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THE EFFECTS OF STOCHASTIC WATER AVAILABILITY ON WATER ALLOCATIONS IN UTAH

by

Gustavo A. Martinez Gerstl

A thesis submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Economics

Approved:

UTAH STATE UNIVERSITY
Logan, Utah

1982

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This paper is dedicated to my wife, Elba, in appreciation of her support and encouragement in learning about Economics.

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Gustavo A. Martinez Gerstl

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure

ABSTRACT

The Effects of Stochastic Water Availability on

Water Allocations in Utah

by

Gustavo A. Martinez Gerstl, Master of Science

Utah State University, 1982

Major Professor: Dr. John E. Keith
Department: Economics

A methodology to estimate stochastic surface water flows was developed and applied to a case study area using chance constrainted programming model. The results were analyzed as to the effects on different areas of production in Utah.

(44 pages)

CHAPTER I

INTRODUCTION

The interest demonstrated in developing Utah's energy resources (Snyder et al., 1981; Keith and Snyder, 1981) has brought to light certain issues which are of general importance to the state and of specific importance to public policy planners. It is critical to examine the effects certain energy development proposals might have on the air and water quality, water availability for other uses and on agricultural production. It is of interest to examine what effects reduced water supply has on water use, since water is the constraint that may or may not be binding on the development of the energy resources, but it is a constraint that is uncertain in its supply due to natural uncontrollable and unpredictable causes from year to year.

In this study, a methodology was developed and used to study the variations in water availability and to relate these variations to changes in agricultural and energy production and environmental externalities. The results should provide a basis for the formulation of public policies that would optimize the state's development of its energy and agricultural resources.

To accommodate both the state of Utah and private firms, the water management system should embody a strategy for efficiently or equitably apportioning available water under conditions of uncertainty. All hydrological phenomena are subject to random variations in quantity with some probability for periodic water shortage.

These shortages might prevent the satisfaction of the entitlements of all water rights holders. An understanding of the probability inherent in satisfying a water right (physical security) is necessary so that investment risks (whether public or private) can be properly evaluated.

The firm, if it is to embark on a long run production in the State of Utah, in an activity that uses water extensively as an input, will be interested in determining what is the probability of obtaining needed water and the acquisition cost at different probabilities. Depending upon the importance of water cost relative to the operation cost for the firm, it will decide whether to obtain its water either through buying senior water rights or by filing for unappropriated water or by a mixture of both. Together with other environmental requirements, a model that incorporates probabilities of water availability provides the firm with a summary of its needs and those of other users to determine whether to establish in Utah. If the firm does choose to operate in the state, it can decide on the best means of obtaining its water requirements.

The use of water is supervised by the state. The state's responsibilities for the use of water are mentioned in the following quote from Treleave (1977, p. 388):

> The state must superimpose controls upon the initiation of uses, the exercise of water rights, the division of water among users, and the reallocation of water rights to new users as needs change. A modern water law system must not only promote the welfare of water users, it must accomplish the state's social and economic objectives, coordinate private activities with state projects, protect the interests of the

> public in common uses and environmental values, and integrate the activities of individual and corporate users into comprehensive state water plans for water development and management.

In Utah, the primary responsibilities in this area are detailed in the Utah Code Annotated, and the Division of Water Rights is assigned to carry out the above objectives. A model of allocations could provide the state engineer, and other planners, with insight into the effects of water availability on optimal resource use. To accomplish this under conditions of uncertainty, the allocation model can be modified by incorporating probability constraints. This would provide some quantitative results with respect to the optimal water allocations.

Problem Statement

The logical extension of the model developed by Snyder et al., (1981), which determined the optimal allocation of surface water resources between energy and agriculture production is to introduce probabilistic levels of surface water availability. The changes in allocation of water and the effects upon the environment (salinity and air pollution) should provide insights into water management options. In addition, the model should give some indication of optimal operating rules under varying surface water availabilities.

Objectives

The purpose of this study is to determine the effect of variability of water supplies on water use in Utah. More specifically, the objectives are to:

1. Obtain the necessary data to determine the surface water availability in each of the H.S.U.s

2. Develop a model for fitting the data to a probabilistic distribution

3. Develop and run the computer programs to obtain the probabilistic levels of surface water availability for each H.S.U. A comparison is done with the actual data and the calculated probabilistic levels in each H.S.U.

4. Review the results obtained with the probabilistic surface water allocations against the base model.

CHAPTER II

REVIEW OF LITERATURE

Stochastic Approaches

Two major concerns were identified: stochastic programming and hydrologic data fitting. The first is relevant when optimizing under uncertainty, and the latter refers to the fitting of the water data to aprobability density function.

Among other stochastic programming techniques, chance constrained programming lends itself well to the problem at hand. This is due to the ease with which a large model such as the one developed by Snyder et al. (1981) can be modified with this technique. Chance constrained programming as developed by Charnes and Cooper (1959, 1963) and described by Wagner (1975) and Hillier and Lieberman (1967) can be applied in a simplified way. Given a problem:

$$\text{Maximize} \sum_{j=1}^{n} c_j x_j \tag{1}$$

subject to

$$\sum_{j=1}^{k} a_{ij} x_j = b_i \qquad \text{for } i = 1, \ldots, n \tag{2}$$

(first stage)

and

$$P\left[\sum_{j=1}^{k} a_{ij} x_j \leq b_i \right] \geq p_i \qquad \text{for } i = n+1, \ldots, m \tag{3}$$

(chance constraints)

and

all $x_i \geqq 0$,

where c_j are the objective function coefficients, x_j are the problem variables, a_{ij} are the constraint coefficients, b_i are the right hand side values and p_i is the probabilities that the ith constraint will be satisfied. There are j variables and i constraints. The chance constraints can be substituted by the deterministic equivalents:

$$\sum_{j=1}^{k} a_{ij}x_j \leqq B_i \qquad \text{for } i = n+1, \ldots, m \tag{4}$$

where B_i is the largest number satisfying

$$P\,[b_i \leqq B_i\,] \geqq p_i \tag{5}$$

This gives a linear model that can be solved through the usual techniques. One of the problems with this technique is the inability to cope with excesses in the availability of B_i as no indication is given for the allocation of the extra amount of resource. This approach has been used successfully for a nonlinear, seasonal-stochastic model for water by Bishop and Narayanan (1977).

Hydrologic data fitting was the other concern. Haan (1979) and Salas et al. (1980) have worked extensively in this area. They examined various approaches and probability density functions for their applicability. As seen in the previous section, this is of interest in calculating the B_i (surface water availability) with a given probability. In choosing a probability density function, some thought has to be given on the availability of practical techniques for estimating its parameters. Detailed explanations are given in Kendall and Stuart (1979) on the parameter estimation techniques examined in this study: maximum likelihood estimation and method of moments.

Existing Base Model

The base model to be chance constrained with the surface water availabilities was developed by Snyder et al. (1981). This is a conceptual model of a multiple-product firm for which the optimal input and output allocations were determined in a region that is constrained by resource availabilities and/or policy constraints.

In the specific case examined by Snyder et al. (1981), a programming model was developed for Utah to determine the optimal allocation of water between agriculture and energy production. This was done with specific environmental policy constraints on air and water quality in effect as relating to environmental quality restrictions and coal source restrictions. In addition, coal mining and transportation costs were included.

For this base model, the surface water availabilities in each of the two seasons (January-June and July-December) and the surface water availabilities including agricultural use are listed in Table 1. These are mean values for the HSUs.

TABLE 1

AVERAGE SEASONAL SURFACE WATER AVAILABILITIES BY HSU

HSU	Season 1 Jan-June	Season 2 July-Dec
	ac ft x 10^3	ac ft x 10^3
1	424.85	188.15
2	519.37	413.63
3	445.78	320.06
4	273.00	265.69
5	196.60	213.40
6	41.30	37.70
7.1	2,216.60	1,148.80
7.2	166.74	92.91
7.3	685.39	360.09
7.4	314.08	168.81
7.5	296.85	286.64
7.W	21.00	9.00
8.1	122.45	79.45
8.2	4,829.70	1,820.20
9	1,427.70	714.25
10	173.49	70.12
WY	1,114.23	682.97
CY	967.00	483.50
CW	354.20	177.15

CHAPTER III

MODEL DEVELOPMENT

In order to develop the chance constrained surface water availabilities, a theoretical model for their probability distribution had to be constructed. This was done in two parts. First, the measured headwater stream flows were normalized to the measured water. Second, budgets to account for the nonmeasurable inflows downstream and the normalized surface water flows were fitted to a probability density function.

By normalizing the headwaters to the average surface water availability, the variability in the headwaters was extended to the whole basin. Since gauging stations downstream reflect the consumptive use of any user upstream, it is extremely complicated to determine what annual variations were due to natural causes and to other voluntary uses of the water. In addition, all offstream inflows in the basin are hard to measure as all records of their occurrence (precipitation) are averaged over broad areas (climatological study units or CSUs) that have no boundary resemblance with the HSUs. (In fact, one CSU encompasses several HSUs.) (Jeppson et al., 1968)

Therefore, the extension of the headwater variability over the rest of the basin will yield an approximation that will probably be superior to any calculated figure arrived at through the integration of climatological data over the area of the HSU below the headstream measuring stations.

Data Normalization

For the i^{th} HSU, the total measured headwaters is

$$TH_{ik} = \sum_{j=1}^{n} h_{ijk} \tag{6}$$

where h_{ij} is the i^{th} stream flowing into the HSU in year k and season i. This TH_{ik} is related to the measured water budget (WB_i) for the HSU through the expected value of TH_i and a parameter γ_i that will account for nonmeasured headwaters and other runoffs into the HSU:

$$E(WB_i) = (1 + \gamma_i)E(TH_i) = E[(1 + \gamma_i)TH_i] \tag{7}$$

In the best of cases, γ_i will be low, and, in general, we would expect that:

$$0 < \gamma_i < 1$$

In none of the HSUs in Utah do we get $\gamma_i < 0$. Given Equation (7), we also can obtain the variance of the water availability

$$V(WB_i) = E[(1 + \gamma_i)TH_i - E[(1 + \gamma_i)TH_i]]^2$$

$$= (1 + \gamma_i)^2 E[TH_i - E(TH_i)]^2 \tag{8}$$

At this point, we have two descriptors of the water availability (mean and variance) and the surface water availabilities normalized for the sample years in each HSU:

$$x_{ik} = (1 + \gamma_i)TH_{ik} \tag{9}$$

Data Fitting

In fitting the observed data for surface water availability to a probability density function, certain characteristics of the sample have to be determined. Among these are the range of the data, skewness, mean, and variance. Continuous distribution functions as the normal, lognormal, gamma, Weibull, and Gumbel are used in practice (Salas et al. 1980).

The normal distribution is widely used when certain conditions hold such as zero skew, symmetry, and tails that asymptotically approach zero as x approaches large and small values (Haan 1979). Given that the data is bound at the low end ($x_i \geqq 0$), this might not be a very suitable distribution, particularly if the variance is large. This distribution can be used on skewed data if the data is transformed. Transformation is often done by using a lognormal - 2 distribution with

$$y_i = \log(x_i) \qquad (10)$$

where y_i is normally distributed with mean μ_y and variance σ_y^2. If the biases in the sample mean and variance are small, this is a good approach; but if they are highly biased, this is not a good approach (Salas et al. (1980). In the latter case, it is preferable to model the skewed series with the appropriate distribution.

For extreme value distributions on bounded series ($x_i \geqq 0$), the Gumbel and Weibull distributions are used. The Weibull is used for minimum values, and the minimum values from a lognormal follow this distribution closely. The Gumbel is an extreme value distribution and is used for maximum or minimum streamflow values (Haan 1979). These distributions are generally fitted with extreme values in the sample and are not usually suited for overall modelling of the time series.

A particular form of the Weibull that is widely used in hydrology (Haan 1979) is the gamma distribution. This is a two-parameter distribution. If necessary, a nonzero lower bound different from zero, can be used, making it a three-parameter distribution. The gamma distribution has several advantages such as assumption of a lower bound ($x_i \geq 0$), asymmetric distribution around the mode (positively skewed), a wide variety of shapes depending on the two parameters (α and β), and a wide acceptance for use in annual or semiannual hydrological data (Haan 1979). There also is a transformation of gamma distribution data into a symmetrical distribution given by:

$$y = \sqrt{x} \tag{11}$$

but this is not an exactly normal distribution (Salas et al. 1980). Additionally, if x_i is replaced by $x_i - c$, a three-parameter gamma distribution with lower bound c results. Since the use of only one distribution to model all the HSUs' surface water availabilities is anticipated, the gamma seems to fit adequately in most cases.

In using the gamma distribution, we assume that the surface water availability x in each HSU has the density function:

$$f(x;\ \alpha,\ \beta) = \begin{cases} \dfrac{1}{\Gamma(\alpha)\beta^{\alpha}}\ x^{\alpha-1}\ e^{-x/\beta} & \text{for } x > 0 \\ & \alpha,\ \beta > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Then, for a desired probability level for the surface water availability, x*,

$$\int_0^{x^*} f(x;\ \alpha,\ \beta)dx = p \qquad 0 < p \leqslant 1 \tag{12}$$

where p is the desired area under the tail of the distribution. This equation is also expressed as:

$$F(x^*;\ \alpha,\ \beta) = p \tag{13}$$

or by using the inverse function,

$$x^* = F^{-1}(\alpha,\ \beta,\ p) \tag{14}$$

With this expression, x* can be calculated when α and β are known. Since α and β are unknown, the alternative is to estimate α and β from which a point estimate for x* is obtained.

Parameter Estimation

There are various methods to estimate α and β. Two methods that are widely used are the maximum likelihood and the method of moments (Haan 1979).

The maximum likelihood estimators are not unbiased; however, as the number of observations increases (n tends to ∞), they become asymptotically unbiased. In addition, maximum likelihood estimators are sufficient and consistent; and if an efficient estimator exists, the maximum likelihood estimator, after correction for bias, will be efficient. The method of moments will equate the first m moments of the distribution to the first m sample moments. Then the resultant m equations can be solved for m unknown parameters. Since only two parameters (α and β) are to be estimated, the first two moments have to be calculated. The method of moments will not always produce the same estimates for the parameters as the method of maximum likelihood. However, it is not always possible to obtain the maximum likelihood of estimators except through iterative numerical solutions. The accuracy of the method of moments can suffer if the moments are long. If a sample from the population is used, the estimates are not the most efficient (Kendall and Stuart 1979).

By assuming we have n random observations, $x_1, \ldots, x_n$, then their joint probability function is $\phi_x(x, \alpha, \beta)$, and the likelihood function is:

$$L(\alpha, \beta) = \prod_{i=1}^{n} \phi_x(x_i; \alpha, \beta) \tag{15}$$

Given that x is gamma distributed with parameters α and β, the joint density function, $\phi(\hat{\alpha}, \hat{\beta})$, would be asymtotically normally distributed so that

$$\phi(\hat{\alpha}, \hat{\beta}) \sim N \left[\begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \begin{bmatrix} \sigma_\alpha^2 & \sigma_{\alpha\beta}^2 \\ \sigma_{\alpha\beta}^2 & \sigma_\beta^2 \end{bmatrix}^{-1} \right] \tag{16}$$

where $$\sigma_{\alpha}^{2} = -E\left(\frac{\partial^{2}\text{Log } L}{\partial\ \alpha^{2}}\right) \tag{17}$$

$$\sigma_{\alpha\beta}^{2} = -E\left(\frac{\partial^{2}\text{Log } L}{\partial\ \alpha\ \partial\ \beta}\right) \tag{18}$$

$$\sigma_{\beta}^{2} = -E\left(\frac{\partial^{2}\text{Log } L}{\partial\beta^{2}}\right) \tag{19}$$

and L is the likelihood function. In this case,

$$L = \left|\frac{1}{\Gamma(\alpha)\beta^{\alpha}}\right|^{n} \prod_{i=1}^{n} (x_i^{\alpha-1} e^{-x_i/\beta}) \tag{20}$$

By obtaining the first order conditions with respect to α and β, we obtain the parameter estimates $\hat{\alpha}$ and $\hat{\beta}$. In practice, the expression used is:

$$\frac{\partial\ \ln\ L}{\partial\alpha} = \frac{L_{\alpha}^{'}}{L} \tag{21}$$

and

$$\frac{\partial\ \ln\ L}{\partial\ \beta} = \frac{L_{\beta}^{'}}{L} \tag{22}$$

where $L > 0$.

Now we obtain the maximum likelihood estimate of x^* by the invariance property:

$$\hat{x}^* = F^{-1}(\hat{\alpha},\ \hat{\beta},\ p) \tag{23}$$

since $\hat{x}^*$ is a MLE. Therefore, under general conditions, $\hat{x}^*$ is a consistent estimator of x^*. Thus,

$$E(\hat{x}^*) = F^{-1}(\alpha,\ \beta,\ p) \tag{24}$$

As the number of observations (n) tends to infinity, the variance of $\hat{x}^*$ becomes asymptotically zero.

This method is not used in the empirical model because of the difficulty in estimating $\hat{\alpha}$ and $\hat{\beta}$ and analytically differentiating the gamma function where α is unknown. Although the maximum likelihood estimation is the preferred method (Haan 1979), there are cases where it is more practical to use the method of moments even though it may not be the most efficient method (Kendall and Stuart 1979).

For this second method, a moment-generating function is defined. Then the first two moments are evaluated for t = 0 and equated to the sample moments.

The moment generating function (MGF) is given by the following:

$$M_x(t) = E(e^{tx}) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \int_0^{\infty} e^{tx} x^{\alpha-1} e^{-x/\beta} dx$$

$$= \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \int_0^{\infty} e^{-x(-t + 1/\beta)} x^{\alpha-1} dx \tag{25}$$

By manipulating this equation (Appendix A), the first and second ordinary moments can be evaluated at t = 0. The first ordinary moment is:

$$\left. \frac{d\, M_x(t)}{dt} \right|_{t=0} = M_1 = \alpha\beta(1-\beta t)^{-\alpha-1} = \alpha\beta \tag{26}$$

and the second ordinary moment is:

$$\left.\frac{d^2M_x(t)}{dt^2}\right|_{t=0} = M_2 = \alpha\beta^2(\alpha+1)(1-\beta t)^{-x-2} = \alpha\beta^2(\alpha+1) \quad (27)$$

Setting M_1 and M_2 equal to the sample moments, then

$$M_1 = \hat{\mu} = \hat{\alpha}\hat{\beta} \quad (28)$$

and

$$\hat{\sigma}^2 = M_2 - M_1^2 \quad (29)$$

The derivation of the variance equation[1] leads to

$$\hat{\sigma}^2 = \hat{\alpha}^2\hat{\beta}^2 + \hat{\alpha}\hat{\beta}^2 - \hat{\alpha}^2\hat{\beta}^2 = \hat{\alpha}\hat{\beta}^2 \quad (30)$$

By simultaneously solving equations (28) and (30), we obtain the estimates of $\hat{\alpha}$ and $\hat{\beta}$ for use in the gamma distribution as follows:

$$\hat{\alpha} = \frac{\hat{\mu}}{\hat{\beta}} \quad (31)$$

Since,

$$\beta^2 = \frac{\hat{\sigma}}{\hat{\alpha}} = \frac{\hat{\sigma}^2}{\frac{\hat{\mu}}{\hat{\beta}}} = \frac{\hat{\sigma}^2\hat{\beta}}{\hat{\mu}} \quad (32)$$

then

$$\hat{\beta} = \frac{\hat{\sigma}^2}{\hat{\mu}} \quad (33)$$

and, by substituting into equation (31),

---

[1]The sample's first and second moments are:

$M_1 = \sum_i x_i/n$; $M_2 = \sum_i x_i^2/n$; and $\hat{\sigma}^2 = \Sigma x_i^2/n - (\Sigma x_i/n)^2$.

Therefore, equation (29) is valid.

$$\hat{\alpha} = \frac{\hat{\mu}^2}{\hat{\sigma}^2} \tag{34}$$

Given a vector of desired probabilities, we can use equation (13) to determine

$$F(x^*;\ \hat{\alpha},\ \hat{\beta}) = p_m \qquad \text{for } m = 1,\ .\ .\ .\ ,\ M \tag{35}$$

By expansion

$$\frac{1}{\Gamma(\hat{\alpha})\hat{\beta}^{\hat{\alpha}^2}} \int_0^{x^*} x^{\hat{\alpha}-1} e^{-x/\hat{\beta}} dx = p_m \qquad \text{for } m = 1,\ .\ .\ .\ ,\ M \tag{36}$$

where the left hand side is the incomplete gamma function.

The incomplete gamma distribution can be transformed into a three-parameter distribution by the addition of a lower bound component. There are three possibilities for c: it can be zero or the two-parameter case; it can be calculated; and it can be the sample low flow (x_{min}). The latter alternatives might produce a better fit whenever the sample data are not close to zero.

CHAPTER IV

EMPIRICAL MODEL

Study Area Description

In the original model, Utah was divided into various Hydrological Study Units (HSU's). These are defined in Table 2 (Snyder et al. 1981) and they are also described in Fig. 1. They form part of two major drainages: the Colorado River Basin and the Great Basin.

TABLE 2

HYDROLOGICAL STUDY UNITS IN UTAH

HSU No.	Basin Name	Drainage
1	Western Desert	Great Basin
2	Bear River	Great Basin
3	Ogden River	Great Basin
4	Jordan River	Great Basin
5	Sevier River	Great Basin
6	Cedar Beaver	Great Basin
7.1	Green River	Colorado River
7.2	Uintah River	Colorado River
7.3	Lake Fork	Colorado River
7.4	Rock Creek	Colorado River
7.5	Headwaters of Strawberry and Duchesne R.	Colorado River
7.W	White River in Utah	Colorado River
8.1	Price River	Colorado River
8.2	West of Colorado and East of Wasatch	Colorado River
9	South and East of Colorado River	Colorado River
10	Virgin River	Colorado River
WY	Wyoming Inflow	Colorado River
CY	Colorado Yampa	Colorado River
CW	Colorado White	Colorado River

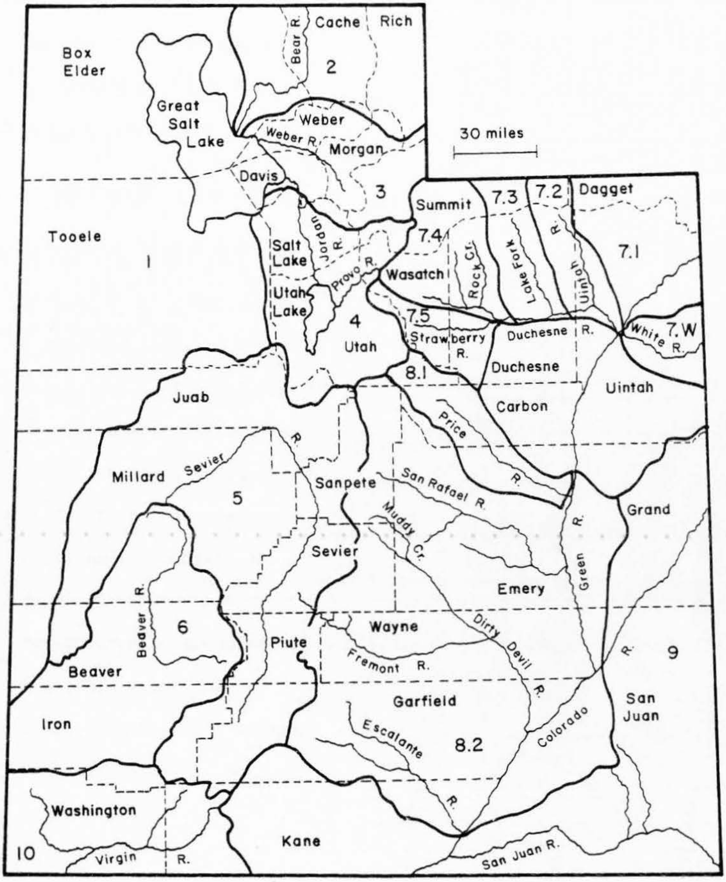

Figure 1. County boundaries, major drainage systems, and hydrologic study units of Utah.

Additionally, the state is divided into four economic regions: The Wasatch Front (HSUs 1, 2, 3, and 4), the Southwest (HSUs 5, 6, and 10), the Uintah Basin (HSUs 7.1, 7.2, 7.3, 7.4, 7.5, and 7.W), and the Southeast (HSUs 8.1, 8.2, and 9). These regions generally correspond to county boundaries, particularly with respect to economic activity.

Data Collection

There are various sources of data for surface water availability but the primary source is the United States Geological Service (USGS) streamflow data, collected at or on stream gauging stations in each drainage. These data are readily available for most streams for a varying number of years at each station. The daily measurements reflect the precipitation less existing use upstream of the station. In addition these data are the original sources of the surface water availability budgets for the HSU's as defined by King et al. (1972). He added consumptive use to the existing flows and then compared for returns to groundwater to obtain estimates of average water availabilities. Given the needs of this study, the primary data source was the USGS streamflow data tape (WATSTORE) for the state of Utah, which covers both the Colorado River drainage and the Great Basin drainage.

Empirical Model Development

The estimation of the model from actual stream flow data was done in various steps. The first step was to extract the headstream flow data for each HSU from the USGS data tape. This was done in order to create a data file for each HSU. The second step was to accumulate the data by season and normalize it against the average surface water availabilities. At this stage, some descriptive statistics are also calculated. The final step is to calculate the probability levels for each HSU by season and then compare the actual data against these levels to obtain the observed probabilities. The last step was repeated under the various assumptions with respect to the intercept for the distribution. A flowchart of the system is shown in Fig. 2 and it shows the three programs that correspond to the steps above mentioned. To preserve the integrity of the calculations in this last step, the subroutine MDGAM from the IMSL library was used to calculate the incomplete gamma function. The observed probabilities will indicate any gross abnormality in fit.

Probabilistic Water Availabilities

For all the HSU's (except 1 and 4) the best overall fit was obtained by using a lower bound defined by the observed low flow. The availabilities were obtained for probabilities of 85%, 90%, and 95%, and are shown in Table 3 for the two seasons. For HSU 1 (Western Desert) there was not enough measured data to account for the measured water budget. Given the nature of the basins (arid, extensive, and subject to wide variations in rainfall over the basin), the average was assumed to be the best measure available. In HSU 4 (Jordan River) the surface water availability is so highly

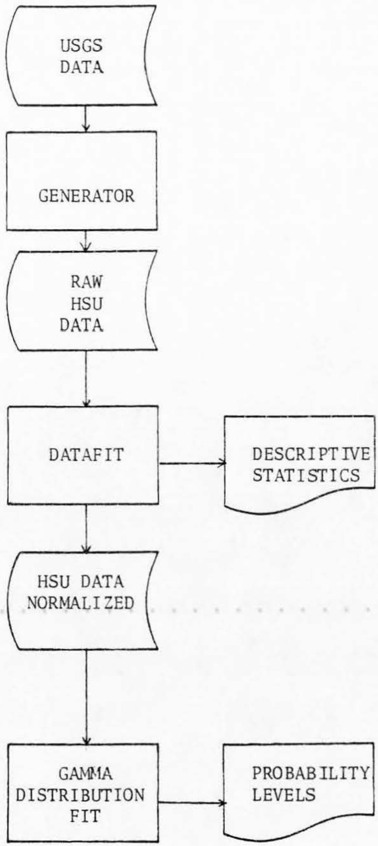

Figure 2. System flow chart.

TABLE 3

PROBABILISTIC SEASONAL SURFACE WATER AVAILABILITIES

BY HSU IN UTAH (ACRE/FEET)

HSU	SEASON 1 (Jan-Jun)			SEASON 2 (Jul-Dec)		
	85%	90%	95%	85%	90%	95%
2	337210.	305143.	261619.	280956.	256891.	223907.
3	216960.	183642.	141265.	238393.	222640.	200633.
5	103440.	89154.	70651.	154378.	143215.	127709.
6	16898.	13731.	9869.	18660.	15858.	12278.
7.1	13663.	10337.	6579.	12410.	10815.	8726.
7.2	117229.	108039.	95351.	67859.	63092.	56456.
7.3	542198.	513427.	427734.	230242.	207615.	177011.
7.4	194441.	174025.	146612.	68496.	55550.	39791.
7.5	199298.	181741.	157739.	214183.	200920.	181466.
7.W	11835.	10366.	8433.	5489.	4896.	4103.
8.1	59580.	51440.	40880.	21234.	16086.	10263.
8.2	1231670.	893040.	52280.	1160030.	1045240.	890060.
9	658370.	550310.	414540.	342368.	288769.	220949.
10	51922.	39144.	24769.	43679.	39149.	33060.
CW	101305.	75536.	46891.	58409.	54077.	29686.
CY	357608.	283685.	194773.	169435.	132694.	89594.
WY	640798	563848.	462177.	403930.	357742.	296340.

regulated that the measured water budget would be available under most circumstances.

CHAPTER V

SUMMARY OF ALLOCATION EFFECTS

By comparing the base model (Snyder et al. 1981) and the chance constrained models, significant effects were identified. The base solution for the model was obtained using average surface water availabilities. In addition, nondegradation policies dictated the maximum salinity levels established in 1972. These levels are consistent with the Federal Water Pollution Control Act Amendments (PL 92-500), the Colorado River Basin Salinity Control Act (PL 92-320) and the Colorado River Salinity Forum recommendations. Treatment of return flows from agriculture was the only mitigating possibility for irrigated agriculture. The treatments considered were sprinkler irrigation and canal lining. Franklin (1982) indicated that publicly financed salinity controls could efficiently be implemented to reduce the impact of salinity restrictions on agriculture. However, with privately financed treatments, agricultural production was constrained in HSUs 1, 5, 7.4, 9 and 10 by the salinity levels. Salinity analysis performed on the model confirmed that salinity and water were critical constraints on agricultural production.

A new base model solution with no salinity level requirements (Base NSC) was obtained. There were some important differences between this solution and the previous solution (Base). The agricultural land presently under irrigation (Class I, II, III, and IIIP) was increased in most cases to the current maximums. In addition the amount of irrigation (full or partial) was also increased. Accompanying these increases was the drop in the shadow price for water, to zero in all HSUs except 5, 6 and 8.1. Because agricultural production is the marginal use of water, that is, the value of marginal product for water is lower than for energy producers, electrical production did not change.

Upon restricting the surface water availability in the chance constrained model (85% probability level) with the salinity constraints in place, the solution becomes unfeasible. This is caused by the inability in the model to reduce the salt loading sufficiently to meet the standards by treatment of agricultural return flows or retirement of land. The natural loading is not reduced proportionately to the decrease in water availabilities (Jeppson et al. 1968). This causes the salt concentration to rise more than the elimination of agricultural loading can compensate for. Thus treatment or retention could not meet standards. Clearly the lower the availabilities, the more constraining the salinity standards are.

Only average salinity levels are expected to be maintained over a long period of time (20 years). The relaxation of these constraints when water availability is reduced, is expected and necessary. However the base case solution with no salinity constraints (Base NSC) was needed to separate the effects of salinity constraint relaxation from those of water reduction in the chance constrained models (85%, 90%, and 95% probabilities).

The chance constrained model solutions were compared to the base model with no salinity constraints (Base NSC). With reductions in surface water availabilities (85%, 90% and 95% probabilities) there is no decrease in irrigated acres with the exception of HSUs 10 and CW. A closer examination of the solutions show that instead of reducing the acreage under irrigation, reduced application in some HSUs (full to partial or one season only) occurs. As a result the foregone profits from decrements to water supply increase as availability decreases, as seen in the increasing shadow price. Table 4 shows the base case solution, the base case with no salinity constraint (Base NSC) solution and the differences between this last solution and the chance constrained solutions.

As water is reduced, the shadow price stays at zero with the exception of HSUs 5, 6, 7W, CW, 7.4, 8.1 and 9 (Table 5). This is to be expected because of the lack of treatment costs and it confirms that agricultural land, even when marginally profitable, will be under some form of irrigation when water is available or salinity standards are relaxed.

Another possible change as surface water availabilities are reduced, is to increase water storage capacity. Storage transfers early runoff to the second season. With one exception this does not happen because agricultural profits at the margin are not large enough to pay for the construction of storage facilities and electrical producers can purchase the existing water rights by paying higher than the agricultural shadow prices. In HSU 8.1 (Price River) 620 and 6443 acre/feet are indicated with the 90% and 95% probability model solutions, respectively. This is expected as the second

TABLE 4

CHANGES IN PRESENTLY IRRIGATED AGRICULTURAL LAND (ACRES) BY HSU IN UTAH

HSU		Base	Base NSL	85% NSL	90% NSL	95% NSL
1	Western Desert	13803.	40000.			
2.	Bear River	212000.	237548.			
3	Ogden River	144366.	144366.			
4	Jordan River	179478.	179478.			
5	Sevier River	272200.	282701.			
6	Cedar Beaver	71500.	75866.			
7.1	Green River	4600.	4600.			
7.W	Uintah River	0.	0.			
7.2	Lake Fork	21000.	21000.			
7.3	Rock Creek	36000.	36000.			
7.4	Headwaters of Strawberry and Duchesne Rivers	27911.	27911.			
7.5	White River/Utah	20000.	20000.			
8.1	Price River	17944.	18000.			
8.2	West of Colorado and East of Wasatch	51510.	62500.			
9	South and East of Colorado R.	9585.	11442.			
10	Virgin River	0.	20300.			(659.)
WY	Wyoming inflow	184116.	251185.			
CY	Colorado Yampa	36374.	36374.			
CW	Colorado White	5753.	22371.	(5099.)	(5503.)	(8664.)

TABLE 5

SHADOW PRICE OF WATER

HSU		Season 1 (January-June)				Season 2 (July-December)			
		Base Case	85%	90%	95%	Base Case	85%	90%	95%
5	Sevier River	4.41	5.27	5.27	5.27				
6	Cedar Beaver	6.13	6.13	6.13	6.13				
7W	Uintah River							6.34	19.87
CW	Colorado White							6.34	19.87
7.4	Headwaters of Strawberry and Duchesne Rivers						0.74	7.78	9.14
8.1	Price River	1.4	2.26	2.26	2.26	1.4	26.28	34.08	34.09
9	South and East of Colorado River				4.76				

season shadow prices for water in HSU 8.1 are quite high, compared to the other HSUs (Table 5).

Electrical production does not change from the base case when the salinity constraints are relaxed (Base NSC) but with the water availability reductions in the chance constrained models, there is some shifting of production. The 85% probability level has a shift out of Western Box Elder to the California plants (Barstow and Cadiz) (Table 6) and some smaller shifts. The reduction in profit due to the loss of irrigated agriculture in HSU 1 is sufficiently high to make the Barstow-Cadiz plants more profitable using New Mexico coal than the Box Elder plants using Utah coal. These shifts are the result of a small difference in electrical generation profitability among the four plants which is offset by a small loss in agricultural profits. Whether such a shift would occur in reality is questionable. However, the similarity of electrical generation profitability among the plants is itself of interest. The 90% profitability level has only a minor adjustment between Warner Valley and Northwest Box Elder and the 95% model has no shifts in production sites. This last result is to be expected since electrical producers are not the marginal users. Their value of the marginal product of water allows them to acquire senior water rights, which have a high probability of being satisfied, at prices in excess of their value in agricultural production.

The previous results indicate that water reduction will not have much effect, given a relaxation of nondegradation policies. Certain procedures (electrical and other energy) are able to pay a

TABLE 6

CHANGES IN ELECTRICAL PRODUCTION (MWH)

	Plant	Base	Base NSC	85% NSC	90% NSC
7	East Juab	10735200.	10735200.	46800	46800.
8	East Basin	665780.	665780.		
9	Sanpete Sevier	2690040.	2690040.		
10	Warner Valley	2817149.	2817149.	(309223.)	(72006.)
11	Western Box Elder	1752000.	1752000.	(1687016.)	(1687016.)
12	Northwest Box Elder	3832398.	3832398.	243532.	6305.
15	Northeast Millard	5693816.	5693816.		
16	Milford-Black Rock	2944668.	2944668.		
17	Iron County	864578.	864573.		
18	Southwest Emery	750887.	750887.		
19	West Carbon	2295393.	2295393.		
20	East Carbon	1721545.	1721545.		
21	S. W. Emery	1147696.	1147696.		
22	East Grand	210220.	210220.		
N1	Harry Allen	723440.	723440.		
NM1	Star Lake	34063.	34063.	(124.)	(124.)
C1	Barstow	419629.	419629.	979134.	979134.
C2	Cadiz	6590086.	6590086.	707564.	707564.
W1	Kemmerer	3190997.	3190997.	19228.	19228.

high price for all their water needs since the net revenues from their production are high. The models indicate that to reduce environmental requirements as needed is the most reasonable policy. To require the same standards under water reduction conditions would reduce agricultural production drastically. Without the environmental standards total output decreases as water is reduced, but the maximum reduction is less than the one per cost of total profit in the base case. It can be concluded that water reduction is a manageable situation that should not result in undue loss of output.

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APPENDIX

Derivation of the First and Second Ordinary Moments for the Gamma Distribution

Given the moment generating function for the gamma distribution as:

$$M_x(t) = E(e^{tx}) \tag{37}$$

$$= \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \int_0^{\infty} e^{tx} x^{\alpha-1} e^{-x/\beta}\, dx \tag{38}$$

$$= \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \int_0^{\infty} e^{-x(-t+1/\beta)} x^{\alpha-1}\, dx \tag{39}$$

if we set

$$z = x(\frac{1}{\beta} - t)$$

then

$$dz = (\frac{1}{\beta} - t)dx$$

and by substitution into equation (39)

$$M_x(t) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \int_0^{\infty} e^{-z} \left[\frac{z}{(1/\beta - t)}\right]^{\alpha-1} \frac{dz}{(1/\beta - t)} \tag{40}$$

$$= \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \int \frac{e^{-z} z^{\alpha-1}}{(1/\beta - t)^{\alpha}}\, dz \tag{41}$$

$$= \frac{1}{\Gamma(\alpha)\beta^{\alpha}(1/\beta - t)^{\alpha}} \int_0^{\infty} e^{-z} z^{\alpha-1}\, dz \qquad (42)$$

and as by definition:

$$\frac{1}{\Gamma(\alpha)\beta^{\alpha}} \int_0^{\infty} e^{-z} z^{\alpha-1}\, dz = 1 \qquad (43)$$

then equation (42) will collapse into

$$M_x(t) = \frac{1}{(1 - \beta t)^{\alpha}} = (1 - \beta t)^{-\alpha} \qquad (44)$$

and then the first ordinary moment evaluated at t = 0 will be:

$$M_1 = \left.\frac{d\, M_x(t)}{dt}\right|_{t=0} = \alpha\beta(1 - \beta t)^{-\alpha-1} = \alpha\beta \qquad (45)$$

and the second ordinary moment, also evaluated when t = 0, is:

$$M_2 = \left.\frac{d\, M_x(t)}{dt}\right|_{t=0} = \alpha\beta^2(\alpha+1)(1-\beta t)^{-\alpha-2} = \alpha^2\beta^2 + \alpha\beta^2 \qquad (46)$$