

US/WELLS^D

Extraction/Injection Well System for
Optimal Groundwater Management

Beta Test Release 1.0
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User's Manual

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The authors do not warrant the US/WELLS software for any specific purpose and do not assume any liability resulting from use of the software. The US/WELLS formulation includes commonly reported procedures and algorithms for simulating groundwater well systems. However, misapplication of US/WELLS may involve parameter combinations that violate assumptions or may require interpretations beyond the limits of various algorithms. The resulting incorrect or inaccurate simulations and numerical instabilities cannot be anticipated by the developers. US/WELLS is designed to compute optimal rates of extraction and injection and resulting optimal heads and gradients. Because of uncertain knowledge of groundwater aquifer parameters and site descriptors, one cannot be certain that predicted optimal heads and gradients will actually occur in the simulated field. However, a chance-constrained formulation is provided to help with that situation.

This publication and the US/WELLS software might contain technical inaccuracies. Changes are periodically made to both and are incorporated in new releases.

Comments and helpful criticism are appreciated. For the present, technical support of the US/WELLS software is provided via telephone or written responses by:

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INTRODUCTION

Presented is a computer program, US/WELLS, that is designed to solve several types of groundwater management problems. The acronym US/WELLS stands for Utah State WELL System. This decision-support tool is usable by persons slightly familiar with groundwater hydraulics. It can be valuable for practical management of groundwater systems that satisfy certain criteria.

US/WELLS is a simulation/optimization (S/O) model. S/O models, generally, calculate optimal groundwater pumping strategies for user-specified settings and management objectives. US/WELLS combines: (1) detailed simulation of the effect of extraction or injection of groundwater on resulting hydraulic heads and gradients and (2) operations research model formulation and solution to determine the optimal distribution of extraction and/or injection in space and time. US/WELLS consists of two modules. The first, the simulation module, is available in two different formulations, deterministic and stochastic. The simulation module uses analytical solutions to determine the influence of extraction or injection at specified well locations on the groundwater system. The second module, the optimization module, employs linear, quadratic, or non-linear programming to determine the optimal magnitudes of extraction and injection rates for the specified locations.

ASSUMPTIONS

US/WELLS is most suitable for homogeneous isotropic confined aquifers. However, the effect of anisotropy of the hydraulic conductivity can be approximately considered. Furthermore, the model can be used for unconfined aquifers by cycling. This will be explained later in greater detail. Only a single layer aquifer can be considered. The effect of multiple wells is addressed using superposition, which assumes that the system is linear. The wells are assumed to penetrate the entire depth of the aquifer. Entrance losses to the wells are neglected.

The effect of a river that is in hydraulic connection with the aquifer is addressed using the image well theory. Depletion from the river, due to extracting water from the aquifer via wells, is evaluated using an analytical solution. The analytical solution considers that the river flows in a straight course which extends for a considerable distance both upstream and downstream from any well location. The river can represent a constant head boundary (such as a lake.) US/WELLS does not consider the effect of nearby interfering impervious boundaries.

For the stochastic module, the drawdown at any point in the groundwater system is assumed to follow a gaussian (normal) distribution.

US/WELLS uses GAMS (1988) to solve the optimization problem. A demonstration version of GAMS is provided as a part of the package. This version can solve problems having up to 1000 non-zeros in the optimization matrix. Versions able to address larger problems are also available.

INSTALLATION

To install US/WELLS on a hard disk, follow these steps:

1. Insert the diskette named US/WELLS (1) in the appropriate drive.
2. Make this drive the current drive. For example by typing A: or B:
3. At the DOS prompt, type the following

GO (Source Drive) (Destination Drive) (Destination Directory)

The installation program will prompt the user to insert the second diskette, named US/WELLS (2), when it is ready.

For example, the command

GO B C USWELLS

can be issued to install US/WELLS from diskettes in drive B to the directory named USWELLS on the hard disk (drive C.) The installation program will create the specified directory under the root directory of the hard disk. Then, the program will install US/WELLS on the created directory. US/WELLS will need about 2 Mega Bytes of free space on the hard disk.

TUTORIAL

To run US/WELLS, the user must first make the directory in which US/WELLS is stored the current directory. Then he must issue the following command

DOIT (/d or /s) (input file name) (output file name)

The switch (/d or /s) is used to determine which model is to be used. /d means that the deterministic version is to be used and /s means the stochastic version is to be used. The program is not case sensitive. That is, either /d or /D can be used.

To run the tutorial, make the US/WELLS directory current and use the following command

DOIT /d TRYME.DAT RESULTS.OUT

In response, the model runs the deterministic version using the data in the TRYME.DAT file and creates a file named RESULTS.OUT for output of results. Figure (1) shows the contents of the file RESULTS.OUT. Appendix (A) includes the input file format and Appendix (B) includes the output file explanation.

The data in TRYME.DAT poses a simple simulation problem. This problem does not actually require optimization. This can be easily realized from the fact that the lower and upper bounds on extraction and injection rates are equal. This is done here to speed processing and to demonstrate that US/WELLS can be used for computation of simple drawdown prediction problems as well as optimization problems.

VALUE OF OBJECTIVE FUNCTION -330000.00				
OPTIMAL EXTRACTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	20000.00	20000.00	20000.00	-1.000
2	10000.00	10000.00	10000.00	-1.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	20000.00	20000.00	20000.00	-10.000
2	10000.00	10000.00	10000.00	-10.000
=====				
OPTIMAL INJECTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	0.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	0.00	0.000
=====				
OPTIMAL HEADS AT OBSERVATION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	67.70	80.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	67.13	80.00	0.000
=====				
OPTIMAL HEADS AT EXTRACTION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	61.53	80.00	0.000
2	50.00	69.02	80.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	60.95	80.00	0.000
2	50.00	68.44	80.00	0.000
=====				
OPTIMAL HEADS AT INJECTION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	68.72	80.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	68.15	80.00	0.000

FIGURE (1) Contents of RESULTS.OUT

||

00070

00100

01100

00100

|

||

BACKGROUND THEORY

THE DETERMINISTIC SIMULATION MODULE

The Theis well function is used to predict the influence of extracting or injecting a unit pumping rate on the groundwater system for two time periods. The duration of the two time periods can differ. By using a shorter time step initially and a very long time step later, the user can simulate both transient and eventual steady state conditions in the planning era. appropriate use of the weighting coefficients (discussed in the optimization module) can permit emphasizing either of the two periods.

The use of the Theis analytical solution is chosen for several reasons. The analytical solution is simple, does not require as much data as finite difference or finite element models, and requires less computer memory and processing time.

An analytical expression for evaluation of the well function is used. The analytical expression can be found in *Clarke* (1987). This expression is used because it gives an accurate approximation to the well function.

In the case where a river exists in the study area, image well theory is used. Also, an analytical solution (*Glover and Balmer*, 1954) is used to evaluate the river depletion resulting from extraction of water from the aquifer. The term "river depletion" is explained as the decrease in discharge from the aquifer to the river plus the increase in recharge from the river to the aquifer caused by extraction of water via a well. The simulation module calculates the response of river depletion to either extraction or injection from any well. During the process of computing an optimal strategy, the total rate of river depletion for each time period is forced to be between user-specified bounds.

THE STOCHASTIC SIMULATION MODULE¹

US/WELLS can also use a stochastic version of the Theis equation (*Aly and Peralta*, unpublished study). In the stochastic version, the hydraulic conductivity of the aquifer material is represented by an average value and a standard deviation. The hydraulic conductivity is considered to follow a lognormal distribution. A level of reliability is then specified (for example 90%). The level of reliability states the model's confidence in the resulting heads. If it is desired to lower the water level then the heads are estimated such that there is at least 90% chance that the actual heads are less than the heads calculated by the optimization module.

The stochastic module does not consider the anisotropy of the hydraulic conductivity. Nor is it capable of considering the effect of a river in the study area.

¹ US/WELLS^D only includes the deterministic version.

THE OPTIMIZATION MODULE

The objective function of the optimization module in US/WELLS is generally applicable and easily used for a variety of situations. The user can select either a linear or a quadratic form. The linear objective function is given as,

Minimize

$$\sum_{x=1}^{x=2} [W_{E,x} \sum_{j=1}^J E_{j,x} + W_{I,x} \sum_{k=1}^K I_{k,x}]$$

where,

$W_{E,x}$ and $W_{I,x}$ = Cost coefficient or Weight assigned to extraction (E) or Injection (I) rates in the x^{th} time period; \$ per L^3/T or dimensionless

$E_{j,x}$ and $I_{k,x}$ = Extraction (E) or injection (I) rate at well j (or k) in the x^{th} time period; L^3/T

J and K = Number of extraction (J) or injection (K) wells

Subject to

1. Hydraulic gradient between any gradient control pair of wells at any time period must be within user-specified bounds. This can ensure that water is moving only in the desired direction. The maximum value can differ for each gradient control pair and time period. This constraint is useful, for example, when US/WELLS is used for groundwater contaminant plume immobilization or for any situation where hydraulic gradient control is desired.
2. Extraction or injection rate at any well must be within user-specified bounds (lower and upper limits.)
3. Hydraulic head at any injection, extraction, or observation well must be within user-specified lower and upper bounds. For example, a lower bound may be used to maintain adequate saturated thickness. An upper bound may be used to prevent surface flooding or to eliminate the need for pressurized injection. These lower and upper bounds can differ for different locations. The bounds are the same for both time periods.
4. Total import or export of water can be controlled to be within a user-specified range. The user can also completely prevent import or export of water or both. If no import or export of water is allowed, the total optimal extraction must equal the total optimal injection.

5. Depletion from the river must be within user-specified bounds (lower and upper limits.) This is only applicable if a river exists in the considered system.
6. For the stochastic module, constraint (3) is changed such that the probability that the actual change in head at any point in the groundwater system is greater than the change calculated by model is at least equal to the reliability level specified by the user.

THE UNCONFINED AQUIFER CASE

The difficulty of modelling an unconfined aquifer arises from the fact that the saturated thickness of the aquifer changes with extraction or injection. Thus, the transmissivity of the aquifer changes and the assumption of system linearity can become invalid. The following procedure describes the use of US/WELLS for unconfined aquifers.

1. Consider the saturated thickness at any point to equal the initial saturated thickness.
2. Run US/WELLS.
3. Compare the resulting optimal heads (and their saturated thicknesses) with the values used in step 1. If the difference in transmissivity is within 10% and the difference in the optimal pumping values is less than 5% then quit. Otherwise compute the saturated thickness at any point to be equal to that resulting from the optimal head, and go to step 2.

DISCUSSION OF THE OBJECTIVE FUNCTION

The objective function shown above is linear. US/WELLS can, optionally, use a quadratic objective function. That is to minimize

$$\text{where, } \sum_{x=1}^{x=2} [WW_{E,x} \sum_{j=1}^J E_{j,x} H_{j,x} + W_{E,x} \sum_{j=1}^J E_{j,x} + W_{I,x} \sum_{k=1}^K I_{k,x}]$$

$H_{j,x}$ = dynamic lift. The difference between ground surface elevation and optimal potentiometric head resulting at extraction well j at the end of the x^{th} time period; L

$WW_{E,x}$ = weight assigned to the power used for extraction in the x^{th} time period; \$ per L^4/T .

The weighting factors can be used to emphasize different criteria and different time periods. For example, assume a problem of minimizing the total extraction using the linear objective function. If the second time period is chosen to be much longer than the first time period and the weights assigned to extraction and injection in the second time period are larger than those used for the first time period, then the solution will tend to minimize steady state extraction/injection rates and less attention will be given to the short-term transient rates.

If the intent is to maximize steady extraction subject to bounds on heads, then a weight of zero can be given to both extraction and injection in the first time period and injection in the second time period. For example, US/WELLS will formulate the objective function to minimize

$$-1 * \sum_{j=1}^J E_{j, t_2}$$

EXAMPLES

US/WELLS is applicable to a variety of groundwater management problems. The following examples show some potential uses of US/WELLS.

Contaminant Plume Immobilization and Extraction (Example A)

US/WELLS can be used to determine the optimal time-varying sequence of extraction and injection of water in pre-specified locations needed for immobilizing a groundwater contaminant plume. In this example, the user specifies potential locations of extraction and injection wells around the contaminant plume. US/WELLS will then determine the extraction/injection rates from different wells and for different time periods. If the user cannot decide if a certain well should be used for extraction or injection, he can locate one of each at the same location. US/WELLS will then determine either an extraction or an injection rate, or neither, for that location.

In this example, 4 extraction wells are placed outside the contaminant plume in order to achieve immobilization of the plume in the first time period. In the second time period, 3 extraction wells are placed inside the plume in order to extract the contaminated water from the plume. The first group of extraction wells are inactive in the second time period while the second group of extraction wells are inactive in the first time period. This strategy is only for illustrative purposes. One can, as well, capture the plume using the internal extraction wells in the first time period.

For this situation, the objective function can be either to minimize the extraction/injection rates needed (linear) or to minimize the hydraulic power used for lifting water (quadratic.) In either case, different weights can be assigned to emphasize any time period. In this example, the quadratic objective function is used with higher weights for the second time period.

The gradient constraint is very important in this situation. Gradient control pairs should be placed around the perimeter of the plume to assure that final hydraulic gradients are towards the center of the plume. An optional constraint assumes that neither export nor import of water is allowed.

Figure (2) shows the hypothetical study area and the proposed well system. Input data format is shown in Appendix (A). Figure (3) shows the input data file used for this problem. Figure (4) shows US/WELLS output. An explanation of output file is in Appendix (B). The verification of one of the marginal values is also shown in Appendix (B). The units of the output are the same units used for output. For this example the pumping rates are in m^3 per day. The input units are meters and days. See the comment on units in Appendix (A.)

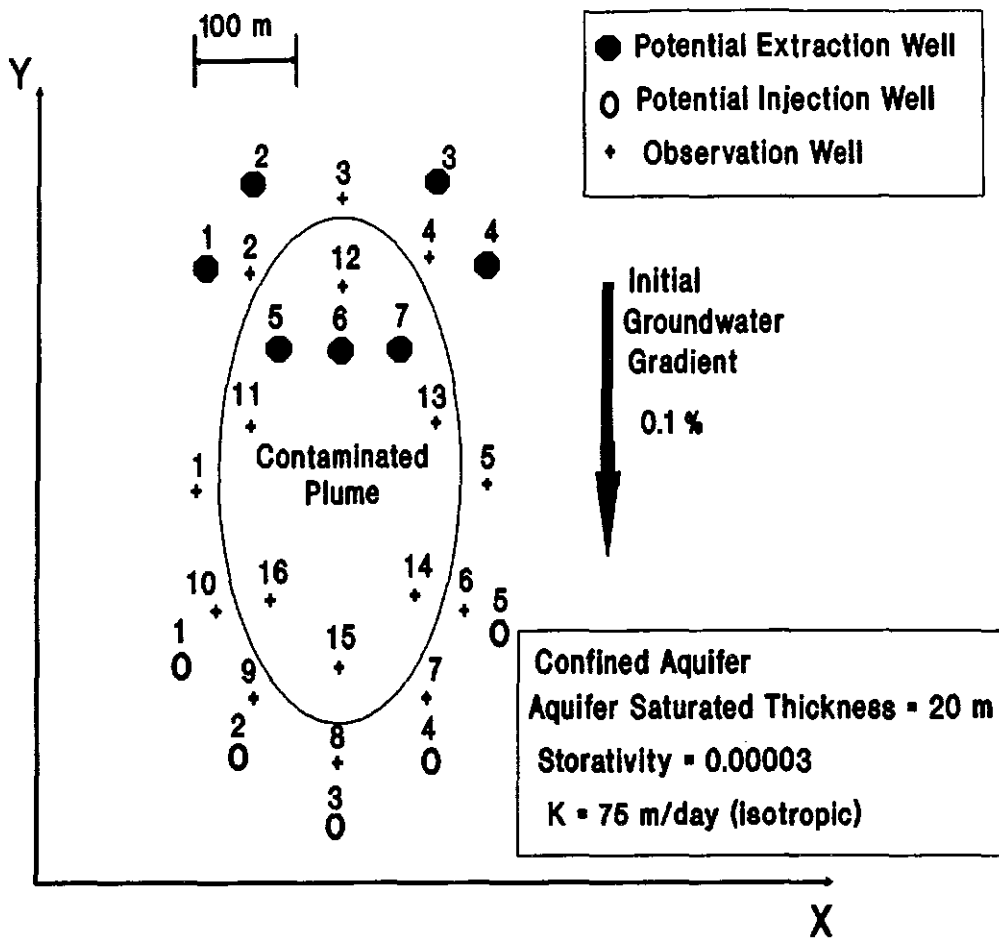


FIGURE 2. The Hypothetical Study Area for Example A

7	5	16	8										
1	160	470	20	30	40	35.88	35.88	35.88	0	900	0	0	0.101
2	210	530	20	30	40	35.92	35.92	35.92	0	900	0	0	0.101
3	390	530	20	30	40	35.92	35.92	35.92	0	900	0	0	0.101
4	440	470	20	30	40	35.88	35.88	35.88	0	900	0	0	0.101
5	230	400	20	30	40	35.80	35.80	35.80	0	0	0	900	0.101
6	300	400	20	30	40	35.80	35.80	35.80	0	0	0	900	0.101
7	370	400	20	30	40	35.80	35.80	35.80	0	0	0	900	0.101
1	150	180	20	30	40	35.58	35.58	35.58	0	900	0	900	0.101
2	200	100	20	30	40	35.50	35.50	35.50	0	900	0	900	0.101
3	300	60	20	30	40	35.45	35.45	35.45	0	900	0	900	0.101
4	400	100	20	30	40	35.50	35.50	35.50	0	900	0	900	0.101
5	450	180	20	30	40	35.58	35.58	35.58	0	900	0	900	0.101
1	150	300	20	30	40	35.70	35.70	35.70	1				
2	210	460	20	30	40	35.86	35.86	35.86	1				
3	300	510	20	30	40	35.92	35.92	35.92	1				
4	390	460	20	30	40	35.86	35.86	35.86	1				
5	450	300	20	30	40	35.70	35.70	35.70	1				
6	420	210	20	30	40	35.61	35.61	35.61	-1				
7	390	140	20	30	40	35.55	35.55	35.55	-1				
8	300	100	20	30	40	35.45	35.45	35.45	-1				
9	210	140	20	30	40	35.55	35.55	35.55	-1				
10	180	210	20	30	40	35.61	35.61	35.61	-1				
11	200	350	20	30	40	35.75	35.75	35.75	1				
12	300	450	20	30	40	35.86	35.86	35.86	1				
13	200	350	20	30	40	35.75	35.75	35.75	1				
14	370	220	20	30	40	35.62	35.62	35.62	-1				
15	300	150	20	30	40	35.57	35.57	35.57	-1				
16	230	220	20	30	40	35.62	35.62	35.62	-1				
0.00003		4	1000		100		50						
75	75		0										
0	0												
1	11		0	0.01		0	0.01						
3	12		0	0.01		0	0.01						
5	13		0	0.01		0	0.01						
6	14		0	0.01		0	0.01						
7	14		0	0.01		0	0.01						
8	15		0	0.01		0	0.01						
9	16		0	0.01		0	0.01						
10	16		0	0.01		0	0.01						
1	1		1		10		10		10				
2	1												
0	0		0										
0	0		0		0								
0	0		0										

FIGURE 3. US/WELLS Input File for Example A

FIGURE 4. US/WELLS Output File for Example A

VALUE OF OBJECTIVE FUNCTION 334668.1				
OPTIMAL EXTRACTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	745.42	900.00	0.000
2	0.00	447.60	900.00	0.000
3	0.00	448.71	900.00	0.000
4	0.00	747.86	900.00	0.000
5	0.00	0.00	0.00	0.000
6	0.00	0.00	0.00	0.000
7	0.00	0.00	0.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	0.00	81.955
2	0.00	0.00	0.00	81.627
3	0.00	0.00	0.00	81.605
4	0.00	0.00	0.00	81.913
5	0.00	426.53	900.00	0.000
6	0.00	883.77	900.00	0.000
7	0.00	428.90	900.00	0.000
=====				
OPTIMAL INJECTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	211.66	900.00	0.000
2	0.00	328.89	900.00	0.000
3	0.00	900.00	900.00	-45.342 < == = explained in appendix B
4	0.00	900.00	900.00	0.000
5	0.00	49.04	900.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	900.00	132.584
2	0.00	293.53	900.00	0.000
3	0.00	900.00	900.00	-4.3E+2
4	0.00	545.67	900.00	0.000
5	0.00	0.00	900.00	132.583
=====				
OPTIMAL HEADS AT OBSERVATION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	30.00	35.69	40.00	0.000
2	30.00	35.54	40.00	0.000
3	30.00	35.60	40.00	0.000
4	30.00	35.55	40.00	0.000
5	30.00	35.70	40.00	0.000
6	30.00	35.79	40.00	0.000
7	30.00	35.92	40.00	0.000
8	30.00	35.88	40.00	0.000
9	30.00	35.84	40.00	0.000
10	30.00	35.77	40.00	0.000
11	30.00	35.65	40.00	0.000
12	30.00	35.60	40.00	0.000
13	30.00	35.65	40.00	0.000
14	30.00	35.79	40.00	0.000
15	30.00	35.88	40.00	0.000
16	30.00	35.77	40.00	0.000

SECOND TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	30.00	35.62	40.00	0.000
2	30.00	35.62	40.00	0.000
3	30.00	35.68	40.00	0.000
4	30.00	35.62	40.00	0.000
5	30.00	35.63	40.00	0.000
6	30.00	35.66	40.00	0.000
7	30.00	35.75	40.00	0.000
8	30.00	35.74	40.00	0.000
9	30.00	35.71	40.00	0.000
10	30.00	35.64	40.00	0.000
11	30.00	35.56	40.00	0.000
12	30.00	35.54	40.00	0.000
13	30.00	35.56	40.00	0.000
14	30.00	35.66	40.00	0.000
15	30.00	35.74	40.00	0.000
16	30.00	35.64	40.00	0.000

=====

OPTIMAL HEADS AT EXTRACTION WELLS

FIRST TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	30.00	35.09	40.00	0.000
2	30.00	35.29	40.00	0.000
3	30.00	35.29	40.00	0.000
4	30.00	35.09	40.00	0.000
5	30.00	35.61	40.00	0.000
6	30.00	35.62	40.00	0.000
7	30.00	35.61	40.00	0.000

SECOND TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	30.00	35.69	40.00	0.000
2	30.00	35.72	40.00	0.000
3	30.00	35.73	40.00	0.000
4	30.00	35.70	40.00	0.000
5	30.00	35.24	40.00	0.000
6	30.00	34.90	40.00	0.000
7	30.00	35.25	40.00	0.000

=====

OPTIMAL HEADS AT INJECTION WELLS

FIRST TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	30.00	35.90	40.00	0.000
2	30.00	36.03	40.00	0.000
3	30.00	36.46	40.00	0.000
4	30.00	36.47	40.00	0.000
5	30.00	35.83	40.00	0.000

SECOND TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	30.00	35.65	40.00	0.000
2	30.00	35.88	40.00	0.000
3	30.00	36.33	40.00	0.000
4	30.00	36.08	40.00	0.000
5	30.00	35.67	40.00	0.000

OPTIMAL HYDRAULIC GRADIENTS					
FIRST TIME PERIOD					
From	To	L.Bound	Optimal	U.Bound	Marginal
1	-> 11	0.00000	0.00055	0.01000	0.000
3	-> 12	0.00000	0.00003	0.01000	0.000
5	-> 13	0.00000	0.00019	0.01000	0.000
6	-> 14	0.00000	0.00000	0.01000	1.17E+7
7	-> 14	0.00000	0.00157	0.01000	0.000
8	-> 15	0.00000	0.00000	0.01000	3.26E+7
9	-> 16	0.00000	0.00087	0.01000	0.000
10	-> 16	0.00000	0.00000	0.01000	1.17E+7
SECOND TIME PERIOD					
From	To	L.Bound	Optimal	U.Bound	Marginal
1	-> 11	0.00000	0.00082	0.01000	0.000
3	-> 12	0.00000	0.00241	0.01000	0.000
5	-> 13	0.00000	0.00027	0.01000	0.000
6	-> 14	0.00000	0.00014	0.01000	0.000
7	-> 14	0.00000	0.00115	0.01000	0.000
8	-> 15	0.00000	0.00000	0.01000	2.88E+8
9	-> 16	0.00000	0.00081	0.01000	0.000
10	-> 16	0.00000	0.00000	0.01000	0.000

FIGURE 4. US/WELLS Output File for Example A

Contaminant Plume Stochastic Immobilization and Extraction (Example B)

In this example, the same situation discussed in example (A) is considered. The only change is that the random variability of the aquifer material is considered and the reliability level for the resulting optimal heads is 90%. That is, at any extraction well, where a lower bound is imposed on head, there is at least a 90% chance that the actual field head is higher than the optimal calculated head. At any injection well, where an upper bound on head is used, there is at least a 90% chance that the actual field head is lower than the optimal calculated head. At any observation well, where it is desired to change the head, there is at least a 90% chance that the actual change in head is larger than the optimal calculated head.

Figure (5) shows the input data file used for this problem. The new data include the standard deviation of the hydraulic conductivity and the desired reliability level. Figure (6) shows US/WELLS output.

3	5	16	8									
1	230	400	20	30	40	35.80	35.80	35.80	0	0	0	2000 0.5
2	300	400	20	30	40	35.80	35.80	35.80	0	0	0	2000 0.5
3	370	400	20	30	40	35.80	35.80	35.80	0	0	0	2000 0.5
1	150	180	20	30	40	35.58	35.58	35.58	0	0	0	2000 0.5
2	200	100	20	30	40	35.50	35.50	35.50	0	0	0	2000 0.5
3	300	60	20	30	40	35.45	35.45	35.45	0	0	0	2000 0.5
4	400	100	20	30	40	35.50	35.50	35.50	0	0	0	2000 0.5
5	450	180	20	30	40	35.58	35.58	35.58	0	0	0	2000 0.5
1	150	300	20	30	40	35.70	35.70	35.70	1			
2	210	460	20	30	40	35.86	35.86	35.86	1			
3	300	510	20	30	40	35.92	35.92	35.92	1			
4	390	460	20	30	40	35.86	35.86	35.86	1			
5	450	300	20	30	40	35.70	35.70	35.70	1			
6	420	210	20	30	40	35.61	35.61	35.61	-1			
7	390	140	20	30	40	35.55	35.55	35.55	-1			
8	300	100	20	30	40	35.45	35.45	35.45	-1			
9	210	140	20	30	40	35.55	35.55	35.55	-1			
10	180	210	20	30	40	35.61	35.61	35.61	-1			
11	200	350	20	30	40	35.75	35.75	35.75	1			
12	300	450	20	30	40	35.86	35.86	35.86	1			
13	200	350	20	30	40	35.75	35.75	35.75	1			
14	370	220	20	30	40	35.62	35.62	35.62	-1			
15	300	150	20	30	40	35.57	35.57	35.57	-1			
16	230	220	20	30	40	35.62	35.62	35.62	-1			
0.00003		4	1000		100		50					
75	75		0									
0	0											
1	11	0		-0.01		0	0.01					
3	12	0		-0.01		0	0.01					
5	13	0		-0.01		0	0.01					
6	14	0		-0.01		0	0.01					
7	14	0		-0.01		0	0.01					
8	15	0		-0.01		0	0.01					
9	16	0		-0.01		0	0.01					
10	16	0		-0.01		0	0.01					
0	0		0		1		10		10			
2	1											
0	0		0									
0	0		0		0							
30	2e-6		0.1									

FIGURE 5. US/WELLS Input File for Example B

FIGURE 6. US/WELLS Output File for Example B

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FIGURE 6. US/WELLS Output File for Example B

Groundwater Pumping Supply Management (Example C)

Consider the situation in which a well owner wishes to know how much he/she can pump from his/her well without causing any harmful consequences. For example, the resulting drawdowns at two neighboring extraction wells are not to exceed prespecified values. Also, the depletion of flow from the river flowing by the farm should not exceed a specified rate. Assume that the aquifer is unconfined.

US/WELLS can be used with the objective of maximizing steady state extraction from this well. For this purpose, a weight of zero is assigned to injection in both time periods and to extraction in the first time period. An index of -1 is to be assigned to the objective function to change it from minimize to maximize. The fixed extraction rate (i.e. Q) from the other two wells can be assured within US/WELLS by assigning Q to both upper and lower bounds of extraction at both wells.

At least one injection well has to be specified for US/WELLS. In this case, to prevent US/WELLS from injecting water into the aquifer, the upper and lower bounds on injection at this well are set to zero. Also, at least one gradient control pair of observation wells has to be specified to US/WELLS. However, the user can, optionally, disable the gradient control constraint by giving any negative value for the upper limit on gradient.

Because this is an unconfined aquifer (a non-linear system), cycling is employed. Figure (7) shows the hypothetical study area. Figures (8) and (9) show the input data file and US/WELLS output file for the first cycle, respectively. Figures (10) and (11) show the input data file and US/WELLS output file for the second cycle, respectively.

This example shows how US/WELLS is used for an unconfined aquifer. Figure (10) shows the revised input file for the second cycle. The final optimal heads resulting from the optimal pumping strategy, obtained in cycle 1, are used as initial saturated thicknesses for the input file in the second cycle. It can be easily recognized that the resulting optimal heads from the second cycle are only slightly different from those resulting from the first cycle. Furthermore, optimal pumping values changed less than 1%. We can stop cycling. However, if greater accuracy is desired, cycling can be continued.

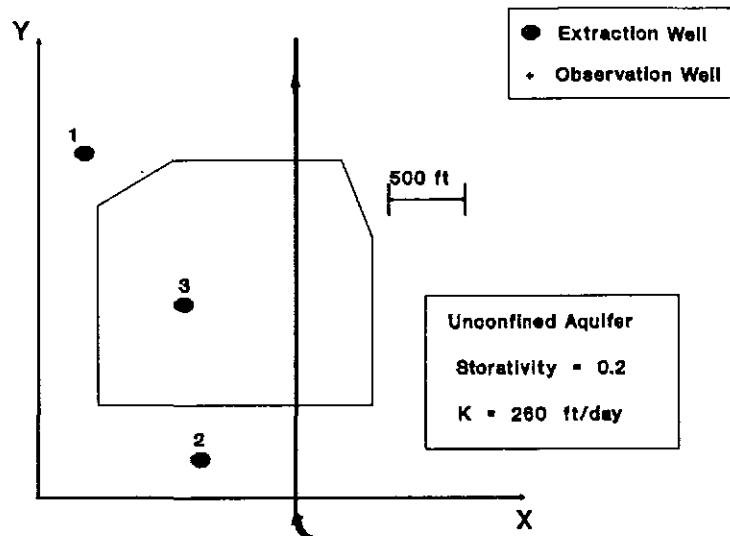


FIGURE 7. The Hypothetical Study Area for Example C

3	1	2	1											
1	375	3100	64.4	50	80	64.4	64.4	64.4	6000	6000	6000	6000	0.5	
2	1000	375	71.0	50	80	71.0	71.0	71.0	5000	5000	5000	5000	0.5	
3	1125	375	67.2	50	80	67.2	67.2	67.2	0	8000	0	8000	0.5	
1	100	100	70.0	50	80	70.0	70.0	70.0	0	0	0	0	0.5	
1	120	120	69.0	50	80	69.0	69.0	69.0	1					
2	130	130	69.0	50	80	69.0	69.0	69.0	1					
0.2	3	200	1000	90										
260	260	0												
-1	-1													
1	2	0	-1	0	-1									
0	0	0	1	0	1									
1	-1													
1	1750	220												
0	14000	0	14000											
0	0	0												

FIGURE 8. US/WELLS Input File for Example C (First Cycle)

FIGURE 9. US/WELLS Output File for Example C (First Cycle)

VALUE OF OBJECTIVE FUNCTION -16029.75				
OPTIMAL EXTRACTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	6000.00	6000.00	6000.00	0.001
2	5000.00	5000.00	5000.00	0.001
3	0.00	0.00	8000.00	0.001
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	6000.00	6000.00	6000.00	-0.113
2	5000.00	5000.00	5000.00	-0.015
3	0.00	5029.75	8000.00	0.000
=====				
OPTIMAL INJECTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	0.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	0.00	0.000
=====				
OPTIMAL HEADS AT OBSERVATION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	68.99	80.00	0.000
2	50.00	68.99	80.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	68.91	80.00	0.000
2	50.00	68.91	80.00	0.000
=====				
OPTIMAL HEADS AT EXTRACTION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	63.98	80.00	0.000
2	50.00	70.68	80.00	0.000
3	50.00	67.12	80.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	63.90	80.00	0.000
2	50.00	70.54	80.00	0.000
3	50.00	66.73	80.00	0.000

OPTIMAL HEADS AT INJECTION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	69.99	80.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	69.91	80.00	0.000
=====				
OPTIMAL RATES OF STREAM DEPLETION				
FIRST TIME PERIOD				
L.Bound	Optimal	U.Bound	Marginal	
0.00	1882.00	14000.00	0.000	
SECOND TIME PERIOD				
L.Bound	Optimal	U.Bound	Marginal	
0.00	14000.00	14000.00	-1.092	

FIGURE 9. US/WELLS Output File for Example C (First Cycle)

```

3 1 2 1
1 375 3100 63.90 50 80 64.4 64.4 64.4 6000 6000 6000 6000 0.5
2 1000 375 70.54 50 80 71.0 71.0 71.0 5000 5000 5000 5000 0.5
3 1125 375 66.73 50 80 67.2 67.2 67.2 0 800 0 8000 0.5

1 100 100 69.99 50 80 70.0 70.0 70.0 0 0 0 0 0.5

1 120 120 68.91 50 80 69.0 69.0 69.0 1
2 130 130 68.91 50 80 69.0 69.0 69.0 1

0.2 3 200 1000 90

260 260 0

-1 -1

1 2 0 -1 0 -1

0 0 0 10 0 10

1 -1

1 1750 220

0 14000 0 14000

0 0 0

```

FIGURE 10. US/WELLS Input File for Example C (Second Cycle)

FIGURE 11. US/WELLS Output File for Example C (Second Cycle)

VALUE OF OBJECTIVE FUNCTION -16035.15				
OPTIMAL EXTRACTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	6000.00	6000.00	6000.00	0.001
2	5000.00	5000.00	5000.00	0.001
3	0.00	0.00	8000.00	0.001
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	6000.00	6000.00	6000.00	-0.114
2	5000.00	5000.00	5000.00	-0.016
3	0.00	5035.15	8000.00	0.000
=====				
OPTIMAL INJECTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	0.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	0.00	0.000
=====				
OPTIMAL HEADS AT OBSERVATION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	68.99	80.00	0.000
2	50.00	68.99	80.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	68.91	80.00	0.000
2	50.00	68.91	80.00	0.000
=====				
OPTIMAL HEADS AT EXTRACTION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	63.98	80.00	0.000
2	50.00	70.68	80.00	0.000
3	50.00	67.12	80.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	63.89	80.00	0.000
2	50.00	70.53	80.00	0.000
3	50.00	66.73	80.00	0.000

OPTIMAL HEADS AT INJECTION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	69.99	80.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	69.91	80.00	0.000
=====				
OPTIMAL RATES OF STREAM DEPLETION				
FIRST TIME PERIOD				
L.Bound	Optimal	U.Bound	Marginal	
0.00	1871.93	14000.00	0.000	
SECOND TIME PERIOD				
L.Bound	Optimal	U.Bound	Marginal	
0.00	14000.00	14000.00	-1.092	

FIGURE 11. US/WELLS Output File for Example C (Second Cycle)

**Groundwater Pumping Supply Management
Under Salt Water Intrusion Conditions (Example D)**

In this example, the same situation discussed in example (C) is considered. The only change is that a salt water body exists in the study area. It is required in this case that hydraulic gradient toward the salt water body is not less than a prespecified value. For this purpose, gradient control pairs of observation wells are located along the salt water body boundary.

Figure (12) shows the hypothetical study area and the placement of the gradient control pairs. Figures (13) and (14) show the input data file and US/WELLS output file for the first cycle, respectively. Figures (15) and (16) show the input data file and US/WELLS output file for the second cycle, respectively.

This example also involves cycling since the aquifer is unconfined. After the second cycle, the difference in the optimal heads is small and the difference in the optimal pumping value is less than 1%. We can stop cycling. Again if greater accuracy is desired, cycling must be continued.

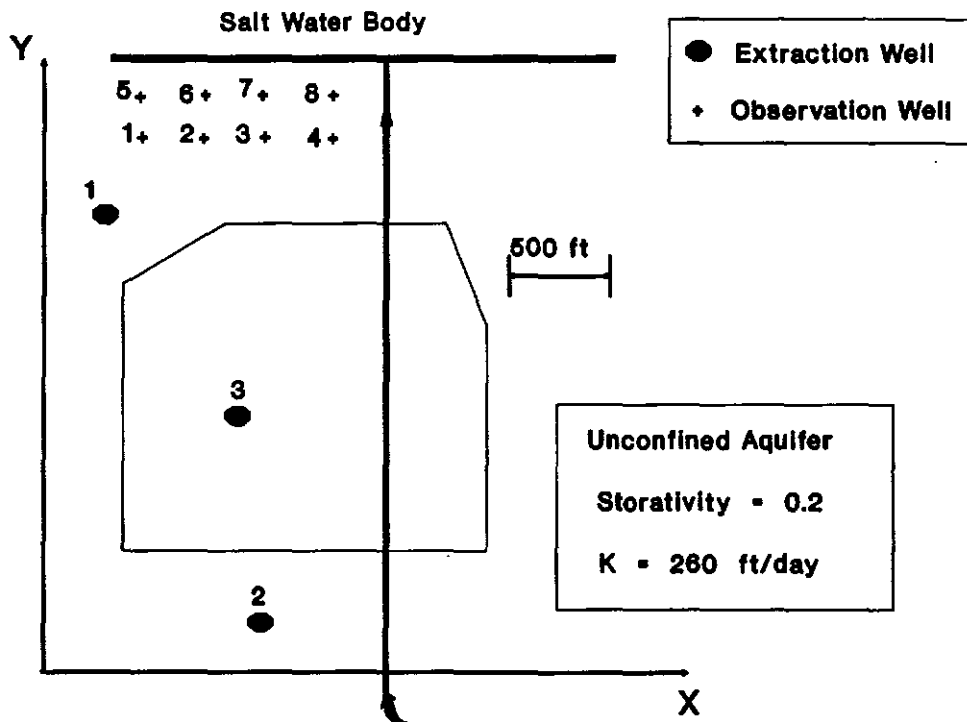


FIGURE 12. The Hypothetical Study Area for Example D

```

3 1 8 4
1 375 3100 74.4 50 80 74.4 74.4 74.4 6000 6000 6000 6000 0.5
2 1000 375 72.1 50 80 72.1 72.1 72.1 5000 5000 5000 5000 0.5
3 1125 375 71.2 50 80 71.2 71.2 71.2 0 8000 0 8000 0.5

1 100 100 75.0 50 80 75.0 75.0 75.0 0 0 0 0 0.5

1 600 1300 69.9 50 80 69.9 69.9 69.9 1
2 800 1300 69.8 50 80 69.8 69.8 69.8 1
3 1000 1300 68.5 50 80 69.5 69.5 69.5 1
4 1200 1300 68.0 50 80 69.0 69.0 69.0 1
5 600 1350 69.7 50 80 69.7 69.7 69.7 1
6 800 1350 69.6 50 80 69.6 69.6 69.6 1
7 1000 1350 69.3 50 80 69.3 69.3 69.3 1
8 1200 1350 68.8 50 80 68.8 68.8 68.8 1

0.2 3 200 1000 90
260 260 0
-1 -1
1 5 0.0035 0.01 0.0035 0.01
2 6 0.0035 0.01 0.0035 0.01
3 7 0.0035 0.01 0.0035 0.01
4 8 0.0035 0.01 0.0035 0.01

0 0 0 1 0 1
1 -1
1 1750 220
0 14000 0 14000
0 0 0

```

FIGURE 13. US/WELLS Input File for Example D (First Cycle)

FIGURE 14. US/WELLS Output File for Example D (First Cycle)

VALUE OF OBJECTIVE FUNCTION -15929.18				
OPTIMAL EXTRACTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	6000.00	6000.00	6000.00	0.001
2	5000.00	5000.00	5000.00	0.001
3	0.00	0.00	8000.00	0.001
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	6000.00	6000.00	6000.00	-0.101
2	5000.00	5000.00	5000.00	-0.017
3	0.00	4929.18	8000.00	0.000
=====				
OPTIMAL INJECTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	0.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	0.00	0.000
=====				
OPTIMAL HEADS AT OBSERVATION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	69.89	80.00	0.000
2	50.00	69.79	80.00	0.000
3	50.00	69.49	80.00	0.000
4	50.00	68.99	80.00	0.000
5	50.00	69.69	80.00	0.000
6	50.00	69.59	80.00	0.000
7	50.00	69.29	80.00	0.000
8	50.00	68.79	80.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	69.82	80.00	0.000
2	50.00	69.72	80.00	0.000
3	50.00	69.43	80.00	0.000
4	50.00	68.94	80.00	0.000
5	50.00	69.62	80.00	0.000
6	50.00	69.52	80.00	0.000
7	50.00	69.23	80.00	0.000
8	50.00	68.75	80.00	0.000

OPTIMAL HEADS AT EXTRACTION WELLS

FIRST TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	74.04	80.00	0.000
2	50.00	71.79	80.00	0.000
3	50.00	71.12	80.00	0.000

SECOND TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	73.96	80.00	0.000
2	50.00	71.65	80.00	0.000
3	50.00	70.76	80.00	0.000

OPTIMAL HEADS AT INJECTION WELLS

FIRST TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	74.99	80.00	0.000

SECOND TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	74.92	80.00	0.000

OPTIMAL RATES OF STREAM DEPLETION

FIRST TIME PERIOD

L.Bound	Optimal	U.Bound	Marginal
0.00	2012.84	14000.00	0.000

SECOND TIME PERIOD

L.Bound	Optimal	U.Bound	Marginal
0.00	14000.00	14000.00	-1.089

OPTIMAL HYDRAULIC GRADIENTS

FIRST TIME PERIOD

From	To	L.Bound	Optimal	U.Bound	Marginal
1	-> 5	0.00350	0.00399	0.01000	0.000
2	-> 6	0.00350	0.00398	0.01000	0.000
3	-> 7	0.00350	0.00398	0.01000	0.000
4	-> 8	0.00350	0.00398	0.01000	0.000

SECOND TIME PERIOD

From	To	L.Bound	Optimal	U.Bound	Marginal
1	-> 5	0.00350	0.00397	0.01000	0.000
2	-> 6	0.00350	0.00396	0.01000	0.000
3	-> 7	0.00350	0.00394	0.01000	0.000
4	-> 8	0.00350	0.00395	0.01000	0.000

FIGURE 14. US/WELLS Output File for Example D (First Cycle)

```

3 1 8 4

1 375 3100 73.96 50 80 74.4 74.4 74.4 6000 6000 6000 6000 0.5
2 1000 375 71.65 50 80 72.1 72.1 72.1 5000 5000 5000 5000 0.5
3 1125 375 70.76 50 80 71.2 71.2 71.2 0 8000 0 8000 0.5

1 100 100 74.92 50 80 75.0 75.0 75.0 0 0 0 0 0.5

1 600 1300 69.82 50 80 69.9 69.9 69.9 1
2 800 1300 69.72 50 80 69.8 69.8 69.8 1
3 1000 1300 68.43 50 80 69.5 69.5 69.5 1
4 1200 1300 68.94 50 80 69.0 69.0 69.0 1
5 600 1350 69.62 50 80 69.7 69.7 69.7 1
6 800 1350 69.52 50 80 69.6 69.6 69.6 1
7 1000 1350 69.23 50 80 69.3 69.3 69.3 1
8 1200 1350 68.75 50 80 68.8 68.8 68.8 1

0.2 3 200 1000 90

260 260 0

-1 -1

1 5 0.0035 0.01 0.0035 0.01
2 6 0.0035 0.01 0.0035 0.01
3 7 0.0035 0.01 0.0035 0.01
4 8 0.0035 0.01 0.0035 0.01

0 0 0 1 0 1

1 -1

1 1750 220

0 14000 0 14000

0 0 0

```

FIGURE 15. US/WELLS Input File for Example D (Second Cycle)

FIGURE 16. US/WELLS Output File for Example D (Second Cycle)

VALUE OF OBJECTIVE FUNCTION -15935.50				
OPTIMAL EXTRACTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	6000.00	6000.00	6000.00	0.001
2	5000.00	5000.00	5000.00	0.001
3	0.00	0.00	8000.00	0.001
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	6000.00	6000.00	6000.00	-0.102
2	5000.00	5000.00	5000.00	-0.017
3	0.00	4935.50	8000.00	0.000
=====				
OPTIMAL INJECTION RATES				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	0.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	0.00	0.00	0.00	0.000
=====				
OPTIMAL HEADS AT OBSERVATION WELLS				
FIRST TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	69.89	80.00	0.000
2	50.00	69.79	80.00	0.000
3	50.00	69.49	80.00	0.000
4	50.00	68.99	80.00	0.000
5	50.00	69.69	80.00	0.000
6	50.00	69.59	80.00	0.000
7	50.00	69.29	80.00	0.000
8	50.00	68.79	80.00	0.000
SECOND TIME PERIOD				
Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	69.82	80.00	0.000
2	50.00	69.72	80.00	0.000
3	50.00	69.43	80.00	0.000
4	50.00	68.94	80.00	0.000
5	50.00	69.62	80.00	0.000
6	50.00	69.52	80.00	0.000
7	50.00	69.23	80.00	0.000
8	50.00	68.75	80.00	0.000

OPTIMAL HEADS AT EXTRACTION WELLS

FIRST TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	74.03	80.00	0.000
2	50.00	71.79	80.00	0.000
3	50.00	71.12	80.00	0.000

SECOND TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	73.96	80.00	0.000
2	50.00	71.65	80.00	0.000
3	50.00	70.76	80.00	0.000

OPTIMAL HEADS AT INJECTION WELLS

FIRST TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	74.99	80.00	0.000

SECOND TIME PERIOD

Well No	L.Bound	Optimal	U.Bound	Marginal
1	50.00	74.92	80.00	0.000

OPTIMAL RATES OF STREAM DEPLETION

FIRST TIME PERIOD

L.Bound	Optimal	U.Bound	Marginal
0.00	2000.26	14000.00	0.000

SECOND TIME PERIOD

L.Bound	Optimal	U.Bound	Marginal
0.00	14000.00	14000.00	-1.089

OPTIMAL HYDRAULIC GRADIENTS

FIRST TIME PERIOD

From	To	L.Bound	Optimal	U.Bound	Marginal
1	-> 5	0.00350	0.00399	0.01000	0.000
2	-> 6	0.00350	0.00398	0.01000	0.000
3	-> 7	0.00350	0.00398	0.01000	0.000
4	-> 8	0.00350	0.00398	0.01000	0.000

SECOND TIME PERIOD

From	To	L.Bound	Optimal	U.Bound	Marginal
1	-> 5	0.00350	0.00397	0.01000	0.000
2	-> 6	0.00350	0.00396	0.01000	0.000
3	-> 7	0.00350	0.00394	0.01000	0.000
4	-> 8	0.00350	0.00396	0.01000	0.000

FIGURE 16. US/WELLS Output File for Example D (Second Cycle)

Appendix A. INPUT FILE FORMAT

The input data file can be prepared using any text editor. A description of the file contents follows. Data is read in free format so that any number of blank spaces and blank lines might be placed anywhere in the data file. This should enhance the clarity of the input data. It is recommended, though not necessary, to write the information of each well in a single line and to insert at least one blank line between different groups of lines.

1st Line :

[no. of extraction wells] [no. of injection wells] [no. of observation wells] [no. of gradient control pairs]

2nd Group of Lines :

Extraction Wells Definition

(one line per extraction well)

[well number] [X coordinate] [Y coordinate] [aquifer saturated thickness]
[minimum allowed head] [maximum allowed head] [initial head] [nonoptimal head at end of the first time period] [nonoptimal head at end of the second time period]
[minimum extraction rate for the first time period] [maximum extraction rate for the first time period] [minimum extraction rate for the second time period] [maximum extraction rate for the second time period]

3rd Group of Lines :

Injection Wells Definition

(one line per injection well)

[well number] [X coordinate] [Y coordinate] [aquifer saturated thickness]
[minimum allowed head] [maximum allowed head] [initial head] [nonoptimal head at end of the first time period] [nonoptimal head at end of the second time period]
[minimum injection rate for the first time period] [maximum injection rate for the first time period] [minimum injection rate for the second time period] [maximum injection rate for the second time period]

4th Group of Lines :

Observation Wells Definition

(one line per observation well)

[well number] [X coordinate] [Y coordinate] [aquifer saturated thickness]
[minimum allowed head] [maximum allowed head] [initial head] [nonoptimal head at end of the first time period] [nonoptimal head at end of the second time period] [index to show if water level is required to be dropped or raised]

5th Line :

[storativity] [Duration of first time period] [Duration of second time period] [Unit pumping rate] [Elevation of the ground surface]

6th Line :

[maximum hydraulic conductivity] [minimum hydraulic conductivity]
[angle between X-axis and K_{max}]

7th Line :

[maximum ratio of excess injection] [maximum ratio of excess extraction]

8th Group of Lines :

Gradient control pairs of observation wells
(one line per pair)

[no. of first well] [no. of second well] [minimum allowed gradient at end of the first time period] [maximum allowed gradient at end of the first time period] [minimum allowed gradient at end of the second time period] [maximum allowed gradient at end of the second time period]

9th Line :

[weight assigned to extraction in the first time period] [weight assigned to injection in the first time period] [weight assigned to hydraulic power in the first time period] [weight assigned to extraction in the second time period] [weight assigned to injection in the second time period] [weight assigned to hydraulic power in the second time period]

10th Line :

[index to show if linear (1) or quadratic (2) objective function is desired]
[index to show if the objective function is a minimization (1) or maximization (-1)]

11th Line :

[index to show if a river exists (1) or does not exist (0)] [X coordinate of the river]
[water surface elevation in the river]

12th Line :

[minimum allowed river depletion rate in the first time period] [maximum allowed river depletion rate in the first time period] [minimum allowed river depletion rate in the second time period] [maximum allowed river depletion rate in the second time period]

13th Line :

[standard deviation of the transmissivity] [standard deviation of the storativity] [level of reliability]

Units :

Any compatible units can be used. However, angles have to be in radians. For example, the following sets of units can be used

Coordinates, heads, saturated thickness, elevations	: meters
Time	: days
Unit Pumping	: cubic meters per day
Hydraulic Conductivity	: meters per day

OR

Coordinates, heads, saturated thickness, elevations	: feet
Time	: days
Unit Pumping	: cubic feet per day
Hydraulic Conductivity	: feet per day

Consequently, US/WELLS output will be in the same units.

Notes :

1. All given X and Y coordinates must be relative to the same fixed system of coordinates. It is not important where the user decides to place the intersection of the X and Y axes.
2. The user can place an extraction and an injection well at the same location. The optimization module will select whether this well will be extraction, injection, or neither. This is useful in situations when it is difficult to decide if a well, or a group of wells, should extract or inject water.
3. The nonoptimal head (specified in lines 2, 3, and 4) is the head at the specific location that will result if the optimal pumping strategy is not implemented. This allows US/WELLS to simulate transient groundwater flow conditions. If the groundwater flow is at steady state, the three nonptimal heads will be the same for each location.
4. The index at the end of the 4th line is an index of 1 if water level is required to be dropped at the observation well. A value of -1 is given if water level is required to be raised at the observation well. This index is only used by the stochastic model in order to use the correct chance constraint at observation wells. This index is not considered by the deterministic model. However, it has to be specified for both models.
5. The ground surface elevation (specified in line 5) is the average ground surface elevation throughout the study area. More accurately, it is the average ground surface elevation only at the locations of extraction wells.
6. The unit pumping rate, specified in the 5th line, is used by the simulation module to generate the system response coefficients matrix. Any value can be used for the unit pumping rate. The resulting optimal extraction and injection rates do not depend on the value of the unit pumping rate specified. However, the larger the unit pumping rate used, the smaller will be the round-off error of the response coefficients. For nonlinear optimization problems, the closer the value of the unit pumping to the optimal pumping rates, the faster convergence occurs.
7. If the aquifer is isotropic, the user should specify the same values for both K_{\max} and K_{\min} . In this case, any value for the angle between K_{\max} and the X-axis will be neglected. However, it is recommended to use a value of zero for the angle.
8. It is important that the angle between the X-axis and the direction of maximum hydraulic conductivity be measured in RADIANS. The angle is positive in the counter-clockwise direction.

9. The ratios mentioned as input in line (7) are explained as follows. The first (R1) is the ratio of maximum allowed difference between total injection and total extraction (I - E) to the total extraction (E) in any time period (ratio of imported water.) The second (R2) is the ratio of maximum allowed difference between total extraction and injection (E - I) to the total extraction (E) in any time period (ratio of exported water.)

R1 and R2 can be shown by the equations

$$\frac{I - E}{E} \leq 1 + R1$$

$$\frac{E - I}{E} \leq 1 + R2$$

For example, if up to 20% of the extracted water can be exported, rather than injected, and no import of water is allowed (E > I), the ratios will be 0 and 0.2, respectively. If neither export nor import of the water is allowed (E=I), the ratios will be 0 and 0.

10. If it is not required to constrain the import or export ratios discussed previously, a value of -1 is to be given to both ratios in the 7th line.
11. Gradient control pairs are to be specified such that the slope is positive if the optimal head in the first well is to be higher than that in the second well. If a negative value is given as the maximum allowed gradient for any time period, US/WELLS will not consider the gradient control constraint for that time period.
12. The indices mentioned in the 10th line are explained as follows. The first index shows which objective function is desired. A value of 1 means that the linear objective function is to be used. A value of 2 means that the quadratic objective function will be used. The second index shows if the objective function is a minimization (a value of 1 is used) or a maximization (a value of -1 is used.)
13. If a constant head boundary exists in the study area, the river index specified in line 11 will have a value of 1. In this case, the upper bounds on river depletion (line 12) are set to be any arbitrary large numbers.
14. The minimum input requirements are 13 lines. This implies that at least one extraction well, one injection well, one observation well, and one pair of gradient control wells must be specified. If there is neither extraction nor injection wells in the study area, dummy wells have to be used for both. For a dummy well, both upper and lower bounds of extraction, or injection, rate are set to zero. If gradient control is not required in the study area, any negative value can be given to the maximum gradient.

15. If the stochastic module is used. The uncertainty in aquifer parameters is represented in the form of the standard deviations of both the hydraulic conductivity and the storativity. In this case, the previously given values for the two parameters are AVERAGE values. The aquifer must be isotropic. The stochastic module uses the value given for the maximum hydraulic conductivity as the average value. The value of the minimum hydraulic conductivity is not considered by this module.
16. For this version, The maximum number of extraction wells in a problem is 25. The maximum number of injection wells is also 25. The maximum number of observation wells is 35. Any number of gradient control pairs can be specified as long as the number of observation wells does not exceed 35. Problem size is , however, constrained by the number of non-zero elements in the optimization matrix (as explained in Appendix C).

Appendix B. EXPLANATION OF OUTPUT

The output file includes the optimal values of the decision and state variables used in the problem. It also shows, for each variable, the lower bound (L. Bound), the upper bound (U. Bound), and the marginal.

The marginal is defined as the value by which the objective function will improve if the tight bound is changed one unit. If a variable's optimal value is not equal to either lower or upper bound, its marginal will be zero. That is, the marginal will only have a value if the optimal value is equal to one of the bounds. In this case, the marginal shows the improvement of the value of the objective function resulting from relaxing this bound by one unit. For example, In example (A), the output file (Figure 4) shows that the marginal of the optimal injection rate in the first time period at injection well (3) is (-45.342). The objective function's value was (334668.1). If the upper bound on injection in the first time period is relaxed by one unit, at the mentioned well. That is the new upper bound is (901) instead of (900), the value of the objective function will be (334622.7). The enhancement in the objective function's value is equal to $334622.7 - 334668.1 = -45.4$, which is equal to marginal value reported in US/WELLS output file.

If the problem is infeasible, US/WELLS writes a warning statement in the output file. For example, the following warning statement may appear in the output file

###INFEASIBLE head (first time period) at observation well 1

Using any text editor, the user can "search" for the sign "###" in the output file to check if any infeasibility is found.

**Appendix C. CALCULATION OF THE NUMBER OF NON-ZEROS
IN THE OPTIMIZATION MATRIX**

The number of the non-zero coefficients in the optimization matrix can be calculated using the NONZ utility which comes with US/WELLS. To use this utility, the user must make the directory in which US/WELLS is stored the current directory then the following command is issued,

NONZ

NONZ is an interactive program, it calculates the number of non-zeros after the user responds to some questions.

INDEX

- Analytical 4, 5, 9
- Anisotropy 5, 9
- Bound 8, 10, 16-19, 26, 27, 29, 30,
33, 34, 36, 37, 46
 - lower 7, 9-11, 19, 24, 44,
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 - upper 7, 10, 11, 19, 24, 44,
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 - extraction 1, 2, 4, 7-13, 16,
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33, 34, 36, 37, 40,
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 - image 5, 9
 - injection 1, 4, 7-13, 16, 17,
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