Scattering Computations from Canonical Geometries and Their Accuracy

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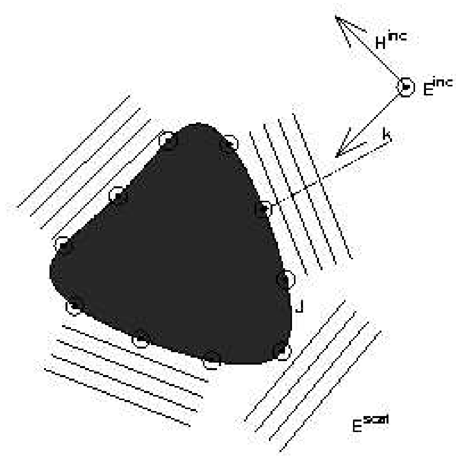
***Abstract*— An overview of the computation of electromagnetic scattering is presented. Issues of solution convergence and future work in error analysis is also discussed.**

I. Introduction

ELECTROMAGNETIC scattering problems arise in satellite communication systems, high-speed circuits, antenna design, remote sensing, radar, imaging, and stealth technology, among others. Simple scattering problems consist of the following scenario:



An object is bombarded with electromagnetic radiation, represented by the incident electric ($\vec{E}^{inc}$) and magnetic ($\vec{H}^{inc}$) field vectors. The direction of the illuminating source is indicated by the vector $\vec{k}$. Most design problems require knowledge of how much energy reflects off the object, and in which directions that energy goes (depicted by parallel lines in the drawing). This reflecting or scattering property of the object is described mathematically by the radar cross-section (RCS) and is usually denoted by $\sigma(\vec{k}^i, \vec{k}^s)$, where $\vec{k}^i$ describes the direction of the incident field (source) and $\vec{k}^s$ describes the direction of outgoing radiation (scattering).

A general scattering problem can be completely described by an integral equation relating the incident field to the electric current it induces on the object. For convenience, we write this equation abstractly as

$$\mathcal{L}\vec{J} = -\hat{n} \times \vec{E}^{inc} \tag{1}$$

where $\hat{n}$ is a unit vector normal to the surface at every point and we have assumed that the object is a perfect electrical conductor (PEC). If this equation can be solved for the electric current $\vec{J}$, then the RCS is easily found from the current.

Unfortunately, many useful scattering problems have no closed form solution, that is, the above equation cannot be solved "by hand". Thus in design, an object's RCS must be simulated by computer. This can be done by seeking an approximate solution to Eq. (1) through a numerical method called the method of moments.

II. The Method of Moments

The method of moments comprises the following: Represent the surface current as a weighted sum of N basis functions, $\vec{f}_n(\vec{r})$:

$$\vec{J}(\vec{r}) \approx \sum_{n=1}^{N} c_n \vec{f}_n(\vec{r}) \tag{2}$$

Here, the basis functions are known and selected to imitate the expected, physical nature of the current. The c_n are constants that must be determined. Define an inner product

$$\langle \vec{f}, \vec{g} \rangle = \int_S \vec{f}^*(\vec{r}) \cdot \vec{g}(\vec{r})\, ds \tag{3}$$

Using Eq. (1), a matrix equation may be generated. We write

$$\overline{\overline{Z}}\bar{c} = \bar{b} \tag{4}$$

where $\overline{\overline{Z}}$ is the moment matrix given by

$$Z_{mn} = \langle \vec{t}_m(\vec{r}), \mathcal{L}\vec{f}_n(\vec{r}) \rangle \tag{5}$$

and

$$b_m = \langle \vec{t}_m(\vec{r}), \vec{E}^i(\vec{r}) \rangle \tag{6}$$

The $\vec{t}_m$ are testing functions used to enforce boundary conditions and are often chosen to equal the basis functions (Galerkin testing). We can then solve for $\bar{c}$ and reconstruct the surface current, given by Eq. (2).

The method of moments is therefore a very powerful technique that reduces the solution of a complicated (and otherwise unsolvable) EM scattering problem to the solution of a linear system. Calculating the RCS of an object therefore pools electromagnetic principles with various mathematical techniques (the calculus of variations, linear algebra, functional analysis, numerical integration) and engineering design (object modeling, computational algorithms, etc.)

Before an RCS computed with the method of moments can be used in design, a measure of the solution's accuracy is desired. The focus of our research is to quantify solution errors and determine their causes.

III. Two-Dimensional Studies

For simplicity it is often assumed that a scattering object is translationally invariant in one or two dimensions. For example a very long cylinder may be assumed to be infinitely long. Under this assumption, the cylinder is entirely described by its cross section. The integral equation (1) and hence the implementation of the method of moments also becomes significantly simpler to implement and analyze.

We have performed an in depth study of the accuracy of the method of moments for solving two dimensional scattering problems [1]. The basis of our approach was to apply the method of moments manually to scattering from a circular cylinder (an infinitely long rod with circular cross-section). Because of the scatterer's symmetry, the moment method solution can be analyzed analytically, exposing key mathematical expressions that determine final solution quality. We then extended our theoretical findings to the set of scatterers shown in Figure (1) and table (I) [1], which were also found to behave (in the error sense) very similarly to when the cross-section is a perfect circle. For example, figure (2) shows the current backscattering amplitude errors for three particular implementations. (The RCS is proportional to the squared magnitude of the scattering amplitude.) We observe error trends that are very consistent with our theoretical results.

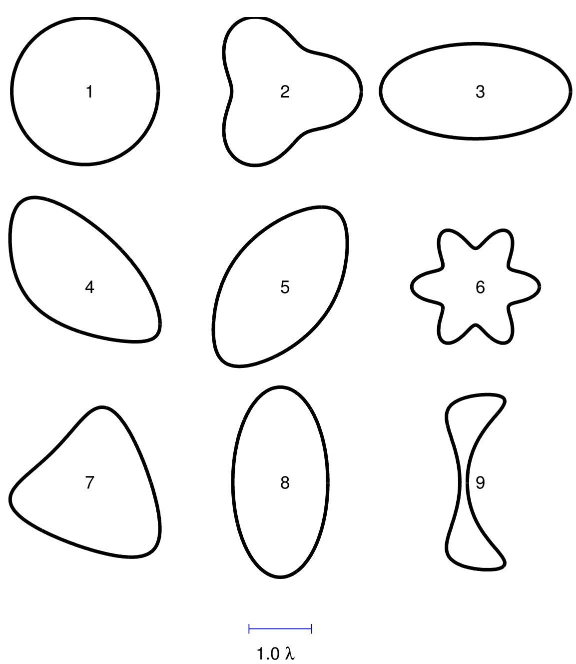


Fig. 1. Scatterers described by Table I. Scatterer 1 is a circular cylinder. All scatterers have a perimeter of approximately 7.4λ.

TABLE I

Parametric equations for nine different scatterers, the first being a circle. Figure 1 shows the geometries of these scatterers. The markers in the third column are used in Fig. 2.

	$x(t)$	$y(t)$	
1	$1.1698\cos t$	$1.1698\sin t$	●
2	$0.2588\cos t(4+\cos 3t)$	$0.2588\sin t(4+\cos 3t)$	○
3	$1.5173\cos t$	$0.7586\sin t$	×
4	$1.4237\sin(\cos 2t)$	$1.4237\cos(-2\cos t+\sin t)$	+
5	$1.2715\sin[\cos(t+1)]$	$1.2715\cos t$	*
6	$0.2040\cos t(4+\cos 6t)$	$0.2040\sin t(4+\cos 6t)$	□
7	$1.1963\cos(t+0.5\cos t)$	$1.1963\sin(t+0.5\sin t)$	▹
8	$0.7586\cos t$	$1.5173\sin t$	▽
9	$-0.4659\cos(2+3\cos t)$	$1.3977\sin t$	△

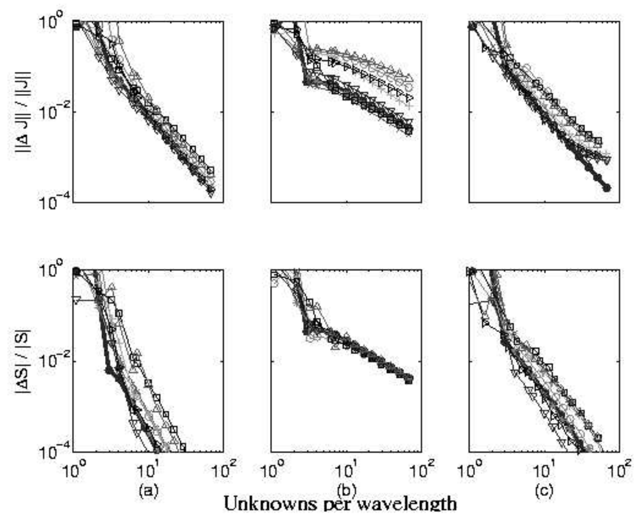


Fig. 2. Relative current and backscattering amplitude errors for the scatterers shown in Fig. 1. We have: (a) ideal implementation, (b) an otherwise ideal implementation with single point integration, and (c) an otherwise ideal implementation with a flat-facet mesh. Current convergence rates are second, first, and second order. Backscattering amplitude convergence rates are third, first, and second order, respectively.

IV. Extending to 3D Problems

In three-dimensions (where the scatterer geometry has no dimension assumed to be infinite), the scattering problem complicates significantly. RCS calculation from even moderately sized objects can take several hours to run. Reducing the runtime of such computations is the subject of much current and exciting research.

Since its introduction in [2], the RWG basis function is often preferred above other basis functions for the method of moments because it is "divergence-free" and preserves continuity of the final solution (in some sense). An RWG basis is defined if the scatterer surface may be modeled as a union of triangular patches, as is illustrated for two canonical scatterers in figures (3) and (4). Note that the triangular patches for both these scatterers are nearly the same size. This is accomplished through the Riemannian metric, which describes distances on a surface. It is commonly held that meshes where the patches are equilateral produce more accurate results for the same computational investment.

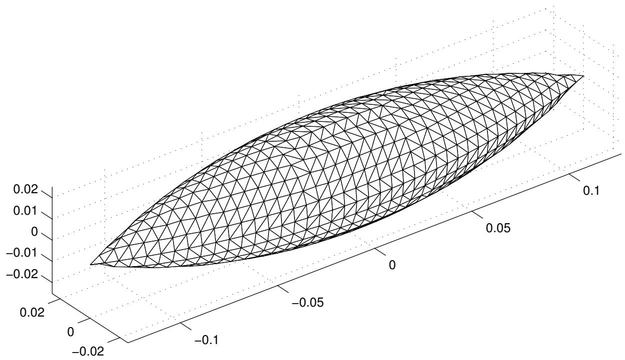


Fig. 3. Canonical scatterer geometry termed the "ogive" [3]. This geometry is used to validate codes because its sharp points are potentially difficult to model.

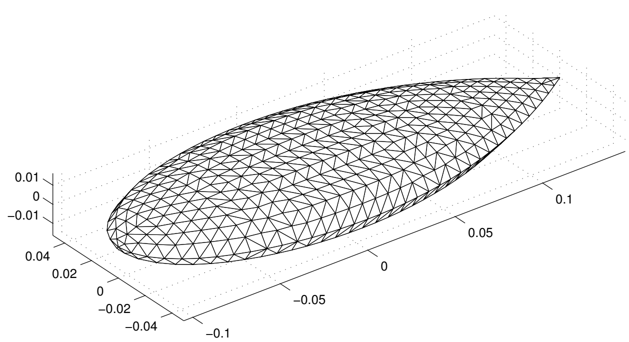


Fig. 4. Canonical scatterer geometry called the "NASA almond" [3]. This scatterer is interesting because of its sharp edges and sharp point, local features that have strong effect on the RCS.

V. Code Verification

Three-dimensional EM solvers are considered state-of-the-art. We have written a versatile 3D moment method code and are using it to perform an error analysis similar to the 2D study. To validate our code, we compared it to published, measured results produced by the Electromagnetic Code Consortium (EMCC) [3]. A comparison of our results to those of the EMCC are shown in figures (6) and (7) at the end of this paper. It is clear that our code produces comparable results to those of the EMCC.

Figure (5) shows the decrease in solution error for backscatter from the ogive. We see from this figure that the error is first order, that is, the solution error decreases at the same rate as the number of unknowns (number of basis functions). This is consistent with our findings in 2D studies.

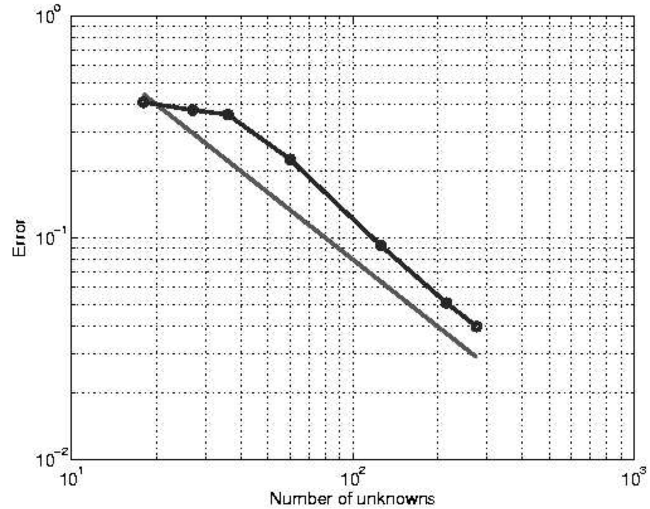


Fig. 5. Numerical experiment demonstrating solution convergence for the ogive. The dots indicate the error in the RCS (backscatter) as the mesh is refined. The solid line is a first order line for reference.

VI. Sobolev Theory

Functional analysis is a branch of mathematics that classifies functions according to their properties (smoothness, integrability, etc.) This discipline also provides each class of functions a measure with which to determine the "distance" between its elements. Electric currents belong to the class of functions known as Sobolev spaces. Sobolev theory provides a measure for electric currents that can be used to determine the "distance" between two currents. The mathematics literature proves that, given unlimited computation time, the "distance" between a moment method solution and the exact solution will converge to zero, that is, that the moment method converges in the Sobolev norm. A product of these proofs is an estimate of the solution error (for finite computation time) in terms of its Sobolev norm. These estimates are not widely used in applications partly because the physical meaning of the Sobolev norm is unknown. Motivated by the historical relationship of Sobolev norms to measures of energy, we have conjectured that the Sobolev norm is equivalent (in the strict mathematical sense) to a function of the forward scattering amplitude (an important EM parameter). We have proved this hypothesis for the case of TM-polarized fields incident on a PEC circular cylinder [4].

VII. Future Work

Future work will be a continuation of our current error analysis of 3D scattering computations and an extension of our previous work in Sobolev theory.

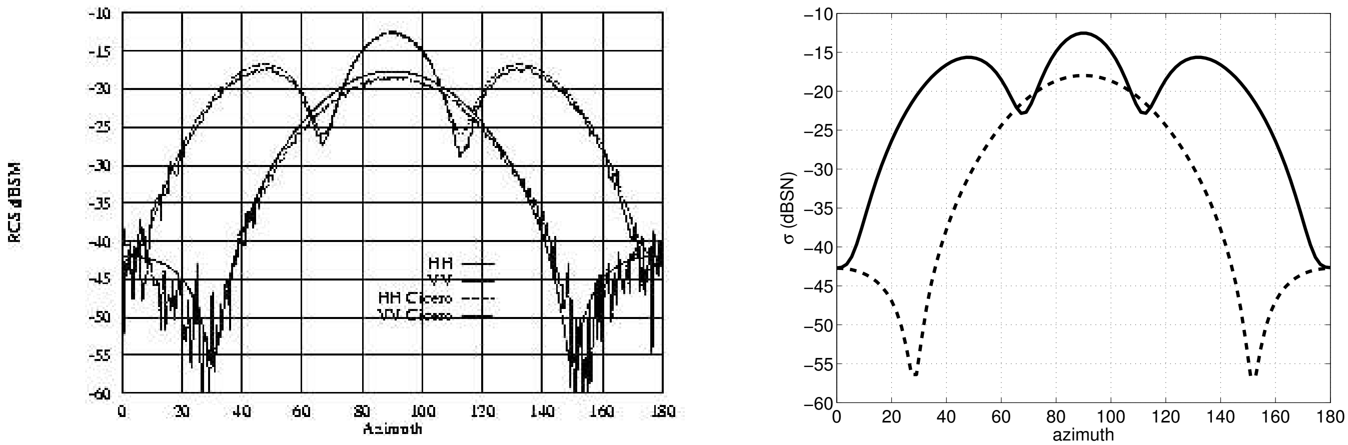


Fig. 6. RCS for the ogive in figure (3). The figure on the left [3] has measured and theoretical RCS values for both horizontal and vertical polarizations as a function azimuth angle. The figure on the right was computed using our moment method code. There is a close agreement between the two figures.

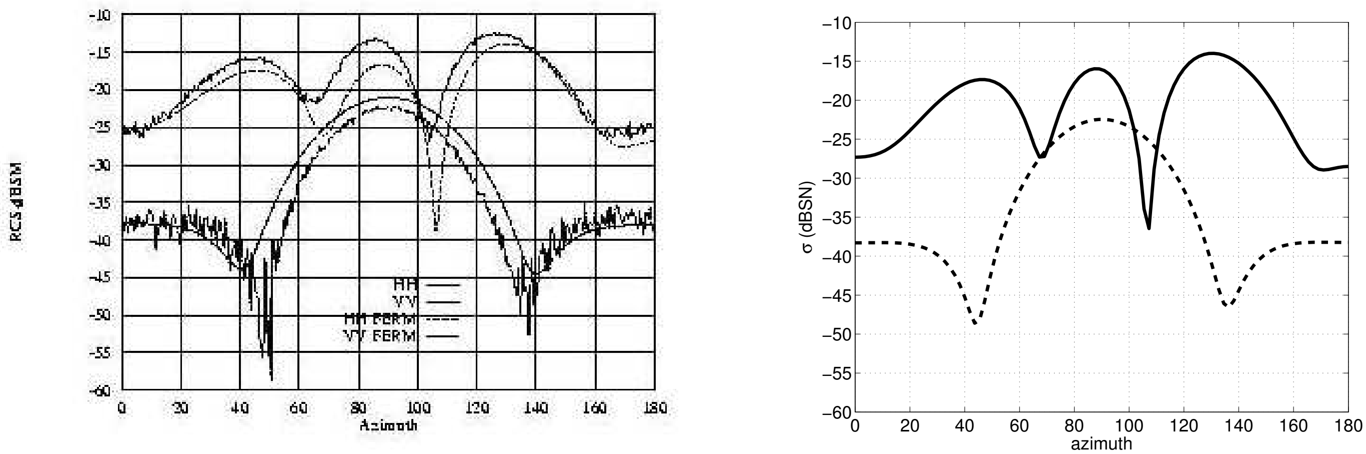


Fig. 7. RCS for the NASA almond in figure (4). The figure on the left [3] has measured and theoretical RCS values for both horizontal and vertical polarizations as a function azimuth angle. The figure on the right was computed using our moment method code. There is a close agreement between the two figures.

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