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MODELLING FOR POTENTIOMETRIC SURFACE MANAGEMENT OF MULTILAYER AQUIFER SYSTEMS

by

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Correction

SUMMARY:
A model for optimally managing ground-water quality and quantity under steady and transient conditions in confined and unconfined aquifers is presented. Alternative volumetrically optimal steady-state strategies are shown. Discussed is use of the model for management of transient conditions.

KEYWORDS: Groundwater, Optimization, Contamination

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ABSTRACT

Assuring the long-term availability of groundwater of adequate quality and quantity frequently requires the implementation of appropriate ground-water and conjunctive water management strategies. Presented is a model for developing optimal strategies for an multilayer aquifer in which stream-aquifer interflow is affected by the potentiometric surface and ground-water use. The model is applied to the Salt Lake Valley. Discussed is the use of pumping to control: 1) potential migration of non-point source agricultural contaminants between aquifer layers and 2) the movement of a mile-long plume caused by mining waste.

INTRODUCTION

Most of Salt Lake City and Salt Lake County are underlain by alluvial deposits comprising a two-layer aquifer system (Figure 1). The lower layer is a primary source of water for urban use. Waddell (1987) indicated that withdrawal from Salt Lake Valley increased from 107,000 ac-ft/year during 1964-68 to an average of 117,000 ac-ft/year for 1969-82. Meanwhile, ground-water levels declined from 5 to 15 ft in the southeastern part of the valley. In some parts of the valley a projected decline of 40 to 60 ft is expected within the next 30 years if the 1982 pumping increases by 65,000 ac-ft. About 75% of the projected increase in pumping will be derived from a reduction in flow in the Jordan River and its tributaries.

There is also concern because of existing contamination in both the upper and lower aquifers. A large plume of sulfates and dissolved solids is moving from the western edge of the area toward the Jordan River. There are isolated industrial plumes in the upper aquifer. Pesticides used in agricultural and urban areas can potentially migrate from the upper aquifer to the principal lower layer. Unless an appropriate ground-water management strategy is implemented (causing the evolution of a suitable potentiometric surface in both aquifers) the following problems might result.

1. A satisfactory sustainable yield will not be guaranteed. Therefore the reliability on ground water will be questionable for the rapidly growing population in Salt Lake Valley (Salt lake County population increased by 35% from 1970 to 1980).

2. Users of water from the Jordan River and its tributaries might face a severe water shortage.

3. Many existing wells, especially those pumping in the shallow unconfined aquifer, might become inoperable.

4. A significant decline in the water table will make pumping more expensive and increase costs of water to purchasers.

5. Some existing water rights might not be satisfied. Water

quantity problems can be caused by ignoring water quality problems. In Salt Lake Valley, in 1986, contamination of shallow ground water was detected at six sites. Eleven privately owned wells and one public well were closed.

6. Excessive pumping in the northern part of the valley can result in salt water intrusion from the Great Salt Lake.

To prevent these problems planners need a reliable tool for developing desirable management strategies. Presented here is an operations research type of ground-water management model that computes optimal water use strategies, subject to specified physical and managerial constraints.

MODEL FORMULATION

The objective function maximizes total ground-water extraction, Z, for a planning period of K time steps and a system of M cells, including $\bar{O}$ pumping cells.

$$\mathbf{Max\ Z} = \sum_{k=1}^{K} \sum_{\bar{o}=1}^{\bar{O}} g_{\bar{o},k} \qquad [1]$$

where

$g_{\bar{o},k}$ = pumping withdrawals (+) from cell $\bar{o}$ during time step k, $[L^3T^{-1}]$;

Constraints and bounds are those described below. First, is the 3-D finite difference approximation of the transient flow equation. Fluxes are positive for discharge from the aquifer and negative for recharge to the aquifer.

$$\mathbf{f(h,\Delta x,\Delta y,T)} = \frac{S_{i,j,l}\,\Delta x_j\,\Delta y_i}{\Delta t_k}\left(h_{\bar{o},k} - h_{\bar{o},k-1}\right) + q^{b}_{\bar{o},k} + q_{\bar{o},k} + q^{s}_{\bar{o},k} + q^{c}_{\bar{o},k} \qquad \text{for } \bar{o} \in M,\ k \in K \qquad [2]$$

where

$$f(h,\Delta x,\Delta y,T) = 2\,\Delta y_i \frac{T_{i,j,l}\,T_{i+1,j,l}}{T_{i,j,l}\,\Delta x_{j+1} + T_{i,j+1,l}\,\Delta x_j}\,(h_{i+1,j,l,k} - h_{i,j,l,k}) +$$

$$2\,\Delta x_j \frac{T_{i,j,l}\,T_{i,j+1,l}}{T_{i,j,l}\,\Delta y_{i+1} + T_{i,j+1,l}\,\Delta y_i}\,(h_{i,j+1,l,k} - h_{i,j,l,k}) +$$

$$\frac{\Delta x_j\,\Delta y_i}{\dfrac{\Delta z_l}{2\,Kz_{i,j,l}} + \dfrac{\Delta z_l}{2\,Kz_{i,j,l}}}\,(h_{i,j,l+1,k} - h_{i,j,k,l}) +$$

$$2\,\Delta y_i \frac{T_{i-1,j}\,T_{i,j+1}}{T_{i,j,l}\,\Delta x_{j+1} + T_{i,j-1,l}\,\Delta x_j}\,(h_{i-1,j,l,k} - h_{i,j,l,k}) +$$

$$2\,\Delta x_j \frac{T_{i,j,l}\,T_{i,j-1,l}}{T_{i,j,l}\,\Delta y_{i-1} + T_{i,j-1,l}\,\Delta y_i}\,(h_{i,j-1,l,k} - h_{i,j,k,l}) +$$

$$\frac{\Delta x_j\,\Delta y_i}{\dfrac{\Delta z_l}{2\,Kz_{i,j,l}} + \dfrac{\Delta z_{l-1}}{2\,Kz_{i,j,l-1}}}\,(h_{i,j,l-1,k} - h_{i,j,k,l}) \qquad [3]$$

$T_{i,j,l}$ = transmissivity for cell i,j,l, $[L^2\,T^{-1}]$;

h = potentiometric surface head, [L];

$S_{i,j,l}$ = storage coefficient or specific yield;

$\Delta x_j, \Delta y_i, \Delta z_l$ = cell size in x , y and z directions, of cell ō located in row i, column j and layer l, [L];

Δt_k = duration of time step k, [T];

$h_{\bar{o},k}$ = average potentiometric head in cell ō at end of time step k, [L];

$q^b_{\bar{o},k}$ = known flow across the boundaries of the study area (i.e. bedrock recharge that is not a function of head, $[L^3T^{-1}]$;

$q_{\bar{o},k}$ = $g_{\bar{o},k} + q^t_{\bar{o},k} + q^z_{\bar{o},k} + q^l_{\bar{o},k,l}$

= distributed discharge from (+) or recharge to (-) the aquifer in cell ō in time step k, that is a function of ground water or surface water management, $[L^3T^{-1}]$;

$q^t_{\bar{o},k}$ = distributed discharge from evapotranspiration (+) or recharge by accretion (-) to the aquifer in cell ō and time step k, $[L^3T^{-1}]$;

$q^l_{\bar{o},k,l}$ = flow reduction to account for the change in the layer condition type (from confined to unconfined (for 2nd layer)

$q^{z}_{\bar{o},k}$ = flow across a boundary. The flow is a function of that boundary's fixed head and adjacent heads, $[L^3T^{-1}]$;

$q^{s}_{\bar{o},k}$ = saturated flow between the aquifer and a river or drain, $[L^3T^{-1}]$;

$q^{c}_{\bar{o},k}$ = saturated flow between the aquifer and general head boundary cells, $[L^3T^{-1}]$;

Bounds include those on pumping, head and recharge from the Great Salt Lake (equations 4, 5 and 6, respectively).

$$g^{L}_{\bar{o},k} \leq g_{\bar{o},k} \leq g^{U}_{\bar{o},k} \qquad [4]$$

$$h^{L}_{\bar{o},k} \leq h_{\bar{o},k} \qquad [5]$$

$$q^{z}_{\bar{o},k} \leq (q^{z}_{\bar{o},k})^{U} \qquad [6]$$

where L and U denote lower and upper bounds, respectively, on superscripted variables.

Expressions describing evapotranspiration, stream-aquifer interflow, interflow to the Jordan river and general head boundary conditions (equations 7-10, respectively) are also included.

$$q^{t}_{\bar{o},k} = E_{\bar{o}}\,\Delta x_j\,\Delta y_i \qquad \text{for } hs_{\bar{o}} \leq h_{\bar{o},k}$$

$$q^{t}_{\bar{o},k} = \frac{E_{\bar{o}}\,\Delta x_j\,\Delta y_i\,(h_{\bar{o},k} - (hs_{\bar{o}} - ds_{\bar{o}}))}{ds_{\bar{o}}} \qquad \text{for } hs_{\bar{o}} - ds_{\bar{o}} < h_{\bar{o},k} \leq hs_{\bar{o}}$$

$$q^{t}_{\bar{o},k} = 0 \qquad \text{for } h_{\bar{o},k} < hs_{\bar{o}} - ds_{\bar{o}} \qquad [7]$$

$$q^{s}_{\bar{o},k} = \Gamma_{\bar{o}}(h_{\bar{o},k} - \sigma_{\bar{o},k}) \qquad \text{for saturated flow}$$

$$q^{s}_{\bar{o},k} = \Gamma_{\bar{o}}(B_{\bar{o}} - \sigma_{\bar{o},k}) \qquad \text{for unsaturated flow} \qquad [8]$$

$$q^{c}_{\bar{o},k} = \Gamma_{\bar{o}}(h_{\bar{o},k} - hc_{\bar{o},k}) \qquad [9]$$

$$q^{\ell}_{\bar{o},k,L+1} = Cv_{\bar{o},k}\,(TOP_{\bar{o},L+1} - h_{\bar{o},k}) \quad 4 \qquad h_{\bar{o},k} \leq TOP_{\bar{o},L+1}$$

$$q^{\ell}_{\bar{o},k,L} = Cv_{\bar{o},k}\,(h_{\bar{o},k} - TOP_{\bar{o},L}) \qquad h_{\bar{o},k} \leq TOP_{\bar{o},L}$$

$Cv_{\bar{o},k}$: vertical conductance, $TOP_{\bar{o},L}$: Top of layer

$$R^{L}_{r} \leq \left(\sum q^{s}_{\bar{o},k}\right)_{r} \qquad [10]$$

where

$E_{\bar{o}}$ = potential evapotranspiration in cell $\bar{o}$, [L];

$hs_{\bar{o}}$ = potentiometric surface elevation below which the evapotranspiration rate begins to decrease, [L];

$ds_{\bar{o}}$ = extinction depth in cell $\bar{o}$, [L];

$\Gamma_{\bar{o}}$ = hydraulic conductance of the stream-aquifer interconnection, (including any clogging layer), $[L^2T^{-1}]$;

$\sigma_{\bar{o},k}$ = elevation of the free water surface in the river, [L];

$B_{\bar{o}}$ = bottom of the river in cell $\bar{o}$ [L];

r = the index number of a reach

R^{L}_{r} = lower bound on discharge from the aquifer to the river for reach r $[L^3T^{-1}]$

In order to simulate the effect on contaminant plume movement or to insure an acceptable concentration in given locations, two other constraints are used. These are a 2-D Galerkin finite element approximation of the unsteady state solute transport equation and bounds on concentration (equations 11 and 12, respectively).

$$[D]\{C\} + [A]\{C\} + [P]\left(\frac{\delta C}{\delta t}\right) = \{b\} \qquad [11]$$

$$C_{\bar{o},k} \le C^{U}_{\bar{o},k} \qquad [12]$$

$C_{o,k} = CT_{o,k} + OC_{o,k} - UC_{\bar{o},k}$

$\left\{ \begin{array}{l} OC \rightarrow \text{over Achi} \\ UC \rightarrow \text{under Achievment} \end{array} \right.$

where

$\{C\}$ = column matrix of nodal concentrations

$\left(\frac{\delta C}{\delta t}\right)$ = column matrix of the time derivative of the nodal concentrations

$C^{U}_{\bar{o},k}$ = upper bound on concentration in node o cell o during time period $k[ML^{-3}]$

The square coefficient matrices [D],[A],and [P] are square coefficient matrices corresponding to the dispersion, and time dependent terms in the solute transport equation. The column matrix {b} corresponds to boundary conditions.

PRESENT AND ANTICIPATED MODEL CAPABILITIES

The developed model has or is gaining the following options.

a. The model has all the steady and unsteady three-dimensional flow simulation options ascribed to the USGS MODFLOW model (McDonald and Harbaugh, 1984), including head response to evapotranspiration, stream-aquifer interflow, discharge from wells and drains, recharge, boundary conditions, etc. The model has been tested by comparison with MODFLOW for the study area and hypothetical areas.

b. The model can also use Muskingum routing in the rivers and treat river stage as a variable, as described by Peralta et al (1990). (MODFLOW assumes that river stage is known.)

c. It can simulate steady or unsteady two-dimensional solute transport via finite element method for part of a study area or for the entire area. This option is still being tested.

d. The model can compute optimal strategies while using all the simulation options mentioned in a-c. above. For example, transport equation can be used as constraints to limit future concentrations at prespecified locations.

e. It can perform simulation or optimization using either linear or nonlinear constraining equations. Linear problems are those in which assumed values, such as transmissivity, do not change significantly with head. Linear constraints are adequate for optimization of linear systems.

Nonlinear problems occur if utilized equations are nonlinear. These can arise when describing flow in an unconfined aquifer, evapotranspiration, stream-aquifer interflow, or solute transport. (Note that three equations are used to compute evapotranspiration and two are used for stream-aquifer interflow.) Chance constrained optimization problems are also nonlinear.

When the linear model is applied to a nonlinear system, the model is used repeatedly until convergence of the solution is obtained. For example, heads from one optimization are used to compute transmissivity, evapotranspiration and interflow, and these are then used as knowns in the next run of the optimization model. When the nonlinear model is applied to a nonlinear problem, evapotranspiration and stream aquifer interflow are treated as unknown variables and are solved for. Cycling is still used because transmissivity is assumed known. Whenever this cycling has been done, solutions have converged (for example, transmissivities assumed to exist in a time step are eventually appropriate for the time step they are used for in the optimization model).

f. In addition to the deterministic optimization options mentioned above, the model is coded to perform steady or unsteady state chance-constrained optimization formulations. However, the chance-constraints on head and pumping still need testing.

g. The model can be adapted to another study area with minor changes (It has recently been applied to perform both steady-state simulation and optimization to a larger area of about 4000 cells).

APPLICATION AND DISCUSSION

Volumetrically Optimal Strategies

Table I shows the model-computed steady-state volume balances that will ultimately result from three scenarios. Case 1 illustrates what will occur if current ground-water withdrawals are continued (the greatest difference in any of the 1086 cells in head computed by our model and MODFLOW was less than 0.1 ft). Cases 2 and 3 show the long-term results of two optimal pumping strategies computed by the model.

In both cases 2 and 3: 1) the lower and upper bounds on pumping are respectively 0.8 and four times the current pumping value (except in cells with a moratorium on further development, where current pumping was used as the upper bound), 2) flow from the Great Salt Lake toward the aquifer cannot increase above historic values (thus maintaining the wetlands), 3) total flow from the aquifer to the Jordan River and tributaries has to be at least 50 percent of current values (thus avoiding dewatering the rivers), 4) heads in the upper layer are not permitted to drop below the base of that layer.

The only difference between cases 2 and 3 involves the lower bounds on head. In case 2, heads in the lower layer are unconstrained. In case 3, heads cannot drop more than 10 feet below the elevations of case 1.

Comparing the pumping of cases 1 and 2 shows that sustainable pumping can be increased from 158 to 224 cfs. Tightening the drawdown constraint in case 3 reduces pumping by 10 cfs below that of case 2.

It is clear that most of the possible increase in pumping will be accompanied with a reduction in discharge from aquifer to streams. In case 2, 87% of the pumping increase comes from reduction in flow to the river. In case 3, the percentage is 89%. Increased pumping is also attended by a reduction in discharge to the Great Salt Lake.

In the optimal strategies, heads in the first layer did not drop significantly. Therefore, evapotranspiration was not greatly affected.

Because of the lack of field data describing the relation between potentiometric surface head and discharge from springs, the model could not very accurately depict that flow. Regardless, discharge from springs is considered to be relatively insignificant here. If field data is available, it can be fairly easily included in the model.

Preventing Ground-water Contaminant Migration

One can employ constraints to assure that, if physically possible, contaminants do not migrate from one layer to another in a cell. This can be done when using both the steady and unsteady optimization modules. For example, the upper layer in the Salt Lake Valley is more subject to contamination by pesticides, leaking underground storage tanks and urban contaminants than is the lower layer. By using a constraint to force head in the lower layer to be not less than head in the upper layer, only upward flow can exist. Of course, sometimes this might require permitting more ground-water pumping in some cells than had been originally desired. Alternatively, one may need to use recharge in the lower layer.

As previously mentioned, the model can include constraints which limit the future concentration that will result in a particular cell and layer at a particular point in time. Since it takes such a long time to evolve into a steady-state concentration field, only transient solute transport equations are very useful for this task. Using this option can also require the use of more pumping or recharge than had been originally planned. This is especially true in a situation like the Great Salt Lake where gradients are steep.

SUMMARY

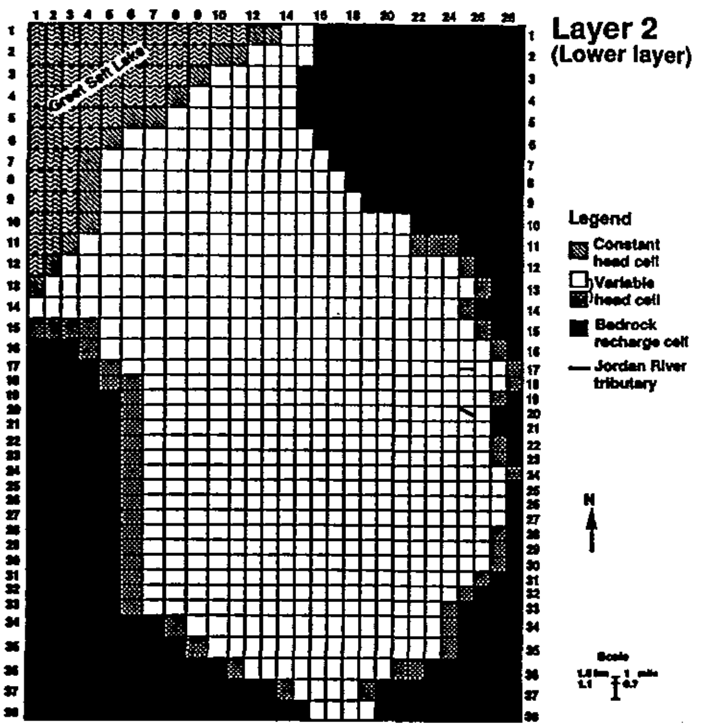
A model for optimizing ground-water yield planning in multilayer aquifer systems is presented. The model has steady and transient three-dimensional flow and two-dimensional solute transport simulation capabilities. It is suitable for application to confined and unconfined aquifer systems. The model will compute optimal strategies that will satisfy future management goals in terms of heads, flows and concentrations. It can be used to plan for the optimal management of both ground-water quantity and quality in many aquifer systems.

Table I. Summary of Results of Possible Steady Pumping Strategies

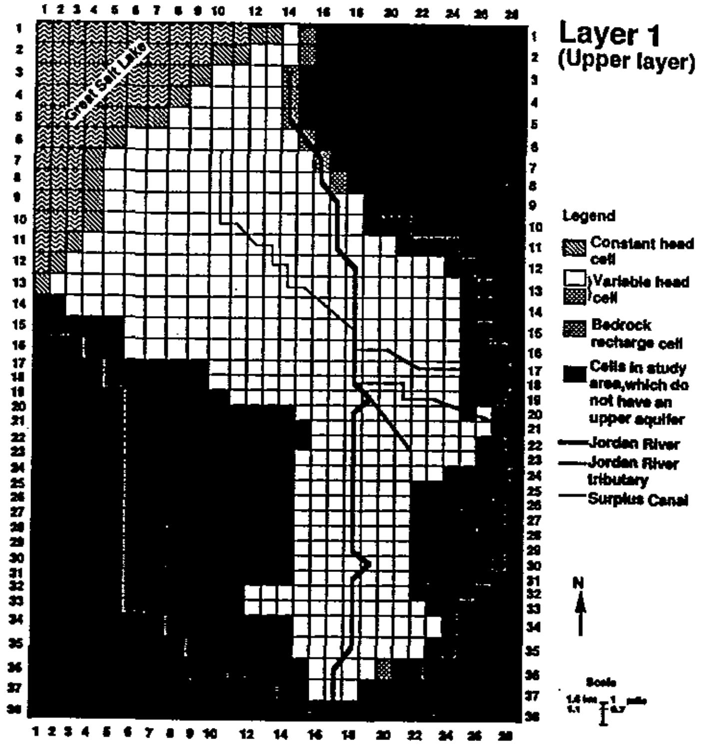
Option	Recharge to aquifer (cfs)			Discharge from aquifer (cfs)		
	case 1	case 2	case 3	case 1	case 2	case 3
Pumping	—	—	—	158.3	224.2	214.1
Precipitation	96.4	96.4	96.4	—	—	—
Bedrock recharge	208.8	208.8	208.8	—	—	—
Irrigation and seepage	167.1	167.1	167.1	—	—	—
Stream aquifer Interflow	0.7	1.3	1.1	182.5	125.1	133.0
General Head Interflow	0.5	0.5	0.5	2.8	2.6	2.5
Great Salt Lake Interflow	1.1	1.0	1.0	10.9	3.9	5.8
Springs	—	—	—	46.4	46.4	46.4
ET	—	—	—	74.0	73.2	73.5
Total	474.6	475.2	475.0	474.8	475.4	475.2

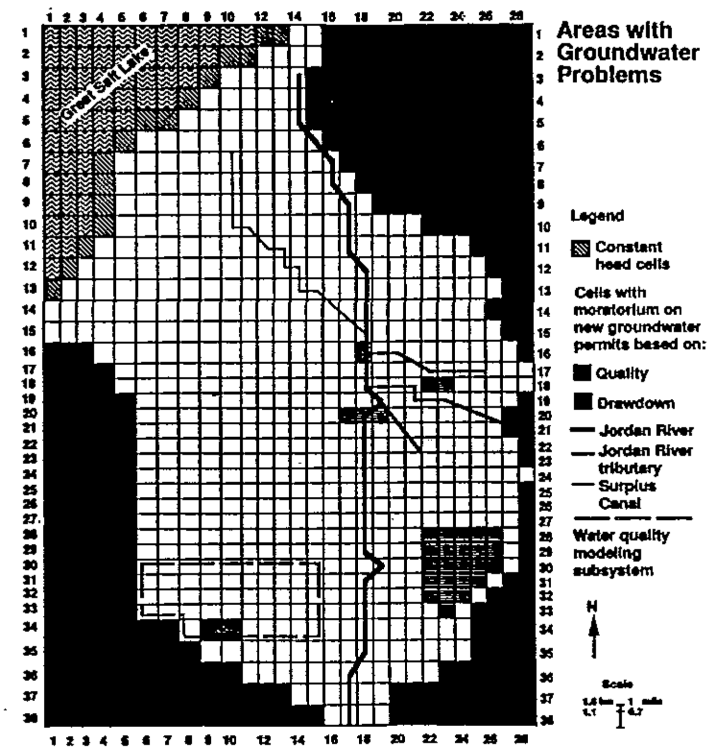
Figure 1. Discretization and characteristics of the Study Area

(a)



(b)





(c)

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