

# Characterization of high-power rocket and jet noise using near-field acoustical holography

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Structural fatigue, hearing damage, and community disturbances are all consequences of rocket and jet noise, especially as they become more powerful. Noise-reduction schemes require accurate characterization of the noise sources within rocket plumes and jets. Near-field acoustical holography (NAH) measurements were made to visualize the sound field in the jet exhaust region of an F-22 Raptor. This is one of the largest-scale applications of NAH since its development in the 1980s. A scan-based holographic measurement was made using a 90-microphone array with 15 cm regular grid spacing, for four engine power settings. The array was scanned through 93 measurement positions, along three different planes in a region near 7 m from the jet centerline and 23 m downstream. In addition, 50 fixed reference microphones were placed along the ground 11.6 m from the jet centerline, spanning 30.8 m. The reference microphones have been used to perform virtual coherence on the measurement planes. Statistically-optimized NAH (SONAH) has been used to backpropagate the sound field to the source region for low frequencies, and to identify jet noise characteristics. Ground reflection interference and other non-ideal measurement conditions must be dealt with. Details relating to jet coherence lengths and their relation to reference microphone requirements will be discussed. Preliminary results of this ongoing work will be presented. [Work supported by Air Force SBIR.]

## INTRODUCTION

Accurate characterization of the spatial distribution of noise sources within a rocket plume or a jet provides insight into physical noise generation mechanisms in the turbulent flow field. This characterization can help lead to reduction schemes of the noise that can cause structural damage to spacecraft or aircraft and shuttle launch pad structures, causes significant hearing loss for military personnel, and is a disturbance to communities.

We wish to use an acoustical inverse method to localize noise sources within the jet of a full-scale military aircraft. The process discussed in this work can apply as well to rocket noise, as both have similar noise-generation mechanisms and radiation properties. Methods other than acoustical inverse methods have been employed to localize noise sources within jets and identify their physical turbulent generation mechanisms, such as particle image velocimetry and hot-wire anemometry, but these methods are not practical for the hot, fast flows of full-scale military jets. Computational models are available that can simulate turbulent flow and can relate flow structures to acoustically radiated waves,

but the processing times for such models are impractical for realistic flow conditions. We explore acoustical inverse methods, particularly near-field acoustical holography (NAH), because they employ a non-intrusive measurement of the sound field outside of the flow field, and then use the wave nature of sound to obtain sound field information at or within the source.

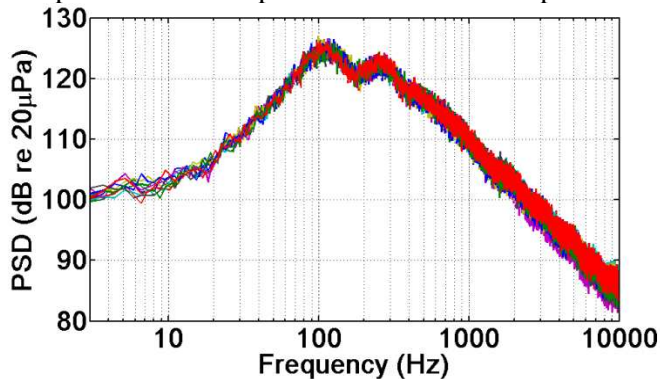
This work is part of a larger ongoing project to identify the sources of noise within a jet using NAH. Before performing NAH to localize sources, a coherent measurement plane is required. This requirement will be met through the use of fixed reference microphones and a partial field decomposition (PFD). Guidelines for the number of reference microphones necessary to perform PFD on measurements made near a jet are unclear in current literature. The main purpose of this paper is to demonstrate two complementary methods for determining the reference microphone requirements, one that can be performed simply, and before acoustical holographic measurements are made, and the other that is performed after the measurement. Therefore, jet noise source localization results will be reserved for future publications.

Section I of this paper will discuss the properties of sound radiation from jets and other aeroacoustic sources. Section II will discuss acoustical inverse methods for sound source localization, and in particular, will introduce the method of near-field acoustical holography. In section III the process of performing NAH on a jet will be outlined. Section IV will explain an important process for determining *a priori* certain requirements for performing NAH. Section V will give details of the physical experiment on a full-scale military aircraft and present results, and conclusions will be presented in Section VI.

## I. SOUND RADIATION FROM AEROACOUSTIC SOURCES

While rigorous analyses have been performed on the radiation characteristics of common sources such as vibrating plates, the noise radiation from a jet is not well-understood. For jets on high-power military aircraft, the radiation is particularly complicated. The noise spectra measured near a high-power jet are dominated by very low frequencies, on the order of a couple hundred Hertz. Typical spectra will follow the trends of those shown in Fig. 1. These spectra were calculated from near-field sound pressures measurements of an F-22 Raptor, approximately 12 m from the jet centerline. Note that the noise is broadband with peak frequencies around 100-150 Hz. These spectra with the characteristic “haystack” shape of jet noise, are probably generated by both small and large turbulent structures within the flow field that couple acoustically with the surrounding medium.

Many noise sources, especially those with characteristic lengths larger than a wavelength, do not radiate like simple sources. For example, a large vibrating plate will have significantly different radiation properties than will a point source. We may consider a vibrating plate as a distribution of radiating monopoles. The phase relationships between each monopole on the



**Figure 1** Power spectral density measured at ten different times at about 12 m from the centerline of an F-22 at afterburner engine conditions. The sound power peaks between 100 and 150 Hz, and is highly stationary.

plate will not be random, but fixed. This fixed-phase relationship causes all the point sources that make up the plate to be coupled in such a way as to generate acoustic radiation into the surrounding fluid very different from that which would be generated by a similar distribution of monopoles all vibrating independently. This fixed-phase relationship may be described by the *correlation* between each monopole.<sup>1</sup> The correlation between two signals describes the degree to which the two signals are related. If the two signals are perfectly related, then the correlation coefficient will have a value of unity. Two fully independent signals will have a correlation coefficient of zero. Because plate vibration is structural, it is a fully correlated source. Signals that are somewhat related will have a correlation coefficient somewhere in between zero and unity. It is well established that aeroacoustic sources are partially correlated over finite distances, and therefore radiate somewhat coherently.<sup>2-3</sup> The correlation lengths within a jet tend to increase with a decrease in frequency. High frequencies radiate from compact regions and are monopole-like, but the low frequencies that dominate jet noise radiate from larger, non-compact regions and are more highly correlated over larger distances. The exact correlations of the radiating sources within the flow are difficult to measure directly or to simulate computationally.

The radiation of spatially-correlated sources may be described with a sum of multiple wave functions, each with a unique wavenumber. For a given frequency of vibration, certain wave functions will radiate into the far field, while others will decay away exponentially. These two types of waves are referred to as radiating and evanescent waves, respectively. The energy of evanescently-decaying waves remains in the near field of the source. Both radiating and evanescent waves are important contributions to characterizing the source, so both types of waves must be measured if we are to fully determine source radiation mechanisms in a jet. If measurements are made in the far field, evanescent waves will not be detected. Thus, near-field measurements are necessary for accurate source localization. The sound field in the vicinity of a jet definitely contains both radiating and evanescent waves, but the details of the individual wave number contributions are unclear.

## II. ACOUSTICAL INVERSE METHODS

There are several acoustical inverse methods that have been employed to localize jet noise sources including the acoustic telescope technique, the acoustic mirror, the polar correlation technique, and beamforming.<sup>4-7</sup> Typically, these methods assume a source distribution of uncorrelated point monopoles each

completely unrelated to the adjacent monopole. For example, beamforming utilizes an array of microphones and, based on the speed of sound, delays each signal by the proper amount for a given “look” direction, or an assumed angle of incidence. The signals are then summed. For a source truly coming from that direction, the signals will add coherently and describe the source. This method will fail for a finite distribution of partially-correlated sources.<sup>4</sup> This is one reason that beamforming tends to give accurate results for localizing high frequencies within a jet, but not low frequencies. These methods are also typically performed with measurements in the far field, and thus do not capture the evanescent waves necessary to fully characterize the source. Measurements made in the far field are also limited to source reconstructions of one-half wavelength. Efforts have been made to modify beamforming algorithms for near-field measurements, and to account for spatially non-compact sources.<sup>4</sup> However, we seek to use (NAH) as an alternative noise-source-localization technique.

The basic theory of NAH is that, from a two-dimensional hologram measurement in the near-field of a noise source, the three-dimensional sound field properties such as pressure, particle velocity, and intensity may be reconstructed in the source region.<sup>8</sup> Measurement in the near-field captures some of the evanescent waves and allows for a more accurate reconstruction. The reconstruction is not limited to a resolution of one-half wavelengths. NAH makes no assumptions about the spatial correlation of the source, and can perform well for spatially extended sources, particularly at low frequencies.

NAH was developed in the 1980s for measuring the vibrations of solid structures. It has only been applied to jet noise within the past decade or so, and has been rather limited.<sup>9</sup> Lee and Bolton<sup>10-11</sup> successfully performed NAH on a laboratory-scale subsonic cold jet with about a 1 cm nozzle, surrounding the jet exhaust region with 32 microphones. Applying NAH to jets on military aircraft is a large jump, and requires a more rigorous approach. In the following section the process of NAH is outlined.

### III. THE PROCESS OF NEAR-FIELD ACOUSTICAL HOLOGRAPHY

NAH requires a coherent measurement over the hologram to propagate the sound field in toward the source. This means that there must be a fixed-phase relationship between every point on the hologram. There are two ways to achieve this. Sound pressures may be measured simultaneously using an array of reference microphones that spans beyond the source region. This kind of measurement is impractical for large sources,

such as high-power jets, if a high-resolution is desired. For this work a patch-and-scan measurement is used. A small dense array of microphones is scanned over the hologram surface. The discontinuities in phase information between scans may be accounted for with an array of fixed microphones that measure sound pressures simultaneously with each scan, which may then be used to tie together the phases in a process called partial field decomposition (PFD).

In PFD, cross spectral matrices between the reference microphones and the array microphones for each scan are used to decompose the measurement hologram into a set of independent, but mutually coherent partial fields. These partial fields form an orthogonal basis set for the sound field. Summing these partial fields on an intensity basis will return the total measured hologram surface magnitude.

Several PFD methods exist. In this work, the virtual coherence method is used.<sup>12-13</sup> This PFD process performs a singular value decomposition (SVD) on the signals measured by the reference microphones. This generates an orthogonal basis of “virtual references”, each one containing information from all the individual physical reference signals. The singular values that describe the strength of each of these virtual references are sorted in descending order. The measurement hologram is then decomposed into partial fields, each of which is fully correlated with one virtual reference. Therefore, the partial fields are also sorted by strength. This is mathematically the “ideal” decomposition, since as much of the sound field as is possible is packed into the first partial fields.

The total number of partial fields that come out of the decomposition will equal the number of reference microphones. The first partial fields will contain information relevant to the source, and the rest will contain lower amplitude noise. Therefore, a sufficient number of partial fields must be selected to reconstruct the source, and the rest discarded. Returning to the example of a vibrating plate, the entire source is correlated. Only one partial field will contain relevant information. Consequently, only one reference microphone is needed to perform PFD. A sound field generated by  $N$  independent sources will require  $N$  reference microphones, and will be decomposed into  $N$  partial fields. More reference microphones may be used, producing more partial fields, but only the first  $N$  will contain useful information. If the number of sources is unknown, the singular values of the SVD on the reference microphones may be observed. For  $N$  independent sources, there will be a sharp drop from the singular value  $N$  to the  $N+1$  singular value. For a jet, the number of independent sources is unclear. The singular values tend to decrease somewhat steadily and monotonically (see

Figs. 8-9). The number of partial fields and the minimum number of reference microphones required to fully measure the source must be determined. The virtual coherence method provides a way to determine this number. This method is where virtual coherence gets its name.

For a chosen frequency we calculate the cross spectral matrix containing cross spectra between each virtual reference, one containing the cross spectra between each reference microphone, and a third one containing cross spectra between each virtual reference signal and each measured hologram microphone signal. These are, respectively,  $\mathbf{C}_{vv}$ ,  $\mathbf{C}_{pp}$ , and  $\mathbf{C}_{vp}$ . Here, a subscript  $v$  denotes a virtual reference, and a subscript  $p$  denotes a hologram measurement position. The virtual coherence between the  $i$ th virtual reference and the  $j$ th measurement position in each scan is given by

$$\gamma_{j,i}^2 = \frac{|\mathbf{C}_{vipj}|^2}{\mathbf{C}_{vivi}\mathbf{C}_{pjpj}} \quad (1)$$

For perfect coherence between the same frequency of two signals,  $\gamma_{j,i}^2 = 1$ , and a value of zero would denote no relation. To select the number of partial fields used for NAH, this virtual coherence is summed over the first  $R$  elements of  $i$ , iteratively increasing  $R$  until the coherence criterion is met, namely

$$\sum_{i=1}^R \gamma_{j,i}^2 \geq \text{coherence criterion} \quad (2)$$

Once the coherence criterion is reached for every measurement position  $j$  in a scan, the  $R$  value is the necessary number of partial fields for that scan. The median of these  $R$  values is selected as the number of partial fields that are processed using NAH. In practice, a coherence of unity is nearly impossible to achieve. We have therefore chosen a coherence criterion of 0.9. This corresponds to a signal-to-noise ratio of approximately 10 dB by the relation

$$\text{SNR} = 10 \log \left( \frac{\gamma^2}{1 - \gamma^2} \right) \quad (3)$$

Where the numerator in the log function represents the coherent power, and the denominator corresponds to noise, or incoherent power.

Once the appropriate number of partial fields has been determined, each one is propagated individually toward the source, using NAH algorithms. The specific method employed in this work is statistically-optimized

near-field acoustical holography (SONAH).<sup>14-18</sup> We choose SONAH because it avoids the windowing effects by avoiding the direct use of a spatial DFT operation on the measurement surface when the measurement aperture does not extend far beyond the source region. A larger-than-source aperture is not feasible for measurement of a jet that is on the order of tens of meters long. This algorithm breaks up each partial field into a set of plane-wave functions. Then, it propagates these wave functions to a reconstruction surface using a transfer matrix that describes sound wave propagation between each of the geometrical locations of the hologram and reconstruction surfaces. After each partial field has been propagated to the reconstruction surface, the reconstructed partial fields are added on an intensity basis, giving the estimated total sound field at that location. For the jet, we will propagate from the measured hologram in towards the source as close as possible.

#### IV. COHERENCE LENGTHS

The above-outlined method for determining the necessary number of partial fields is not actually very useful for determining the necessary number of reference microphones required *a priori*, as it requires already having sufficient reference microphones to perform. It would therefore be useful to use some other easily-measured jet noise property to give a reference microphone guideline before a full NAH experimental attempt is made.

In 2009, Gardner<sup>19</sup> published a thesis giving a guideline for determining  $R$  which could theoretically be performed with only two microphones (although it would be more practical with several). For a given frequency, the coherence between one reference microphone and the entire array is calculated, giving a plot like that in Fig. 10. The coherence between the chosen microphone and itself is, of course, unity, and the coherence tends to decrease moving away from the chosen reference location. We define a local coherence length  $L_c$  as the physical distance over which the coherence drops from unity to 0.5. This value is assigned to the location and frequency of the chosen microphone. For the reference microphones away from the aperture edge, there will be a coherence length defined on both sides, which will result in two values. When this occurs, the average of these two values is determined to be the coherence length. We may determine the coherence length over a range of frequencies, and over the reference microphone array aperture, giving a plot like that shown in Fig. 11. This can visually give sense of coherence lengths in the vicinity of the jet.

For a given frequency, we can average the local coherence lengths across the aperture. Gardner<sup>19</sup> showed

through numerical simulation that *two microphones per coherence length* in the reference array, regardless of frequency, give the minimum error in source reconstruction using SONAH. Including more reference microphones does not improve results. Thus, by calculating the coherence lengths near the jet for the frequencies of interest, the minimum number of reference microphones required may be determined before performing NAH. This also provides an *a priori* feasibility test for performing NAH on large complicated sources.

#### IV. EXPERIMENTAL RESULTS

##### A. Experiment

In July 2009, near-field measurements of the jet on a Lockheed Martin/Boeing F-22 Raptor were taken at Holloman Air Force Base in New Mexico. A 5 by 18 array of G.R.A.S 1/4" microphones, with 0.15 cm spacing, scanned an approximately 2 m by 24 m region as near to the jet as would not cause the microphones to clip (see Figs. 2-3). This was repeated for three more measurement planes some distance further from the jet. In addition, 50 fixed reference microphones were placed on the ground with 0.6 m spacing, spanning more than 30 m, (shown in Fig. 4). Measurements were repeated for four engine conditions ranging from idle to full afterburner. Figure 5 shows the overall sound pressure levels measured in relation to the aircraft location for the afterburner engine condition. There were a total of more than 6000 measurement positions, making this the largest-scale acoustic measurement of a high-power jet ever performed.

##### B. Virtual Coherence

Virtual coherence was performed on the



**Figure 2** Blue Ridge Research and Consulting 90-microphone array, scanning the near field of the jet on an F-22 Raptor.

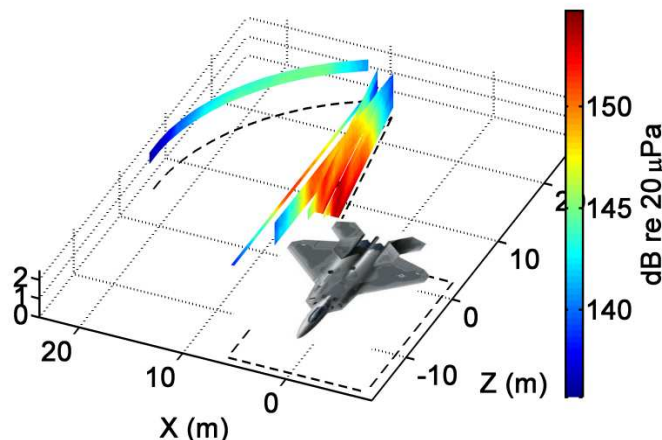


**Figure 3** An example sound pressure level map overlaid with the jet photo at the approximate measurement location.

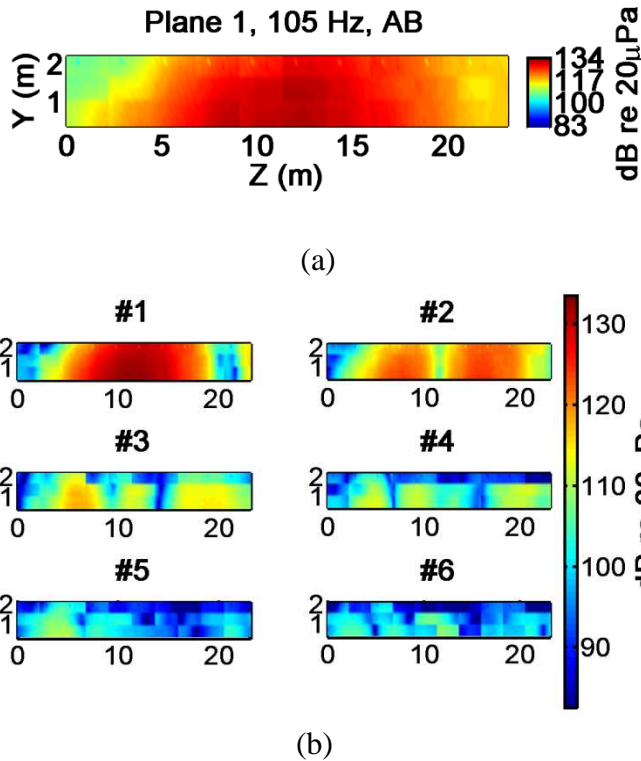


**Figure 4** Fifty reference microphones were placed on the ground 12 m from the jet centerline, which measured sound pressures simultaneously with each scan.

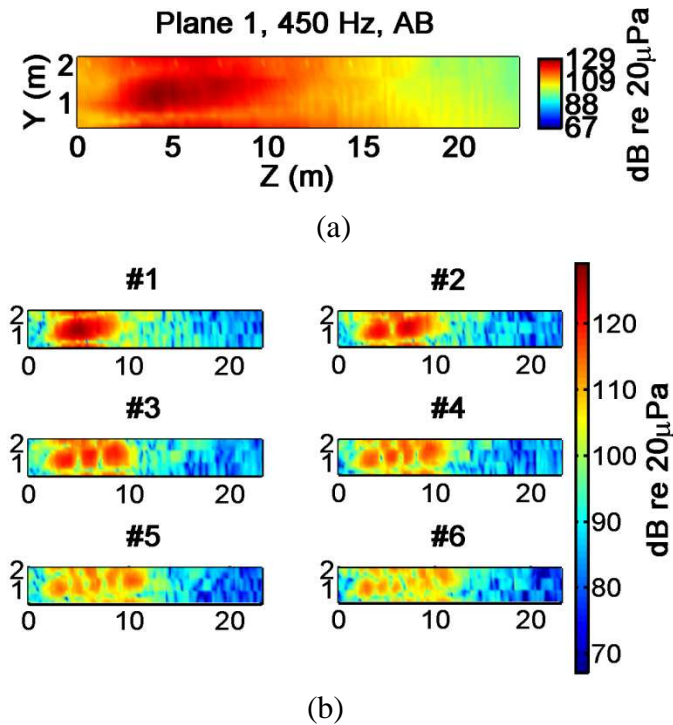
measured hologram data. Results are shown for afterburner engine conditions measured at the closest plane for the peak frequency, 105 Hz, and for 450 Hz in Figs. 6-7. Parts (a) in each figure are the total measured sound pressure levels. Parts (b) show the first six independent partial fields after performing virtual coherence. For each frequency individually, parts (a) and (b) are on the same color scale. Both the color scales in the two figures span approximately 62 dB. Note that the



**Figure 5** Overall sound pressure levels measured in the jet vicinity for afterburner engine conditions.



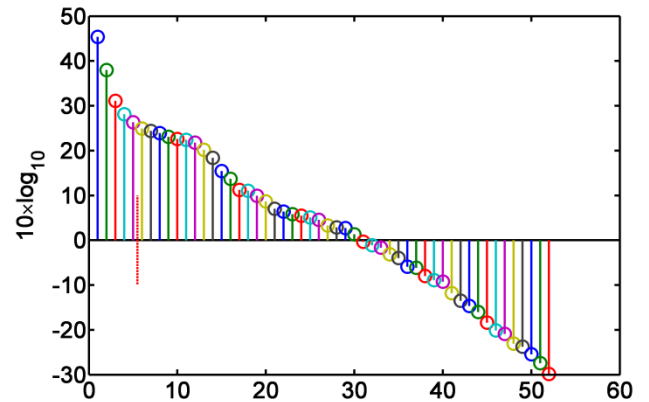
**Figure 6** Measurement and virtual coherence results for 105 Hz: (a) SPL at hologram; (b) first six partial fields after PFD using virtual coherence.



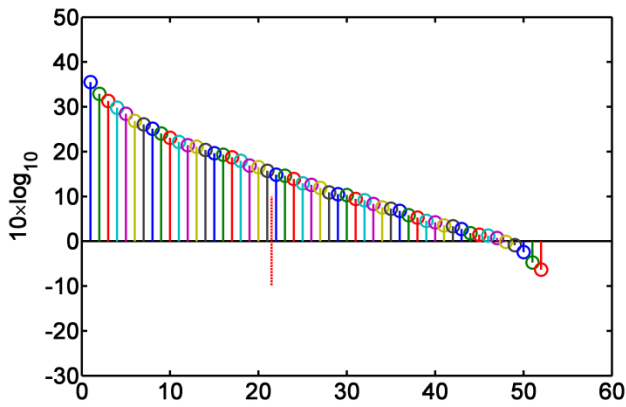
**Figure 7** Measurement and virtual coherence results for 450 Hz: (a) SPL at hologram; (b) first six partial fields after PFD using virtual coherence.

difference between the peak levels for the first and sixth partial fields for the 105 Hz case is approximately 30 dB. However, for the 450 Hz case, the peak difference is only around 10 dB. This demonstrates the trend that, as frequency decreases, more relative energy is contained in the first partial fields, suggesting that fewer partial fields will be required to fully determine jet noise. In other words, there are “fewer” independent sources within the jet at low frequencies than high.

After calculating the virtual coherence such that the coherence criterion of 0.9 is met for each scan, the appropriate number of partial fields is determined for both frequencies. In Figs. 8-9 we compare the singular values of the virtual references for each frequency. As is typical of the singular values for measured jet noise, the trend is a monotonic, steady decrease. However, we note an important difference between the low and high frequency, namely that the first several singular values in the 105 Hz case decrease rapidly, and then the slope decreases to a steady monotonic descent. This is consistent with the partial fields shown above, and also suggests that only a few partial fields are required to meet the coherence criterion, and thus characterize the source



**Figure 8** Singular values of  $C_{rr}$  after SVD for the 105 Hz case. The number of singular values before the red dashed line represent the number of partial fields required to meet the coherence criterion.

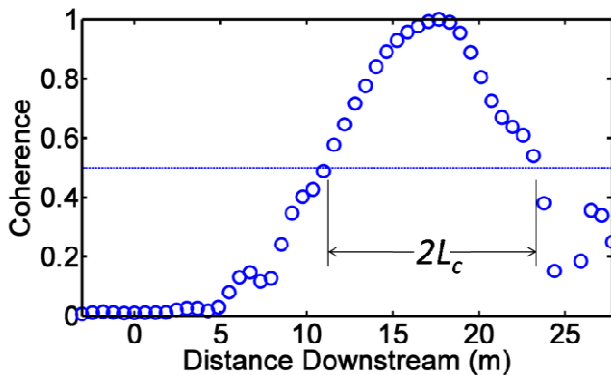


**Figure 9** Singular values of  $C_{rr}$  after SVD for the 450 Hz case. The number of singular values before the red dashed line represent the number of partial fields required to meet the coherence criterion.

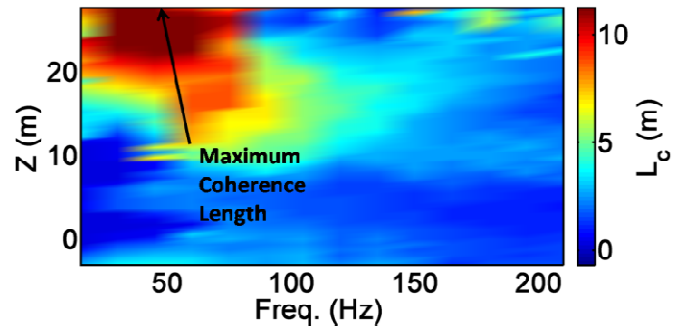
for lower frequencies. The partial field cutoff point is designated by a dashed red line on each figure. It turns out that, while 21 partial fields will determine the measured sound field for the 450 Hz case, only 5 partial fields are sufficient at 105 Hz, which is the peak frequency. For certain low frequencies and engine conditions, as few as two or three partial fields are enough!

### C. Coherence Length Analysis

The coherence lengths at the reference microphone array give additional insight into why so few partial fields are needed for low frequencies. Figure 10 shows the coherence between a reference microphone 18 m downstream of the jet nozzle and all the other reference microphones for the 105 Hz afterburner case. At this location and frequency we calculate a coherence length of approximately 6 m. If we repeat this calculation over a range of frequencies and across the entire reference aperture, we obtain the results shown in Fig. 11. The



**Figure 10** Coherence measured between the reference microphone 18 m downstream of the jet nozzle, and all other reference microphones, for the 105 Hz afterburner case. The coherence length  $L_c$  is determined to be the average distance over which coherence drops from 1 to 0.5.



**Figure 11** Coherence lengths plotted against position on the reference array and frequency. The maximum coherence length exceeds 16 m and occurs 27 m downstream of the jet nozzle, and at 45 Hz.

trend of increasing coherence length with decreasing frequency is consistent with the fact that fewer reference microphones are necessary for lower frequencies, according to the two-reference-microphone-per-coherence-length criterion. The remarkably high coherence lengths towards the downstream end of the jet seem to follow the trend of Tam's two-source jet noise model.<sup>2</sup>

For 105 Hz, the mean coherence length over the entire aperture is 3.4 m. This suggests that the minimum number of reference microphones required to perform NAH on this jet at 105 Hz is 18. However, we have shown that, according to the coherence criterion, as few as 5 reference microphones may be able to characterize the source. At most frequencies and engine conditions we have explored, the number of reference microphones required is usually fewer than the number determined by the two-reference-per-coherence-length guideline. The true number of required reference microphones is likely somewhere in between, since the coherence criterion requirement is somewhat idealized, and the singular value decomposition can always be improved by adding extra reference microphones. By increasing the coherence criterion (and consequently the SNR) the estimated reference microphone requirement will increase.

### VI. CONCLUSIONS

We have shown two complementary methods of determining the number of reference microphones necessary to perform PFD on measured hologram data for jet noise source localization. The requirement that two reference microphones per coherence length must be used is a simple, *a priori* estimation if near-field coherence lengths are known, and is probably an overestimation. This requirement can be verified and compared to the reference microphone requirements determined using the virtual coherence method, a post-measurement assessment. To ensure a high-fidelity reconstruction, it is recommended that

the higher, two reference microphone per coherence length guideline be followed.

Most importantly, it is evident that a feasible number of reference microphones may be used to fully determine the near-field sound pressure of a full-scale jet on a military aircraft, and that NAH is a viable jet noise source localization technique, even for such large and complicated sources. Future work will present jet noise source reconstructions determined using SONAH, including an analysis of optimization techniques and of insights into the physical noise generation mechanisms in high-power jets.

- <sup>1</sup> J. S. Bendat and A. G. Piersol, *Random Data: Analysis and Measurement Procedures*, (John Wiley & Sons, Inc., New York, 2000).
- <sup>2</sup> C. K. W. Tam, N.N. Pastouchenko, R.H. Schlinker, "Noise source distribution in supersonic jets," *J. Sound Vib.* **291**, 192-201 (2006).
- <sup>3</sup> A. Michalke, "Some remarks on source coherence affecting jet noise," *J. Sound Vib.* **87** (1) 1-17 (1983).
- <sup>4</sup> S. R. Venkatesh, D. R. Polak and S. Narayanan, "Beamforming algorithm for distributed noise source localization and its application to jet noise", *AIAA J.* **41** 1238-1246 (2003).
- <sup>5</sup> H. V. Fuchs, "On the application of acoustic 'mirror', 'telescope', and 'polar correlation' techniques to jet noise source location," *J. Sound Vib.* **58** (1) 117-126 (1978).
- <sup>6</sup> J. Billingsley and R. Kinns, "The acoustic telescope", *J. Sound Vib.* **48** 485-510 (1976).
- <sup>7</sup> M. J. Fisher, M. Harper-Bourne and S. A. L. Glegg, "Jet engine noise source location: The polar correlation technique", *J. Sound Vib.* **51** 23-54 (1977).
- <sup>8</sup> J. D. Maynard, E. G. Williams, and Y. Lee, "Nearfield acoustic holography: I. Theory of generalized holography and the development of NAH," *J. Acoust. Soc. Am.* **78** (4) 1395-1413 (1985).
- <sup>9</sup> M. Lee, J. S. Bolton, and L. Mongeau, "Application of cylindrical near-field acoustical holography to the visualization of aeroacoustic sources," *J. Acoust. Soc. Am.* **114** (2) 842-858 (2003).
- <sup>10</sup> M. Lee and J. S. Bolton, "Scan-based near-field acoustical holography and partial field decomposition in the presence of noise and source level variation," *J. Acoust. Soc. Am.* **119** (1) 382-393 (2006).
- <sup>11</sup> M. Lee, J. S. Bolton, "Source characterization of a subsonic jet by using near-field acoustical holography," *J. Acoust. Soc. Am.* **121** (2) 967-977 (2007).
- <sup>12</sup> H. -S. Kwon and J. S. Bolton, "Partial field decomposition in nearfield acoustical holography by the use of singular value decomposition and partial coherence procedures," *Proc. NOISE-CON* **98** 649-654 (1998).
- <sup>13</sup> H. -S. Kwon, Y. -J. Kim, and J. S. Bolton, "Compensation for source nonstationarity in multireference, scan-based near-field acoustical holography," *J. Acoust. Soc. Am.* **113** (1) 360-368 (2003).
- <sup>14</sup> D. Long, J. Peters, and M. Anderson, "Evaluating turbofan exhaust noise and source characteristics from near field measurements," 15th AIAA/CEAS Aeroacoustics Conference, Miami, FL, AIAA Pap. 209-3214 (2009).
- <sup>15</sup> R. Steiner and J. Hald, "Near-field acoustical holography without the errors and limitations caused by the use of spatial DFT," *Int. J. Sound Vib.* **6** (2) 83-89 (2001).

- <sup>16</sup> J. Hald, "Patch near-field acoustical holography using a new statistically optimal method," *Proc. INTER-NOISE 2003*, 2203-2210 (2003).
- <sup>17</sup> Y. T. Cho, J. S. Bolton, and J. Hald, "Source visualization by using statistically optimized near-field acoustical holography in cylindrical coordinates," *J. Acoust. Soc. Am.* **118** (4), 2355-2364 (2005).
- <sup>18</sup> J. Hald, "Basic theory and properties of statistically optimized near-field acoustical holography," *J. Acoust. Soc. Am.* **125** (4), 2105-2120 (2009).
- <sup>19</sup> M. Gardner, "Scan-based near-field acoustical holography on partially correlated sources", Master's thesis, Brigham Young University, Provo, UT (2009).