A Suggested Mathematics Curriculum for Preparation of Teachers of Modern Secondary School Mathematics in Utah

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A SUGGESTED MATHEMATICS CURRICULUM FOR PREPARATION OF TEACHERS OF MODERN SECONDARY SCHOOL MATHEMATICS IN UTAH

by

Harold Nolan Phillips

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H. Nolan Phillips

Harold Nolan Phillips

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ABBREVIATIONS OF ORGANIZATIONS

AAAS	American Association for the Advancement of Science
AACTE	American Association of Colleges for Teacher Education
BSCS	Biological Science Curriculum Study
CBA	Chemical Bond Approach Project
CCTSM	Cooperative Committee on the Teaching of Science and Mathematics
Chem Study	Chemical Education Materials Study
CUPM	Committee on the Undergraduate Program in Mathematics
MAA	Mathematical Association of America
NCTM	National Council of Teachers of Mathematics
NSF	National Science Foundation
PSSC	Physical Science Study Committee
SMSG	School Mathematics Study Group
UEA	Utah Educational Association
UICSM	University of Illinois Committee on School Mathematics
UMMaP	University of Maryland Mathematics Project

ABSTRACT

A Suggested Mathematics Curriculum for Preparation of

Teachers of Modern Secondary School Mathematics

in Utah

by

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Utah State University, 1967

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Department: Secondary Education

"New math" has drastically changed secondary mathematics and the demands on the secondary mathematics teacher. The changes and effects of changes were studied with emphasis on suggested programs in teacher preparation.

Questionnaires were given to one hundred four secondary mathematics teachers in Utah. Fifty-eight were returned, of which fifty were usable. The questionnaire contained twenty-six mathematics courses offered to mathematics education majors in Utah universities. The teachers indicated which courses were valuable to them in teaching secondary school mathematics. Rank order correlation coefficients were calculated among subgroups of the questionnaire to determine internal consistency. All coefficients were above the 1 per cent significance level. The first fifteen courses listed in rank order according to

the percentage of teachers who felt each course was valuable are: college algebra, trigonometry, analytic geometry, differential calculus, modern algebra, methods for secondary mathematics teachers, mathematics for secondary school teachers, foundations of mathematics, integral calculus, number theory, history of mathematics, foundations of geometry, solid geometry, logic, and foundations of algebra.

On the basis of the courses generally recommended for prospective modern mathematics teachers by nationally interested groups and the results of the evaluations of courses by Utah mathematics teachers, the following program in mathematics was proposed for prospective mathematics teachers in Utah.

Mathematics education majors should take:

College Algebra (or equivalent)

Trigonomentry (or equivalent)

Analytic Geometry

Differential Calculus

Abstract Algebra (at least one course)

College Geometry (at least one course other than Analytic Geometry)

Mathematics for Secondary School Teachers

Methods course (may be taken under the Department of Education)

After completing this basic program, teachers intending to teach grades seven, eight, or nine should choose three or more courses from the following:

Foundations of Mathematics

Additional courses in Abstract Algebra

Additional courses in College Geometry

(other than Analytic Geometry)

Number Theory

Logic

History of Mathematics

Probability and Statistics

A teacher intending to teach grades ten, eleven, or twelve should complete integral calculus and choose three or more courses from the following:

Foundations of Mathematics

Additional courses in Abstract Algebra

Additional courses in College Geometry

(other than Analytic Geometry)

Number Theory

Logic

History of Mathematics

Probability and Statistics

Additional Calculus courses

(85 pages)

INTRODUCTION AND PROBLEM STATEMENT

The past decade (1957-1967) has ushered in what many educators feel is the largest scale educational revolution in the history of United Stated education. Beginning with the adoption of "new math" in public schools, many new programs were initiated in other fields. Some programs, such as the Chemical Education Materials Study (Chem Study), Chemical Bond Approach Project (CBA), Physical Science Study Committee (PSSC), and the Biological Science Curriculum Study (BSCS), have changed primarily the approach to the subject matter, while others, such as the new math and new English programs, have changed or added much new terminology as well as changing the approach and structure. This study will be concerned with the new mathematics program and the secondary school mathematics teacher in the state of Utah.

The changes in the new mathematics curriculum are so extensive in both new vocabulary and content structure that they place new and very different demands on classroom teachers. As the initial presentation of material to the students in the classroom is of great importance to their motivation, understanding, and concept formation, it is of great importance that the teacher be well prepared to function properly in his role. The success of the new mathematics is highly dependent upon the skill and ability of the teacher to present the program to the students in the spirit in which the program was created and structured (Adler, 1966). For this reason, most of the study groups responsible for the new programs in mathematics have suggested a revised university curriculum which they hope will allow the teacher to attain a preparation specifically adapted to the functioning of their program in the secondary school classroom.

The proposals for revision of teacher education in mathematics have received varied degrees of acceptance from the universities. They were made by the study groups in anticipation of teacher needs. They are evaluated by university staff members to determine a curriculum which, by the staff members' judgement, will give the teachers a satisfactory background to teach "modern math."

In both cases (study groups' and staff members' suggestions), the proposed curriculum is based on theoretical anticipation of the needs of a secondary mathematics teacher. Regardless of how competent the makers of these proposed programs may be, the programs remain unsubstantiated theoretical projections until tested in some way to evaluate their effect.

Thus the problem is one of uncertainty: Everyone concerned wants the best possible preparation for the teacher in the classroom, but none is certain that present programs are giving the teacher the preparation he needs and wants.

One way to determine whether the present university mathematics programs are meeting teaching needs is to determine if teachers, after teaching modern mathematics and experiencing the needs imposed upon them in the classroom situation, would choose the same curriculum, or if they would choose another to give them the effective preparation they need.

OBJECTIVES

This study will pursue three objectives in bringing together the necessary information to suggest an ideal undergraduate mathematics program for the secondary mathematics education major in Utah.

The first objective is to gather and compare curricula for teachers of mathematics which have been suggested by the study groups preparing the "new math" materials for use in public schools and by other recognized authorities.

The second objective is to compare the evaluations given to university mathematics courses by teachers now actively engaged in teaching mathematics in Utah's secondary schools to the evaluations given the same courses by the study groups and authorities.

The third objective is to determine a university mathematics program for secondary mathematics teachers in Utah based upon their evaluation of university courses coupled with suggestions of study groups. This proposed program should give the secondary mathematics teachers of Utah the preparation they need and want.

Definition

For the purposes of this study, the word "course" is used to mean a unit of instruction rather than a series of studies leading to a degree.

REVIEW OF LITERATURE

Striving for Excellence and Change

In man's continual search for excellence and improvement, mathematics has been one of the most useful tools. Great societies of all times have respected and promoted mathematics within their cultures. Perhaps at no other time in history has mathematics found so much use as today in our space age of computers, astronauts, and systematization. In addition to the changes in mathematics resulting from a natural striving for excellence, there are many changes, additions, and adaptations caused by the needs for and uses of mathematics. These continual changes come about in various ways. Some result from the extension of applications of mathematics. Some result from refinement, some for philosophical reasons, and some from new ideas which may be completely revolutionary in nature. Much change simply results from growth.

Due, probably, to a combination of these causes, mathematics has undergone many changes in recent years. Joseph Landin (1963) indicates that the number of new developments reported in mathematics each year has increased sixfold from 1940 to 1961. Flanagan (1965) claims there has been greater change in mathematics and mathematics education in the past twenty-five years than in the previous two hundred years.

The tremendous growth and change and many new developments have caused much concern about mathematics and mathematical systems. This concern has in turn raised much interest in mathematics education and in the teacher of mathematics.

Educators, too, are striving for excellence and improvement in their profession, which means that even if the subject taught remained inert, there would be continual change in the teaching of the subject. This change, combined with the change of a growing and progressing subject such as mathematics, requires constant adaptation. The changes in United States mathematics teacher education up to 1958 can be interpreted to a degree from Tables 1 and 2 taken from data by John A. Schumaker (1961).

> A tendency throughout the period was for new courses to be added to offerings without other courses being dropped. There was agreement only on the freshman course and calculus; no other course was specifically required by more than one-third of the institutions in any of the selected academic years. (Schumaker, 1961, p. 417)

Read (1966) indicates that the greatest change in mathematics education has occurred in the past ten years in what he calls "the great reform movement." The validity of this statement should become evident as the changes in mathematics education in the past ten years are discussed on the following pages.

Reasons for Change

Several factors have contributed to the changes in mathematics education in recent years. The technological explosion following World War II created a definite awareness of the inconsistency between school mathematics programs and the needs of our changing scientific society. This awareness was accentuated in the United States when the U. S. S. R. became the first nation to

Table 1. Per cent[a] of institutions requiring mathematics courses of prospective teachers majoring in mathematics in selected years

Courses required	1920-21	1928-29	1936-37	1943-44	1950-51	1957-58
College Algebra	78	82	84	88	85	82
Solid Geometry	25	18	21	19	18	17
Trigonometry	69	86	73	84	88	82
Analytical Geometry	91	84	80	89	86	85
Differential Calculus	88	94	90	97	98	100
Integral Calculus	81	92	88	91	97	98
History of Mathematics	6	16	12	9	13	12
Statistics	0	2	11	11	9	10
Methods for Secondary Math Teachers	0	0	0	5	2	5
Foundation of Algebra	3	2	0	3	4	11
Foundation of Geometry	3	0	0	4	2	6
Math for Secondary School Teachers	9	4	9	8	4	7
Theory of Equations	19	22	15	17	22	25
Projective Geometry	6	2	1	0	2	1
Advanced Calculus	16	18	10	9	12	9
Differential Equations	22	20	16	17	22	11

[a] Percents are based on thirty-two institutions for 1920-21, forty-nine for 1928-29, eighty-one for 1936-37, 103 for 1943-44, 121 for 1950-51, and 133 for 1957-58.

Table 2. Per cent[a] of teacher-education institutions offering mathematics courses in selected years

Courses offered	1920-21	1928-29	1937-37	1943-44	1950-51	1957-58
College Algebra	93	83	87	90	92	87
Solid Geometry	68	58	52	62	59	47
Trigonometry	98	88	92	96	94	89
Analytical Geometry	94	91	89	93	94	87
Differential Calculus	98	98	99	99	100	100
Integral Calculus	92	92	98	99	100	100
Number Theory	11	10	11	17	31	34
History of Mathematics	44	45	48	50	50	52
Matrices	4	3	2	4	5	18
Probability	11	8	13	14	17	26
Statistics	24	41	53	62	65	72
Methods for Secondary Math Teachers	0	4	10	35	34	57
Modern Algebra	7	7	6	8	16	39
Projective Geometry	29	31	40	37	37	35
Math for Secondary School Teachers (Teacher's Course)	22	17	16	16	11	18
Foundation of Algebra	6	6	11	11	19	29
Foundation of Geometry	8	6	8	8	13	28
Foundation of Math (Set Theory)	0	1	3	5	9	6
Numerical Analysis	0	1	1	4	9	24
Theory of Equations	47	57	68	70	82	79
Topology	0	0	1	2	4	11
Advanced Calculus	40	42	49	53	64	68
Differential Equations	64	64	74	78	84	90
Functions of Real and Complex Variables	13	15	13	10	19	24

[a] Percents are based on eighty-five institutions for 1920-21, 114 for 1928-29, 126 for 1936-37, 133 for 1943-44, 140 for 1950-51, and 140 for 1957-58.

orbit a satellite—a feat which is becoming symbolic of modern technological achievement. Immediately following the first satellite of 1957, study groups were organized and financed to accelerate the proposed curriculum changes previously anticipated by some educators in the United States. Landin (1963) attempts to summarize the shortcomings of traditional mathematics into four classes. The shortcomings are the following:

> 1. It failed to provide adequate computational facilities. Many entering /university/ freshman could not add, subtract, multiply or divide.
> 2. It failed to give the student an adequate conceptual background. They had little or no idea of proof and mathematical structure.
> 3. It failed to serve science and technology. Students couldn't apply it. They had no concept of how to apply mathematics to other fields.
> 4. It failed to make use of knowledge of learning processes which make the mathematics classroom a valuable experience. (Landin, 1963, p. 369)

Some authors feel that one of the more important reasons for these failures is that mathematics has a structure, and needs to be taught as such. Mathematics should be understood. In the past it has been taught as a set of techniques—as rules to be memorized. The mechanics of mathematics were taught, but mathematics itself was not (Haag, 1964; Gager, 1962; Mayor et al., 1960). Mayor, who worked on the University of Maryland Mathematics Project (UMMaP), which began in 1957, stated the two main objectives of UMMaP to be: (1) to clarify language used in mathematics to make it meaningful and precise and (2) to teach the structure of mathematics.

According to Landin (1963), both the University of Illinois Committee on School Mathematics (UICSM) and the School Mathematics Study Group (SMSG) have programs in mathematics that are based on a structural approach, in the hope that it will increase understanding and aid in concept formation.

In any case, the awareness of the failings in mathematics education and the problems created by them were pronounced enough that many of the United States' mathematicians and educators became involved with the idea of improving the situation. They joined forces to help create what Read (1966) calls "the great reform movement" in education.

Among their efforts is one which has caused much discussion, The Cambridge Conference Report on School Mathematics.

Cambridge Conference Report on School Mathematics

In the spring of 1962, a few mathematicians from Cambridge and some representatives of the National Science Foundation met by invitation of Professors J. Zacharias and W. J. Martin. Their purpose was to discuss the state of mathematics instruction in public schools. In this informal meeting, a decision was made to organize a conference to discuss the goals of school mathematics.

A steering committee consisting of Edward Begle, Jerome Bruner, Andrew Gleason, Mark Kac, William Ted Martin, Edwin Moise, Mina Rees, Patrick Suppes, Stephen White, and S. S. Wilks, was enlisted. The conference began June 18, 1963, in Cambridge, Massachusetts, under the sponsorship of the National Science Foundation. The following list of prominent men participated in the conference:

Maurice Auslander, Professor of Mathematics, Brandeis University
Edward Begle, Professor of Mathematics, Stanford University
R. Creighton Buck, Professor of Mathematics, University of Wisconsin
George F. Carrier, Professor of Applied Mathematics, Harvard University
Julian Cole, Professor of Applied Mathematics, California Institute of Technology
Robert B. Davis, Professor of Mathematics, Syracuse University
Robert P. Dilworth, Professor of Mathematics, California Institute of Technology
Bernard Freidman, Professor of Mathematics, University of California
H. L. Frisch, Bell Telephone Laboratories
Andrew Gleason, Professor of Mathematics, Harvard Universtiy
Peter J. Hilton, Professor of Mathematics, Cornell University
J. L. Hodges, Professor of Statistics, University of California
S. Koenig, IBM Watson Laboratories
C. C. Lin, Professor of Mathematics, Massachusetts Institute of Technology
Earle L. Lomon, Professor of Physics, Massachusetts Institute of Technology
William Ted Martin, Professor of Mathematics, Massachusetts Institute of Technology
Edwin E. Moise, Professor of Mathematics Education, Harvard University
Fredrick Mosteller, Professor of Statistics, Harvard University
Henry O. Pollak, Director of Mathematics Research, Bell Telephone Laboratories
Mina Rees, Dean, City University of New York
Max M. Schiffer, Professor of Mathematics, Stanford University
George Springer, Professor of Mathematics, University of Kansas
Patrick Suppes, Professor of Mathematics, Stanford University
A. H. Taub, Professor of Mathematics, University of Illinois
S. S. Wilks, Professor of Statistics, Princeton University
Jerrold R. Zacharias, Professor of Physics, Massachusetts Institute of Technology (Davis, 1963, p. 6)

These men represented several areas of pure and applied mathematics, statistics, physics, and chemistry. Their proposals were bold and ambitious, but would be of great value if they can be achieved. The boldness and value are perhaps best expressed by the writers of the report in the following excerpt:

The subject matter which we are proposing can be roughly described by saying that a student who has worked through the full thirteen years of mathematics in grades K to 12 should have a level of training comparable to three years of top-level college training today; that is, we shall expect him to have the equivalent of two years of calculus, and one semester each of modern algebra and probability theory. At first glance this seems to be totally unrealistic; yet we must remember that, since the beginning of this century, there has been about a three-year speed-up in the teaching of mathematics. Of course, one cannot argue that such steps can be taken indefinitely, but it is comforting to realize that the proposed changes are no more radical on their face than changes which have actually taken place within the memory of many.

Since the amount of time to be spent of mathematics will certainly not increase in the face of the additional effort now being focussed on the sciences in elementary schools, and the mean level of native ability of students probably does not change appreciably in periods shorter than geological, it is clear that the inclusion of more content at the top must be compensated by the omission of something else. There are a few topics whose omission has been frequently signalled over the recent past, the most obvious being the numerical solution of triangles. Dropping these will not release three years, however. We propose to gain three years through a new organization of the subject matter and the virtually total abandonment of drill for drill's sake, replacing the unmotivated drill of classical arithmetic by problems which illustrate new mathematical concepts.

Lest there by any misunderstanding concerning our viewpoint, let it be stated that <u>technical proficiency in arithmetic calculation and algebraic manipulation is essential to the study of mathematics</u>. However the means of imparting such skill need not rest on methodical drill. We believe that entirely adequate technical practice can be woven into the acquisition of new concepts. But our belief goes farther. It is not merely that adequate practice <u>can</u> be given along with more mathematics; we believe that this is the only truly effective way to impart technical skills. Pages of drill sums and repetitious 'real-life' problems have less than no merit; they impede the learning process. We believe that arithmetic as it has been taught in grade schools until quite recently has such a meagre intellectual content that the oft-noted reaction against the subject is not an unfortunate rebellion against a difficult subject, but a perfectly proper response to a preoccupation with triviality.

We are not saying that some drill problems may not be appropiate for the individual student whose technical skill is behind, but we do believe that this should be the exception not the rule. We are definitely opposed to the view that the main objective is arithmetic proficiency and that new interesting concepts are being introduced primarily to sugar-coat the bitter pill of computational practice.

A mere recital of the topics proposed for the future curriculum does scant justice to our goals. Familiarity is our real objective. We hope to make each student in the early grades truly familiar with the structure of the real number system and the basic ideas of geometry both synthetic and analytic. On this firm foundation we believe a very solid mathematical superstructure can be erected which will make the pupils familiar with the ideas of calculus, algebra and probability. The primary school program should be understandable by virtually all students; it should lead to a level of competence well above that of the general population today. As pupils advance through junior and senior high school we must expect that fewer and fewer will elect mathematics; consequently we have attempted to build first in the directions most suitable for those who take mathematics only a few years after grade school. Of particular importance is an elementary feeling for probability and statistics. Although there was considerable difference of opinion on this point (see Section 6) many felt that a nodding acquaintance with the Calculus had the next priority. The strongest argument for its early inclusion was one of general education: liberal education requires the contemplation of the works of genius, and the Calculus is one of the grandest edifices constructed by mankind.

The conference felt that mathematics is a subject of great humanistic value: its importance to the educated man is almost as great as its importance to many teachnical specialists. (Davis, 1963, p. 8-10)

It is hoped that the students would also understand what mathematics is and what it is not; its uses and its limitations. Mathematics is a growing subject, and should be understood as such.

Many inefficient thought patterns of everyday life may be modified by the study of mathematics. The Report claims that modestly endowed students are able to recreate large parts of mathematics from a few basic ideas. Concepts such as set, function, transformation group, and isomorphism can be introduced in elementary form to young children.

Use of a spiral method in teaching mathematics in grades K through 12 is suggested, helping the students to build upon concepts (such as those just mentioned) and giving the student an increasingly sophisticated comprehension of the concepts.

> Similarily we view the problems of language, notation, and symbolism. It is unquestionably possible to obscure a subject by introducing too much special terminology and symbolism; but we feel that most errors of this sort in fact cover an inadequate understanding of the subject matter. The function of language is to communicate. In mathematics its function is to communicate with extraordinary precision; it is inevitable therefore that mathematics requires some special terminology. Special terms are good or bad exactly according to their effectiveness in communication, and the same applies to special notations and symbols.
>
> . . . Mathematics is, to a large extent, a process of organizing data. Through symbolization and the precise formulation of new concepts, large blocks of information are brought within the grasp of the mind. (Davis, 1963, p. 13)

The conference members suggest much more devotion to inequalities. They feel that the almost complete devotion to problems of equality have led students to the misconception that mathematics deals only with exact answers and exact laws. The ability of mathematics to deal effectively with both qualitative and uncertain relationships should be brought out and emphasized.

The conference members also feel that students should understand the following:

> . . . Mathematics per se does nothing directly for even the classical, exact disciplines of physics and astronomy. Only after a model of the real world has been formulated does mathematics enter the picture. Every application of mathematics depends on a model, and the value of a deduction is more an attribute of the model than it is of mathematics. We believe that students can be made aware of the distinction between the real world and its various mathematical models. . . . (Davis, 1963, p. 15)

A further recommendation made by the report was that each topic be approached intuitively, and through as many different intuitive approaches as possible. Rigor is important in mathematics, but should not be overdone. Every effort should be made to foster independent and creative thinking. (Davis, 1963)

The previous discussion of the Cambridge Conference Report on Mathematics gives an idea of some of the general goals and attitudes proposed by a distinguished group of men who are greatly concerned about mathematics education. They made specific suggestions and outlined a program K-12 to accomplish the goals they set forth. The purpose of the above discussion is to present the spirit and attitude of the Cambridge Conference Report, since it is one of the major influences on recent mathematics education developments. In all probability, its influence will enjoy great tenure.

Comments on the Cambridge Conference Report

The proposals of the Cambridge Conference have received both negative and positive criticism, most of which remains verbal opinions. A few people are turning to research to prove either the possibility or impossibility of accomplishing the goals set forth. One such study began in the fall of 1965 at Nova High School, Fort Lauderdale, Florida. A series of conferences were held "to explore the feasibility of implementing a long-range curriculum development project for a non-graded, K-12 school based on the recommendations of the Cambridge Conference on School Mathematics." (Foster, 1966, p. ii)

These conferences resulted in a report which goes beyond the Cambridge Conference Report in suggesting the specific requirements of an educational system which are needed to implement an optimal curriculum in mathematics for all students. The suggestions include such things as teacher training, materials production, information processing, research, and evaluation, as well as a system for integrating these component parts into the daily operation of a school (Foster, 1966).

Nova High School is now carrying out this project with financial help from the Cooperative Research Program of the Office of Education, U. S. Department of Health, Education, and Welfare. The early phases of the project have been successful, and an eighty-page report (Cooperative Research Project #S405) has been published by the Cooperative Research Program for others interested in similar projects (Foster, 1966).

Adler (1966) is quite confident that the goals of the Cambridge Report are attainable before 1990 if there is adequate preparation today, if teachers and prospective teachers are brought up in the spirit of the Report. He states four reasons for his beliefs.

> 1. Children can learn more than we think they can.
> 2. The transition from one stage of learning to the next can be accelerated by a better curriculum and better teaching.
> 3. Early use of the concepts of mathematical structure accelerates learning by simplifying and unifying the subject matter.
> 4. Changes like those proposed by the [Cambridge] report have already been tried successfully. (Adler, 1966, p. 214-215)

Although the Cambridge Conference Report suggestions are not yet fully implemented, many of the Report's suggestions have been included in newer secondary school programs.

Both Ferguson (1964) and Genise (1960) report very favorable results in working with programs based upon ideas similar to those presented in the Cambridge Conference. From their comments it appears that the "new math" results in better university preparation inasmuch as college students with backgrounds in modern mathematics feel more comfortable in university mathematics courses than do others. Ferguson indicated that students taught under the SMSG program did as well on traditional tests as students taught under traditional programs. Naturally, the SMSG students did much better on modern mathematics tests than did students having only a traditional mathematics background.

Ferguson also felt that one of the best effects of the new program was that poor mathematics students seemed to show the greatest improvement. Many of them "came alive" to the world of mathematics under the new program.

Both Ferguson and Genise claim there is more motivation and interest in "new math" programs than in traditional programs. Perhaps this is partially due to the Hawthorne effect, but since it continued over a period from 1957 to 1964, it seems doubtful that the "newness" of the program is the fundamental reason.

Haag (1964, p. III), in the preface of his book, Structure of Algebra, which is written specifically for the training of teachers to teach modern mathematics writes: "The so-called 'new approach' to the teaching of mathematics is no longer an experiment. In this country [U. S. A.] and abroad, there is widespread approval and use of materials that reflect the new thinking about mathematics education."

Implications

The acceptance of the ideas and goals presented in the Cambridge Conference Report mean three things—new approach, new materials, and new terminology.

New approach

H. P. Fawcett (1960, p. 420), past President of the National Council of Teachers of Mathematics (NCTM), made the following statement about the new approach in his summary of the NCTM Policy Conference in Chicago, April, 1960:

> No student will be guided toward an understanding of mathematical method through teaching procedures which feast his memory and starve his reason. The beauty of mathematical structure will be forever denied to those who continue to sit in classrooms where mathematics is taught only as a tool subject and routine drill is emphasized. (Fawcett, 1960, p. 420)

Haag explains one of the advantages of teaching structure in his statement:

> In all the new thinking there appears to be a common theme: Mathematics must be based on understanding. In the past, mathematics was too often presented as a set of techniques and rules to be memorized; as soon as a rule was forgotten, the manipulative skill developed with the rule was lost also.
>
> Now the trend is toward the teaching of ideas, and it is felt that skills should evolve out of the ideas. (Haag, 1964, p. III)

Landin (1963), Ferguson (1964), and others emphasize the change in approach indicated above by Fawcett and Haag. The main theme from all is "understand the structure of mathematics."

In accomplsihing this purpose, the trend is toward teaching mathematics as an axiomatic system, which it is, and emphasizing deductive reasoning as much as possible. Many exercises are based on inductive reasoning, especially in grades and situations where deductive processes over-challenge students. Memorization of rules and techniques, the "follow my example" kind of problem solving, and the "use it because it works" procedures of traditional mathematics have little place in the new approach to mathematics.

The purpose of the new approach to mathematics is well expressed in the preface of the 1966 edition of the Scott-Foresman beginning algebra test. A part of it reads:

> Today's students know that arithmetic is not just a collection of unrelated rules to be memorized and applied. They have learned that there are sensible reasons to explain what works and what doesn't work in computation. They also know a great deal about algebra, even before they begin a course in algebra. By contrast, many students in the past came to believe that algebra (and arithmetic) consisted of a great many disconnected ideas. It is our hope that in this book you will see that all ideas of algebra can be made to depend upon a few simple and basic principles and

> that you will observe that algebra is a system of related ideas, rather than a collection of special rules to be learned.
>
> As you use this book you will gain insight into the structure of algebra and develop skill in using algebraic methods. You will also learn to think about mathematical ideas precisely and to express the ideas clearly in the special language of algebra. (Van Engen, et al., 1966, p. 3)

New materials

The new materials are not completely new ideas and concepts in mathematics; in fact, most of them are over a hundred years old, and some were even the basic ideas used by earliest mathematicians in developing the number systems we use today. The newness results from their adoption for use in education. Introducing these basic concepts of mathematical structure and operations in public schools is new to both teachers and students.

The new materials are needed to facilitate accession of the objectives of the new approach in teaching.

Both Landin (1963) and Ferguson (1964) suggest that the materials need to be changed and reorganized to rectify the failings of traditional mathematics and to conform to the needs of today.

New terminology

The use of new materials has been accompanied by a conversion to new terminology which is capable of expressing the ideas and concepts of the new materials precisely and clearly. As previously mentioned, clarity and precision of mathematical language are needed to obtain the objectives of the

Cambridge Conference Report. They have been included as important objectives in the "new math" programs by most authors already cited in support of the new approach.

A comparison of the indexes of a 1966 edition and a 1952 edition of beginning algebra texts revealed a great difference in terminology. The 1966 edition is Algebra by Henry Van Engen, Maurice L. Hartung, Harold C. Trimble, Emil J. Berger, and Ray W. Cleveland, published by Scott Foresman and Company. The 1952 edition is Algebra for Problem Solving by Julius Freilich, Simon L. Berman, and Elsie Parker Johnson, published by Houghton Mifflin Company.

The following is a list of terms in the 1966 index which represent ideas or concepts of modern mathematics and which were not in the 1952 index:

abundant number
additive inverse
biconditional
binary operation
Cartesian product
closed half plane
closed interval
closure property
column vector
commutative group
commutative property
compliment of a set
completeness property
compound conditions
compound statements
conditionals
contradiction
converse
convex sets
density property
difference property
disjoint sets
disjunction
distributive property
domain
element of a set
empty set
enumerable sets
equivalent conditions
equivalent sets
existence property of square roots
field
finite number system
finite set
group
half-open interval
half planes
identity element
identity element property
"if . . . then" (connective)
indirect proof
induction
infinite set
intersection of sets
inverse (additive or multiplicative)
inverse property

logic
mapping
math induction
math systems
    commutative group
    field
    group
    number system
    ordered field
    vector space
natural numbers
one-to-one correspondence
one-to-one function
open half plane
open interval
order properties
ordered field
ordered pairs
ordered triples
parameter
proof (by induction, indirect, direct)
proper subset
properties
    closure
    commutative
    completeness
    density
    difference
    distributive
    of an equality
    of a field
    identity element
    inverse
    reflexive
    replacement
    symmetric
    transitive
relation
sets
    compliments of
    denumerable
    disjoint
    elements of
    empty
    equal
    equivalent
    finite
    infinite
    intersection of
    members of
    of ordered pairs
    proper subset of
    solution
    subsets of
    union of
    well-defined
simple condition
symmetric property
transformation
transitive property
truth value
union of sets
well-defined properties

New Demands for Teacher Preparation

The vast changes in approach, materials, and terminology in secondary school mathematics place new and very different demands on the teacher. Much concern has been expressed over the ability of teachers to present the new programs, and of the preparation that the teacher needs to have the background to teach the programs in the spirit in which they were created.

In speaking for NCTM on the subject, H. P. Fawcett makes the following explanation:

> Our dedication to the improvement of mathematics education in America throws the spotlight, not only on the curriculum, but also on the classroom teacher. The quality of the curriculum is important, but no mathematics program will ever be any better than the faculty responsible for it. The curriculum is not a disembodied force which in some unique and mysterious manner moves through the classrooms of America, stirs the imagination of our students, enriches their mathematical insights and develops their highest potential. To achieve such desirable outcomes, a teacher is needed, and the real curriculum includes those methods and procedures by which he brings meaning and significance to the mathematical content covered. . . .
>
> If we are serious in our purpose of improving the teaching of mathematics in America, we must be concerned with the quality of the classroom teacher in both the elementary and secondary schools, which means that we must be concerned with teacher education and certification. (Fawcett, 1960, p. 420)

Included in the summary of the 1960 NCTM Policy Conference was the observation that curricula for the education of mathematics teachers are receiving more attention now than ever before.

One of four goals of an eighteen-month study financed by the Carnegie Corporation and administered by the American Association for the Advancement of Science is to "develop procedures whereby representation of logically interested organizations and state certification and accreditation officials may work together effectively in the development of teacher preparation programs, and to prepare suggested guides for program approval by state certification officers." (Fawcett, 1960, p. 420)

Another concerned group is the Commission of Mathematics of the College Entrance Examinations Board. The Commission thinks that the secondary school mathematics curriculum should be a principal determining factor of teacher education curricula (Schumaker, 1961).

Schumaker (1961, p. 422) feels "there is also a great need for an investigation of the effectiveness of present teacher-education programs in meeting the needs of teachers in modern comprehensive high schools."

Some writers feel that new courses in mathematics should be offered which better fill the needs of secondary mathematics teachers (Kinsella, 1960). Some ideas used in secondary school curricula may not be included in any presently offered university courses.

Lloyd Morrisett (1966, p. 127) feels that evaluation of programs should be a continual process since "curriculum construction, teacher training, the teacher, and research are inextricably linked in the process of education." Morrisett's theme seems to be that changing patterns in curricula require continual evaluation of all factors concerned, from the curriculum creators to the curriculum absorbers (students).

In any case, the preparation of secondary mathematics teachers in recent years has developed many problems because of the change in secondary and college mathematics (Kinsella, 1960).

> They [teachers] discover that they must either learn the new skills and knowledge required by the changes that are adopted or be seriously handicapped or totally obsolete. For example, a teacher who makes no effort to prepare for the new math

> program will be seriously handicapped in teaching arithmetic to elementary students. (Costa, 1966, p. 11)

Ferguson (1964) feels that it is more difficult for teachers to make the change to new math than for the students. Bruce E. Meserve (1966), in speaking to administrators and mathematics teachers says:

> Help your teachers and colleagues to understand what the new approaches to the teaching of mathematics are, and how to use these approaches effectively in their classrooms. This is not a matter simply of adopting a new textbook. No matter how good the book is, teachers need, and can be most effective with, materials that they thoroughly understand and can teach with confidence. In other words, I firmly believe that some teachers are more effective with so-called obsolete textbooks that they understand than with new texts that they do not trust. The problem also cannot be solved by sending one or two teachers to an institute or inservice program. Enough teachers must be influenced to affect the outlook of all teachers. (Meserve, 1966, p. 524)

Some administrators, convinced of the superiority of new math programs, expect an overnight change to modern mathematics simply by adopting a new program. This leads to teacher frustration and confusion. The teachers are not familiar with the complete content of the new programs and cannot become familiar with them by simply reading through the text (Fawcett, 1960).

Curriculum reforms remain strictly academic if they are not simultaneously concerned with the training of teachers who are to bring about the recommended reforms. The success or failure of a curriculum is in the hands of the classroom teacher (Delessert, 1966).

It seems quite clear that the new school mathematics programs require training somewhat different than for traditional programs. The new programs have the potential of being much superior to traditional programs, but their success depends on the classroom teacher. As stated by Kinsella (1960, p. 27), " . . there is little doubt that the preparation of secondary school mathematics teachers will have to be modified."

A question of great importance is how they should be modified.

This question has bothered many people, especially those most closely responsible for the effects of modification. NCTM has made proposals, as have other nationally interested mathematical organizations. The study groups creating the new programs have made recommendations. The Committee on Undergraduate Programs in Mathematics (CUPM) of the Mathematics Association of America has devoted much time, money, and effort to answer this question and to recommend the properly modified program. Even the national government has shown considerable interest in the question.

Most of the recommendations are made in anticipation of the needs the teachers are expected to experience in teaching the new math programs. They are, for the most part, theoretical projections which need confirmation through research. Very recently, some research has been done to either validate or discount the validity of the proposals and suggestions. Before reviewing this most recent research, it is fitting to review the proposals and suggestions for modification of teacher education. In doing so, it should be of interest, and perhaps of importance also, to observe any trends or consistencies.

New Mathematics Teacher Education Programs

Kinsella (1960) makes some general suggestions as to how teacher education should be modified for the years following 1963. Because of his belief that new courses in mathematics should be offered by universities to allow a better preparation, Kinsella names no specific courses. His suggestions follow:

> 1. provide for a knowledge of the logical foundations and important properties of the natural number, integers, rational numbers, irrational numbers, complex numbers, cardinal numbers, ordinal numbers.
> 2. provide for attention to the structure of algebras.
> 3. provide for a knowledge of different kinds of geometry.
> 4. provide for a background in probability and statistics.
> 5. provide for a command of the calculus and the associated function concept.
> 6. provide experience with applications of mathematics to the physical and social sciences.
> 7. provide for a reading knowledge of the history of mathematics.
> 8. provide for experience in integrating mathematics.
> (Kinsella, 1960, p. 31-32)

With similar general objectives in mind, Landin (1963) proposes a university mathematics curriculum which he feels is suitable to the SMSG and UICSM programs. His proposed program contains

College Algebra	3 semester hours
Trigonometry	2 semester hours
Analytic Geometry	3-4 semester hours
Calculus	6-8 semester hours
Advanced Geometry of the Circle and Triangle	3 semester hours
Structure of Arithmetic	3-5 semester hours
Abstract Algebra (Structure of Algebra and Foundations of Algebra)	6 semester hours
Advanced Analytic Geometry	3 semester hours
Introduction to Higher Analysis	6 semester hours

He suggests also some supporting courses in science. In presenting the above list, Landin noted that both college algebra and trigonometry need upgrading on the university level. He feels that analytical geometry is but a ghost of what it should be and, therefore, proposed the advanced analytical geometry to compensate for it. Abstract algebra was stressed most strongly since it develops concepts of structure and proof—groups, fields, rings, logic, and precise definitions. Introduction to higher analysis was suggested to give the teacher a sufficient background to teach elementary calculus or lay a foundation for it.

Estes (1961) recommended a curriculum for prospective teachers of modern mathematics programs which was endorsed by the Board of Governors of the Mathematics Association of America. His recommendations include:

Analytic Geometry	3 semester hours
Calculus	6 semester hours
Structure of Algebra	3 semester hours
Linear Algebra	3 semester hours
Foundations of Geometry	3 semester hours
Structure of Geometry	3 semester hours
Probability	3 semester hours
Statistics	3 semester hours
Advanced Electives	6 semester hours
Number Theory	
Real Variables	
History of Mathematics	
Topology	
Numerical Analysis	

Jones (1962) in a report on new curriculum patterns for mathematics teachers published in the American Association of Colleges for Teacher Education (AACTE) Yearbook, suggests the following curriculum for teachers of algebra and geometry in the seventh, eighth, ninth, and tenth grades:

Analtyic Geometry

Calculus

Abstract Algebra
(Foundation and Structure)

Structure of Geometry

Probability and Statistics

A course containing logic and
set theory

For high school teachers teaching ninth through twelfth grades, Jones recommended additional courses in algebra and geometry.

A report from the sub-committee on teacher certification of the Cooperative Committee on the Teaching of Science and Mathematics (CCTSM) which is part of the American Association for the Advancement of Science (AAAS) suggested the following curriculum as being a minimum for teacher certification:

Analysis (Analytic Geometry and Calculus)	12 semester hours
Probability	3 semester hours
Statistics	3 semester hours
Abstract Algebra	3 semester hours
Geometry	3 semester hours

Applied Mathematics
(Numerical Analysis and Linear Programming) 6 semester hours

Foundation of Mathematics 3 semester hours

Supporting science courses were recommended (Estes, 1961).

CUPM is composed of forty-three university professors, most of whom are well known in their field. The panel on teacher training consists of the following:

E. G. Begle, Stanford University
Roy Dubisch, University of Washington
Mary Folsom, University of Miami
W. T. Guy, Jr., University of Texas
Clarence E. Hardgrove, Northern Illinois University
P. S. Jones, University of Michigan
John L. Kelley, University of California, Berkeley
John G. Kemeny, Dartmouth College
E. R. Kolchin, Columbia University
Bruce E. Meserve, University of Vermont
Edwin E. Moise, Harvard University
George Springer, Indiana University
Rothwell Stephens, Knox College
Henry Van Engen, University of Wisconsin
Stephen S. Willoughby, New York University
Gail S. Young, Tulane University
(CUPM, 1966, p. iv)

CUPM printed recommendations for the training of teachers of mathematics in 1961. The recommendations were revised in August, 1964, and again in December, 1966. The recommendations as revised in December, 1966, are divided according to the different levels of teaching. For the teachers of the elements of algebra and geometry (grades 7 through 10), the following is recommended:

> Prospective teachers should enter this program ready for a mathematics course at the level of a beginning course in analytic geometry and calculus (requiring a minimum of

three years in college preparatory mathematics). It is recognized that many students will need to correct high school deficiencies in college. However, such courses as trigonometry and college algebra should not count toward the fulfillment of minimum requirements at the college level. Their college mathematics training should then include:

(A) Three courses in elementary analysis (including or presupposing the fundamentals of analtyic geometry). This introduction to analysis should stress basic concepts. However, prospective teachers should be qualified to take more advanced mathematics courses requiring a year of calculus, and hence calculus courses especially designed for teachers are normally not desirable.

(B) Four other courses: a course in abstract algebra, a course in geometry, a course in probability from a set-theoretic point of view, and one elective. One of these courses should contain an introduction to the language of logic and sets. The Panel strongly recommends that a course in applied mathematics or statistics be included. (CUPM, 1966, p. 8-9)

For the teachers of high school mathematics, CUPM recommends:

(1) Three courses in analysis (analytic geometry and calculus).

(2) Two courses in abstract algebra. These courses should include linear algebra, groups, rings, and fields.

(3) Two courses in geometry beyond analytic geometry. These courses are for a higher understanding of high school geometry.

(4) Two courses in probability and statistics. It is recommended that these courses be based on calculus.

(5) A course in computer science.

(6) Two upper class courses. Recommended courses are topology, number theory, history of mathematics, or introduction to real variables (CUPM, 1966).

For the teachers of advanced high school programs, a Master's Degree is suggested. At least two-thirds of the courses for the Master's program should be in mathematics. These courses should include two courses of theoretical analysis (CUPM, 1966).

Evaluation of Proposed Curriculum Programs

The following table was constructed for the purpose of summarizing the recommended programs in undergraduate mathematics for prospective secondary school mathematics teachers. The index shown indicates the relative support given a specific course. An index of 1.000 indicates complete support, while .000 indicates no support. The following code and point system was used to determine the index:

X means course was suggested; one point.

P means course is a prerequisite for a suggested course; one point.

$\frac{1}{2}$ X means one out of two courses is suggested; one half point.

SX means course fulfills requirements of a course described in suggestions; one point.

SE means course is suggested as an elective. If it is suggested as one out of two possible—one-half point. If it is one out of four possible, where two should be taken—one-half point.

The index is found by dividing the total number of points given a course by seven (the number of possible points if suggested by each source making recommendations). The index may be changed to indicate per cent support by multiplying by one hundred.

Table 3. Recommended mathematics programs for secondary school teachers

	Landin	Estes	Jones (grades 7-10)	Jones (grades 9-12)	Sub-committee of CCTSM	Grades 7-10 CUPM	Grades 9-12 CUPM	Index
College Algebra[a]	X	P	P	P	P	P	P	1.000
Trigonometry[a]	X	P	P	P	P	P	P	1.000
Analytic Geometry	X	X	X	X	X	X	X	1.000
Calculus	X	X	X	X	X	X	X	1.000
Abstract Algebra[b]	X	X	X	X	X	X	X	1.000
Structure of Arithmetic	X							.143
Foundations of Geometry	X	X		X	$\frac{1}{2}$X	$\frac{1}{2}$X	X	.714
Structure of Geometry		X	X	X	$\frac{1}{2}$X	$\frac{1}{2}$X	X	.714
Advanced Analysis	X							.143
Probability		X	X	X	X	X	X	.857
Statistics		X	X	X	X	SE	X	.786
Number Theory		SE					SE	.143
Introduction to Real Variables		SE					SE	.143
History of Mathematics		SE					SE	.143
Topology		SE					SE	.143
Numerical Analysis		SE			X			.212
Linear Programming					X		SX	.286
Foundations of Mathematics				SX	X	SE		.357

[a] Or appropriate high school background

[b] Includes Foundations of Algebra and Structure of Algebra

College algebra and trigonometry are either recommended or required as prerequisites for all sets of recommendations. In some cases, equivalent high school courses may be substituted.

Analytic geometry, calculus, and abstract algebra were each recommended unanimously. The next most popularly recommended course is probability, followed by statistics.

There was unanimous consent that a geometry course should be taught, but no definite preference for foundations of geometry or structure of geometry was shown. This may be the reason they follow probability and statistics. Had both been included under one heading, "college geometry," this classification would have preceded probability and statistics.

Beyond this point, general support for any one course is not strong enough to claim the course is generally receommended. Courses in this category are: foundations of mathematics, linear programming, numerical analysis, topology, history of mathematics, number theory, structure of arithmetic, introduction to real variables, and advanced analysis.

Listing the courses in order of decreasing index yields the following sequence:

Index	Course
1.000	College Algebra (or equivalent)
	Trigonometry (or equivalent)
	Analytic Geometry
	Calculus
	Abstract Algebra (foundation and structure)

Index (con't.)	Course (Contd.)
.857	Probability
.786	Statistics
.714	Foundations of Geometry
	Structure of Geometry
.357	Foundations of Mathematics
.286	Linear Programming
.212	Numerical Analysis
.143	Topology
	Number Theory
	History of Mathematics
	Introduction to Real Variables
	Advanced Analysis
	Foundations of Arithmetic

In looking for general trends of support, and consistencies of recommendations, it is obvious that courses with an index of 1.000 are unanimously accepted as a necessary preparatory courses for secondary mathematics teachers. Indexes of .700 and above show general acceptance as a needed course, whereas indexes less than .500 show no general acceptance, and would indicate under normal circumstances that less than 50 per cent of the sources recommended that course.

The trend shown by the above chart and index listing is for strong support of the following courses:

College Algebra (or equivalent)

Trigonometry (or equivalent)

Analytic Geometry

Calculus

Abstract Algebra (foundation and structure)

Probability

Statistics

Foundations of Geometry

Structure of Geometry

There is a notable break in the size of indexes (.714 to .357) after the geometry courses, indicating some disagreement as to the need for usefulness of these classes with lower indexes. It is possible that, although these classes may be useful, there are several and it would be impractical to require all of them of prospective teachers. In any case, none of the following courses had universal support of the sources used:

Foundations of Mathematics

Linear Programming

Numerical Analysis

Topology

Number Theory

History of Mathematics

Introduction to Real Variables

Advanced Analysis

Foundations of Arithmetic

On the basis of the recommendations reviewed, the author suggests that courses with an index greater than .700 are generally considered as necessary for adequate preparation to teach modern mathematics in secondary schools. Those courses with an index less than .500 are generally considered as suggested electives for work beyond the necessary courses, but have no universal support. These results will later be compared with the courses Utah secondary mathematics teachers feel are most useful in their classroom teaching experiences.

Recent Educational Research (Three Doctoral Dissertations)

Two doctoral dissertations were completed in 1964, suggesting possible undergraduate programs in mathematics for prospective mathematics teachers. Both suggested programs were based on their authors' studies of changes in secondary school mathematics curricula and suggestions by national groups interested in mathematics teacher education. No analysis for the formulation of these two programs was indicated.

Stephens suggested the following:

> (a) Four courses in elementary analysis (or three four-semester-hour courses).
> (b) A two course sequence in algebra.
> (c) Two courses in geometry.
> (d) A two-course sequence in probability and statistics . . .
>
> In addition, the senior high school teacher should have:
> (e) a two-course sequence in the foundations of mathematics, including mathematical logic.
> (f) Two courses in applied mathematics.
> (g) A two course sequence in advanced analysis.

> In addition, elective courses should be available for those students who can fit them into their schedules. These should include theory of numbers, topology, and the history of mathematics. (1964, p. 130-131)

Berg recommends the following for a major teaching assignment in secondary school mathematics in Oklahoma:

> (A) a minimum of 10 semester hours in mathematical analysis (analytic geometry and calculus);
> (B) a minimum of 6 semester hours in modern algebra (beyond college algebra);
> (C) a minimum of 6 semester hours in contemporary geometry (beyond analytic geometry);
> (D) a minimum of 3 semester hours in probability-statistics; and
> (E) a minimum of 7 semester hours of electives in mathematics, not all chosen from only one of the other four areas. (1964, p. 173-174)

In 1965, R. H. Annis completed a doctoral dissertation on the applicability of university courses to actual classroom teaching. For Annis' study, he used ninety graduates of the University of North Dakota who had graduated in mathematics education in the previous ten years and who had taught mathematics in secondary schools. He asked teachers to rate the applicability of each course listed on a scale one through five, where five meant very applicable and one meant the course had little or no application to secondary school mathematics. Annis obtained a rank order correlation of .91 for ratings by teachers who have and who have not taught modern mathematics. He obtained a .91 rank order correlation also for the ratings by teachers with more than five years' experience and those with less than five years' experience (Annis, 1965).

The degree of applicability given university mathematics courses by the secondary mathematics teachers was in some ways quite different than the proposals previously discussed. The first twenty courses included in Annis' study are listed below in rank order:

1. College Algebra
2. Algebraic Structures
3. Linear Algebra
4. Trigonometry
5. Teacher's Course in Mathematics
6. History of Mathematics
7. Analytic Geometry
8. College Geometry
9. Elementary Statistics
10. Theory of Equations
11. Theory of Probability
12. General Astronomy
13. Non-Euclidian Geometry
14. Differential Calculus
15. Integral Calculus
16. Vector Analysis
17. Mathematical Theory of Statistics
18. Differential Equations
19. Advanced Calculus
20. Intermediate Calculus

METHOD

One of the objectives of this study was to compare the evaluations of university courses by Utah teachers to undergraduate programs suggested for teacher preparation. For this purpose, a questionnaire was prepared and distributed to approximately 104 teachers now actively engaged in mathematics education in Utah. Approximately eighty teachers were given questionnaires at the Utah Council of Teachers of Mathematics general session for secondary school teachers which was part of the Utah Education Association's (UEA) annual teacher's convention in Salt Lake City, Utah, on October 7, 1966. This meeting was attended by Utah mathematics teachers from throughout the state of Utah. The majority of them, however, were from school districts in or near Salt Lake City. An additional twenty-one teachers from Logan and Cache County school districts, who did not attend the convention, were later given questionnaires, as were three other teachers contacted by the investigator. Fifty-eight questionnaires were received between October, 1966 and May, 1967. Of these fifty-eight, seven were deleted because of incompleteness and one because of ambiguity. The remaining fifty were used, as appropriate, to formulate the tables used in showing the results.

The questionnaire (see Appendix) contained twenty-six courses of university mathematics which are taught in Utah universities as undergraduate courses. Teachers were asked to indicate whether they felt the course has been or would be of value to them in teaching secondary school mathematics. To gain some idea of the degree of value, teachers were also asked to indicate the five courses they felt were most valuable in teaching secondary mathematics. They were also asked if they felt a course should be part of the requirements for a teaching major in mathematics.

The questionnaire also asked for name, date, and school district. It was designed to show whether the teacher's school had a modern mathematics program, how long that program had been in effect, and how many years the teacher had been teaching. It also indicated courses most frequently taught, and whether the teacher graduated with a mathematics major or minor.

RESULTS

The findings of this study are expressed in the form of tables which show the teachers' reactions to each course according to the information asked for. Tables 4, 5, and 6 are concerned with all teachers who submitted acceptable questionnaires. Tables 7, 8, and 9 are concerned only with those teachers who have a teaching major or minor in mathematics or a science-mathematics composite major, and who have taught modern mathematics. (There are thirty-six such teachers who submitted questionnaires). Tables 10, 11, and 12 are concerned with teachers who do not have a teaching major or minor in mathematics or who have not taught modern mathematics (There are fourteen such teachers who submitted questionnaires, of which only one was placed in this category for not having taught modern mathematics).

The data in Tables 4, 7, and 10 are concerned with whether the course has been or would be valuable in teaching secondary school mathematics. Tables 5, 8, and 11 contain data relating to the degree of value of the courses. It asked teachers to check the five courses they felt to be the most valuable in teaching secondary school mathematics. The evaluations shown on Tables 6, 9, and 12 are to indicate the courses Utah teachers feel should be part of the requirements for a teaching major in secondary mathematics education.

All teachers did not complete all parts of the questionnaires. In such cases, only the parts completed were used. For this reason, each table indicates

Table 4. The value of courses in teaching secondary mathematics by all teachers (46 completed this category)

	A	B	C
College Algebra	39	84.8	1
Solid Geometry	21	45.7	12.5
Trigonometry	37	80.5	2
Analytic Geometry	31	67.4	3
Differential Calculus	28	60.9	4
Integral Calculus	23	50.0	10
Number Theory	23	50.0	10
Logic	20	43.5	14.5
History of Mathematics	23	50.0	10
Matrices	12	26.3	19
Probability and Statistics	19	41.3	16
Methods for Sec. Math. Teach.	25	54.4	6
Modern Algebra	27	58.7	5
Modern Geometry	17	36.8	17
Math for Sec. School Teach.	24	52.2	7.5
Foundations of Math (set theory)	24	52.2	7.5
Foundation of Algebra	20	43.5	14.5
Foundation of Geometry	21	45.7	12.5
Analysis	7	15.2	23.5
Theory of Equations	9	19.6	20
Topology	3	6.5	26
Linear Algebra	13	28.4	18
Projective Geometry	8	17.4	21
Functions of Real and Complex Variables	7	15.2	23.5
Advanced Calculus	7	15.2	23.5
Differential Equations	7	15.2	23.5

Key: (A) Number of teachers who feel course is valuable in teaching secondary school mathematics

(B) Per cent of teachers who feel course is valuable in teaching secondary school mathematics

(C) Rank order

Table 5. The five most valuable courses in teaching secondary school mathematics by all teachers (48 completed this category)

	A	B	C
College Algebra	34	70.9	2
Solid Geometry	12	25.0	7
Trigonometry	35	72.9	1
Analytic Geometry	25	52.1	3
Differential Calculus	12	25.0	7
Integral Calculus	6	12.5	15
Number Theory	12	25.0	7
Logic	8	16.7	12.5
History of Mathematics	7	14.8	14
Matrices	0	0.0	24
Probability and Statistics	5	10.4	16.5
Methods for Sec. Math Teach.	15	31.2	5
Modern Algebra	20	41.7	4
Modern Geometry	11	22.9	10
Math for Sec. School Teach.	11	22.9	10
Foundations of Math (set theory)	11	22.9	10
Foundation of Algebra	8	16.7	12.5
Foundation of Geometry	5	10.4	16.5
Analysis	1	2.1	20
Theory of Equations	3	6.3	18
Topology	1	2.1	20
Linear Algebra	0	0.0	24
Projective Geometry	0	0.0	24
Functions of Real and Complex Variables	0	0.0	24
Advanced Calculus	1	2.1	20
Differential Equations	0	0.0	24

Key: (A) Number of teachers who feel course is one of the five most valuable courses in teaching secondary school mathematics.

(B) Per cent of teachers who feel course is one of the five most valuable in teaching secondary school mathematics

(C) Rank order

Table 6. Courses recommended as teaching major requirements by all teachers (36 completed this category)

	A	B	C
College Algebra	33	82.5	1.5
Solid Geometry	23	57.5	7.5
Trigonometry	33	82.5	1.5
Analytic Geometry	30	75.0	3
Differential Calculus	26	65.0	4
Integral Calculus	24	60.0	6
Number Theory	16	40.0	16
Logic	17	42.5	14
History of Mathematics	21	52.5	10
Matrices	7	17.5	19
Probability and Statistics	16	40.0	16
Methods for Sec. Math Teach.	25	62.5	5
Modern Algebra	23	57.5	7.5
Modern Geometry	21	52.5	10
Math for Sec. School Teach.	21	52.5	10
Foundations of Math (set theory)	18	45.0	13
Foundation of Algebra	16	40.0	16
Foundation of Geometry	19	47.5	12
Analysis	3	7.5	22.5
Theory of Equations	10	25.0	18
Topology	1	2.5	25.5
Linear Algebra	5	12.5	20
Projective Geometry	3	7.5	22.5
Functions of Real and Complex Variables	3	7.5	22.5
Advanced Calculus	1	2.5	25.5
Differential Equations	3	7.5	22.5

Key: (A) Number of teachers who feel course should be part of requirements for a teaching major

(B) Per cent of teachers who feel course should be part of requirements for a teaching major

(C) Rank order

Table 7. The value of courses in teaching secondary school mathematics by teachers who have a mathematics teaching major or minor or a science-mathematics composite and who have taught modern mathematics (33 answered this category)

	A	B	C
College Algebra	28	85.0	1.5
Solid Geometry	14	42.4	16.5
Trigonometry	28	85.0	1.5
Analytic Geometry	24	72.7	3
Differential Calculus	20	60.6	6.5
Integral Calculus	20	60.6	6.5
Number Theory	20	60.6	6.5
Logic	16	48.4	13.5
History of Mathematics	18	54.5	11
Matrices	12	36.4	18
Probability and Statistics	16	48.4	13.5
Methods for Sec. Math Teach.	20	60.6	6.5
Modern Algebra	22	66.7	4
Modern Geometry	17	51.5	12
Math for Sec. School Teach.	19	57.6	9.5
Foundations of Math (set theory)	19	57.6	9.5
Foundation of Algebra	14	42.4	16.5
Foundations of Geometry	15	45.5	15
Analysis	7	21.2	23
Theory of Equations	7	21.2	23
Topology	4	12.1	26
Linear Algebra	11	33.3	19
Projective Geometry	8	24.2	20
Functions of Real and Complex Variables	7	21.2	23
Advanced Calculus	7	21.2	23
Differential equations	7	21.2	23

Key: (A) Number of teachers who feel course is valuable in teaching secondary school mathematics

(B) Per cent of teachers who feel course is valuable in teaching secondary school mathematics

(C) Rank order

Table 8. The five most valuable courses in teaching secondary school mathematics by teachers who have a mathematics teaching major or minor, or a mathematics-science composite (36 answered this category)

	A	B	C
College Algebra	23	64.0	1
Solid Geometry	7	19.7	11
Trigonometry	22	61.2	2
Analytic Geometry	19	52.8	3
Differential Calculus	8	22.5	10
Integral Calculus	5	14.0	15
Number Theory	9	25.0	7.5
Logic	6	16.9	12.5
History of Mathematics	4	11.2	17
Matrices	0	0.0	23
Probability and Statistics	5	14.0	15
Methods for Sec. Math Teach.	10	28.8	5
Modern Algebra	15	41.7	4
Modern Geometry	9	25.0	7.5
Math for Sec. School Teach.	9	25.0	7.5
Foundations of Math (set theory)	9	25.0	7.5
Foundation of Algebra	6	16.9	12.5
Foundation of Geometry	5	14.0	15
Analysis	0	0.0	23
Theory of Equations	3	8.4	18
Topology	0	0.0	23
Linear Algebra	0	0.0	23
Projective Geometry	0	0.0	23
Functions of Real and Complex Variables	0	0.0	23
Advanced Calculus	1	2.8	19
Differential Equations	0	0.0	23

Key: (A) Number of teachers who feel course is one of the five most valuable courses in teaching secondary school mathematics

(B) Per cent of teachers who feel course is one of the five most valuable courses in teaching secondary school mathematics

(C) Rank order

Table 9. Courses recommended as teaching major requirements by teachers who have a mathematics teaching major or minor, or a mathematics-science composite (31 completed this category)

	A	B	C
College Algebra	26	83.8	1.5
Solid Geometry	17	54.8	9.5
Trigonometry	26	83.8	1.5
Analytic Geometry	24	77.4	3
Differential Calculus	21	67.7	5
Integral Calculus	20	64.5	6
Number Theory	13	41.9	16.5
Logic	14	45.2	13.5
History of Mathematics	17	54.8	9.5
Matrices	4	12.9	19
Probability and Statistics	13	41.9	16.5
Methods for Sec. Math Teach.	22	71.0	4
Modern Algebra	18	58.1	7
Modern Geometry	17	54.8	9.5
Math for Sec. School Teach.	17	54.8	9.5
Foundations of Math (set theory)	14	45.2	13.5
Foundation of Algebra	14	45.2	13.5
Foundation of Geometry	14	45.2	13.5
Analysis	2	6.5	23
Theory of Equations	9	29.1	18
Topology	0	0.0	26
Linear Algebra	3	9.7	21
Projective Geometry	3	9.7	21
Functions of Real and Complex Variables	1	3.2	24.5
Advanced Calculus	1	3.2	24.5
Differential Equations	3	9.7	21

Key: (A) Number of teachers who feel course should be part of requirements for a teaching major in mathematics

(B) Per cent of teachers who feel courses should be part of requirements for a teaching major in mathematics

(C) Rank order

Table 10. The value of courses in teaching secondary school mathematics by teachers who do not have mathematics majors or minors or who have not taught modern mathematics (13 answered this category)

	A	B	C
College Algebra	11	84.5	1
Solid Geometry	7	53.5	4.5
Trigonometry	9	69.2	2
Analtyic Geometry	7	53.8	4.5
Differential Calculus	8	61.6	3
Integral Calculus	2	15.4	17
Number Theory	3	23.0	13.5
Logic	4	30.8	11
History of Mathematics	5	38.4	8.5
Matrices	0	0.0	23.5
Probability and Statistics	3	23.0	13.5
Methods for Sec. Math Teach.	5	38.4	8.5
Modern Algebra	5	38.4	8.5
Modern Geometry	0	0.0	23.5
Math for Sec. School Teach.	3	23.0	13.5
Foundations of Math (set theory)	5	38.4	8.5
Foundation of Algebra	3	23.0	13.5
Foundation of Geometry	6	46.2	6
Analysis	1	7.7	19.5
Theory of Equations	2	15.4	17
Topology	1	7.7	19.5
Linear Algebra	2	15.4	17
Projective Geometry	0	0.0	23.5
Functions of Real and Complex Variables	0	0.0	23.5
Advanced Calculus	0	0.0	23.5
Differential Equations	0	0.0	23.5

Key: (A) Number of teachers who feel course is valuable in teaching secondary school mathematics

(B) Per cent of teachers who feel course is valuable in teaching secondary school mathematics

(C) Rank order

Table 11. The five most valuable courses in teaching secondary school mathematics by teachers who do not have mathematics majors or minors or have not taught modern mathematics (9 answered this category)

	A	B	C
College Algebra	7	77.8	1.5
Solid Geometry	6	66.7	2.5
Trigonometry	7	77.8	1.5
Analytic Geometry	6	66.7	2.5
Differential Calculus	5	55.6	6
Integral Calculus	2	22.2	15.5
Number Theory	3	33.3	10.5
Logic	3	33.3	10.5
History of Mathematics	4	44.4	7.5
Matrices	0	0.0	24
Probability and Statistics	3	33.3	10.5
Methods for Sec. Math Teach.	3	33.3	10.5
Modern Algebra	5	55.6	6
Modern Geometry	2	22.2	15.5
Math for Sec. School Teach.	1	11.1	19.5
Foundations of Math (set theory)	4	44.4	7.5
Foundation of Algebra	2	22.2	15.5
Foundation of Geometry	5	55.5	6
Analysis	1	11.1	19.5
Theory of Equations	1	11.1	19.5
Topology	1	11.1	19.5
Linear Algebra	2	22.2	15.5
Projective Geometry	0	0.0	24
Functions of Real and Complex Variables	0	0.0	24
Advanced Calculus	0	0.0	24
Differential Equations	0	0.0	24

Key: (A) Number of teachers who feel course is one of the five most valuable courses in teaching secondary school mathematics

(B) Per cent of teachers who feel course is one of the five most valuable courses in teaching secondary school mathematics

(C) Rank order

Table 12. Courses recommended as teaching major requirements by teachers who do not have a mathematics major or minor or who have not taught modern mathematics (12 teachers answered this category)

	A	B	C
College Algebra	11	91.6	2
Solid Geometry	5	41.6	5
Trigonometry	12	100.0	1
Analytic Geometry	6	50.0	3
Differential Calculus	4	33.3	7
Integral Calculus	1	8.3	16
Number Theory	3	25.0	8.5
Logic	2	16.7	12
History of Mathematics	3	25.0	8.5
Matrices	0	0.0	22
Probability and Statistics	0	0.0	22
Methods for Sec. Math Teach.	5	41.6	5
Modern Algebra	5	41.6	5
Modern Geometry	2	16.7	12
Math for Sec. School Teach.	2	16.7	12
Foundations of Math (set thoery)	2	16.7	12
Foundation of Algebra	2	16.7	12
Foundation of Geometry	0	0.0	22
Analysis	1	8.3	16
Theory of Equations	0	0.0	22
Topology	1	8.3	16
Linear Algebra	0	0.0	22
Projective Geometry	0	0.0	22
Functions of Real and Complex Variables	0	0.0	22
Advanced Calculus	0	0.0	22
Differential Equations	0	0.0	22

Key: (A) Number of teachers who feel course should be part of requirements for a teaching major in mathematics

(B) Per cent of teachers who feel course should be part of requirements for a teaching major in mathematics

(C) Rank order

the number of teachers correctly completing the section of the questionnaire appropriate to that table.

The internal consistency of the data is noticably high. (With twenty-six items, rho = .515 is significant at the .01 level.) The rank difference correlation coefficient for the value of courses and courses recommended from Tables 4 and 6 (all teachers in survey) is .94. For Tables 7 and 9 (teachers with major or minor), it is .92. Both of these are above the .01 significance level, showing a high correlation between courses the teachers found valuable in teaching, and those they proposed for teaching major requirements in mathematics. For Tables 10 and 12 (teachers without major or minor), the rank difference correlation coefficient was .832, showing less consistency than the more professionally prepared groups. The standard equation

$$\text{rho} = 1 - \frac{6 \sum D^2}{n(n^2-1)}$$

D = rank difference

n = number of items ranked

was used to find the correlation coefficients.

The rank difference between courses valued, and courses recommended by Utah teachers was greatest for number theory, modern geometry, and foundations of mathematics. These three courses were ranked higher for value to teaching than as recommended courses for major teaching requirements.

The correlation coefficients for the tables for valuable courses and for the five most valuable courses were not quite as high, but still showed significant consistency (.01 level). For Tables 4 and 5, rho = .90; for 7 and 8, rho = .90; and for 10 and 11, rho = .94.

Courses with the greatest rank differences were solid geometry, integral calculus, history of mathematics, matrices, and modern geometry. Each of these courses ranked much lower as one of the five most valuable courses than for having value in teaching secondary school mathematics. In this category it was the teachers without teaching majors of minors in mathematics who were more consistent.

The rank difference correlation coefficient for the value of courses; between all teachers and teachers having majors or minors (Tables 4 and 7) is .96; between all teachers and teachers without majors and minors (Tables 4 and 10), .87; and between teachers with majors and minors and teachers without majors and minors (Tables 7 and 10), .75. The first two coefficients are expected to be high, since the second set of teachers in each case is a subset of all teachers, to which it is being compared. The third coefficient (.75) is probably more revealing than the other two. It indicates a modest but noticeable degree of difference in value between teachers with professional preparation and experience teaching modern mathematics and those without.

Courses having the greatest rank difference are solid geometry, integral calculus, matrices, modern geometry, foundations of geometry, and topology. Integral calculus, matrices, and modern geometry were ranked higher by teachers with teaching majors and minors. Solid geometry, foundations of geometry, and topology were ranked high by teachers without majors and minors.

It should be noted that the number of teachers without teaching majors and minors in mathematics who returned questionnaires is relatively small (thirteen out of fifty teachers with usable questionnaires) and may not accurately represent the large number of teachers in Utah who teach mathematics without a major or minor in the field. For this reason, the remainder of this study will deal primarily with the questionnaires of the two larger sets of teachers: those with mathematics teaching majors or minors and experience in modern mathematics, and all teachers.

For ease of comparison, the following tables were prepared:

Table 13 shows courses listed in rank order according to their value to Utah mathematics teachers as taken from Tables 4 and 7.

Table 14 shows courses listed in rank order according to their evaluation as being one of the five most valuable courses to Utah mathematics teachers as taken from Tables 5 and 8.

Table 15 shows courses listed in rank order as recommended for teaching major requirements in mathematics by Utah teachers, taken from Tables 6 and 9.

By listing the courses on Table 3 (page 32) in rank order, it is possible to calculate a rank difference correlation coefficient (rho) for those courses and the courses on Table 13 (page 54). This would present some idea of the correlation between nationally recommended courses (Table 3) and courses Utah mathematics teachers feel are valuable in teaching secondary school mathematics (Table 13).

Table 13. Rank order of courses according to their value in teaching secondary school mathematics

A		B	
1	College Algebra	1.5	College Algebra
2	Trigonometry	1.5	Trigonometry
3	Analytic Geometry	3	Analytical Geometry
4	Differential Calculus	4	Modern Algebra
5	Modern Algebra	6.5	Differential Calculus
6	Methods for Secondary Mathematics Teachers	6.5	Integral Calculus
7.5	Mathematics for Secondary School Teachers	6.5	Number Theory
7.5	Foundations of Mathematics	6.5	Methods for Secondary Mathematics Teachers
10	Integral Calculus	9.5	Foundations of Mathematics
10	Number Theory	9.5	Mathematics for Secondary School Teachers
10	History of Mathematics	11	History of Mathematics
12.5	Foundations of Geometry	12	Modern Geometry
12.5	Solid Geometry	13.5	Logic
14.5	Logic	13.5	Probability and Statistics
14.5	Foundations of Algebra	15	Foundations of Geometry
16	Probability and Statistics	16.5	Foundations of Algebra
17	Modern Geometry	16.5	Solid Geometry
18	Linear Algebra	18	Matrices
19	Matrices	19	Linear Algebra
20	Theory of Equations	20	Projective Geometry
21	Projective Geometry	23	Analysis
23.5	Differential Equations	23	Theory of Equations
23.5	Advanced Calculus	23	Advanced Calculus
23.5	Analysis	23	Differential Equations
23.5	Functions of Real and Complex Variables	23	Functions of Real and Complex Variables
26	Topology	26	Topology

Key: (A) All teachers

(B) Teachers who have a teaching major or minor in mathematics and have taught modern mathematics

Table 14. Rank order of courses considered to be in the five most valuable courses in teaching secondary school mathematics

A		B	
1	Trigonometry	1	College Algebra
2	College Algebra	2	Trigonometry
3	Analytic Geometry	3	Analytical Geometry
4	Modern Algebra	4	Modern Algebra
5	Methods for Secondary Mathematics Teachers	5	Methods for Secondary Mathematics Teachers
7	Number Theory	7.5	Number Theory
7	Differential Calculus	7.5	Modern Geometry
7	Solid Geometry	7.5	Mathematics for Secondary School Teachers
10	Modern Geometry	7.5	Foundations of Mathematics
10	Foundations of Mathematics	10	Differential Calculus
10	Mathematics for Secondary School Teachers	11	Solid Geometry
12.5	Foundations of Algebra	12.5	Logic
12.5	Logic	12.5	Foundations of Algebra
14	History of Mathematics	15	Integral Calculus
15	Integral Calculus	15	Probability and Statistics
16.5	Probability and Statistics	15	Foundations of Geometry
16.5	Foundations of Geometry	17	History of Mathematics
18	Theory of Equations	18	Theory of Equations
20	Analysis	19	Advanced Calculus
20	Topology	23	Matrices
20	Advanced Calculus	23	Analysis
24	Matrices	23	Topology
24	Linear Algebra	23	Linear Algebra
24	Projective Geometry	23	Projective Geometry
24	Differential Equations	23	Differential Equations
24	Functions of Real and Complex Variables	23	Functions of Real and Complex Variables

Key: (A) All teachers
(B) Teachers who have a teaching major or minor in mathematics and who have taught modern mathematics

Table 15. Rank order of courses recommended for a teaching major in secondary school mathematics

A		B	
1.5	College Algebra	1.5	College Algebra
1.5	Trigonometry	1.5	Trigonometry
3	Analytic Geometry	3	Analytical Geometry
4	Differential Calculus	4	Methods for Secondary
5	Methods for Secondary		Mathematics Teachers
	Mathematics Teachers	5	Differential Calculus
6	Integral Calculus	6	Integral Calculus
7.5	Modern Algebra	7	Modern Algebra
7.5	Solid Geometry	9.5	Mathematics for Secondary
10	History of Mathematics		School Teachers
10	Modern Geometry	9.5	Modern Geometry
10	Mathematics for Secondary	9.5	History of Mathematics
	School Teachers	9.5	Solid Geometry
12	Foundations of Geometry	13.5	Logic
13	Foundations of Mathematics	13.5	Foundations of Mathematics
14	Logic	13.5	Foundations of Algebra
16	Number Theory	13.5	Foundations of Geometry
16	Probability and Statistics	16.5	Number Theory
16	Foundations of Algebra	16.5	Probability and Statistics
18	Theory of Equations	18	Theory of Equations
19	Matrices	19	Matrices
20	Linear Algebra	21	Linear Algebra
22.5	Analysis	21	Projective Geometry
22.5	Projective Geometry	21	Differential Equations
22.5	Functions of Real and	23	Analysis
	Complex Variables	24.5	Advanced Calculus
22.5	Differential Equations	24.5	Functions of Real and
25.5	Topology		Complex Variables
25.5	Advanced Calculus	26	Topology

Key: (A) All teachers

(B) Teachers who have a teaching major or minor in mathematics and who have taught modern mathematics

In doing so, any course which did not appear on both tables was deleted. Both differential calculus and integral calculus on Table 13 were considered as one course (calculus) and the questionnaires were recounted to find the number of teachers who felt either course was valuable. Modern algebra and foundations of algebra were classified together as a course in abstract algebra, and questionnaires were recounted as for calculus (see Appendix).

The above mentioned procedure yielded fourteen courses which were usable in calculating the rank difference correlation coefficient (rho). The coefficient calculated by this method is .79. Since there was some difference in the methods used to obtain the rank listings, and all courses of the two rank listings did not correspond originally, this is a crude measurement. Still, it does give a rough idea of the correlation and similarity of the two listings.

CONCLUSIONS AND SUMMARY

Evaluation of Results

There is a fairly high correlation between courses included in nationally recommended programs of undergraduate mathematics for teacher preparation and courses which were most frequently valued in teaching secondary school mathematics by Utah mathematics teachers. The rank order correlation coefficient (rho) is .79, using fourteen courses, which is significant at the .01 level.

Even with the high correlation coefficient, there are some differences which warrant attention. The courses most commonly recommended nationally, as presented on page 34, are:

College Algebra (or equivalent)

Trigonometry (or equivalent)

Analytic Geometry

Calculus

Abstract Algebra

Probability

Statistics

Foundations of Geometry

Structure of Geometry

College algebra, trigonometry, and analytic geometry were valued most frequently by Utah mathematics teachers, as well as being most frequently recommended in national programs. These three courses seem to have been the core of mathematics study by beginning students for a long period of time as indicated previously (page 7). In some cases, these courses are being taught sufficiently well in high schools to allow students to begin collegiate work in calculus. If the goals of modern mathematics are realized, this will soon be the general case.

Differential calculus was consistently rated high by Utah teachers, but integral calculus was not. Integral calculus was ranked fifteenth (out of twenty-six courses) as one of the five most valuable classes by all teachers and also by teachers with math majors or minors (Table 14). It appears that although a fair share of Utah mathematics teachers (50 per cent on Table 4 and 60.6 per cent on Table 7) value integral calculus, few of them (12.5 per cent on Table 5 and 14 per cent on Table 8) feel it is one of the five most valuable courses. Several other courses such as modern algebra, modern geometry, number theory, logic, foundations of mathematics, history of mathematics, and solid geometry were more frequently chosen as one of the five most valuable courses.

The need for integral calculus in teaching seventh, eighth, ninth, and possibly tenth grade mathematics is questionable. Usually, those teachers teaching advanced high school mathematics are the ones who would find the greatest value for integral calculus.

It may be worthwhile to investigate the possibility of including integral calculus in required programs only as an optional course, especially if it is a barrier, preventing some prospective mathematics teachers from continuing in mathematics.

A course in abstract algebra (modern algebra) was ranked fourth and fifth by all groups in frequency of being chosen as a valuable course and as one of the five most valuable courses in teaching secondary school mathematics. The consistency of Utah teachers in choosing modern algebra as a valuable course harmonizes well with the recommended programs in mathematics.

Probability and statistics was given little support by Utah mathematics teachers (41 per cent felt it a valuable course; 10.4 per cent felt it to be one of the five most valuable courses: Tables 4 and 5). It was ranked sixteenth and 13.5th as a valuable course by all teachers, and by only those with mathematics majors and minors respectively. It was ranked 16.5th and fifteenth as one of the five most valuable courses by the same two groups respectively. (See Tables 13 and 14.) Several courses were consistently ranked above probability and statistics by all groups in all categories. (See Tables 13, 14, and 15.) These courses, with the exception of courses already discussed in this section, are:

Methods for Secondary Mathematics Teachers

Mathematics for Secondary School Teachers

Foundations of Mathematics

Number Theory

College Geometry (Modern Geometry or Foundations of Geometry)

Logic

History of Mathematics (ranked lower in one of six cases)

In view of other courses more frequently chosen in all categories (see Tables 13, 14, and 15), there appears to be little justification to make probability and statistics a required course for a mathematics teaching major, as suggested in the proposed undergraduate programs (pages 27-30), since other courses ranked higher are not required.

University geometry courses (modern geometry, foundations of geometry, and solid geometry) fluctuated somewhat in their rankings by different groups and in different categories. Although there was no consistent evaluation given to any one specific geometry, the tables indicate that teachers generally felt that a university geometry course other than analytic geometry is valuable. Somewhat the same situation existed among the recommendations of mathematics programs discussed earlier (page 33). It may be worthwhile to investigate the indecisiveness concerning which geometry is most valuable or useful, or even if a course could be designed especially to fit the needs of a high school teacher.

Based upon the results presented in the tables, the following courses have sufficient support to justify consideration of their inclusion in a teacher preparation program for secondary school mathematics:

Mathematics for Secondary School Teachers

Methods for Secondary School Teachers

Number Theory

Foundations of Mathematics

Logic

History of Mathematics

This list does not include courses previously discussed. The first two courses in this list were ranked in the fifth, sixth, seventh, or eighth places in all categories. Number theory had an average rank position of 7.9 on the tables indicating value, but was lower on the table of recommended courses for a teaching major. The last three courses in the above list were generally ranked in the upper half (one through fourteen) and carried 50 per cent or more of the support on Tables 4, 6, 7, and 9.

Proposed Program

Taking into consideration the goals and objectives of the Cambridge Conference Report, the recommendations made for prospective mathematics teachers, and the results of this study among Utah mathematics teachers, it is difficult for any one curriculum to fit the needs of all secondary mathematics teachers in Utah. Some courses, such as integral calculus, necessary for teachers of advanced high school mathematics have very little value to the junior high school mathematics teacher. Time and effort may be very poorly used if prospective teachers of seventh, eighth, and ninth grade basic mathematics, beginning algebra, and geometry are required to prepare to teach advanced algebra, trigonometry, and beginning calculus.

Because of the higher degree of rigor and efficiency necessary to properly teach modern mathematics programs, it may be wise to specialize, more than has been done in the past, for the area in which a teacher plans to teach. It is possible that a less-demanding, but better-directed program for prospective teachers of seventh, eighth, and ninth grade mathematics could prepare them more fully for their actual classroom teaching demands and at the same time encourage more students to major in mathematics education. It is beyond the scope of this study to deal with the problems of specialization in secondary mathematics education, but for the reasons stated above, a two-phase program is suggested. The first phase is to be completed by all prospective teachers of secondary school mathematics, and the second phase according to the area in which the prospective teacher plans to teach.

According to the three factors mentioned at the beginning of this section (Cambridge Conference goals, recommended programs, and the results of this study), prospective secondary mathematics teachers of Utah should take the following courses for a mathematics teaching major:

College Algebra (or equivalent)

Trigonometry (or equivalent)

Analytic Geometry

Differential Calculus

Abstract Algebra (at least one course)

College Geometry (at least one course other than Analytic Geometry)

Mathematics for Secondary School Teachers

Methods course (may be taken under the Department of Education)

After completing this basic program, teachers intending to teach grades seven, eight, or nine should choose three or more courses from the following:

Foundations of Mathematics

Additional courses in Abstract Algebra

Additional courses in College Geometry

(other than Analytic Geometry)

Number Theory

Logic

History of Mathematics

Probability and Statistics

A teacher intending to teach grades ten, eleven, or twelve should complete integral calculus and choose three or more courses from the following:

Foundations of Mathematics

Additional courses in Abstract Algebra

Additional courses in College Geometry

(other than Analytic Geometry)

Number Theory

Logic

History of Mathematics

Probability and Statistics

Additional Calculus courses

Annis (1965) recommended that a special program for junior high school teachers be made available. The curriculum suggested above makes some allowance for the differing needs of junior high school teachers and is still in general harmony with the Cambridge Conference goals, the recommended programs for teacher preparation, and the evaluations of Utah mathematics teachers questioned in this study. The greatest disagreement between factors was the strong support given probability and statistics by those recommending undergraduate programs in mathematics as opposed to the small number of Utah secondary mathematics teachers who found the course to be valuable in teaching mathematics. Another noticeable discrepancy between factors originated out of the need for teachers to be prepared to teach introductory courses in calculus, as suggested by the Cambridge Conference. Integral calculus was recommended in the undergraduate programs cited, but few teachers valued it. In the suggested curriculum above, those teachers who need integral calculus will have it, and those teaching the more elementary courses, who have little use for it, would not be forced to take it.

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APPENDIX

Copy of the questionnaire submitted to Utah secondary mathematics teachers

Name:______________________________

Date:______________________________

School district:______________________

Has your school adopted a modern math program? Yes____ No____

If so, how many years has it been in effect?__________________

How many years have you been teaching math?________________

Circle math courses you teach most frequently: Basic math, practical math, business math, algebra I, algebra II, algebra III, geometry, trigonometry, other__________________, ____________________.

Did you graduate with a math major or minor? Yes____No____

Comments:

Courses taught in Utah Universities.	If course has been taken give approx. year.	Opinion of course instruction:a, b, c, d, f.	Feel course has been (would be) valuable in teaching sec. math.	Feel course should be part of sec. math education major requirements.	Check 5 courses you feel to be most valuable to sec. math teaching.
College Algebra					
Solid Geometry					
Trigonometry					
Analytic Geometry					
Differential Calculus					
Integral Calculus					
Number Theory					
Logic					
History of Mathematics					
Matrices					
Probability and Statistics					
Methods for Sec. Math Teach.					
Modern Algebra					
Modern Geometry					
Math for Sec. School Teach.					
Foundations of Math (set theory)					
Foundation of Algebra					
Foundation of Geometry					
Analysis					
Theory of Equations					
Topology					
Linear Algebra					
Projective Geometry					
Functions of Real and Complex Variables					
Advanced Calculus					
Differential Equation					

Data used to find rank order coefficient for Tables 3 and 13 (See pages 56-57).

Courses	Rank Order from Table 3	Rank Order from Table 13
College Algebra	3	1
Trigonometry	3	2
Analtyic Geometry	3	3
Calculus	3	5
Number Theory	12	7.5
History of Mathematics	12	7.5
Probability and Statistics	6	10
Abstract Algebra	3	4
Modern Geometry	7.5	11
Foundations of Mathematics	9	6
Foundations of Geometry	7.5	9
Analysis	12	12.5
Topology	12	14
Functions of Real and Complex Variables	12	12.5

rho = .794

(With 14 items, rho = .715 is significant at the .01 level.)

Questionnaire Used by Annis (Annis, 1965, p. 48). (See pages 37-38)

Please use this from to evaluate your college mathematics courses for their applicability to your teaching.

My mathematics teaching generally falls into: (check one or more).

______ Group A: 7th, 8th, and 9th grade mathematics.
______ Group B: 9th grade algebra and plane geometry.
______ Group C: Algebra II and higher mathematics.

Opposite each college mathematics course (that you have taken) place a number from 1 to 5 (see rating scale) as you feel that course has value for teacher preparation for your group. More than one group may be used for ratings.

5 Great application
4 Above average application
3 Average application
2 Below average application
1 Little or no application

	Group A	Group B	Group C
Refresher Course in Mathematics	______	______	______
College Algebra	______	______	______
Solid Geometry	______	______	______
Trigonometry	______	______	______
Analytic Geometry	______	______	______
Business Mathematics	______	______	______
Mathematics of Investment	______	______	______
Differential Calculus	______	______	______
Integral Calculus	______	______	______
Elementary Statistics	______	______	______
Teachers' Course in Mathematics	______	______	______
College Geometry	______	______	______
Non-Euclidian Geometry	______	______	______
Advanced Analytic Geometry	______	______	______
Intermediate Calculus	______	______	______
General Astronomy	______	______	______
Algebraic Structures	______	______	______
Linear Algebra	______	______	______
Applied Mathematics	______	______	______
Reading Course in Mathematics	______	______	______
Theory of Probability	______	______	______
History of Mathematics	______	______	______
Theory of Equations	______	______	______
Advanced Plane Analytic Geometry	______	______	______
Solid Analytic Geometry	______	______	______
Differential Equations	______	______	______
Vector Analysis	______	______	______
Mathematical Theory of Statistics	______	______	______
Advanced Calculus	______	______	______

Table 10.--A Summary of the Applicability Ratings of College Mathematics Courses by All Teachers in the Study.

Course Title	Group A	Group B	Group C
College Algebra	(44) 4.09	(49) 4.57	(44) 4.68
Trigonometry	(38) 3.11	(44) 3.57	(46) 4.57
Analytic Geometry	(38) 2.97	(44) 3.48	(45) 4.13
Differential Calculus	(38) 2.18	(43) 2.30	(46) 3.20
Integral Calculus	(38) 2.13	(43) 2.28	(46) 3.11
Elementary Statistics	(12) 3.00	(13) 3.00	(13) 3.54
Teachers Course in Mathematics	(30) 3.47	(32) 3.72	(29) 3.93
College Geometry	(17) 3.00	(26) 3.77	(20) 3.40
Non-Euclidian Geometry	(5) 2.20	(5) 2.80	(7) 3.14
Intermediate Calculus	(8) 1.25	(12) 1.67	(14) 1.93
General Astronomy	(14) 3.00	(14) 2.86	(10) 2.80
Algebraic Structures	(10) 3.80	(14) 3.86	(16) 4.50
Linear Algebra	(11) 4.82	(12) 3.42	(11) 3.82
Theory of Probability	(11) 3.00	(13) 2.54	(20) 3.45
History of Mathematics	(17) 3.65	(24) 3.54	(15) 3.80
Theory of Equations	(22) 2.68	(27) 2.44	(30) 3.60
Differential Equations	(29) 1.79	(38) 1.82	(38) 2.16
Vector Analysis	(9) 2.22	(13) 2.23	(19) 2.42
Mathematical Theory of Statistics	(12) 1.67	(10) 2.00	(13) 2.92
Advanced Calculus	(8) 1.25	(9) 1.44	(16) 2.06

(Annis, 1965, p. 25)

Note: Numbers in parentheses are number of teachers rating the course. Index numbers may be interpreted by referring to rating instructions on Annis' questionnaire (page 73).

Table 9. --Comparison of Ratings by Teachers Who Have and Have Not Taught Modern Mathematics.

Course Title	Teachers Who Have Taught Modern Mathematics	Teachers Who Have Not Taught Modern Mathematics
College Algebra	(70) 4.29	(67) 4.63
Trigonometry	(67) 2.44	(61) 2.41
Analytic Geometry	(67) 3.52	(60) 3.48
Differential Calculus	(67) 2.57	(60) 2.62
Integral Calculus	(67) 2.52	(60) 2.55
Elementary Statistics	(23) 3.04	(15) 3.40
Teachers' Course in Mathematics	(47) 3.55	(44) 3.86
College Geometry	(39) 3.54	(24) 3.29
Non-Euclidean Geometry	(10) 2.80	(7) 2.71
Intermediate Calculus	(20) 2.00	(14) 1.21
General Astronomy	(21) 2.76	(16) 3.06
Algebraic Structures	(26) 4.35	(14) 3.64
Linear Algebra	(25) 3.80	(11) 3.73
Theory of Probability	(28) 3.25	(16) 2.75
History of Mathematics	(28) 3.50	(28) 3.79
Theory of Equations	(46) 3.20	(33) 2.91
Differential Equations	(59) 1.59	(46) 1.91
Vector Analysis	(27) 2.56	(14) 1.86
Mathematical Theory of Statistics	(23) 2.30	(12) 2.08
Advanced Calculus	(19) 1.95	(14) 1.36

(Annis, 1965, p. 24)

Note: Numbers in parentheses are number of teachers rating the course. Index numbers may be interpreted by referring to rating instructions on Annis' questionnaire (page 73).

VITA

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Master of Arts

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