Income Distribution Effects of Water Quality Controls: An Economic Approach

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INCOME DISTRIBUTION EFFECTS OF WATER QUALITY CONTROLS:
AN ECONOMETRIC APPROACH

by

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TABLE OF CONTENTS

		Page
ACKNOWLEDGEMENTS		ii
LIST OF TABLES		iv
LIST OF FIGURES		v
ABSTRACT		vi
Chapter		
I.	INTRODUCTION	1
	Study Objectives	1
II.	THE DISTRIBUTION MEASURES	3
	The Lognormal Densities	5
	The Gamma Density	9
	The Beta Density	11
	The Empirical Estimation of Income Distribution	11
III.	THE THEORETICAL AND EMPIRICAL MODELS	29
	The Production Sector	29
	The Distribution Sector	32
	The Model	39
IV.	THE WATER QUALITY INDICES	42
V.	RESULTS AND CONCLUSIONS	48
	Data Collection	48
	Empirical Results	50
	Summary and Conclusions	61
LITERATURE CITED		64
APPENDIX		71
A.	Parameters Estimation Program of Lognormal Densities	72
B.	Parameters Estimation Program of Gamma and Beta Densities	76
C.	1960 Factor Analysis for SMSA's	81
D.	1970 Factor Analysis for SMSA's	85
E.	Distribution Parameters and the Lorenz Curve	90
VITA		92

LIST OF TABLES

Table		Page
1	1960 Income Parameters for each Distribution Function and Gastwirth Bounds	14
2	1970 Income Parameters for each Distribution Function and Gastwirth Bounds	21
3	Two Groups of SMSA's by Factor Analysis	54
4	Empirical Results of the Coefficient Signs	57

LIST OF FIGURES

Figure		Page
1	Skewed distribution of income distribution	5
2	Effects of waste disposal	34
3	The empirical model	40

ABSTRACT

Income Distribution Effects of Water Quality Controls:

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The imposition of water quality controls may affect the economy chiefly by altering aggregate production and changing the factor payments. These two effects could not only reallocate resources among production possibilities, but also could change the distribution of benefits of production among members of the society.

This study attempted to provide a workable theory to establish an empirical test of the impacts of water quality controls on family income distribution. It consists of two separate areas: first, to analyze methodologies of measuring income distribution changes, and, second, to develop a theoretical model that is useful for empirical tests of the impacts of different water quality controls.

A number of alternative probability density functions have been proposed as models of personal income distribution. The lognormal, displaced lognormal, gamma, and beta distribution

functions were considered as appropriate methodologies, since each allows more productive power for income distribution as suggested in the past literature. Detailed information on income distribution can be extracted from the approximations of the distribution functions.

One of the objectives of the research was to evaluate the different methodologies for usefulness. The Gastwirth bounds for Gini coefficient were used as the test of goodness of fit; the beta density was clearly superior to the other densities for the SMSA data.

Next, a theoretical model was constructed, emphasizing the production sector and the distribution sector. Water quality controls were introduced in the production process as a negative input. Water quality data were collected for all states, and indices of quality were estimated using analysis of variance techniques. The equilibrium conditions in commodity and factor markets generated the first impacts of water quality controls on total output and factor payments in the economy.

The specific assumption was made as a theoretical bridge connecting family income distribution and factor payments in the distribution sector. It was assumed that a family's income equals total payments received from owned labor and capital in the production process. Thus, changes in factor payments and total output were included in the distribution equations. Water quality controls would, therefore, effect family income distribution through changes in total output and changes in factor payments.

The simultaneous equation regression results for 172 SMSA's were not conclusive. It appeared that water quality parameter may effect the wage rate and total output, if the parameter was not, in fact, a surrogate for other excluded variables in the system. The effect of wage changes on income distribution was not significant, but changes in total output appeared to be the most significant variable in the distribution equations.

In an attempt to account for the many variables which might be expected to effect income distribution, factor analysis was performed on the SMSA's. Two groups of SMSA's were identified, and regressions were performed for these groups. Results from these regressions were similar in sign to the results from the 172 observation regressions, although many of the coefficents were not significant.

Interpreting the results of the research was somewhat difficult, even though some results did appear consistent among all regressions. It does appear that there is some evidence to indicate that water quality controls lead to less equal family income distribution. Better data are required for more complete and accurate analysis.

The principle thrust of the study was to develop a model to organize the complexity of economic causality with respect to income distribution change and water quality policy. It appeared that this type of systematic econometric approach can be fruitful in analyzing income distribution change.

(93 pages)

CHAPTER I

INTRODUCTION

The imposition of environmental constraints on economic activity has heightened people's interest in the consequent impacts of these new policies. Most people generally acknowledge that changes in environmental control affect the economy chiefly by altering aggregate production. However, environmental policies may have two possible effects: changing resource allocations among production possibilities and changing the distribution of the benefits of production among members of society. Many of the standard economic tools are structured to analyze allocation effects. The study of distributional changes also has a rather long history, but only recently have tools with strong analytic capability been suggested in the literature. The past studies have been theoretical, rather than empirical. This study attempts to provide some methodologies for empirical analysis of the distributional impacts of water quality controls.

Study Objectives

The primary objective of the proposed research directs itself toward two separate areas: first to analyze methodologies of measuring income distribution changes, and, second, to select appropriate methodology and empirically test the hypothesis that there are significant distribution impacts from water quality controls. In order to achieve these objectives, several steps will be accomplished in the following sequential order:

1. To determine comparable economic and demographic units in cross section and in time series.
2. To estimate income distributions in each of the units, using different methods of inequality measurement.
3. To evaluate the estimation efficiencies of different inequality measurements.
4. To establish a theoretical relationship among the income inequality, water quality indices, and measurable socio-economic variables.
5. To develop indices from the water quality data which had a broad range of variables.
6. To develop an econometric model from the theoretical relationships in order to test the hypotheses for significance empirically.
7. To apply the tools, if they are efficient, to water quality policies to appraise its effect on income distribution changes which might occur.

CHAPTER II

THE DISTRIBUTION MEASURES

Sen (1973) discussed measures of inequality that have been proposed in the literature. He pointed out the strengths and weaknesses of different measures. He concluded that inequality is not easily represented by a single measure.

Inequality can be viewed in relative terms, viz., as a departure from some notion of an appropriate distribution. It is not only a measure of dispersion but also as a measure of the bargaining process between different income classes. In a normative sense, the "right" distribution of income based on "need" and "appropriations." Sen pointed out this normative assumption, and separated the measures of income distribution into two categories: 1) Those using statistical measures of relative variation of income to measure the extent of dispersion in an objective way are the positive indices of inequality, such as the Gini Coefficient, variance, and coefficient of variation, 2) Those indices that try to measure inequality based on some normative notion, such as Dalton's measure, Atkinson's measure, and Theil's entropy index.[1]

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[1]Dalton's measure is based on a comparison between actual levels of aggregate utility and the level of utility that would be obtained, if income were equally distributed. Atkinson's measure was the concept of equally distributed equivalent income. See Champernowne (1952) for more detail.

The appropriate approach for the study of changes in distribution due to water quality controls is clearly the former. The income distribution measures most often utilized in the analysis of policy effects have been the Gini, Pietra, or Theil's Entropy measure. The Gini Coefficient, which is derived from the Lorenz curve, is insufficient in that Lorenz curve which cross and have very different distributional characteristics may have identical Gini coefficient. Theil's entropy is based on the concept in Thermodynamics, which is proposed to measure disorder or randomness for particles. One disadvantage of using the Theil's entropy is that the proportion of families in different income ranges cannot be predicted from the index. All of these indices suffer from a lack of a unique relationship between the index and the actual income distribution.

Several authors have suggested alternative measuring methods based on probability density functions which have parameters which relate to both the mean and skewness of a density function (Champernowne, 1974). Metcalf (1972) utilized lognormal and displaced lognormal distributions to estimate the Lorenz curve. The latter function has the property that the distribution need not necessarily be symmetric about the mean, as would be expected of a Lorenz curve. Two of the Pearson family of curves have also been suggested: the gamma density function (Salem and Mount, 1974) and the beta density function (Thurow, 1973). All these functions have two parameters which relate mean, variances, skewness, and kurtosis, allowing a more complete description of the Lorenz curve.

The Lognormal Densities

Census data indicate that income distribution is positively skewed in that mean is greater than the median (See Figure 1). Thus, it is likely that income is more closely approximated by a lognormal curve than a normal curve.

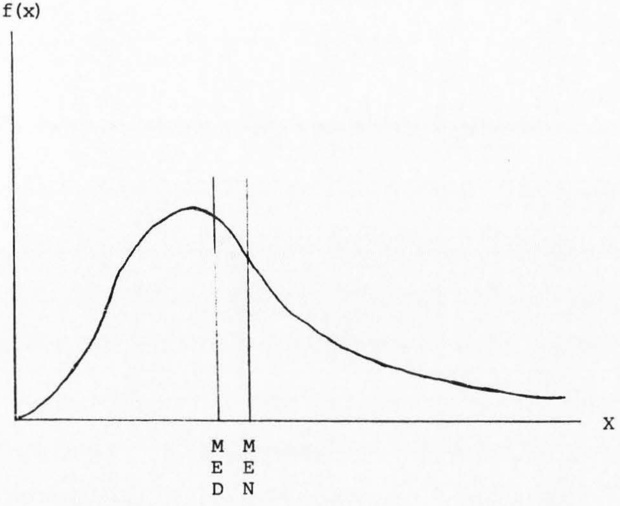

Figure 1. Skewed distribution of income distribution

The distribution of family income may be approximated by a two parameter lognormal distribution function,

$$\Lambda\left(\chi \mid \alpha \cdot \beta\right) = \frac{1}{\chi\beta\sqrt{2\pi}} \exp\left\{-\frac{\{\log\chi - \log\alpha\}^2}{2\beta^2}\right\}$$

$\chi > 0$, $\alpha > 0$, and $\beta > 0$

or a three parameter displaced lognormal distribution function,

$$f \{ \chi \mid C, \alpha, \beta \} = \Lambda \{ (\chi - C) \mid \alpha, \beta \}$$

The variable χ is defined as the income level, $\Lambda(\chi)$ and $f(\chi)$ are the percentage of families attaining that income level. The density functions involve three parameters, α, β and c, which must be estimated from data. Various measures of distribution equality from the two functions are then obtainable.[2]

The parameter α, which is the natural log of the geometric mean of χ, should equal the natural log of the median of the actual distribution is the two parameter lognormal. Since the income distribution in the SMSA's is skewed, often dramatically, the three parameter lognormal may be a more desirable estimation. The third parameter of the lognormal distribution, c, will indicate the extent of the log transform of the skewed data.[3]

The curve fitting procedures include the computation of mean income in each of the income groups. The midpoint is chosen as the mean income for the first income group; the mean income of the open end interval is obtained by fitting a Pareto curve to the data.

Pareto's mathematical formulation is widely used as the basis for estimating the mean for the open-end of an income distribution.

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[2] Aitchison, J. and Brown, J.A.C., The Lognormal Distribution with special reference to its uses in economics, Cambridge: Cambridge University Press, 1973. Chapter 2.

[3] The two parameter lognormal is simply a special case of the three parameter one, wherein the skewness, or third parameter, is zero.

For a discussion of fitting a Pareto curve to the open-end interval see U.S. Bureau of the Census (1965). Due to the assumed geometric nature of the income distribution, the mean income of each of the remainder of the groups is computed from the geometric mean of the lower and upper bounds.

The overall mean income of the population (μ) is estimated by:

$$\mu = \frac{\sum_i \mu_i f_i}{\sum_i f_i}$$

where μ_i is the mean income of group i and f_i is the number of families in income group i.

The method of quantiles is used to estimate the parameters. It is more efficient to take median, 10% decile, and 90% decile as the three quantiles. If B, M, and S denote the estimators of three parameter lognormal functions[4], the determining simultaneous equations are:

$$\begin{aligned} DEC &= B + e^{m-1.28S} \\ XMED &= B + e^{m} \\ DEC9 &= B + e^{m + 1.28S} \end{aligned}$$

where DEC = 10% decile,
XMED = median,
DEC9 = 10% decile.

For further discussion of estimation procedure, see Aitchison and Brown (1973), Chapter 6. The computer program is listed in Appendix A.

Metcalf (1972) applied the displaced lognormal distribution function to postwar United States income data. He states that

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[4]Again, for the case of two parameter lognormal, B = 0.

"...A cursory examination of U.S. income data for any year reveals that the actual distribution is positively skewed, contrary to the symmetry of a normal distribution...The coefficients of skewness and Kurtosis are both positive, indicating a departure from normality." He, then, chooses the lognormal distribution model. He indicates the statistical failure of the descriptive power of the lognormal distribution model. In the discussion of the rejection of a simple lognormal distribution, he found that it is unlikely that the displacements are random variations about a zero mean.

Given the empirical assertion that f (χ) is positively skewed, f $(\ln \chi)$ overcorrects for the positive skewness. Clearly, there exists some value of $c > o$ such that the transformation f $(\ln \{\chi - c\})$ has zero skewness. Thus, it is possible to find a value of c such that the distribution possesses the desired degree of skewness. Metcalf suggested using the displaced lognormal to improve the fit, and he accepted it as analytic tractable.

Nevertheless, Salem and Mount (1974) rejected the displaced lognormal distribution as an alternative approximation of income distribution, due to the difficulty of relating the parameters to an inequality measure. They indicated that "...Even though the displaced lognormal provides a good fit to the data, there are two serious drawbacks that reduce its usefulness as a model of income distribution..."[5] The two drawbacks involve the statistical properties and economic interpretations of the three parameters. Testing hypotheses about the parameters is difficult, since the statistical properties of the estimators are unknown. Furthermore, skewness depends upon both β and C; hence, the economic interpretation of the parameters is no longer straightforward. They concluded that gamma density may be a better functional form to describe changes in the distribution of income.

The Gamma Density

The gamma distribution may be defined:

$$g\ (\chi \mid \alpha \cdot \beta) = \frac{\beta^{\alpha}}{\Gamma\{\alpha\}}\ \chi^{\alpha-1}\ e^{-\beta\chi}$$

where $o < \chi < \infty$, α and β are positive parameters, and

$$\Gamma\ (\alpha) = \int_{o}^{\infty} e^{-u}\ u^{\alpha-1}\ du$$

is the gamma function.

Salem and Mount found that the two parameters can be directly related to indicators of inequality and scale respectively, and the two parameters are easy to estimate.

Assume that all the family incomes (χ) are multiplied by a constant k, namely $Y=k\chi$, as would happen under Gibrat's Law of proportionate growth. The density function of Y is g (Y), and the cumulative distribution function is G (Y), where

$$g\ (Y) = \frac{d}{dY}\ G\ (Y)$$

by definition.

G (Y) and g (Y) can be related to F (X) and f (x) in the following equations:

---

[5] See Glenn, A.B.Z. and Mount, T.D. "A Convenient Descriptive Model of Income Distribution: The Gamma Density," Econometrica 42 (6), November, 1974, 1115-1127.

$$
\begin{aligned}
G\ (Y) &= P\ \{Y \leq y\} \\
&= P\ \{k\chi \leq y\} \\
&= P\ \{\chi \leq y/k\} \quad \text{(assume } k \leq 0) \\
&= \int_0^{y/k} f\ (\chi)\ dx \\
&= F\ (y/k) - F\ (0) \\
&= F\ (y/k) \\
g\ (Y) &= \frac{d}{dY}\ G(Y) \\
&= \frac{d}{dY}\ F(y/k) \\
&= f\ (y/k)\ \frac{dx}{dY} \\
&= \frac{1}{k}\ f\ (\frac{y}{k}) \\
&= \frac{1}{k}\ \frac{\beta^{\alpha}}{\Gamma(\alpha)}\ (\frac{y}{k})^{\alpha-1}\ e^{-\beta\cdot\ \frac{y}{k}} \\
&= \frac{(\beta/k)^{\alpha}}{\Gamma(\alpha)}\ y^{\alpha-1}\ e^{-(\frac{\beta}{k})y} \\
&= f\ (y \mid \alpha, \frac{\beta}{k})
\end{aligned}
$$

It is clear, then, that α is not directly related to the scale change in income, but is related to the skewness, kertosis, and variance. It has been shown that the Gini (Salem and Mount, 1974), Theil's entropy (Salem and Mount, 1974), and Pietra (McDonald and Jensen, 1976) indices are functions of α only. Thus, the non-uniqueness of these inequality measures is obvious. McDonald and Jensen (1976), indicate that maximum likelihood estimators have smaller sample biases than method of moments estimators in most cases. Thus, maximum likelihood technique will be used to estimate the two parameters. The computer program is developed in Appendix B for both gamma and beta densities.

The Beta Density

The final distribution form to be examined is the beta density function, as suggested by Thurow (1973).

The beta function has the form

$$f(\chi_{,}\sigma_{,}\rho) = \frac{\Gamma(\sigma\ \rho)}{\Gamma(\sigma)\ \ \Gamma(\rho)}\ \chi^{\sigma-1}\ \ (1-\chi)^{\ \rho-1}$$

where $o < \chi < 1$, $\rho > o$, and $\sigma > o$.

The relationship between the beta function and the three indices of inequality is currently under study. No specific relationship has been determined, nor has it been demonstrated that the maximum likelihood estimators have smaller sample basis than the method of moments. HOwever, since the beta and gamma functions are members of the same Pearson family of distributions, the use of maximum likelihood estimators appears arranged. Since no direct maximum likelihood estimators of the beta function exist, a Newton-Raphson approximation is used.

The Empirical Estimation Of Income Distribution

The major problem in estimating the income distribution parameters for each of the chosen functions were: first, to choose the method of estimating the mean of the highest (unbounded) income class, and second, to estimate the parameters of each function corresponding to the income distribution data collected from the 1960 and 1970 census data for every Standard Metropolitan Statistical Area (SMSA) in the nation.

As discussed, the Pareto-Levy law was used by the U.S. Bureau of the Census (1965), to estimate the mean of the unbounded upper income group. This law states:

> ...The upper ranges of the income distribution could be described by a curve of the general type, $Y = AX^{-v}$, where X is the income size and Y is the number of persons have that, or a larger, income.

Graphically the curve would appear as a straight line in its logarithmic form. While the law is difficult to use for lower income levels[6], it is a reasonable approximation of higher income group. Since the only income group requiring estimation is the open-end highest income group, the law should be appropriate.

A related difficulty occurs when the beta function is used for income distribution estimation. Since the beta function is a finite distribution function of scaled incomes (that is, scaled between 0 and 1), it has a maximum income implicit in its estimation. The mean income of the open-ended interval derived from the Pareto-Levy law, is the mid-point of the interval. The formula used is:

$$UD = B + 2(X-B)$$

where UD = maximum income;

B = upper limit of the interval preceding the open-ended interval; and

X = mean income of the open-ended interval.

Incomes are divided by UD to satisfy the scale (0 to 1). Distribution data were collected from the 1960 and 1970 Census of population (U.S. Bureau of the Census, 1963, 1973, 1974, 1975) and from data available in the 1972 County and City Data Book (1972).

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[6] See for references, R.G.D. Allen, Mathematical Analysis for Economists, (London: MacMillan), 1974, pp. 407-408 and L.R. Klein, An Introduction to Econometrics, (Prentice Hall, Inc.) 1962, pp. 152-153.

Since only grouped data of the family income of SMSA's are available, it is assumed that every member of the particular income group receives the same income, measured by the midpoint of that group. Thirteen income groups are used for 1960 and 1970 data to make comparisons possible.

Computer programs were developed to estimate the parameter of the displaced lognormal, gamma, and beta density functions from these data. The programs can be found in appendices. The estimation of these parameters are given in Table 1 and 2 for each SMSA.

One of the objectives of the research was to evaluate the different methodologies for usefulness. The Gastwirth indices were used as the test.

Gastwirth (1971, 1972, 1974) suggested a method of estimation of the Gini coefficient with group data that does not require any assumption about the fundamental form of income distribution. The method yields upper and lower bounds for the Gini coefficient. A test can be performed by relating the Gini coefficient generated by each of the estimation techniques to the Gastwirth bounds. The Gastwirth bounds are also indicated in Tables 1 and 2 for each SMSA.

Gastwirth and Smith (1972) have found that the lognormal and displaced lognormal functions fail this test consistently. The Gini coefficients granted by the lognormal and displaced lognormal functions fell outside the Gastwirth bounds in every SMSA, as can be seen in Tables 1 and 2. The gamma distribution

Table 1. 1960 income parameters for each distribution function and Gastwirth bounds.

S.M.S.A. (60)	Gamma α	Gamma β	Gamma Gini	Beta α	Beta θ	Beta Gini	Lognormal M	Lognormal V	Lognormal Gini	Displaced Lognormal C	Displaced Lognormal M	Displaced Lognormal V	Displaced Lognormal Gini	Gastwirth GU	Gastwirth GL
Abilene, Tex	1.91	0.00031	0.27	1.59	13.40	0.39	8.42	0.634	0.43	15046.25	9.94	0.030	0.10	0.39	0.38
Akron, Ohio	2.59	0.00034	0.24	2.07	12.46	0.34	8.73	0.500	0.38	5793.73	9.45	0.096	0.17	0.32	0.32
Albany, Ga	1.68	0.00031	0.29	1.43	13.17	0.41	8.25	0.756	0.46	435973.30	13.00	0.000	0.00	0.41	0.40
Albany, NY	2.32	0.00033	0.25	1.88	11.51	0.36	8.62	0.557	0.40	4907.84	9.33	0.120	0.19	0.35	0.34
Albuquerque, NM	2.08	0.00028	0.26	1.68	10.36	0.37	8.64	0.617	0.42	3049.67	9.14	0.197	0.25	0.37	0.36
Allentown, NJ	2.44	0.00036	0.24	1.97	15.50	0.36	8.60	0.495	0.38	2645.49	9.05	0.160	0.22	0.34	0.33
Amarillo, Tex	2.02	0.00028	0.26	1.62	13.83	0.39	8.61	0.572	0.41	3395.37	9.17	0.160	0.22	0.38	0.37
Ann Arbor, Mich	2.16	0.00026	0.26	1.70	9.09	0.37	8.76	0.602	0.42	6381.78	9.52	0.106	0.18	0.36	0.35
Ashville, NC	1.58	0.00028	0.29	1.34	17.51	0.43	8.26	0.749	0.46	69607.93	11.22	0.002	0.02	0.43	0.42
Atlanta, Ga	1.77	0.00025	0.28	1.44	10.54	0.40	8.54	0.706	0.45	12081.93	9.84	0.054	0.13	0.40	0.40
Atlantic City, NJ	1.86	0.00030	0.27	1.55	13.20	0.40	8.43	0.671	0.44	3765.05	9.10	0.156	0.22	0.39	0.38
Austin, Tex	1.64	0.00025	0.29	1.35	12.34	0.42	8.43	0.722	0.45	10672.78	9.72	0.057	0.13	0.42	0.42
Bakersfield, Cal	2.05	0.00030	0.26	1.69	11.35	0.38	8.56	0.628	0.42	4331.86	9.24	0.150	0.22	0.37	0.36
Baltimore, Md	2.12	0.00029	0.26	1.71	11.24	0.37	8.64	0.603	0.42	5341.23	9.38	0.119	0.19	0.36	0.36
Baton Rouge, La	1.83	0.00027	0.28	1.51	10.32	0.39	8.53	0.707	0.45	26230.18	10.41	0.017	0.07	0.39	0.39
Bay City, Mich	2.55	0.00037	0.24	2.11	14.51	0.34	8.62	0.510	0.39	6787.51	9.47	0.079	0.16	0.32	0.32
Beaumont, Tex	1.90	0.00029	0.27	1.59	13.44	0.39	8.50	0.694	0.44	101891.40	11.59	0.001	0.02	0.37	0.37
Billings, Mont	2.40	0.00034	0.24	1.94	11.83	0.35	8.64	0.538	0.40	2919.45	9.11	0.173	0.23	0.34	0.33
Binghampton, NY	2.69	0.00037	0.23	2.15	12.55	0.34	8.69	0.470	0.37	2728.32	9.13	0.162	0.22	0.32	0.32
Birmingham, Ala	1.59	0.00026	0.29	1.34	11.85	0.42	8.38	0.802	0.47	22986.08	10.27	0.019	0.08	0.42	0.41
Boston, Mass	2.21	0.00027	0.25	1.72	10.44	0.37	8.75	0.553	0.40	1131.30	8.95	0.277	0.29	0.36	0.36
Bridgeport, Conn	2.56	0.00033	0.24	2.01	10.80	0.34	8.75	0.509	0.39	1333.11	8.98	0.239	0.27	0.33	0.32
Brockton, Mass	2.93	0.00043	0.22	2.37	15.80	0.32	8.65	0.428	0.36	1684.84	8.95	0.175	0.23	0.31	0.30
Buffalo, NY	2.51	0.00034	0.24	2.01	11.69	0.35	8.69	0.519	0.39	3619.00	9.22	0.147	0.21	0.33	0.32

Table 1 (continued)

S.M.S.A. (60)	Gamma α	β	Gini	Beta σ	ρ	Gini	Lognormal M	V	Gini	Displaced Lognormal C	M	V	Gini	Gastwirth GU	GL
Canton, Ohio	2.46	0.00035	0.24	2.00	13.50	0.35	8.64	0.519	0.39	4314.13	9.27	0.122	0.19	0.33	0.33
Cedar Rapids, Iowa	2.17	0.00029	0.26	1.75	13.95	0.37	8.66	0.572	0.41	8911.42	9.65	0.063	0.14	0.35	0.35
Champaign, Ill	1.99	0.00028	0.27	1.61	11.66	0.39	8.57	0.613	0.42	1586.44	8.90	0.278	0.29	0.38	0.38
Charlotte, SC	1.48	0.00027	0.30	1.28	11.66	0.43	8.22	0.897	0.50	6196.04	9.28	0.104	0.18	0.43	0.42
Charleston, W Va	1.82	0.00028	0.28	1.53	10.75	0.39	8.48	0.751	0.46	8837.78	9.60	0.076	0.15	0.38	0.37
Charlotte, NC	1.68	0.00024	0.29	1.36	11.83	0.42	8.53	0.710	0.45	9005.45	9.64	0.074	0.15	0.42	0.41
Chicago, Ill	2.21	0.00025	0.25	1.72	9.17	0.37	8.82	0.586	0.41	4733.57	9.40	0.132	0.20	0.36	0.35
Cleveland, Ohio	2.19	0.00026	0.25	1.72	10.46	0.37	8.77	0.579	0.41	4503.90	9.35	0.136	0.21	0.36	0.35
Colo Springs, Colo	2.24	0.00035	0.25	1.84	13.44	0.36	8.52	0.556	0.40	4928.00	9.29	0.110	0.19	0.36	0.35
Columbia, SC	1.58	0.00028	0.29	1.35	12.86	0.42	8.27	0.805	0.47	4341.02	9.09	0.142	0.21	0.42	0.42
Columbus, Ohio	2.11	0.00028	0.26	1.69	11.72	0.38	8.67	0.598	0.42	5923.99	9.44	0.108	0.18	0.36	0.36
Corpus Christi, Tex	1.52	0.00025	0.30	1.29	12.68	0.43	8.33	0.834	0.48	5573.95	9.28	0.125	0.20	0.43	0.42
Dallas, Tex	1.70	0.00023	0.28	1.37	10.64	0.41	8.58	0.719	0.45	10122.59	9.73	0.069	0.15	0.41	0.41
Davenport, Ill	2.43	0.00033	0.24	1.97	12.44	0.35	8.68	0.541	0.40	4629.03	9.32	0.120	0.19	0.33	0.33
Dayton, Ohio	2.36	0.00031	0.25	1.90	11.46	0.35	8.70	0.559	0.40	7808.39	9.59	0.079	0.16	0.34	0.33
Decatur, Ill	2.29	0.00033	0.25	1.86	12.38	0.36	8.59	0.566	0.41	3787.33	9.18	0.145	0.21	0.34	0.34
Denver, Colo	2.22	0.00029	0.25	1.76	11.93	0.37	8.70	0.559	0.40	7045.42	9.54	0.088	0.17	0.36	0.35
Des Moines, Iowa	2.27	0.00030	0.25	1.80	12.21	0.37	8.69	0.546	0.40	5637.69	9.42	0.106	0.18	0.35	0.35
Detroit, Mich	1.92	0.00027	0.27	1.59	9.53	0.38	8.57	0.710	0.45	5608.80	9.37	0.133	0.20	0.37	0.37
Dubuque, Iowa	2.12	0.00031	0.26	1.75	11.57	0.37	8.56	0.638	0.43	14259.58	9.93	0.036	0.11	0.35	0.35
Duluth, Minn	2.40	0.00039	0.24	2.02	16.64	0.35	8.49	0.522	0.39	16704.51	10.01	0.020	0.08	0.34	0.33
El Paso, Tex	1.93	0.00031	0.27	1.60	12.39	0.39	8.46	0.635	0.43	1867.05	8.86	0.259	0.28	0.39	0.38
Erie, Pa	2.42	0.00037	0.24	1.99	13.80	0.35	8.55	0.531	0.39	4876.22	9.27	0.102	0.18	0.33	0.33
Eugene, Ore	2.40	0.00035	0.24	1.95	13.49	0.35	8.61	0.521	0.39	11287.62	9.78	0.044	0.12	0.34	0.33
Evansville, Ind	1.81	0.00030	0.28	1.52	14.48	0.40	8.41	0.690	0.44	68441.12	11.21	0.002	0.03	0.39	0.39
Fargo, ND	2.36	0.00035	0.25	1.92	14.41	0.36	8.58	0.518	0.39	3960.26	9.20	0.129	0.20	0.35	0.34

Table 1 (continued)

S.M.S.A. (60)	Gamma α	Gamma β	Gini	Beta α	Beta β	Gini	Lognormal M	Lognormal V	Gini	Displaced Lognormal C	Displaced Lognormal M	Displaced Lognormal V	Gini	Gastwirth GU	Gastwirth GL
Fitchburg, Mass	2.72	0.00041	0.23	2.21	14.79	0.33	8.60	0.456	0.37	2054.21	8.96	0.180	0.24	0.32	0.32
Ft Lauderdale, Fla	1.58	0.00024	0.29	1.29	12.08	0.43	8.42	0.737	0.46	8026.10	9.53	0.081	0.16	0.43	0.43
Ft Wayne, Ind	2.38	0.00031	0.25	1.91	12.21	0.36	8.70	0.543	0.40	5444.29	9.41	0.107	0.18	0.34	0.33
Ft Worth Tex	1.97	0.00030	0.27	1.63	13.60	0.39	8.51	0.641	0.43	17237.50	10.07	0.027	0.09	0.38	0.37
Fresno, Cal	1.80	0.00027	0.28	1.48	11.83	0.40	8.50	0.696	0.44	3825.89	9.15	0.176	0.23	0.40	0.39
Gary, Ind	2.64	0.00035	0.23	2.12	11.38	0.33	8.71	0.498	0.38	3194.27	9.20	0.151	0.22	0.32	0.31
Grand Rapids, Mich	2.31	0.00031	0.25	1.85	12.96	0.36	8.67	0.536	0.40	4781.14	9.33	0.119	0.19	0.35	0.34
Great Falls, Mont	2.40	0.00035	0.24	1.93	12.74	0.35	8.61	0.522	0.39	3470.33	9.16	0.146	0.21	0.34	0.34
Green Bay, Wis	2.42	0.00034	0.24	1.95	14.65	0.36	8.63	0.512	0.39	2361.46	9.03	0.173	0.23	0.34	0.33
Greensboro, NC	1.81	0.00027	0.28	1.47	14.03	0.41	8.50	0.642	0.43	16609.20	10.04	0.027	0.09	0.40	0.40
Greenville, SC	1.77	0.00031	0.28	1.49	15.43	0.41	8.33	0.692	0.44	5724.57	9.26	0.089	0.17	0.40	0.39
Hamilton, Ohio	2.43	0.00033	0.24	1.97	11.80	0.35	8.68	0.548	0.40	8673.08	9.65	0.067	0.15	0.33	0.33
Harrisburg, Pa	2.41	0.00035	0.24	1.97	14.50	0.35	8.60	0.519	0.39	2830.92	9.07	0.161	0.22	0.34	0.33
Hartford, Conn	2.50	0.00029	0.24	1.92	10.70	0.35	8.83	0.496	0.38	1181.52	9.01	0.235	0.27	0.34	0.33
Honolulu, Ha	2.05	0.00025	0.26	1.62	8.65	0.38	8.75	0.633	0.43	1267.18	8.98	0.286	0.29	0.37	0.36
Houston, Tex	1.75	0.00024	0.28	1.43	11.14	0.41	8.58	0.716	0.45	14064.25	9.95	0.044	0.12	0.40	0.39
Huntington, Ky	1.74	0.00030	0.28	1.49	13.29	0.40	8.33	0.753	0.46	208923.70	12.28	0.000	0.01	0.39	0.39
Indianapolis, Ind	2.18	0.00028	0.26	1.74	10.71	0.37	8.70	0.585	0.41	6211.11	9.48	0.103	0.18	0.36	0.35
Jackson, Mich	2.39	0.00033	0.24	1.93	12.84	0.35	8.67	0.532	0.39	7697.98	9.57	0.074	0.15	0.34	0.33
Jacksonville, Fla	1.84	0.00029	0.27	1.53	13.14	0.40	8.45	0.685	0.44	30247.55	10.50	0.011	0.06	0.39	0.38
Jersey City, NJ	2.68	0.00038	0.23	2.17	10.99	0.33	8.64	0.496	0.38	2652.82	9.08	0.172	0.23	0.32	0.31
Kalamazoo, Mich	2.34	0.00031	0.25	1.86	12.17	0.36	8.70	0.543	0.40	4525.55	9.32	0.127	0.20	0.34	0.34
Kansas City, Kan	2.11	0.00028	0.26	1.69	12.13	0.38	8.66	0.591	0.41	3536.91	9.20	0.165	0.23	0.36	0.36
Kenosha, Wis	2.78	0.00035	0.23	2.20	11.26	0.33	8.77	0.476	0.37	2620.96	9.15	0.170	0.23	0.31	0.30
Knoxville, Tenn	1.73	0.00030	0.28	1.49	12.98	0.40	8.33	0.758	0.46	6102.50	9.31	0.099	0.18	0.40	0.39

Table 1 (continued)

S.M.S.A. (60)	Gamma			Beta			Lognormal			Displaced Lognormal				Gastwirth	
	α	β	Gini	σ	ρ	Gini	M	V	Gini	C	M	V	Gini	GU	GL
Lake Charles, La	1.95	0.00032	0.27	1.66	14.81	0.39	8.41	0.646	0.43	33730.61	10.59	0.008	0.05	0.38	0.37
Lancaster, Pa	2.29	0.00034	0.25	1.86	13.60	0.36	8.58	0.539	0.40	2848.40	9.07	0.166	0.23	0.35	0.34
Las Vegas, Nev	2.32	0.00029	0.25	1.83	9.82	0.36	8.76	0.561	0.40	4004.70	9.31	0.146	0.21	0.35	0.34
Lewiston, Me	2.69	0.00045	0.23	2.20	17.06	0.34	8.48	0.435	0.36	5667.53	9.30	0.066	0.14	0.33	0.32
Lawton, Ohio	2.03	0.00037	0.26	1.71	15.44	0.38	8.34	0.600	0.42	18418.65	10.07	0.017	0.07	0.37	0.37
Lexington, Ky	1.63	0.00024	0.29	1.35	12.98	0.42	8.46	0.741	0.46	12217.14	9.81	0.048	0.12	0.42	0.41
Lima, Ohio	2.16	0.00033	0.26	1.81	14.73	0.37	8.52	0.592	0.41	21807.26	10.24	0.017	0.07	0.35	0.35
Lincoln, Neb	2.39	0.00036	0.24	1.94	14.20	0.26	8.58	0.511	0.39	3159.93	9.10	0.151	0.22	0.34	0.34
Little Rock, Ark	1.77	0.00030	0.28	1.48	12.58	0.40	8.37	0.706	0.45	26746.12	10.39	0.012	0.06	0.40	0.39
Loraine, Ohio	2.70	0.00038	0.23	2.21	13.56	0.33	8.65	0.489	0.38	4906.99	9.33	0.102	0.18	0.31	0.31
Los Angeles, Cal	2.06	0.00024	0.26	1.62	9.42	0.38	8.77	0.622	0.32	4974.09	9.40	0.135	0.21	0.37	0.36
Lowell, Mass	2.87	0.00043	0.23	2.34	14.08	0.32	8.62	0.444	0.36	5540.58	9.36	0.083	0.16	0.31	0.30
Lubbock, Tex	1.74	0.00025	0.28	1.40	11.57	0.41	8.51	0.676	0.44	7056.92	9.49	0.095	0.17	0.41	0.41
Lynchburg, Va	1.75	0.00029	0.28	1.49	16.17	0.41	8.37	0.710	0.45	73611.93	11.28	0.002	0.02	0.40	0.39
Macon, Ga	1.78	0.00030	0.28	1.51	12.01	0.40	8.39	0.726	0.45	38411.58	10.70	0.008	0.05	0.39	0.39
Manchester, NH	2.57	0.00039	0.24	2.08	15.62	0.35	8.59	0.467	0.37	3958.44	9.19	0.109	0.18	0.33	0.33
Memphis, Tenn	1.56	0.00026	0.30	1.31	14.31	0.43	8.35	0.786	0.47	134181.50	11.85	0.001	0.01	0.43	0.42
Meriden, Conn	3.02	0.00041	0.22	2.41	12.93	0.32	8.73	0.429	0.36	1504.40	9.00	0.189	0.24	0.30	0.29
Miami, Fla	1.61	0.00024	0.29	1.32	11.56	0.42	8.46	0.757	0.46	18792.59	10.13	0.027	0.09	0.42	0.42
Midland, Tex	1.88	0.00022	0.27	1.47	9.47	0.40	8.77	0.644	0.43	10427.84	9.79	0.066	0.14	0.40	0.39
Milwaukee, Wis	2.59	0.00032	0.24	2.02	11.87	0.34	8.79	0.488	0.38	2246.87	9.13	0.182	0.24	0.33	0.32
Minneapolis, Minn	2.36	0.00032	0.25	1.89	11.73	0.36	8.67	0.538	0.40	3973.96	9.25	0.145	0.21	0.34	0.34
Mobile, Ala	1.78	0.00030	0.28	1.52	13.47	0.40	8.38	0.726	0.45	30420.38	10.50	0.011	0.06	0.39	0.39
Montgomery, Ala	1.48	0.00025	0.30	1.26	11.76	0.43	8.31	0.864	0.49	41881.26	10.78	0.007	0.05	0.44	0.43
Muncie, Ind	2.22	0.00033	0.25	1.84	13.70	0.36	8.55	0.582	0.41	11613.63	9.79	0.044	0.12	0.35	0.34

Table 1 (continued)

S.M.S.A (60)	Gamma α	Gamma β	Gamma Gini	Beta σ	Beta [illegible]	Beta Gini	Lognormal M	Lognormal V	Lognormal Gini	Displaced Lognormal C	Displaced Lognormal M	Displaced Lognormal V	Displaced Lognormal Gini	Gastwirth GU	Gastwirth GL
Muskegon, Mich	2.58	0.00038	0.24	2.15	15.62	0.34	8.60	0.514	0.39	20780.23	10.21	0.016	0.07	0.32	0.31
Nashville, Tenn	1.63	0.00025	0.29	1.35	12.11	0.42	8.45	0.738	0.46	3514.52	9.09	0.184	0.24	0.42	0.41
New Bedford, Mass	2.28	0.00039	0.25	1.92	18.04	0.36	8.44	0.536	0.40	28224.66	10.43	0.008	0.05	0.35	0.34
New Britain, Conn	3.08	0.00041	0.22	2.43	13.00	0.32	8.75	0.411	0.35	438.64	8.85	0.255	0.28	0.30	0.30
New Haven, Conn	2.15	0.00027	0.26	1.69	11.32	0.38	8.73	0.568	0.41	2078.03	9.07	0.220	0.26	0.36	0.36
New Orleans, La	1.63	0.00025	0.29	1.34	12.35	0.42	8.44	0.739	0.46	8498.00	9.57	0.077	0.16	0.42	0.42
New York, NY	1.92	0.00023	0.27	1.50	9.40	0.39	8.72	0.642	0.43	2819.63	9.16	0.212	0.26	0.39	0.38
Newark, NJ	2.06	0.00023	0.26	1.58	8.42	0.38	8.82	0.615	0.42	2268.12	9.15	0.220	0.26	0.37	0.37
Newport News, Va	2.20	0.00033	0.25	1.83	11.98	0.36	8.54	0.592	0.41	9029.87	9.64	0.064	0.14	0.36	0.35
Norfolk, Va	1.56	0.00027	0.30	1.35	11.95	0.42	8.32	0.871	0.49	191565.00	12.19	0.000	0.01	0.41	0.40
Odessa, Tex	2.35	0.00034	0.25	1.94	13.72	0.36	8.61	0.560	0.40	10811.13	9.76	0.047	0.12	0.34	0.33
Ogden, Ut	2.91	0.00041	0.22	2.38	13.49	0.32	8.67	0.453	0.37	3382.97	9.18	0.134	0.20	0.30	0.30
Oklahoma City, Okla	1.86	0.00027	0.27	1.53	13.37	0.40	8.53	0.653	0.43	8171.30	9.57	0.076	0.15	0.39	0.38
Orlando, Fla	1.71	0.00027	0.28	1.42	13.24	0.41	8.43	0.694	0.44	11699.62	9.78	0.049	0.12	0.41	0.41
Paterson, NJ	2.45	0.00028	0.24	1.86	8.93	0.35	8.86	0.519	0.39	2095.24	9.16	0.196	0.25	0.34	0.33
Peoria, Ill	2.34	0.00032	0.25	1.91	12.38	0.36	8.65	0.561	0.40	3961.90	9.24	0.141	0.21	0.34	0.33
Philadelphia, Pa	2.20	0.00039	0.25	1.75	10.74	0.37	8.68	0.579	0.41	2796.35	9.13	0.194	0.24	0.36	0.35
Phoenix, Ariz	1.91	0.00026	0.27	1.55	11.77	0.39	8.60	0.658	0.43	8321.27	9.60	0.079	0.16	0.38	0.38
Pittsburgh, Pa	2.22	0.00031	0.25	1.79	11.84	0.37	8.61	0.568	0.41	2359.60	9.03	0.211	0.25	0.35	0.35
Portland, Me	2.29	0.00034	0.25	1.87	16.38	0.37	8.56	0.516	0.39	-823.53	8.28	0.582	0.41	0.35	0.35
Portland, Ore	2.15	0.00029	0.26	1.74	12.85	0.37	8.65	0.582	0.41	13185.78	9.90	0.041	0.11	0.36	0.35
Provo, Ut	2.46	0.00040	0.24	2.05	12.78	0.34	8.50	0.528	0.39	5015.96	9.26	0.099	0.18	0.33	0.33
Raleigh, NC	1.56	0.00026	0.30	1.32	12.13	0.42	8.32	0.820	0.48	90587.25	11.48	0.002	0.02	0.42	0.42
Reno, Nev	2.14	0.00024	0.26	1.65	9.87	0.38	8.81	0.574	0.41	3242.89	9.25	0.170	0.23	0.37	0.36
Richmond, Va	1.88	0.00026	0.27	1.52	11.86	0.40	8.60	0.656	0.43	8665.64	9.63	0.075	0.15	0.39	0.38

Table 1 (continued)

S.M.S.A. (60)	Gamma			Beta			Lognormal			Displaced Lognormal				Gastwirth	
	α	β	Gini	σ	ρ	Gini	M	V	Gini	C	M	V	Gini	GU	GL
Roanoake, Va	1.80	0.00028	0.28	1.50	15.88	0.41	8.44	0.671	0.44	13443.85	9.96	0.028	0.09	0.39	0.39
Rochester, NY	2.47	0.00030	0.24	1.93	10.06	0.35	8.79	0.526	0.39	5050.57	9.42	0.116	0.19	0.34	0.33
Rockford, Ill	2.49	0.00033	0.24	2.00	12.12	0.35	8.71	0.528	0.39	5126.68	9.38	0.109	0-18	0.33	0.32
Sacramento, Cal	2.57	0.00032	0.24	2.02	10.42	0.34	8.79	0.507	0.39	3848.60	9.30	0.140	0.21	0.33	0.32
Saginow, Mich	2.27	0.00033	0.25	1.86	14.57	0.36	8.60	0.563	0.40	9580.66	9.67	0.055	0.13	0.34	0.34
St. Joseph, MO	2.15	0.00035	0.26	1.79	15.05	0.37	8.47	0.580	0.41	7578.46	9.47	0.065	0.14	0.36	0.35
St. Louis, Ill	2.02	0.00037	0.26	1.64	11.81	0.38	8.63	0.631	0.43	4635.49	9.30	0.139	0.21	0.37	0.36
Salt Lake City, Utah	2.40	0.00033	0.24	1.91	13.07	0.36	8.68	0.510	0.39	1827.64	9.00	0.213	0.26	0.34	0.34
San Anjelo, Tex.	1.54	0.00036	0.30	1.28	15.12	0.43	8.33	0.728	0.45	18457.28	10.08	0.021	0.08	0.44	0.43
San Antonio, Tex	1.66	0.00028	0.29	1.40	15.20	0.42	8.35	0.718	0.45	11712.40	9.75	0.043	0.12	0.42	0.41
San Bernadino, Cal	2.18	0.00032	0.26	1.80	11.51	0.36	8.56	0.594	0.41	3996.08	9.20	0.151	0.22	0.36	0.35
San Diego, Cal	2.03	0.00027	0.26	1.65	10.40	0.38	8.66	0.648	0.43	4966.45	9.35	0.136	0.21	0.37	0.36
San Francisco, Cal	2.13	0.00025	0.26	1.67	9.64	0.37	8.78	0.605	0.42	4779.83	9.38	0.136	0.21	0.36	0.36
San Jose, Cal	2.40	0.00028	0.24	1.87	9.12	0.35	8.82	0.553	0.40	5037.88	9.43	0.120	0.19	0.34	0.33
Santa Barbara, Cal	2.02	0.00024	0.26	1.59	9.93	0.38	8.75	0.615	0.42	3388.60	9.23	0.180	0.24	0.38	0.37
Savannah, Ga	1.82	0.00032	0.28	1.55	12.39	0.39	8.34	0.702	0.45	122743.50	11.76	0.001	0.02	0.39	0.39
Scranton, Pa	2.19	0.00039	0.25	1.85	16.05	0.37	8.38	0.568	0.41	5273.94	9.23	0.084	0.16	0.35	0.35
Seattle, Wash	2.38	0.00029	0.25	1.88	10.89	0.36	8.76	0.535	0.39	7254.95	9.58	0.084	0.16	0.34	0.34
Shreveport, La	1.54	0.00025	0.30	1.30	11.50	0.43	8.33	0.819	0.48	40041.95	10.74	0.007	0-05	0.43	0.43
Sioux, Iowa	1.92	0.00029	0.27	1.58	14.00	0.39	8.50	0.635	0.43	6611.72	9.42	0.087	0.17	0.38	0.38
Sioux Falls, SD	2.28	0.00036	0.25	1.89	13.09	0.36	8.52	0.563	0.40	5648.23	9.34	0.096	0.17	0.35	0.34
South Bend, Ind	2.55	0.00034	0.24	2.05	13.21	0.34	8.71	0.504	0.38	6387.18	9.49	0.086	0.16	0.33	0.32
Spokane, Wash	2.30	0.00033	0.25	1.87	12.97	0.36	8.62	0.541	0.40	4009.27	9.22	0.143	0.21	0.35	0.34
Springfield, Mo	1.90	0.00033	0.27	1.59	15.29	0.39	8.34	0.634	0.43	6556.65	9.34	0.073	0.15	0.39	0.38
Springfield, Ohio	2.36	0.00036	0.25	1.96	13.77	0.35	8.56	0.551	0.40	12157.49	9.82	0.039	0.11	0.34	0.33

Table 1 (continued)

S.M.S.A. (60)	Gamma			Beta			Lognormal			Displaced Lognormal				Gastwirth	
	α	β	Gini	σ	p	Gini	M	V	Gini	C	M	V	Gini	GU	GL
Steubenville, Ohio	2.51	0.00037	0.24	2.07	12.53	0.34	8.60	0.535	0.39	8892.25	9.63	0.057	0.13	0.32	0.32
Stockton, Cal	1.98	0.00029	0.27	1.63	11.75	0.38	8.55	0.646	0.43	4354.46	9.23	0.146	0.21	0.37	0.37
Syracuse, NY	2.35	0.00032	0.25	1.88	11.25	0.36	8.67	0.550	0.40	4473.08	9.31	0.131	0.20	0.34	0.34
Tacoma, Wash	2.22	0.00032	0.25	1.83	13.59	0.37	8.59	0.563	0.40	5028.39	9.32	0.119	0.19	0.35	0.35
Tampa, Fla	1.62	0.00028	0.29	1.36	14.87	0.42	8.30	0.719	0.45	592379.60	13.30	0.000	0.00	0.43	0.42
Topeka, Kan	2.24	0.00032	0.25	1.83	13.96	0.37	8.59	0.551	0.40	8465.74	9.60	0.066	0.14	0.35	0.35
Trenton, NJ	2.18	0.00027	0.26	1.71	10.04	0.37	8.74	0.578	0.41	2187.30	9.09	0.219	0.26	0.36	0.35
Tuscon, Ariz	1.98	0.00029	0.27	1.62	13.53	0.39	8.55	0.616	0.42	7282.62	9.51	0.082	0.16	0.38	0.37
Tulsa, Okla	1.72	0.00024	0.28	1.41	12.20	0.41	8.53	0.705	0.45	10399.80	9.72	0.062	0.14	0.41	0.40
Tusclaoosa, Ala	1.43	0.00028	0.31	1.24	12.08	0.44	8.15	0.913	0.50	1939.71	8.63	0.304	0.30	0.44	0.43
Tyler, Tex	1.48	0.00025	0.30	1.25	12.43	0.44	8.28	0.836	0.48	61567.61	11.12	0.003	0.03	0.44	0.44
Utica, NY	2.45	0.00036	0.24	2.00	11.75	0.35	8.60	0.538	0.40	5841.81	9.40	0.096	0.17	0.33	0.33
Washington, D.C.	2.21	0.00025	0.25	1.71	7.86	0.36	8.84	0.591	0.41	11170.88	9.86	0.058	0.13	0.36	0.35
Waterbury, Conn	2.83	0.00036	0.23	2.22	11.80	0.33	8.77	0.450	0.36	976.92	8.95	0.235	0.27	0.31	0.31
Waterloo, Iowa	2.50	0.00034	0.24	2.02	14.67	0.35	8.67	0.507	0.39	6032.08	9.44	0.088	0.17	0.33	0.32
W. Palm Beach, Fla	1.43	0.00022	0.31	1.19	14.00	0.45	8.36	0.786	0.47	18211.64	10.09	0.026	0.09	0.46	0.45
Wichita, Kan	2.24	0.00031	0.25	1.80	13.46	0.37	8.65	0.552	0.40	4330.08	9.28	0.129	0.20	0.35	0.35
Wilkesburg, Pa	2.09	0.00038	0.26	1.79	17.57	0.37	8.33	0.596	0.41	6693.85	9.34	0.065	0.14	0.36	0.36
Wilmington, Del	2.05	0.00025	0.26	1.60	9.43	0.38	8.74	0.614	0.42	5364.25	9.43	0.125	0.20	0.37	0.37
Worcester, Mass	2.47	0.00035	0.24	1.98	14.20	0.35	8.64	0.495	0.38	2205.62	9.02	0.187	0.24	0.34	0.33
Yorktown, Pa	2.42	0.00037	0.24	1.99	15.13	0.35	8.55	0.517	0.39	5852.38	9.35	0.077	0.16	0.34	0.33
Younston, Ohio	2.45	0.00034	0.24	1.99	12.47	0.35	8.65	0.530	0.39	2821.85	9.10	0.172	0.23	0.33	0.33

Table 2. 1970 income parameters for each distribution function and gastwirth bounds.

S.M.S.A. (70)	Gamma α	Gamma β	Gamma Gini	Beta σ	Beta ρ	Beta Gini	Lognormal M	Lognormal V	Lognormal Gini	Displaced Lognormal C	Displaced Lognormal M	Displaced Lognormal V	Displaced Lognormal Gini	Gastwirth GU	Gastwirth GL
Abiline, Tex	1.99	0.00022	0.27	1.53	7.14	0.38	8.83	0.644	0.43	1450.74	9.04	0.338	0.32	0.38	0.37
Akron, Ohio	2.53	0.00020	0.24	1.81	4.97	0.33	9.21	0.549	0.40	-1846.42	8.84	0.725	0.45	0.33	0.31
Albany, Ga	1.72	0.00018	0.28	1.34	5.48	0.40	8.82	0.817	0.48	738.81	8.92	0.559	0.40	0.40	0.39
Albany, NY	2.43	0.00020	0.24	1.75	5.05	0.34	9.18	0.561	0.40	-2951.78	8.46	1.220	0.57	0.34	0.32
Albuquerque, NM	1.90	0.00018	0.27	1.43	4.80	0.38	8.99	0.739	0.46	1836.50	9.25	0.390	0.34	0.38	0.37
Allentown, NJ	2.74	0.00024	0.23	1.96	5.93	0.33	9.15	0.486	0.38	-539.56	9.06	0.446	0.36	0.32	0.31
Amarillo, Tex	2.18	0.00021	0.26	1.63	5.86	0.36	9.00	0.616	0.42	-451.87	8.90	0.572	0.41	0.36	0.35
Ann Arbor, Mich	2.30	0.00016	0.25	1.62	4.18	0.35	9.33	0.603	0.42	9.33	0.60	3.481	0.00	0.35	0.33
Ashville, NC	1.97	0.00022	0.27	1.53	6.28	0.38	8.83	0.689	0.44	206.38	8.82	0.531	0.39	0.38	0.37
Atlanta, Ga	2.04	0.00016	0.26	1.49	4.51	0.37	9.16	0.690	0.44	-2340.97	8.50	1.259	0.57	0.37	0.35
Atlantic City, NJ	1.90	0.00018	0.27	1.43	5.57	0.38	8.98	0.697	0.45	-2782.05	7.55	3.061	0.78	0.39	0.38
Austin, Tex	1.87	0.00016	0.27	1.40	5.16	0.39	9.04	0.712	0.45	-1020.60	8.79	0.808	0.47	0.39	0.38
Bakersfield, Cal	2.00	0.00019	0.27	1.51	5.12	0.37	8.96	0.689	0.44	2814.40	9.34	0.305	0.30	0.38	0.36
Baltimore, Md	2.17	0.00018	0.26	1.59	4.70	0.36	9.15	0.655	0.43	-2199.38	8.59	1.076	0.54	0.36	0.34
Baton Rouge, La	1.85	0.00017	0.27	1.40	4.65	0.38	9.01	0.772	0.47	354.55	9.00	0.607	0.42	0.38	0.37
Bay City, Mich	2.58	0.00022	0.24	1.88	5.24	0.33	9.15	0.538	0.40	-939.72	8.94	0.590	0.41	0.33	0.31
Beaumont, Tex	2.05	0.00020	0.26	1.56	5.47	0.37	8.98	0.693	0.44	547.82	9.02	0.492	0.38	0.36	0.35
Billings, Mont	2.25	0.00021	0.25	1.68	5.90	0.36	9.01	0.593	0.41	159.16	9.01	0.484	0.38	0.36	0.34
Binghampton, NY	2.47	0.00022	0.24	1.80	5.21	0.34	9.10	0.552	0.40	-1928.07	8.61	0.947	0.51	0.34	0.32
Birmingham, Ala	1.76	0.00018	0.28	1.37	5.70	0.39	8.87	0.788	0.47	34.66	8.79	0.684	0.44	0.39	0.38
Boston, Mass	2.19	0.00016	0.25	1.56	4.50	0.36	9.27	0.617	0.42	-3426.44	8.30	1.627	0.63	0.37	0.34
Bridgeport, Conn	2.56	0.00019	0.24	1.79	4.63	0.33	9.30	0.536	0.40	-3572.31	8.49	1.212	0.56	0.34	0.31
Brockton, Mass	2.90	0.00024	0.22	2.07	5.32	0.31	9.21	0.482	0.38	-2118.40	8.84	0.656	0.43	0.31	0.29
Buffalo, NY	2.46	0.00021	0.24	1.79	5.27	0.34	9.15	0.559	0.40	-2748.01	8.40	1.276	0.58	0.34	0.32

Table 2 (continued)

S.M.S.A. (70)	Gamma			Beta			Lognormal			Displaced Lognormal				Gastwirth	
	r	β	Gini	σ	ρ	Gini	M	V	Gini	C	M	V	Gini	GU	GL
Canton, Ohio	2.72	0.00023	0.23	1.96	5.75	0.32	9.15	0.501	0.38	-2442.00	8.62	0.875	0.49	0.32	0.30
Cedar Rapids, Iowa	2.70	0.00023	0.23	1.95	5.37	0.32	9.17	0.510	0.39	1872.04	9.36	0.293	0.30	0.32	0.30
Champaign, Ill	2.10	0.00017	0.26	1.48	4.89	0.37	9.14	0.630	0.43	-152.81	9.09	0.530	0.39	0.37	0.36
Charlotte, SC	1.56	0.00017	0.30	1.25	4.94	0.41	8.76	0.975	0.51	2033.07	9.05	0.453	0.37	0.40	0.39
Charleston, W Va	1.96	0.00020	0.27	1.50	5.50	0.37	8.92	0.705	0.45	1105.93	9.06	0.459	0.37	0.37	0.36
Charlotte, NC	2.04	0.00018	0.26	1.51	5.17	0.37	9.09	0.663	0.44	-49.01	9.04	0.549	0.40	0.37	0.36
Chicago, Ill	2.26	0.00016	0.25	1.61	4.29	0.35	9.29	0.623	0.42	-2537.41	8.71	0.965	0.51	0.35	0.33
Cleveland, Ohio	2.23	0.00017	0.25	1.61	4.49	0.35	9.23	0.642	0.43	-2778.70	8.50	1.253	0.57	0.35	0.33
Colo Springs, Colo	2.15	0.00020	0.26	1.60	5.55	0.36	9.01	0.636	0.43	-88.89	8.99	0.521	0.39	0.36	0.35
Columbia, SC	1.86	0.00018	0.27	1.42	5.39	0.38	8.93	0.755	0.46	-950.49	8.62	0.920	0.50	0.39	0.37
Columbus, Ohio	2.28	0.00019	0.25	1.66	4.97	0.35	9.15	0.607	0.42	-1442.38	8.84	0.742	0.46	0.35	0.33
Corpus Christi, Tex	1.78	0.00018	0.28	1.38	5.47	0.39	8.85	0.780	0.47	2299.85	9.20	0.358	0.33	0.40	0.38
Dallas, Tex	2.02	0.00016	0.26	1.48	4.85	0.37	9.15	0.670	0.44	-495.43	9.02	0.605	0.42	0.38	0.36
Davenport, Ill	2.46	0.00021	0.24	1.80	5.28	0.34	9.14	0.567	0.41	1072.80	9.26	0.350	0.32	0.33	0.32
Dayton, Ohio	2.54	0.00020	0.24	1.81	4.71	0.33	9.23	0.552	0.40	-2935.95	8.56	1.079	0.54	0.33	0.31
Decatur, Ill	2.35	0.00020	0.25	1.72	5.37	0.35	9.14	0.583	0.41	-539.46	9.02	0.538	0.40	0.34	0.33
Denver, Colo	2.32	0.00019	0.25	1.67	4.91	0.35	9.19	0.586	0.41	-2527.79	8.59	1.059	0.53	0.35	0.33
Des Moines, Iowa	2.46	0.00020	0.24	1.76	5.13	0.34	9.18	0.539	0.40	-1413.92	8.88	0.696	0.44	0.34	0.33
Detroit, Mich	2.33	0.00017	0.25	1.66	4.18	0.34	9.29	0.614	0.42	-2528.08	8.71	0.970	0.51	0.34	0.32
Dubuque, Iowa	2.47	0.00022	0.24	1.81	5.66	0.34	9.12	0.547	0.40	1939.98	9.32	0.310	0.31	0.34	0.32
Duluth, Minn	2.52	0.00025	0.24	1.90	6.44	0.34	8.98	0.536	0.40	1216.51	9.10	0.326	0.31	0.33	0.32
El Paso, Tex	1.86	0.00020	0.27	1.43	5.69	0.39	8.85	0.732	0.45	895.94	9.01	0.468	0.37	0.39	0.38
Erie, Pa	2.55	0.00024	0.24	1.87	6.39	0.34	9.07	0.518	0.39	-1316.83	8.78	0.646	0.43	0.33	0.32
Eugene, Ore	2.29	0.00021	0.25	1.70	5.79	0.35	9.04	0.591	0.41	-73.55	8.98	0.513	0.39	0.35	0.34
Evansville, Ind	2.25	0.00022	0.25	1.69	5.79	0.35	8.99	0.604	0.42	36.81	8.95	0.519	0.39	0.35	0.34
Fargo, ND	2.32	0.00021	0.25	1.70	5.65	0.35	9.08	0.570	0.41	-205.93	9.02	0.515	0.39	0.35	0.34

Table 2 (continued)

S.M.S.A. (70)	Gamma α	Gamma β	Gamma Gini	Beta q	Beta p	Beta Gini	Lognormal M	Lognormal V	Lognormal Gini	Displaced Lognormal C	Displaced Lognormal M	Displaced Lognormal V	Displaced Lognormal Gini	Gastwirth GU	Gastwirth GL
Fitchburg, Mass	2.47	0.00021	0.24	1.80	5.61	0.34	9.13	0.552	0.40	-1347.10	8.85	0.659	0.43	0.34	0.32
Ft Lauderdale, Fla	1.76	0.00014	0.28	1.31	5.15	0.40	9.09	0.731	0.45	-409.94	8.98	0.661	0.43	0.41	0.40
Ft Wayne, Ind	2.75	0.00022	0.23	1.94	5.33	0.32	9.22	0.492	0.38	-2199.12	8.83	0.703	0.45	0.32	0.30
Ft Worth, Tex	2.29	0.00020	0.25	1.67	5.12	0.35	9.11	0.601	0.42	-1239.32	8.84	0.717	0.45	0.35	0.33
Fresno, Cal	1.86	0.00018	0.27	1.41	5.29	0.39	8.94	0.721	0.45	55.90	8.91	0.624	0.42	0.39	0.38
Gary, Ind	2.60	0.00021	0.24	1.88	4.82	0.32	9.19	0.557	0.40	-1332.33	8.92	0.622	0.42	0.32	0.30
Grand Rapids, Mich	2.54	0.00021	0.24	1.82	5.40	0.34	9.18	0.532	0.39	-1981.61	8.77	0.778	0.47	0.33	0.31
Great Falls, Mont	2.27	0.00022	0.25	1.69	6.23	0.36	9.01	0.586	0.41	348.21	9.06	0.428	0.36	0.35	0.34
Green Bay, Wis	2.61	0.00022	0.24	1.89	6.18	0.33	9.16	0.515	0.39	-764.11	9.02	0.484	0.38	0.33	0.31
Greensboro, NC	2.06	0.00019	0.26	1.54	5.74	0.37	9.02	0.657	0.43	3739.92	9.46	0.245	0.27	0.37	0.36
Greenville, SC	2.10	0.00021	0.26	1.61	6.01	0.36	8.93	0.659	0.43	-911.94	8.66	0.749	0.46	0.36	0.35
Hamilton, Ohio	2.53	0.00022	0.24	1.85	5.14	0.33	9.14	0.555	0.40	-1371.93	8.86	0.659	0.43	0.33	0.31
Harrisburg, Pa	2.46	0.00021	0.24	1.79	5.56	0.34	9.12	0.555	0.40	-3537.70	7.96	2.006	0.68	0.34	0.32
Hartford, Conn	2.55	0.00018	0.24	1.77	4.45	0.33	9.34	0.541	0.40	-4467.28	8.01	2.130	0.70	0.34	0.31
Honolulu, Ha	2.04	0.00014	0.26	1.47	3.92	0.36	9.28	0.706	0.45	2791.15	9.53	0.280	0.29	0.37	0.35
Houston, Tex	2.04	0.00017	0.26	1.51	4.87	0.37	9.11	0.685	0.44	160.07	9.09	0.522	0.39	0.37	0.35
Huntington, Ky	1.94	0.00021	0.27	1.52	6.13	0.38	8.83	0.710	0.45	1717.90	9.05	0.354	0.33	0.38	0.36
Indianapolis, Ind	2.45	0.00020	0.24	1.76	5.02	0.34	9.18	0.559	0.40	4.97	9.16	0.441	0.36	0.34	0.32
Jackson, Mich	2.46	0.00020	0.24	1.77	5.05	0.34	9.18	0.560	0.40	-1911.66	8.76	0.821	0.48	0.34	0.32
Jacksonville, Fla	1.79	0.00017	0.28	1.38	5.22	0.39	8.92	0.806	0.47	1320.66	9.10	0.458	0.37	0.39	0.37
Jersey City, NJ	2.25	0.00020	0.25	1.68	4.97	0.35	9.05	0.636	0.43	-890.25	8.85	0.669	0.44	0.35	0.33
Kalamazoo, Mich	2.45	0.00019	0.24	1.75	4.94	0.34	9.22	0.554	0.40	-2725.35	8.60	1.046	0.53	0.34	0.32
Kansas City, Kan	2.32	0.00019	0.25	1.68	5.08	0.35	9.16	0.592	0.41	-2152.13	8.68	0.922	0.50	0.35	0.33
Kenosha, Wis	2.82	0.00024	0.23	2.04	5.52	0.32	9.15	0.490	0.38	-75.62	9.12	0.407	0.35	0.31	0.29
Knoxville, Tenn	1.84	0.00019	0.27	1.42	5.82	0.39	8.87	0.742	0.46	64.38	8.81	0.647	0.43	0.39	0.38

Table 2 (continued)

S.M.S.A. (70)	Gamma α	Gamma β	Gamma Gini	Beta q	Beta p	Beta Gini	Lognormal M	Lognormal V	Lognormal Gini	Displaced Lognormal C	Displaced Lognormal M	Displaced Lognormal V	Displaced Lognormal Gini	Gastwirth CU	Gastwirth CL
Lake Charles, La	1.88	0.00020	0.27	1.47	5.40	0.38	8.86	0.763	0.46	2204.91	9.16	0.359	0.33	0.38	0.36
Lancaster, Pa	2.55	0.00022	0.24	1.85	5.93	0.34	9.12	0.528	0.39	-3170.45	8.25	1.406	0.60	0.33	0.32
Las Vegas, Nev	2.31	0.00019	0.25	1.67	4.60	0.34	9.18	0.604	0.42	1756.66	9.38	0.323	0.31	0.35	0.33
Lewiston, Me	2.44	0.00025	0.24	1.84	8.30	0.35	8.95	0.503	0.38	-553.32	8.73	0.476	0.37	0.35	0.34
Lawton, Okla	1.94	0.00022	0.27	1.53	6.33	0.38	8.77	0.728	0.45	2440.38	9.13	0.256	0.28	0.37	0.36
Lexington, Ky	2.07	0.00018	0.26	1.54	5.14	0.37	9.06	0.651	0.43	-1318.17	8.73	0.860	0.49	0.37	0.36
Lima, Ohio	2.56	0.00023	0.24	1.88	5.62	0.33	9.08	0.536	0.40	-1172.69	8.83	0.639	0.43	0.33	0.31
Lincoln, Neb	2.48	0.00022	0.24	1.79	5.66	0.34	9.11	0.522	0.39	-791.46	8.95	0.576	0.41	0.34	0.33
Little Rock, Ark	1.94	0.00020	0.27	1.49	5.90	0.38	8.90	0.703	0.45	892.05	9.02	0.456	0.37	0.38	0.37
Loraine, Ohio	2.89	0.00024	0.22	2.07	5.35	0.31	9.20	0.487	0.38	-1379.85	8.97	0.528	0.39	0.31	0.29
Los Angeles, Cal	1.97	0.00015	0.27	1.44	4.37	0.37	9.19	0.704	0.45	-1288.77	8.87	0.797	0.47	0.38	0.36
Lowell, Mass	2.80	0.00023	0.23	2.01	4.89	0.31	9.19	0.514	0.39	-2855.50	8.55	1.008	0.52	0.31	0.29
Lubbock, Tex	1.81	0.00017	0.28	1.38	5.95	0.39	8.93	0.717	0.45	-269.96	8.88	0.633	0.43	0.40	0.39
Lynchburg, Va	2.17	0.00021	0.26	1.64	6.13	0.36	8.97	0.610	0.42	-17.06	8.92	0.522	0.39	0.36	0.35
Macon, Ga	1.89	0.00019	0.27	1.45	5.13	0.38	8.92	0.733	0.45	-293.97	8.79	0.730	0.45	0.38	0.37
Manchester, NH	2.54	0.00023	0.24	1.86	5.95	0.34	9.09	0.529	0.39	-1429.23	8.78	0.683	0.44	0.33	0.32
Memphis, Tenn	1.69	0.00017	0.29	1.31	5.30	0.40	8.89	0.828	0.48	1646.72	9.12	0.451	0.37	0.40	0.39
Meriden, Conn	2.92	0.00024	0.22	2.07	5.07	0.31	9.22	0.480	0.38	74.91	9.22	0.368	0.33	0.31	0.29
Miami, Fla	1.64	0.00014	0.29	1.24	4.96	0.41	9.03	0.816	0.48	-1964.99	8.35	1.531	0.62	0.42	0.41
Midland, Tex	1.96	0.00015	0.27	1.43	4.69	0.38	9.15	0.695	0.44	1430.80	9.32	0.386	0.34	0.38	0.37
Milwaukee, Wis	2.59	0.00020	0.24	1.83	4.94	0.33	9.24	0.531	0.39	-2076.31	8.82	0.757	0.46	0.33	0.31
Minneapolis, Minn	2.62	0.00019	0.24	1.83	4.85	0.33	9.29	0.514	0.39	-3795.06	8.30	1.527	0.62	0.33	0.31
Mobile, Ala	1.73	0.00019	0.28	1.37	5.89	0.40	8.79	0.819	0.48	2950.36	9.21	0.296	0.30	0.39	0.38
Montgomery, Ala	1.62	0.00017	0.29	1.27	5.27	0.41	8.82	0.857	0.49	1687.99	9.09	0.457	0.37	0.41	0.40
Muncie, Ind	2.40	0.00022	0.24	1.76	5.77	0.35	9.08	0.564	0.40	-1105.70	8.84	0.645	0.43	0.34	0.33

Table 2 (continued)

S.M.S.A. (70)	Gamma α	β	Gini	Beta r	p	Gini	Lognormal M	V	Gini	Displaced Lognormal C	M	V	Gini	Gastwirth GU	GL
Muskegon, Mich	2.55	0.00023	0.24	1.88	5.67	0.33	9.08	0.544	0.40	-1313.88	8.77	0.689	0.44	0.33	0.31
Nashville, Tenn	1.96	0.00018	0.27	1.48	5.50	0.38	9.01	0.696	0:44	1341.57	9.18	0.408	0.35	0.38	0.37
New Bedford, Mass	2.22	0.00022	0.25	1.68	5.99	0.36	8.96	0.617	0.42	-1229.42	8.59	0.830	0.48	0.35	0.34
New Britain, Conn	2.85	0.00023	0.23	2.01	5.00	0.31	9.24	0.485	0.38	-1382.81	9.00	0.545	0.40	0.31	0.29
New Haven, Conn	2.09	0.00016	0.26	1.52	4.53	0.36	9.21	0.678	0.44	-2761.58	8.46	1.333	0.59	0.37	0.35
New Orleans, La	1.60	0.00015	0.29	1.25	5.04	0.41	8.90	0.892	0.50	-605.79	8.62	1.007	0.52	0.41	0.40
New York, NY	1.74	0.00013	0.28	1.29	4.19	0.39	9.18	0.805	0.47	-1150.26	8.86	0.852	0.49	0.40	0.39
Newark, NJ	2.02	0.00014	0.26	1.45	4.20	0.37	9.30	0.676	0.44	-3702.15	7.99	2.344	0.72	0.38	0.36
Newport News, Va	2.17	0.00020	0.26	1.63	4.84	0.35	9.03	0.674	0.44	978.74	9.17	0.408	0.35	0.35	0.33
Norfolk, Va	1.89	0.00019	0.27	1.46	5.12	0.38	8.92	0.776	0.47	-345.61	8.79	0.704	0.45	0.37	0.36
Odessa, Tex	2.38	0.00022	0.25	1.77	6.09	0.35	9.03	0.570	0.41	1978.97	9.28	0.287	0.30	0.34	0.33
Ogden, Ut	2.49	0.00022	0.24	1.81	5.03	0.33	9.12	0.552	0.40	-1902.35	8.69	0.855	0.49	0.34	0.32
Oklahoma City, Okla	2.07	0.00019	0.26	1.54	5.33	0.37	9.04	0.654	0.43	-919.25	8.81	0.734	0.46	0.37	0.36
Orlando, Fla	1.94	0.00018	0.27	1.46	5.21	0.38	8.98	0.703	0.45	-1620.64	8.51	1.107	0.54	0.38	0.37
Paterson, NJ	2.31	0.00015	0.25	1.60	4.23	0.35	9.39	0.578	0.41	-4738.25	7.90	2.487	0.74	0.36	0.34
Peoria, Ill	2.60	0.00021	0.24	1.86	5.34	0.33	9.19	0.521	0.39	-1103.20	8.98	0.567	0.41	0.33	0.31
Philadelphia, Pa	2.22	0.00018	0.25	1.61	4.76	0.35	9.18	0.627	0.42	583.96	9.23	0.422	0.35	0.35	0.34
Phoenix, Ariz	2.09	0.00018	0.26	1.54	5.05	0.37	9.09	0.652	0.43	555.54	9.14	0.467	0.37	0.37	0.35
Pittsburgh, Pa	2.34	0.00021	0.25	1.71	5.66	0.35	9.10	0.575	0.41	-2210.18	8.54	1.040	0.53	0.35	0.33
Portland, Me	2.42	0.00022	0.24	1.78	5.85	0.34	9.07	0.558	0.40	-2651.01	8.31	1.304	0.58	0.34	0.33
Portland, Ore	2.32	0.00019	0.25	1.68	5.16	0.35	9.15	0.587	0.41	-2144.92	8.61	1.002	0.52	0.35	0.33
Provo, Ut	2.24	0.00024	0.25	1.73	6.41	0.35	8.89	0.607	0.42	-297.72	8.73	0.540	0.40	0.35	0.34
Raleigh, NC	1.97	0.00017	0.27	1.47	4.95	0.37	9.04	0.706	0.45	1529.61	9.24	0.398	0.34	0.38	0.36
Reno, Nev	2.27	0.00017	0.25	1.61	4.68	0.35	9.23	0.590	0.41	-1243.89	8.96	0.666	0.44	0.36	0.34
Richmond, Va	2.18	0.00019	0.26	1.60	5.16	0.36	9.11	0.627	0.42	-2067.62	8.58	1.044	0.53	0.36	0.34

Table 2 (continued)

S.M.S.A. (70)	Gamma			Beta			Lognormal			Displaced Lognormal				Gastwirth	
	α	β	Gini	σ	ρ	Gini	M	V	Gini	C	M	V	Gini	GU	GL
Roanoke, Va	2.28	0.00021	0.25	1.69	5.94	0.35	9.03	0.580	0.41	-1678.41	8.60	0.893	0.50	0.36	0.34
Rochester, NY	2.54	0.00019	0.24	1.78	4.43	0.33	9.29	0.546	0.40	-3241.36	8.53	1.171	0.56	0.33	0.31
Rockford, Ill	2.59	0.00021	0.24	1.87	5.12	0.33	9.20	0.542	0.40	327.77	9.21	0.393	0.34	0.32	0.30
Sacramento, Cal	2.17	0.00018	0.26	1.59	4.63	0.35	9.12	0.648	0.43	-1472.17	8.77	0.842	0.48	0.36	0.34
Saginow, Mich	2.40	0.00020	0.24	1.75	4.77	0.34	9.17	0.602	0.42	-776.44	8.98	0.593	0.41	0.33	0.31
St Joseph, Mo	2.27	0.00024	0.25	1.73	6.84	0.36	8.92	0.584	0.41	-493.35	8.74	0.560	0.40	0.35	0.34
St Louis, Mo	2.21	0.00018	0.25	1.62	4.94	0.35	9.14	0.634	0.43	1164.00	9.26	0.391	0.34	0.35	0.34
Salt Lake City, Ut	2.38	0.00020	0.25	1.73	5.59	0.35	9.12	0.569	0.41	-2752.74	8.39	1.259	0.57	0.35	0.33
San Angelo, Tex	1.82	0.00019	0.28	1.40	6.59	0.40	8.84	0.707	0.45	-293.73	8.74	0.659	0.43	0.40	0.39
San Antonio, Tex	1.79	0.00018	0.28	1.38	5.65	0.39	8.87	0.767	0.46	77.64	8.85	0.632	0.43	0.40	0.38
San Bernadino, Cal	1.99	0.00018	0.27	1.50	5.00	0.37	9.00	0.695	0.44	-852.44	8.77	0.785	0.47	0.37	0.36
San Diego, Cal	1.99	0.00017	0.27	1.48	4.63	0.37	9.09	0.721	0.45	-47.59	9.05	0.560	0.40	0.37	0.35
San Francisco, Cal	2.09	0.00015	0.26	1.51	4.14	0.36	9.26	0.682	0.44	-2522.07	8.60	1.151	0.55	0.37	0.34
San Jose, Cal	2.49	0.00018	0.24	1.74	4.20	0.33	9.33	0.564	0.40	1360.86	9.43	0.312	0.31	0.34	0.31
Santa Barbara, Cal	2.17	0.00018	0.26	1.58	4.60	0.36	9.14	0.631	0.43	-2261.46	8.54	1.163	0.55	0.36	0.34
Savannah, Ga	1.79	0.00018	0.28	1.39	5.46	0.39	8.86	0.780	0.47	-545.54	8.61	0.887	0.49	0.39	0.38
Scranton, Pa	2.47	0.00025	0.24	1.85	7.03	0.34	8.96	0.529	0.39	-1210.04	8.67	0.627	0.42	0.32	0.33
Seattle, Wash	2.51	0.00019	0.24	1.77	4.59	0.33	9.28	0.551	0.40	-2716.54	8.71	0.926	0.50	0.34	0.31
Shreveport, La	1.71	0.00018	0.28	1.34	5.54	0.40	8.83	0.810	0.48	1645.01	9.07	0.436	0.36	0.40	0.39
Sioux City, Iowa	2.16	0.00020	0.26	1.61	6.18	0.37	9.00	0.597	0.42	554.58	9.05	0.461	0.37	0.37	0.35
Sioux Falls, SD	2.41	0.00023	0.24	1.80	5.97	0.34	9.02	0.565	0.40	1483.28	9.19	0.335	0.32	0.34	0.32
South Bend, Ind	2.55	0.00022	0.24	1.84	5.64	0.34	9.15	0.526	0.39	-519.06	9.04	0.498	0.38	0.33	0.32
Spokane, Wash	2.19	0.00020	0.25	1.63	5.47	0.36	9.04	0.614	0.42	-1350.39	8.67	0.862	0.49	0.36	0.34
Springfield, Mo	2.37	0.00020	0.25	1.72	5.37	0.35	9.15	0.561	0.40	-1436.72	8.86	0.697	0.44	0.37	0.36
Springfield, Ohio	2.49	0.00022	0.24	1.83	5.36	0.33	9.10	0.561	0.40	-1508.73	8.76	0.736	0.46	0.33	0.31

Table 2 (continued)

S.M.S.A. (70)	Gamma α	Gamma β	Gamma Gini	Beta p	Beta q	Beta Gini	Lognormal M	Lognormal V	Lognormal Gini	Displaced Lognormal C	Displaced Lognormal M	Displaced Lognormal V	Displaced Lognormal Gini	Gastwirth GU	Gastwirth GL
Steubenville, Ohio	2.53	0.00024	0.24	1.90	6.01	0.33	9.04	0.552	0.40	-83.49	8.96	0.464	0.37	0.33	0.31
Stockton, Cal	1.98	0.00018	0.27	1.49	5.11	0.37	9.02	0.706	0.45	-263.27	8.91	0.651	0.43	0.37	0.36
Syracuse, NY	2.35	0.00019	0.25	1.70	51.2	0.35	9.15	0.584	0.41	-2542.92	8.51	1.139	0.55	0.35	0.33
Tacoma, Wash	2.19	0.00019	0.25	1.62	5.18	0.36	9.08	0.631	0.43	-1184.22	8.80	0.755	0.46	0.36	0.34
Tampa, Fla	1.88	0.00019	0.27	1.43	6.17	0.39	8.88	0.687	0.44	-936.86	8.62	0.849	0.49	0.40	0.39
Topeka, Kan	2.38	0.00021	0.25	1.75	5.82	0.35	9.08	0.562	0.40	-732.25	8.93	0.567	0.41	0.35	0.33
Trenton, NJ	2.21	0.00017	0.25	1.58	4.75	0.36	9.24	0.609	0.42	-3405.14	8.26	1.663	0.64	0.36	0.34
Tuscon, Ariz	1.95	0.00018	0.27	1.46	5.37	0.38	8.99	0.692	0.44	-361.34	8.89	0.661	0.43	0.38	0.37
Tulsa, Okla	2.04	0.00019	0.26	1.53	5.52	0.37	9.03	0.659	0.43	1815.71	9.26	0.364	0.33	0.37	0.36
Tuscaloosa, Ala	1.61	0.00018	0.29	1.28	5.70	0.41	8.74	0.863	0.49	1067.56	8.90	0.523	0.39	0.41	0.40
Tyler, Tex	1.93	0.00020	0.27	1.48	6.03	0.38	8.89	0.692	0.44	2122.25	9.19	0.343	0.32	0.38	0.37
Utica, NY	2.48	0.00022	0.24	1.82	5.60	0.34	9.08	0.548	0.40	-2229.97	8.54	0.991	0.52	0.34	0.32
Washington, D.C.	2.02	0.00013	0.26	1.43	3.82	0.37	9.38	0.691	0.44	-3949.34	8.38	1.835	0.66	0.38	0.36
Waterbury, Conn	2.55	0.00020	0.24	1.81	4.86	0.33	9.24	0.545	0.40	-2185.53	8.84	0.734	0.46	0.33	0.31
Watterton, Iowa	2.45	0.00022	0.24	1.79	5.74	0.34	9.11	0.546	0.40	1532.28	9.26	0.339	0.32	0.34	0.32
W. Palm Beach, Fla	1.56	0.00013	0.30	1.18	5.02	0.42	9.04	0.819	0.48	-1722.43	8.50	1.318	0.58	0.44	0.43
Wichita, Kan	2.26	0.00021	0.25	1.68	5.65	0.35	9.05	0.609	0.42	-204.23	9.00	0.509	0.39	0.35	0.34
Wilkesburg, Pa	2.35	0.00025	0.25	1.79	7.30	0.35	8.92	0.566	0.41	2589.22	9.24	0.213	0.26	0.35	0.33
Wilmington, Del	2.30	0.00018	0.25	1.65	5.05	0.35	9.20	0.591	0.41	-3534.69	8.11	1.883	0.67	0.35	0.33
Worcester, Mass	2.65	0.00022	0.23	1.89	5.29	0.33	9.19	0.507	0.39	-2182.74	8.75	0.796	0.47	0.33	0.31
Yorktown, Pa	2.63	0.00023	0.23	1.91	6.25	0.33	9.12	0.497	0.38	-3198.49	8.20	1.452	0.61	0.33	0.31
Youngstown, Ohio	2.58	0.00022	0.24	1.88	5.28	0.33	9.15	0.542	0.40	-2032.52	8.70	0.836	0.48	0.33	0.31

function also consistently fails the Gastwirth test for each SMSA. The beta function produces Gini coefficient which generally fall between the upper and lower Gastwirth bounds, and even when the Gini is outside the bounds, the coefficient is substantially closer to one or the other of the bounds than any of the other functional forms. The Gastwirth test indicates that, for the purpose of this study, the beta is the appropriate estimation function.

CHAPTER III

THE THEORETICAL AND EMPIRICAL MODELS

In an attempt to measure the impact of water quality controls on family income distribution, there are two sectors presented in the model: the production sector and the distribution sector.

The Production Sector

It is useful to consider effluent emission as simply one more input in the production process. The theory of production is concerned with the optimum allocation of factors of production (including the effluent emission) that minimizes the total cost for each output. Thus, define c (y, w, r, e) as the cost function[7] which will yield the minimum cost at which output y can be produced given factor prices w (wage), r (rental rate of capital), and e (cost of waste discharge). Suppose that the output price (p) and factor supplies (L and K) are exogenously determined. Under competitive conditions, the producer achieves an optimal output by setting his marginal cost equal to the exogenous output price, i.e.

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[7] Duality principles in the theory of cost and transformation functions have been developed in detail by Hall (1973) and McFadden (1975). However, Shephard (1970) established this dual determination of production functions from cost curves, Uzawa (1964) formulated explicitly the conditions for cost curves that are derived from neoclassical production process by a minimization of total cost.

1) $$C_y\ (y, w, r, e) = \frac{\partial\ c\ (y, w, r, e)}{\partial y} = p$$

gives the equilibrium condition in the commodity market.

The partial derivative of the cost function with respect to the price of a factor yields the derived demand for that factor.

2) $$C_w\ (y, w, r, e) = \frac{\partial\ c\ (y, w, r, e)}{\partial w} \quad \text{and}$$

3) $$C_r\ (y, w, r, e) = \frac{\partial\ c\ (y, w, r, e)}{\partial r}$$

are the demand functions for labor and capital. Since factor supplies are assumed to be determined exogenously, the wage and rental rates are determined in the factor market. This market equates factor demands and supplies, in that:

4) $$C_w\ (y, w, r, e) = L \qquad \text{and}$$

5) $$C_r\ (y, w, r, e) = K$$

Now, a system of three simultaneous equations, (1), (4), and (5), with three unknowns, y, w, and r, can be solved implicitly as follows:

6) $$y = f\ (L, K, P, e)\ ,$$

7) $$w = g\ (L, K, P, e)\ , \text{ and}$$

8) $$r = h\ (L, K, P, e)$$

Consider the effect of the imposition of water quality controls on production. The competitive conditions tend to change factor prices and the output level. The rates of change may not be equal.

The total differentials of equations (1), (4), and (5) can be written as:

9) $$dp = C_{yy}\ dy + C_{yw}\ dw + C_{yr}\ dr + C_{ye}\ de$$

10) $dL = C_{wy}\ dy + C_{ww}\ dw + C_{wr}\ dr + C_{we}\ de$

11) $dK = C_{ry}\ dy + C_{rw}\ dw + C_{rr}\ dr + C_{re}\ de$

All the variables are first differences. The three endogenous variables, dy, dw, and dr, are functions of the four exogenous variables, dp, dL, dK, and de.

12) $dy = F\ (dp, dL, dK, de)$

13) $dw = G\ (dp, dL, dK, de)$

14) $dr = H\ (dp, dL, dK, de)$

The data limitations are significant in the analysis. An additional assumption to simplify the three-equation system to two equations with fewer variables will make the analysis more tractable.

It is assumed that $dr = 0$; that is dp, dL, dK, and de are interdependent mathematically.

$$H\ (dp, dL, dK, de) = 0$$

Thus, dp can be written in terms of dL, dK, and de[8], so that a system of two equations can be constructed:

15) $dy = F\ (dL, dK, de)$

16) $dw = G\ (dL, dK, de)$

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[8]The assumption implies that the ratio of payments for capital to capital stock is constant over time. The elimination of the price variable, while possibly detrimental to the analysis, is necessary because data for price indices for all SMSA's does not exist. Only for a few selected SMSA's are these indices published. With accurate price data, the three equation model could be utilized.

The two equations describe the impacts of water quality controls on changes in output and wage rate. These equations form the production sector of the model. The other sector of the model deals with the distribution effects of dy and dw.

The Distribution Sector

Friedman (1953), Becker and Chiswick (1966) have made attempts to connect the functional distribution of income with the personal distribution of income. Newhouse (1969) also developed a more operational theory to predict income distribution among areas. He focused his attention on the industry mix as a variable of crucial importance in determining income distribution. For this study, family income is assumed to be the crucial consideration.

A rather specific concept of family income is used: a family's income equals total payments received from owned labor and capital in the production process. It is assumed that there is only one competitive wage rate and rental rate in the model. The labor and capital inputs are homogeneous. The distribution of labor and capital is different from family to family. Thus,

17) $Y_i = WL_i + rk_i$ where

y_i = income of family i

W = wage rate

L_i = labor of family i

r = rental rate

K_i = capital of family i

It is this definition of income -- the sum of factor payments to a family -- which is used.

Equation (17) implies that

$$\sum_i Y_i = W \sum_i L_i + r \sum_i K_i$$

Thus, the sum of family income can be calculated from w, r, the sum of labor factors, and the sum of capital factors. Given that w and r are competitively determined, the distribution of Y_i is a function of w, r, the distribution of L_i, and the distribution of K_i. Assume that Y_i, L_i, and K_i each exhibit some distribution function. Let I_p, Ld and Kd be vectors of parameters of density functions that describe the distribution of family income, labor factors, and capital factors, then

$$I_p = f\ (w, r, Ld, Kd)$$

The total differential of this function can be written as:

18) $$dI_p = g\ (dw, dr, dLd, dKd)$$

Thus, the change in I_p is a function of changes in w, r, Ld, and Kd.

To aid in the analysis, a rather simple model of water quality control impact on functional income redistribution is constructed.

Assume that there is an aggregate production function with labor input (L), capital input (K), and a homogeneous input which will be called waste disposal (e).[9] Therefore:

---

[9] Waste disposal is defined as the production residual or a negative input which is the untreated portion of disposal. For further discussion, see Dwight R. Lee, "Efficiency of Pollution Taxation and Market Structure," Journal of Environmental Economics and Management 2, pp. 69-72 (1975).

$$q_1 = q\ (L,\ K,\ e)$$

where q is continuous, twice differentiable, and strictly convex as a result of diminishing marginal rate of technical substitution between inputs. The equation is actually a surface showing all possible combinations of different inputs capable of producing a given level of output, namely q_1. Given that e is exogenously determined, specific isoquants can be graphed in two dimensions as shown in Figure 2.

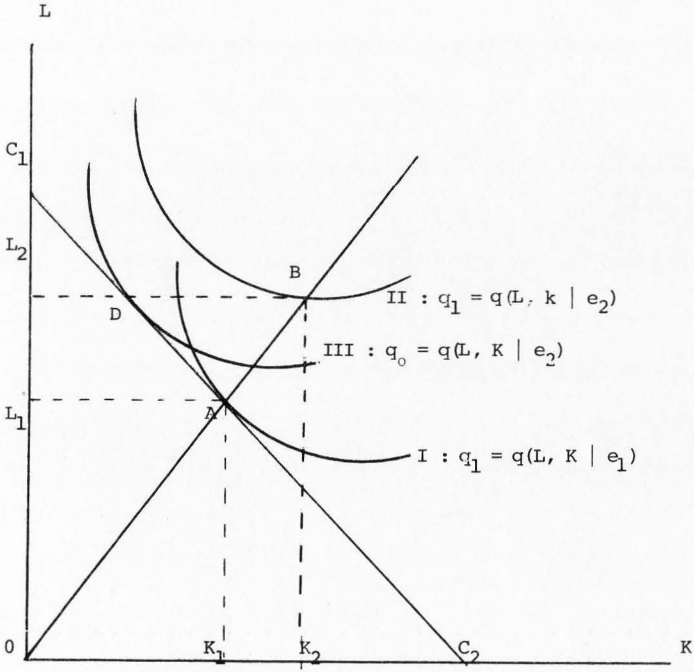

Figure 2. Effects of waste disposal

Isoquant I and isoquant II are the same output level, namely q_1, but with different waste disposal requirements. Note that $e_2 < e_1$. The additional inputs, L_1L_2 and K_1K_2, at point B are used for reducing the amount of waste from e_1 to e_2. In other words, the shift of isoquant I to isoquant II is due to the additional cost of reducing waste effluent. However, the two isoquants may not be parallel to each other, since the marginal rate of technical substitution between L and K changes from A to B. Isoquant III satisfies the cost constraint as isoquant I, but with $e_2 < e_1$ and $q_0 < q_1$.

The cost constraint is defined as:

$$C = wL + rK$$

Total cost (C) equals the sum of wage payment and rental payment of capital. The straight line tangent to isoquant I and isoquant III, shown in Figure II, is the cost constraint. The triangle $\Delta 0C_1C_2$ is the maximum feasible cost for production of the society. Optimum production is achieved where the cost constraint is tangent to the isoquant. Thus, the optimum output level is lowered from $q_1 = q\ (L, K \mid e_1)$ to $q_0 = q\ (L, K \mid e_2)$, if the environment (effluent) requirements are raised from e_1 to e_2. The input combinations for point A and point D are different, although the total cost stays the same. One of the impacts of effluent regulation is to change the factor shares of production. Water quality control can have either a positive or a negative distributional impact on input factors.

Equation (18) describes the relationship between changes in family income distribution and changes in factor distributions between families:

$$dI_p = g\ (dW,\ dr,\ dLd,\ DKd)$$

The empirical problem is measuring the distribution of labor and capital among families.

Backer and Chiswick (1966) explained why income distributions take various shapes, yet their approach cannot predict the distribution of factors among families. Newhouse's model (1969) indicates the direction to be pursued. Based on the assumption of a constant industry wage structure, he estimated the proportion of jobs in every income class in each industry.

The following linear model provides a possibility to measure the distribution of labor and capital indirectly.

Define the indentity

$$J_n = A_{Ln}\, b_L + A_{kn}\, b_k$$

where J_n = the percentage of families with relatively few labor or capital forces in the nth area

A_{Ln} = the percentage of families with labor income in the nth area

A_{kn} = the percentage of families with capital income in the nth area

b_L = the percentage of families with relatively few labor factors

b_k = the percentage of families with relatively few capital factors

Note that b_L (the percentage of relatively low labor income families) and b_k (the percentage of relatively low capital income families) are good approximations for Ld and Kd since wage rate and rental rate are exogenous. Changes in the distribution of labor and capital factors between families may change the relative percentage of families with different amounts of factors. A regression analysis of J_n on A_{Ln} and A_{kn} will give simultaneous estimators of b_L and b_k. Thus:

$$b_L = l\ (J_n, A_{Ln}, A_{kn}) \ ,$$

and $$b_k = k\ (J_n, A_{Ln}, A_{kn}) \ .$$

Furthermore, A_{Ln} can be approximated by a fraction formed from wages and salaries as the numerator and total value-added as the denominator. A_{kn} will be approximated by $1-A_{Ln}$ for simplicity. This assumption implies that the average productivity of labor per unit equals the average productivity of capital per unit for the family. Note that it is not necessarily true that the productivities of capital and labor are equal. The average units of labor and capital per family may be different. Thus, the total differentials of b_L and b_k can be simplified as:

$$db_L = l_o\ (dJ_n, dA_{Ln})$$

and $$db_k = k_o\ (dJ_n, dA_{Ln})$$

Using db_L and db_k to approximate dLd and dKd, given that db_L and db_k are functions of dJ_n and dA_{Ln}, equation (18) is equivalent to:

19) $$dI_p = h\ (dw, dr, dJ_n, dA_{Ln})$$

Furthermore, changes in the percentage of families with relatively few labor or capital factors would intuitively coincide with changes in the total output. The former is not easy to measure, while the latter is usually available from regional data. Therefore, it is convenient to substitute dy for dJ_n in equation (19).

Thus:

20) $$dI_p = h\ (dw, dr, dy, da)^{10}$$

Equation (20) is an empirically useful construct. Changes in the distribution of income can be explained by changes in factor payments, changes in total output and changes in factor share. Theoretically, the distribution of family income is dependent on labor income, capital income, total value-added, and the share of labor or capital income.

Changes of family income distribution can be measured by shifts of Lorenz curve[11] of income distribution. The Lorenz curve corresponding to any random variable X (family income level) with cumulative distribution function F (X) and finite mean $\mu = \int x dF\ (X)$ is defined to be $L\ (p) = \mu^{-1} \int_0^p F^{-1}\ (t) dt$ $0 \leq p \leq 1$. Note that L (p) is the fraction of total income that the holders of the lowest pth fraction of income possess. dI_p is helpful in describing changes of the cumulative distribution function F (X). If income distributions in all areas have the same functional form and with the same parameters, the value of all statistical measures of inequality would everywhere be the same.

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[10]For simplicity, dA_{Ln} is written da.

[11]A general definition of the Lorenz curve see Kendall and Stuart (1969), and Gastwirth (1971). The standard definition mathematically is written as,

$$p = F\ (X) = \int_0^X f\ (t) dt,$$

and $$L\ (p) = \phi\ (X)^{\circ} = \frac{1}{\mu} \int_0^X tf\ (t) dt,$$

where $$F^{-1}\ (p) = X$$

But, a statistical distribution with different parameters will predict different forms of distribution, though the Gini coefficient or quantiles may be no different.

Thus, changes in the parameters of the cumulative distribution function, F (X), are used as a measure of changes in distribution of family income.

For the case of beta density, $d\sigma$ and $d\rho$ are the elements of dI_p, and equation (20) can be rewritten as two equations.

21) $d\sigma = \sigma\ (dw, dy, da, d\rho)$

22) $d\rho = \rho\ (dw, dy, da, d\sigma)$

Note that, since the two parameters in the distribution function are functionally related, changes in each parameter is included in the equation for the other. Since dr is assumed to be zero, equations (21) and (22) do not include dr as an exogenous variable.

The Model

Several empirical problems were encountered in the research effort. Data for important variables were missing so that surrogate variables consistent with the available data had to be selected. The water quality data had a broad range of variables, so that indices had to be developed for each state. The data limitations were significant in the analysis.

The structural equations were developed from production to distribution hypotheses and from the relationships between appropriate variables and the distribution parameters.

The empirical model as proposed is in four equations:

$$dw = G\ (dL,\ dK,\ de)$$

$$dy = F\ (dL,\ dK,\ de)$$

$$d\sigma = \sigma\ (dw,\ dy,\ da,\ d\rho)$$

$$d\rho = \rho\ (dw,\ dy,\ da,\ d\sigma)$$

dw, dy, $d\sigma$, and $d\rho$ are endogenous variables; dL, dK, de, da are exogenous variables in the model. The model may be also represented by a structural flow chart as in Figure 3.

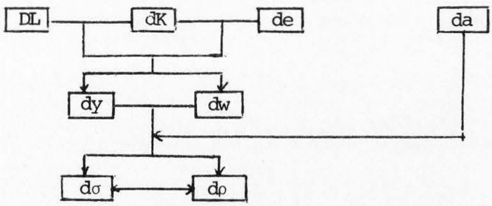

Figure 3. The empirical model

The exogenous variables are on the first row, the second and third rows are for endogenous variables. Note that all the endogenous variables have input flows from exogenous and/or endogenous variables. The structural flow provides a logic of the theory to evaluate the distribution impacts of water quality controls.

The model is assumed to be linear, so that the equation system is:

23) $$dw = a_1 + b_1\ dL + c_1\ dk + d_1\ de + E_1$$

24) $$dy = a_2 + b_2\ dL + c_2\ dk + d_2\ de + E_2$$

25) $d\sigma = a_3 + b_3\ dw + c_3\ dy + d_3\ da + m_3\ d\ \rho + E_3$

26) $d\rho = a_4 + b_4\ dw + c_4\ dy + d_4\ da + m_4\ d\ \sigma + E_4$

where a_1, b_1, c_1, d_1, and m_j stand for the parameters to be estimated (i = 1, 2, 3 and 4, j = 3, 4). E_i stands for the stochastic disturbance for four equations.

So far as the identification problems are concerned, equation (23) and (24) are over-identified and equation (25) and (26) are just-identified. Thus, indirect least squares estimation yields results which may not be consistent. Two stage least squares method (2SLS) is a very useful all-purpose technique for simultaneous model, and the parameters estimated are consistent.

CHAPTER IV

WATER QUALITY INDICES

The remaining modeling problem is to empiricise water quality controls in order to quantify de in this model. Since cost data are not available for various levels of quality constraint over all SMSA's for all industries, it is assumed that the level of cost are monotonically related to the levels, or strengthening, of water quality standards. Thus, indices of water quality controls are proxies for de.

THE WATER QUALITY INDICES

Water quality controls in each state exist for five different classifications of uses: agricultural, industrial, recreational, fishery, and municipal. Each classification has specific controls or levels for 14 different criteria.[12] These classifications can be treated as a series of treatments in an analysis of variance, for which the experimental design is written mathematically as:

$$Y_{ij} = \mu + \alpha_i + e_{ij}$$

where i is the classification $(i = 1,\ldots,5)$

and j is the criteria $(j = 1,\ldots,14)$

and Y_{ij} = the jth criteria for the ith classification

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[12]These criteria include temperature, BOD, specific metals, dissolved oxygen, phosphorus, nitrogen, etc.

μ = population mean

α_i = the adjustment for water quality for the ith classification

e_{ij} = disturbance term

The entire model in matrix form can be written:

$$
\underset{(70 \times 1)}{\begin{pmatrix} Y_{11} \\ \vdots \\ Y_{1,14} \\ Y_{21} \\ \vdots \\ Y_{2,14} \\ Y_{31} \\ \vdots \\ Y_{3,14} \\ Y_{41} \\ \vdots \\ Y_{4,14} \\ Y_{51} \\ \vdots \\ Y_{5,14} \end{pmatrix}}
=
\underset{(70 \times 6)}{\begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
1 & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & 1 & 0 & \vdots & \vdots & \vdots \\
\vdots & 0 & 1 & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & 1 & 0 & \vdots & \vdots \\
\vdots & \vdots & 0 & 1 & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & 1 & 0 & \vdots \\
\vdots & \vdots & \vdots & 0 & 1 & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 1 & 0 \\
\vdots & \vdots & \vdots & \vdots & 0 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}}
\cdot
\underset{(6 \times 1)}{\begin{pmatrix} \mu \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{pmatrix}}
+
\underset{(70 \times 1)}{\begin{pmatrix} e_{11} \\ \vdots \\ e_{1,14} \\ e_{21} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ e_{5,14} \end{pmatrix}}
$$

The first equation, $Y_{1,1} = \mu + \alpha_1 + e_{11}$ might be interpreted as the temperature (j=1) of agricultural water (i=1), where the temperature cannot exceed the mean water temperature, plus α_1, the adjustment for agricultural water quality control levels, and an unobservable disturbance term. $Y_{1,1}$ is not an absolute term; rather, it is the ratio of the criterion divided by its mean among 50 states. The model is linear with all X_{kj} equal to one or zero.

Clearly, the ranks of the matrices are:

$$R(X) = 6, \qquad \text{and} \qquad R(X'X) < 6,$$

hence (X'X) is a singular matrix. Regression analysis cannot be performed to estimate parameters. Assume:

$$\mu_1 = \mu + \alpha_1$$
$$\mu_2 = \mu + \alpha_2$$
$$\mu_3 = \mu + \alpha_3$$
$$\mu_4 = \mu + \alpha_4$$
$$\mu_5 = \mu + \alpha_5$$

in order to reduce the 6 parameters to 5 by linear combination. $X\beta$ can be estimated since $R(X) = 5$. The following useful relations can be derived:

$$Y = x\beta + e$$

$$E(Y) = E(X\beta + e) = X\beta + E(e) = X\beta$$

Y is a linear unbiased estimate of $X\beta$

where $\quad Y = X\hat{\beta}$

$X'X\beta$ is also an estimable function. Therefore, the linear unbiased estimate of $X'X\beta$ is $X'Y$.

$$X'Y = \begin{pmatrix} \sum_i \sum_j Y_{ij} \\ \sum_j Y_{1j} \\ \sum_j Y_{2j} \\ \sum_j Y_{3j} \\ \sum_j Y_{4j} \\ \sum_j Y_{5j} \end{pmatrix} = \begin{pmatrix} Y_{..} \\ Y_{1.} \\ Y_{2.} \\ Y_{3.} \\ Y_{4.} \\ Y_{5.} \end{pmatrix}$$

Furthermore,

$$\begin{aligned} E(Y_{1}.) &= E(Y_{1,1} + Y_{1,2} + \dots + Y_{1,14}) \\ &= 14\ (\mu+\alpha_1) \\ &= 14\ \mu_1 \\ E(Y_{2}.) &= E(Y_{2,1} + Y_{2,2} + \dots + Y_{2,14}) \\ &= 14\ (\mu+\alpha_2) \\ &= 14\ \mu_2 \\ E(Y_{3}.) &= E(Y_{3,1} + Y_{3,2} + \dots + Y_{3,14}) \\ &= 14\ (\mu+\alpha_3) \\ &= 14\ \mu_3 \\ E(Y_{4}.) &= E(Y_{4,1} + Y_{4,2} + \dots + Y_{4,14}) \\ &= 14\ (\mu+\alpha_4) \\ &= 14\ \mu_4 \\ E(Y_{5}.) &= E(Y_{5,1} + Y_{5,2} + \dots + Y_{5,14}) \\ &= 14\ (\mu+\alpha_5) \\ &= 14\ \mu_5 \end{aligned}$$

Thus, choose

$$\hat{\mu}_1 = \frac{\sum_j Y_{1j}}{14} \quad ,$$

$$\hat{\mu}_2 = \frac{\sum_j Y_{2j}}{14} \quad ,$$

$$\hat{\mu}_3 = \frac{\sum_j Y_{3j}}{14} \quad ,$$

$$\hat{\mu}_4 = \frac{\sum_j Y_{4j}}{14} \quad , \qquad \text{and}$$

$$\hat{\mu}_5 = \frac{\sum_j Y_{5j}}{14} \quad ,$$

as the five indices of water quality for five different water uses.

These are the simple arithmetic means of the relative stringency of each status controls, derived from calculated observations. The empirical model used only $\hat{\mu}_2$, standard for industry, as variable.

CHAPTER V

RESULTS AND CONCLUSIONS

There were two main objectives of this research: First, to test the various distribution functions in order to determine which was more appropriate for estimating income distribution changes; and second, to examine the impacts of water quality controls on income distribution using an empirical model relating the parameters of the chosen distribution function to variables which were expected to influence income distribution, including water quality controls. Once the beta function was selected as the appropriate form and the water quality indices were generated, an empirical test of the theoretical model was devised.

Data Collection

Data were collected for all SMSA's from several sources. The data for the income distributions and the variables in the empirical model, excluding water quality parameters, were obtained from the 1960 and 1970 Census of Population (U.S. Bureau of the Census, 1963, 1973, 1974, 1975) and the 1970 City and County Data Book (Inter-University Consortium, 1972). Some data were not compatible as between years, so that original date tapes were obtained from the Bureau of the Census and the data were reorganized in order that compatibility was achieved. For example, income distribution groupings were different from 1960 to 1970, and it was necessary to utilize the more precise groupings for 1960 data from the data tapes in order to construct a 13-group distribution for 1960 comparable to the 1970 data. The SMSA's were then grouped in order to compare 1960 and 1970 classification. One hundred seventy-two SMSA's were listed in both years with little or no change in spatial designations from 1960 to 1970. Several SMSA's were eliminated in that either the SMSA's were created between the two years, or the 1960 SMSA had been significantly enlarged or combined with other SMSA's.

Data for water quality controls were collected from the Regional offices of the Environmental Protection Agency. Compilations of each state's quality requirements were available from most regions. The Central Region data were collected from each state's legal documents concerning water quality parameters. A final aggregation of water quality standards by state and use type was made and where only qualitative parameters existed for standards (criteria or classification) adjustments were made to reflect average or similar quantitative parameters for other states. Each SMSA in a given state is assumed to be subject to that state's standards. Local standards were not available for SMSA's.

The enforcement of these water quality standards was not fully implemented by 1970. Not until 1972 and 1973 did water quality controls actually become widely applied. However, it is assumed that industries and other producers reacted to these controls as if enforcement was extant in all cases. The expectation of enforcement was likely incorporated into industrial management plans, since the passage of PL 92-500 and its amendments were indicative of future requirements. As long as businesses acted as if these controls were a fact, the impact is identical. Not until the 1980 Census will a full test of the impact be possible, since annual data for income distribution for SMSA's is not available.

Empirical Results

Empirical results were mixed and somewhat difficult to interpret. However, some areas for further research are suggested.

The initial results of the empirical test were generated from two stage least squares regression using the 172 SMSA's as the sample. Results are:

$$dw = \underset{(2.15139)}{46.9636} + \underset{(2.06815)}{46.4261}dL + \underset{(0.376160)}{0.285903}dk + \underset{(1.79224)}{0.0675268}de + e_1$$

D-W = 1.9951

$$dy = \underset{(11.0147)}{4.5593} + \underset{(12.4476)}{5.29848}dL + \underset{(18.0338)}{0.259906}dK - \underset{(-1.94071)}{0.00138651}de + e_2$$

D-W = 1.9847

$$d\sigma = \underset{(2.64887)}{8.31796} - \underset{(-0.221938)}{0.136232}dw + \underset{(2.4035)}{6.57754}dy + \underset{(0.302029)}{0.014609}da + \underset{(2.04246)}{3.22756}d\rho + e_3$$

D-W = 2.2500

$$d\rho = -2.57717 + 0.042209dw - 2.03793dy - 0.00452633da + 0.309831d\sigma + e_4$$
$$\quad (2.1073) \quad (0.21215) \quad (-1.73873) \quad (-0.286802) \quad (2.04246)$$

$D\text{-}W = 2.2500$

Numbers in parenthesis are values of the student t-Statistic, and D-W is the Durbin-Watson Statistic.

A statistical problem exists with regard to the interpretation of the t-statistic of the two stage least squares (2SLS) estimators. For single equation models, the distribution of coefficient estimator is normally distributed and the t value can be derived from the assumption of a normal distribution of the stochastic disturbance term. Since the small sample properties of simultaneous equation systems are unknown, except for the most simple cases (two equations, two or three unknowns), it is assumed that these sample sizes are sufficiently large to approach asymptotic distribution.[13] Further, it is doubtful that the distributions which have been generated for the simple cases for testing hypotheses asymptotically approach the t-distribution. Thus, the signficance of the t-statistics is doubtful. However, the common practice in the literature is to treat the results as if a student's t was appropriate, and is the approach used in this regression analysis.

One empirical problem was perhaps more critical. It is clear

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[13] 2SLS estimator of the parameter vector is consistent and asymptotically normally distributed. See Henri Theil, Principles of Econometrics (John Wiley & Sons, Inc. 1971), pp. 497–499.

that the 1960-1970 decade was one in which broad public programs and defense expenditures increased enormously, and public policy changes in many ways which might have affected changes in income distribution more than water quality controls. Among these policies would be tax changes and public expenditure shifts. In order to eliminate as many of these compounding factors as possible, the SMSA's were grouped, using factor analysis, into more or less homogenous factors. In order to group the SMSA's, a Q-type analysis was required. This analysis uses a transposed matrix, so that the SMSA's become the factors which are grouped while the variables, which are normally grouped in a factor analysis, become the independent observations or cases. Because the SMSA's were considered as the variables and exhibited a wide variation in the demographic characteristics, which were the cases, standardization for several of the demographic characteristics were required. Standardization was performed prior to transposing the matrix.

The number of cases on which the Q analysis was performed exceeded the number of SMSA's in both 1960 and 1970. One hundred and twenty-nine characteristics were identified as relevant to the factoring of SMSA's. In order to perform the statistical procedures, the number of cases must exceed the number of variables, similar to the conditions required for a solution to multiple equation systems. It was necessary, therefore, to divide the SMSA's into smaller groups. This was done on the basis of population. For 1960, a division was made between SMSA's over and under

250,000 population. For 1970, four divisions were made based on population: under 150,000; 150,001 to 250,000; 250,001 to 500,000; and over 500,000. Data for all the characteristics were taken from the Census of Populations for each year and from the City and County Data Book.

The groupings were picked from the Rotated Factor Matrix factors with an element greater than the absolute value of .50 with relatively low loadings on other factors. If an SMSA seemed to load on more than one factor it was eliminated from the analysis. The rotation was based on the verimax criterion, was orthogonal, and used the correlation matrix. The trace of that matrix was the squared multiple correlation coefficients. A listing of the results of the factor analysis for 1960 and 1970 may be found in appendix C and D.

Compilation of SMSA's which remained in the same factor for both 1960 and 1970 was accomplished. The results were not usable, since no more than seven to ten such SMSA's could be found in any one factor. Since the number of variables in the regression equations exceeded the number of observations, a further consolidation of SMSA's was required. The consolidation was performed by eliminating some of the population breakdown for 1970, and combining the factors, so that population groupings for both years were two: over 250,000 and under 250,000. Factor analysis in these two categories yielded two groups with 20 and 16 observations (SMSA's). Table 3 is a list of these SMSA's by population group.

Table 3 Two groups of SMSA's by factor analysis

GROUP 1	GROUP 2
Birmingham, Ala.	Baton Rouge, La.
Cleveland, Ohio	Bay City, Mich.
Columbia, S. C.	Cedar Rapids, Iowa
Davenport, Ill.	Charlotte, S. C.
Detroit, Mich.	Corpus Christi, Tex.
Houston, Tex.	Decatur, Ill.
Huntington, W. V.	Jackson, Mich.
Jacksonville, Fla.	Kalamazoo, Mich.
Knoxville, Tenn.	Kenosha, Wis.
Memphis, Tenn.	Lexington, Ky.
Milwaukee, Wis.	Little Rock, Ark,
Minneapolis, Minn.	Macon, Ga.
Mobile, Ala.	Meriden, Conn.
New Orleans, La.	Montgomery, Ala.
Newark, N. J.	Muncie, Ind.
Norfolk, Va.	Savannah, Ga.
Paterson, N. J.	
Rochester, N. Y.	
San Antonio, Tex.	
San Francisco, Calif.	

Regressions were run on these two groups; results of these four regressions are:

Group 1:

$$dw = \underset{(2.95727)}{226.040} + \underset{(2.89463)}{227.771dL} - \underset{(-0.629111)}{1.78205dk} + \underset{(0.158857)}{0.012274de} + e_1$$

D-W = 1.8514

$$dy = \underset{(3.11627)}{5.34166} + \underset{(3.46337)}{6.11155dL} + \underset{(3.60908)}{0.229265dk} - \underset{(-0.800246)}{0.0013866de} + e_2$$

D-W = 2.2177

$$d\sigma = \underset{(-0.0677277)}{-257.065} + \underset{(0.0678556)}{10.6816dw} - \underset{(-0.0670327)}{193.143dy} - \underset{(-0.0689206)}{68.5915da} - \underset{(-0.129725)}{2.0884d\rho} + e_3$$

D-W = 1.9492

$$d\rho = \underset{(-0.0825492)}{-123.092} + \underset{(0.0814406)}{5.11471dw} - \underset{(-0.0814365)}{92.4837dy} - \underset{(-0.0844671)}{32.8441da} - \underset{(-0.129725)}{0.478835d\sigma} + e_4$$

D-W = 1.9492

Group 2:

$$dw = \underset{(2.09227)}{164.573} + \underset{(2.06146)}{167.607dL} - \underset{(-1.05591)}{3.2500dk} + \underset{(0.119158)}{0.0187123de} + e_1$$

D-W = 1.7338

$$dy = \underset{(1.98311)}{3.35136} + \underset{(2.24128)}{3.91518dL} + \underset{(6.55653)}{0.433578dk} -$$

$$\underset{(-1.87033)}{0.00631038de} + e_2$$

D-W = 1.8728

$$d\sigma = \underset{(-0.107055)}{-3.26151} + \underset{(0.55197)}{1.38029dw} - \underset{(-0.0281348)}{0.603032dY} -$$

$$\underset{(-0.309605)}{2.66631da} + \underset{(0.183512)}{0.632859d\rho} + e_3$$

D-W = 1.5055

$$d\rho = \underset{(0.0742601)}{5.15362} - \underset{(-0.149562)}{2.18105dw} + \underset{(0.0254715)}{0.952869dY} +$$

$$\underset{(0.135429)}{4.21312da} + \underset{(0.183512)}{1.58013d\sigma} + e_4$$

D-W = 1.5055

The interpretation of the results is somewhat difficult, even though some results do appear consistent among all regressions. The results for three different sample sizes are summarized in Table 4. The first column is for dependent variables, and the first row is for independent variables. All signs for the parameters are listed in the table. For the first two equations, the results are fairly consistent except for dk in dw. For 172 SMSA and Group 1, the coefficients for dk are not significant although they are different in sign.

dL is significant in explaining dw and dY. The higher the labor productivity is, the higher the wage rate and the output level are. Like dL, dk has a positive effect on dY; nevertheless dk may have a negative effect on dw. Raising up the rental rate of capital may possibly contract the wage rate of labor input.

Table 4 Empirical results of the coefficient signs

Equation Set	C	dL	dK	de	dw	dy	da	dσ	dρ	Sample Size
dw	+	+	(+)	+						172
	+	+	(−)	(+)						20
	+	+	$-^{a}$	(+)						16
dy	+	+	+	−						172
	+	+	+	(−)						20
	+	+	+	−						16
dσ	+				(−)	+	(+)		+	172
	(−)				(+)	(−)	(−)		(−)	20
	(−)				(+)	(−)	(−)		(+)	16
dρ	−				(+)	−	(−)	+		172
	(−)				(+)	(−)	(−)	(+)		20
	(+)				(−)	(+)	(+)	(+)		16

Note: The signs in parenthesis were not significant from t-test. Significant levels are interpresuted as $t > |1.7|$

a. Significant at $t > |1.0|$

Regressions in Group 2 showed this result.

The coefficients of de for 172 SMSA's were significant. After reducing the probability that de's were proxies for other variables, the results were different. For Group 1 (with 20 SMSA's), none of the coefficients for de is significant; for Group 2 (with 16 SMSA's), de is significant only in explaining dY and the effect is negative in sign. It is reasonable that the higher quality the policy demanded the lower the output would turn to be.

Most of the coefficients for the rest of the two equations, $d\sigma$ and $d\rho$, were not significant. This could be seen in Table 4. Note that dY was significant in the equation for $d\sigma$ and $d\rho$ and different in sign. Thus, raising up the quality standards seemed to equalize the distribution of income through the reduction in total output.

$d\sigma$ and $d\rho$ were positively correlated. Thurow (1972) pointed out, increase in the first parameter of the beta distribution, $d\sigma$, leads to a less equal distribution. Increase in the second parameter, $d\rho$, leads to a more equal distribution, <u>ceterus paribus</u>.[14] Thus, changes in distribution could involve changes in both of the two parameters.

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[14] This is true only if the estimated Lorenz curve falls below the 45 degree equal distribution line. If the curve lies above that line the opposite is true. The estimations for the beta parameters for the data indicate that the Lorenz curve is, in all cases, below the 45 degree line. See appendix E for examples.

The wage variable was consistently not significant in the beta distribution parameters' estimates. Significance of the industrial controls in the wage regression equation has no effect on income distribution.

Two single equation regressions[15] were also run for both $d\sigma$ and $d\rho$. Results were:

For 172 observations:

$$d\sigma = \underset{(3.5767)}{48.5239} + \underset{(3.59847)}{49.0454dL} - \underset{(-0.992422)}{0.486286dk} - \underset{(-3.99408)}{0.091472de} + \underset{(0.543517)}{0.0125824da} + e_1$$

D-W = 1.8515

$$d\rho = \underset{(0.634562)}{4.76825} + \underset{(0.766692)}{5.92465dL} - \underset{(-2.42924)}{0.626845dk} - \underset{(-1.8608)}{0.0235944de} + \underset{(0.435599)}{0.00538978da} + e_2$$

D-W = 2.1036

For Group 1:

$$d\sigma = \underset{(1.76487)}{199.52} + \underset{(1.72163)}{202.802dL} - \underset{(-0.80698)}{3.56391dk} - \underset{(-1.31718)}{0.116525de} + \underset{(0.0660757)}{0.18808da} + e_1$$

D-W = 2.172

$$d\rho = \underset{(-2.42541)}{-118.93} - \underset{(-2.35789)}{120.926dL} + \underset{(1.45077)}{2.85695dk} + \underset{(1.03557)}{0.0437597de} - \underset{(-0.312148)}{0.414005da} + e_2$$

[15] All exogenous variables were taken as independent variables, while $d\sigma$ and $d\rho$ were dependent variables. Ordinary least squares method was used to estimate the coefficients for each of the two equations.

$D\text{-}W = 1.6530$

For Group 2:

$$d\sigma = \underset{(2.88985)}{248.200} + \underset{(2.82495)}{257.677dL} - \underset{(-1.52739)}{6.27415dk} - \underset{(-0.137059)}{0.0178135de} - \underset{(-1.38411)}{3.12838da} + e_1$$

$D\text{-}W = 1.8915$

$$d\rho = \underset{(0.667199)}{43.7808} + \underset{(0.681117)}{46.9425dL} - \underset{(-0.80839)}{2.0594dk} - \underset{(-0.831851)}{0.0603245de} - \underset{(-0.44608)}{0.628161da} + e_2$$

$D\text{-}W = 2.2411$

The results have the same signs and significant variables as the simultaneous equation esimations. Of course, these equations are somewhat misleading in that the simultaneity of their determination is lost, and therefore, results are somewhat ambiguous.

Given the reservations concerning both the conceptual model's structure and the interpretation of the empirical results, policy prescriptions appear rather inappropriate. Clearly, there have been changes in the income distribution in SMSA's from 1960 to 1970, as indicated by the changes in the beta distribution functions' parameters. Without a more extensive data collection, the casuality of these changes is not easy to test, even though water quality controls are significant variables in some of the regressions.

SUMMARY AND CONCLUSIONS

In general, there is a need for much more study of the problems concerned with income distribution. The efficiency and equity in the optimal supply of environmental quality is of broad interest nowadays. This study tried to examine empirically 1) the analytical tools as suggested as estimators of income distribution and 2) their applicability to a economic model of water quality controls.

The lognormal, displaced lgonormal, gamma, and beta distribution functions were considered as appropriate approximations for income distribution functions. The Gastwirth upper and lower bound test for Gini coefficient was applied as a fitness measure and the beta function was clearly superior to the other forms from the SMSA data.

A simultaneous equation econometric model was constructed, based upon hypotheses about production and distribution. Water quality controls were introduced to the model as a negative input in the production process. Based on the duality principle in the theory of cost and transformation function, a cost function, which yields the minimum cost given the output level and factor prices, was defined. Equilibria in commodity and factor markets were also assumed. Thus, a theoretical bridge connecting the water quality policy with output level and factor payments was completed.

Factor incomes were assumed to be the basis of the family income, and payments to labor and capital were used in the model. The link between family income and factor payments is the pricing of factors of production and the distribution of benefits of factors between families.

The model indicated changes in factor prices and total output resulting from the imposition of water quality controls on production. Meanwhile, the consequent effects on family income distribution from those changes in factor payments and output level were tested in a simultaneous equation system.

The simultaneous equation regression results are not significantly conclusive about the effects of water quality controls on income distribution. It does appear that water quality parameter may effect the wage payment and total output, if the parameter was not in fact a surrogate for other excluded variables in the economic system. The effect of wage changes on income distribution was not significant. Changes in total output appeared to be the most significant in the distribution equations. Theoretically, increases in factor inputs should increase the output level; in the empirical test, changes in output were positively related to changes in labor and capital inputs. Furthermore, the output elasticity of labor seemed greater than that of capital as implied by the coefficients estimated.

Results also indicated that changes in labor supply affected changes in wage rate, but changes in capital supply did not.

Changes in capital supply may indirectly affect the family income distribution through changes in output. Specifically, increases in capital supply may lead to increases in output and less equal distributoin of income. The imposition of environmental constraints may decrease output, and cause a more equal distribution of income as a result. While these results are not intuitively obvious, some are similar to those obtained by Thurow, and most of the coefficient signs appear consistent among regressions. Single equation regressions of the exogenous variables on the distribution parameters yielded similar results.

Better data are required for more complete and accurate analysis. In order to draw more positive conclusions about specific impacts of water quality policy, the model could be applied to areas in which detailed industrial and distribution data are available.

The principle thrust of the research was to develop a model which would provide a systematic analysis of the impact of water quality policy. The methodology used does provide a means to organize the complexity of economic causality with respect to income distribution change. Factor parameters, total output, water policy, family income, and other variables were included in an economic model of income distribution which was subject to econometric analysis. It appears that this type of systematic econometric approach can be fruitful in analyzing income distribution change. Further research in regions where detailed data are available is indicated as an additional test of the methodology.

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APPENDICIES

Appendix A. The following program was used to generate the parameters of the lognormal and displaced lognormal distribution:

```
$WATFIV            04409BANNERWATERQ,TIME=300,PAGES=300
      SUBROUTINE NDTR(Z,P,DD)
      AX=ABS(Z)
      T=1.0/(1.0+.2316419*AX)
      D=0.3989423*EXP(-Z*Z/2.0)
      P=1.0-D*T*((((1.330274*T-1.821256)*T+1.781478)*T-0.3565638)*T+
     10.3193815)
      IF(Z)1,2,2
    1 P=1.0-P
    2 RETURN
      END

      CHARACTER*20 OHS,XHS,UHS,VHS,WHS
      DIMENSION THS(500),AHS(500),BHS(500),CHS(500),DHS(500),EHS(500),
     XFHS(500),GHS(500),HHS(500),SHS(500),PHS(500),QHS(500),RHS(500),
     XD(500),XMED(500),MOD(500),OHS(500),XHS(500),UHS(500),VHS(500),
     XWHS(500)
      REAL M,LAHY,LBHY,LCHY,LDHY,LEHY,LFHY,LGHY,LX,LAHS,LBHS,LCHS,LDHS,
     1LEHS,LFHS,LGHS,LD,J,MOD,LHHY,LSHY,LPHY,LQHY,LRHY,LHHS,LSHS,LPHS,
     XLQHS,LRHS
      AHY=500
      BHY=1500
      CHY=2500
      DHY=3500
      EHY=4500
      FHY=5500
      GHY=6500
      HHY=7500
      SHY=8500
      PHY=9500
      QHY=12500
      RHY=20000
      A=15000
      Y=25000
      XNLA=ALOG(A)
      XNLB=ALOG(Y)
      W=0
      G=1.28
      DO 10 I=1,500
      READ(5,1,END=3)THS(I),AHS(I),BHS(I),CHS(I),DHS(I),EHS(I),FHS(I),
     XGHS(I),HHS(I),SHS(I),PHS(I),QHS(I),RHS(I),D(I),XMED(I),MOD(I),
     XOHS(I),XHS(I),UHS(I),VHS(I),WHS(I)
    1 FORMAT(9X,F8.0,8F7.0,/,9X,F8.0,6F7.0,5A4)
      C=RHS(I)+D(I)
      XNLC=ALOG(C)
      XNLD=ALOG(D(I))
      XV=(XNLC-XNLD)/(XNLB-XNLA)
      X=(Y*XV)/(XV-1)
      LAHS=500
      LBHS=1000
      LCHS=2000
      LDHS=3000
      LEHS=4000
      LFHS=5000
      LGHS=6000
      LHHS=7000
      LSHS=8000
      LPHS=9000
      LQHS=10000
      LRHS=15000
```

```
      LD=25000
      UAHS=999
      UBHS=1999
      UCHS=2999
      UDHS=3999
      UEHS=4999
      UFHS=5999
      UGHS=6999
      UHHS=7999
      USHS=8999
      UPHS=9999
      UQHS=14999
      URHS=24999
      UD=X+(X-25000)
      LAHY=ALOG(LAHS)
      LBHY=(ALOG(UBHS)+ALOG(LBHS))/2
      LCHY=(ALOG(UCHS)+ALOG(LCHS))/2
      LDHY=(ALOG(UDHS)+ALOG(LDHS))/2
      LEHY=(ALOG(UEHS)+ALOG(LEHS))/2
      LFHY=(ALOG(UFHS)+ALOG(LFHS))/2
      LGHY=(ALOG(UGHS)+ALOG(LGHS))/2
      LHHY=(ALOG(UHHS)+ALOG(LHHS))/2
      LSHY=(ALOG(USHS)+ALOG(LSHS))/2
      LPHY=(ALOG(UPHS)+ALOG(LPHS))/2
      LQHY=(ALOG(UQHS)+ALOG(LQHS))/2
      LRHY=(ALOG(URHS)+ALOG(LRHS))/2
      LX=ALOG(X)
      SLX=LX**2
      E=(LAHY*AHS(I))+(LBHY*BHS(I))+(LCHY*CHS(I))+(LDHY*DHS(I))+
     X(LEHY*EHS(I))+(LFHY*FHS(I))+(LGHY*GHS(I))+(LHHY*HHS(I))+
     X(LSHY*SHS(I))+(LPHY*PHS(I))+(LQHY*QHS(I))+(LRHY*RHS(I))+(LX*D(I))
      M=E/THS(I)
      V=(1/(THS(I)-1))*((AHS(I)*(LAHY-M)**2)+(BHS(I)*(LBHY-M)**2)+
     X(CHS(I)*(LCHY-M)**2)+(DHS(I)*(LDHY-M)**2)+(EHS(I)*(LEHY-M)**2)+
     X(FHS(I)*(LEHY-M)**2)+(GHS(I)*(LGHY-M)**2)+(HHS(I)*(LHHY-M)**2)+
     X(SHS(I)*(LSHY-M)**2)+(PHS(I)*(LPHY-M)**2)+(QHS(I)*(LQHY-M)**2)+
     X(RHS(I)*(LRHY-M)**2)+(D(I)*(LX-M)**2))
      AX=EXP((2*M)+V)
      AN=EXP(V)-1
      AMCD=EXP(M-V)
      AMED=EXP(M)
      AMEN=EXP(M+(V/2))
      AVAR=AX*AN
      AD=AMED-XMED(I)
      W=W+1
      ASKW=(AN**1.5)+(3*(AN**.5))
      AKUR=(AN**4)+(6*(AN**3))+(15*(AN**2))+(16*AN)
      F=(AHY*AHS(I))+(BHY*BHS(I))+(CHY*CHS(I))+(DHY*DHS(I))+(EHY*EHS(I))
     1+(FHY*FHS(I))+(GHY*GHS(I))+(HHY*HHS(I))+(SHY*SHS(I))+(PHY*PHS(I))+
     X(QHY*QHS(I))+(RHY*RHS(I))+(X*D(I))
      XMEN=F/THS(I)
      Z=(V**.5)/(2**.5)
      CALL NDTR(Z,P,D)
      AIND=(2*P)-1
      ACOF=2*(AMEN)*AIND
      IF(AHS(I).GT.(THS(I)+1)/10)GO TO 20
      IF(AHS(I)+BHS(I).GT.(THS(I)+1)/10)GO TO 30
      IF(AHS(I)+BHS(I)+CHS(I).GT.(THS(I)+1)/10)GO TO 40
      IF(AHS(I)+BHS(I)+CHS(I)+DHS(I).GT.(THS(I)+1)/10)GO TO 50
      IF(AHS(I)+BHS(I)+CHS(I)+DHS(I)+EHS(I).GT.(THS(I)+1)/10)GO TO 60
```

```
      IF(AHS(I)+BHS(I)+CHS(I)+DHS(I)+EHS(I)+FHS(I).GT.(THS(I)+1)/10)
     1GO TO 70
      IF(AHS(I)+BHS(I)+CHS(I)+DHS(I)+EHS(I)+FHS(I)+GHS(I).GT.(THS(I)+1)
     1/10)GO TO 80
 20   DEC=0+(((THS(I)+1)/10-0)/AHS(I))*1000
      GO TO 16
 30   DEC=LBHS+(((THS(I)+1)/10-AHS(I))/BHS(I))*1000
      GO TO 16
 40   DEC=LCHS+(((THS(I)+1)/10-(AHS(I)+BHS(I)))/CHS(I))*1000
      GO TO 16
 50   DEC=LDHS+(((THS(I)+1)/10-(AHS(I)+BHS(I)+CHS(I)))/DHS(I))*1000
      GO TO 16
 60   DEC=LEHS+(((THS(I)+1)/10-(AHS(I)+BHS(I)+CHS(I)+DHS(I)))/EHS(I))
     1*1000
      GO TO 16
 70   DEC=LFHS+(((THS(I)+1)/10-(AHS(I)+BHS(I)+CHS(I)+DHS(I)+EHS(I)))/
     1FHS(I))*1000
      GO TO 16
 80   DEC=LGHS+(((THS(I)+1)/10-(AHS(I)+BHS(I)+CHS(I)+DHS(I)+EHS(I)+
     1FHS(I)))/GHS(I))*1000
 16   IF(.9*THS(I).GT.THS(I)-D(I))GO TO 100
      IF(.9*THS(I).GT.THS(I)-(D(I)+RHS(I)))GO TO 110
      IF(.9*THS(I).GT.THS(I)-(D(I)+RHS(I)+QHS(I)))GO TO 120
      IF(.9*THS(I).GT.THS(I)-(D(I)+RHS(I)+QHS(I)+PHS(I)))GO TO 130
100   DEC9=LD+(((9*THS(I)+9)/10-(THS(I)-D(I)))/D(I))*(UD-LD)
      GO TO 29
110   DEC9=LRHS+(((9*THS(I)+9)/10-(THS(I)-D(I)-RHS(I)))/RHS(I))*10000
      GO TO 29
120   DEC9=LQHS+(((9*THS(I)+9)/10-(THS(I)-D(I)-RHS(I)-QHS(I)))/QHS(I))*
     X5000
      GO TO 29
130   DEC9=LPHS+(((9*THS(I)+9)/10-(THS(I)-D(I)-RHS(I)-QHS(I)-PHS(I)))/
     XPHS(I))*1000
 29   H=DEC/XMED(I)
      J=DEC9/XMED(I)
      B=XMED(I)*((H*J-1)/(2-H-J))
      D1=ALOG((XMED(I)+B)/((H*XMED(I))+B))
      D9=ALOG(((J*XMED(I))+B)/(XMED(I)+B))
      BM=ALOG(XMED(I)+B)
      BV=(D1/G)**2
      BX=EXP((2*BM)+BV)
      BN=EXP(BV)-1
      BMOD=EXP(BM-BV)-B
      BMEN=EXP(BM+(.5*BV))-B
      BMED=EXP(BM)-B
      BVAR=BX*BN
      BSKW=(BN**1.5)+(3*(BN**.5))
      BKUR=(BN**4)+(6*(BN**3))+(15*(BN**2))+(16*BN)
      Z=(BV**.5)/(2**.5)
      CALL NOTR(Z,P,DD)
      BIND=(2*P)-1
      BCOF=2*BMEN*BIND
 17   WRITE(6,2)OHS(I),XHS(I),JHS(I),VHS(I),WHS(I)
  2   FORMAT('1',5A4,/,' ',20('-'))
      WRITE(6,4)X
  4   FORMAT('0','X=',3X,F15.2)
      WRITE(6,7)AD
  7   FORMAT('0','AD  =',F15.2)
      WRITE(6,8)XMED(I)
  8   FORMAT('0','XMED=',F15.2,15X)
```

```
      WRITE(6,9)XMEN
    9 FORMAT('0','XMEN=',F15.2)
      WRITE(6,5)M
    5 FORMAT('0','M=',3X,F15.4)
      WRITE(6,6)V
    6 FORMAT('0','V=',3X,F15.2)
      WRITE(6,47)AMOD
   47 FORMAT('0','AMOD=',F15.2)
      WRITE(6,48)AMED
   48 FORMAT('0','AMED=',F15.2)
      WRITE(6,49)AMEN
   49 FORMAT('0','AMEN=',F15.2)
      WRITE(6,51)AVAR
   51 FORMAT('0','AVAR=',F17.2)
      WRITE(6,12)ASKW
   12 FORMAT('0','ASKW=',F15.4)
      WRITE(6,13)AKUR
   13 FORMAT('0','AKUR=',F17.2)
      WRITE(6,14)AIND
   14 FORMAT('0','AIND=',F15.4)
      WRITE(6,15)ACOF
   15 FORMAT('0','ACOF=',F15.2)
      WRITE(6,18)DEC
   18 FORMAT('0','DEC=',1X,F15.2)
      WRITE(6,19)DEC9
   19 FORMAT('0','DEC9=',F15.2)
      WRITE(6,31)B
   31 FORMAT('0','B=',3X,F15.2)
      WRITE(6,32)BM
   32 FORMAT('0','BM=',2X,F15.4)
      WRITE(6,33)BV
   33 FORMAT('0','BV=',2X,F15.4)
      WRITE(6,34)BMOD
   34 FORMAT('0','BMOD=',F15.2)
      WRITE(6,35)BMED
   35 FORMAT('0','BMED=',F15.2)
      WRITE(6,36)BMEN
   36 FORMAT('0','BMEN=',F15.2)
      WRITE(6,37)BVAR
   37 FORMAT('0','BVAR=',F15.2)
      WRITE(6,38)BSKW
   38 FORMAT('0','BSKW=',F15.2)
      WRITE(6,39)BKUR
   39 FORMAT('0','BKUR=',F15.2)
      WRITE(6,42)BIND
   42 FORMAT('0','BIND=',F15.4)
      WRITE(6,41)BCOF
   41 FORMAT('0','BCOF=',F15.2)
      WRITE(7,240)M,V,ASKW,AKUR,AIND,OHS(I),XHS(I),UHS(I),VHS(I),WHS(I),
     XB,BM,BV,BSKW,BKUR,BIND,OHS(I),XHS(I),UHS(I),VHS(I),WHS(I)
  240 FORMAT (5F10.5,10X,5A4/F12.5,2F9.5,3F10.5,5A4)
   10 CONTINUE
    3 STOP
      END
$EXEC
```

<u>Appendix B</u>. The following program was used to generate the parameters of the gamma and beta densities.

```
$WATFIV        24409READING,TIME=300,PAGES=300
      SUBROUTINE DGTG(X,PHYY,PPHY)
      PHYY=ALOG(X)-1/(2*X)-1/(12*(X**2))+1/(120*(X**4))-1/(252*(X**6))+
     X1/(240*(X**8))
      PPHY=1/X+1/(2*(X**2))+1/(6*(X**3))-1/(30*(X**5))+1/(42*(X**7))-
     X1/(30*(X**9))
      RETURN
      END

      SUBROUTINE G(X,GAA)
      GAA=2.5066281*(X**(X+0.5))*EXP(-X)*(1+1/(12*X)+1/(288*(X**2))-
     X139/(51840*(X**3)))
      RETURN
      END

      DIMENSION THS(500),AHS(500),BHS(500),CHS(500),DHS(500),EHS(500),
     XFHS(500),GHS(500),HHS(500),OHS(500),PHS(500),QHS(500),RHS(500),
     XD(500),SHS(500),SSS(500 ),RRR(500 ),PPP(500),OOO(500)
      REAL M,LAHY,LBHY,LCHY,LDHY,LEHY,LFHY,LGHY, LHHY,LOHY,LPHY,LQHY,
     XLRHY,LX,LAHS,LBHS,LCHS,LDHS,LEHS,LFHS,LGHS,LHHS,LOHS,LPHS,LQHS,
     XLRHS,J
      DO 10 I=1,500
      READ(5,1,END=3)THS(I),AHS(I),BHS(I),CHS(I),DHS(I),EHS(I),FHS(I),
     XGHS(I),HHS(I),OHS(I),PHS(I),QHS(I),RHS(I),D(I),SHS(I),SSS(I),RRR(I
     X),PPP(I),OOO(I)
1     FORMAT(9X,F8.0,8F7.0/9X,F8.0,4F7.0,14X,5A4)
      AHY=500
      BHY=1500
      CHY=2500
      DHY=3500
      EHY=4500
      FHY=5500
      GHY=6500
      HHY=7500
      OHY=8500
      PHY=9500
      QHY=12500
      RHY=20000
      LAHY=ALOG(AHY)
      LBHY=ALOG(BHY)
      LCHY=ALOG(CHY)
      LDHY=ALOG(DHY)
      LEHY=ALOG(EHY)
      LFHY=ALOG(FHY)
      LGHY=ALOG(GHY)
      LHHY=ALOG(HHY)
      LOHY=ALOG(OHY)
      LPHY=ALOG(PHY)
      LQHY=ALOG(QHY)
      LRHY=ALOG(RHY)
      A=15000
      B=25000
      XNLA=ALOG(A)
      XNLB=ALOG(B)
      C=PHS(I)+D(I)
      XNLC=ALOG(C)
      XNLD=ALOG(D(I))
      XV=(XNLC-XNLD)/(XNLB-XNLA)
      X=(B*XV)/(XV-1)
      LX=ALOG(X)
```

```
      M=(AHY*AHS(I)+BHY*BHS(I)+CHY*CHS(I)+DHY*DHS(I)+EHY*EHS(I)+
     XFHY*FHS(I)+GHY*GHS(I)+HHY*HHS(I)+OHY*OHS(I)+PHY*PHS(I)+QHY*QHS(I)
     X+RHY*RHS(I)+X*D(I))/THS(I)
      Y=M*THS(I)
      W=(   LAHY*AHS(I)+LBHY*BHS(I)+LCHY*CHS(I)+LDHY*DHS(I)+LEHY*EHS(I)+
     XLFHY*FHS(I)+LGHY*GHS(I)+LHHY*HHS(I)+LOHY*OHS(I)+LPHY*PHS(I)+
     XLQHY*QHS(I)+LRHY*RHS(I)+LX*D(I))/THS(I)
      CW=ALOG(M)-W
      EA=1/(2*CW)
   16 YEA=EA+10
      CALL  DGTG(YEA,PHYY,PPHY)
      DG=  PHYY-1/(EA+9)-1/(EA+8)-1/(EA+7)-1/(EA+6)-1/(EA+5)-1/(EA+4)-
     X1/(EA+3)-1/(EA+2)-1/(EA+1)-1/EA
      TG=PPHY+1/((EA+9)**2)+1/((EA+8)**2)+1/((EA+7)**2)+1/((EA+6)**2)+
     X1/((EA+5)**2)+1/((EA+4)**2)+1/((EA+3)**2)+1/((EA+2)**2)+
     X1/((EA+1)**2)+1/(EA**2)
    4 XNR=(ALOG(EA)-DG-CW)/(1/EA-TG)
      EA=EA-XNR
      AXNR=ABS(XNR)
      IF(AXNR.GE.0.0010)GO TO 16
      EB=EA/M
      AEA=EA+1
      CALL  G(AEA,GAA)
      GAM=GAA/(AEA*EA)
      BEA=AEA-0.5
      CALL  G(BEA,GAA)
      GH=GAA/BEA
      CEA=AEA+1
      CALL  G(CEA,GAA)
      GG=GAA/(CEA*AEA)
      AIND=0.3989423*GH/GG
      FB=((EB**EA)/GAM)*(BHY**(EA-1))*EXP(-EB*BHY)*1000
      FA=((EB**EA)/GAM)*(AHY**(EA-1))*EXP(-EB*AHY)*1000
      FC=((EB**EA)/GAM)*(CHY**(EA-1))*EXP(-EB*CHY)*1000
      FD=((EB**EA)/GAM)*(DHY**(EA-1))*EXP(-EB*DHY)*1000
      FE=((EB**EA)/GAM)*(EHY**(EA-1))*EXP(-EB*EHY)*1000
      FF=((EB**EA)/GAM)*(FHY**(EA-1))*EXP(-EB*FHY)*1000
      FG=((EB**EA)/GAM)*(GHY**(EA-1))*EXP(-EB*GHY)*1000
      FH=((EB**EA)/GAM)*(HHY**(EA-1))*EXP(-EB*HHY)*1000
      FO=((EB**EA)/GAM)*(OHY**(EA-1))*EXP(-EB*OHY)*1000
      FP=((EB**EA)/GAM)*(PHY**(EA-1))*EXP(-EB*PHY)*1000
      FQ=((EB**EA)/GAM)*(QHY**(EA-1))*EXP(-EB*QHY)*5000
      FR=((EB**EA)/GAM)*(RHY**(EA-1))*EXP(-EB*RHY)*10000
      FX=((EB**EA)/GAM)*(X**(EA-1))*EXP(-EB*X)*2*(X-25000)
      SFA=LAHY*FA+LBHY*FB+LCHY*FC+LDHY*FD+LEHY*FE+LFHY*FF+LGHY*FG+LX*FX
     X+LHHY*FH+LOHY*FO+LPHY*FP+LQHY*FQ+LRHY*FR
      GM=EXP(SFA)
      AMEN=EA/EB
      AMOD=(EA-1)/EB
      AMED=AMOD+2/(3*EB)
      AVAR=EA/(EB**2)
      ASKW=2/SQRT(EA)
      AKUR=6/EA
      ACOF=2*AMEN*AIND
      R=AMEN/GM
      UD=X+(X-25000)
      AHY=500/UD
      BHY=1500/UD
      CHY=2500/UD
      DHY=3500/UD
```

```
      EHY=4500/UD
      FHY=5500/UD
      GHY=6500/UD
      HHY=7500/UD
      OHY=8500/UD
      PHY=9500/UD
      QHY=12500/UD
      RHY=20000/UD
      X=X/UD
      M=(AHY*AHS(I)+BHY*BHS(I)+CHY*CHS(I)+DHY*DHS(I)+EHY*EHS(I)+
     XFHY*FHS(I)+GHY*GHS(I)+HHY*HHS(I)+OHY*OHS(I)+PHY*PHS(I)+QHY*QHS(I)
     X+RHY*RHS(I)+X*D(I))/THS(I)
      V=(((AHY-M)**2)*AHS(I)+((BHY-M)**2)*BHS(I)+((CHY-M)**2)*CHS(I)+
     X((DHY-M)**2)*DHS(I)+((EHY-M)**2)*EHS(I)+((FHY-M)**2)*FHS(I)+
     X((GHY-M)**2)*GHS(I)+((HHY-M)**2)*HHS(I)+((OHY-M)**2)*OHS(I)+((PHY-
     XM)**2)*PHS(I)+((QHY-M)**2)*QHS(I)+((RHY-M)**2)*RHS(I)+((X-M)**2)*
     XD(I))/(THS(I)-1)
      P=((M**2)*(1-M)/V)-M
      Q=(M*(1-M)/V-1)*(1-M)
   34 YP=P+10
      YQ=Q+10
      CALLDGTG(YP,PHYY,PPHY)
      DGP=PHYY   -1/P-1/(P+1)-1/(P+2)-1/(P+3)-1/(P+4)-1/(P+5)-1/(P+6)-
     X1/(P+7)-1/(P+8)-1/(P+9)
      TGP=PPHY+1/(P**2)+1/((P+1)**2)+1/((P+2)**2)+1/((P+3)**2)+
     X1/((P+4)**2)+1/((P+5)**2)+1/((P+6)**2)+1/((P+7)**2)+
     X1/((P+8)**2)+1/((P+9)**2)
      CALL  DGTG(YQ,PHYY,PPHY)
      DGQ=PHYY-1/Q-1/(Q+1)-1/(Q+2)-1/(Q+3)-1/(Q+4)-1/(Q+5)-1/(Q+6)-
     X1/(Q+7)-1/(Q+8)-1/(Q+9)
      TGQ=PPHY+1/(Q**2)+1/((Q+1)**2)+1/((Q+2)**2)+1/((Q+3)**2)+
     X1/((Q+4)**2)+1/((P+5)**2)+1/((Q+6)**2)+1/((Q+7)**2)+
     X1/((Q+8)**2)+1/((Q+9)**2)
      YPQ=P+Q+10
      CALL  DGTG(YPQ,PHYY,PPHY)
      DGPQ=PHYY-1/(P+Q)-1/(P+Q+1)-1/(P+Q+2)-1/(P+Q+3)-1/(P+Q+4)-1/(P+Q+5
     X)-1/(P+Q+6)-1/(P+Q+7)-1/(P+Q+8)-1/(P+Q+9)
      TGPQ=PPHY+1/((P+Q)**2)+1/((P+Q+1)**2)+1/((P+Q+2)**2)+
     X1/((P+Q+3)**2)+1/((P+Q+4)**2)+1/((P+Q+5)**2)+1/((P+Q+6)**2)+
     X1/((P+Q+7)**2)+1/((P+Q+8)**2)+1/((P+Q+9)**2)
      FP=TGP-TGPQ
      FQ=-TGPQ
      FFP=-TGPQ
      FFQ=TGQ-TGPQ
      U= AHY**(AHS(I)/THS(I))* BHY**(BHS(I)/THS(I))* CHY**(CHS(I)/THS(I)
     X)* DHY**(DHS(I)/THS(I))* EHY**(EHS(I)/THS(I))* FHY**(FHS(I)/THS(I)
     X)* GHY**(GHS(I)/THS(I))* HHY**(HHS(I)/THS(I))* OHY**(OHS(I)/THS(I)
     X)* PHY**(PHS(I)/THS(I))* QHY**(QHS(I)/THS(I))* RHY**(RHS(I)/THS(I)
     X)* X**(D(I)/THS(I))
      W=((1-AHY)**(AHS(I)/THS(I)))*((1-BHY)**(BHS(I)/THS(I)))*
     X    ((1-CHY)**(CHS(I)/THS(I)))*((1-DHY)**(DHS(I)/THS(I)))*
     X    ((1-EHY)**(EHS(I)/THS(I)))*((1-FHY)**(FHS(I)/THS(I)))*
     X    ((1-GHY)**(GHS(I)/THS(I)))*((1-HHY)**(HHS(I)/THS(I)))*
     X    ((1-OHY)**(OHS(I)/THS(I)))*((1-PHY)**(PHS(I)/THS(I)))*
     X    ((1-QHY)**(QHS(I)/THS(I)))*((1-RHY)**(RHS(I)/THS(I)))*
     X((1-X)**(D(I)/THS(I)))
    LIMIT OF 19 CONTINUATION CARDS EXCEEDED
      F=ALOG(U)
      FF=ALOG(W)
      F=DGP-DGPQ-F
```

```
      FF=DGQ-DGPQ-FF
      J=FP*FFQ-FQ*FFP
      DTP=(F*FFQ-FF*FQ)/J
      DTQ=(FP*FF-FFP*F)/J
      ADTP=ABS(DTP)
      ADTQ=ABS(DTQ)
      P=P-DTP
   99 Q=Q-DTQ
      IF(ADTP.GT.0.0010) GO TO 34
      IF(ADTQ.GT.0.0010) GO TO 34
      AP=P+1
      CALL G(AP,GAA)
      GAMPP=GAA/AP
      GAMP=GAMPP/P
      AQ=Q+1
       CALL G(AQ,GAA)
      GAMQ=GAA/(AQ*Q)
      BP=2*AP
      CALL G(BP,GAA)
      GAM2P=GAA/(BP*(2*P+1)*2*P)
      BQ=2*AQ
      CALL G(BQ,GAA)
      GAM2Q=GAA/(BQ*(2*Q+1)*2*Q)
      BPQ=2*AP+2*AQ
      CALL G(BPQ,GAA)
      GAM2PQ=GAA/(BPQ*(BPQ-1)*(BPQ-2)*(BPQ-3)*(BPQ-4))
      APQ=AP+AQ
      CALL G(APQ,GAA)
      GAMPQ=GAA/(APQ*(APQ-1)*(APQ-2))
      BIND=2*(GAMPQ**2)*GAM2P*GAM2Q/(GAMP*GAMPP*GAMQ*GAMQ*GAM2PQ)
      S=GAMPQ/(GAMP*GAMQ)
      FA=S*(AHY**(P-1))*((1-AHY)**(Q-1))*1000/UD
      FB=S*(BHY**(P-1))*((1-BHY)**(Q-1))*1000/UD
      FC=S*(CHY**(P-1))*((1-CHY)**(Q-1))*1000/UD
      FD=S*(DHY**(P-1))*((1-DHY)**(Q-1))*1000/UD
      FE=S*(EHY**(P-1))*((1-EHY)**(Q-1))*1000/UD
      FF=S*(FHY**(P-1))*((1-FHY)**(Q-1))*1000/UD
      FG=S*(GHY**(P-1))*((1-GHY)**(Q-1))*1000/UD
      FH=S*(HHY**(P-1))*((1-HHY)**(Q-1))*1000/UD
      FO=S*(OHY**(P-1))*((1-OHY)**(Q-1))*1000/UD
      FP=S*(PHY**(P-1))*((1-PHY)**(Q-1))*1000/UD
      FQ=S*(QHY**(P-1))*((1-QHY)**(Q-1))*5000/UD
      FR=S*(RHY**(P-1))*((1-RHY)**(Q-1))*10000/UD
      FX=S*(X**(P-1))*((1-X)**(Q-1))*2*(1-25000/UD)
      SFA=FA*ALOG(AHY)+FB*ALOG(BHY)+FC*ALOG(CHY)+FD*ALOG(DHY)+
     XFE*ALOG(EHY)+FF*ALOG(FHY)+FG*ALOG(GHY)+FX*ALOG(X)
     X+FH*ALOG(HHY)+FO*ALOG(OHY)+FP*ALOG(PHY)+FQ*ALOG(QHY)+FR*ALOG(RHY)
      GN=EXP(SFA)
      BMEN=P/(P+Q)
      BMOD=(P-1)/(P+Q-2)
      BMED=(3*(P-1)*(P+Q-2)+4*P+2*Q-6)/(3*(P+Q-2)*(P+Q))
      BVAR=(P*Q)/((P+Q)*(P+Q)*(P+Q+1))
      BSKW=(((P+Q+1)**(1/2))/(SQRT(P)*Q*SQRT(Q)))*((P+1)*(P+2)*(P+Q)*
     X(P+Q)/(P+Q+2)-3*P*(P+1)*(P+Q)+2*P*P*(P+Q+1))
      BKUU=(P+1)*(P+2)*(P+3)*((P+Q)**3)*(P+Q+1)/((P+Q+2)*(P+Q+3)*P)-
     X4*(P+1)*(P+2)*(P+Q)*(P+Q)*(P+Q+1)/(P+Q+2)+6*P*(P+1)*(P+Q)*(P+Q+1)-
     X3*P*P*(P+Q+1)*(P+Q+1)
      BKUR=BKUU/(Q*Q)-3
      BCOF=2*BMEN*BIND
      RR=BMEN/GN
```

```
      WRITE(6,21)SHS(I),SSS(I),RRR(I),PPP(I),QQQ(I)
   21 FORMAT('1',5A4,/,' ',15('-'))
      WRITE(6,22)EA,P
   22 FORMAT('0','EA=',2X,F15.2,15X,'P=',3X,F15.2)
      WRITE(6,23)EB,Q
   23 FORMAT('0','EB=',2X,F15.2,15X,'Q=',3X,F15.2)
      WRITE(6,24)AIND,BIND
   24 FORMAT('0','AIND=',F15.2,15X,'BIND=',F15.2)
      WRITE(6,25)GM,GN
   25 FORMAT('0','GM=',2X,F15.2,15X,'GN=',2X,F15.2)
      WRITE(6,26)AMEN,BMEN
   26 FORMAT('0','AMEN=',F15.2,15X,'BMEN=',F15.2)
      WRITE(6,27)AMOD,BMOD
   27 FORMAT('0','AMOD=',F15.2,15X,'BMOD=',F15.2)
      WRITE(6,28)AMED,BMED
   28 FORMAT('0','AMED=',F15.2,15X,'BMED=',F15.2)
      WRITE(6,29)AVAR,BVAR
   29 FORMAT('0','AVAR=',F15.2,15X,'BVAR=',F15.2)
      WRITE(6,30)ASKW,BSKW
   30 FORMAT('0','ASKW=',F15.2,15X,'BSKW=',F15.2)
      WRITE(6,31)AKUR,BKUR
   31 FORMAT('0','AKUR=',F15.2,15X,'BKUR=',F15.2)
      WRITE(6,32)ACOF,BCOF
   32 FORMAT('0','ACOF=',F15.2,15X,'BCOF =',F15.2)
      WRITE(6,33)R,RR
   33 FORMAT('0','R=',3X,F15.2,15X,'RR=',2X,F15.2)
      WRITE(6,35) Y,M
   35 FORMAT('0','Y=',3X,F15.2/'0','M= ',F20.6)
      WRITE(7,240)EA,EB,AIND,ASKW,AKUR,R,SHS(I),SSS(I),RRR(I),PPP(I),
     XQQQ(I),P,Q,BIND,BSKW,BKUR,RR,Y,SHS(I),SSS(I),RRR(I),PPP(I),QQQ(I)
  240 FORMAT(F4.2,1X,F7.5,1X,F4.2,1X,3F5.2,27X,5A4/6F5.2,2X,F15.2,13X,5A
     14)
   10 CONTINUE
    3 STOP
      END
```

1EXEC

Appendix C. 1960 Factor Analysis for SMSA's.

1960 SMSA's-Over 250,000

Factor 1 (21)	Factor 2 (20)	Factor 3 (11)
Atlanta, Ga.	Akron, Ohio	Charleston, W.V.
Beaumont, Tex.	Canton, Ohio	Denver, Colo.
Birmingham, Ala.	Chicago, Ill.	El Paso, Tex.
Buffalo, N.Y.	Davenport, Iowa	Flint, Mich.
Charlotte, N.C.	Dayton, Ohio	Huntington, W.V.
Chattanooga, Tenn.	Ft. Lauderdale, Fla.	Johnstown, Pa.
Cleveland, Ohio	Grand Rapids, Mich.	Sacramento, Calif.
Columbia, S.C.	Greensboro, N.C.	San Jose, Calif.
Erie, Pa.	Lansing, Mich.	Utica, N.Y.
Jacksonville, Fla.	Los Angeles, Calif.	Washington, D.C.
Findervill, Tenn.	Miami, Fla.	Wilkes-Barre, Pa.
Memphis, Tenn.	New York, N.Y.	
Milwaukee, Wisc.	Peoria, Ill.	
Mobile, Ala.	New York, N.Y.	
Nashville, Tenn.	Salt Lake City, Ut.	
Newark, N.J.	San Francisco, Calif.	
Norfolk, Va.	Syracuse, N.Y.	
Patterson, N.J.	Toledo, Ohio	
Rochester, N.Y.	Wilminton, Del.	
San Antonio, Tex.	Youngstown, Ohio	
Shreveport, La.		

Continued. SMSA's Over 250,000

Factor 5 (6)	Factor 6 (5)	Factor 7 (8)
Bridgeport, Conn.	Baltimore, Md.	Ft. Worth, Tex.
Hartford, Conn.	Portland, Ore.	Miami, Fla.
New Haven, Conn.	Seattle, Wash.	Orlando, Fla.
Providence, R.I.	Spokane, Wash.	Phoenix, Ariz.
Springfield, Mass.	Tacoma, Wash.	San Bernandino, Calif.
Worcester, Mass.		San Diego, Calif.
		Tampa, Fla.
		Tuscon, Ariz.

Factor 8 (6)	Factor 9 (7)	Factor 10 (4)
Albuquerque, N.M.	Dallas, Tex.	
Allentown, Pa.	Fresno, Calif.	
Erie, Pa.	Houston, Tex.	Jersey City, N.J.
Harrisburg, Pa.	Kansas City, Mo.	Newark, N.J.
Lancaster, Pa.	Oklahoma City, Okla.	Paterson, N.J.
Reading, Pa.	Tulsa, Okla.	Trenton, N.J.
	Wichita, Kan.	

Factor 11 (4)	Factor 12 (2)	
Bakersfield, Calif.	Cincinnati, Ohio	
Columbus, Ohio	Louisville, Ky.	
Gary, Ind.	**Cities not Grouped (7)**	
Indianapolis, Ind.	Albany, N.Y.	Detroit, Mich.
No Cities	Boston, Mass.	Honolulu, Haw.
Factor 4, 13, 14	Des Moines, Iowa	New Orleans, La.
	Omaha, Neb.	

Continued. 1960 SMSA's under 250,000

Factor 1 (22)

Austin, Tex.
Bay City, Mich.
Charleston, S.C.
Decatur, Ill.
Porham, N.C.
Hamilton, Ohio
Jackson, Mich.
Jackson, Mass.
Kenosha, Wisc.
Lima, Ohio
Little Rock, Ark.
Lorain, Ohio
Montgomery, Ala.
Muncie, Ind.
Muskegum, Mich.

Factor 2 (17)

Albany, Ga.
Atlantic City, N.J.
Billings, Mont.
Evansville, Ind.
Fargo, N.D.
Great Falls, Mont.
Huntsville, Ala.
Lawton, Okla.
Midland, Tex.
Norwalk, Conn.
Odessa, Tex.
Ogden, Ut.
Scranton, Pa.
Sious Falls, S.D.
Wheeling, W.V.
York, Pa.

Factor 4 (12)

Brokton, Mass.
Falls River, Mass.
Fitchburg, Mass.
Lawrence, Mass.
Lewiston, Maine
Lowell, Mass.
Manchester, N.H.
New Bedford, Mass.

Factor 3 (14)

Altoona, Pa.
Cedar Rapids, Iowa
Ft. Smith, Ark.
Ft. Wayne, Ind.
Gadsden, Ala.
Lynchburg, Va.
Madison, Wisc.
Monroe, La.
Rockford, Ill.
San Angelo, Tex.
South Bend, Ind.
Texarkana, Tex.
Tuscaloosa, Ala.
Tyler, Tex.

Factor 5 (4)

St. Joseph, Mo.
Sioux City, Iowa
Springfield, Mo.
Terre Haute, Ind.

New Britany, Conn.
Pittsfield, Mass.
Portland, Maine
Waterbury, Conn.

Continued. Under 250,000

Factor 6 (4)

Durham, N.C.
Greensboro, N.C.
Greenville, S.C.
Winston-Salem, N.C.

Factor 7 (2)

Lincoln, Neb.
Topeka, Kan.

Factor 8 (2)

Macon, Ga.
Newport-News, Va.

Factor 10 (2)

Brownsville, Tex.
Stanford, Conn.

Factor 11 (2)

Ann Arbor, Mich.
Champaign, Ill.

Factor 9 (3)

Colorado Springs, Colo.
Las Vegas, Nev.
Reno, Nev.

Factor 13 (1)

Galveston, Tex.

Factor 14 (1)

Amarillo, Tex.

Factor 12 (2)

Santa Barbara, Calif.
West Palm Beach, Fla.

Factor 15 (3)

Abilene, Tex.
Amarillo, Tex.
Wichita Falls, Tex.

Cities not Grouped

Ashville, N.C.
Augusta, Ga.
Baton Rouge, La.
Columbus, Ga.
Corpus Christi, Tex.
Eugene, Ore.
Green Bay, Wisc.
Kalamazoo, Mich.
Lake Charles, La.
Waco, Tex.
Laredo, Tex.
Lexington, Ky.
Lubbuck, Tex.
Mariden,Conn.
New London, Conn.
Pensecola, Fla.
Roanoke, Va.
Saginow, Mich.
Springfield, Ill.
Stockton, Calif.

No Cities Factors 16 & 12

Appendix D. SMSA's 1970 Over 500,000

Rotated Factor Scores

Factor 1 (16)	Factor 2 (7)	Factor 3 (7)	Factor 4 (5)
4 Anaheim	3 Allentown, N.J.	8 Boston	11 Cinncinati
7 Birmingham	5 Atlanta, Ga.	14 Dallas	24 Honolulu
12 Cleveland	9 Buffalo	19 Ft. Worth	36 Indianapolis
22 Greensboro	46 Pittsburgh	25 Houston	31 Louisville
23 Hartford	48 Providence	29 Kansas City	52 St. Louis
27 Jacksonville	60 Springfield	41 Oklahoma City	
32 Memphis	68 Washington D.C.	45 Phoenix	
35 Minneapolis	Factor 6 (2)	Factor 7 (2)	Factor 8 (4)
36 Nashville	47 Portland, Ore.	6 Baltimore	13 Columbus
37 New Orleans	59 Seattle	49 Richmond	16 Denver
38 Newark	Factor 9 (5)	Factor 10 (3)	42 Omaha
40 Norfolk	10 Chicago	51 Sacramento	53 Salt Lake
43 Paterson	17 Detroit	55 San Bernadino	
54 San Francisco	30 Los Angeles	56 San Diego	
57 San Francisco	38 New York		
58 San Jose	44 Philadelphia		
Factor 5 (12)		Factor 11 (1)	Factor 12 (0)
1 Arbor	33 Miami	34 Milwaukee	
15 Dayton	50 Rochester		
18 Ft. Lauderdale	61 Syracuse		
20 Gary	62 Tampa		
21 Grand Rapids	63 Toledo		
28 Jersey City	65 Youngstown		

Continued. SMSA's 1970 Between 250,000 & 500,000 (60)

Factor 1 (20)

3 Augusta, Ga.
6 Baton Rouge, La.
9 Bridgeport, Conn.
10 Canton, Ohio
11 Charleston, S.C.
14 Columbia, S.C.
15 Corpus Christi, Tex.
16 Davenport, Iowa
19 El Paso, Tex.
27 Jackson, Miss.
29 Knoxville, Tenn.
33 Little Rock, Ark.
36 Mobile, Ala.
37 New Haven, Conn.
41 Peoria, Ill.
43 Rockford, Ill.
46 Shreveport, La.
47 South Bend, Ind.
51 Trenton, N.J.
59 Worchester, Mass.

Factor 4 (3)

18 Duluth, Minn.
49 Stockton, Calif.
54 Utica, N.Y.

Factor 2 (16)

1 Albuquerque, N.M.
4 Austin, Tex.
13 Chattanooga, Tenn.
24 Greenville, S.C.
26 Huntington, W.V.
28 Johnstown, Pa.
30 Lancaster, Pa.
32 Las Vegas, Nev.
35 Madison, Wisc.
40 Oxnard, Calif.
42 Reading, Pa.
44 Salinas, Calif.
45 Santa Barbara, Calif.
52 Tuscon, Ariz.
57 Wilkes-Barre, Pa.
60 York, Pa.

Factor 5 (4)

5 Bakersfield, Calif.
7 Beaumont, Tex.
23 Fresno, Calif.
56 Wichita, Kan.

Not Factored (7)

2 Appleton, Wisc.
8 Binghampton, Pa.
20 Erie, Pa.
22 Fort Wayne, Ind.
31 Lansing, Mich.
38 New Port News, Va.
58 Wilmington, Del.

Factor 3 (6)

12 Charlotte, N.C.
25 Harrisburg, Pa.
34 Lorain, Ohio
39 Orlando, Fla.
53 Tulsa, Okla.
55 W. Palm Beach, Fla.

Factor 6 (2)

13 Des Moines, Iowa
21 Flint, Mich.

Factor 7 (2)

43 Spokane, Wash.
50 Tacoma, Wash.

Factor 8 (0)

Factor 9 (0)

Factors 10-13 (0)

Continued. SMSA's 1970 Between 150,000 & 250,000 (50)

Factor 1 (8)

3 Beoran, Mass.
13 Ft. Smith, Ark.
19 Lawrence, Mass.
23 Lowell, Mass.
37 Salem, Ore.
42 Springfield, Mo.
46 Terre Haute, Ind.
49 Waterbury, Conn.

Factor 2 (6)

2 Atlantic City, N.J.
17 Huntsville, Ala.
30 New Bedford, Mass.
34 Raleigh, N.C.
40 Scranton, Pa.
50 Wheeling, W.V.

Factor 3 (7)

20 Lexington, Ky.
21 Lima, Ohio
29 Muskegon, Mich.
33 Racine, Wisc.
36 Saginaw, Mich.
43 Springfield, Ohio
45 Steubenable, Ohio

Factor 4 (4)

22 Lincoln, Neb.
26 McAllen, Tex.
41 Springfield, Ill.
42 Topeka, Kan.

Factor 5 (3)

27 Medesto, Calif.
38 Santa Rosa, Calif.
48 Vallejo, Calif.

Factor 6 (7)

4 Cedar Rapids, Iowa
8 Columbus, Ga.
10 Eugene, Ore.
18 Kalamazoo, Mich.
25 Macon, Ga.
28 Montgomery, Ala.
39 Savannah, Ga.

Factor 7 (1)

11 Evansville, Ind.

Factor 8 (3)

7 Colorado Springs, Co.
12 Fayetteville, N.C.
44 Stanford, Conn.

Factor 9 (1)

35 Roanoke, Va.

Factor 10 (3)

1 Ann Arbor, Mich.
5 Champaign, Ill.
9 Durham, N.C.

Factor 11 (1)

14 Galveston, Tex.

Factor 12 (1)

15 Green Bay, Wisc.

Not Factored

6 Charleston, W.V.
16 Hamilton, Ohio
29 Lubbuck, Tex.
31 New London, Conn.
32 Pensacola, Fla.

Continued. SMSA's 1970 Under 150,000

Factor 1 (12)

5 Anderson, Ind.
7 Bay City, Mich.
14 Bryan, Tex.
17 Decatur, Ill.
25 Jackson, Mich.
26 Kenosha, Wisc.
31 Laredo, Tex.
36 Mansfield, Ohio
40 Muncie, Inc.
42 New Britian, Conn.
56 San Angelo, Tex.
64 Vineland, N.J.

Factor 4 (6)

1 Abilene, Tex.
2 Albany, Ga.
3 Altoona, Pa.
12 Bristof, Conn.
16 Danbury, Conn.
41 Nashua, N.H.

Factor 10 (3)

66 Waterloo, Iowa
67 Wichita Falls, Tex.
68 Wilmington, N.C.

Factor 2 (6)

10 Bloomington, Ind.
13 Brownsville, Tex.
15 Columbia, Mo.
23 Gainsville, Fla.
29 Layfayette, Ind.
60 Tallahasse, Fla.

Factor 5 (7)

21 Fitchburg, Mass.
28 Layfayette, La.
30 Lake Charles, La.
37 Meriden, Conn.
39 Monroe, La.
49 Pittsfield, Mass.
52 Tuscaloosa, Ala.

Factor 8 (5)

22 Gadsen, Ala.
46 Owensburo, Ky.
48 Pine Bluff, Ark.
61 Texakana, Tex.
63 Tyler, Tex.

Factor 11 (1)

4 Amarillo, Tex.

Factor 3 (3)

18 Dubque, Iowa
27 La Crosse, Wisc.
59 Sioux, Falls, S.D.

Factor 6 (6)

19 Fall Rider, R.I.
33 Lewiston, Maine
35 Manchester, N.H.
38 Midland, Tex.
44 Odessa, Tex.
50 Portland, Maine

Factor 7 (3)

55 St. Joseph, Mo.
58 Sioux City, Iowa
65 Waco, Tex.

Factor 9 (3)

Norwalk, Conn.
52 Pueblo, Colo.
53 Reno, Nev.

Factor 12 (5)

8 Billings, Mont.
11 Boise, Id.
20 Fargo, N.D.
24 Great Falls, Mont.
54 Rochester, N.Y.

Factor 13 (2)	Factor 14 (2)	Not Factored
34 Lynchburg, Va.	45 Ogden, Utah	6 Ashville, N.C.
47 Petersburg, Va.	57 Sherman, Tex.	9 Biloxi, Miss.
		32 Lawton, Okla.
		51 Provo, Utah

Appendix E. Distribution Parameters and the Lorenz Curve

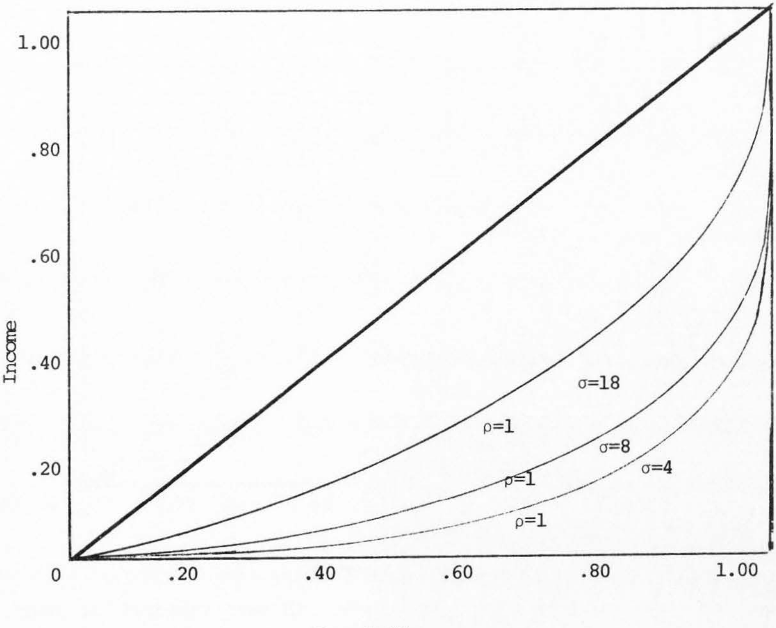

Chart I Extreme Values for σ when $\rho = 1$

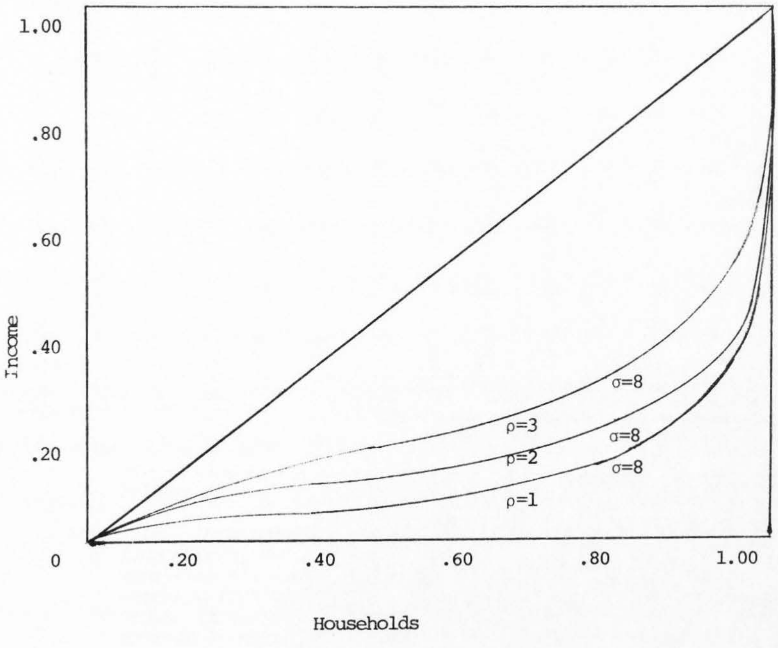

Chart II Extreme Values for ρ when σ = 8.

VITA

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