/38

MANAGEMENT OF IRRIGATION AND DRAINAGE SYSTEMS:
INTEGRATED PERSPECTIVES

Proceedings of the 1993 NATIONAL CONFERENCE
ON IRRIGATION AND DRAINAGE ENGINEERING
Park City, Utah
July 21-23, 1993

Sponsored by the
Irrigation and Drainage Division
of the American Society of Civil Engineers

Edited by Richard G. Allen ***and Christopher M.U. Neale***

Published by the
American Society of Civil Engineers
345 East 47th Street
New York, New York 10017-2398

OPTIMAL CONTAMINANT PLUME MANAGEMENT WITH US/WELLS

Alaa H. Aly[1] and *Richard C. Peralta*[2]

Abstract

A micro-computer based software package developed at Utah State University for computing optimal pumping strategies for well systems (US/WELLS) is demonstrated. US/WELLS is used to determine the optimal time-varying sequence of extraction and injection rates when only limited data is available. The software determines the extraction/injection rates, in pre-specified locations, needed for immobilizing and/or extracting a groundwater contaminant plume. In the optimization problem, the objective function can be either to minimize the extraction/injection rates needed (linear) or to minimize the hydraulic power used for lifting water (quadratic). In either case, different weights can be assigned to emphasize any time period. Gradient control pairs of observation wells are placed around the perimeter of the plume to assure that final hydraulic gradients are toward the center of the plume.

Introduction

US/WELLS is a simulation/optimization (S/O) model. US/WELLS combines: (1) detailed simulation of the effect of extraction or injection of groundwater on resulting hydraulic heads and gradients and (2) operations-research model formulation and solution to determine the optimal distribution of extraction and/or injection in space and time.

US/WELLS consists of two modules. The first, the simulation module, is available in two different formulations, deterministic and stochastic. The simulation module uses analytical solutions to determine the influence of extraction or injection at specified well locations on the groundwater system. The second, the optimization module, employs linear, quadratic, or non-linear programming to determine the optimal magnitudes of extraction and injection rates for the specified locations. In this paper, only the deterministic model is presented and discussed.

Assumptions

The simulation module in US/WELLS uses the Theis (1935) equation to determine transient effect of pumping on groundwater hydraulic head. Because of the use of the Theis equation, US/WELLS is most suitable for homogeneous isotropic confined aquifers. However, the effect of anisotropy of the hydraulic conductivity can be approximately considered. Furthermore, the model can be used for unconfined aquifers by cycling. This will be explained later in greater detail. Only a single layer aquifer can be considered. The effect of multiple wells is addressed using superposition, which assumes that the system is linear. The wells are assumed to penetrate the entire depth of the aquifer. Entrance losses to the wells are neglected.

The effect of a river that is in hydraulic connection with the aquifer is addressed using image well theory. Depletion from the river, due to extracting water from the aquifer via wells, is evaluated using an analytical solution (*Glover* and *Balmer*, 1954). The analytical solution considers that the river flows in a straight course which extends for a considerable distance both upstream and downstream from any well location. The river can represent a constant head boundary (such as a lake.) US/WELLS does not consider the effect of nearby interfering impervious boundaries.

US/WELLS employs GAMS (*Brooke* et. al., 1988) to formulate the optimization problem. MINOS (*Murtagh* and *Saunders*, 1987) is chosen to solve the optimization problem because it has been effective for a wide range of groundwater management applications.

The Simulation Module

The Theis well function is used to predict the influence of extracting or injecting a unit pumping rate on the groundwater system for two time periods. The duration of the two time periods can differ. By using a shorter time step initially and a very long time step later, the user can simulate both transient and eventual steady state conditions in the planning era. Appropriate use of the weighting coefficients (discussed in the optimization module) can permit emphasizing either of the two periods.

The use of the Theis analytical solution is chosen for several reasons. The analytical solution is simple, does not require as much data as finite difference or finite element models, and requires less computer memory and processing time.

An analytical expression is used to evaluate the well function (*Clarke*, 1987). This is used because it gives an accurate approximation to the well function.

In the case where a river exists in the study area, an analytical solution (*Glover* and *Balmer*, 1954) is used to evaluate the river depletion resulting from extraction of water from the aquifer. The term "river depletion" is explained as the decrease in discharge from the aquifer to the river plus the increase in recharge from the river to the aquifer caused by extraction of water via a well. The simulation module calculates the response of river depletion to either extraction or injection from any well. During the process of computing an optimal strategy, the total rate of river depletion for each time period is forced to be between user-specified bounds.

The Optimization Module

The objective function of the optimization module in US/WELLS is generally applicable and easily used for a variety of situations. The user can select either a linear or a quadratic form. The linear objective function is given as,

---

[1] Research Assistant, Dept. of Biological and Irrigation Eng., Utah State Univ., Logan, Utah.

[2] Professor, Dept. of Biological and Irrigation Eng., Utah State Univ., Logan, Utah.

$$MINIMIZE: \quad \sum_{x=1}^{2} \left[C_{E,x} \sum_{j=1}^{J} E_{j,x} + C_{I,x} \sum_{k=1}^{K} I_{k,x} \right]$$

where,

$C_{E,x}$ and $C_{I,x}$ = Cost coefficient or Weight assigned to extraction (E) or Injection (I) rates in the x^{th} time period; $ per L^3/T or dimensionless

$E_{j,x}$ and $I_{k,x}$ = Extraction (E) or injection (I) rate (positive) at well j or k in the x^{th} time period; L^3/T

J and K = Number of extraction (J) or injection (K) wells

Subject to

$$E^L_{j,x} \le E_{j,x} \le E^U_{j,x} \tag{1}$$

$$I^L_{k,x} \le I_{k,x} \le I^U_{k,x} \tag{2}$$

$$h^L_\delta \le h_{\delta,x} \le h^U_\delta \tag{3}$$

$$h_{\delta,x} = h^{non}_{\delta,x} - \sum_{t=1}^{x} \sum_{j=1}^{J} \delta^h_{\delta,j}(x,t) \frac{E_{j,t}}{P_{unit}} + \sum_{t=1}^{x} \sum_{k=1}^{K} \delta^h_{\delta,k}(x,t) \frac{I_{k,t}}{P_{unit}} \tag{4}$$

$$G^L_{\delta_1,\delta_2,x} \le G_{\delta_1,\delta_2,x} \le G^U_{\delta_1,\delta_2,x} \tag{5}$$

$$G_{\delta_1,\delta_2,x} = \frac{h_{\delta_1,x} - h_{\delta_2,x}}{L_{\delta_1,\delta_2}} \tag{6}$$

$$D^L_x \le D_x \le D^U_x \tag{7}$$

$$D_x = \sum_{t=1}^{x} \sum_{j=1}^{J} \delta^D_j(x,t) \frac{E_{j,t}}{P_{unit}} - \sum_{t=1}^{x} \sum_{k=1}^{K} \delta^D_k(x,t) \frac{I_{k,t}}{P_{unit}} \tag{8}$$

$$\frac{\sum_{k=1}^{K} I_{k,x} - \sum_{j=1}^{J} E_{j,x}}{\sum_{j=1}^{J} E_{j,x}} \le R_1 \tag{9}$$

$$\frac{\sum_{j=1}^{J} E_{j,x} - \sum_{k=1}^{K} I_{k,x}}{\sum_{j=1}^{J} E_{j,x}} \le R_2 \tag{10}$$

where,

D_x = river depletion rate at time period x; L^3/T
$G_{\delta_1,\delta_2,x}$ = hydraulic gradient between locations δ_1 and δ_2 at time period x; L/L
$h_{\delta,x}$ = hydraulic head at head control location δ at time period x; L
$h_{\delta,x}^{non}$ = unmanaged head at location δ at time period x; L
L_{δ_1,δ_2} = distance between locations δ_1 and δ_2; L
P_{unit} = unit pumping used to generate the influence coefficients; L^3/T
R_1 = maximum allowed difference between total injection and total extraction, defined as a fraction of total extraction (maximum ratio of imported water); dimensionless
R_2 = maximum allowed difference between total extraction and total injection, defined as a fraction of total extraction (maximum ratio of exported water); dimensionless
$\delta^h_{\delta,j}(x,t)$ = influence of unit pumping at extraction location j in time period t on head at location δ at time period x; L per L^3/T
$\delta^h_{\delta,k}(x,t)$ = influence of unit pumping at injection location k in time period t on head at location δ at time period x; L per L^3/T
$\delta^D_j(x,t)$ = influence of unit pumping at extraction location j in time period t on depletion from the river at time period x; L^3/T per L^3/T
$\delta^D_k(x,t)$ = influence of unit pumping at injection location k in time period t on depletion from the river at time period x; L^3/T per L^3/T
L and U = superscripts denoting Lower and Upper bounds, respectively;
δ = subscript denoting a head control location. This includes all extraction and injection wells in addition to head control locations.
x = subscript denoting time period;

Equations 1 and 2 state that extraction or injection rate at any well must be within user-specified bounds (lower and upper limits).

Equation 3 states that hydraulic head at any injection, extraction, or observation well must be within user-specified lower and upper bounds. For example, a lower bound may be used to maintain adequate saturated thickness. An upper bound may be used to prevent surface flooding or to eliminate the need for pressurized injection. These lower and upper bounds can differ for different locations. The bounds are the same for both time periods. Equation 4 is the summation expression used to evaluate the head at location δ at time period x.

Equation 5 states that hydraulic gradient between any gradient control pair of head control locations at any time period must be within user-specified bounds. This can ensure that water is moving only in the desired direction. The bounds can be different for different time periods. This constraint is useful, for example, when US/WELLS is used for groundwater contaminant plume immobilization or for any situation where hydraulic gradient control is desired.

Equation 7 states that depletion from the river must be within user-specified bounds (lower and upper limits.) This is only applicable if a river exists in the considered system. Equation 8 is the summation expression used to evaluate the river depletion at the end of time period x.

Equations 9 and 10 state, respectively, that total import and export of water can be controlled to be within a user-specified range. The optimization module can optionally prevent import or export of water or both. If no import or export of water is allowed, the total optimal extraction must equal the total optimal injection ($R_1 = R_2 = 0$).

The Unconfined Aquifer Case

The difficulty of modelling an unconfined aquifer arises from the fact that the saturated thickness of the aquifer changes with extraction or injection. Thus, the transmissivity of the aquifer changes and the assumption of system linearity can become invalid. The following procedure describes the use of US/WELLS for unconfined aquifers.

1. Consider the saturated thickness at any point to equal the initial saturated thickness.
2. Run US/WELLS.
3. Compare the resulting optimal heads (and their saturated thicknesses) with the values used in step 1. If the difference in transmissivity is within 10% and the difference in the optimal pumping values is less than 5% then quit. Otherwise compute the saturated thickness at any point to be equal to that resulting from the optimal head, and go to step 2.

Discussion of the Objective Function

The objective function shown above is linear. US/WELLS can, optionally, use a quadratic objective function. That is; to minimize

$$\sum_{x=1}^{2} [\ CC_{E,x} \sum_{j=1}^{J} E_{j,x}\, H_{j,x} + C_{E,x} \sum_{j=1}^{J} E_{j,x} + C_{I,x} \sum_{k=1}^{K} I_{k,x}]$$

where,

$H_{j,x}$ = dynamic lift. The difference between ground surface elevation and optimal potentiometric head resulting at extraction well (just outside the casing of the well) j at the end of the x^{th} time period; L

$CC_{E,x}$= weight assigned to the power used for extraction in the x^{th} time period; $ per L^4/T.

The weighting factors can be used to emphasize different criteria and different time periods. For example, assume a problem of minimizing the total extraction using the linear objective function. If the second time period is chosen to be much longer than the first time period and the weights assigned to extraction and injection in the second time period are larger than those used for the first time period, then the solution will tend to minimize steady state extraction/injection rates and less attention will be given to the short-term transient rates.

If the intent is to maximize steady extraction subject to bounds on heads, then a weight of zero can be given to both extraction and injection in the first time period and injection in the second time period. For example, US/WELLS will formulate the objective function to minimize

$$-1 * \sum_{j=1}^{J} E_{j,t_2}$$

Application of US/WELLS to a Contaminant Plume Management Problem

US/WELLS can be used to determine the optimal time-varying sequence of extraction and injection of water in pre-specified locations needed for immobilizing a groundwater contaminant plume. In this example, the user specifies potential locations of extraction and injection wells around the contaminant plume. US/WELLS will then determine the extraction/injection rates from different wells and for different time periods. If the user cannot decide if a certain well should be used for extraction or injection, he can locate one of each at the same location. US/WELLS will then determine either an extraction or an injection rate, or neither, for that location.

In this problem, 4 extraction wells are placed outside the contaminant plume in order to achieve immobilization of the plume in the first time period. In the second time period, 3 extraction wells are placed inside the plume in order to extract the contaminated water from the plume. The first group of extraction wells are inactive in the second time period while the second group of extraction wells are inactive in the first time period. This strategy is only for illustrative purposes. It is quite feasible to capture the plume using the internal extraction wells in the first time period.

For this situation, the objective function can be either to minimize the extraction/injection rates needed (linear) or to minimize the hydraulic power used for lifting water (quadratic.) In either case, different weights can be assigned to emphasize any time period.

The gradient constraint is very important in this situation. Gradient control pairs should be placed around the perimeter of the plume to assure that final hydraulic gradients are towards the center of the plume. An optional constraint assumes that neither export nor import of water is allowed.

Figure (1) shows the hypothetical study area and the proposed well system.

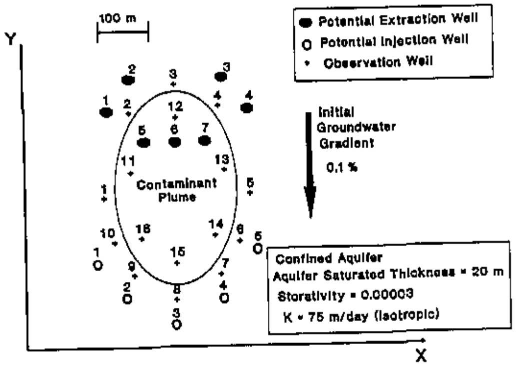
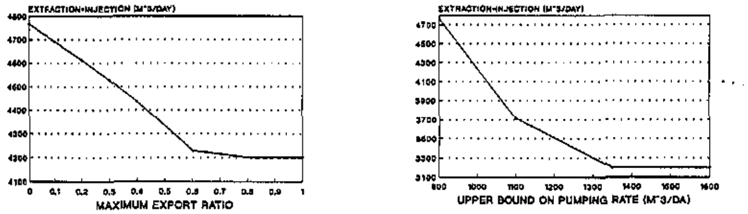


Figure 1. The hypothetical study area for the contaminant plume management problem

Several scenarios have been tested. When the linear objective function is used, the optimization matrix includes 933 non-zero elements. It includes 947 non-zero elements when the quadratic objective function is used. In the following discussion, the linear objective function is used.

Figure 2(a) shows the effect of changing the maximum allowed export ratio (Equation 10) on the total optimal pumping. For this problem, as the maximum export ratio increases, total optimal pumping decreases. This is explained by the fact that when no export of water is allowed, the total injection is increased only to prevent export of water. When some export of water is allowed, the optimal injection decreases until the export ratio becomes about 0.8. At this point, the optimal injection is needed to control the hydraulic gradients. Further allowed export of water will not improve the optimal strategy. An export ratio of 1.0 will mean that no injection is needed. However, when the maximum export ratio is 0.8 or higher, the optimal pumping strategy does not change because some injection have to be used to minimize total pumping. When the maximum import ratio is increased, the optimal pumping strategy does not change because no import of water is desired for this problem.

Figure 2(b) shows the effect of changing the upper bound on pumping (Equations 1 and 2) on the total optimal groundwater pumping (extraction+injection) needed. When the upper limit on pumping is 900 m^3/day, 4 extraction wells and 4 injection wells are needed in the first time period. When the upper limit on pumping is increased, only 1 extraction well and 1 injection well are needed in the first time periods. Total optimal pumping decreases as the upper limit on pumping increases.



(a)Effect of Export Ratio (b)Effect of Upper Bound

Figure 2. Changes in Total Extraction+Injection

Sensitivity Analysis

It is important to notice that an optimal pumping strategy predicted by US/WELLS is sensitive to the value of the hydraulic conductivity. For example, when the hydraulic conductivity is reduced by one half, total optimal pumping is reduced from 4766 to 1493 m^3/day (69% reduction). The optimal pumping strategy is not as sensitive to the value of the specific yield. An increase in the specific yield from 3x10^{-5} to 3x10^{-3} results in increasing total optimal pumping from 4766 to 4786 m^3/day (0.4% increase).

When the quadratic objective function is used, total extraction+injection increased from 4766 to 4780 m^3/day. However the distribution of the pumping wells differs considerably between the quadratic and the linear objective function's strategies.

Summary

A simulation/optimization model is presented. Several constraints and two forms of the objective function can be used for different groundwater management problems. The model is used to solve a contaminant plume management problem. Results are presented for different scenarios.

References

Brooke, A., Kendrick, D., and Alexander, M. 1988. "GAMS: A USER'S GUIDE," The Scientific Press.

Clarke, D. 1987. "Microcomputer programs for groundwater studies," Elsevier Science Publishers.

Glover, R.E. and Balmer, G.G., River depletion resulting from pumping a well near a river, *Transactions of the American Geophysical Union*, Vol. 35, No. 3, June 1954.

Murtagh, B.A. and Saunders, M.A. 1987. "MINOS 5.1 User's Guide," Report SOL 83-20R, December 1983, revised January 1987, Stanford University.

Theis, C. V., The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage, *Transactions of the American Geophysical Union*, 16, 519-524, 1935

138

LLOYD C. FOWLER
385 VIA EL ENCANTADOR
SANTA BARBARA CA 93111

805-964-5430

26 October 1992

Alaa H. Aly
Richard C. Peralta
Department of Biological and Irrigation Engineering
Utah State University
Logan UT 84322-4105

Thank you for submitting an abstract of a paper on "Optional Contaminant Plume Management with US/WELLS" to be presented at the National Conference on Irrigation and Drainage Engineering in Park City Utah, to be held 21-23 July 1993. Your paper has been accepted and will be part of a Session on "Ground Water Management."

Now is the time to begin to prepare your paper. It will be printed in the advance Proceedings of the Conference. ASCE headquarters has mailed, or will soon mail, you instructions regarding the preparation of your paper. The page limit will be eight pages, single spaced. A copy of the final draft of the paper must be given to Rick Allen, Program Chairman, located in your Department at Utah State University, by 15 January 1993. Please send me a copy at the same time. The paper will be reviewed and comments returned to you by 28 February 1993. Final camera ready papers must be received by Rick Allen prior to 6 April 1993.

I look forward to receipt of your paper and to your presentation at the Conference.

Lloyd C Fowler

Lloyd C. Fowler, P. E.
Ground Water Management
Session Contact

cc Rick Allen