC^0) in the first cycle loop equal to starting heads (STRT) which are initially guessed or given.

In the ICG:

2. Run external ICG: run an external Influence Coefficient Generator (ICG) using heads of the unconfined aquifer in the former (n-1 th) cycle (HFC^{n-1}).

In the management model:

3. Read influence coefficients: read influence coefficients which are generated by ICG in step 2.
4. Formulate (start of cycling loop): using heads (HFC^{n-1}) in the former (n-1 th) cycle, estimate transmissivities

and hydraulic conductances and determine which linear segment of Type 3 external flow is applied.

5. Solve: using the MINOS LP solver, solve the linear model which includes superposition (Equation 13), the embedded flow equation (Equation 2), Type 2 external flow, and LP formulas of Type 3 external flows which are linearized in the former steps. In the model, an initial value of H is set equal to HFC^{n-1}.
6. Compare and converge (end of cycle): compare optimal values of variables such as head and pumping rate in the current (n th) cycle and those in the former (n-1 th) cycle. If the difference between the optimal solutions of two consecutive cycles satisfies certain criteria which indicates the convergence of variables, then go to step 8; otherwise, go to step 7.
7. Replace: optimal values of all variables in the former (n-1 th) cycle are replaced with those in the current (n th) cycle. Go back to step 2 and continue through step 6.
8. Optimal solution: stop the cycle, and the true optimal solutions are found.

Alternative R3

Solution procedure of Alternative 3 is the same as Alternatives R1 and R2 except for step 7, which is described below in two parts:

7a Run external Pre-ICG: using optimal pumping rate in the current (n th) cycle and heads in the former (n-1 th) cycle HFC^{n-1}, run Pre-ICG to estimate heads which are necessary to recompute the transmissivities (T) and conductances (CR and CC) of the unconfined aquifer for the next (n+1 th) cycle optimization.

7b Replace: optimal solutions of heads and variables in the former cycle are replaced with heads resulted from Pre-ICG and optimal solutions in the current (n th) cycle.

In summary, solution procedures and formulas in all the management models are shown in Table 2. The embedding method models do not use external programs such as the ICG. On the other hand, R1 and R2 use ICG only, and R3 uses both the ICG and the Pre-ICG. Among the response matrix models, R3 is the best model because it needs the least memory. The use of the Pre-ICG enables this model to only compute heads of interest.

Model Application

The sample problem is addressed for a hypothetical three-layer aquifer system using all six alternatives. The aquifer system has the following complex characteristics: (1) multilayer, (2) unconfined and confined aquifers, and (3) Type 3 external flow.

Hypothetical, Three-Layer Aquifer System

Consider the hypothetical three-layer aquifer system of Figure 4 (McDonald and Harbaugh, 1988). The upper layer is unconfined, the middle and lower layers are confined, separated from each other by aquitards. The aquifer is square measuring 75,000 ft on a side, and is discretized into 625 cells (3 layers x 15 rows x 15 columns). Flow within the aquitards is not simulated, but vertical flow between the layers is computed using vertical conductances. Flow into the system is through infiltration from precipitation. Flow leaves the system via six pumping wells, drains, and the sea, represented by a constant head boundary. Initial heads in Layer 1 range from zero at a constant boundary to 178.90 ft at both corners furthest from the sea.

Description of Scenario

The problem objective is to maximize total sustainable (steady-state) groundwater pumping subject to hydraulic constraints. Six pumping cells are located in the lowest layer. Upper and lower bounds on pumping rates are 16 cfs and 4 cfs, respectively. The lower bound on head at the pumping cells is 30 ft above sea level. To prevent salt water intrusion from the sea, the lower bound on flow across the constant head boundary (q^c) is set to 0.0. (Since inflow is negative and outflow is positive, this prevents inflow).

Model Formulation

The embedding models are formulated as shown in Table 3. Response matrix model formulations are described below (Table 4):

Alternative R1. The embedded flow equation (Equation 2) is applied to all cells of the upper layer, 15 constant boundary cells on the west side of the middle layer, and six pumping cells (Figure 5(a)). Since Equation 2 simulates vertical flow between the upper and middle layers, superposition (Equation 13) is used in all cells of the middle layer and in 15 cells on the west side of the lower layer. Also, heads in cells surrounding pumping cells are computed with Equation 13.

Alternative R2. The embedded flow equation (Equation 2) is applied to constant head cells of the upper and middle layers (Figure 5(b)). Heads of cells next to the constant head cells in the middle layer, (2,1,2) to (2,15,2), and all heads of the upper layer are calculated using Equation 13.

Alternative R3. Heads of the unconfined aquifer needed to estimate transmissivity in the next cycle are estimated using the Pre-ICG. Only heads in cells containing external flows (gp, q^d, and q^c) are estimated with Equation 13 (Figure 5 (c)).

Results

Initially assumed heads are 0.0 ft in all cells. This initial guess is intentionally chosen to be far from the optimal head to rigorously test the models' ability to always reach the same optimal solution. For E1 and E2, the optimization continues cyclically until the largest absolute difference between heads for two consecutive cycles is less than 0.001 ft. This requires six cycles. Response matrix model RM is also cycled six times. The resulting optimal aquifer water budgets are summarized in Table 5. The optimal potentiometric head in Layer 3 is shown in Figure 6.

The fully nonlinear model (E3) calculates the same solution as the other models, even when radically different initial guesses are chosen. Global optimality seems to be obtained.

Computational Accuracy

Because E3 does not use any linearization before beginning the solution, it solves the nonlinear flow system most accurately of all the models. E1 and E2 achieve the same optimal results as E3 by cycling. The final optimal solutions in the response matrix models also are virtually identical to those of E3. However, the computational accuracy of a response matrix model depends on how appropriately the influence coefficients are generated with external simulation models. In the sample problem, 1 cfs is used as a unit pumping and the following SIP parameters are specified: (1) the error criteria: 0.0001 ft, (2) the acceleration parameter: 1.0, (3) the maximum number of iterations: 200, (4) the seed: 0.001, (5) the number of iteration parameters 100, and (6) the head change criteria: 1.0. The ICG needs about 30 iterations to generate a set of influence coefficients for one unit pumping. The number of significant figures also affects the accuracy. To obtain the optimal values acceptably close to those of E3, the influence coefficients have four digits after the decimal point (i.e., 2.2345 (ft s/ft^3)).

The total absolute difference between heads for two consecutive cycles (TDHC) was 0.186 ft for 494 heads in R1, 0.082 ft for 276 heads in R2, and 0.005 for 89 heads in R3 (Table 6). Other combinations of SIP parameters and unit pumping or use of other solvers may converge more quickly and yield more accurate results than those obtained here. However, searching for the best combination of SIP parameters and unit pumping involves trial and error. In the sample problem, there are no significant differences of the computational efficiency among the response matrix models. Generally, R1 might be more accurate than R2 and R3 because heads of the unconfined aquifer are estimated using the flow equation (Equation 2). Since R3 uses two external simulation models, its errors in computing heads might be greater, unless its unit pumping and SIP parameters were well chosen.

Computational Efficiency

Because MINOS itself has no fixed limit on the size of a problem, a limiting factor is the amount of main storage available on a particular machine and CPU time which is shared for a decision-maker (Brooke et al., 1988). Therefore, it is important to know a priori the size of an optimization scheme required to implement a particular modelling approach for a specific aquifer problem.

The number of equations, variables, and nonzero elements indicates the size of the optimization model. A coefficient related to a linear term is a linear, nonzero element (otherwise, a nonlinear, nonzero element). The number of equations and variables equals the number of rows and columns in the solved matrix, respectively. However, unless most cells are pumping cells and locations of head constraint, most of the matrices are sparse. In fact, most elements in the matrices are zero. To avoid occupying main storage with such a large number of zeros, GAMS/MINOS uses one large array to store only nonzero elements in main storage (Brooke et al., 1988). If the nonlinear formula is involved, additional memory is required. The number of nonlinear elements shows the degree of nonlinearity.

The number of equations, variables, and nonzero elements can be predicted by counting the number of constraints and the number of variables and coefficients in those constraints. For example, in E2, using the embedding method, out of 685 equations, there are 675 flow equations (Equation 2), because each cell contains its own flow equation. The remaining 10 equations include 9 drain discharge equations (Equation 7a), and 1 objective function (Equation 1). Of 721 variables, there are 675 heads (H), 9 drain discharge (q^d), 30 fluxes (q^c) at constant head cells, 6 pumping (gp), and 1 objective value (obj). The total number of nonzero elements equals 4,165, including 4,095 hydraulic conductances (CR, CC, and CV), 30 coefficients for q^c's, 9 for q^d's, 6 for gp's in the flow equation (Equation 2), 9 linear and 9 nonlinear nonzero elements in the drain discharge equation (Equation

7b: DNLP formula) and 7 in the objective function (Equation 1).

In R2, a specific cell contains either (1) the flow equation (Equation 2) or (2) the superposition (Equation 13). Of the 286 equations, 30 (2x15) are Equation 2, 246 (1x15x14+2X15x1+6) are Equation 13, and 9 are drain discharge equations, and 1 is a objective function. Out of 1,929 nonzero elements, there are 7 (1+Ngp, Ngp: a number of pumping wells) in the objective function, 161 hydraulic conductances, and 30 coefficients for q^c's in the flow equation (Equation 2), 9 for q^d's in the drain discharge equation (Equation 6a), and (1+Ngp) x (a number of cells with Equation 13: 1x15x14+2x15X1+6=246) = 1,722.

A program for estimating the number of equations, variables, and nonzero elements, even for irregular shape aquifers, is developed. This program reads data files and counts those numbers for all six alternative models using several kinds of indicators and geohydrological parameters.

Table 6 compares alternative models with respect to computational resource requirement. This requirement includes the number of equations, variables, and nonzero elements, required memory, consumed CPU time, and cycles to convergence. Some numbers will not change even if the model is run on different machines. On the other hand, required memory and CPU time will vary depending on the machine. We used a VAX 5420. The required CPU time is the total CPU time for six cycles including the time for generating influence coefficients.

In overview, E3 needs the most memory because of its nonlinearity. R3 needs the least memory because it uses superposition and does not compute heads not needing constraint. On the other hand, E3 needs the least total CPU time because it avoids cycles. R3 needs the most CPU time because it cycles and uses two external FORTRAN programs.

Prediction of Model Size

In the sample problem having six pumping cells, the response matrix models need less memory than the embedding method models. However, this is not always the result. Memory requirements are situation dependent and can be predicted based on the number of nonzero elements required for the models (Peralta et al., 1991b). The number of nonzero elements is very dependent upon the number of pumping cells and cells requiring head constraint.

In this case, different situations are considered by increasing the number of pumping cells. In comparison, equations for estimating nonzero elements by increasing the number of pumping cells for the hypothetical area system are shown in Table 6.

In the embedding models, every cell contains the flow equation. Adding a pumping variable to an existing cell adds two linear nonzero elements, one in the flow equation and one in the objective function.

In the response matrix models, required heads, except for

VHCc in R1, R2, and R3 and VHCu in R3, are calculated by summation using influence coefficients (Equation 13). In general, nonzero elements are added according to an arithmetic series:

$$INC_{nz} = \frac{1}{2} NP_t \{N_h + (NP_t - 1) d\} \quad (14)$$

where

INC_{nz} increase in number of nonzero elements.

NP_t total number of pumping well cells.

d 2 for R2 and R3, 0 to 8 for R1 depending on the location of a pumping cell.

N_h number of cells which are VHCf, VHCc, VHCs, VHCb, VHCo, and VHCu.

This increment can be reduced somewhat if a pumping cell is also a VHCf, VHCc, VHCu, or VHCs cell. If pumping cells are installed in the confined aquifer of this hypothetical area, R1 needs the least memory if the problem has 1 to 41 pumping cells. However, in the later case, the ICG should have to be rerun 41 times. On the other hand, E1 needs the least memory if there are more than 42 pumping cells.

Summary and Conclusions

Alternative steady-state groundwater simulation/optimization models for a multilayer, nonlinear, aquifer system are presented. The models are demonstrated for a rectangular, hypothetical, unconfined/confined aquifer system. The models' objective is to maximize sustained-yield pumping. The constraints include a steady-state, quasi-three-dimensional flow equation and a drain discharge equation. The variables are heads, pumping rate, flux across constant head boundary, and drain discharge. The models are compared with regard to computational accuracy and efficiency. Conclusions are:

1. The E3 fully nonlinear embedding model can compute a correct optimal pumping strategy for an unconfined aquifer without recomputing transmissivities. All other embedding and response matrix models require cycling to recompute transmissivity. The model describes the nonlinear flow system by expressing transmissivities of the unconfined aquifer as a function of heads.
2. The E1 (fully linear) and E2 (nonlinear except for transmissivity) embedding models use cycling to achieve the same solution as the E3 model. These require more solution time but less computer memory.
3. The R1, R2, and R3 models use the principle of superposition instead of the embedding approach. These models can handle external flows via nonsmooth functions as well as transmissivity in an unconfined aquifer. Normal response matrix models cannot solve such nonlinear flow systems because the above terms are not represented by linear equations. This difficulty is overcome by using cycling and linear influence coefficients generated by a modified McDonald and Harbaugh model (MODFLOW). The accuracy of the optimal solutions depends on how accurately influence coefficients can be computed using

the external simulation model.

4. For the tested scenario, the fully nonlinear model (E3) computes the same optimal solution as the other models. It suggested that global optimality is obtained. The tested aquifer system is complex and nonlinear. System components include (1) an unconfined layer where transmissivity is a function of head, (2) drain discharge described by a nonsmooth function, and (3) three-layer system (675 cells).

5. In the sample problem containing only six pumping cells, the response matrix model (RM) requires less memory than the embedding models. However, if many heads and external flows must be constrained and many potential pumping cells exist, the embedding models are preferred to the response matrix models because of computational efficiency and the ease of obtaining an accurate solution. For the tested system, if more than 42 cells (about 10% of all cells) have pumping potential decision variables, the E1 fully nonlinear embedding model needs the least computer memory. Otherwise, the response matrix model (R3) requires the least memory.

6. In overview, if there is enough available computer memory, the E3 fully nonlinear model is preferred to other models because it can directly achieve the optimal solution of the nonlinear flow system. However, it always needs more memory than the other embedding models. If there is not enough memory, the E1 fully linear

embedding model or the R3 response matrix model needs the least memory.

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Table 1. Equations Used for Different Types of Cells

	Models		
Type of cells	R1	R2	R3
CHC	Eq.2	Eq.2	(Eq.2)[a]
VHCc	Eq.13	Eq.13	(Eq.13)[a]
VHCf	Eq.2	Eq.13	(Eq.13)[a]
VHCs	Eq.13	-	-
VHCb	Eq.13	Eq.13	Eq.13
VHCo	Eq.13	Eq.13	(Eq.13)[a]
VHCu	Eq.2	Eq.13	(Eq.13)[a]

[a]indicates the equation is used if the system has that kind of cell.

Table 2. Summary of Solving Procedures and Formulas for Alternative Models

Original Eq. /Models	Transmissivity		External flows	
	Confined	Unconfined	Type 2	Type 3
Original Eq.	Constant	Nonlinear	Linear	Nonsmooth
E1	Constant	LP&Cycle[b]	LP[a]	LP&Cycle[b]
E2	Constant	LP&Cycle[b]	LP[a]	DNLP[d]
E3	Constant	NLP[c]	LP[a]	DNLP[d]
R1,R2,and R3	Constant	LP&Cycle[b]	LP[a]	LP&Cycle[b]

[a]LP means a linear equation.
[b]LP&Cycle means a linear equation but it is linearized in the former step and needs cycling to address the nonlinearity of the original equation.
[c]NLP means a nonlinear equation.
[d]DNLP means a equation for nonlinear programming with discontinuous derivatives.

Table 3. Embedding Models

	Model components	Models E1	E2	E3
1.	Objective function	Eq.1 (LP)	Eq.1 (LP)	Eq.1 (LP)
2.	Constraints			
	Flow equation	Eq.2 (LP)	Eq.2 (NLP)	Eq.2 (NLP)
	Hydraulic conductances: CC and CR	(C[a])	(C[a])	Eqs.3a,3b (NLP)
	Drain discharge	Eq.5a (LP)	Eq.5b (DNLP)	Eq.5b (DNLP)
3.	Bounds			
	Head of the upper layer		$H \geq -150$ ft	
	Head at the pumping cell		$H \geq 30$ ft	
	Pumping rate		4 cfs $\leq gp \leq$ 16 cfs	
	Flux across constant boundary	$q^c \geq 0.0$	$q^c \geq 0.0$	$q^c \geq 0.0$
	Discharge from drain[b]	-	$q^d \geq 0.0$	$q^d \geq 0.0$
4.	Variable declaration			
	Positive		gp	gp
	Default (free)		h, q^d, q^c	h, q^d, q^c, CC, CR
	Free		obj	obj[c]
5.	MINOS solver	LP	DNLP	DNLP
6.	Cyclic Procedure	Yes	Yes	No

[a]C means constant in a cycle.
[b]When the DNLP solver is used, appropriate bounds should be specified on every variable.

Table 4. Response Matrix Models

	Model components	R1	R2	R3
		Models		
A.	Pre-ICG	No	No	Yes
B.	ICG	Yes	Yes	Yes
C.	Management model			
1.	Objective function	Eq.1 (LP)	Eq.1 (LP)	Eq.1 (LP)
2.	Constraints			
	Flow equation Eq.2 (LP)/ Summation for head Eq.13 (LP)			
	Hydraulic conductances	(C)	(C)	(C)
	for CHC	Eq. 2	Eq. 2	Eq. 2
	for VHCc	Eq.13	Eq.13	Eq.13
	for VHCf	Eq. 2	Eq.13	Eq.13
	for VHCs	Eq.13	Eq.13	Eq.13
	for VHCu	Eq. 2	Eq.13	-
	Drain discharge	Eq.5a (LP)	Eq.5a (LP)	Eq.5a (LP)
3.	Bounds			
	Head of the upper layer		$H \geq -150$ ft	
	Head at the pumping cell of the lower layer		$H \geq 30$ ft	
	Pumping rate		4 cfs $\leq gp \leq$ 16 cfs	
	Flux across constant head boundary	$q^c \geq 0.0$	$q^c \geq 0.0$	$q^c \geq 0.0$
4.	Variable declaration			
	Positive		gp	
	Default (free)		h, q^d, q^c	
	Free		obj	
5.	MINOS solver	LP	LP	LP
6.	Cyclic Procedure	Yes	Yes	Yes

Table 5. Computed Steady-State Water Budgets of the Aquifer

	Recharge/ Discharge (cfs)	Models			
		E1,E2,E3	R1	R2	R3
A.	Recharge to the aquifer	157.500	157.500	157.500	157.500
	Precipitation	157.500	157.500	157.500	157.500
	From the sea	0.000	0.000	0.000	0.000
B.	Discharge from the aquifer	157.500	157.502	157.501	157.501
	Pumping wells	69.030	69.032	69.032	69.031
	Drain	36.745	36.745	36.744	36.745
	To the sea	51.725	51.725	51.725	51.725
C.	Discrepancy (A-B)	0.000	-0.002	-0.001	-0.001

Table 6. Summary of Computational Statistics

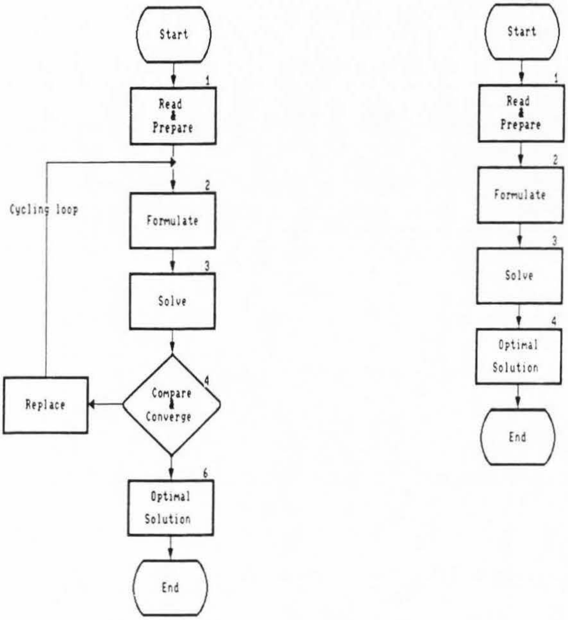
Item	Models					
	E1	E2	E3	R1	R2	R3
A. Number of nonzero elements	4158	4165	7585	3211	1929	622
linear	4158	4156	4606	3211	1929	622
nonlinear	0	9	2979	0	0	0
B. Number of equations	685	685	1330	504	286	99
C. Number of variables	721	721	1396	540	322	135
D. Memory (Mbytes)	0.40	0.46	1.31	0.29	0.19	0.06
E. Cycles	6	6	1	6	6	6
F. Total CPU time (min:sec)	3:05	3:50	2:41	5:34	5:01	6:15
G. Convergence in the sixth cycle						
LDHC[a] (ft)	less than 0.001		-	0.005	0.003	0.001
TDHC[b] (ft)	0.060	0.013	-	0.186	0.082	0.005
H. Largest head difference between E3 and other models	less than 0.001		-	0.003	0.003	0.002

[a]LDHC is the largest absolute difference between heads for two consecutive cycles.
[b]TDHC is the total absolute difference between heads for two consecutive cycles. E1, E2, and E3 estimate heads at 625 cells. R1, R2, and R3 estimate heads at 493 cells, 246 cells, and 88 cells, respectively.

TABLE 7. Equations for Estimating Number of Nonzero Elements

Alternatives	NZ_t = NZ_0 + increments
Embedding Method models	
E1	$4146 + 2NP_t$
E2	$4153 + 2NP_t$
E3	$7573 + 2Np_t$
Response Matrix Approach models	
R1	$1652+0.5NP_t\{482+(NP_t-1)a\}$
R2	$441+0.5NP_t\{486+(NP_t-1)2\}$
R3	$254+0.5NP_t\{112+(NP_t-1)2\}$

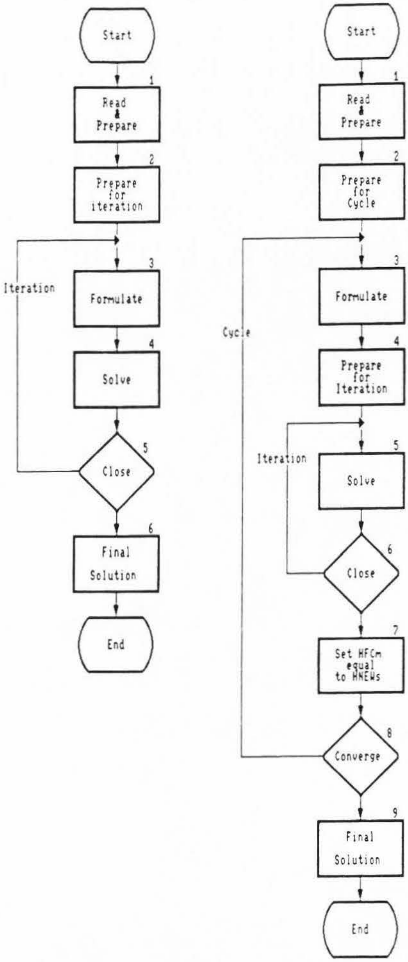
NZ_t = total number of nonzero elements.
NP_t = total number of pumping well cells.
NZ_o = number of nonzero elements with no pumping cells.
a in R1 = 2 to 8 depending on the location of a pumping cell.



(a) Alternative E1&E2

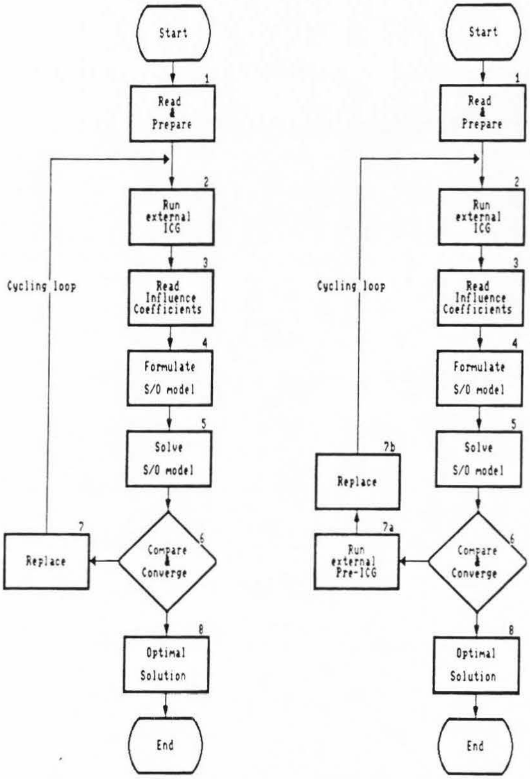
(b) Alternative E3

Fig. 1. Flow charts of solving procedure for the embedding models.



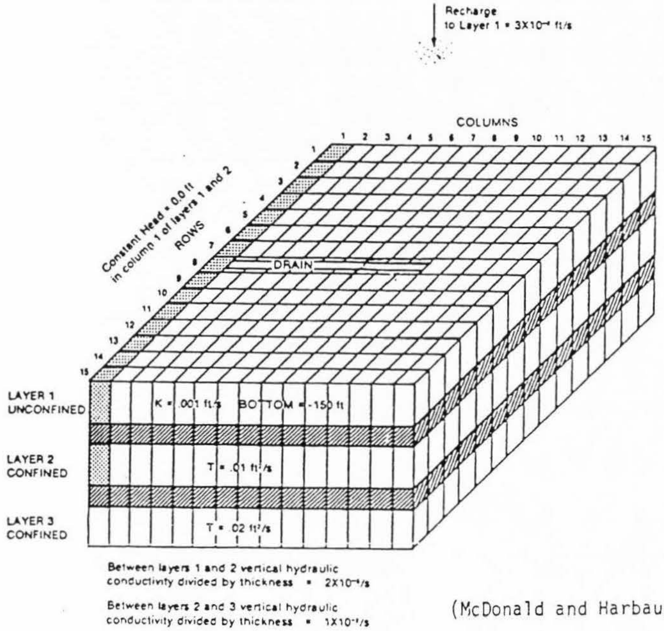
(a) McDonald & Harbaugh Model (b) Modified McDonald & Harbaugh Model

Fig. 2 Summary of solution procedures for the original and modified McDonald and Harbaugh models.



(a) Alternative R1&R2 (b) Alternative R3

Fig. 3 Flow charts of solution procedures for the response matrix models.



(McDonald and Harbaugh, 1984)

Fig. 4. Hypothetical three-layer aquifer system.

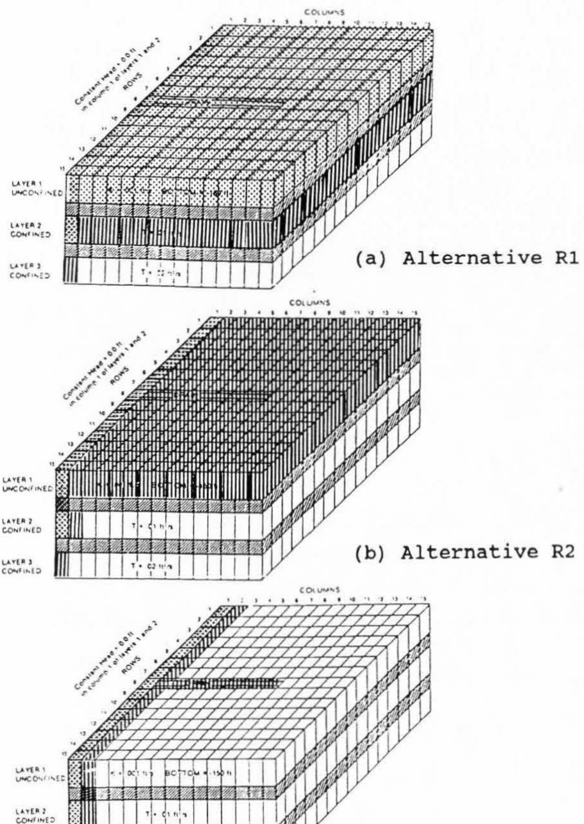
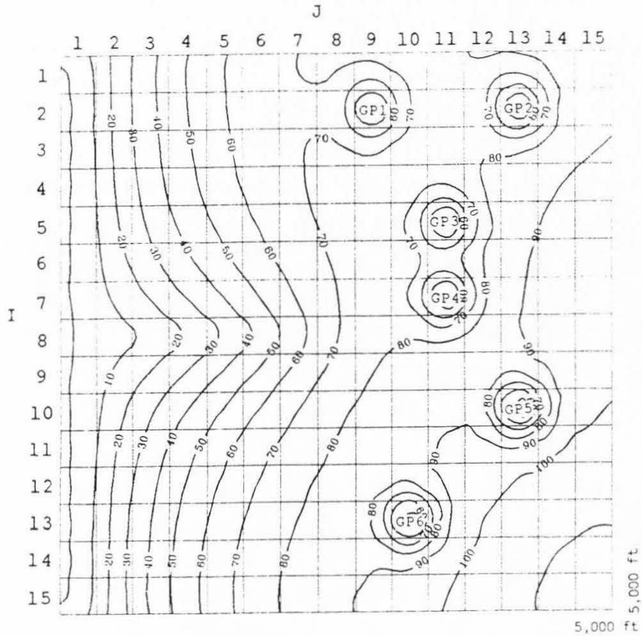


Fig. 5. Head computation for the response matrix models.



LEGEND

Variable head cell

Variable head cell with pumping

Pumping well	Discharge (cfs)
GP1	9.396
GP2	11.294
GP3	10.368
GP4	10.868
GP5	13.842
GP6	13.267

Fig. 6. Potentiometric heads in layer 3 (the lower layer).

CHAPTER III

PERENNIAL GROUNDWATER YIELD PLANNING FOR THE EAST SHORE AREA, UTAH

Computer models are developed for computing optimal perennial groundwater withdrawal strategies for the East Shore Area bordering the Great Salt Lake in Utah. The underlying aquifer has confined or unconfined layers. Both embedding and response matrix approaches are tested and compared. Historically, it has been difficult to incorporate simulation of an unconfined aquifer and many external flow equations described by nonsmooth functions within linear programming models. The presented response matrix model, which normally assumes system linearity, overcomes this difficulty by using cycling and influence coefficients generated with a modified McDonald and Harbaugh model. In this groundwater flow simulation model, the above nonlinear terms are treated linearly. The embedding model contains quasi-three-dimensional finite-difference forms of the groundwater flow equation as constraints. To achieve a stable optimal solution, the completely linearized formulation is cyclically optimized. The embedding model is preferred in this study because of its flexibility and ability to handle more linear and nonlinear geohydrological variables for a specified amount of memory. Using the embedding model, optimal, spatially distributed, sustainable, annual groundwater pumping rates are computed for alternative future scenarios. Strategy results are then verified using external steady-state and transient simulation. This study demonstrates applicability of the embedding approach for optimizing perennial-yield planning of large, complex aquifers.

INTRODUCTION

Long-term planning and management decisions can be facilitated by using combined simulation and optimization models which optimize steady (sustainable) groundwater extraction rates. Such regional groundwater planning models are constructed for the East Shore Area of Utah. There, the water demand for municipal and industrial use (M&I) is increasing due to urbanization. Increased groundwater extraction will decrease flow from flowing (artesian) wells.

This study started from applying a linear version of the USUGWM, developed by Gharbi et al.[10], to the East Shore Area aquifer system (three-layer, 4,880 cells). The USUGWM is the first embedding model to successfully optimize groundwater pumping for a large, complex, and nonlinear system. When the linear USUGWM is applied to a nonlinear system, heads known from the previous cycle are used to compute transmissivity and to select the linear segment of a nonsmooth function. The model is cyclically optimized until the values of variables do not change with the cycles. However, since the discretized system of this study area is extremely large and contains around 2,000 nonsmooth functions, the initial embedding model faced the following problems. The model contains about 40,000 nonzero elements and 12,000 single equations and variables. Using the previous version of MINOS on the VAX 6250, it took around 30 cycles and totaled around 12 hr CPU time to perform one optimization on the average. In cells containing nonsmooth functions, the bounds on head in the current cycle are limited within those in the previous cycle. In this process, the solutions are sometimes declared to be infeasible during cycles even if the feasible solutions exist.

Because the embedding model always needs a specific amount of memory, the response matrix model can be an alternative. However, it is difficult to satisfy the system linearity while accurately representing the above nonlinear problems.

The objectives of this study are: (1) to improve the modelling approach originally presented in the USUGWM to directly achieve an optimal solution without many cycles, (2) to develop the response matrix model to be suitable for nonlinear flow systems containing nonsmooth functions as well as transmissivity in an unconfined aquifer, and (3) to apply the appropriate model to develop perennial-yield pumping strategies for the study area.

Three management scenarios and their variations are implemented. After applying and comparing both the embedding and response matrix approaches for one scenario, the embedding approach was selected for one subsequent application. The major reason was its greater ability to handle numerous external flows as variables in the optimization scheme. Perennial-yield pumping strategies are computed for alternative future scenarios to demonstrate the flexible abilities of the embedding model. This model can help future water resource planning for the East Shore Area.

RELEVANT RESEARCH

A common management goal in arid and semi-arid regions is to fully utilize water resources to produce economic and social benefits. A groundwater management plan should satisfy specified objectives while considering the physical constraints of the aquifer system as well as legal and economic constraints. For the last two decades, groundwater development and conservation problems have been increasingly addressed using combined simulation and optimization (S/O) models. These combined models predict the behavior of a given aquifer and determine the best management strategy for the specified objectives and constraints.

Previous researchers have tackled a variety of groundwater management problems using several techniques. In general, most flow management models assumed system linearity. However, most real aquifer systems are complex and have nonlinear flow processes. Thus, there exists a need for an approach which can conveniently and accurately handle the common, nonlinear flows. Published research most relevant for this effort is cited below.

S/O models are frequently classified as using either the embedding approach or the response matrix approach, based on how groundwater head response to hydraulic stress is simulated in the model (see Ref. 12). The embedding approach incorporates finite-difference or finite-element approximations of the groundwater flow equation directly within the model as constraints. This approach provides considerable information, such as optimal potentiometric head and pumping rate in each cell simultaneously for the whole area and for all time steps. Because of the numerical difficulties with optimization algorithms resulting from the large dimensionality[12,30,31], the embedding approach was generally used for small scale, steady-state models. However, it has been more recently applied to larger scale problems. Cantiller et al.[5] used the embedding approach to develop a strategy for the conjunctive use of surface and groundwater for 13,000 square miles of the Mississippi alluvial, one-layer, large-scale aquifer system with 1,595 cells.

Gharbi et al.[10] used the embedding approach in the USU Groundwater Management Model (USUGWM) dealing with the 1,086 cell, two-layer (unconfined/confined), large-scale aquifer system underlying the Salt Lake Valley of Utah. In order to solve nonlinearities of unconfined flow, evapotranspiration, and aquifer-stream interflow, a cycling procedure was used. Before cycling begins, nonlinear formulas are linearized or quasi-linearized. Then optimization is performed. Because the optimization model uses a linear surrogate to a nonlinear formula, the model needs to be solved repeatedly until the values of variables updated in each repetition converge. This procedure has been used for several groundwater management models (e.g., Danskin and Gorelick[7], Peralta and Killian[23], Tung[29], and Willis and Yeh[33]). In general, the steady-state embedding approach has been most useful for long-term perennial groundwater yield planning in an area where most cells contain pumping and many heads must be constrained. An alternative to the embedding approach is the response matrix approach, which is most commonly used for transient operational models. The response matrix approach uses superposition to compute heads and is appropriate for linear systems. Many researchers have used the response matrix approach for large-scale transient models. It does not require equations for all cells and time steps. It can calculate aquifer response at specified locations only. This reduces the need for computer memory. However, a preliminary simulation to generate influence coefficients using an external simulation model is necessary. Thus, any change in an aquifer parameter can require regenerating influence coefficients (see Refs. 12 and 24 for details). Influence coefficients are also termed discrete kernels[22,15], technological functions[2], algebraic technological functions[18], and response functions[32,29].

The Boussinesq equation for saturated groundwater flow is linear for a confined aquifer but is nonlinear for an unconfined aquifer in which a saturated thickness varies significantly with head. The principle of superposition through influence coefficients cannot be applied to such a nonlinear system without adaptive measures or assumptions. Several researchers (e.g., Maddock[19], Heidari[14], Illangasekare and Morel-Seytoux[16], Danskin and Gorelick[7], Willis and Yeh[33], and Elwell and Lall[9]) have addressed this problem while using the response matrix approach. In this study, a different approach using cycling is demonstrated for perennial-yield planning in the East Shore Area aquifer system. This approach addresses the nonlinearity of flows described by nonsmooth functions as well as that of unconfined flow.

"Perennial yield" is defined as the maximum quantity of water that can be continuously withdrawn from a groundwater basin without adverse effects[1]. A "perennial-yield pumping strategy" is a specific pattern of spatially distributed pumping that causes the evolution and maintenance of an appropriate potentiometric surface. Thus a perennial-yield pumping strategy assures a certain amount of water to the user over a long time period. Such a perennial-yield pumping strategy can be computed using a steady-state S/O model. Knapp and Feinerman[17] endorsed the usefulness of computing optimal steady-state solutions.

If steady pumping is implemented and maintained, the potentiometric head of the aquifer will reach a certain level and, once achieved, will be maintained forever (discounting seasonal and daily changes, and assuming other recharge and boundary conditions remain constant).

Based on the above review, none of the response matrix models explicitly address external flows described by nonsmooth functions such as evapotranspiration. Such flows are commonly assumed to be known (fixed) or their nonsmooth nature is ignored.

The discretized aquifer system of this study contains more cells than others reported in the literature. In this study, both the embedding and response matrix approaches are improved in their ability to address external flows described by nonsmooth functions.

THE STUDY AREA

The East Shore Area, located north of Salt Lake City, is bounded by the Wasatch Front to the East and the Great Salt Lake to the West (Fig. 1). It is about 40 miles long and 3 to 20 miles wide, covering about 450 square miles. The population of the East Shore Area has tripled with the growth of agriculture, industry, and business during the last 40 years[25]. That portion of the study area from Willard to Farmington is the northern part of the most densely populated area in Utah.

To meet the increasing water demand in the area, the Weber Basin Project was implemented in 1952. This project utilizes the streamflow of the Weber River and the Ogden River with six dams and reservoirs and about 67 miles of conveyance systems. The project was designed to supply a total of 212,800 acre-ft per year, 162,800 acre-ft for irrigation and 50,000 acre-ft for municipal and industrial (M&I) use. The Weber Basin Conservancy District (Weber Basin W.C.D.) has since supplied water to this area. Recently, the United States Bureau of Reclamation (USBR) and the Weber Basin W.C.D.[28] proposed that the 33,000 acre-ft per year of water stored in Willard Reservoir should be converted from irrigation to M&I use.

Groundwater has been utilized for M&I use, irrigation, stock, watering, and domestic purposes in the area. Irrigated agriculture is the main user of the water and is mainly supplied from the Weber River. About 70% of the M&I use of water is supplied by groundwater[26,27]. Due to the rapid urbanization in the area for the last 20 years, the demand for M&I water has increased markedly, but the demand for irrigation water has been relatively constant. This trend is expected to continue. Groundwater use in 1969 and 1988 is shown in Fig. 2.

The groundwater reservoir is a three-layer aquifer system. The upper layer is shallow and unconfined, the middle layer is partially unconfined, and the lower layer is deeply unconfined in the mountain side and confined near the Great Salt Lake. The generalized profile of the aquifer system in the East Shore Area is shown in Fig. 3. Along the mountain side, large pumping wells are utilized for municipal and industrial use[4]. Near the shore, the potentiometric heads of the middle and lower aquifers are above the ground surface. In addition, many flowing wells provide water for agriculture, wetlands, and biota.

Groundwater levels in the East Shore Area have declined for more than 40 years. The decline exceeds 50 ft in the vicinity of Hill Air Force Base due to the increasing withdrawal of groundwater (Fig. 4). There was no significant decline of water quality of the aquifer between prior to 1970 and after 1980. Groundwater in most of the area is suitable for any use. However, groundwater in some areas, where chloride concentration exceeds 250 mg/l, is not recommended for public supply use and cannot be extensively developed[6]. Another concern about potential groundwater quality deterioration by agricultural pesticide use in the area has been recently reported[8]. The contamination hazard results because of the proximity of the water table to the ground surface, soil permeability and composition, and chemicals.

Although a large amount of groundwater has been pumped near the mountains, water still moves upward through leakage from the underlying layers to the shallow and unconfined aquifer on the agricultural lands near the lake shore. Outflow from the aquifer into the Great Salt Lake still occurs[6].

The groundwater reservoir is expected to be able to contribute to the increasing demand for water in the East Shore Area. However, the following problems may result from improper groundwater management:

1. Pumping cost might increase or wells might become

inoperable due to declining water levels.

2. Some flowing wells might not produce the flow needed for agriculture, wetlands, and wildlife.
3. Conflict among water users might cause societal problems.
4. Salt or brackish water might intrude from the Great Salt Lake.
5. Pesticides and insecticides on agricultural lands might degrade groundwater quality.

To address the above concerns, a combined model will be used to develop groundwater strategies for the study area. In that process, several innovations will be presented.

AQUIFER SIMULATION

Governing flow equation

A quasi-three-dimensional groundwater flow equation[20,11] for the multilayer system can be written as

$$\frac{\partial}{\partial x}(T_{xx}\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(T_{yy}\frac{\partial h}{\partial y}) + VC_{l+1}\ (h_{l+1} - h_l) + VC_l\ (h_{l-1} - h_l) = W \tag{1}$$

where

T_{xx} transmissivity along x coordinate axis (L^2/T);

T_{yy} transmissivitiy along y coordinate axis (L^2/T);

h potentiometric head or water table (L);

W volumetric flux per unit area and represents external flow (L/T);

VC_{l+1} hydraulic conductance between the upper layer l+1 and the layer l (L^2/T);

VC_l hydraulic conductance between the layer l and the lower layer l-1 (L^2/T);

USGS simulation model

Clark et al.[6] applied the McDonald and Harbaugh (MODFLOW) model to that part of the East Shore Area aquifer system from one mile north of Centerville to one mile north of Willard. Using geohydrological data and historical water-level and pumping records, they performed a steady-state calibration for conditions in 1955. Then they performed a transient calibration from 1955 to 1985. The results include the spatial distribution of transmissivities, storage coefficients, and several kinds of hydraulic conductances. After verification of the simulation model, the predictive simulations were performed for 1985 to 2005. The normal recharge condition of 107,000 acre-ft or less-than-normal climatical condition of 100,000 acre-ft is assumed. By the year 2005, groundwater withdrawal rates are assumed to be twice the average of the 1980-1984 annual pumping from M&I wells of 23,400 acre-ft (a 25% increases each 5 years). Predicted are groundwater level declines of 35 ft and 50 ft in the pumping center near the Hill Air Force Base (Hill A.F.B.), assuming normal recharge conditions and less-than-normal recharge conditions, respectively. A decrease or a cessation of discharge from flowing wells was also predicted.

General description of the USGS model is summarized as follows:

Layered system. Consistent with the generalized profile of the aquifer (Fig. 3), the USGS model consists of three layers. Layer 1 represents the upper, shallow, unconfined aquifer. This layer involves quasi-three-dimensional saturated flow under water table conditions, discharge from drains and flowing wells, evapotranspiration, and upward inflow from the underlying aquifer to the Great Salt Lake. Transmissivity of Layer 1 is treated as a function of head.

Layer 2, the middle layer, is partially unconfined and includes the "Sunset aquifer." Layer 3 represents the lowest aquifer which is deeply unconfined near the mountain side and confined under the rest of the entire area. Most of the large pumping wells for M&I use penetrate the "Delta aquifer" which is a principle part of the lower layer. In Layers 2 and 3, these transmissivities are assumed to be constant, even in the unconfined zone (no data for the case of the aquifer is available). The quasi-three-dimensional saturated flow under pressure, constant recharge, and discharge from flowing and pumping wells are simulated. Flow within aquitards between the aquifers is not simulated, but vertical flow through the aquitards is simulated.

Model discretization. The discretizations and cell types for Layers 1, 2, and 3 are shown in Figures 4, 5, and 6, respectively. A block-centered, finite-difference cell with a size length varying from 0.5 mile to 1.0 mile is used. The grid consists of 36 columns and 67 rows. The smallest active cells, representing 0.25 square mile, are used in the pumping center near the Hill A.F.B. The largest active cells, containing 0.5 square mile, are primarily used in the Great Salt Lake. The number of each different type of cell is summarized in Table 1.

Boundary condition. The area is assumed to be surrounded by no-flow boundaries in every direction. On the west side, a general-head boundary is used to permit upward inflow from the underlying layers into the Great Salt Lake. It is assumed that this boundary condition will not change in the future.

Hydrogeological parameters. The distribution of hydrological parameters are determined based on the aquifer-test data. For example, transmissivity of Layer 3 ranges from less than 2,500 ft^2/day in the western part to 100,000 ft^2/day in the pumping center near Hill A.F.B.

EMBEDDING SIMULATION/OPTIMIZATION (S/O) MODEL: A MODIFIED VERSION of USUGWM

Model formulation

Most simply, the S/O model is formulated to maximize the perennial-yield groundwater pumping rate subject to the physical aquifer system. However, alternative management goals, involving political equity, tradeoffs between types of water users, and environmental protection, are also considered. Thus one additional objective function and several constraints are used. The model is written in the General Algebraic Modeling System (GAMS) language[3].

Optimization is performed with the MINOS[21] LP solver using an advanced simplex method.

Objective function. The objective function of the model is to maximize total groundwater extraction.

$$maximize \quad z = \sum_{o=1}^{N} gp_o \tag{2}$$

where

gp_o groundwater pumping in a cell o, (L^3/T);

N total number of cells with pumped wells.

Groundwater flow equation. The steady-state, finite-difference form of the quasi-three-dimensional groundwater flow equation (Eq. 1)[20] is contained directly as a constraint for every cell. Using the same form of equation permits validating the simulation abilities of the S/O model by using MODFLOW.

$$CR_{l,i,j+1/2}(h_{l,i,j+1}-h_{l,i,j})+CR_{l,i,j-1/2}(h_{l.i.j-1}-h_{l.i,j})$$
$$+CC_{l,i+1/2,j}(h_{l,i+1,j}-h_{l,i,j})+CC_{l,i-1/2,j}(h_{l,i-1,j}-h_{l,i,j})$$
$$+CV_{l+1/2,i,j}(h_{l+1,i,j}-h_{l,i,j})+CV_{l-1/2,i,j}(h_{l-1,i,j}-h_{l,i,j})$$
$$= \Sigma^{N}{}_{n=1}\, q^{*}{}_{i,j,k,n} \tag{3}$$

where

$$CR_{l,i,j+1/2}=2dx_j(T^j{}_{l,i,j}T_{jl,i,j+1})/(T^j{}_{l,i,j}dy_{i+1}+T^j{}_{l,i,j+1}dy_i)$$

$$CC_{l,i+1/2,j}=2dy_i(T^i{}_{l,i,j}T^i{}_{l,i+1,j})/(T^i{}_{l,i,j}dx_{j+1}+T^i{}_{l,i+1,j}dx_j)$$

$$CV_{l+1/2,i,j}=dx_jdy_i/\{(dz_l/2Kz_{l,i,j})+(dz_{l+1}/2Kz_{l+1,i,j})\}$$

$h_{l,i,j}$ potentiometric head, (L);

l,i,j layer, row, column indices of a finite different cell;

CR,CC hydraulic conductance (harmonic averages of transmissivities) along x,y axes, (L^2/T);

CV vertical conductance between the nodes, (L^2/T);

$T_{l,i,j}$ transmissivity of a cell, (L^2/T); Transmissivity of unconfined layer is a function of head (T=kh). Transmissivity of confined layers is constant.

d_x,d_y,d_z cell sizes in layer l, row i, and column j, (L);

$Kz_{l,i,j}$ vertical hydraulic conductivity, (L^2/T);

$q^*_{l,i,j,n}$ (n th) external flow term in a cell, (L^3/T).

As in MODFLOW, several external flows are involved in the model as constraints.

Known constant recharge (q^r). The 1970-1984 average annual recharge rate of 10,700 acre-ft (normal climatic condition) is applied in the recharge area along the Wasatch Front (Figures 5 and 6). This includes bedrock recharge, unsaturated seepage from the Weber and Ogden Rivers, main canal seepage, precipitation, and irrigation seepage.

Pumping and flowing wells. Based on USGS work[6], about 5,900 wells have been constructed in the East Shore Area, including those in the city of Bountiful. There are 200 large diameter pumping wells for industrial and municipal use, 1,200 small diameter pumping wells for domestic, stock, and irrigation use, and 4,500 flowing wells for mainly irrigation use. Of the 4,500 flowing wells, 1,200 flow continuously and 1,800 are controlled by a pump or a valve. In addition, about 800 wells have been plugged or unused until 1985. There are also 700 wells which have ceased to flow because of a decrease in artesian pressure. The total annual discharge from wells in the area averaged about 54,000 acre-feet for 1969-1984. Of the total discharge, 52% was extracted by large pumping wells, 41% was from continuous flowing wells, 3-6% from controlled flowing wells, and 2-4% from small diameter pumping wells.

Pumping wells (gp): The 1970-1984 average annual pumping rate of 23,400 acre-ft is considered via bounds in the S/O model. The existing pumping wells for M&I use are located at 61 cells in the middle and lower layers (Figures 5 and 6).

Flowing wells (q^f): To properly estimate the change in discharge from flowing wells on agricultural lands (Figures 5 and 6) and link it to the steady-state simulation and LP technique, discharge from the flowing wells is newly formulated as

$$q^f_{1,i,j} = \Gamma^f_{1,i,j}\,(h_{1,i,j}-h^{gs}_{1,i,j}) \quad \text{for } h_{1,i,j} \geq h^{gs}_{1,i,j}$$
$$= 0 \quad \text{for } h_{1,i,j} < h^{gs}_{1,i,j} \qquad (4)$$

where

Γ^f coefficient describing reduction in discharge rate of the flowing wells per 1 foot head decline, (L^2/T);

h^{gs} ground surface, (L).

Flow through general head boundary (q^g). Flow between the underlying aquifer and the Great Salt Lake is represented using a general-head boundary (Fig. 5).

$$q^g_{l,i,j} = \Gamma^g_{l,i,j} (h_{l,i,j} - h^{ls}_{l,i,j}) \qquad (5)$$

where

Γ^g hydraulic conductance between the aquifer and the general boundary head cell, (L^2/T);

h^{ls} water level of the Great Salt Lake, (L).

Evapotranspiration (q^e). Evapotranspiration on the agricultural or undeveloped lands of the upper layer (Fig. 5) is formulated as a function of the water table elevation.

$$\begin{aligned} q^e_{l,i,j} &= E_o\, dx_j dy_i && \text{for } h_{l,i,j} \geq h^s_{l,i,j} \\ &= E_o\, dx_j dy_i \{h_{l,i,j} - (h^s_{l,i,j} - d_{l,i,j})\}/d_{l,i,j} && \text{for } h^s_{l,i,j} - d_{l,i,j} \leq h_{l,i,j} < h^s_{l,i,j} \\ &= 0 && \text{for } h_{l,i,j} < h_{sl,i,j} - d_{l,i,j} \qquad (6) \end{aligned}$$

where

E^o potential evapotranspiration, (L/T);

h^s potentiometric surface elevation below which evapotranspiration decreases, (L);

d extinction depth, (L).

Drain discharge (q^d). There is considerable discharge from artificial and natural drains on the agricultural and undeveloped lands along the shore side (Fig. 5). This discharge is simulated as saturated flow using a function of the water table elevation.

$$q^{d}_{l,i,j} = \Gamma^{d}_{l,i,j}(h_{l,i,j}-B^{d}_{l,i,j}) \quad \text{for } h_{l,i,j} \geq B^{d}_{l,i,j}$$
$$= 0 \quad \text{for } h_{l,i,j} < B^{d}_{l,i,j} \qquad (7)$$

where

Γ^d hydraulic conductance between the aquifer and drains, (L^2/T);

B^d bottom elevation of the drains, (L).

Vertical flow reduction (q^{rd}). Eq. 3 overestimates the amount of vertical flow between layers when the lower layer becomes unconfined. In such cases, vertical flow must be reduced using Eq. 8. In this area, this correction (reduction) in flow only involves flow between the middle and lowest layers.

$$q^{rd}_{l,i,j} = -CV_{l,i,j}(E^{top}_{l+1,i,j}-h_{l,i,j}) \quad \text{for } h_{l+1,i,j}<E^{top}_{l+1,i,j}$$
$$= 0 \quad \text{for } h_{l+1,i,j} \geq E^{top}_{l+1,i,j} \qquad (8)$$
$$(q^{rd}_{l,i,j}=-q^{rd}_{l+1,i,j})$$

where

E^{top}_{l+1} elevation of the top of layer l+1, (L).

Bounds on variables. Bounds on pumping and head are described as

$$gp^{L}_{l,i,j} \leq gp_{l,i,j} \leq gp^{U}_{l,i,j} \qquad (9)$$
$$h^{L}_{l,i,j} \leq h_{l,i,j} \leq h^{U}_{l,i,j} \qquad (10)$$

where

L and U notation of upper and lower bounds.

Difficulties in using the fully linearized formulas

The steady-state finite-difference form of the quasi-three-dimensional groundwater flow equation (Eq. 3) for the East Shore area contains (1) nonlinearity in an unconfined aquifer, where transmissivity is not constant but is a function of head and (2) nonsmooth functions of head consisting of two or three linear segments--evapotranspiration (q^e), discharge from flowing wells (q^f), drain discharge (q^d), and vertical flow reduction due to desaturation (q^{rd}).

These terms cannot be solved with the LP technique directly. Following the procedure of USUGWM[10], the above terms are linearized first using known heads from the former cycle. Then, to reach the solution of the nonlinear system, the linearized model is rerun (cycled) until variable values do not change with the cycles.

A model for the East Shore Area can be formulated without making major changes to the USUGWM originally applied to the Salt Lake Valley[11]. Necessary changes include adding expressing for flowing artesian wells. In the original USUGWM, transmissivity is linearized in a cycle by substituting a known head (HFC) in the former cycle for an unknown head (H) in the current cycle. However, the large number of nonsmooth functions describing q^e, q^d, q^f, and q^{rd} in the East Shore Area make it difficult to achieve feasible solutions for each cycle. When the linearized formulas of nonsmooth functions in the original USUGWM are used, the following problems occur:

1. The feasible solution is declared to be infeasible if

initial guesses of head are far from the optimal heads.

2. If the problem is not infeasible, it takes many cycles to achieve the true optimal solution.

3. The model behaves as if multiple optimal solutions exist--some of which are significantly smaller in magnitude than others.

In the presented modified USUGWM, the formulas and solving procedure for nonsmooth functions are improved to address the above problems.

Comparison of the original and modified USUGWMs

The linearized formula and solving procedure of the original and modified USUGWM are compared below:

Linearized formula. For example, an original drain discharge equation is described as Eq. 7. In the model, discharge, i.e., groundwater pumping, is a positive value, and recharge is a negative value. Since q^d is external flow leaving from drains (discharge), q^d should be 0 for h < bottom elevation of drain. Otherwise, it should be positive (Fig. 8(a)).

In both the original and improved USUGWMs, the linear segment is selected based on head HFC^{n-1} known from the previous cycle. Drain discharge, q^d, is computed as

$$q^d_{l,i,j} = \Gamma^d_{l,i,j}(H^n_{l,i,j} - B^d_{l,i,j}) \quad \text{for } HFC^{n-1}_{l,i,j} > B^d_{l,i,j} \qquad (11a)$$

$$= 0 \quad \text{for } HFC^{n-1}_{l,i,j} \le B^d_{l,i,j} \qquad (11b)$$

where

HFC^{n-1} known head in the previous (n-1 th) cycle.

H^n unknown head in the current (n th) cycle.

As a result, q^d becomes either a simple linear equation or zero in each cycle. However, a major difference is in the bounds applied to H^n based on HFC^{n-1}. In the original USUGWM, the bounds limit H^n to the range (linear segment) it occurred in the former (n-1) cycle (Fig. 8(b)). In the modified USUGWM, H^n is either a free variable if $HFC^{n-1} > H^d$ or equals zero if $HFC^{n-1} < H^d$ (Fig. 8(c)). This permits MINOS the freedom to solve. By the end of cycling, all head below the drain bottom correctly have q^d's of zero. How this difference affects the solution procedure is described below.

Solution procedure. Assume variable head cells containing drains in a discretized aquifer system. Initial heads are above the drain bottoms while some optimal heads are below the drain bottoms.

The original USUGWM: Since the initial guesses of head are above the drain bottoms (Figures 8(a) and 8(d)), both the original and the improved models use Eq. 11a in the first cycle. However, if the drain discharge is declared as a positive variable (bounded to be nonnegative), then the solved problem here can be infeasible in some cases. (Because this positive declaration is akin to trying to force $q^d > 0.0$ or h > drain bottom at every cell with a drain, it might be infeasible). If the solution is feasible, the original model forces some heads to be at the elevation bottom in the first cycle (Fig. 9(b)), and the optimal solution in this case is smaller than the true optimal solution. In the next cycle, the heads fall below the drain bottoms because Eq. 11b is used for computation (Fig. 9(c)). Thus if the initial guess of head is not far from the optimal solution, meaning that the model is not expected to face the infeasibility mentioned above, the model can reach the true optimal solution after cycling. However, whenever heads fall below the drain bottoms, heads reach the drain bottoms first. Thus it takes many cycles to reach the true optimal solution.

The modified USUGWM: Drain discharge is allowed to be negative temporally during cycling, but it becomes either zero or a positive value as subsequent cycles converge. In the first cycle, some heads fall below the drain bottoms, and the drain discharge becomes negative (Fig. 9(e)). In this case, the optimal pumping is larger than the true optimal pumping because the model behaves as if recharge occurred from the drain. In the next cycle, q^d's are zero at these cells since Eq. 11b is used instead of Eq. 11a. Here, the negative values disappear (Fig. 9(f)). Thus the model can reach the true optimal solution faster without having the problems which occur in the original USUGWM.

RESPONSE MATRIX SIMULATION/OPTIMIZATION (S/O) MODEL

The principle of superposition cannot be used for unconfined aquifer systems without certain assumptions since the governing groundwater flow equation (Eq. 1) is nonlinear for such systems. Even if the aquifer system is confined or the saturated thickness is great enough that linearity can be assumed but if it contains significant external flows described by nonsmooth functions such as drain discharge, the assumption of linearity is also violated when head moves from one linear segment to another linear segment (Fig. 8(a)).

The basic idea for addressing these nonlinearities is the same as in the embedding model except that superposition rather than embedding is used to compute heads. To satisfy the assumption of linearity through convergence and to permit the application of the response matrix (superposition) approach to nonlinear systems, the following approach is used.

Generating influence coefficients

The McDonald and Harbaugh (MODFLOW) model can be used as the Influence Coefficient Generator (ICG) for the linear system, even if the system is multilayered, because vertical flow terms, described as $CV(h_{l+1,i,j}-h_{l,i,j}) + CV(h_{l-1,i,j}-h_{l,i,j})$, are linear. However, this model cannot be used directly as the ICG for the nonlinear system.

In MODFLOW, the nonlinearities described above are solved using heads known from the former (m-1 th) iteration. Here, we use the Strong Implicit Procedure (SIP) for solving a large system of simultaneous linear equations by iteration.

Transmissivity of the unconfined aquifer is linearized by using heads ($HNEW^{m-1}$) known from the former (m-1 th) iteration to compute hydraulic conductances CR, CC for the current (m th) iteration. As a result, CR and CC are assumed constants. Similarly, any external flow consisting of two or three linear segments is linearized based on heads ($HNEW^{m-1}$) known from the former (m-1) iteration.

$$q^d_{l,i,j} = \Gamma^d_{l,i,j}(HNEW_{l,i,j} - B^d_{l,i,j}) \quad \text{for } HNEW^{m-1}_{l,i,j} > B^d_{l,i,j} \qquad (12a)$$

$$= 0 \quad \text{for } HNEW^{m-1}_{l,i,j} < B^d_{l,i,j} \qquad (12b)$$

where

HNEW unknown head in the current iteration

Therefore, q^d is described as either a simple linear equation or zero in each iteration. Then, SIP solves the linear equation (Eq. 3). Many iterations are usually required to converge to a solution.

Since we are using MODFLOW to generate influence coefficients, we must emulate the above process for compatibility between the management model and MODFLOW. A cycle in the development of influence coefficients and computation of the optimal strategy will be similar to the effect of a single iteration in MODFLOW. The approach is to use the same assumptions in developing influence coefficients and in computing the optimal strategy. Some of the assumed equation segments of Type 3 external flows will be wrong. However, they will be corrected by cycling just as MODFLOW assumes and corrects these equations by iteration.

Construction of the ICG required three actions: First, the McDonald and Harbaugh model is modified with respect to transmissivity in the upper, unconfined aquifer, drain discharge, evapotranspiration, discharge from flowing wells, and vertical flow reduction. The "Pre-ICG" is designed to perform the steady-state simulation through solving the flow equation (Eq. 3) repeatedly. This equation is linearized in each cycle by substituting head known from the former cycle rather than from the former iteration as described above.

Second, the simulation ability of the Pre-ICG is verified by comparing the simulation results with those of the MODFLOW including a flowing well subroutine (Appendix A).

Third, the Pre-ICG is designed to compute two kinds of steady-state influence coefficients (Appendix B).

h^{um}_o unmanaged head describing average steady-state head response over a cell only to known constant stresses (q^r: bedrock recharge, precipitation, etc. and these stresses do not include current nonoptimal pumping) (L^3/T);

$\delta_{o,m}$ influence coefficient describing the average head response over a cell only to a unit stress in a pumping cell m, (L^3/T).

Model formulation

In the response matrix S/O model, the same objective function and bounds on pumping are used as the embedding model. However, bounds on head are set only at necessary cells, and the following superposition expression is used as constraints to compute heads at those cells.

$$h_o = h^{um}{}_o + \sum_{m=1}^{M} \delta_{o,m}\, q_m \qquad (13)$$

where

h_o average potentiometric head in cell, (L);

q_m stress of pumping in a cell *m*, (L^3/T).

PRELIMINARY APPLICATION SCENARIO TO RESPONSE MATRIX S/O MODEL

Objectives of this section are (1) to demonstrate how required memory can be reduced using the response matrix approach for some scenarios and (2) to compare the applicability of the embedding and response matrix models to the East Shore Area study. As shown in Table 2, both models are formulated to determine the maximum sustained yield from the 61 cells, which contain the existing M&I use pumping wells installed in the middle and lower layers. Flow charts in Fig. 10 compare the solution procedures. Both models are repeatedly optimized until variables do not change with the cycles. However, in the response matrix model, two external simulations (ICG and Pre-ICG) are involved in the cycle.

Bounds on variables

Bounds on pumping. The lower bound on pumping is the current withdrawal rate for all the existing pumping cells. For most cells, the upper bound on pumping is twice the current withdrawal rate. Exceptions are the 12 cells containing the Weber Basin W.C.D. and Hill A.F.B. wells. There, existing well capacities are the upper bounds on pumping (Appendix C).

Bounds on head in specific pumping cells. In the 12 cells containing the Weber Basin W.C.D. and Hill A.F.B. wells where large pumping has occurred, the maximum allowable drawdown is 20 ft below 1985 head.

Bounds on head of the unconfined aquifer. Heads in cells of the upper-shallow, unconfined aquifer are not allowed to fall below the base of the layer. In the embedding model, the bounds on head are easily set for all cells (1,270 cells) of the upper, unconfined aquifer since every cell contains the flow equation. Thus there is no increase of required memory resulting from setting bounds on variables.

In the response matrix model, it is impractical to set the bounds on head for 1,270 cells. Sixty-one pumping cells x 1,270 cells = 77,470 influence coefficients would result in a huge memory allocation. For this preliminary testing, it is assumed that if head in the cell where the saturated thickness in 1985 is the thinnest does not fall below the base of the aquifer layer, then heads in any other cells will not fall below the geological bottom. Thus only one head located at layer 1, row 19, column 25 (1,19,25) is computed with 61 influence coefficients (δ) and unmanaged head (h^{um}) and is bounded in the management model. Post-optimization simulation verifies that no other cells are completely dewatered either (although undesirable drawdowns might occur).

Computation of head with Pre-ICG

Cycling requires estimating heads in an unconfined aquifer and in cells containing nonsmooth functions (for q^e, q^d, q^f, and q^{rd}) as input for the ICG in the next cycle. In this preliminary test, heads only in 13 cells are computed in the management model using Eq. 13. The Pre-ICG computes other heads in the current cycle using heads in the former (n-1 th) cycle and optimal pumping rates in the current (n th) cycle.

Results from embedding and response matrix S/O models

Heads in 1985 are used as the initial guesses. Optimal pumping rates and computed heads from both models are almost identical. If more effort were made to identify a better combination of SIP parameters, the results between the models might be even closer. However, that would require more iterations of the ICG and more CPU time in generating influence coefficients. Table 3 compares computational resource required by both models. We used the VAX 5240. The response matrix model uses less than 6% of the memory required by the embedding model in every cycle. In terms of the required CPU time, the embedding model requires 103 minutes for the first cycle but only about 4 minutes after the second cycle. The response matrix model needs 8 to 13 minutes for every cycle, including running two external simulation models. Since both models need ten cycles to converge, the total CPU time is slightly less for the response matrix model. However, if any new bounds or constraints require new influence coefficients generation, then the response matrix model could need more total CPU time than the embedding model.

Selection of S/O model for subsequent optimizations

In this study area, existing pumping wells are located at 61 cells. Most commonly, lower bounds on head are proposed at pumping cells. This assumes that the maximum drawdown occurs at a pumping cell. If this assumption is used for scenarios considering only the existing pumping as in this preliminary scenario, the response matrix model looks better than the embedding model because it uses less memory despite the need for regenerating influence coefficients for any changes of bounds and constraints.

However, that approach might not be appropriate here. The maximum drawdown always occurs between wells near the mountains and the mountains in Layers 2 and 3 (Fig. 14). Furthermore, we cannot specify a location where the maximum drawdown might occur. Thus we propose tight lower bounds on head (maximum drawdown) in the entire city zone for subsequent management scenarios (discussed in the next section). In addition, we propose to permit pumping in many more cells. For this situation, the response matrix model is not practical. It would require too many simulations to generate influence coefficients. Also, too many influence coefficients would be needed in constraint equations. This results because this is a steady-state optimization, and most of the concern is about heads in confined layers. Pumping in one lowest layer all affects steady heads at most other middle and lowest layer cells. Thus, the memory requirement would be huge for an optimization. In the embedding model, such bounds can be easily set using the same amount of memory as in the model without the bounds.

In conclusion, the response matrix model is a viable alternative to the embedding model for steady-state optimizations if constraints and bounds on variables do not need to be specified to many locations. At this stage of the study, it was difficult to specify how many potential pumping cells and head constraints would be needed. Because of its flexibility and easy adaptability, the embedding model was selected for subsequent optimization.

USE OF EMBEDDING S/O MODEL FOR PERENNIAL-YIELD PUMPING STRATEGIES

The results of alternative future scenarios are compared. Due to the rapid urbanization in the area over the last 20 years, the demand for M&I water has increased markedly, but the demand for irrigation water, which is mainly obtained from the Weber River, has not increased much. Those trends are expected to continue. Common assumptions for all scenarios are: (1) it is more important to extract water for M&I use than to have flowing wells for agricultural use, and (2) it is desirable that optimal pumping not be less than current pumping in any cell.

The study area is divided among the 25 water entities of Davis, Weber, and Box Elder counties. These entities are a city or group thereof served by a single local public supplier or a wholesaler, Weber Basin W.C.D. (Fig. 11).

In overview, scenario 1 is the nonoptimal scenario. For the other scenarios, optimal sustainable annual groundwater pumping rates are computed using the modified version of the USUGWM. In scenario 2, the model maximizes the total sustainable pumping rate from the 61 cells containing wells currently pumping for M&I use. If existing wells cannot supply water of sufficient quantity and quality, one approach to meet the increasing water demand is to install new, large, pumping wells. The S/O model can help choose appropriate locations from many candidate pumping cells. In scenarios 3 and 4, this ability is demonstrated. Table 4 summarizes model formulations for the different scenarios. Appendix D shows computation results for these scenarios: (1) steady-state water budgets for the entire aquifer, and (2) the distribution of pumping and flowing discharge among water entities.

Bounds on pumping and head for management scenarios

The following bounds on head and pumping are considered for all management scenarios:

Bounds on head. To avoid or minimize problems resulting from unacceptable drawdowns of the middle and lower layers where flowing and pumping wells are installed, the lower bounds on head of those layers are set as

$$h^{city}_{2,i,j} \leq h^{city}_{2,i,j} \text{ in } 1985 - D^L \qquad (14a)$$

$$h^{city}_{3,i,j} \leq h^{city}_{3,i,j} \text{ in } 1985 - D^L \qquad (14b)$$

where

h^{city} heads at cells within the city zones (= the water entity limits) as shown in Fig. 11.

D^L maximum acceptable cell drawdown.

The lower bound on head in Layer 1 is the aquifer bottom.

$$h_{1,i,j} > \text{Bottom}_{1,i,j} \qquad (15)$$

Bounds on pumping. The lower bound on pumping is the current pumping rate for all existing wells. Upper bounds on pumping are usually based on well capacity or water requirements. In this model, for 12 cells containing Weber Basin W.C.D. and Hill A.F.B. wells, the well capacities are used as the upper bounds. These well capacities far exceed the current withdrawal rates. For other existing pumping wells, the upper bound is a multiple of the current pumping.

Scenario 1: nonoptimal scenario

The simulation option of the embedding method is used to predict the additional water-level declines that will ultimately result from continuing current withdrawals from flowing and pumping wells. It takes eight cycles for convergence (using 1985 heads as the initial guesses).

Scenario 2a: pumping from existing wells

In this scenario, the model maximizes total perennial-yield pumping in the 61 cells where pumping wells for M&I use currently exist. In most cells, except for the 12 cells that contain Weber Basin W.C.D and Hill A.F.B. wells, the upper bound on pumping is twice the current withdrawal rate. The maximum allowable drawdown in the entire city zone is 20 ft, so the lower bound on head is 20 ft below 1985 heads.

Computed steady-state water budgets. The total optimal pumping rate increases 50% to 48.4 cfs from current pumping (Fig. 12). The increase in pumping causes a decline of water levels in the upper unconfined aquifer and potentiometric heads in the middle and lower confined aquifer. This decline decreases the discharge from flowing wells and drains, upward inflow to the Great Salt Lake, and evapotranspiration. Their decreases in discharge are 25% and 12%, 6%, and 3% of the nonoptimal discharge, respectively.

Spatial distribution of pumping and flowing discharge. In Davis county, pumping increases in all water entities except for South Weber and totals 14.6 cfs, which is 90% of the regional pumping increase (Appendix D). On the other hand, in Weber county, pumping increases only 1.6 cfs in two water entities, which are West Weber and Roy. The total discharge of pumping and flowing wells decreases 3.3 cfs compared with the nonoptimal scenario. The decrease in the flowing discharge is greatest in Syracuse, West Point, and

West Weber.

Scenario 2b: effects by changing bounds on pumping and head

To analyze its effect on optimal pumping, the model is also run for different sets of lower and upper bounds on pumping and maximum allowable drawdown (Appendix E).

Upper bound on pumping. In most cells, except for those 12 cells that contain Weber Basin W.C.D. and Hill A.F.B. wells, the upper bound on pumping is varied: four, six, and ten times the current withdrawal rate. Other bounds are the same as in scenario 2a. In scenario 2a, the upper bound on pumping is twice the current pumping. By increasing the upper bound on pumping from twice to ten times the current pumping, the optimal sustainable pumping rate increases by 3.3 cfs to 51.7 cfs as shown in Table 5.

Lower bound on pumping. The lower bound on pumping is varied: 95%, 90% and, 80% of the current withdrawal rate for all existing pumping wells, while other bounds are the same as in scenario 2a. By releasing the lower bound on pumping from 100% of that in scenario 2a to 80% of the current pumping, the optimal sustainable pumping rate increases by 4.1 cfs to 52.5 cfs (Table 5).

Maximum allowable drawdown: The maximum allowable drawdown inside the city zone is varied: 15 ft, 25 ft, 30 ft, and 40 ft, while other bounds are the same as in scenario 2a. The problem is infeasible using 15 ft bound because heads near North Ogden fall below more than 15 ft simply to maintain the current pumping rate for all existing wells. When the lower bound on pumping is released to 70% of the current pumping rate for all existing cells, an optimal solution is found. In cases of 25 ft, 30 ft, and 40 ft, optimal sustainable pumping rates are 9.5 cfs, 13.1 cfs, and 19.4 cfs greater than that of scenario 2a, respectively (Table 5). The model is more sensitive to the increase of the maximum allowable drawdown than to the changes of the lower and upper bounds on pumping.

Scenario 2c: trade-off between pumping and flowing discharge

If pumping for M&I use increases in the urban area along the Wasatch Front mountains, then discharge from flowing wells on the agricultural lands will decrease. A conflict over water may occur between irrigation users and M&I users. There exists a tradeoff between pumping discharge for M&I use and flowing discharge for irrigation use. To consider the trade-off, the following constraint is added to the constraints of scenario 2a. The total discharge from the flowing wells for each water entity should meet or exceed a specified proportion of the discharge in the nonoptimal scenario.

$$\sum_{i=1}^{N^f} q^f \geq r \; [\sum_{i=1}^{N^f} unmanaged \; q^f] \qquad (16)$$

where

r parameter represents a fraction of total discharge of the nonoptimal scenario for each water entity.

q^f discharge from flowing wells in a cell, (L^3/T);

N^f total number of cells containing flowing wells for each water entity.

The model is run using various values of parameter (r). As the value of r decreases, the total optimal sustainable pumping rate increases, and the total discharge from the flowing wells decreases almost linearly (Fig. 13). This curve can be considered to be the pareto optimum between the objective of maximizing pumping and maximizing free flow from artesian wells.

Scenario 3a: pumping from proposed wells along irrigation conveyance system

If the results of implementing the strategy of scenario 2 are unsatisfactory, additional groundwater can be developed by installing new pumping wells along the existing water conveyance system. There are 17 main irrigation conveyance systems including that of the Weber Basin Project. Potential additional pumping cells exist in all water entities except for Centerville, which includes none of the 17 irrigation conveyance systems. In this scenario, candidate sites for new pumping wells are located in 75 cells in the lower aquifer along the main irrigation conveyance systems. These sites are advantageous in having relatively high pressure for distributing water for M&I use (due to their relatively higher elevations) and the ease with which pumping groundwater can be placed in the conveyance system. The objective function is to maximize total groundwater pumping from the existing and proposed wells (61+75=136 cells). Constraints and bounds on head and existing wells are the same as in scenario 2a--lower and upper bounds on pumping in new candidate cells are 0 and 1,000 gpm (1.114 cfs), respectively.

Computed steady-state water budgets. Total optimal pumping rate is 179% of the current pumping rate, while discharge from flowing wells, drain discharge, evapotranspiration, and upward inflow to the Great Salt Lake are 58 %, 85%, 95%, and 90% of the nonoptimal rates, respectively (Fig. 12). Discharge from flowing wells ceased at 245 out of the original 813 flowing well cells (Table 6). The area, where flowing wells cease to flow, expands from the mountain side where potentiometric heads of the lower and middle layers are originally close to the ground surface (Fig. 3).

Spatial distribution of pumping and flowing discharge. Regional optimal pumping is 9.2 cfs greater than that of scenario 2a. There is discharge in 24 new pumping cells (Table 7). The spatial distribution of pumping differs from scenario 2a (Appendix D). The increase in pumping concentrates in Syracuse, West Point, and West Weber. There, the aquifer is not intensively developed and new pumping cells line the Layton canal. The net increase of total pumping and flowing discharge is unequally distributed and increases in only five water entities (Table. 8).

Scenario 3b: assuring total discharge from wells

In this scenario, we assume that a reduction in water from flowing wells can be compensated for using water from newly installed pumping wells along the main canals in each water entity. While the objective function and bounds on head and pumping are the same as in scenario 3a, the following constraint is considered to address this scenario. The total supply of groundwater from either pumping wells or flowing wells for each water entity should meet or exceed that in the nonoptimal scenario (for all entities having current pumping or candidate pumping).

$$\sum_{i=1}^{N} (gp + q^f) \geq \sum_{i=1}^{N} nonoptimal\ (gp + q^f) \qquad (17)$$

The optimal sustainable groundwater pumping rate decreases 4.2 cfs from scenario 3a to 53.4 cfs, while the flowing well discharge increases 2.4 cfs to 23.1 cfs. By assuring the total discharge from both pumping and flowing wells, total discharge from wells for all water entities except for Centerville, in which no pumping cells exist, are more than zero as shown in Table 8. However, the spatial distribution of the increase in pumping is generally the same as in scenario 3a--concentrated in Syracuse, West Point, and West Weber.

Scenario 4a: pumping from proposed wells within water entities

In this scenario, an attempt is made to determine the potential for additional groundwater development at all cells of the lower layer inside city limits, with exception of the low development potential areas. The excluded areas are the low lands below 4,215 ft along the Great Salt Lake (lake level: 4,200 ft) and the area containing high TDS expanding from the east of Ogden to Plain City (Hansen Allen & Luce, Inc.[13]). We assume here that each water entity will have to develop the groundwater reservoir under its own area and meet its own water demand with groundwater as much as possible. The objective function is to maximize total groundwater pumping from the existing and proposed well sites (61+785=846 cells), while constraints and bounds on head and pumping wells are the same as in scenario 3a.

Drawdowns in 78 cells were 20 ft of the maximum drawdown. It is still impractical to use the response matrix model even if the tight bounds on head could be specified only for these cells. A huge memory allocation of 846 potential pumping cells x 78 cells = 65,988 influence coefficients would result. Furthermore, the ICG must rerun 846 times to generate influence coefficients for unit pumping.

Computed steady-state water budgets. The total optimal pumping rate increases to 205% of the current pumping rate while discharge from flowing wells, drain, evapotranspiration, upward inflow to the Great Salt Lake from the underlying aquifer decreases to 47%, 79%, 94%, and 82% of the nonoptimal rate, respectively (Fig. 12). The intrusion of salt water from the Great Salt Lake at 24 cells totals 0.052 cfs. Discharge to the lake at another 425 cells totals 17.7 cfs. No downward inflow from the Great Salt Lake is recognized in the other scenarios (except for scenario 2b2 and 2b3, shown in Appendix E, in which downward inflow totals 0.003 cfs).

Spatial distribution of pumping and flowing discharge. The pumping increase is mostly concentrated in newly proposed pumping cells. Of the 785 newly proposed pumping cells, the model chose to pump at 81 cells. These are distributed in the northwestern part of West Weber and along the shore of the Great Salt Lake in Davis county, such as in Syracuse, West Point, Kaysville, Farmington, and Centerville (Table 7).

Scenario 4b: preventing salt water intrusion

To prevent the intrusion of salt water from the Great Salt Lake, the following bound in all cells with general head boundary is added to the constraints in scenario 4a.

$$q^{g}_{1,i,j} \geq 0.0 \qquad (18)$$

The resulting tradeoff to prevent any lake water downflow to the aquifer is a 1.7 cfs decrease in regional pumping (Fig. 12). The spatial distribution of new pumping wells in West Weber differs from that of scenario 4a. The number of new pumping cells in West Weber decreases from 29 cells to 14 cells (Table 7). Thus there are 127 cells with nonzero pumping (61 existing and 66 new proposed pumping cells).

Scenario 4c: egalitarian goal

The total pumping of 67.9 cfs in scenario 4a indicates the physical development potential from the entire aquifer for the specified bounds on head and pumping. However, the pumping increases in only prespecified areas. Further changing the bounds on pumping and head will not permit much more regional change even if different sets of bounds on pumping and head are used for this scenario. Such a strategy cannot be adopted for economic and egalitarian reasons. In this scenario, an attempt to develop a more egalitarian pumping strategy is performed. If future excess in groundwater extractions is allocated to water entities in proportion to their area and the withdrawal must occur within their boundaries, then less sustainable pumping is possible.

This is accomplished by setting the following objective function and constraints; other constraints are the same as in scenario 4a. The objective function is to maximize a ratio (r) of increased pumping to an assumed upper limit on pumping.

$$\text{maximize } r \tag{19}$$

For each water entity, the ratio (rw) is constrained:

$$rw = \frac{AD}{ULDP} \tag{20}$$

where

AD = additional development (optimal-current) pumping

ULDP = upper limit of development potential pumping

= (areal size ratio of each water entity to the whole water entity limits) x (maximum additional sustained yield of the whole water entity limits)

This ratio (rw) should be the same for all water entities based on the egalitarian goal.

$$r = rw \tag{21}$$

The maximum additional total sustained yield is 33.7 cfs since total perennial-yield in scenario 4a is 65.9 cfs and the total of the current pumping rates is 32.2 cfs. Table 9 shows area, areal ratio, ULDP, and optimal additional development of pumping (AD) across water entities. The optimal ratio is 0.28. The ratio is low because withdrawal from all water entities of Weber county, except for West Weber, is restricted due to their drawdowns. If the maximum allowable drawdown for these areas can be relaxed, the ratio will be improved significantly.

Vertical water movement between layers

On the agricultural lands near the Great Salt Lake, the water table of the shallow and unconfined aquifer is lower than heads of the underlying layers allowing water to move upward through leakage (Fig. 3). In this condition, groundwater contaminants--pesticides and insecticides--remain in the shallow aquifer. However, the downward movement of low-quality groundwater of the shallow aquifer to the confined aquifer may occur by the large-scale withdrawal from the underlying confined aquifers.

Table 10 summarizes upward and downward movement of water between the upper and middle layers (Layers 1 & 2) and between the middle and lower layers (Layers 2 & 3). As additional groundwater development increases, downward flow from the middle layer to the lower layer increases significantly. In scenario 4a, which is the most developed case, the downward flow occurs in 227 cells of the 1,644 cells and totals 5.593 cfs. On the other hand, the downward movement from the upper layer to the middle layer--the deterioration of water quality being the main concern--is not significant. In scenario 4a, the downward movement occurs only in 10 cells and totals only 0.082 cfs. As long as additional groundwater is pumped primarily from the lowest layer, significant downward flow from the uppermost layer will not occur. However, the model does not consider a seasonal fluctuation of head such as extreme drawdowns resulting from pumping in the summer. This may cause intrusion or low quality water from the upper shallow aquifer. Therefore, a more detailed investigation of groundwater water quality problems is appropriate for setting bounds on head.

Conclusion for tested scenarios is that here is not much close of contaminants moving to lower levels. However, contaminants can enter the major where they are unconfined near the mountains.

Declines of potentiometric heads in the lower layer

For all scenarios, the decline of potentiometric heads exceeds 50 ft near North Ogden (outside of the city zone). For the nonoptimal case (scenario 1), no significant decline of heads occurs in the pumping center in the vicinity of the Hill A.F.B. (Fig. 14). For the optimal management scenarios, two typical patterns in decline of heads are found. One results from maximizing pumping from existing pumping wells (scenario 2). The other results from maximizing pumping from existing and/or newly proposed wells (scenarios 3 and 4). Figures 15 and 16 show the drawdown contours for scenario 2b (maximum allowable drawdown = 30 ft) and scenario 4b (maximum allowable drawdown = 20 ft), respectively. In both scenarios, optimal pumping rates are about twice the current pumping. In the vicinity of the Hill A.F.B., for scenario 2b, the declines of head are 25 ft to 30 ft. On the other hand, for scenario 4b, the declines are only 5 to 10 ft.

Validation of optimal solutions

Steady-state flow simulation. The flow simulation ability of the S/O model is confirmed by comparing optimal heads with heads simulated to results from optimal pumping values. Heads were simulated using a McDonald and Harbaugh model in which a flowing well subroutine is added. Optimal pumping rates from scenarios 1, 2a, 3a, and 4a are used as input data for this comparison. Both models estimate almost identical heads, discharge from flowing wells and drains, evapotranspiration, and general-head boundary interflow (Appendix A). The absolute value of the maximum difference between simulated heads obtained from the two models does not exceed 0.02 feet in any cell.

Evolution of head to the optimal steady-state. To trace the evolution of heads to the optimal steady-state, transient 50-year simulations using optimal pumping strategies for the above scenarios are performed. The McDonald and Harbaugh model is run to get transient solutions for five ten-year stress periods in which each stress period is divided into four time steps. Heads in 1985 calibrated by USGS are used as initial heads. At each time step, total absolute differences (TAD) between transient heads and optimal steady-state heads are calculated and plotted as shown in Fig. 17. The time required to achieve the optimal steady-state heads depends on how far an initial head is from an optimal solution. If we assume that heads reach the optimal steady-state when TAD attains 200 ft (average difference between optimal head and attained head of of 200 ft for 4,880 cells = 0.04 ft), then the head evolution era are 11, 20, 30, and 40 years in duration for scenarios 1, 2a, 3a, and 4a, respectively.

Global optimality. Since the problems are highly complex and nonlinear, it is necessary to confirm global optimality of solutions (even though global optimality of the LP solution to the linear surrogate problem is guaranteed). By allowing variables such as evapotranspiration, drain discharge, and flowing wells to be negative in each cycle, the model can converge to the stable solution even if the initial guess is far from the optimal solution. Therefore, we assume here that the global optimality is guaranteed if the optimal solution does not increase by changing the starting point-an initial guess of the optimal solution which is either close to or far from the optimal solution. For confirmation, the model is run for scenario 2a using different sets of the initial guess, in which the furthest one is a set of variables including heads in 1985 and the closest one is scenario 2b having four times the current pumping as the upper bound. In all cases, optimal solutions vary by no more than 0.01% from each other. In conclusion, the optimal solution computed by the S/O model can be considered to be very close to the global optimal. How close one gets depends on the convergence criterion used for stopping cycling.

SUMMARY AND CONCLUSIONS

The development and use of a cycling procedure for applying embedding and response matrix approaches to an extremely large, complex, nonlinear/linear aquifer system are presented and tested. The addressed groundwater reservoir in the East Shore Area of Utah is discretized into 4,880 finite-difference cells in the model. The cycling procedure involves repeating the optimization of linearized forms of nonlinear flow equations to reach the true optimal solution. The solved problem is large and nonlinear since the upper, unconfined (nonlinear) aquifer is discretized into 1,274 cells. Also involved are 2,123 nonsmooth functions describing discharge from flowing wells, drain discharge, and evapotranspiration. To facilitate both approaches for this aquifer system, new developments include:

1. The linear version of USUGWM is improved by completely linearizing nonsmooth functions. The model uses the embedding approach and treats transmissivity and nonsmooth functions linearly in each cycle. This improvement enables the USUGWM to converge to a stable optimal solution in any initial guess in a wide range. The modified version of the USUGWM has around 40,000 nonzero elements, 12,000 single equations and variables. The previously reported disadvantage of the embedding model is mainly computational difficulty resulting from its large dimensionality. This study shows that the embedding model can solve such a huge nonlinear system.
2. To correctly represent the above nonlinear system while satisfying the principle of superposition, the response matrix model uses cycling and linear influence coefficients generated using a modified McDonald and

Harbaugh (MODFLOW) model. In the modified MODFLOW, the above nonlinear system is treated linearly in each cycle. The linear segments of nonsmooth functions are selected based on head known from the previous cycle. Some of the selected linear segments of the nonsmooth functions are wrong. However, they will be corrected through cycling just as MODFLOW corrects equation assumptions through iteration. In the management model, only heads of interest are computed using superposition. After optimization, the modified MODFLOW computes other heads, which are necessary to implement the next cycle (to select the linear segments of nonsmooth functions and to compute transmissivity in an unconfined aquifer). This model is the first response matrix S/O model which has the same steady-state simulation abilities of MODFLOW.

After comparison between response matrix and embedding S/O models for a preliminary scenario, its embedding model is selected for further use. Selection is based on its ability to address large number and potential pumping cells.

Four groups of scenarios are tested. All management scenarios consider pumping from 61 existing pumping cells and/or many other potential pumping cells. Some scenarios constrain discharge from flowing wells at 813 cells. The embedding model, a modified version of the USUGWM, can compute the perennial-yield pumping rate for the presented scenarios. Other scenarios can be run for different bounds, constraints, and objective functions to better suit management needs.

The general conclusions for the tested scenarios are as follows:

1. The groundwater reservoir can be developed physically to meet the increasing demand of water for M&I use in the East Shore Area. In the tested scenarios, the largest sustainable pumping yield is 205% of the current pumping. However, the additional development potential relies heavily on groundwater underlying agricultural lands near the lakeshore. There, much groundwater currently discharges by itself through flowing artesian wells.
2. An increase of pumping for M&I use will almost linearly decrease the discharge from flowing wells for irrigation use.
3. For computed pumping strategies that allow to develop groundwater in the lowest aquifer, a large amount of low quality water in the upper, shallow aquifer will not intrude into the fresh water in the underlying confined aquifers.
4. In this model, a uniform maximum allowable drawdown is used for the entire study area. More pumping could be obtained by permitting more drawdown in some locations. However, determining what is acceptable requires detailed were beyond the scope of this study.

The models presented here are useful for reconnaissance-level perennial-yield planning of a large, complex, unconfined/confined aquifer system. For this purpose, the embedding model is preferred because of its flexibility in changing sets of bounds and constraints, numbers of pumping cells, and its ability to handle numerous external flows. This flexibility permits planners to readily consider pumping and drawdown consequences in many locations, and to change locations of interest. This is helpful to planners who cannot easily a priori specify all which might result from development and the locations where these problems might occur. On the other hand, the response matrix model is a valuable alterative. It can require less memory if the number (proportion) if pumping cells and cells requiring head constraint are not large.

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Table 1. Number of finite-difference cells

Type of cells	Layer 1 (Upper)	Layer 2 (Middle)	Layer 3 (Lower)	Total
Active cells	1274	1644	1962	4880
Cells with pumping wells	0	10	51	61
Cells with flowing wells	0	402	411	813
Cells with ET[a]	708	0	0	708
Cells with drain	602	0	0	602
Cells with GHB[b]	449	0	0	449

[a]ET means evapotranspiration
[b]GHB means general head boundary

Table 2. Comparison of model formulation: the embedding and response matrix approaches for preliminary problem

	Components	Equation and or definition: Embedding	Equation and or definition: Response Matrix
A.	External simulation model		
1.	Pre-ICG	-	Yes
2.	ICG	-	yes
B.	Management model		
1.	Objective function	2(LP)	2(LP)
2.	Constraints		
	Flow equation	3(LP)	-
	Flowing wells	4(LP)	-
	General head boundary	5(LP)	-
	Evapotranspiration	6(LP)	-
	Drain discharge	7(LP)	-
	Vertical flow reduction	8(LP)	-
	Head computation	-	13(LP)
3.	Bounds		
	Heads		
	Layer 1	$H_{1,i,j} \geq Bot_{1,i,j}$	$H_{1,19,25} \geq Bot_{1,i,j}$
	at 12 WBWCD & Hill AFB	$H_{1,i,j} \geq H'^{85}_{1,i,j} - 20$ ft	
	Pumping		
	12 WBWCD & Hill AFB wells	$gp^c \leq gp \leq gp^{cap}$	
	Other existing wells	$gp^c \leq gp \leq 2 \times gp^c$	
4.	Variable declaration		
	Positive	gp	gp
	Default (free)	$h, q^d, q^g, q^f, q^e, q^{rd}$	h
	Free	objective value	objective value
5.	MINOS solver	LP	LP
6.	Cyclic Procedure	Yes	Yes

gp^c means current pumping rate
gp^{cap} means well capacity

Table 3. Computational requirements of the embedding and response matrix models for preliminary problem

Items	Embedding	Response Matrix
Equations	12433	14
Variables	12521	102
Nonzero elements	46565	895
Required Memory (Mbytes)	7.04	0.4
CPU time		
1st cycle	103 min.	8 min.
after 1st cycle	about 4 min.	8 to 13 min.

Table 4. Summary of model formulations for various scenarios

Components/scenarios	Equation/definition						
	2a	2c	3a	3b	4a	4b	4c
1. Objective function							
Maximizing total gp	2	2	2	2	2	2	-
Egalitarian goal	-	-	-	-	-	-	19
2. Constraints							
Flow equation	3	3	3	3	3	3	3
Flowing wells	4	4	4	4	4	4	4
General head boundary	5	5	5	5	5	5	5
Evapotranspiration	6	6	6	6	6	6	6
Drain discharge	7	7	7	7	7	7	7
Vertical flow reduction	8	8	8	8	8	8	8
Tradeoff between gp&qf	-	16	-	-	-	-	-
Assuring net withdrawal	-	-	-	17	-	-	-
Excess/potential	-	-	-	-	-	-	20&21
3. Bounds							
Prevent salt water	-	-	-	-	-	18	-
Heads							
Layer 1	$H_{1,i,j} \geq Bot_{1,i,j}$						
Layer 2	$H^{city}_{2,i,j} \geq H^{'85}_{2,i,j}-20$ ft						
Layer 3	$H^{city}_{3,i,j} \geq H^{'85}_{3,i,j}-20$ ft						
Pumping							
Number of Existing & candidate locations	61	61	136	136	846	846	846
Bounds							
12 WBWCD & Hill AFB wells	$gp^c \leq gp \leq gp^{cap}$						
Other existing wells	$gp^c \leq gp \leq 2 \times gp$						
Newly proposed wells	$0 \leq gp \leq 1{,}000$ gpm						
4. Variable declaration							
Positive	gp						
Default (free)	$h, q^d, q^g, q^f, q^c, q^{rd}$						
Free	obj						
5. MINOS solver	LP						
6. Cycling procedure	YES						

Table 5. Computed water budgets for Scenario 2b's

(a) Total optimal pumping (cfs)

	Multiple of current pumping	gp^U 1	2	4	6	10
	1	32.21	<u>48.40</u>	49.98	51.26	51.75
gp^L	0.95	-	50.67	-	-	-
	0.90	-	51.74	-	-	-
	0.80	-	52.49	-	-	-
	Drawdown (ft)					
	20	-	<u>48.40</u>	-	-	-
D^L	30	-	57.92	-	-	-
	35	-	61.54	-	-	-
	40	-	67.82	-	-	-

(2) Total discharge from flowing wells (cfs)

	Multiple of current pumping	gp^U 1	2	4	6	10
	1	35.95	<u>27.13</u>	26.33	25.41	25.05
gp^L	0.95	-	26.03	-	-	-
	0.90	-	25.47	-	-	-
	0.80	-	25.12	-	-	-
	Drawdown (ft)					
	20	35.95	<u>27.13</u>	-	-	-
D^L	30	-	22.29	-	-	-
	35	-	20.71	-	-	-
	40	-	18.23	-	-	-

(3) Total of other discharge (Et, drain, and GHB) (cfs)

	Multiple of current pumping	gp^U 1	2	4	6	10
	1	80.23	<u>72.86</u>	72.08	71.72	71.59
gp^L	0.95	-	71.69	-	-	-
	0.90	-	71.18	-	-	-
	0.80	-	70.78	-	-	-
	Drawdown (ft)					
	20	-	<u>72.86</u>	-	-	-
D^L	30	-	68.18	-	-	-
	35	-	66.14	-	-	-
	40	-	62.34	-	-	-

gp^U is a upper bound on pumping, multiple of current pumping.
gp^L is a lower bound on pumping, multiple of current pumping.
D^L is a maximum allowable drawdown under 1985 head.

Table 6. Change in flow of flowing wells

Flowing condition	Number of cells			
	Scenarios			
	1	2a	3a	4a
Decrease or Cease	436	705	705	792
Cease	143	188	245	311
Increase	377	108	108	21
No change	0	0	0	0
Total	813	813	813	813

Table 7. Spatial distribution of existing and additional pumping cells across the water entities

Water entities	Number of cells with existing pumping wells	Number of cells with additional pumping wells: Scenarios 3a	3b	4a	4b	4c
Davis County	26	18	19	52	52	11
Centerville	0	0	0	2	0	1
Clearfield	3	0	0	0	0	1
Clinton	1	0	0	0	1	1
Farmington	3	1	1	14	13	1
Fruit Heights	1	2	3	0	0	1
Hill Field	6	0	0	0	0	0
Kaysville	0	0	1	14	15	1
Layton	4	0	1	1	2	1
So. Weber	3	0	0	0	0	1
Sunset	1	0	0	0	0	1
Syracuse	2	7	6	14	14	1
West Point	2	8	5	7	8	1
Weber County	31	6	9	29	14	11
Ogden	3	0	0	0	0	1
No. Ogden	8	0	0	0	0	0
Pleasant View	2	0	1	0	0	1
Harrisville	0	0	1	0	0	1
Farr West	2	0	1	0	0	1
Plain City	0	0	1	0	0	1
So. Ogden	2	0	0	0	0	1
Riverdale	4	0	0	0	0	1
Roy	1	0	0	0	0	1
Washington T	2	0	0	0	0	1
Uintah	0	0	0	0	0	1
West Weber	7	6	5	29	14	2
Box Elder County	4	0	1	0	0	1
Willard City	4	0	1	0	0	1
Total	61	24	26	81	66	23

Table 8. Pumping and Flowing Well Discharge for Water Entities for Scenarios 3a and 3b

Water entities	Scenario 3a Δgp (cfs)	Scenario 3a Δq^f (cfs)	Scenario 3a $\Delta total$ (cfs)	Scenario 3b Δgp (cfs)	Scenario 3b Δq^f (cfs)	Scenario 3b $\Delta total$ (cfs)
Davis County	15.725	-6.183	9.542	14.353	-5.737	8.616
Centerville	-	-0.042	-0.042	-	-0.042	-0.042
Clearfield	0.000	-	0.000	0.000	-	0.000
Clinton	0.000	-	0.000	0.000	-	0.000
Farmington	1.309	-0.397	0.912	1.277	-0.396	0.881
Fruit Heights	0.965	-	0.965	0.977	-	0.977
Hill Field	0.000	-	0.000	0.000	-	0.000
Kaysville	0.000	-0.176	-0.176	0.169	-0.169	0.000
Layton	0.000	-0.398	-0.398	0.389	-0.389	0.000
So. Weber	0.000	-	0.000	0.000	-	0.000
Sunset	0.000	-	0.000	0.000	-	0.000
Syracuse	6.717	-3.002	3.715	6.442	-2.814	3.628
West Point	6.734	-2.168	4.566	5.099	-1.927	3.172
Weber County	9.643	-8.714	0.929	6.803	-6.803	0.000
Ogden	0.000	-0.047	-0.047	0.041	-0.041	0.000
No. Ogden	0.000	-0.050	-0.050	0.053	-0.053	0.000
Pleasant View	0.000	-0.048	-0.048	0.063	-0.063	0.000
Harrisville	0.000	-0.092	-0.092	0.098	-0.098	0.000
Farr West	0.000	-0.140	-0.140	0.154	-0.154	0.000
Plain City	0.000	-0.196	-0.196	0.173	-0.173	0.000
So. Ogden	0.000	-	0.000	0.000	-	0.000
Riverdale	0.000	-	0.000	0.000	-	0.000
Roy	0.000	-	0.000	0.000	-	0.000
Washington T	0.000	-	0.000	0.000	-	0.000
Uintah	0.000	-	-	0.000	-	0.000
West Weber	9.643	-8.141	1.502	6.220	-6.220	0.000
Box Elder County	0.000	-0.008	-0.008	0.011	-0.011	0.000
Willard City	0.000	-0.008	-0.008	0.011	-0.011	0.000
Out of city zone	-	-0.268	-0.268	-	-0.245	-0.245
Total	25.368	-15.173	10.195	21.885	-13.182	8.728

Δ means change in discharge (increase or decrease) from discharge of the nonoptimal scenario to optimal discharge in the management scenario.

Table 9. Additional development of pumping under the egalitarian goal: scenario 4c

Water entities	Area ($mile^2$)	Aerial ratio (cfs)	ULDP (cfs)	Pumping SC1[a] (cfs)	SC4C[b] (cfs)	AD (cfs)
<u>Davis County</u>	<u>98.875</u>	<u>0.415</u>	<u>13.968</u>	<u>22.627</u>	<u>26.540</u>	<u>3.913</u>
Centerville	2.625	0.012	0.391	-	0.110	0.110
Clearfield	7.500	0.033	1.116	0.246	0.559	0.313
Clinton	5.750	0.025	0.856	0.017	0.257	0.240
Farmington	8.625	0.038	1.283	0.686	1.045	0.359
Fruit Heights	3.000	0.013	0.446	0.035	0.160	0.125
Hill Field	9.750	0.043	1.451	6.248	6.655	0.407
Kaysville	10.000	0.044	1.488	-	0.417	0.417
Layton	25.750	0.114	3.831	3.456	4.529	1.073
So. Weber	5.500	0.024	0.818	11.603	11.832	0.229
Sunset	1.000	0.004	0.149	0.067	0.109	0.042
Syracuse	11.750	0.052	1.748	0.175	0.665	0.490
West Point	2.625	0.012	0.391	0.094	0.204	0.110
<u>Weber County</u>	<u>118.500</u>	<u>0.523</u>	<u>17.632</u>	<u>9.271</u>	<u>14.251</u>	<u>4.980</u>
Ogden	22.750	0.100	3.385	0.043	0.991	0.948
No. Ogden	3.750	0.017	0.558	0.976	1.132	0.156
Pleasant View	7.750	0.034	1.153	0.200	0.503	0.303
Harrisville	4.000	0.018	0.595	-	0.107	0.107
Farr West	4.250	0.019	0.632	0.051	0.228	0.177
Plain City	4.000	0.018	0.595	-	0.167	0.167
So. Ogden	5.750	0.025	0.856	0.595	0.835	0.240
Riverdale	4.000	0.018	0.595	4.298	4.465	0.167
Roy	7.000	0.031	1.042	0.835	1.127	0.292
Washington T	2.000	0.009	0.298	0.775	0.858	0.083
Uintah	1.500	0.007	0.223	-	0.062	0.062
West Weber	51.750	0.229	7.700	1.498	3.655	2.157
<u>Box Elder County</u>	<u>14.000</u>	<u>0.062</u>	<u>2.809</u>	<u>0.309</u>	<u>0.893</u>	<u>0.584</u>
Willard City	14.000	2.500	2.809	0.309	0.893	0.584
<u>Total</u>	<u>226.375</u>	<u>1.000</u>	<u>33.683</u>	<u>32.207</u>	<u>41.644</u>	<u>9.437</u>

[a]SC1 means scenario 1
[b]SC4C means scenario 4c

Table 10. Vertical water movements

Item	Scenarios			
	1	2a	3a	4a
Layer 1 & 2				
Upward				
volume (cfs)	80.263	72.939	70.122	65.556
number of cells	1273	1268	1268	1264
Dounward				
volume	0.027	0.071	0.081	0.082
number of cells	1	6	6	10
Layer 2 & 3				
Upward				
volume (cfs)	83.001	73.383	69.280	64.664
number of cells	1603	1570	1534	1417
Downward				
volume (cfs)	1.676	2.321	2.398	5.593
number of cells	41	74	110	227

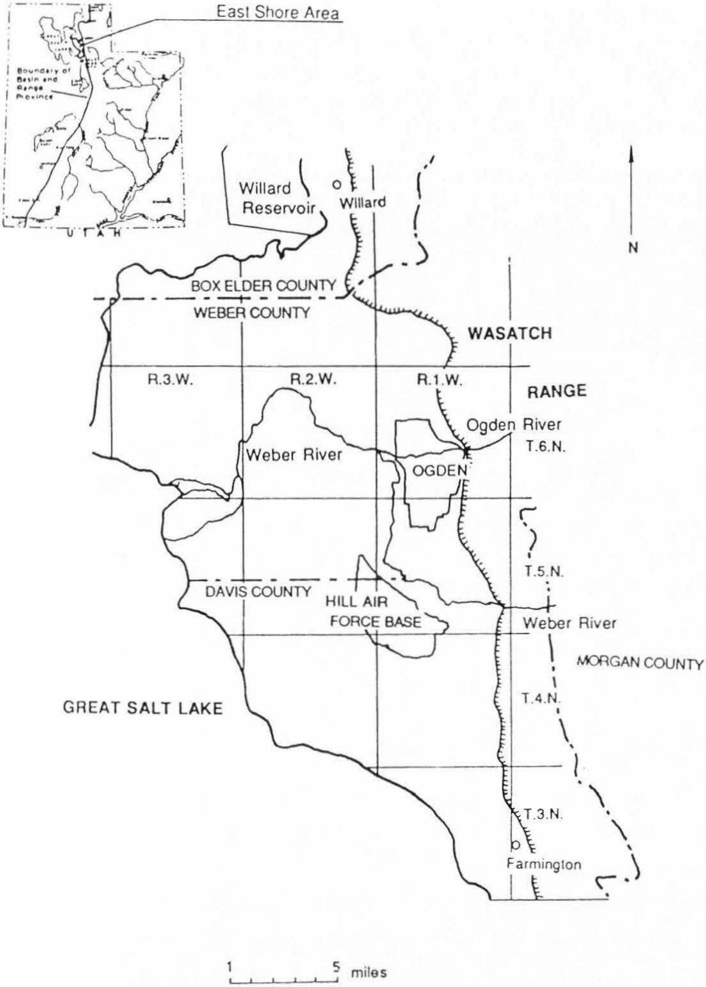


Fig. 1. Map of the East Shore Area, Utah

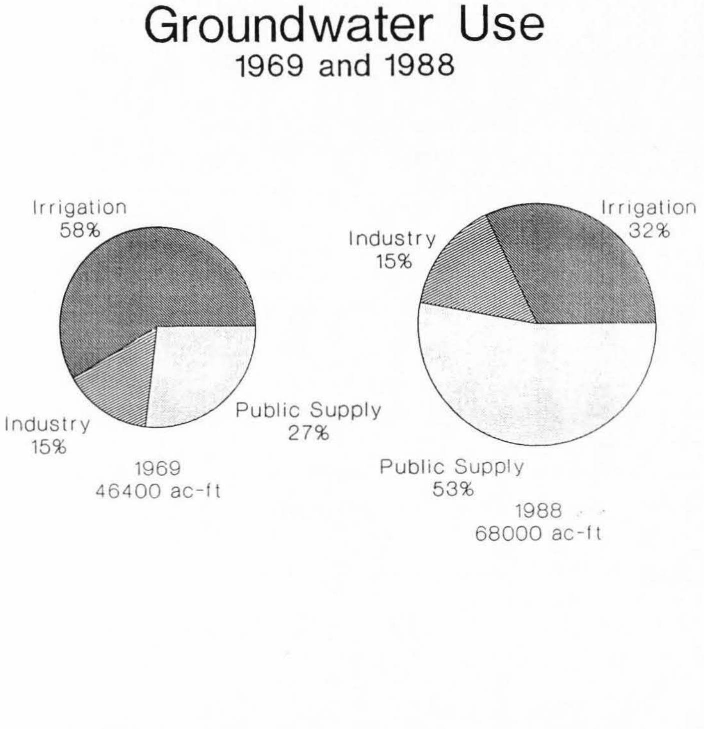


Fig. 2. Groundwater use of the East Shore Area in 1969 and 1988

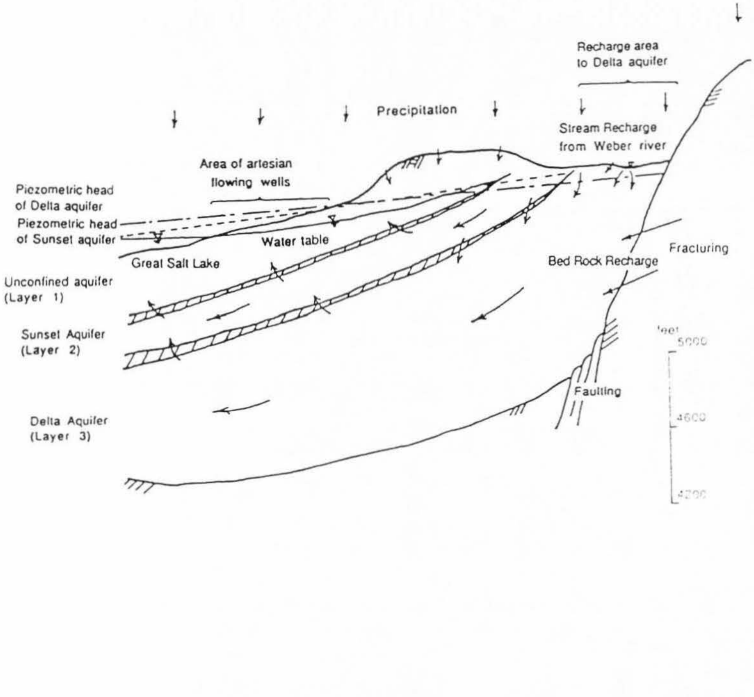


Fig. 3. Generalized profile of the East Shore Area aquifer system

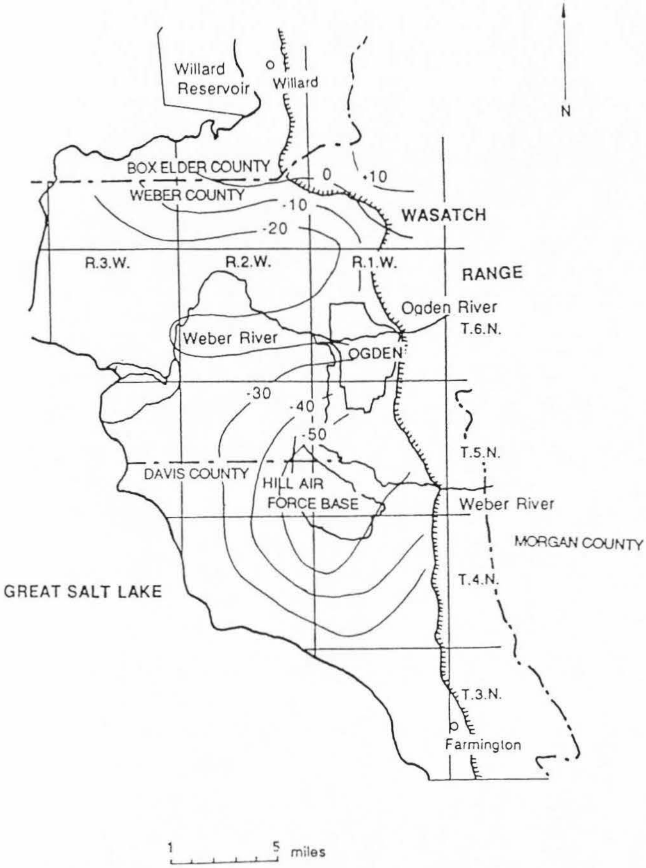


Fig. 4. Change of potentiometric heads of layer 3 (lower layer) from 1955 to 1985

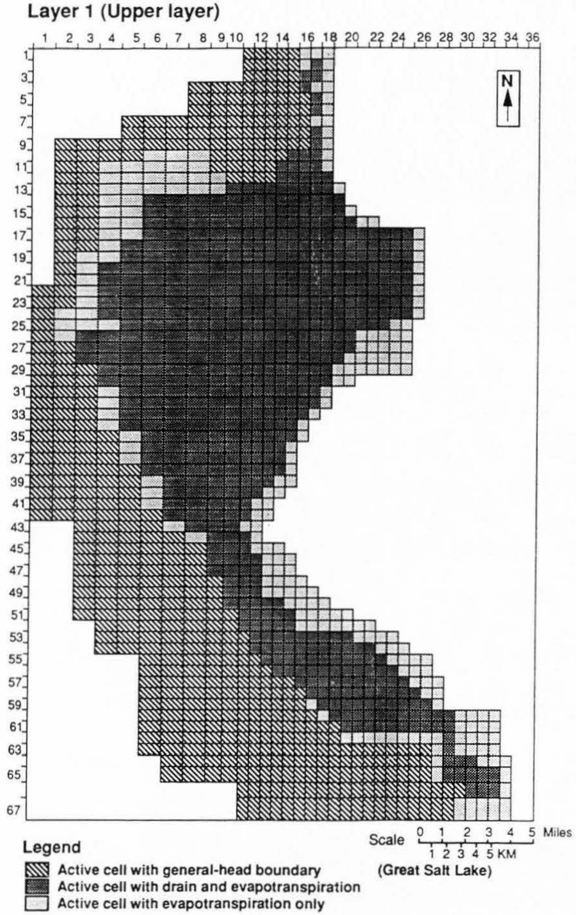


Fig. 5. Discretization of layer 1 (upper layer)

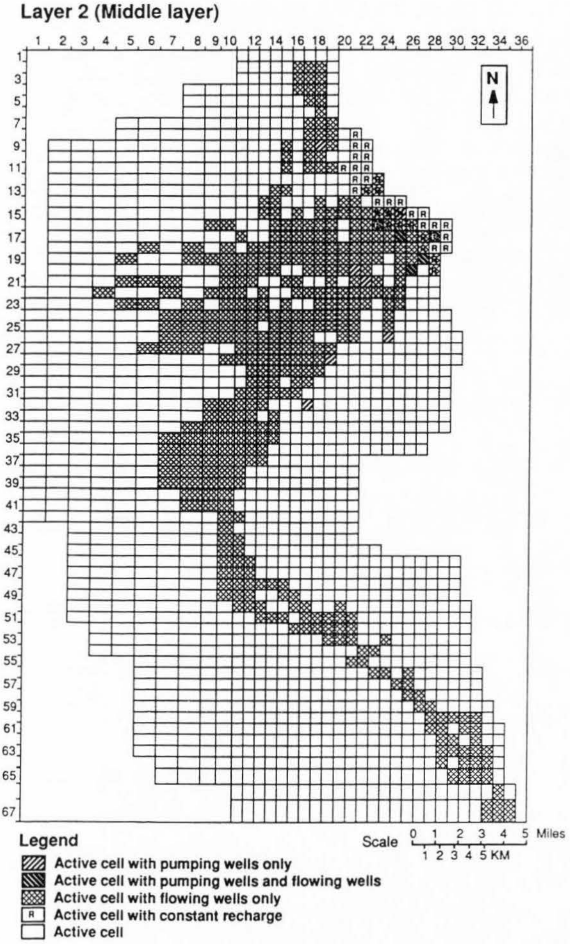


Fig. 6. Discretization of layer 2 (middle layer)

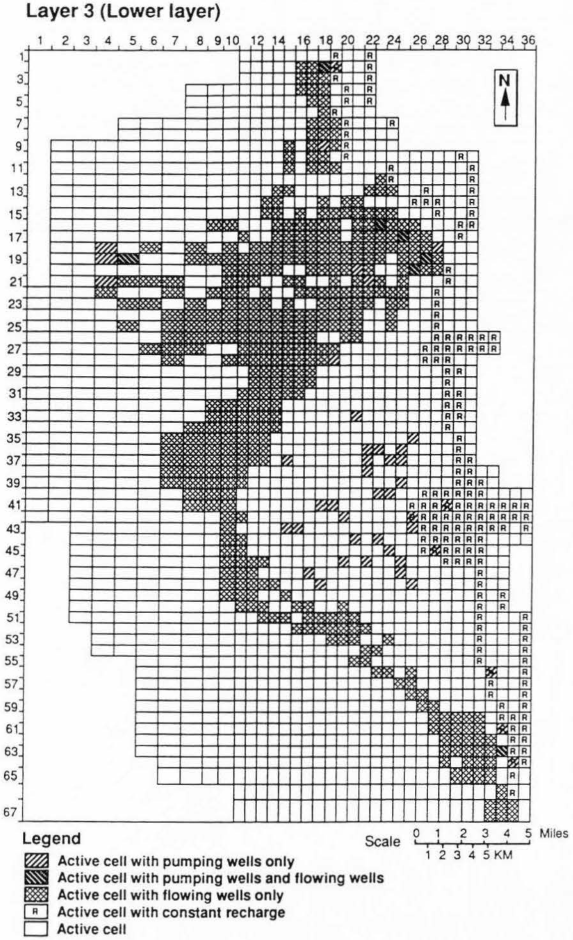
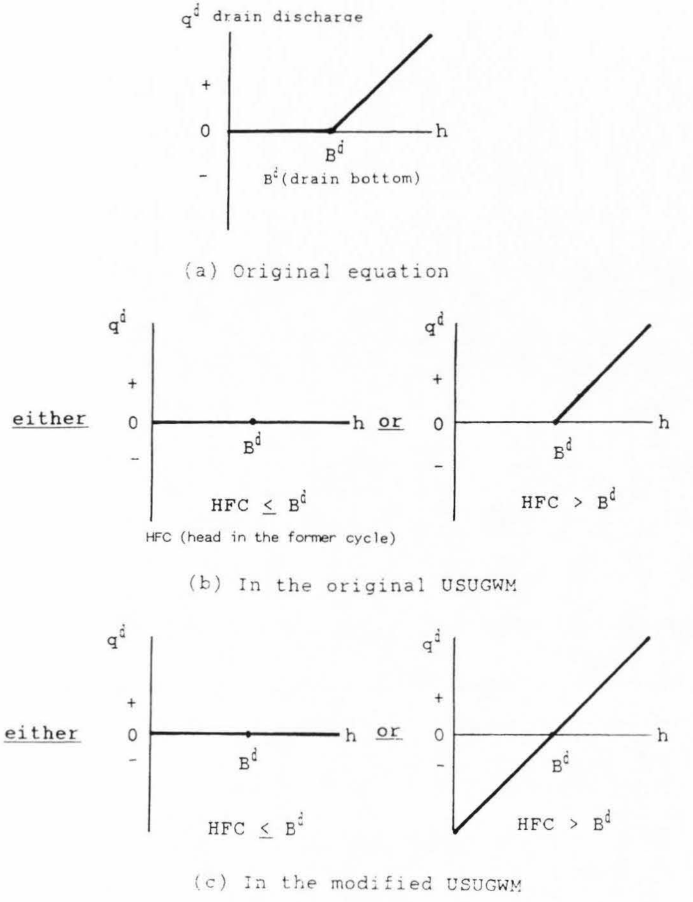


Fig. 7. Discretization of layer 3 (lower layer)



(a) Original equation

(b) In the original USUGWM

(c) In the modified USUGWM

Fig. 8. Linear formulae for discharge from drains

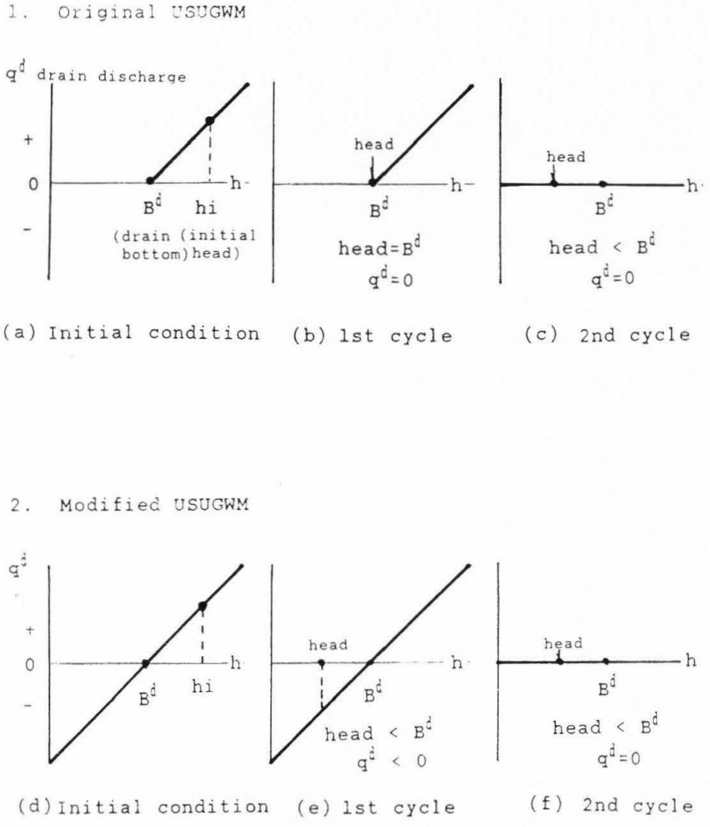
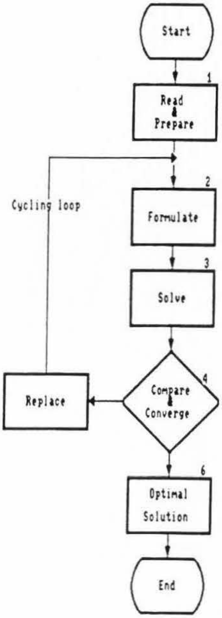
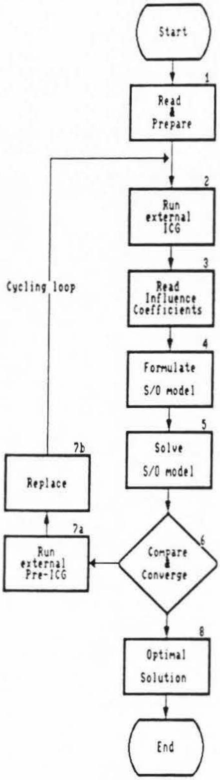


Fig. 9. Solving procedures for discharge from drains

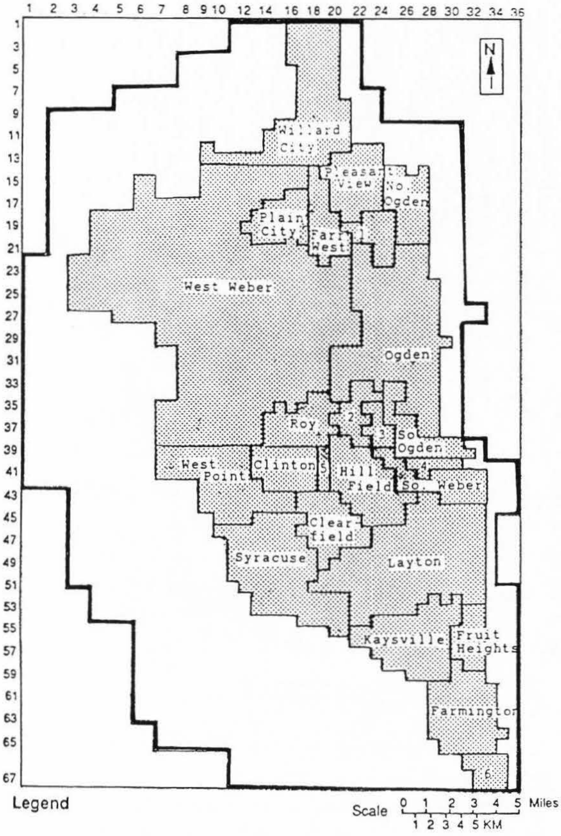


Embedding model



Response Matrix model

Fig. 10. Flow charts of solving procedures for the embedding and response matrix S/O models



Limits of water entities

Boundary of model area

1. Harrisville
2. Riverdale
3. Washington T.
4. Uintah
5. Sunset
6. Centerville

Fig. 11. Boundaries of water entities

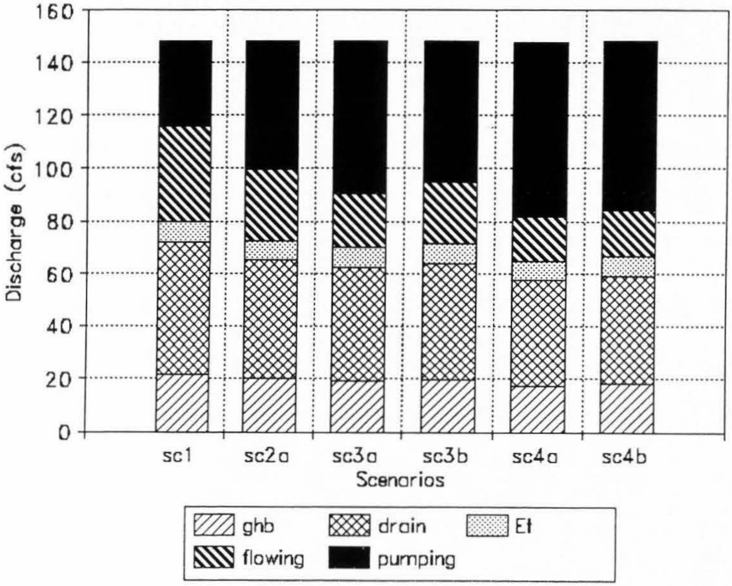


Fig. 12. Discharges for scenarios 1, 2a, 3a, 3b, 4a, and 4b.

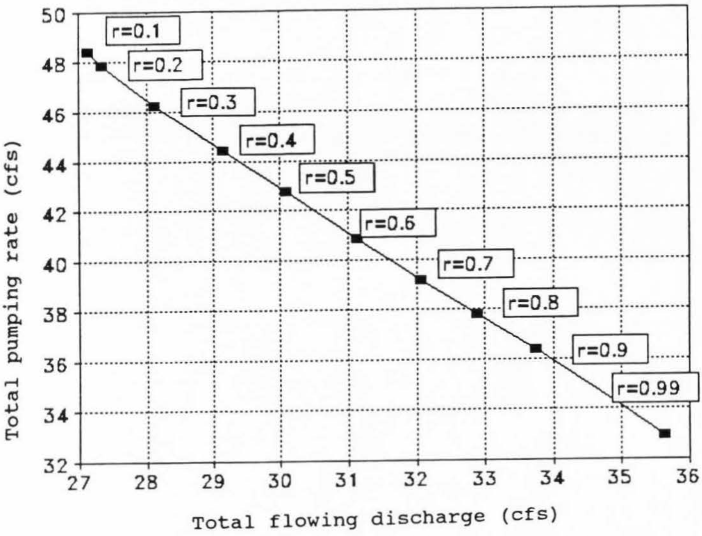
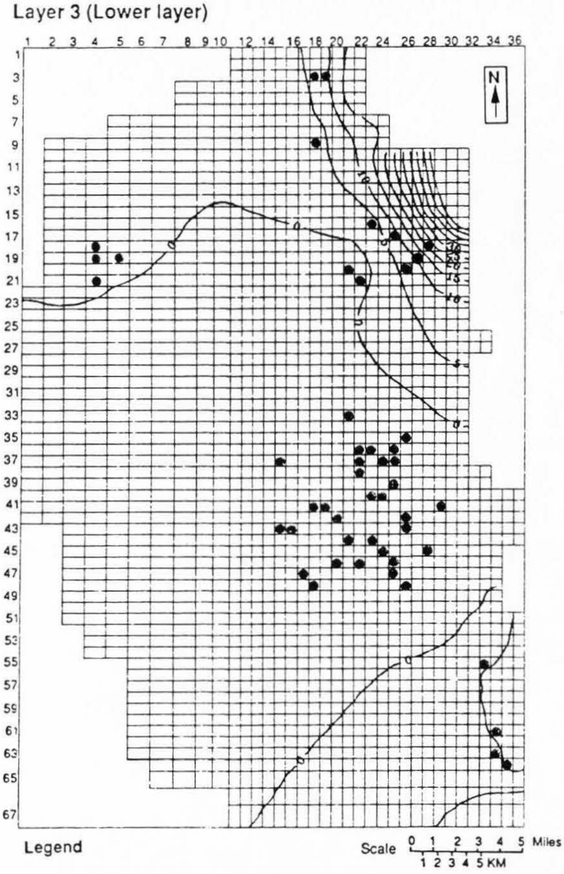
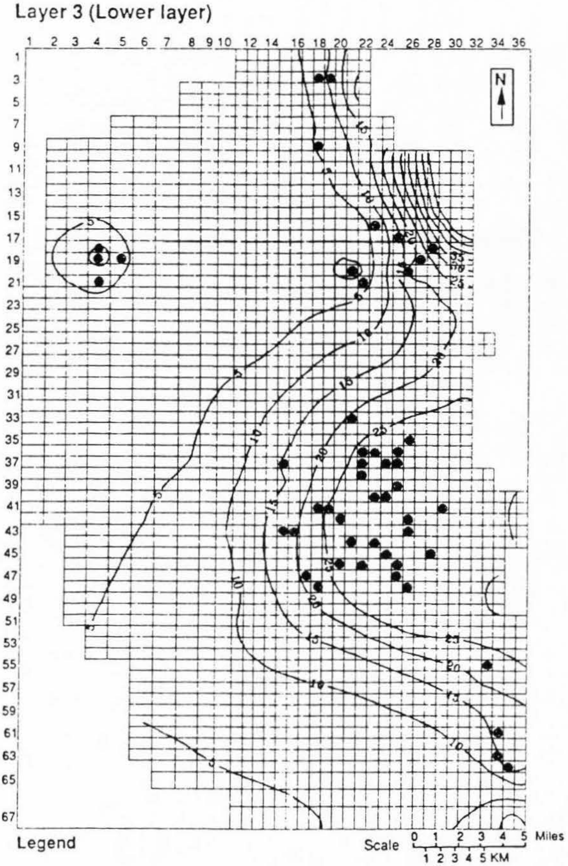


Fig. 13. Tradeoff curve between pumping and flowing wells



Active cell with existing pumping wells
Active cell

Fig. 14. Drawdown contours of scenario 1



Active cell with existing pumping wells

Active cell

Fig. 15. Drawdown contours of scenario 2b (maximum allowable drawdown = 30 ft)

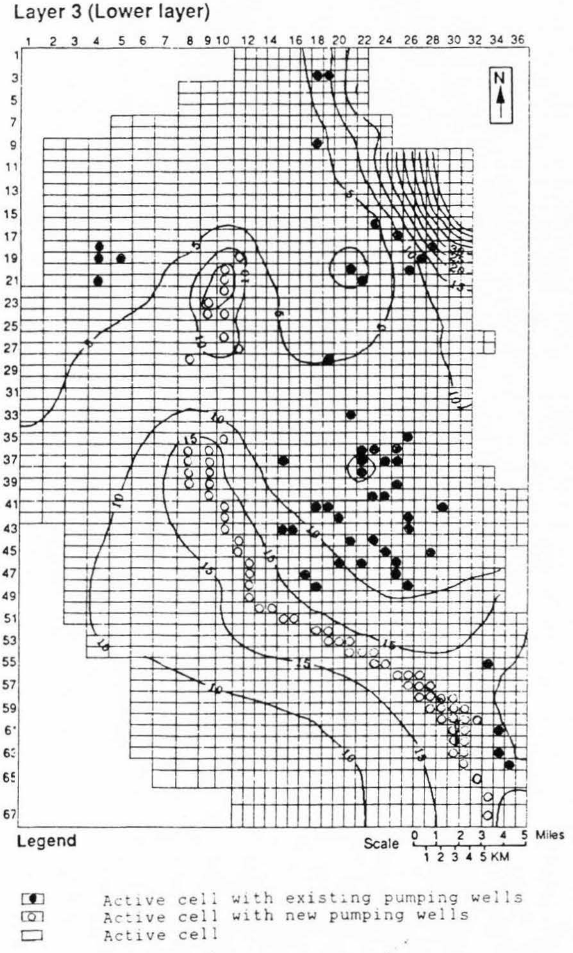


Fig. 16. Drawdown contours of scenario 4b

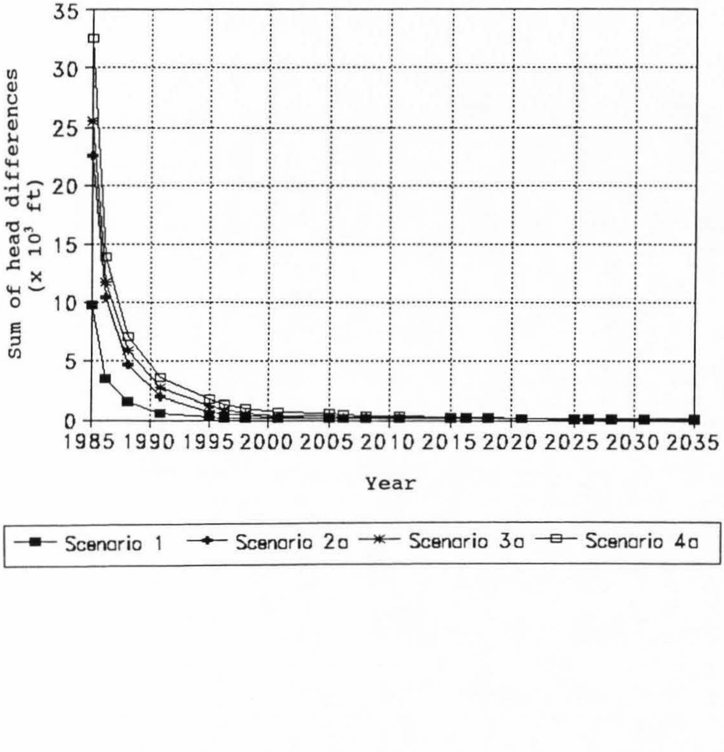


Fig. 17. Evolution of heads to the optimal steady-state

CHAPTER IV

PRELIMINARY DECISION SUPPORT SYSTEM FOR REGIONAL GROUNDWATER MANAGEMENT

Overview of System Structure

A long-term need is a "Decision Support System" (DSS) that will most efficiently optimize regional groundwater extraction. Such a DSS would use the response matrix approach, the embedding approach, or their modifications or combinations, depending on which is most appropriate for the particular situation. The most appropriate method would be the one which yields acceptable, accurate answers while requiring the least computer processing time or memory.

Here, a preliminary DSS is developed. It is useful in developing optimal sustained-yield pumping strategies and has the essential DSS features mentioned above. The system consists of a main DCL command procedure (MAIN) and independent subroutines which are grouped into five program packages. The packages include the following: Predictive Comparison Program Package (PCPP), Embedding Model Package (EMP), Response Matrix Model Package (RMMP), Groundwater Flow Simulation Program Package (GFSPP), and Common Utility Program Package (CUPP).

The PCPP aids a user in deciding which method requires the least computer memory. EMP and RMMP contain management models using the embedding and response matrix approaches, respectively. Those management models are written in GAMS and can be solved with MINOS on computers ranging from a PC-XT to a supercomputer. The system is designed to be transferable to any aquifer system. It assumed that a user is familiar with MODFLOW, FORTRAN, and GAMS. The system is applied to the hypothetical aquifer system in Chapter II (Appendix F) and the East Shore Area aquifer system in Chapter III. The users can prepare input data files and modify programs, if necessary. The user can execute all the programs interactively on the VAX-VMS system.

Overall Design Structure of the System: the Main DCL Command Procedure

The flow chart of the main DCL command procedure (MAIN) is shown in Fig. 1. MAIN calls packages and subroutines to perform the following tasks in order:

General instruction (step 1)

1. Display general model instructions to the user.

Preparation of data (steps 2 to 5)

2. Query the user whether the GAMS tables, which contain pumping and geohydrological data for a given aquifer, have already been made or not.
3. Inform the user that he/she can use JAMFLOW (Tika, 1990) to prepare the GAMS table.
4. Query the user whether he/she is running the model for the first time. If it is the first trial, then go to PREDAT. If it is not the first trial and the user restarts work files (eight files termed "file name.g01" through "file name.g08"), then go to

PREGO.

5. Call PREDAT to compile these data files written in GAMS (COM1_.GMS, IGS_.GMS, and COM2_.GMS). COM1_.GMS contains OPTION statements for MINOS. These statements: set work space, control output of the SOLVE statement, and specify layer, row, and column in the grid for the aquifer. IGS_.GMS contains initial guesses of variables such as head and pumping rate. COM2_.GMS contains tables of pumping and geohydrological data for the aquifer such as hydraulic conductivity, transmissivity, hydraulic conductances, bottom elevation of the drain, etc.
6. Call PREGO, which interrogates the user for the names of work files (filename.g01 through filename.g08), and copies those to work files (d.g01 to d.g08)

Comparison of alternative management models in terms of computational efficiency (steps 7 to 8)

7. Ask the user whether he/she wants to look at a comparison of six alternative simulation/ optimization models with respect to the numbers of rows, columns, and nonzero elements of the matrix of the optimization scheme.
8. Call PCPP to estimate the numbers of rows, columns, and nonzero elements of the matrix for each alternative, and display the results to the user.

Solving the management model (steps 9 to 11)

9. Ask the user which management model package is preferred, EMP or RMMP.

10. Call EMP to run the embedding model (E1, E2, or E3).

11. Call RMMP to run the response matrix model (R1, R2, or R3).

Transient-state simulation (steps 12 to 13)

12. Ask the user whether he/she wants to compare the optimal steady-state heads with heads predicted using transient-state simulation. This permits demonstrating the change (evolution) of heads from their initial values into the optimal steady-state values, in response to continued pumping at the optimal rate.

13. Call GFSPP: In GFSPP, the user can run the McDonald and Harbaugh model to obtain the transient-simulation results, including the time required for heads to evolve to the optimal steady-state.

Predictive Comparison Program Package: PCPP

The DSS identifies the most appropriate head and flow-constraining modelling approach for a given situation. The most appropriate approach is selected based on not only the least computer time or memory but also other factors. However, in this case, the least computer memory is assumed to be the most important factor.

In the evaluation, PCPP estimates the number of equations, variables, and nonzero elements needed for each approach to address the posed aquifer management problem. In essence, the predictive program identifies the equations needed for each cell using known indicators and geohydrological parameters. In the package, pcpp.gms is compiled, and the computation results are displayed on the screen as shown in Table 1.

Embedding Method Package: EMP

EMP contains three alternative management models which use the embedding approach (E1, E2, and E3 presented in Chapter II). The user will select one of the models based on the characteristics of the addressed flow system. For a confined aquifer (Case A), if the flow system does not include any Type 3 external flows, then the cycling procedure is not necessary. For an unconfined aquifer (Case B), if either Alternative E1 or E2 is selected, then cycling is necessary to reach the true optimal solution. If Alternative E3 is selected, the cycling procedure is not necessary.

The EMP flow chart is shown in Fig. 2. The EMP performs its tasks in the following order:

General instruction (step 1)

1. Instruct the user to select one of the three alternative models (E1, E2, and E3).

Formulating the management model (steps 2 to 4)

2. If this is the first optimization, then go to step 3 (Call EMFORM); otherwise, restart the work files (file name.g01 through file name.g08) and go to step 4 (Call BOUND).
3. Call EMFORM: EMFORM reads work files (D.g01 to D.g08) and compiles EMFORM_.GMS, which contains an objective function, flow equations, both linear and nonlinear external flow term equations, nonlinear hydraulic conductances, and a set of bounds on variables such as pumping rate and head.
4. Call BOUND: BOUND is used to change a set of bounds on variables (see BOUND in the CUPP in detail).

Solving the management model (steps 5 to 7)

5. Call EMSOLV: EMSOLV asks the user which alternative management model he/she wants to use to solve the given system and compiles one of emalt1_.gms, emalt2_.gms, and emalt3_.gms. These contain statements using LP solver for Alternative E1 and the DNLP solver for E2 and E3, respectively.
6. Call CRIT: CRIT serves two tasks: (1) replaces, for the next cycle, the transmissivities (TRAN) and hydraulic conductances (CR and CC) of the unconfined aquifer in the current cycle with values computed using new optimal heads; (2) estimates the total absolute difference of heads (TADH) and maximum absolute difference of heads (MADH) between the former (n-1 th) cycle and the current (n th)

cycle, and displays them on the screen.

7. Ask the user whether the cycling procedure is still necessary or not. If the flow system is solved using Alternative E3, or it is completely linear (only confined and no Type 3 external flow terms), then the cycling procedure is not necessary; go to step 8 (Call OUTPUT). If TDAH and MADH satisfy their convergence criteria, then the true optimal solution is found; go to step 8 (call OUTPUT). If not, the cycling procedure is still necessary to reach the true optimal solution; go back to step 5 (call EMSOLV).

Output of optimization results (step 8)

8. Call OUTPUT: OUTPUT serves tasks about outputs of computational results using subroutines in the CUPP (described later in this chapter).

Response Matrix Model Package (RMMP)

RMMP consists of the influence coefficient generator (ICG), the Pre-ICG written in FORTRAN, and the management models (R1, R2, R3) written in GAMS. The flow chart of the RMMP is shown in Fig. 3. The RMMP performs its tasks in the following order:

General Instruction (step 1)

1. Instruct the user one of the three alternative models (R1, R2, and R3). Alternative R3 uses both the ICG and the Pre-ICG while Alternatives R1 and

R2 use only the ICG. If a solved aquifer system consists of only confined aquifer layers and no Type 3 external flows, cycling is unnecessary. Other cases need the cycling procedure to address the nonlinearity.

Formulating the management model (steps 2 to 5)

2. If it is the first optimization, then call IDCELL; otherwise, skip IDCELL.

3. IDCELL identifies cells in which head must be calculated using either the groundwater flow equation (Eq. 2 in Chapter II) and superposition (Eq. 13 in Chapter II). In this process, it follows the types of variable heads and constant head cells defined in Table 1, Chapter II.

4. Call RMFORM: If this is the first run, then the RMFORM asks the user which management model is to be used for solving the given system and compiles rmalt1_.gms, rmalt2_.gms, or rmalt3_.gms. These contain the indicator matrix defined in step 3 for Alternatives R1, R2, and R3, respectively. The RMFORM also compiles rmform_.gms which contains the flow equation (Eq. 2), the superposition equation (Eq. 13), external flows, and a set of bounds on variables.

5. Call BOUND: This is the same as in the EMP.

Solving the management model (steps 6 to 10)

6. Call ICG: Subroutine ICG contains an execution

file of the influence coefficient generator (ICG_.exe) written in FORTRAN. The ICG_.exe generates influence coefficients for cells in which head is computed with Eq. 13 and makes a GAMS file, inf_.gms, containing tables of influence coefficients.

7. Call RMSOLV: RMSOLV compiles optrm_.gms which contains a statement using the LP solver. The management model is solved here.
8. Call CRIT: This is the same as in the EMP.
9. Ask the user whether the cycling procedure is still necessary. If the system is completely linear, cycling is unnecessary; so the processing moves to step 11 (call OUTPUT). If TDAH and MADH satisfy their convergence criteria, the true optimal solution is found. Step 11 follows. If convergence is not yet attained, cycling continues by going back to step 6.
10. CALL PNCYCL: If the user has chosen Alternative R3, implement PREICG. The PREICG executes prewel.gms which makes an input well data file for the modified MODFLOW (preicg.exe) and runs preicg.exe. Program preicg.exe estimates heads for all cells and makes inith.si. If the user has chosen either Alternative R1 or R2, then execute PREHEAD which makes inith.si. The inith.si is an input data file of head for the ICG in the next cycle.

Output of optimization results (step 11)

11. Call OUTPUT: the same as in the EMP.

Groundwater Flow Simulation Program Package (GFSPP)

GFSPP is used to simulate expected system response to implementing a computed critical strategy. For example, it can demonstrate the evolution of heads to the optimal steady state. GFSPP consists of MODFLOW, its data files, optwel.gms, and a TRACE execution file written in FORTRAN. Optwel.gms prepares a MODFLOW input data file of optimal pumping rates. MODFLOW performs the transient-state simulation using the optimal pumping rate for time steps specified by the user. TRACE reads the starting heads and calculates the total differences between simulated heads and the optimal steady-state heads for each time step.

Common Utility Program Package (CUPP)

The Common Utility Program Package contains those subroutines which perform general tasks needed for applying all S/O models. There are five subroutines:

PREGO. PREGO asks the user the name of work files (file name.g01 through file name.g08), which contain all of the previously solved information of a model run. PREGO renames them as work files (d.g01 through d.g08).

PREGOC. PREGOC asks the user the name of work files (data_.g01" through "data_.g01) which contains compiled com1_.gms, igs_.gms, and com2_.gms. These .gms files were saved previously. REGOC renames these eight files as c.g01 through c.g08.

BOUND. BOUND asks the user whether he/she wants to change bounds on variables. If the user answers "Yes," then BOUND compiles cbound.gms which contains another set of bounds on variables specified by the user. This subroutine is useful for a sensitivity analysis in which the model must be run using different sets of bounds on variables.

POSTGO. POSTGO permits renaming files d.g01 through d.g08. It contains all of the information of some model run and renames them as work files called "file name_.g01" to "file name_.g08."

OUTPUT. OUTPUT performs its tasks in the following order:

1. Ask the user whether or not he/she wants to print out all the information of the optimization result. If the user answers "Yes," then go to step 2.
2. Call ROPT. ROPT contains ropt_.gms which creates a file of the optimization result.
3. Ask the user whether he/she wants to save work files of the optimization result. If the user answers "Yes," then go to step 4.
4. Call POSTGO.
5. Ask the user whether or not he/she wants to create data files for drawing contour maps using a graphical software package. SURFER (PC version) is the graphical software package here. If the user

answers "No," then skip this step and return to the MAIN.

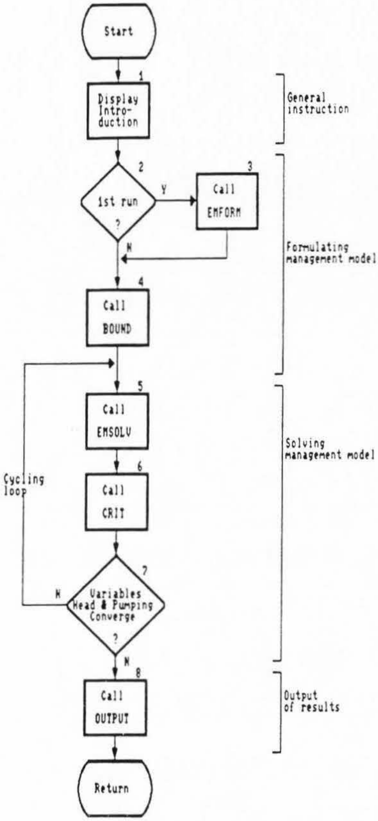
6. Call CONTOUR: CONTOUR contains cont.gms which creates a data file of heads for each layer. The user transfers these files to a PC machine using FTP and runs SURFER.

Table 1. Number of Nonzero Elements, Single Equations, and Single Variables (Example Display)

	Embedding method models		
Indicator	E1	E2	E3
1. Nonzero elements	4158	4165	7585
Linear	4158	4156	4606
Nonlinear	0	9	2979
2. Single equations	685	685	685
3. Single variables	721	721	721
	Response matrix approach models		
Indicator	R1	R2	R3
1. Nonzero elements	3213	1931	622
Linear	3213	1931	622
Nonlinear	0	0	0
2. Single equations	685	685	685
3. Single variables	540	322	135

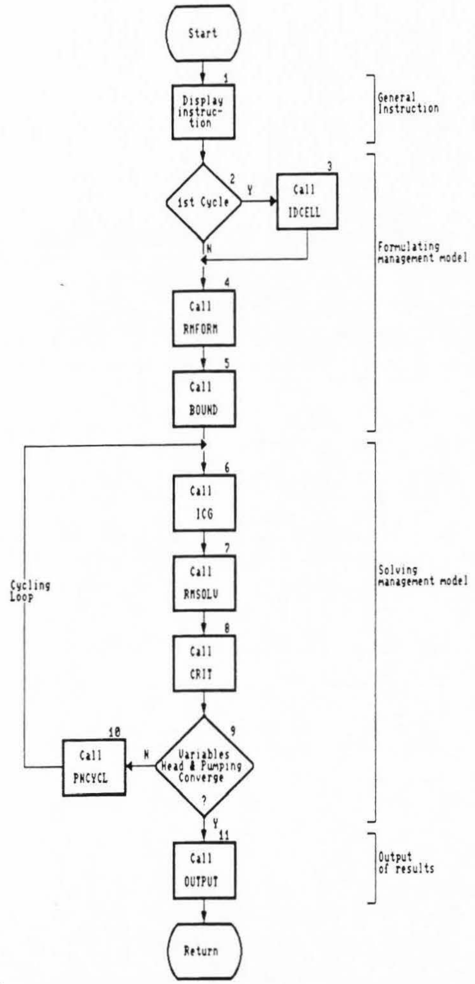
Table 2. Selection of Alternative Management Model

Aquifer system	Type 3 external flows	Choice of Model	Cycling needed
A. Confined			
	No	E1	No
	Yes	E1/E2	Yes/No
B. Unconfined/ Confined	No	E1/E3	Yes/No
	Yes	E1/E2/E3	Yes/Yes/No



If the flow system is solved with E3, or it is completely linear, then cycling is unnecessary.

FIGURE 2. Flow chart of EMMP



In PNCYCL (step 10)
R3 uses ICG and Pre-ICG.
R1&R2 uses only ICG.

FIGURE 3. Flow chart of RMMP

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The main focus of this study is to demonstrate how to most efficiently incorporate a realistic steady-state groundwater flow simulation of a complex, nonlinear aquifer system within an optimization technique. Six alternative modelling approaches for optimizing sustained-yield groundwater planning for a multilayer, unconfined/confined, aquifer system are presented. These approaches utilize an embedding approach, a response matrix approach, and their combinations to represent groundwater flow. All approaches represent: (1) transmissivity in the unconfined aquifer as a function of head either with or without a cycling procedure, (2) quasi 3-D flow in a multilayer system, and (3) external flows such as pumping, general head boundary flux, constant head boundary flux, and flows described by nonsmooth functions (evapotranspiration, stream-aquifer interflow, flow from flowing wells, and drain discharge). The utility of all of the models is compared for a three-layer, nonlinear, hypothetical system. Subsequently, a groundwater planning model (a modified version of the USUGWM) is constructed and applied to compute optimal sustained-yield for alternative scenarios for the East Shore Area, Utah.

Conclusions

1. A fully nonlinear embedding model is presented. This model contains nonlinear equations to more correctly describe an unconfined flow system. It formulates transmissivity with unknown heads and hydraulic conductivities, in contrast to the other models which use heads known from the previous cycle. However, the model requires more computer memory than the others because it is more nonlinear.
2. A previously reported cycling procedure, which is coupled with the embedding method in the USUGWM, is improved in solving nonsmooth functions describing evapotranspiration, drain discharge, and flow from flowing wells. The modified version of USUGWM can achieve a stable optimal solution by solving completely linear formulas in successive cycles. As a result, the global optimality of the solution can be confirmed by changing a starting point in a wide range.
3. A response matrix model, which can reduce required computer memory drastically for some situations, is developed as an alternative to the embedding models. The model uses linear influence coefficients and superposition. Cycling causes the convergence to optimality. The cycling procedure used in the modified version of the USUGWM is

applied to the response matrix model since the principle of superposition can be assured in each cycle. The model is the first response matrix model which addresses all of the steady-state abilities of the McDonald and Harbaugh model, including the nonsmooth processes mentioned above. The computation procedure is automated by linking an external influence coefficient generator, a modified MODFLOW, to selected optimization models.

4. Six alternative steady-state S/O models compute the same optimal strategies for the hypothetical aquifer system, but the fully nonlinear model can compute it directly without cycling. The fully nonlinear model yields the global optimal, as do the other models.

5. Also presented is an automated methodology for comparing the sizes of alternative optimization models. The program computes the number of linear and nonlinear nonzero elements, single equations, and single variables. Output is useful in selecting the most appropriate model for a specific situation.

6. All of the models and the comparison methodology are combined in a single interactive program which is run on a VAX computer under VMS. All computational procedures such as selecting the appropriate model and cycling optimization can be

easily accessed for the use on other study areas.

7. The developed models are useful for a reconnaissance-level study of sustained-yield pumping planning for large, complex, unconfined, or confined aquifer systems. If sufficient computer memory is available, embedding models are preferred for very nonlinear systems having a large portion of cells which must contain pumping variables or bounded heads. The embedding models can handle a large number of external flows and easily bound pumping and head in all cells. The embedding models are also flexible in changing sets of bounds and constraints. On the other hand, response matrix models are valuable alternatives. Memory requirements of the response matrix models are proportional to the number of decision variables, pumping locations, and cells requiring head constraint. If there are not too many of these cells, the response matrix models are preferred to the embedding models.

Recommendations

The following further studies are recommended:

1. The fully nonlinear model was not applied to the East Shore Area. About 10 M byte of memory is required. The model should be tested in another study area such as the Salt Lake Valley (1,086

cells) which is smaller and less complex (in steady-state) than the East Shore Area. The fully nonlinear model should also be tried for the East Shore Area.

2. All the models should have transient-state simulation ability. The original USUGWM has this facility but needs an appropriate initial guess for each time step. The modified USUGWM presented here converges better (despite the values of initial guess) than the initial USUGWM. Thus, better convergence for a transient problem is expected. This will help implement the transient-state optimization.
3. All of the models written in GAMS can be used on any level of machine if the available machine has GAMS and MINOS software. However, the program system developed here, which conducts and regulates all of the program packages, uses VAX-VMS. Thus, PC and Unix versions of the VMS routines should be constructed.
4. There exist groundwater quality problems at Hill Air Force Base. These problems should be linked to regional groundwater management using the S/O models.
5. The modified version of USUGWM should be run for scenarios with different objective functions, bounds, and constraints to better suit water

management needs for the East Shore Area, Utah.

APPENDIXES

Appendix A. Validation of Simulations.

Pre-ICG Simulation Ability

The simulation ability of the Pre-ICG is verified. To do this, a steady-state simulation, using the current pumping rate under normal climatic condition was performed for the East Shore Area. Results were compared with the simulation results of MODFLOW including the flowing well subroutine. The Pre-ICG is cycled eight times until heads do not change with the cycles. Both models predicted almost identical heads, evapotranspiration, discharge from flowing wells and flow through the general head boundary (Table 1). The absolute value of the largest difference between simulated heads of the two models did not exceed 0.01 ft.

Simulation Ability of the Modified Version of the USUGWM

The flow simulation ability of the modified version of the USUGWM is verified by comparing optimal solutions with simulation results obtained from a modified MODFLOW. The utilized MODFLOW has a flowing well subroutine added. For scenarios 1, 2a, 3a, and 4a, this comparison shows both results are almost identical (Table 2).

TABLE 1. Simulation Results of the Modified and Original McDonald and Harbaugh (MODFLOW) Models

Recharge /Discharge (cfs)	Modified Model 1st cycle	Modified Model 8th cycle	Original Model
A. Recharge to the aquifer	148.542	148.388	148.388
Constant recharge	148.388	148.388	148.388
Flowing wells	0.000	0.000	0.000
Evapotranspiration	0.010	0.000	0.000
Drain-aquifer	0.144	0.000	0.000
General head-aquifer	0.000	0.000	0.000
B. Discharge from the aquifer	148.535	148.383	143.381
Pumping wells	32.207	32.207	32.207
Flowing wells	32.752	35.944	35.944
Evapotranspiration	7.913	7.899	7.899
Drain-aquifer	51.122	50.808	50.808
General head-aquifer	21.541	21.525	21.523
C. Discrepancy (A-B)	0.008	0.005	0.007

TABLE 2. Flow Simulation Validity of the Modified Version of the USUGWM

Items	Absolute values of difference			
	SC1[a]	SC2a[b]	SC3a[c]	SC4a[d]
Water budgets (unit: cfs)				
A. Recharge to the aquifer	0.000	0.000	0.000	0.001
Constant recharge	0.000	0.000	0.000	0.000
General head-aquifer	0.000	0.000	0.000	0.001
B. Discharge from the aquifer	0.007	0.004	0.001	0.003
Pumping wells	0.000	0.000	0.000	0.000
Flowing wells	0.001	0.004	0.002	0.003
Evapotranspiration	0.000	0.001	0.000	0.000
Drain discharge	0.000	0.004	0.002	0.004
General head-interflow	0.006	0.004	0.003	0.004
Heads (unit: ft)				
Total difference of head (or drawdown)	8.67	8.06	6.67	12.77
Maximum difference of head	0.01	0.02	0.02	0.02

[a]SC1 means scenario 1.
[b]SC2a means scenario 2a.
[c]SC3a means scenario 3a.
[d]SC4a means scenario 4a.

Appendix B. Generating Influence Coefficients and Results from the Response Matrix S/O model

In general, these influence coefficients are generated in the following order:

1. When h^{um} (unmanaged head) is computed, pumping rate gp is 0.
2. When δ (influence coefficient to unit pumping) is computed, q^r is 0.

However, influence coefficients (δ) cannot be generated without known constant recharge (q^r) and an appropriate set of parameters of the SIP (Table 3); otherwise, the ICG behaves as if many cells dried up for this aquifer system. So the ICG generates h^{um} first and then generates head response to unit pumping with q^r (δ'). Lastly, it subtracts h^{um} from δ' to compute δ.

Table 3. SIP Parameters for the ICG for the East Shore Area Aquifer

SIP parameters	Values
Error criteria	0.001
Acceleration	1.0
Max Iteration	100
Iteration	5
Seed	2.77E-5
Head change	1.0

The computational accuracy of the response matrix S/O model depends on how appropriately influence coefficients are generated with the ICG. In this preliminary problem, 1 cfs is used as a unit pumping with the above SIP parameters.

Table 4 compares the results from both the embedding and response matrix S/O models.

Table 4. Computation Results of the Embedding and Response Matrix S/O models

Wells	Node (layer,row,column)	EM[a]	RMM[b]	Net Difference
A. Pumping rate (cfs)				
Weber Basin W.C.D wells				
North Ogden	(3,20,26)	1.124	1.124	0.000
Riverdale	(3,36,23)	2.235	2.234	0.001
District No. 2	(3,38,22)	5.569	5.569	0.000
Clearfield No.1	(3,42,20)	2.586	2.592	-0.006
Clearfield No.2	(3,46,22)	2.082	2.093	-0.011
South Weber No.1	(3,39,25)	7.165	7.165	0.000
South Weber No.2	(3,42,26)	5.033	5.033	0.000
Layton	(3,46,25)	0.846	0.846	0.000
Hill Air Force Base wells				
No.2 & 3	(3,40,24)	2.714	2.702	0.012
No. 4	(3,43,26)	0.285	0.285	0.000
No. 5	(3,44,23)	1.821	1.794	0.027
No. 6 & 7	(3,40,23)	3.145	3.145	0.000
Sub total of the above cells		34.605	34.582	0.023
Sub total of others		20.164	20.217	-0.053
Total		54.769	54.799	-0.030
B. Head (ft)				
Near North Ogden	(1,19,25)	4295.44	4295.44	0.00
Weber Basin W.C.D wells				
North Ogden	(3,20,26)	4349.85	4349.85	0.00
Riverdale	(3,36,23)	4265.20	4265.00	0.00
District No. 2	(3,38,22)	4265.06	4265.07	-0.01
Clearfield No.1	(3,42,20)	4264.70	4264.70	0.00
Clearfield No.2	(3,46,22)	4275.18	4275.18	0.00
South Weber No.1	(3,39,25)	4277.69	4277.69	0.00
South Weber No.2	(3,42,26)	4286.51	4286.51	0.00
Layton	(3,46,25)	4283.91	4283.91	0.00
Hill Air Force Base wells				
No.2 & 3	(3,40,24)	4276.17	4276.17	0.00
No. 4	(3,43,26)	4287.74	4287.74	0.00
No. 5	(3,44,23)	4277.59	4277.59	0.00
No. 6 & 7	(3,40,23)	4272.59	4272.58	0.01

[a]EMM means the embedding model.
[b]RMM means the response matrix model.

Appendix C. Well Data

Tables 5 and 6 show well data for M&I use pumping wells in the study area. These data are obtained from USGS (1971 and 1990).

TABLE 5. Pumping Capacities for Weber Basin W.C.D. and Hill A.F.B. Wells

Wells	Node (layer,row,column)	Pumping capacity gpm	cfs
Weber Basin W.C.D. wells			
North Ogden	(3,20,26)	750	1.671
Riverdale	(3,36,23)	2500	5.569
District No. 2	(3,38,22)	2500	5.569
Clearfield No.1	(3,42,20)	2500	5.569
Clearfield No.2	(3,46,22)	2500	5.569
South Weber No.1	(3,39,25)	4476	9.971
South Weber No.2	(3,42,26)	5000	11.138
Layton	(3,46,25)	2450	5.458
Hill A.F.B. wells			
No.2 & 3	(3,40,24)	1490	3.319
No. 4	(3,43,26)	1080	2.406
No. 5	(3,44,23)	1000	2.228
No. 6 & 7	(3,40,23)	1412	3.145

Table 6. Well data in Table 7 of Technical Publication No. 35 (1972) and the USGS Predictive Simulation Model (1/4)

Location of Public Water Supplier	Owner	Well Number	Technical Publication No.35 Yield Rate (gpm)	(ac-ft/yr)	Owner	The USGS Predictive Simulation Model Coordinates	Discharge Rate (cfs)	(ac-ft/yr)
DAVIS COUNTY								
CLEARFIELD	Clearfield City	(B-4-1)5CCC-1	1000	1614	Clearfield City	(3,45,24)	0.605	438
					Clearfield City	(3,44,21)	0.144	104
						Sub Total	0.749	543
	WBWCD CLRFLD N1	(B-5-2)36BCC-1	-		WBWCD CLRFLD N1	(3,42,20)	0.556	403
	WBWCD CLRFLD N2	(B-4-1) 7BAA-1	2500	4035	WBWCD CLRFLD N2	(3,46,22)	0.055	40
						Sub Total	0.611	443
CLINTON					Clinton	(3,41,18)	0.017	12
FARMINGTON	Farmington City	(A-3-1)18CCB-1	1780	2873	Farmington	(3,63,34)	0.487	353
	Farmington City	(A-3-1)19CDA-1	900	1453	Farmington	(3,64,35)	0.131	95
	Farmington City	(A-3-1)30CAA-3	60	97	Farmington	(3,61,34)	0.068	49
	Farmington City	(A-3-1)31CDA-2	-				0.686	497
		Sub Total	2740	4422				
FRUIT HEIGHTS					Fruit Heights	(3,56,33)	0.035	25
HILL AFB	Hill AFB No.5	(B-4-1)6DCD-1	1000	1614	Hill AFB No.5	(3,44,23)	1.273	922
	Hill AFB No.3	(B-5-1)29BDB-3	740	1194	Hill AFB No.2 & 3	(3,40,24)	1.842	1334
	Hill AFB No.2	(B-5-1)29BDC-1	750	1211				
	Hill AFB No.7	(B-5-1)30ADA-1	537	867	Hill AFB No.6 & 7	(3,40,23)	1.687	1222
	Hill AFB No.6	(B-5-1)33ADD-1	875	1412				
	Hill AFB No.4	(B-5-1)33CDA-1	1080	1743	Hill AFB No.4		0.285	206
		Sub Total	4982	8041		Sub Total	5.087	3685
KAYSVILLE								
LAYTON	Layton City	(B-4-1)8DCD-1	2380	3841	Layton City	(3,45,28)	0.671	486
	Layton City	(B-4-1)16BDD-1	2400	3874	Layton City	(3,48,26)	0.835	605
		Sub Total	4780	7715	Layton City	(3,47,25)	1.104	800
						Sub Total	2.61	1891
	WBWCD LAYTONA	(B-4-1)8ACD-1	2450	3954	WBWCD LAYTONA	(3,46,25)	0.846	613

Table 6. Well data in Table 7 of Technical Publication No. 35 (1972) and the USGS Predictive Simulation Model (2/4)

Location of Public Water Supplier	Owner	Well Number	Technical Publication No.35 Yield Rate (gpm)	Technical Publication No.35 Yield Rate (ac-ft/yr)	Owner	The USGS Predictive Simulation Model Coordinates	Discharge Rate (cfs)	Discharge Rate (ac-ft/yr)
DAVIS COUNTY								
MUTTON HOLLOW								
SOUTH WEBER					South Weber TWN	(3,41,29)	0.449	325
	WBWCD So Weber 1	(B-5-1)20DDD-2	4476	7224	WBWCD So Weber 1	(3,39,25)	6.121	4434
	WBWCD So Weber 2	(B-5-1)33BAA-2	5000	8070	WBWCD So Weber 1	(3,42,26)	5.033	3646
		Sub Total	9476	15294		Sub Total	11.154	8081
SUNSET	Sunset City	(B-5-2)26DAA-1	1300	2098	Sunset City	(3,41,19)	0.067	49
SYRACUSE	Syracuse City	(B-4-2)10DAA-2	-		Syracuse City	(3,47,17)	0.134	97
					Syracuse City 3	(3,48,18)	0.041	30
						Sub Total	0.175	127
WEST POINT	West Point town	(B-5-2)32DDD-1	950	1533	West Point Town	(3,43,15)	0.027	20
	West Point town	(B-5-2)33DDC-1	-		West Point Town	(3,43,16)	0.067	49
	West Point town	(B-5-2)34CCD-1	1400	2260		Sub Total	0.094	68
		Sub Total						
OTHERS								
Weber Baisin Jo Co.					Freeport Center 1&2	(3,46,20)	0.047	34
WEBER COUNTY								
BONA VISTA	Bona Vista WTRD	(B-6-2)1ACD-6	1450	2340				
HOOPER	Hooper IMPDIS	(B-5-2)16DDA-2	-		Hooper IMPDIS	(3,37,15)	0.835	605

Table 6. Well data in Table 7 of Technical Publication No. 35 (1972) and the USGS Predictive Simulation Model (3/4)

			Technical Publication No.35			The USGS Predictive Simulation Model		
Location of Public			Yield Rate				Discharge Rate	
Water Supplier	Owner	Well Number	(gpm)	(ac-ft/yr)	Owner	Coordinates	(cfs)	(ac-ft/yr)
WEBER COUNTY								
NORTH OGDEN	No Ogden City	(B-7-1)27CBC-1	250	404	No Ogden City	(3,17,28)	0.085	62
	No Ogden City	(B-7-1)27DDC-4	167	270		(2,17,28)	0.085	62
	No Ogden City	(B-7-1)33DBD-2	235	379	No Ogden City	(3,17,25)	0.085	62
		Sub Total				(2,17,25)	0.085	62
					No Ogden City	(3,19,27)	0.085	62
						(2,19,27)	0.085	62
						Sub Total	0.51	369
	WBWCD No Ogden	(B-6-1)4BBD-5	750	1211	WBWCD No Ogden	(3,20,26)	0.233	169
						(2,20,26)	0.233	169
						Sub Total	0.466	338
OGDEN	Ogden CAA Airport	(B-5-2)1DDA-1	230	371	Ogden City	(3,33,21)	0.01	7
PLEASANT VIEW					Pleasant View	(2,16,23)	0.102	74
						(3,16,23)	0.098	71
						Sub Total	0.2	145
RIVERDALE					Riverdale City	(3,36,22)	0.708	513
					Riverdale City	(3,37,22)	0.216	156
					Sub Total		0.924	669
	WBCWCD Riverdale	(B-5-1)18ABB-1	2500	4035	WBWCD Riverdale	(3,36,23)	0.934	677
ROY	Roy City	(B-5-2)14BDC-1	1550	2502				
SOUTH OGDEN	South Ogden CONS DS	(B-5-1)8CCA-1	2145	3462	South Ogden City	(3,36,25)	0.152	110
					S Ogden CONS DS	(3,35,26)	0.443	321
						Sub Total	0.595	431
TAYLOR-W. WEBER	Taylor-W. Weber	(B-5-2)3AAB-1	2400	3874	Taylor-W Weber 1&2	(2,32,17)	0.745	540
	Taylor-W. Weber	(B-5-2)3AAB-2	400	646				
		Sub Total	2800	4519				
WASHINGTON T	(Washington TRCE	(B-5-1)17CBC-1	1900	3067	Washington TRCE	(3,37,24)	0.089	64
	Washington TRCE	(B-5-1)17DDD-1	2500	4035	Washington TRCE	(3,37,25)	0.686	497
		Sub Total	4400	7102		Sub Total	0.775	561

Table 6. Well data in Table 7 of Technical Publication No. 35 (1972) and the USGS Predictive Simulation Model (4/4)

Location of Public Water Supplier	Owner	Well Number	Technical Publication No.35 Yield Rate (gpm)	(ac-ft/yr)	Owner	The USGS Predictive Simulation Model Coordinates	Discharge Rate (cfs)	(ac-ft/yr)
WEBER COUNTY								
OTHERS	GSL M&CC N14	(B-7-3)31AAC-1	56	90	GSL M&C C N1&2	(3,21,4)	0.096	70
	GSL M&CC N15	(B-7-3)31AAC-2	180	291	GSL M&C C N 3-10,11	(3,19,4)	0.438	317
	GSL M&CC N11	(B-7-3)31ADC-1	53	86	GSL M&C C 11,14,15	(3,18,4)	0.148	107
	GSL M&CC N3	(B-7-3)31DAA-1	29	47	GSL M&C C N13	(3,19,5)	0.049	35
	GSL M&CC N4	(B-7-3)31DAA-2	30	48		Sub Total	0.731	530
	GSL M&CC N5	(B-7-3)31DAA-3	40	65				
	GSL M&CC N12	(B-7-3)31DAA-4	69	111				
	GSL M&CC N6	(B-7-3)31DAB-1	29	47				
	GSL M&CC N7	(B-7-3)31DAB-2	72	116				
	GSL M&CC N10	(B-7-3)31DAB-3	65	105				
	GSL M&CC N8	(B-7-3)31DAC-1	28	45				
	GSL M&CC N9	(B-7-3)31DDA-1	28	45				
	GSL M&CC N13	(B-7-3)32CCB-1	90	145				
		Sub Total	769	1241				
					WBWCD DISTRICT2	(3,38,22)	2.44	1768
BOX-ELDER COUNTY								
SOUTH WILLARD	South Willard WTCO	(B-7-2)2CAD-1	-					
WILLARD	Willard City	(B-8-2)23DDA-1	1650	2663	Willard City	(3,3,19)	0.103	75
OTHERS	USBR WRW 1	(B-7-2)16AAA-1	62	100	Willard WTR DEV	(3,3,18)	0.01	7
	USBR WRW 2	(B-7-2)9CDA-1	107	173				

Appendix D. Computation Results for Management Scenarios

Computational results for management scenarios are summarized in terms of: (1) steady-state water budgets of the entire area (Tables 7, 8, and 9) and (2) the spatial distribution of pumping and flowing discharge among the 25 entities (Tables 10, 11).

TABLE 7. Computed Steady-State Water Budgets of the Aquifer for Scenarios 1 and 2a

Recharge/Discharge (cfs)		Scenario 1	Scenario 2a
A.	Recharge to the aquifer	148.388	148.388
	Constant recharge	148.388	148.388
	General head-aquifer	0.000	0.000
B.	Discharge from the aquifer	148.388	148.388
	Pumping wells	32.207	48.397
	Flowing wells	35.945	27.125
	Evapotranspiration	7.899	7.651
	Drain discharge	50.808	44.918
	General head-aquifer	21.529	20.297

TABLE 8. Computed Steady-State Water Budgets of the Aquifer for Scenarios 3a and 3b

Recharge/Discharge (cfs)		Scenario 3a	Scenario 3b
A.	Recharge to the aquifer	148.388	148.388
	Constant recharge	148.388	148.388
	General head-aquifer	0.000	0.000
B.	Discharge from the aquifer	148.388	148.388
	Pumping wells	57.575	53.375
	Flowing wells	20.772	23.148
	Evapotranspiration	7.525	7.637
	Drain discharge	43.135	44.529
	General head-aquifer	19.381	19.700

TABLE 9. Computed Steady-State Water Budgets of the Aquifer for Scenarios 4a and 4b

Recharge/Discharge (cfs)		Scenario 4a	Scenario 4b
A.	Recharge to the aquifer	148.440	148.388
	Constant recharge	148.388	148.388
	General head-aquifer	0.052	0.000
B.	Discharge from the aquifer	148.440	148.388
	Pumping wells	65.890	64.204
	Flowing wells	17.025	17.316
	Evapotranspiration	7.440	7.515
	Drain discharge	40.353	40.906
	General head-aquifer	17.733	18.448

TABLE 10. Pumping and Flowing Well Discharge of Water Entities for Scenarios 1 and 2a

Water entities	Scenario 1 gp (cfs)	Scenario 1 q^f (cfs)	Scenario 1 total (cfs)	Scenario 2a △gp (cfs)	Scenario 2a △q^f (cfs)	Scenario 2a △total (cfs)
Davis County	22.627	11.443	34.070	14.624	-3.844	10.780
Centerville	-	0.501	0.501	-	-0.032	-0.032
Clearfield	0.246	-	0.246	5.705	-	5.705
Clinton	0.017	-	0.017	0.017	-	0.017
Farmington	0.686	2.730	3.416	0.686	-0.276	0.410
Fruit Heights	0.035	-	0.035	0.035	-	0.035
Hill Field	6.248	-	6.248	5.013	-	5.013
Kaysville	-	0.864	0.864	-	-0.170	-0.170
Layton	3.456	0.472	3.928	2.832	-0.404	2.428
So. Weber	11.603	-	11.603	0.000	-	0.000
Sunset	0.067	-	0.067	0.067	-	0.067
Syracuse	0.175	4.394	4.569	0.175	-1.899	-1.724
West Point	0.094	2.482	2.576	0.094	-1.063	-0.969
Weber County	9.271	21.146	30.417	1.566	-4.859	-3.293
Ogden	0.043	0.690	0.733	0.000	-0.042	-0.042
No. Ogden	0.976	2.174	3.150	0.000	-0.049	-0.049
Pleasant View	0.200	1.228	1.428	0.000	-0.038	-0.038
Harrisville	-	1.614	1.614	-	-0.083	-0.083
Farr West	0.051	1.340	1.391	0.000	-0.080	-0.080
Plain City	-	1.037	1.037	-	-0.071	-0.071
So. Ogden	0.595	-	0.595	0.000	-	0.000
Riverdale	4.298	-	4.298	0.000	-	0.000
Roy	0.835	-	0.835	0.835	-	0.835
Washington T	0.775	-	0.775	0.000	-	0.000
Uintah	-	-	-	-	-	-
West Weber	1.498	13.063	14.561	0.731	-4.496	-3.765
Box Elder County	0.309	2.500	2.809	0.000	-0.005	-0.005
Willard City	0.309	2.500	2.809	0.000	-0.005	-0.005
Out of cityzone	-	0.856	0.856	-	-0.112	-0.112
Total	32.207	35.945	68.152	16.190	-8.820	7.370

△ means change in discharge (increase or decrease) from discharge of the nonoptimal scenario to optimal discharge in the management scenario.

TABLE 11. Pumping and Flowing Well Discharge of Water Entities for Scenarios 4a and 4b

Water entities	Scenario 4a			Scenario 4b		
	Δgp (cfs)	Δq^f (cfs)	$\Delta total$ (cfs)	Δgp (cfs)	Δq^f (cfs)	$\Delta total$ (cfs)
Davis County	21.030	-7.398	13.632	20.786	-7.407	13.379
Centerville	0.625	-0.114	0.511	0.626	-0.114	0.512
Clearfield	0.000	-	0.000	0.000	-	0.000
Clinton	0.000	-	0.000	0.000	-	0.000
Farmington	2.176	-0.687	1.489	2.129	-0.687	1.442
Fruit Heights	0.000	-	0.000	0.000	-	0.000
Hill Field	0.000	-	0.000	0.000	-	0.000
Kaysville	0.000	-0.380	-0.380	2.719	-0.381	2.338
Layton	0.456	-0.443	0.013	0.723	-0.445	0.278
So. Weber	0.000	-	0.000	0.000	-	0.000
Sunset	0.000	-	0.000	0.000	-	0.000
Syracuse	7.965	-3.439	4.526	7.870	-3.443	4.427
West Point	6.970	-2.336	4.634	6.719	-2.337	4.382
Weber County	12.652	-11.192	1.460	11.211	-10.896	0.315
Ogden	0.000	-0.046	-0.046	0.000	-0.046	-0.046
No. Ogden	0.000	-0.050	-0.050	0.000	-0.050	-0.050
Pleasant View	0.000	-0.061	-0.061	0.000	-0.060	-0.060
Harrisville	0.000	-0.095	-0.095	0.000	-0.095	-0.095
Farr West	0.000	-0.204	-0.204	0.000	-0.202	-0.202
Plain City	0.000	-0.400	-0.400	0.000	-0.400	-0.400
So. Ogden	0.000	-	0.000	0.000	-	0.000
Riverdale	0.000	-	0.000	0.000	-	0.000
Roy	0.000	-	0.000	0.000	-	0.000
Washington T	0.000	-	0.000	0.000	-	0.000
Uintah	0.000	-	0.000	0.000	-	0.000
West Weber	12.652	-10.336	2.316	11.211	-10.043	1.168
Box Elder County	0.000	-0.020	-0.020	0.309	-0.016	-0.016
Willard City	0.000	-0.020	-0.020	0.000	-0.016	-0.016
Out of city zone	-	-0.311	-0.311	-	-0.310	-0.310
Total	33.683	-18.921	14.762	31.997	-18.629	-13.368

Δ means change in discharge (increase or decrease) from discharge of the nonoptimal scenario to optimal discharge in the management scenario.

Appendix E. Computation Results for Scenarios 2b's

TABLE 12. Scenario Matrix for 2b Series

Multiple/ drawdown (ft)	Upper Bounds on pumping (gp^U): multiple of '80-'84 average				
	1	2	4	6	10
Lower bounds on pumping (gp^L): multiple of '80-'84 average					
1	1	2a[a]	2b4[a]	2b5[a]	2b6[a]
0.95	-	2b1[a]	-	-	-
0.90	-	2b2[a]	-	-	-
0.80	-	2b3[a]	-	-	-
Maximum allowable drawdown (D^L) under 1985 head (ft)					
20	-	2a(1)[b]	-	-	-
30	-	2b7(1)[b]	-	-	-
35	-	2b8(1)[b]	-	-	-
40	-	2b9(1)[b]	-	-	-

[a]Maximum allowable drawdown (DL) is 20 ft.
[b]Lower bound on pumping is 1.0 x current pumping ('80-'84 average).

TABLE 13. Computed Steady-State Water Budgets of the Aquifer for Scenarios 2b1 to 2b6

	SC2b1	SC2b2	SC2b3
	Upper bound on pumping multiple of '80-'84 average		
Recharge/Discharge (cfs)	4	6	10
A. Recharge to the aquifer	148.388	148.391	148.391
Constant recharge	148.388	148.391	148.391
General head-aquifer	0.000	0.003	0.003
B. Discharge from the aquifer	148.388	148.391	143.891
Pumping wells	49.984	51.261	51.745
Flowing wells	26.325	25.413	25.046
Evapotranspiration	7.628	7.594	7.578
Drain discharge	44.428	44.219	44.152
General head-aquifer	20.019	19.905	19.869

	SC2b4	SC2b5	SC2b6
	Lower bound on pumping multiple of '80-'84 average		
Recharge/Discharge (cfs)	0.95	0.90	0.80
A. Recharge to the aquifer	148.388	148.388	148.388
Constant recharge	148.388	148.388	148.388
General head-aquifer	0.000	0.000	0.000
B. Discharge from the aquifer	148.388	148.388	143.888
Pumping wells	50.671	51.743	52.494
Flowing wells	26.025	25.471	25.118
Evapotranspiration	7.593	7.562	7.545
Drain discharge	43.925	43.517	43.208
General head-aquifer	20.174	20.096	20.023

TABLE 14. Computed Steady-State Water Budgets of the Aquifer for Scenarios 2b7 to 2b9

	SC2b7	SC2b8	SC2b9
	Maximum allowable drawdown under 1985 head		
Recharge/Discharge (cfs)	25 ft	30 ft	40 ft
A. Recharge to the aquifer	148.388	148.388	148.388
Constant recharge	148.388	148.388	148.388
General head-aquifer	0.000	0.000	0.000
B. Discharge from the aquifer	148.388	148.388	148.388
Pumping wells	57.916	61.537	67.823
Flowing wells	22.289	20.712	18.233
Evapotranspiration	7.435	7.338	7.139
Drain discharge	41.038	39.399	36.422
General head-aquifer	19.711	19.404	18.772

Appendix F. Program List of Preliminary DSS

```
$!===========================================================================!
$!                        Decision Support System
$!                                  for
$!                Optimal Regional Sustainable Pumping Strategy
$!
$!                           - VMS Version 1.0
$!
$!                           by Shu Takahashi
$!
$!                           May 31,  1992
$!===========================================================================!
$!    setting the DCL commands
$!---------------------------------------------------------------------------!
$     Pr:= Write Sys$output
$     In:= Inquire/nopu
$!---------------------------------------------------------------------------!
$!            MAIN: the main DCL command)                                    !
$!---------------------------------------------------------------------------!
$!
$!MS -----start of MAIN
$   Type Sys$Input
             Decision Support System
                       for
       Optimal Regional Sustainable Pumping Strategy

                  - VMS Version 1.0

          Welcome to the DSS -VMS version 1.0

$!                                                                           !
$!M1 ----- display introduction                                              !
$   pr  ""
$   pr  "Do you want to know an overall structure of the DSS ?"
$   inquire check "Enter Y[ES] to continue"
$   if check .eqs. "Y" then gosub intro
$   pr  ""
$   pr  ""
$!
$!M2 ----- ready GAMS table
$   pr  "Have you already prepared GAMS tables for data ?"
$   inquire check "Enter Y[ES] to continue"
$!
$!M3 ----- display message "go JAMFLOW"
$   if check .eqs. "N" then gosub textjam
$!
$   inquire check "Enter Y[ES] to continue"
$   if .not. check then exit
$!
$!M4 ----- 1st run ?
$   pr  ""
$   pr  ""
$   pr  "Do you start
$   pr  "  a. from the 1st cycle or"
$   pr  "  b. from the other cycle using workfiles which are -.g01 to -.go8 ?" !
$   inquire cycle "Enter a or b"
$!
$!M5 ----- call predat                                                       !
$   if cycle .eqs. "A" then gosub predat
$!
```

```
$!M6 ----- call preg0
$   if cycle .eqs. "B" then gosub preg0
$!
$!M7 ----- look at comparison ?
$   pr ""
$   pr ""
$   pr "Do you compare number of nonzero elemetns, single equations,"
$   pr "and, variables for alternative models ?"
$   inquire check "Enter Y[ES] to continue"
$!
$!M8 ----- call PCPP
$   if check .eqs. "Y" then gosub pcpp
$   pr  ""
$   pr  ""
$   inquire check "Enter Y[ES] to continue"
$   if .not. check then exit
$!
$!M9 ----- select approach EMP or RMMP
$   SELECT:
$   pr ""
$   pr ""
$   pr "Which method will you use"
$   pr "A. Embedding Approach or B. Response Matrix Approach ?"
$   inquire select1 "Enter Number A or B"
$!
$!M10 ----- call EMP
$   if select1 .eqs. "A" then gosub emp
$!
$!M11 ----- call RMMP
$   if select1 .eqs. "B" then gosub rmmp
$
$   inquire check "Enter Y[ES] to continue"
$   if .not. check then exit
$!
$!ME ------ end of MAIN
$ close
$
$!----------------------------------------
$!             subroutines
$!----------------------------------------
$ intro:
$    Type Sys$Input

     Overview of System Structure
        A long-term goal is the DSS that will most efficiently optimize
     regional groundwater extraction.  The DSS uses the embedding approach,
     the response matrix approach, or their modifications, depending on which
     is most appropriate for a particular situation.  The most appropriate
     method is the one which yields acceptable, accurate answers while
     requiring the least computer processing time or memory.

       This version of the preliminary DSS is  constructed for the hypothetical!
     aquifer system problem in Chapter II and consists of the main DCL command !
     (MAIN) and four program packages.  The package includes the following:   !
     Predictive Comparison Program Package (PCPP), Embedding Model Package     !
     (EMP), Response Matrix Model Package (RMMP), and Common Utility Program   !
     Package (CUPP).

       PCPP supports a user to find the model which requires the least memory. !
```

```
     EMP and RMMP contain models using the embedding and the response         !
     matrix approaches, respectively.  All the models involve the quasi-three- !
     dimensional fininte-difference groundwater flow equation (McDonald &     !
     Harbaugh, 1988)                                                          !
                                                                              !
$    return                                                                   !
$
$   textjam:
$    Type Sys$Input

      If you have not prepared GAMS tables including several kinds of
    geohydrological data and indicators yet, you can use JAMFLOW (Tika, 1990)
    to prepare those tables.

$    return
$
$!-----------------------------------------------------------------------------!
$!             PCPP: Predictive Comparison Program Package                    !
$!-----------------------------------------------------------------------------!
$ PCPP:
$   @gams pcpv1.gms  r=c pw=118
$   type pcpv1.cmp
$   return
$
$!-----------------------------------------------------------------------------!
$!             EMP: Embedding Model Package                                   !
$!-----------------------------------------------------------------------------!
$ EMP:
$!
$!ES ----- start of EMP
$    type sys$input

                        Embedding Model Package

                            - VMS Version 1.0

$ fn=1
$!E1 ----- display introduction
$   pr  ""
$   pr  "Do you look at a general instruction for the EMP ?"
$   inquire check "Enter Y[ES] to continue"
$   if check .eqs. "Y" then gosub emintro
$   pr  ""
$   pr  ""
$!
$!E2 ----- 1st run ?
$!E3 ----- call emform
$ if cycle .eqs. "A" then gosub emform
$!
$!E4 ----- call bound
$ gosub bound
$!
$ loop1s:
$!
$!E5 ----- call emsolv  (start of cycle)
$ gosub emsolv
$!
$!E6 ----- call crit
$ gosub critem
$!
```

```
$!E7 ----- variables head & pumping converge ?
$   pr   ""
$   pr   "Do you need another cycling procedure ?"
$   inquire cycle "Enter Y or N"
$!
$!----- need more cycle
$   if cycle .eqs. "Y" then goto loop1e
$!
$!E8 ----- call output (end of cycle)
$   if cycle .eqs. "N" then gosub output
$!
$!EE ----- end of EMP
$   exit
$
$   loop1e:
$   goto loop1s
$
$!---------------------------------------
$!            subroutines
$!---------------------------------------
$ emintro:
$    Type Sys$Input

       Depending on a given flow situation as shown below, the user can
    select one of the three alternatives.

    Selection of alternative management model
    ----------------------------------------------------------------
                          Type 3 external flow terms       Choice of
    Aquire sytem          (nonsmooth function)              Model
    ----------------------------------------------------------------
    A. Confined only
                                not included                  E1
                                 included               E1, E2, or E3
    B. Unconfined/confined
                                not included               E1 or E3
                                 included               E1, E2, or E3
    ----------------------------------------------------------------
$
$    return
$
$  emform:
$  @gams emformv1.gms  r=c s=d pw=118
$  del c.g0*;*
$  return
$
$  emsolv:
$   pr "Which alternative will you use  a. E1,  b. E2, or c. E3 ?"          !
$   inquire select2 "Enter  a, b, or c"
$   pr select2
$  if select2 .eqs. "A" then gosub emalt1
$  if select2 .eqs. "B" then gosub emalt2
$  if select2 .eqs. "C" then gosub emalt3
$  return
$
$  emalt1:
$  @gams emalt1v1.gms r=d s=e pw=118
$  del d.g0*;*
$  return
$
```

```
$  emalt2:
$  @gams emalt2v1.gms r=d s=e pw=118
$  del d.g0*;*
$  return
$
$  emalt3:
$  @gams emalt3v1.gms r=d s=e pw=118
$  del d.g0*;*
$  return
$
$
$!--------------------------------------------------------------------------!
$!             RMMP: Response Matrix Model Package                          !
$!--------------------------------------------------------------------------!
$ RMMP:
$!
$!RS ----- start of RMMP
$    type sys$input

                       Response Matrix Model Package

                            - VMS Version 1.0

$ fn=1
$!
$!R1 ----- display introduction
$   pr  ""
$   pr  "Do you look at a general instruction for the RMMP ?"
$   inquire check "Enter Y[ES] to continue"
$   if check .eqs. "Y" then gosub rmintro
$   pr  ""
$   pr  ""
$!
$!R2 ----- 1st cycle ?
$!R3 ----- call idcell
$ if cycle .eqs. "A" then gosub idcell
$!
$!R4 ----- call rmform
$  gosub rmform
$!
$!R5 ----- bound
$ gosub bound
$ loop2s:
$!
$!R6 ----- call icg
$ gosub icg
$!
$!R7 ----- call rmsolv
$ gosub rmsolv
$!
$!R8 ----- call crit
$ gosub critrm
$!          0
$!R10 ----- call pncycl
$ gosub pncycl
$!
$!R9 ----- variables head & pumping converge ?
$   pr   ""
$   pr   "Do you need another cycling procedure ?"                          !
$   inquire cycle "Enter Y or N"                                            !
```

```
$!
$!
$! ----- need more cycle
$   if cycle .eqs. "Y" then goto loop2e                                      !
$!
$!R11 ----- call output (end of cycling loop)
$   if cycle .eqs. "N" then gosub output                                     !
$!
$!RE ----- end of RMMP
$ exit
$
$ loop2e:
$   fn=fn+1
$  goto loop2s
$
$!---------------------------------------
$!             subroutines
$!---------------------------------------
$ rmintro:
$    Type Sys$Input

      Three alternatives, which can simulate groundwater flow in a complex   !
    nonlinear aquifer system using the principle of superposition, can be
    selected.
      The basic idea in solving the nonlinear flow system is the same as
    Alternative E1 except that superposition rather than embedding is used
    to compute heads.  All Alternatives, R1, R2, and R3, use the influence
    coefficient generator (ICG) while R3 uses another external simulation
    model, Pre-ICG, as shown below.

    Selection of alternative management model
    --------------------------------------------------------------
                                       Combined models
    External simulation models       R1          R2        R3
    --------------------------------------------------------------
    A. ICG                           Yes         Yes       Yes

    B. Pre-ICG                       No          No        Yes
    --------------------------------------------------------------

$ return
$
$ idcell:
$ @gams idcelv1.gms r=c s=d pw=118
$ del c.g0*;*
$ return
$
$ rmform:
$   pr  ""
$   pr "Which alternative will you use  a. R1,  b. R2, or c. R3 ?"           !
$   inquire select3 "Enter  a, b, or c"
$   pr select3
$   if cycle .eqs. "A" then gosub form
$   return
$
$   form:
$   if select3 .eqs. "A" then gosub rmalt1
$   if select3 .eqs. "B" then gosub rmalt2
$   if select3 .eqs. "C" then gosub rmalt3
$   @gams rmformv1.gms r=f s=d pw=118
```

```
$   del f.g0*;*
$   return
$
$ rmalt1:
$  @gams rmalt1.gms r=d s=f pw=118
$  del d.g0*;*
$  return
$
$ rmalt2:
$  @gams rmalt2.gms r=d s=f pw=118
$  del d.g0*;*
$  return
$
$ rmalt3:
$  @gams rmalt3.gms r=d s=f pw=118
$  del d.g0*;*
$  return
$
$ icg:
$     deassign sys$input
$     run icgv1
$     rename for080.dat infsi.gms
$     @gams infsi.gms  r=d s=f pw=118
$     rename inith.si inith.sj
$     return
$
$ rmsolv:
$     @gams optsrm.gms r=f s=e PW=118
$     del f.g0*;*
$     return
$
$ critrm:
$   if select3 .eqs. "A" then gosub critem
$   if select3 .eqs. "B" then gosub critem
$   if select3 .eqs. "C" then gosub critem
$   return
$
$ pncycl:
$   if select3 .eqs. "A" then gosub prehead
$   if select3 .eqs. "B" then gosub prehead
$   if select3 .eqs. "C" then gosub preicg
$   return
$
$ prehead:
$     @gams fmtf94.gms r=d pw=118
$     return
$
$ preicg:
$     @gams prewel.gms r=d pw=118
$     run preicgv1
$     rename for079.dat inith.si
$     return
$
$!----------------------------------------------------------------------!
$!              CUPP: Common Utility Program Package                    !
$!----------------------------------------------------------------------!
$!              subroutines
$!---------------------------------------
$ predat:
$  pr ""
```

```
$  pr  "Do you have workfiles of data which area -----.g01 to -----.g08 ?"
$  pr ""
$   inquire data "Enter Y[ES] or N[O]"
$   if data .eqs. "Y" then goto preg0c
$     @gams com1v1.gms s=a  pw=118
$     @gams igsv10.gms r=a s=b pw=118
$     del a.g0*;*
$     @gams com2v1.gms r=b s=c pw=118
$     del b.g0*;*
$     return
$
$ preg0c:
$    inquire workfile "Enter Workfile of Data(---.g0 files) name"
$    copy 'workfile'.g01 c.g01
$    copy 'workfile'.g02 c.g02
$    copy 'workfile'.g03 c.g03
$    copy 'workfile'.g04 c.g04
$    copy 'workfile'.g05 c.g05
$    copy 'workfile'.g06 c.g06
$    copy 'workfile'.g07 c.g07
$    copy 'workfile'.g08 c.g08
$    return
$
$ preg0:
$    inquire workfile1 "Enter Workfile (---.g0 files) name"
$    copy 'workfile1'.g01 d.g01
$    copy 'workfile1'.g02 d.g02
$    copy 'workfile1'.g03 d.g03
$    copy 'workfile1'.g04 d.g04
$    copy 'workfile1'.g05 d.g05
$    copy 'workfile1'.g06 d.g06
$    copy 'workfile1'.g07 d.g07
$    copy 'workfile1'.g08 d.g08
$    return
$
$ postg0:
$    inquire workfile2 "Enter Workfile (---.g0 files) name"
$    rename d.g01 'workfile2'.g01
$    rename d.g02 'workfile2'.g02
$    rename d.g03 'workfile2'.g03
$    rename d.g04 'workfile2'.g04
$    rename d.g05 'workfile2'.g05
$    rename d.g06 'workfile2'.g06
$    rename d.g07 'workfile2'.g07
$    rename d.g08 'workfile2'.g08
$    return
$
$   bound:
$   pr   ""
$   pr   "Do you want to run the model with another set of bound ?"
$   inquire bound "Enter Y or N"
$   if bound .eqs. "Y" then gosub ubound
$   return
$
$   ubound:
$   @gams bound.gms r=d s=d1 pw=118
$   rename d1.g01 d.g01
$   rename d1.g02 d.g02
$   rename d1.g03 d.g03
$   rename d1.g04 d.g04
```

```
$   rename dl.g05 d.g05
$   rename dl.g06 d.g06
$   rename dl.g07 d.g07
$   rename dl.g08 d.g08
$   return
$
$ critem:
$     @gams critv1.gms   r=e s=d pw=118
$     del e.g0*;*
$     type critv1.put
$     return
$
$   output:
$
$ !  pr   ""
$ !  pr   "Do you need to datafiles for SURFER?"
$ !  inquire contl "Enter Y or N"
$ !  if contl .eqs. "Y" then gosub contour
$
$   pr   ""
$   pr   "Do you need all the optimal results ?"
$   inquire roptl "Enter Y or N"
$   if roptl .eqs. "Y" then gosub ropt
$
$   pr   ""
$   pr   "Do you need to save workfiles of the results ?"
$   inquire savel "Enter Y or N"
$   if savel .eqs. "Y" then gosub postg0
$
$   return
$
$   ropt:
$   @gams roptv1.gms r=d pw=118
$   return
$
$   contour:
$   @gams contv1.gms  r=d pw=118
$   return
```

VITA

Shu Takahashi

Candidate for the Degree of

Doctor of Philosophy

Dissertation: Comparison of Optimization Methodologies for Sustained-Yield Groundwater Pumping Planning in East Shore Area, Utah

Major Field: Biological and Irrigation Engineering

Biographical Information:

Personal Data: Born in Nakaniida, Miyagi, Japan, March 10, 1959, son of Yunosuke and Minako; married Misa Takahashi May 31, 1987; daughter--Natsuko.

Education: Received the Bachelor of Science degree in Agricultural Engineering in 1982 and the Master of Science degree in Environmental Science in 1984 from Hokkaido University, Japan; in 1992 completed the requirements for the Doctor of Philosophy degree, with a major in Irrigation Engineering from Utah State University in Logan, Utah.

Professional Experience: Irrigation Engineer at Nippon Koei Co., Ltd., Japan, 1984-present.

Papers: "Optimal Sustained-Yield Groundwater Pumping Strategy for the East Shore Area." Paper presented at the 1991 ASAE International Summer meeting, Albuquerque, New Mexico. "Optimal Aquifer Planning with Salt Water Boundary." Paper presented at the 1991 ASCE Irrigation and Drainage Division Conference, Honolulu, Hawaii. Co-author of "Embedding and Response Matrix Techniques for Maximizing Steady-State Ground-Water Extraction: Computational Comparison." Ground Water v. 29, no. 3, pp. 357-369, 1991.