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## Staged and Notched Labyrinth Weir Hydraulics

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Dabling, Mitchell R. and Crookston, Brian M., "Staged and Notched Labyrinth Weir Hydraulics" (2012).  
*International Junior Researcher and Engineer Workshop on Hydraulic Structures. 2.*  
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## STAGED AND NOTCHED LABYRINTH WEIR HYDRAULICS

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**ABSTRACT:** Replacement spillways are frequently required to pass revised and larger design storm events. Generally matching the outflow hydrograph of the existing spillway is also a common design requirement. Labyrinth spillways can increase spillway discharge capacity. Staged and notched sections of crest have been used in design to satisfy discharge hydrograph requirements. However, inadequate hydraulic design information is available specific to staged and notched labyrinth weirs. In this study, the flow characteristics of multiple staged and notched labyrinth weir configurations (laboratory-scale) were tested. Head-discharge relationships were evaluated experimentally and compared with computed results using superposition (predicting the discharge over the upper and lower stages separately and summing). The results of this comparison show that, for all configurations tested, the superposition technique estimated actual discharges by approximately  $\pm 10\%$ .

**Keywords:** labyrinth spillways, staged weir, head-discharge relationship, flood impacts

### INTRODUCTION

#### Labyrinth Weirs

Dams are a critical infrastructure component throughout the world. They provide water supply (municipal, agricultural, industrial), flood control, hydropower, navigation, and recreation. The benefits provided by many existing dams are still needed today, with new dams regularly under construction to meet growing needs. However, aging infrastructure, new spillway design flood criteria, and increasing water supply demands often require spillway rehabilitation.

As shown in Figure 1, the geometry of a labyrinth weir can significantly increase the crest length within a given channel width. The additional crest length will generally increase discharge capacity for a given upstream water elevation. As a result of their hydraulic performance, labyrinth weirs have been of interest to practitioners and researchers for many years. A selection of labyrinth weir design publications focused on discharge performance are: HAY and TAYLOR (1970), DARVAS (1971), HINCHLIFF and HOUSTON (1984), LUX and

HINCHLIFF (1985), MAGALHÃES and LORENA (1989), TULLIS et al. (1995), MELO et al. (2002), FALVEY (2003), TULLIS et al. (2007), CROOKSTON (2010), CROOKSTON et al. (2012), and CROOKSTON and TULLIS (2012a,b,c). Labyrinth weirs have been used with great success to increase spillway capacity and manage upstream flooding.

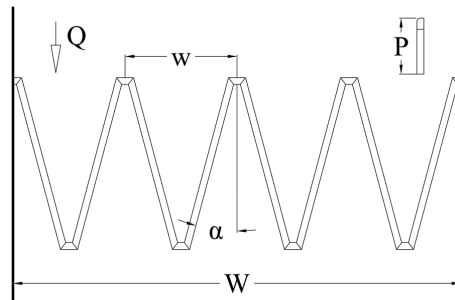


Figure 1 – Labyrinth weir geometric and hydraulic variables

Replacement spillways are frequently required to pass revised and larger design storm events; they are also often required to generally match the outflow hydrograph of the existing spillway. For example, peak outflows from a new spillway may be required to be less than or equal to the existing spillway peak outflows for the more frequent (e.g., 2-, 10-, and 100-year) flood events. The increased hydraulic capacity of a labyrinth spillway can decrease reservoir attenuation and increase peak outflows, which could potentially increase downstream flooding for moderate floods that occur with greater frequency (PAXSON et al. 2011).

#### Staged Labyrinth Spillways

In order to meet spillway peak outflow requirements, a variety of spillway types (e.g., broad-crested weirs, ogee spillways, labyrinth spillways) have been designed and built that feature multiple crest elevations. Such spillways are termed staged or notched spillways. Although the terms ‘notch’ and ‘stage’ have been used interchangeably in conversation and published literature, this paper defines a stage as any portion of the spillway crest set at a different elevation. A notch refers to a low stage with a crest length that is less than the labyrinth sidewall length. A notch or lower stage(s) may be set at the normal pool elevation and convey base flows and runoff from smaller storms (e.g., up to the 100-year event). The higher stage would provide the additional discharge capacity required for the more extreme event (e.g., probable maximum flood). In addition to ‘tuning’ the head-discharge rating curve, notched or multi-staged crests confine base-flows and smaller storm events to a portion of the spillway and, at very low heads,

can thicken the nappe in the lower stage(s) to prevent nappe vibration, and limit algal growth. The recently constructed Lake Townsend Dam (presented in Figure 2) features a 7-cycle staged labyrinth spillway; 2 cycles have a lower stage elevation by approximately 0.3 m.



Figure 2 – Staged labyrinth spillway at Lake Townsend, Greensboro, NC, USA

Although numerous design methods have been published for labyrinth weirs, there is insufficient design information available regarding labyrinth weirs with staged or notched crests. Practicing engineers would benefit from this information, as it would facilitate more accurate stage-discharge relationship estimations. The objective of this study is to investigate the hydraulic performance of notched and staged labyrinth weirs.

## EXPERIMENTAL SETUP

Physical modelling was conducted at the Utah Water Research Laboratory (UWRL) in a gravity-fed rectangular laboratory flume (1.2-m x 14.6-m x 1.0-m deep). A 4-cycle  $15^\circ$  sidewall angle ( $\alpha = 15^\circ$ ) labyrinth weir with a quarter-round crest shape was tested with the following crest stage/notch configurations (see Figure 3): apex notches, one-half sidewall length notch (centered on upstream apex), one staged cycle, and an unmodified labyrinth (constant crest elevation). Staged and notched section depths were 20% of the weir height and featured a quarter-round crest shape. However, due to the size of the apex notches, the crest within those notches was flat-topped. The test matrix is summarized in Table 1.

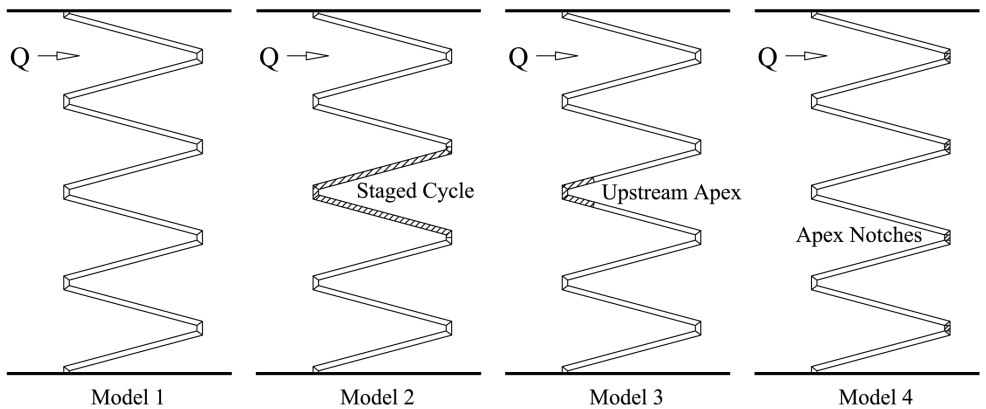


Figure 3 – Schematic of tested weir configurations

Table 1 – Physical model test matrix

Labyrinth Geometry	Model		Stage/Notch Geometry	Crest Shape <sup>†</sup>
	(#)	Description	$l_{stage}$	
$\alpha = 15^\circ, N = 4$	1	No Stage	0.0 mm	QR
$l_{c-cycle} = 995.7 \text{ mm}$	2	Staged Cycle	995.7 mm	QR
$P = 152.4 \text{ mm}$	3	Notched Upstream Apex	232.6 mm	QR
$w = 305.9 \text{ mm}$	4	Apex Notches	18.4 mm x 4	Flat

<sup>†</sup> QR = Quarter Round where  $R_{crest} = 1/2 t_w$

## EXPERIMENTAL RESULTS

### Head-discharge Performance

Eq. 1 was selected to quantify the head-discharge relationship of the tested physical models. It is a common form of the weir equation (HENDERSON 1966) and was used to calculate discharge coefficients for varying flow conditions.

$$Q = \frac{2}{3} C_d L \sqrt{2g} H_t^{3/2} \quad (1)$$

In Eq. (1), Q is flow rate;  $C_d$  is a dimensionless discharge coefficient that varies with weir type, geometry, crest shape, and flow conditions; L is the weir crest length; g is the gravitational acceleration constant; and  $H_t$  is the free-flow (non-submerged) upstream total head measured relative to the weir crest elevation.  $H_t$  was used rather than the piezometric head (h) to account for approach flow velocities. A stilling well equipped with a point gauge readable to  $\pm 0.15 \text{ mm}$

located 6.5P (P is the weir height) upstream of the weir, was used to measure h.  $H_t$  was then calculated as  $h+V^2/2g$ . Approximately 15 to 30 flow measurements were taken for each weir configuration.  $C_d$  values were computed for each measured flow condition and are presented in Figure 5. An empirical curve-fit equation based upon the headwater ratio,  $H_t/P$ , was fit to experimentally determined  $C_d$  values ( $R^2 > 0.995$ ) and is presented as Eq. (2). Corresponding curvefit coefficients are presented in Table 2.

$$C_d = a \left( b \frac{H_t}{P} \right) \left( \frac{H_t}{P} \right)^c + d$$

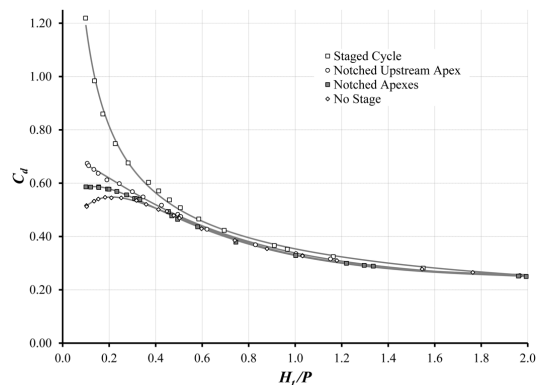


Figure 4 – Head-discharge relationships of tested weirs

Table 2 – Physical model test matrix

Model		Coefficients for Eq. 2			
(#)	Description	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
1	No Stage	1.3400	0.0616	0.5860	0.2489
2	Staged Cycle	0.2617	0.7997	-0.6117	0.1445
3	Notched Upstream Apex	0.6312	0.1701	0.0819	0.2309
4	Notched Apexes	0.9058	0.0976	0.3232	0.2407

Figure 4 presents the head-discharge relationships for each labyrinth model; the experimental setup did not allow flow measurement specific to the staged section. Note in Figure 4 that the experimentally determined  $C_d$  values are greater than  $C_d$  values of the unmodified labyrinth weir for a given  $H_t/P$ . This is partly due to flows concentrated over the low stage, which impacted the discharge performance of the entire spillway model. Because of the additional flow over these notches,  $C_d$  value estimations are higher than those typical of a labyrinth weir with similar cycle geometry and a single crest elevation. This can result in  $C_d$  values being greater than 1.0.

The applicability of the principle of superposition was investigated (total  $Q$  is a function of the sum of  $Q$  over each stage) using the experimental results for comparison. Flow was calculated over the high stage of the labyrinth weir using Eq. 1 with the weir length adjusted to  $L = L_c - l_{stage}$ . To calculate the flow over the notch/stage,  $L = l_{stage}$ .  $H_t$  data and single-stage  $C_d$  were used to calculate flow over each stage. The flow over each stage was calculated independently and then summed to estimate discharge for the models. The percent error was then calculated between the predicted flow rate and the observed experimental results [ $100*(Q_{predicted} - Q_{laboratory})/Q_{laboratory}$ ]. These data are presented in Figure 5.

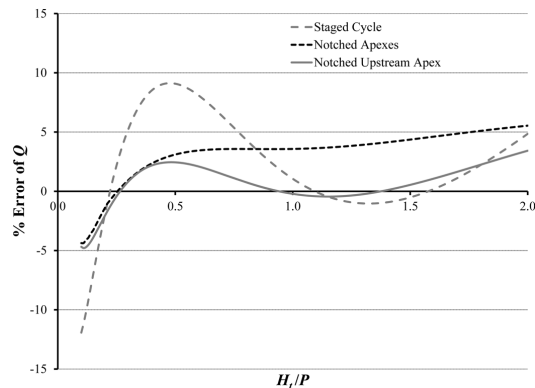


Figure 5 – % Error of  $Q$  at varying levels of  $H_t/P$

Because the apex notches were essentially channelized and flat topped, the  $C_d$  values used were from JOHNSON's (2000) study of flat-topped broad crested weirs. For other weir configurations, calculating flow using a contracted weir equation was evaluated (HAESTAD 2002), but this data set produced larger % error values than using labyrinth  $C_d$  values, therefore the experimental results from the non-staged labyrinth weir from this study were used.

For low levels of upstream head ( $H_t/P < 0.25$ ) the superposition method underestimates the weir flow by up to 15%. At higher levels of head, the accuracy of the superposition method varied from 2% underestimation to 9% overestimation depending on weir configuration and the value of  $H_t/P$ . Flow imbalance over the model resulted in head-discharge relationships and weir coefficients that are not typical of non-staged labyrinth weirs.

## CONCLUSIONS

The results of this study provide design guidance for staged and notched labyrinth weirs and identify the accuracy of using superposition to estimate staged labyrinth weir discharges. This study was performed to increase the understanding of the design and flow characteristics of staged and notched labyrinth weirs. It is anticipated that additional data analysis will result in a more accurate technique to estimate discharge over staged and notched linear weirs and will help practicing engineers design replacement spillways more accurately and efficiently. Recommendations for future research include varying the notch location, length, and depth. Multiple stages at two or more elevations would also be of interest.

## NOMENCLATURE

- $A$  = labyrinth weir sidewall angle;
- $C_d$  = dimensionless discharge coefficient;
- $g$  = gravitational constant;
- $h$  = piezometric head;
- $H_t$  = total upstream head of a weir relative to the crest elevation;
- $L$  = weir centerline crest length;
- $l_{\text{cycle}}$  = weir centerline crest length of one cycle;
- $l_{\text{stage}}$  = centerline crest length of notch/stage;
- $N$  = number of cycles in labyrinth weir;
- $P$  = weir height;
- $Q$  = flow;

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