OPTIMAL GROUNDWATER QUANTITY/QUALITY PLANNING FOR SALT LAKE VALLEY

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Generally, the embedding technique for optimizing groundwater management has been limited to small scale or steady-state groundwater flow management cases. The embedding approach causes sparse, highly structured constraint matrices which have been sometimes difficult to solve. Embedding applications have been limited by lack of: optimization algorithms able to easily address such matrices for large scale problems, and efficient methods for addressing common nonlinearities and converging to a solution. Computer software, USUEM, addresses these issues by: creation of alternative linear, nonlinear, combined and partitioned formulations of the same problem, implemention of an efficient cycling procedure, and good selection of optimization algorithm and computational parameters. The application of the methodology is demonstrated for a large scale study area (Salt Lake Valley) where groundwater quantity and quality management is needed and where the proportion of pumping cells is great.

1. Introduction

Salt Lake Valley lies within the most populated county in the State of Utah. It covers an area of about 500 square miles. The groundwater reservoir (Fig. 1) consists of two unconsolidated aquifer layers of Quaternary age. Sources of recharges include bedrock recharge, seepage from irrigation, precipitation, canals, and creek channels (Hely et al, [1971]). Also, some recharge to the shallow aquifer comes from the upward movement of water from the confined aquifer. Discharges result from pumping, flowing wells, evapotranspiration, seeps, springs, and subsurface flow to the Great Salt Lake and sections of Jordan River and tributaries. The number of wells in the valley is estimated to be more than 12,000. Major groundwater uses are for municipalities, industries, private residences, irrigation and livestock. Almost all pumping is from the lower layer, which has better water quality than the upper layer. Current approved groundwater rights (some of which are not being utilized and are not yet legally perfected rights), have been thought to exceed what the aquifer can satisfactorily provide (Hansen et al, [1989,1990]). Requests for groundwater are expected to increase (Bishop et al, [1988], Waddell et al., [1987a]). In recent years ground-water levels declined 5 to 15 ft in the southeastern part of the Valley. Declines of 40 to 60 ft are projected by the year 2000 (Waddell et al., [1987a]). A significant result will be reduced baseflow from the aquifer to the Jordan River, which transects the valley from South to North.

There is also concern about water quality, especially in the southwestern part of the valley (Waddell et al,[1987b]). A large plume of dissolved solids and sulfates is moving toward wells and the Jordan River. To date, the plume underlies primarily commercial/industrial or agricultural activities and has not impacted significant residential areas (Baskin, [1990]). There are also isolated industrial plumes in the upper layer that can be hazardous if water migrates downward to the lower aquifer.

Lall et al. [1987] developed a response matrix model for the Salt Lake Valley. To reduce computations, they optimized pumping at only 46 cells where pumping is greater than 0.6 cfs (instead of 403 current pumping cells). Their assumption that further pumping should be in wells where the discharge is already high might unnecessarily limit management options. The actual trend is to shift pumping to low discharge cells, and encourage owners to give up or trade some of their water rights to limit pumping to areas of high water extraction. Although informative, that model ignored the effect of pumping on Jordan River flows and other fluxes between the upper aquifer and the external system.

1.1. *The Principal Aquifer (Lower Layer)*

The major groundwater-bearing formation is confined in the northern and central section of the Valley. It is unconfined between the confined portion and the mountains. In some locations, it is more than 2,000 feet thick. This formation is considered to be the second layer in the model. All pumping considered in the model takes place in this layer.

1.2. *The Shallow Unconfined Aquifer (Upper Layer)*

Between the water-bearing formation and the shallow unconfined layer is a 40-100 feet thick semi-confining bed. The thickness of the shallow unconfined aquifer ranges from few to 50 feet. It covers a smaller area than the principal aquifer. Because of its poor water quality, it is seldom used for water supply.

1.3. *Water Quantity Conditions*

The 1989 withdrawal of water from wells in Salt Lake Valley was about 133,000 acre-feet, or 183 cfs (Allen, [1990]). Water levels in the principal aquifer declined in most of the Salt Lake Valley in 1989. Most of the decline was recorded east of both Sandy and Herriman. Currently, the only permit applications to develop groundwater that are being approved in the valley are for single family wells in the county (i.e., away from municipal water supply). In some areas, no new groundwater development is being approved at all.

1.4. *Water Quality Conditions*

Concern about poor water quality exists mainly in the southwestern portion of the valley. A Dames & Moore report [1989] indicates that ground water contains TDS ranging from 500 mg/l to 50,000 mg/l. This results primarily from leachate from mining and industrial activities.

The same report indicates that chloride concentrations vary from 10 mg/l to 900 mg/l. The high chloride concentration is not related to the mining activities. These high concentrations resulted from industrial discharges, the use of chloride salts for roads, geothermal waters, and the leaching of the natural chloride salts from soils by irrigation water.

Sulfate concentration ranges from 10 mg/l to 70,000 mg/l. The area of highest concentration is bounded by the Oquirrh Mountains in the West, the Jordan River in the East, Bingham Creek in the North, and Butterfield Creek in the South. A large sulfate plume is moving from the southwestern part of the valley (tailings area) toward the Jordan River. The main sources of sulfate are Bingham Reservoir, the mine dumps, the old and new evaporation ponds, the cemetery pond, the Lark tailings, the Anaconda tailings and infiltration of irrigation waters.

There are isolated industrial plumes in the upper aquifer (Vitro tailings). Pesticides used in agricultural and urban areas can potentially migrate from the upper aquifer to the principal lower layer.

1.5 *Overview*

In conclusion, both water quality and quantity management are needed in the Salt Lake Valley. Unless an appropriate ground-water management strategy is implemented (causing the evolution of a suitable potentiometric surface in both aquifers) the following problems might result.

1. A satisfactory sustainable groundwater yield will not be guaranteed. Therefore the reliability of ground water will be questionable for the rapidly growing population in Salt Lake Valley
2. Users of surface water from the Jordan River and its tributaries might face a severe water shortage, if the effect of a pumping increase is not well investigated.
3. A significant decline in the water table will make pumping more expensive and increase costs of water to purchasers.
4. Some existing water rights might not be satisfied. Water quantity problems can be caused by ignoring water quality problems. In Salt Lake Valley, in 1986, contamination of shallow ground water was detected at six sites. Eleven privately owned wells and one public well were closed.
5. Excessive pumping in the northern part of the valley can result in salt water intrusion from the Great Salt Lake.

To prevent these problems, planners need a reliable tool for developing desirable management strategies. The model described by Gharbi and Peralta [1992] is used here to compute optimal sustainable groundwater pumping strategies, subject to specified physical and managerial constraints. Managing migration of the large sulfate plume is addressed through finite element transport constraints. Preventing downward migration of the Vitro tailings contamination is addressed by constraining heads. Terms and equations referred to in this paper are defined in the companion paper (Gharbi and Peralta, [1992]).

2. Model Input Data

No optimization model can be developed without first having a calibrated simulation model. Data and discretization from the only available calibrated model of the valley [Waddell et al., 1987a] were utilized in our study. Included are data on bedrock recharge, precipitation, seepage from irrigation and canal stream beds, soil characteristics, and pumping cell locations. As stated by Waddell et al. [1987a] the transmissivities of the unconfined aquifer are assumed to be constant. This assumption is valid when drawdown is relatively small compared to the saturated thickness. In all runs the constraint on drawdown is likely to enforce that previous assumption.

2.1. *Model Discretization*

2.1.1. *Flow*

The study area is bounded on the North by Davis County, to the Northwest by the Great Salt Lake, to the East and Southeast by the Wasatch Front Mountains, to the West and Southwest by the Oquirrh Mountains, and to the South by the Jordan narrows. A block-centered finite-difference formulation with rectangular cells in 38 rows and 28 columns was adopted (Fig. 2). The grid spacing ranges from 0.7 to 1 miles in both rows and columns. Smaller grid spacing was used in areas of heavy pumping and relatively steep gradient.

The numbers of cells of different types are presented in Table 1. The Jordan River, its tributaries and the surplus canal were divided in 8 reaches in which stream/aquifer interflow are separately constrained. This helps avoid computing unrealistic fluxes when the optimization model is applied.

2.1.2. *Transport*

The sub-system where groundwater contaminant concentration is to be managed is within rows 30 to 34 and columns 6 to 15. Sulfate concentrations are not expected to change significantly ouside that area (Dames and Moore, [1989]). This subsystem includes 48 finite-difference cells. Since finite-element nodes are also centers of finite-difference cells, the subsystem includes 48 finite element nodes and 34 rectangular elements (Fig. 3). Also from Dames and Moore, [1989], are estimates of sources of sulfate and corresponding mass flux rates (Table 2). These include recharges from rainfall, bedrock, and irrigation with concentrations of at least 100 ppm sulfate.

The isolated small plume (Vitro tailings area) in cell (16,18) will be addressed through a simple flow control procedure, by constraining head in the upper layer to not exceed head of the lower layer preventing the movement of water of poor quality to the principal aquifer (also making sure that head in the considered cell is lower than of the surrounding cells).

2.2. *Boundary Conditions for the Study Area and Model Assumptions*

The northern and northeastern boundaries are of Dirichlet type, where head is specified, reflecting the Great Salt Lake. To the other compass directions, Neumann conditions, having known recharges or no flow, are specified. Recharge and discharge boundaries are specified along the Jordan river, lower reaches of tributaries and the surplus canal. Discharge through evapotranspiration occurs in the central and northern parts of the upper unconfined layer.

Besides the assumptions used in MODFLOW (McDonald and Harbaugh, [1984]) and SUTRA (Voss, [1984]), the following assumptions are adopted for all management scenarios:

1. The first layer is unconfined, while the second is confined in some locations.
2. Flows are corrected when a second layer cell becomes unconfined.
3. Transmissivity is unchanged within a stress period and is computed at the beginning of each cycle, using the average head for that stress period from the previous cycle. Consequently, transmissivities are known before and during a simulation or optimization cycle.
4. Transmissivities between cells are computed using the harmonic mean formulation.
5. Boundary conditions are assumed constant during the entire planning period (they could be variable if adequate data were available).
6. The pumping from all wells in a single cell is represented by a single distributed discharge value.
7. A quasi 3-D formulation adequately represents flow.
8. The advective-dispersive transport of the sulfate plume is conservatively estimated using a nonreactive 2-D formulation.

3. Model Application

To better manage the future one should know the result of continuing current management (the unoptimized scenario). Comparison between the unoptimized scenario and results of different optimal scenarios is then useful to water managers.

After consultation with USGS personnel, some minor modifications were made to the original data. This was mainly in the constant-head cells locations. Also the original USGS pumping cells location and withdrawal quantities were combined with more recent data (Hansen Allen & Luce Inc., [1990]). The result totaled 158.238 cfs (114,637 ac-ft/year).

To validate USUEM, its flow simulation ability was first compared with that of MODFLOW for the same study area. When continuing current pumping for either steady-state or transient conditions, both models computed essentially the same results (Fig. 4). The greatest difference between simulated heads was less than 0.018 ft. Similarly, transport prediction of USUEM was validated by comparison with SUTRA.

3.1. *Unoptimized Condition Computation*

3.1.1. *Unoptimized Fluxes*

If current pumping is continued for the next 20 years, projected drawdowns in the upper layer are small (less than 4 feet). However, in the lower layer drawdowns as great 40 ft are expected in the southwestern part of the valley (Fig 5). Simulated rates of change in storage decrease with time, showing that the system is approaching some steady-state condition.

3.1.2. *Unoptimized Concentrations*

Figure 6 shows current sulfate concentrations and those projected to result during the next 20 years if current pumping continues. Twenty-five of the 48 subsystem nodes contain pumping as a decision variable. In twenty-two of the 48 total nodes and 7 of the 25 pumping nodes concentrations already exceed the 500 ppm health standard, although the groundwater is still being pumped and used. After 20 years, concentration will exceed the health advisory level in 17 of the 25 pumping nodes.

The increase in concentration is as great as 3127 ppm, in pumping node 34 (31,13). This results from sulfate migration from adjacent node 29 (31,12). A very high sulfate concentration is recorded in node 5 (not a pumping node) because of the continuous source of sulfate from Bingham Reservoir. In some nodes concentrations are decreasing mainly because of the elimination of the sulfate source (closure of evaporation ponds in nodes 29 and 30, (cells (30,13) and (29,13) respectively), or because of mixing with higher quality water (flow across boundaries, rainfall, bedrock and seepage).

The main concern is the movement of sulfate toward pumping wells and the Jordan river. Figure 6 shows the 500 ppm sulfate contour moving to the east where most the pumping is occurring. The sulfate will move about 2 miles in the next 20 years in the eastern part of the subsystem. If current pumping is continued, sulfate concentrations will be a problem in most subsystem pumping nodes. Only the southeastern portion of the study area is expected to continue satisfying the 500 ppm standard.

3.2. *Upper and Lower Bounds Used in the Management Scenarios*

Upper and lower bounds used in scenarios A-D and the sensitivity analysis are summarized in Table 3. Most bounds are expressed as a function of the current conditions (terms having c as subscript describes current conditions). Maintaining current pumping is the same as permitting no future development, i.e. an unoptimized scenario.

Lower bounds on pumping are expressed as a fraction (kpl) of the current pumping in all pumping cells. For kpl = 0.8 the lower bound on pumping is 80% of current pumping. Upper bounds are expressed as a multiple (kpu) of the current pumping (kpu = 1 for cells where there is a current moratorium preventing increased pumping, and kpu = 4 for other pumping cells).

The lower bound on variable heads in the first layer is the base of that layer (Bottl1). In the second layer where most of the pumping is occurring, the maximum drawdown (Maxddown) with respect to the current heads is 20 ft, suggested by David Hansen (personal communication, Hansen, Allen & Luce, Inc.). In each constant head cell, recharge from the Great Salt Lake to the aquifer was not permitted to exceed the maximum recharge rate currently observed in any cell (not more negative than the most negative currently observed recharge).

Total recharge from the Great Salt Lake should not exceed what is currently observed, thus insuring that the increase in pumping will not result in additional influx from the lake to the aquifer. Discharge to the Great Salt Lake is unbounded.

3.3. *Expected Sources of Extra Pumping*

To increase pumping and still keep drawdown acceptable, water must be captured. Inflow to the system is relatively fixed, water might be captured in the following ways.

1. Reducing evapotranspiration (Et). This requires dropping heads in the center and northern part of the first layer where most of the evapotranspiration is taking place. However, that is unlikely to happen because of the constant head constraints and the need to avoid excessive recharge from the Great Salt Lake. As a result, the drawdown might be insufficient to cause a substantial reduction in evapotranspiration.

2. Reducing discharge to the Great Salt Lake. Estimated current discharge through constant head cells is 11.3 cfs. Consequently, an increase in pumping due to a reduction of this component could not exceed that amount. Inflow from the Great Salt Lake is constrained by Eqs. 20, and 21 (Gharbi and Peralta, [1992]).

3. Continuing withdrawal from storage. This happens in proportion to drawdown. However, since pumping is from the confined second layer (where the storage coefficient is small), and the maximum drawdown is 20 ft, the withdrawal from storage might not an important component in the optimal pumping increase.

4. Reducing flow from aquifer to streams. This alternative can permit a substantial increase in pumping. To avoid dewatering the streams this reduction must be limited. In the study, a reduction of 50% in each reach (kr=0.5) is used. Other values of kr are tried in the sensitivity analysis.

3.4. *Expected Improvement in Quality Using Flow Management*

A quick analysis of the data for the subsystem that will be modeled suggests:

1. It is unlikely, with the current location of pumping cells and the utilized bounds on pumping, that sulfate concentrations in all pumping cells within the quality subsystem can be reduced below 500 ppm. Initial concentrations (more than 10,000 ppm at some nodes) are already high and there are continuing sources of sulfate.

2. It is unlikely, with the current configuration of pumping cells, that the gradient could be reversed or changed sufficiently that the contaminant could be channeled away from pumping cells. Because of the steepness of the current hydraulic gradient, significant alterations will require either impractically high extraction or injection, or placing wells north of the plume where there are currently few wells.

3.5. *Scenarios Considered*

In this paper, we will illustrate use of the model for different scenarios, and demonstrate the interaction between quality and quantity management. To most economically solve the subsystem contamination problem, other measures beyond the scope of this study might be taken.

Tested scenarios involve maximizing sustainable pumping, and, in some cases, avoiding the groundwater quality deterioration resulting from management strategy implementation. An implicit goal is that any increase in pumping should not unacceptably affect the users of Jordan River, or cause poor quality water to flow from layer 1 (the upper layer) to layer 2 (the principal aquifer) in selected sites. To attain these goals, and variations thereof, the following Scenarios A-F, are tested.

A. Maximize steady-state pumping.

B. Maximize unsteady-state pumping for a planning period of 20 years, subject to a constraint that the pumping not decrease with time, and pumping at the end of that era be sustainable.

C. Same as scenario B, but including water quality restrictions. The resulting sulfate concentration should not exceed, if possible, the unoptimized concentrations (i.e., the unoptimized concentrations are used as targets). Water of poor quality should not move downward to the principal aquifer in cell 16,18.

D. Same as scenario C, except that 500 ppm (sulfate legal standard) is used as a target instead of the unoptimized concentrations.

E. Same as scenario D, but some bounds on pumping and heads are relaxed. Lower and upper bounds on pumping are .4 and 8 times current pumping and drawdowns up to 40 ft are permitted.

F. Same as scenario D, except that only the easternmost column in the subsystem have target concentrations (500 ppm). This slows plume movement toward the Jordan River. In addition, drawdowns up to 40 ft are permitted.

Scenarios A and B involve only flow management. The models for those scenarios do not include the finite element transport equations. Scenarios C-F combine both quantity and quality management. In all scenarios, decision variables are withdrawal at each pumping cell. In Scenarios A-D bounds are as shown in Table 3. Scenario E differs from Scenario D in that bounds are changed somewhat to improve water quality in the pumping nodes. Scenario F differs from Scenario D in that fewer cells have target concentrations and drawdown can be greater. Compared will be the results from these scenarios and the no-future-development (unoptimized) case of continuing current pumping.

4. Results and Discussion

4.1. *Flow Management*

4.1.1. *Scenario A: Maximize Steady-state Pumping*

This case determines the maximum steady-state (sustainable) pumping that does not produce undesirable heads or flows. It does not attempt to achieve target concentrations. For both model versions, and solvers, the equations describing the objective, constraints, and bounds are reported in Table 4. Since utilized S values are zero, the model only solves for one set of steady-state flow equations.

The model is solved cyclically until the largest absolute difference between heads for two consecutive cycles is less than 0.1 ft (user convergence criteria). These results reflect fluxes at optimal steady-state, not necessarily those occurring at any time in the next 20 years.

The number of cycles required for convergence depends on the initial guess. However, once a feasible solution is found only a few cycles are needed to reach the optimal solution (2 to 3 depending on the users's convergence criteria). Also the time spent in each cycle is affected by the number of equations and variables. The numbers of each for Scenario A are shown in Table 5 for the partitioned and combined LP and DNLP models, respectively (Gharbi and Peralta, 1992).

Both linear and nonlinear formulations were used alternatively. Fluxes computed using the two formulations are within 2% of each other, probably assuring some proximity to global optimality of the solution. Switching (Gharbi and Peralta, [1992]) from the nonlinear to the linear formulation is always problem free. When the linear formulation converged, switching to the nonlinear formulation might give an error resulting from the structure of the Jacobian matrix (a whole row of the Jacobian could be zero at an optimal solution, resulting in the singularity of the Jacobian matrix).

Effect on fluxes. Under the constraints cited above, regional pumping can increase 27 % from 158 cfs (current pumping) to about 201 cfs (Table 6). Only 31% of the 403 pumping cells increase in pumping. In 250 cells (62%) pumping is at its lower bound, and in 113 cells (28%) pumping is at its upper bound. Most cells where pumping increased are near the Jordan River and its tributaries. The net flow from aquifer to streams and general-head boundary cells dropped by 24%, even though a 50% reduction would have been permitted by the constraints. Flow to the Great Salt Lake is reduced by 37% (3.83 cfs). Other fluxes remain similar to current conditions.

Effect on head/drawdown. Drawdowns in the first layer are not restrictive and are not shown here. Fig. 7 shows drawdown contours in the principal aquifer (layer 2), and identifies cells where pumping increases, decreases, or is unchanged. Groundwater flow direction is toward the Jordan River (column 18). Cell (27,10) has the greatest drawdown in the principal aquifer. There, pumping is relatively high (5 cfs) and transmissivity is the lowest in the valley (10 to 25 times lower than the rest of the area).

It is useful to determine which model version and solver are more suitable for this type of problems. Table 5 shows the results of testing 4 version/solver combinations. All 4 tests began with the same initial guesses. Reported are total system times for on an Apollo 4500 workstation with math accelerator and 16 MB of RAM, under the Sys 10.1 operating system. The combined LP formulation required only 67 % as much elapsed time as the slowest formulation, and seems most suitable for this type of model.

4.1.2. *Scenario B: Maximize the Unsteady-state Pumping Subject to a Final Sustainable Pumping After the End of Planning Period*

This scenario is mathematically more rigorous and requires more effort than scenario A. It simulates transient groundwater flow, but not transport. Also, it insures the sustainability of pumping after the planning period. Here, a 20 year planning period. Pumping is considered constant during each stress period. The same bounds and types of equations presented previously (Tables 3 and 4) are used for each stress period. However, the transient form of the flow equation is used during the 20-year planning period, and the steady-state form of the flow equation is used beyond that. To insure monotonically increasing sustainable pumping (pumping will not have to be decreased after the 20 year planning period), the following constraint is added (Eq. 17 from Gharbi and Peralta, 1992):

$$g_{\bar{o},k-1} \leq g_{\bar{o},k} \leq g^{ss}_{\bar{o}} \qquad (1)$$

where $g^{ss}_{\bar{o}}$ = unknown steady groundwater pumping beyond the planning period, $[L^3T^{-1}]$, which is the solution to a set of steady-state flow equations.

The number of equations and variables for this scenario are presented in Table 4. Again, the combined LP model is significantly faster than alternatives.

To speed convergence, pumping and heads obtained from scenario A are used as initial guess for scenario B.

Effect on Fluxes/Drawdown. Pumping occurs in the confined layer where the storage coefficient is relatively small and the change in storage is not very great. The sustainability constraint above limits pumping such that those values are almost identical to those computed in scenario A.

Effect on concentrations. In Scenario B (and A) pumping increased in 13 out of the 25 pumping nodes within the quality subsystem. These nodes are located in the eastern side of quality study area, near the Jordan river. In the remaining 12 nodes, pumping decreased. Concentration increased in 72% of the pumping nodes in the quality subsystem. The highest increase in sulfate concentration resulting from the implementation of scenario B is 501 ppm after 20 years, recorded in cell (30,12) (node 34). In general the increases in concentration in the pumping nodes is not very high (only 5 nodes have an increase greater than 100 ppm), and are smaller to the East and the Jordan River. Sixteen nodes (68%) will have a concentration exceeding 500 ppm (Table 6). These are the same nodes having excessive concentration in the unoptimized scenario. Implementation of the pumping of scenario B will result in a slightly higher concentrations in the pumping nodes, but will not significantly deteriorate conditions in nodes that met the 500 ppm standard under the unoptimized case.

4.2. *Flow and Transport Management*

Flow and transport management scenarios seek an increase in water pumping while attempting to achieve pre-specified water quality. Pumping should be within physical, economic, and legal bounds. Water quality should respect health standards, to the extent possible. The water quality results of optimal pumping should not be worse than the results of continuing nonoptimal pumping.

4.2.1. *Scenario C: Quantity and Quality Management with the Unoptimized Concentrations as a Target*

This scenario illustrates the tradeoff between maximizing pumping and preventing concentrations from exceeding the unoptimized concentrations at control points. It uses the same flow formulation as scenario B. However, an additional term is influential in the objective function (wc is equal to 1 rather than 0). The model also contains the constraints related to contamination (Eqs. 3-6 in Gharbi and Peralta, [1992]). Because target concentrations are the unoptimized values, the additional term attempts to prevent concentrations from exceeding the unoptimized values in pumping nodes. In essence, this scenario answers the question: how much can we increase pumping without increasing the number of pumping nodes that will exceed the sulfate health standard. This requires using about 400 more variables and equations in this scenario than in Scenario B.

Switching between the linear and nonlinear formulation was performed. The converged optimal strategies computed by both forms were very similar. This might give some confidence in the nearness of the solution to global optimality.

Effect on Fluxes/Drawdown. Figure 8 shows drawdowns resulting from the pumping strategy developed when wc equals 1. Total pumping is not quite as great as that from Scenario B. In effect 2% (3 cfs) of pumping are given up to achieve the quality enhancement described below. Otherwise, drawdowns and fluxes are similar to those computed by scenario B (although there are obvious differences which affected transport in the subsystem). Pumping increased (above current pumping) in 109 pumping cells, compared to 124 cells for scenario B.

In the subsystem where quality is modeled, pumping increased in 9 of the 25 pumping cells compared to 13 of 25 for scenario B. Interestingly, the locations of cells having increased pumping are almost the opposite (cells with increasing pumping in one scenario decreased pumping in the other scenario) to what was computed by Scenario B. Pumping in all cells on or near the eastern side of the subsystem decreased in Scenario C, slowing plume movement.

Effect on Quality. For this scenario a wc value of 1 was used. In only one pumping node (node 46, cell (32,14)) was concentration higher (insignificantly) than the unoptimized concentration (only 0.3 ppm, after 2 years). In all other pumping nodes, concentrations never exceeded the unoptimized future concentration during almost the total planning period. Sulfate concentrations (Fig. 9) are less than the unoptimized concentrations in 42 of the 48 subsystem nodes.

The objective to prevent concentrations from exceeding the unoptimized concentrations was achieved with only a slight reduction in pumping (2 cfs) compared to scenario B, and a significant increase (39 cfs) compared to current pumping. That was accomplished through:

- Reducing pumping (mainly in the eastern side of the subsystem).
- Spatiallly redistributing pumping rates.

The result is smaller velocities and slower plume movement.

This scenario showed that the model can be used to compute a significantly enhanced sustainable pumping strategy, without causing concentrations to exceed those of the unoptimized (no future increase in pumping) scenario.

Other values of wc (50 and 100) were also used. These resulted in substantial reduction in pumping with a negligible further reduction in concentrations.

4.2.2. *Scenario D: Quantity and Quality Management using the Standard (500 ppm) as Target Concentrations.*

Effect on Fluxes/Drawdown. Average pumping is 181 cfs (Tab. 6), 23 cfs greater than current pumping, but 19 cfs less than when only pumping is maximized (Scenario B). Optimal pumping is the lowest among the optimal scenarios so far discussed, even though a wc of 1 is used.

Pumping is at its upper bound in only 28 pumping cells, and is at its lower bound in 346 of the pumping cells. Pumping distribution differs from previous scenarios, being concentrated in the center eastern section of the Valley. In the subsystem, pumping decreased in all pumping cells near the Jordan River, and increased in 4 of the 25 pumping cells.

Effect on Quality. The objective to reduce concentrations to the 500 ppm health standard is not achieved. Sixteen of the 25 pumping nodes have concentrations higher than the standard 500 ppm. This is the same number as scenario C, although there is a further slight reduction in the concentrations. Again, reducing pumping (mainly in the eastern side of the subsystem) and redistributing pumping slowed the plume. Though this scenario achieved the lowest concentrations in pumping cells of all previous scenarios, it fell short of its objective. To improve objective attainment, some bounds should be relaxed. However, as is explained later, practical choices are limited.

Tight constraints. In all previous scenarios, constraints and bounds that most prevented the pumping from increasing are the following:

1. Lower bound on recharge from Salt lake Valley: The high negative marginal value associated with the recharge in cell (4,8) in layer 1, suggests that this scenario is very sensitive to the recharge in that cell. Reducing the lower bound and (permitting more recharge from Salt Lake) will result in an increase in pumping. A quick analysis of the effect of this constraint suggests that a small change in this lower bound affects the head in river cells.

2. Constraint on constant head: This constraint is a not a management decision. It is a natural constraint describing the projected mean level of the Great Salt Lake. Assuming that the average level will remain constant for the next 20 years, this constraint shouldn't be relaxed to enhance water quality goal achievement.

3. Lower and upper bounds on pumping: The high number of pumping cells at their upper and lower bounds suggest that pumping can be increased by increasing upper bounds and decreasing lower bounds. These upper and lower bounds reflect management decisions and should be chosen carefully to realistically describe the practical future. Reducing the lower bound on pumping below 80% of current pumping can be politically infeasible.

4. Lower bound on head: This is a tight constraint, but the reported marginal values suggest that this constraint is not as limiting as the previous ones. Also it is important to limit drawdown within an acceptable range to avoid dewatering partially penetrating wells.

5. Constraint on baseflow from aquifer to river: In previous scenarios, this constraint is only tight in 2 or 3 of 8 reaches (numbers 5,6,7). This, and the value of the marginal suggests that this constraint is not very limiting. In fact, in some reaches the recharge is higher than the current recharge.

Unless pumping is permitted in new cells, or injection is permitted, the the chance of increasing pumping and reducing the number of pumping nodes having a concentration higher than 500 ppm is very slim. Nevertheless, in Scenario E bounds are relaxed and improvement is attempted.

4.2.3. *Scenario E: Quantity and Quality Management with the Standard (500 ppm) Concentrations as a Target and Relaxed Bounds.*

This scenario attempts to improve objective attainment by relaxing bounds shown in Table 2. Lower and upper bounds are 0.4 and 8 times the current pumping, respectively at moratorium and normal pumping cells. The maximum drawdown allowed in pumping cells is 40 ft.

Effect on Fluxes/Drawdown. Average computed pumping is 184 cfs, 3 cfs greater than Scenario D (Tab 6). Pumping increases slightly and drawdown increases to 40 ft in the east-central portion of the Valley. This mainly results from relaxing the drawdown constraint.

Pumping increased in 4 of 25 subystem pumping nodes. Among the 304 total pumping cells, 336 are at their lower bounds (10 less than scenario D), and 25 are at their upper bounds (4 more than scenario D), reflecting the ability to increase pumping at desirable locations.

Effect on Quality. Concentrations resulting in all subsystem pumping nodes are lower than in the unoptimized scenario or in Scenarios A-D. However, the same number of nodes (16 of 25) still have a concentration exceeding 500 ppm.

In effect, doubling the upper bounds on pumping and drawdown, and reducing to half the lower bound on pumping resulted only in an increase in pumping of about 3 cfs, and a slight improvement in concentrations. Extraction alone will be unable to reduce concentrations below the standard in the pumping nodes, if only current pumping cells are permitted to pump. However, to slow plume movement toward the Jordan River, a final scenario is tested.

4.2.3. *Scenario F: Slowing the Movement of the Plume Toward Jordan River.*

To achieve the above objective, the objective function (Eq. 1, Gharbi and Peralta, [1992]) is replaced by:

$$\text{Max } Z = \sum_{k=1}^{K} \sum_{\omega=1}^{\Omega} g_{\omega,k} - wc \sum_{kq=1}^{KQ} \sum_{nq=44}^{NQ} c^{+}_{nq,kq} \qquad (2)$$

Where the target concentration of equation 6 (Gharbi and Peralta, [1992]) is still 500 ppm, but the model attempts to achieve the target only in the final column of the subsystem. Reducing concentrations in these nodes will slow the movement of contaminant toward the Jordan River. The maximum allowed drawdown is 40 ft. Standard bounds of Table 2 are used for other variables.

Effect on Fluxes/Drawdown. Average computed pumping is greater than that of Scenarios D and E and current pumping (Tab. 6). Pumping increased in 12% of pumping cells. Drawdowns are most similar to those of Scenario E.

Effect on Quality. As in previous scenarios, concentrations are higher than the standard in 15 of the 25 pumping nodes. Also, optimal concentration exceeds unoptimized concentrations in 20 of 25 pumping nodes. However, the model made progress toward its goal. In the five target nodes of column 15, concentrations are lower than in any other scenario. Tab 6 illustrates how average concentrations in the final column decrease as efforts to reduce concentrations increase (i.e., scenarios C-F).

4.3 *Closure*

There as many possible scenarios as there are possible combinations of bounds and constraints. Scenarios tested above reflect what can be reasonably done to maximize sustainable pumping, and control pumped concentrations. Sustainable pumping can be up to 27% greater than current pumping if water quality is not considered. If water quality is considered, the increase will be somewhat less. Although the model did compute pumping strategies that did not cause pumped water concentrations to exceed those of the nonoptimal scenario, it could not force all wells to achieve the health standard. Achieving greater success in controlling concentrations might require significant new drilling and denial of existing water permits.

5. Sensitivity Analysis

Sensitivity analysis was performed to evaluate the effect of changes in aquifer parameters, bounds and constraints on total regional pumping (Tab 8). Total pumping is somewhat sensitive to most changes, but is quite sensitive to vertical stream-aquifer conductances. Since most increase in pumping comes from reducing baseflow, and these parameters directly affect stream/aquifer interflow, one would expect these conductances to affect pumping significantly. Total pumping is also quite affected by the upper bound on pumping in individual cells. Total sustainable pumping is relatively unaffected by storage coefficient and specific yield.

The sensitivity of Scenario D to assumed dispersivities was also evaluated. Values used initially in Scenarios C-F were, $\alpha_L=30$ ft and $\alpha_T=10$ ft. In four sensitivity analysis run, those values were multiplied by 0.0, 0.5, 2, and 10, respectively. In four cases, computed pumping is relatively unaffected. The number of pumping nodes having concentrations exceeding 500 ppm at 20 years was unchanged, although individual node concentrations did change. The regional model is not very sensitive to dispersivities for the tested scenario.

6. Summary and Conclusions

6.1. *Summary*

An integrated methodology for applying the embedding method to complex nonlinear groundwater management problems is tested. Via the USUEM model, a combination of new linear and nonlinear model formulations are used to successfully develop optimal groundwater pumping strategies for the Salt Lake Valley. This valley contains confined and unconfined aquifer layers, both large and small contaminant plumes and declining water levels. If current pumping continues, water level declines and contaminant migration will make some wells inoperable.

Computed optimal sustainable groundwater pumping can be 127% of current pumping. However, this assumes no special consideration is given to controlling migration of a large contaminant plume. To avoid degrading groundwater quality at pumped wells below that currently projected, the maximum sustainable pumping can be 125% of current pumping. Thus, there is a minor 2%, 3cfs, tradeoff between a purely volumetric goal and achieving both volumetric and quality goals.

The hydraulic gradient near that plume is very steep. Without placing wells in currently nonpumping cells or using injection, it is not practical to prevent some well concentrations from exceeding health standards. However, plume movement toward the Jordan River can be slowed.

An interesting observation is that two flow optimization models (neither of which considered transport) both computed the same optimal strategy, although one was much simpler than the other. A steady-state model gave the same answer as a transient model that also had: (1) terminal (steady-state) constraints and (2) monotonicity constraints which prevented pumping from decreasing with time. This result supports use of steady-state optimization models for regional sustained groundwater yield planning.

The USUEM model contains both linear and nonlinear (discontinuous derivative) embedded finite difference flow equations and finite element solute equations. Here, the temporal discretization for transport was four times that used for flow. Validity of the simulation ability of both linear and nonlinear forms of the model was verified by comparison with MODFLOW and SUTRA. For the same known system stresses and fluxes, USUEM computes the same system responses as those well known simulation models.

Having both linear and nonlinear formulations is useful. It is frequently easier to develop initially optimal solutions using the nonlinear model. Subsequent optimizations proceeded more rapidly using the linear form. After repetetive optimizations, both linear and nonlinear models converged to essentially the same optimal solution.

Another desirable feature is having both partitioned and combined forms of the flow equation. In the partitioned form, each flux that can be described by nonsmooth function (having discontinuous derivative) is represented by a separate equation and variable. In the combined approach there is only one flow equation per cell and only heads and pumping are variables. The partitioned form is more useful in the initial stages of optimization for identifying processes and data that cause constraint violations. The combined form is more useful later because it requires less memory and solves more rapidly.

6.2. *Conclusions*

The embedding technique can be applied successfully to optimizing long-term, reconnaissance scale, planning of large-scale nonlinear groundwater problems. Here, this involves: embedding transient flow and transport equations, utilizing linearized and nonlinear (with discontinuous derivative) versions of those equations, and cycling (reinitializing and repeating the optimization) until a convergence criterion is satisfied.

Having both nonlinear and linear forms of the same problem is a key element of the process. The nonlinear form can be essential for developing an initial feasible or optimal solution. The linear form frequently solves and converges much more rapidly in subsequent optimizations. Both ultimately converge to nearly the same solution, lending confidence to optimality.

The modelling approach should be useful for nonlinear systems where a large proportion of the cells: (1) contain pumping as a decision variable, (2) require head constraint or (3) have fluxes described by nonsmooth functions (discontinuous derivatives). The simulation abilities of this embedding approach are useful for coarse-scale management of groundwater flow and dispersed groundwater contamination. It is assumed that each cell might have many wells and that treating a cell's pumping as if it were uniformly distributed across the cell is appropriate. This approach is not a substitute for the detailed transient management capabilities of the response matrix approach.

The approach should be useful for integrating management of groundwater supply and nonpoint source pollution. The objective function emphasizes both maximizing groundwater pumping and achieving target groundwater qualities. The use of weights in the objective function permits the planner to favor one objective over the other. This makes it easy to determine tradeoffs between goals.

FIGURE 1. Salt Lake Valley Aquifer Characteristics.

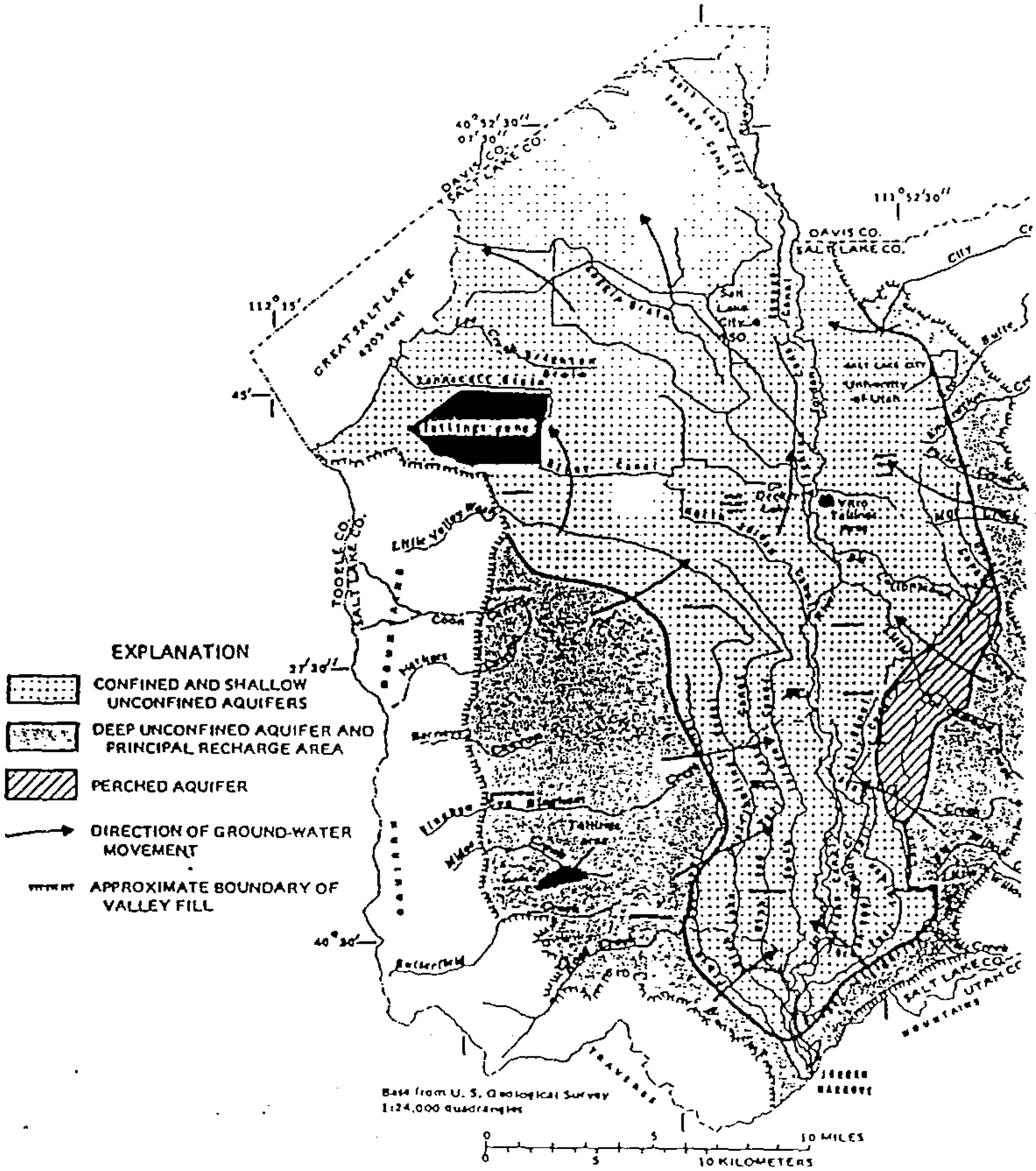


FIGURE 2. Study Area Discretization.

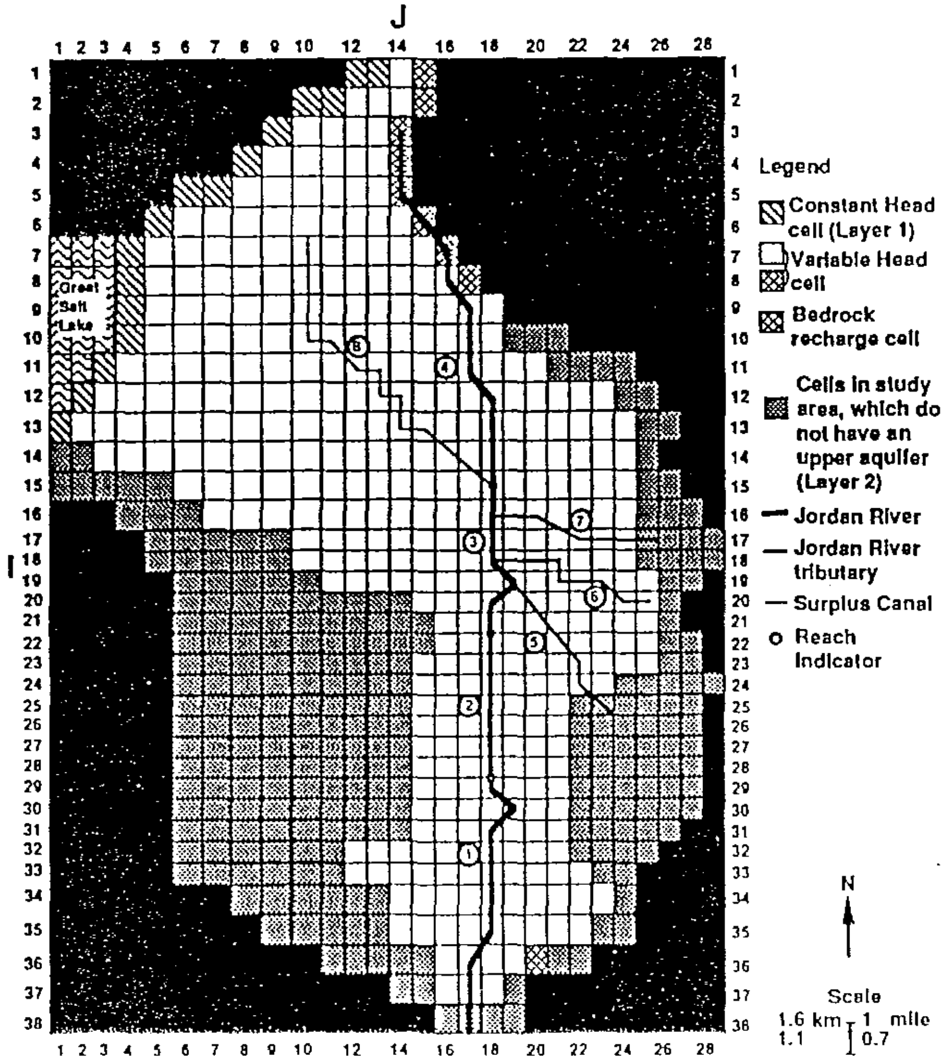


FIGURE 3. Groundwater Contamination Subsystem Discretization

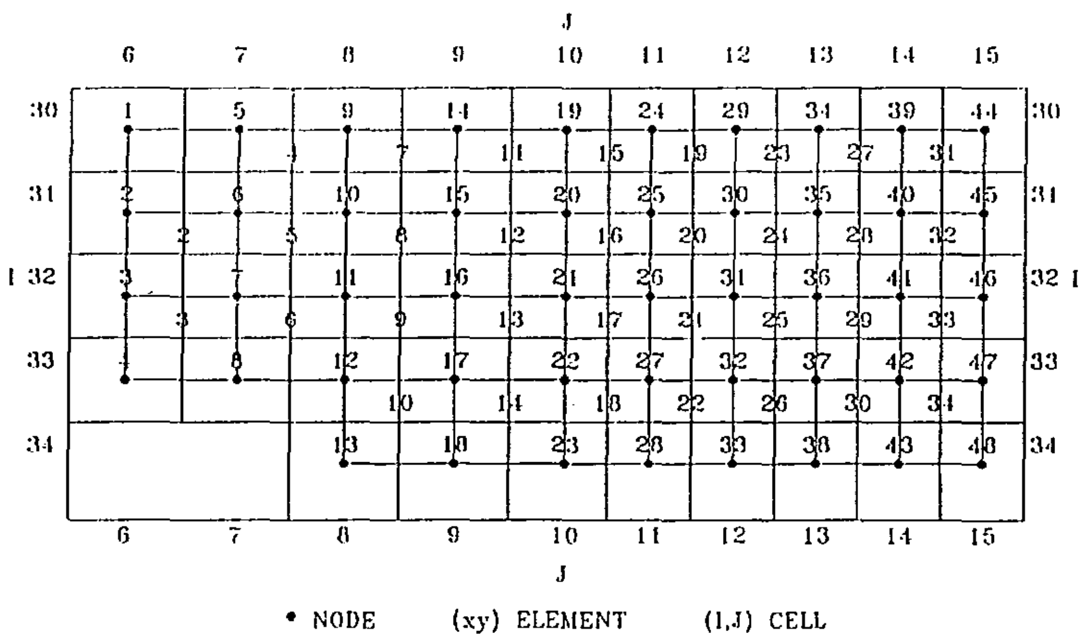


FIGURE 4. Head Contours in Layer 2 after 20 Years of Continuing Current Pumping (the Unoptimized Scenario), (ft above MSL).

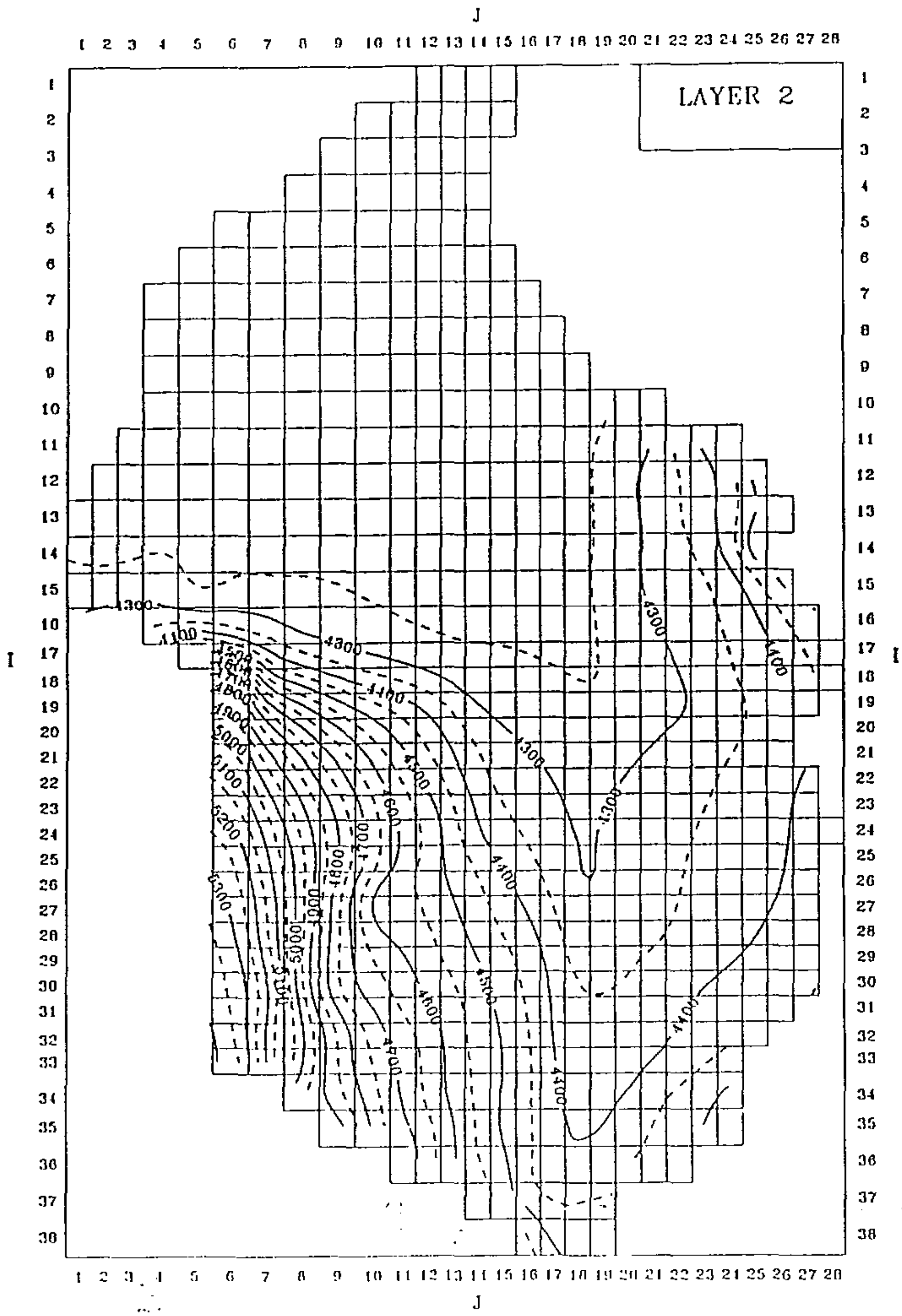


FIGURE 5. Drawdown Contours in Layer 2 after 20 Years of the Unoptimized Scenario, (ft).

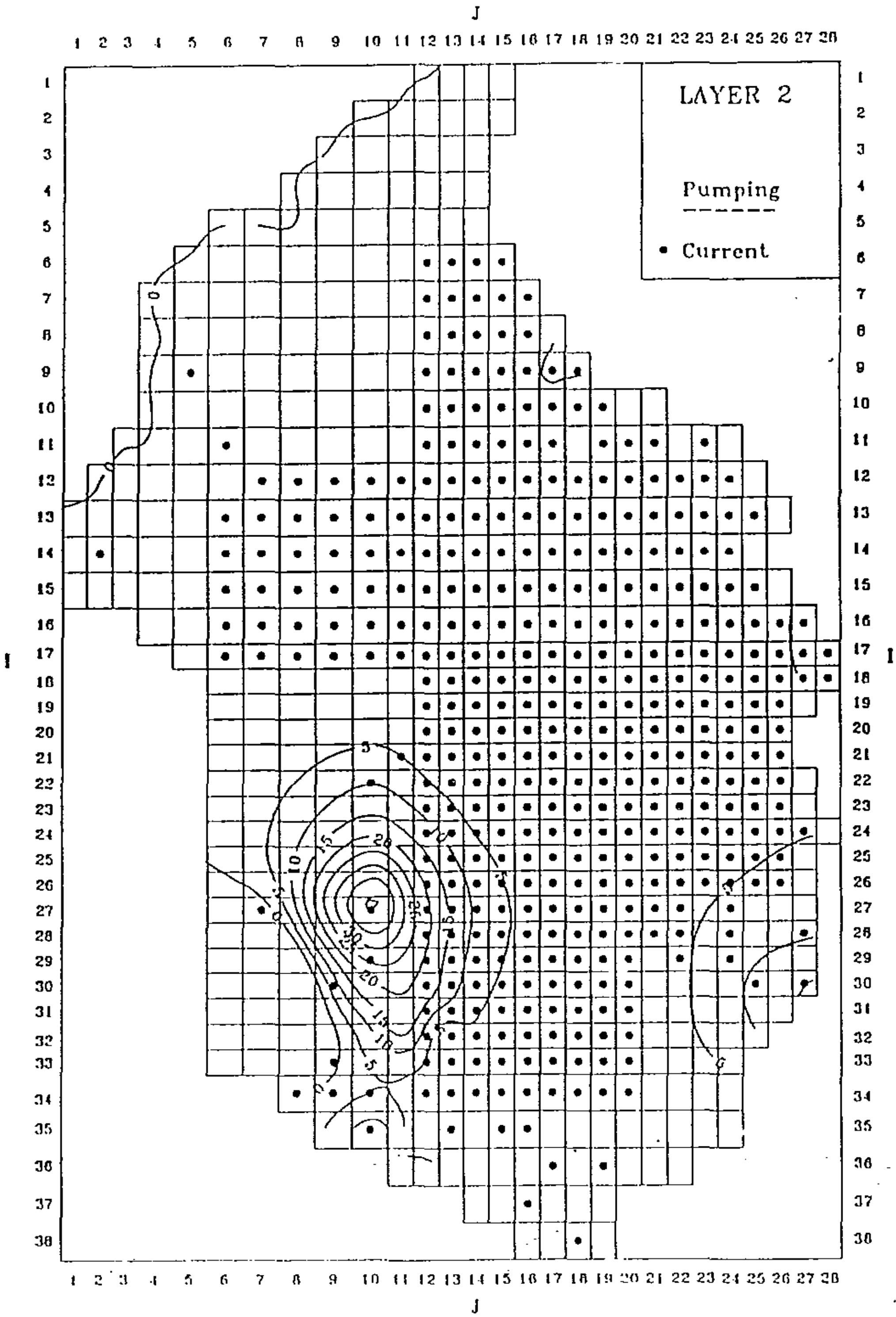
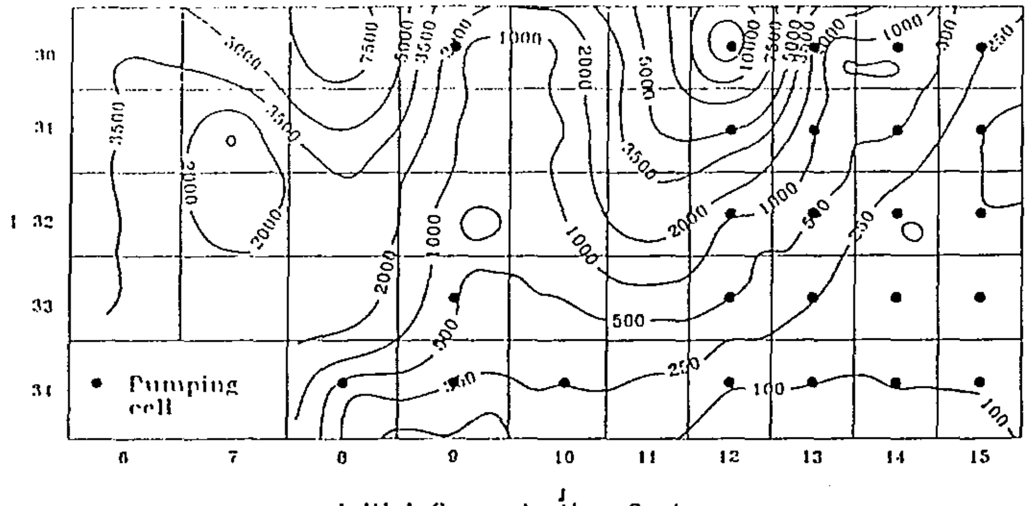
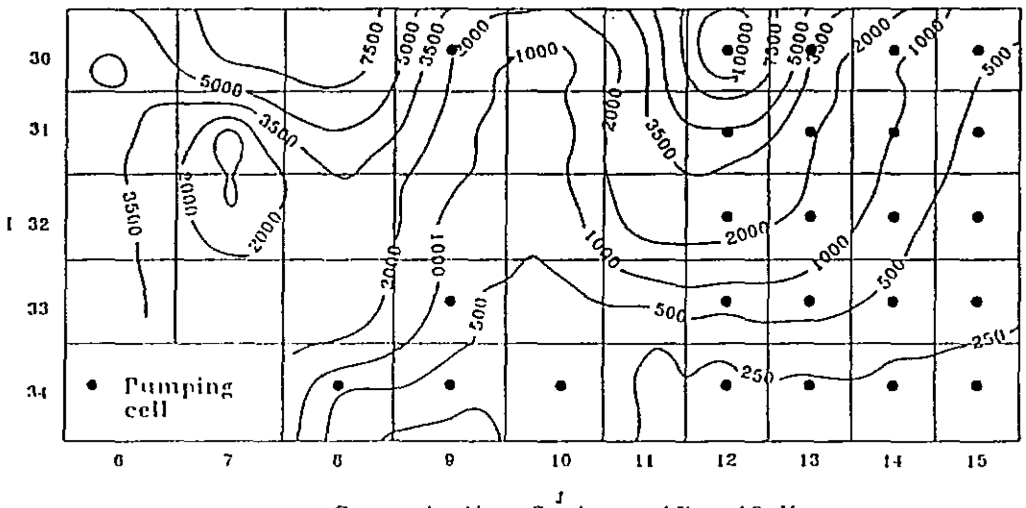


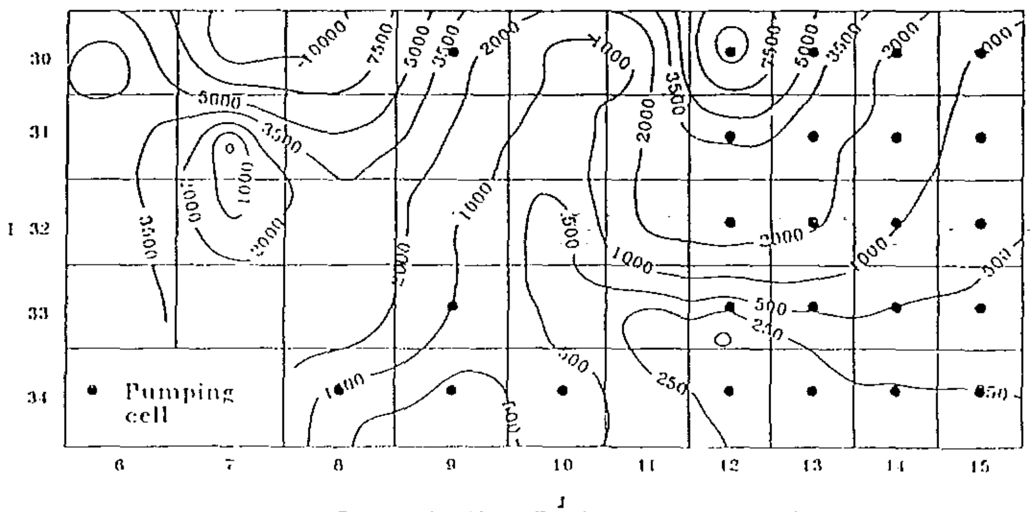
FIGURE 6. Sulfate Concentration Contours in Layer 2 Initially, and after 20 Years of the Unoptimized Scenario, (ppm).



Initial Concentration Contours.



Concentration Contours After 10 Years.



Concentration Contours After 20 Years.

FIGURE 7. Drawdown Contours in Layer 2 after 20 Years of Scenario A, (ft).

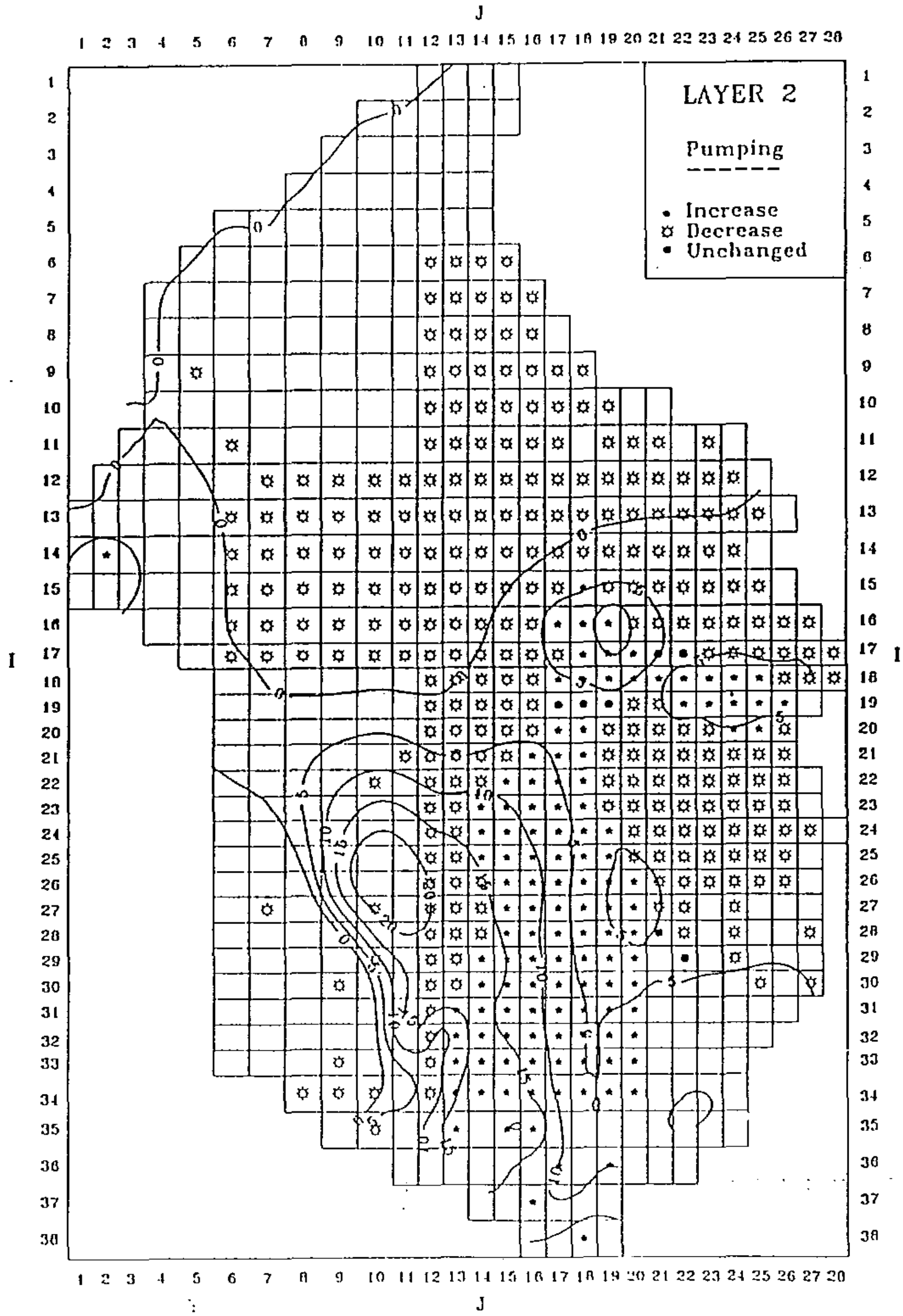


FIGURE 8. Drawdown Contours in Layer 2 after 20 Years of Scenario C, (ft).

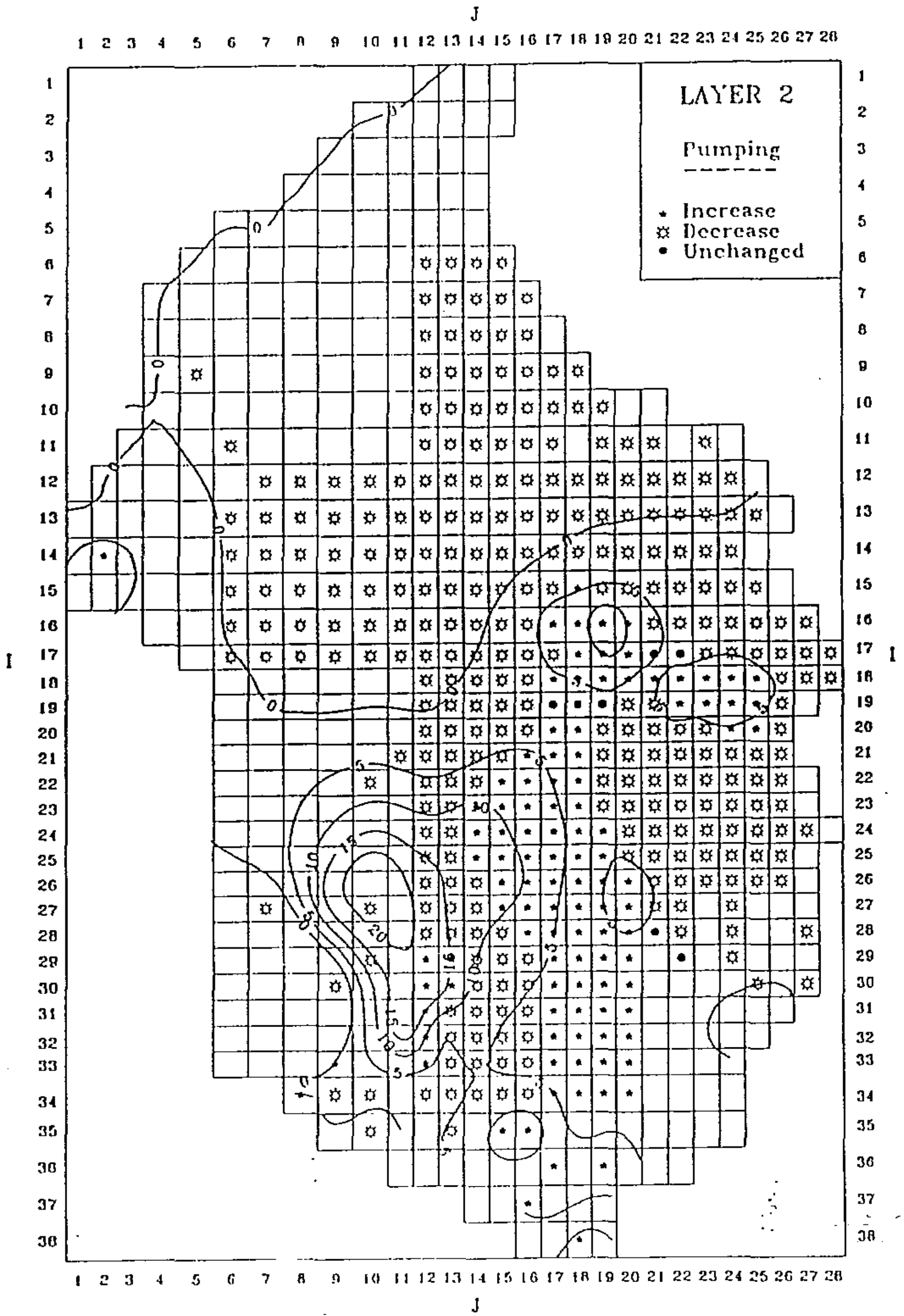
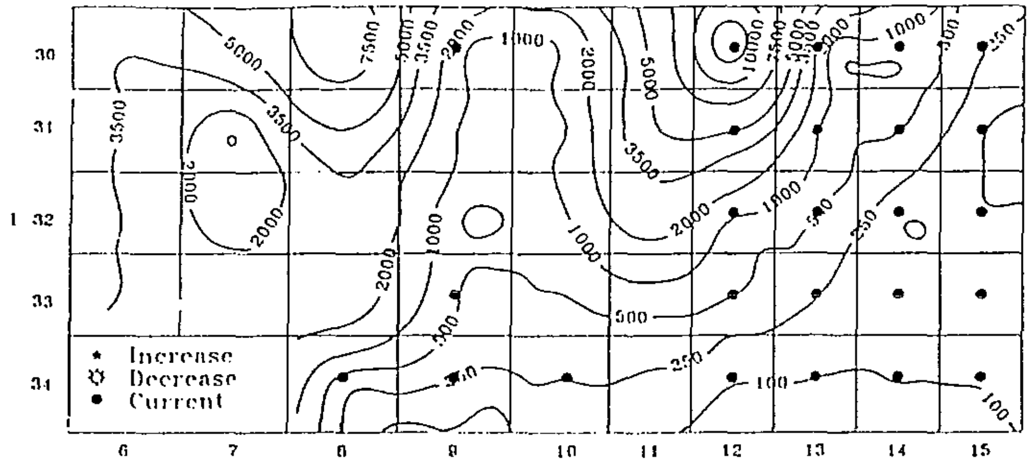
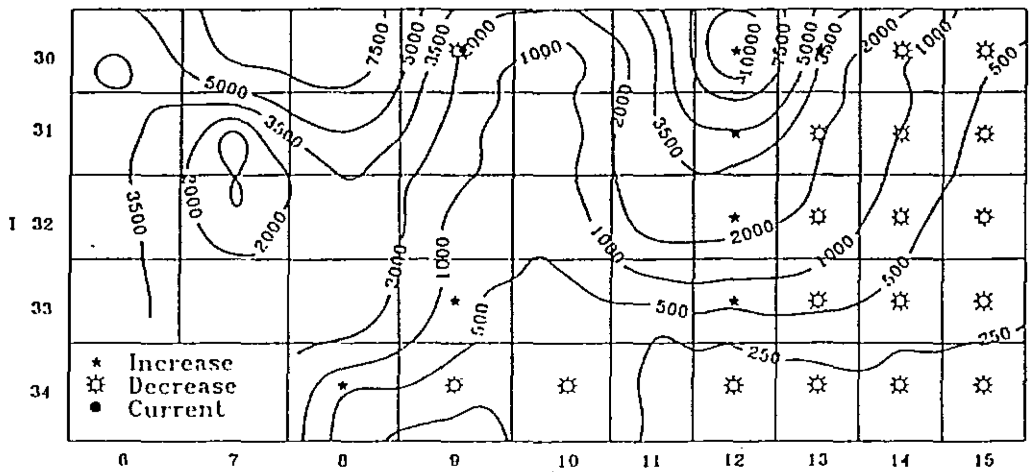


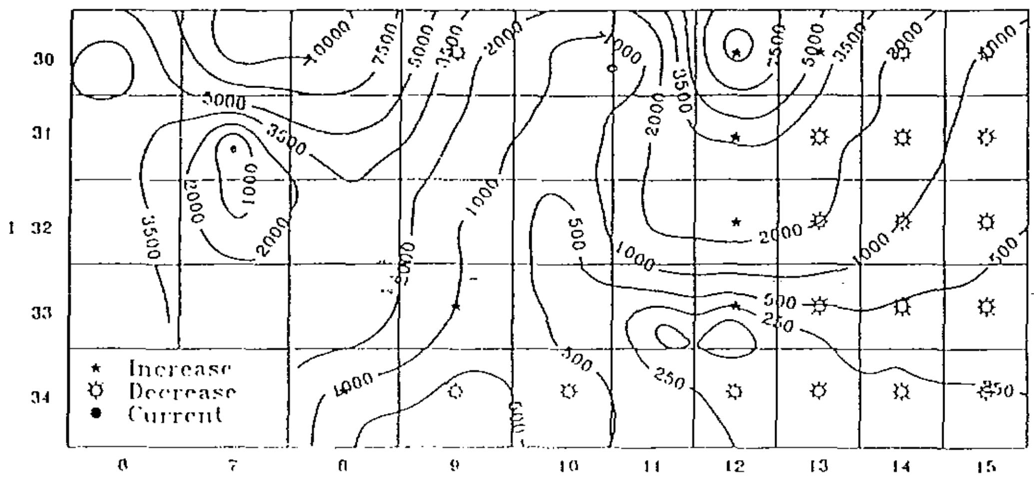
FIGURE 9. Sulfate Concentration Contours in Layer 2 after 20 Years of Scenario C (ppm).



Initial Concentration Contours.



Concentration Contours After 10 Years.



Concentration Contours After 20 Years.

Table 1. Number of Finite-difference Cells of Each Category

Category	Number of cells in each layer		Total number of cells of each category
	Layer 1	Layer 2	
Study area	411	675	1086
River and tributaries	58	2	60
Constant-head	16	16	32
Bedrock recharge	54	2	56
Pumping	0	403	403
General-head	12	0	12
ET	201	0	201

Table 2. Sources of Sulfates and the Corresponding recharges.

Node number	Sulfate Concentration (ppm)	Recharge (cfs)
5	25,000	-2.62
8	3,500	-0.04
11	3,500	-0.04
12	3,500	-0.04
23	1,250	-0.5
26	3,500	-1.2
31	3,500	-1.2

TABLE 3. Upper and Lower Bounds on Variables for Scenarios A-D.

Fluxes	Layer	Lower bound	Upper bound
Pumping	1	no pumping	no pumping
Pumping	2	$g^{L}_{\bar{o},k} = kpl\ g_{\bar{o},c}$	$g^{U}_{\bar{o},k} = kpu\ g_{\bar{o},c}$
Head – Variable head	1	$h^{L}_{\bar{o},k} = Botl1_{\bar{o}}^{(1)}$	$h^{U}_{\bar{o},k} = \infty$
Head – Variable head	2	$h^{L}_{\bar{o},k} = h_{\bar{o},c}\text{-Maxddown}^{(2)}$	$h^{U}_{\bar{o},k} = \infty$
Head – Constant head	1	$h^{L}_{\bar{o},k} = h_{\bar{o},c}$	$h^{L}_{\bar{o},k} = h_{\bar{o},c}$
Head – Constant head	2	$h^{L}_{\bar{o},k} = h_{\bar{o},c}$	$h^{L}_{\bar{o},k} = h_{\bar{o},c}$
Constant-head recharge	1 and 2	$(q^{z}_{\bar{o},k})^{L} = \min(q^{z}_{\bar{o},c})$ $QC_{k}^{L} = \sum_{o-1}^{oc} (q_{\bar{o},c}^{z})$ for cells for which $(q^{z}_{\bar{o},c}) \le 0$	$(q^{z}_{\bar{o},k})^{U} = \infty$ $QC^{U}_{k} = \infty$
Streams and General-head aqui-fer interflow	1 and 2	$R_{\kappa,k}^{L} = kr\ (\sum_{\bar{o}-1}^{N_\kappa} q_{\bar{o},c}^{g})$	$R^{U}_{\kappa,k} = \infty$

(1) $Botl1_{\bar{o}}$ = bottom of layer 1 in cell $\bar{o}$; (2) Maxddown = maximum acceptable drawdown.

TABLE 4. Objective, Constraints, and Bounds Equations for Scenario A.

	Model version	Utilized Equations[a,b]		Comments
		LP solver	DNLP solver	
Objective		1	1	wc = 0
Constraints	partitioned	2,7,9,11,12, 13,15,18, 19,20,21	2,8,10,11,12 14,16,18, 19,20,21	kpl = 0.8 kpu = 4.0 kr = 0.5 kc = 0.5 S = 0 Maxddown = 20 ft
	combined	2,12,18, 19,20,21	2,12,18,19, 20,21	

[a] Equation numbers are those listed and described by Gharbi and Peralta [1992].

[b] When used with the LP solver, Equation 2 includes preselected linear equations for flow processes best described by nonsmooth functions. When used with the DNLP solver, Equation 2 includes nonlinear functions for the same flow processes.

TABLE 5. Numbers of Equations and Variables and Processing Time for Models Which Maximize Sustainable Groundwater Pumping

Scenario, version	Number of Equations	Number of Variables	Elapsed Time (s) LP	Elapsed Time (s) DNLP
A, partitioned	1,369	1,795	1,104	1,429
A, combined	1,096	1,522	956	1,193
B, partitioned	4,524	4,996	16,083	21,401
B, combined	4,091	4,564	14,413	21,012

TABLE 6. Summary of the Results for Tested Scenarios.

Category	SCENARIO						
	Unoptimized	A	B	C	D	E	F
% change in pumping	0	27	27	25	15	16	18
% change in SAI(1)	0	-22	-20	-19	-9	-12	-12
% change in GSLAI(2)	0	-37	-35	-34	-77	-80	-67
% of cells ≤ UNCON(3)	0	N/A	28	100	96	100	20
% of cells ≤ STCONC(4)	32	N/A	32	36	36	36	40

(1) net flow to stream from aquifer,

(2) net flow to Great Salt Lake from aquifer,

(3) percentage of pumping nodes where computed concentrations do not exceed concentrations resulting from unoptimized pumping concentrations.

(4) percentage of pumping nodes where computed concentrations do not exceed the standard 500 ppm concentrations.

TABLE 7. Comparison of Sulfate Concentrations after 20 Years in the Easternmost Column of the Subsystem (Nodes 44, 45, 46, 47, and 48).

Node	Cell	Sulfate concentrations for different scenarios (ppm)					
		Unoptimized	B	C	D	E	F
44	(30,15)	929	978	922	901	833	796
45	(31,15)	778	817	767	740	699	627
46	(32,15)	606	623	589	557	555	463
47	(33,15)	421	430	409	385	379	351
48	(34,15)	241	272	236	205	184	186
Average Concentration of the 5 nodes		595	624	585	558	530	485

TABLE 8. Summary of the Sensitivity Analysis.

Parameter	Percentage variation from values used for Scenario A or B	Percentage change in pumping compared to that from Scenario A or B
Aquifer Parameters		
Storage coefficient/ specific yield[B]	80% to 120%	-0.1% to 1%
Conductances[A]	50% to 150%	11.5% to -36.5%
Hydraulic conductivities/ transmissivites[A]	100% to 120%	0% to 8%
Management Parameters		
Lower bound on pumping[A] Upper bound on pumping[A]	0% to 75% 50% to ∞	13.5% to 9.5% -7.5% to 20.5%
Maximum permitted drawdown[A]	200% to 500%	3% to 4.5%
Minimum acceptable stream-aquifer interflow[A]	40% to 160%	4.5% to -5.5%

[A] Computed using model of Scenario A
[B] Computed using model of Scenario B

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